

Bio-Inspired Optimization of LQR Controllers for Feedback-Linearized Cruise Missiles

1st Subbareddy Chitta

*Assistant Professor, Department of Mechatronics and RA
Parul University
Vadodara, India
subbareddy.nitkkr@gmail.com*

2nd Ramakalyan Ayyagari

*Professor, Department of ICE
National Institute of Technology Tiruchirappalli, India
rkalyn@nitt.edu*

3rd Shreehan Santosh Kate

*Student, Department of ICE
National Institute of Technology Tiruchirappalli, India
shreehan1912@gmail.com*

Abstract—This study investigates the development of an advanced state feedback controller leveraging the LQR framework, enhanced through the Feedback Linearization (FL) technique, with specific emphasis on the control of longitudinal dynamics. A quasi-linearized fourth-order model of the inner loop is obtained from the non-linear model using the FL technique, which facilitates the application of linear state-feedback control methodology. Using the standard Linear Quadratic Regulator (LQR) framework, the optimal control law $u = Kx$ is formulated for the outer loop of the plant and subsequent selection of optimal weighting matrices Q (positive semi-definite) and R (positive definite) proves essential for determining the feedback gain matrix K . Next, to address the challenging problem of selecting appropriate Q and R matrices, meta-heuristic optimization techniques, specifically the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), are proposed to converge toward suitable solutions efficiently. The performance of the non-linear missile model employing FL with LQR, followed by optimization with GA and PSO, is compared with iterative LQR tuning procedures. The effectiveness and advantages of selecting optimal weights using the meta-heuristic approaches have been demonstrated through an extensive simulation analysis. The control strategies are evaluated by analyzing error metrics alongside time-domain specifications.

Index Terms—Nonlinear missile dynamics, Nonlinear Controller, LQR (LQR) Controller, Feedback linearization (FL), Genetic Algorithm (GA), Particle Swarm Optimization (PSO).

I. INTRODUCTION

Modern missiles face increasingly demanding performance requirements that call for exceptional accuracy, attack precision, and efficient operability in extreme conditions. The design of a longitudinal missile autopilot plays a crucial role in achieving these performance objectives. A well-engineered longitudinal autopilot significantly enhances the practicality of precision target engagement [1] [2].

Missile autopilots have traditionally relied on classical control techniques, involving linearization around fixed operating points and a gain-scheduled approach [3] [4]. Though effective, this approach struggles to meet the growing demands for high performance in nonlinear flight regimes, leading to a shift toward nonlinear control strategies for autopilot designs, such as feedback linearization (FL) [5], backstepping control [6], and sliding mode control [7].

Sliding mode and backstepping control techniques can be interpreted as specific implementations of feedback linearization (FL). However, this paper focuses on FL because it allows verification of system controllability, providing deeper insight into the design process. FL achieves an exact transformation of a non-linear system into an equivalent quasi-linear form, without approximations [8], which facilitates the direct application of state-space techniques, allowing optimal control techniques rather than limiting control to only the dominant dynamics. There has been a notable shift in missile control design approaches in recent years, with optimal control solutions gaining significance in modern longitudinal missile autopilot designs [5]. This trend reflects the growing need for sophisticated control strategies to maximize missile performance while operating within complex constraints.

The LQR is known for its elegant formulation of optimal solutions and its implementation of straightforward state linear feedback control laws. The computational efficiency and practical applicability of LQR have made it an indispensable tool for researchers and engineers in the field of automatic control. Despite its merits, the efficacy of LQR control is critically dependent on the judicious selection of weighting matrices for system state variables and input variables. Suboptimal or inappropriate choices of these matrices can lead to solutions that are either ineffective or, in extreme cases, devoid of meaningful control action. Control techniques based on the LQR framework often fail to perform in situations where there are significant variations in system parameters. Furthermore, tuning of LQR controllers relies heavily on manual expertise, which Additionally, tuning of LQR controllers largely relies on manual experience, which could lead to the emergence of suboptimal solutions, which leads degradation of performance in dynamic environments characterized by strong wind disturbances or substantial load changes, potentially leading to reduced control accuracy. The crux of this challenge lies in accurately capturing the nuances of the actual control process through these matrices, a task that often requires multiple iterative refinements to achieve satisfactory performance [9].

Meta-heuristic Algorithms are techniques used for solving different types of optimization problems, both constrained and unconstrained, based on biological laws of evolution, something that traditional or deterministic algorithms are incapable of doing. development in artificial intelligence and soft computing, along with the implementation of bio-inspired algorithms, has been enormous. The GA is a computational method that forms a part of the broader category of evolutionary/meta-heuristic algorithms. Primarily employed for optimization in conventional problem-solving scenarios, GA utilizes Darwinian-inspired operations such as mutation, crossover, and selection to arrive at optimal solutions in the prescribed search space. The Particle Swarm algorithm derives its essence from swarm intelligence. The algorithm's core mechanism involves iterative refinement of the particles' velocities and positions, essentially making PSO one of the most important algorithms for optimization. The movement of each particle is guided by both its personal best-known position and the swarm's most optimal position. This collective intelligence approach guides the swarm toward optimal solutions in the search space.

Meta-heuristic algorithms have been used in the optimization of controller parameters in several applications. In [10], Particle Swarm optimization is leveraged in order to tune the parameters of the Proportional-Integral-Derivative (PID) controller for UAVs whose performance has been evaluated using IAE, ISE, ITAE, and ITSE. The results indicate that PID controllers optimized using Particle Swarm Optimization (PSO) exhibit superior performance compared to conventional PID controllers when subjected to wind disturbances. [11] presents a detailed study of the inner-outer loop architecture along with optimal tuning of the PID controller located in the outer loop, using Particle Swarm Optimization and Genetic Algorithm, thereby enhancing the control of Quadrotors. Reference [12] developed an improved sparrow search algorithm for a quadrotor UAV combined with an active disturbance rejection controller to address the problem of disturbance rejection. The effect on the accuracy of altitude control and stability caused by uncertainties in the plant model and external disturbances has been successfully mitigated by implementing this approach.

This paper proposes a novel method for selecting the optimal weighting matrices Q and R for a LQR control system. These matrices greatly influence the performance and behavior of the LQR, as they balance the trade-off between system performance (like state error) and control effort (like actuator usage). To obtain optimal selection of these weights, we employ advanced optimization techniques—Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). By leveraging these methods, we systematically search the space of possible Q and R values to find those that yield the best control performance according to a predefined objective function (RMSE, ISE, and IAE).

This paper is organized as follows: Section II presents the nonlinear dynamic model of the missile system used in this

paper. In Section III, the nonlinear missile model is linearized using feedback linearization technique, and an LQR controller is subsequently designed. The optimal tuning of the LQR controller using Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is discussed in Section IV. The results obtained from simulations, along with a comparative analysis of the control strategies, are presented and discussed in the subsequent sections.

II. NON-LINEAR MISSILE MODEL

This research uses a non-linear missile model, shown as Fig.1, tailored for the longitudinal autopilot of a skid-to-turn (STT) missile. The longitudinal dynamics (γ , α , q , δ_a , δ_b , δ_c), representing flight path angle in radians, Angle of Attack (AOA) (radians), pitch rate (radian/seconds), tail deflection angle (radians), the tail deflection rate (radian/seconds), the commanded tail deflection angle (radians), are of primary focus here. Additionally, V as velocity and M as Mach number have been used to describe the dynamics of the missile. The objective of this control problem aims to ensure that the missile precisely tracks the commanded trajectory specified by the guidance law. α , q , δ_a , and δ_b have been selected as the state variables, with M and γ being the auxiliary states. In the nonlinear missile model, the output is the angle of attack AOA (α), governed by the input of the commanded tail deflection δ_c . The proposed design aims to facilitate accurate tracking of the specified AOA and simultaneously achieve a convergence time of 1 second while maintaining optimal tracking performance (ISE, RMSE, and IAE). In the literature, this model is commonly adopted as a benchmark for the development and evaluation of longitudinal autopilot system designs [1]. The equations of motion are as follows:

$$\dot{M} = \frac{0.7P_0S}{ma} [M^2 (C_{D0} - C_Z \sin \alpha)] - \frac{g}{a} \sin \gamma \quad (1)$$

$$\dot{\alpha} = \frac{0.7P_0S}{ma} MC_Z \cos \alpha + \frac{g}{aM} \cos \gamma + q \quad (2)$$

$$\dot{\gamma} = -\frac{0.7P_0S}{ma} MC_Z \cos \alpha - \frac{g}{aM} \cos \gamma \quad (3)$$

$$\dot{q} = \frac{0.7P_0Sd}{I_Y} M^2 C_m \quad (4)$$

$$\dot{\delta}_a = \delta_b \quad (5)$$

$$\dot{\delta}_b = -2\xi\omega_a\delta_b - \omega_a^2\delta_a + \omega_a^2\delta_c \quad (6)$$

The normal force and pitch moment are characterized through their respective aerodynamic coefficients:

$$C_Z = a_n \alpha^3 + b_n \alpha |\alpha| + c_n \left(2 - \frac{M}{3}\right) \alpha + d_n \delta$$

$$C_m = a_m \alpha^3 + b_m \alpha |\alpha| + c_m \left(-7 + \frac{8M}{3}\right) \alpha + d_m \delta + e_m q$$

I lists the aerodynamic parameters and coefficients described for a missile operating at an altitude of 20,000 ft [1] [5].

III. CONTROL TECHNIQUES

A. Feedback Linearization

Feedback Linearization is a non-linear control technique that transforms the original system into a new system with a

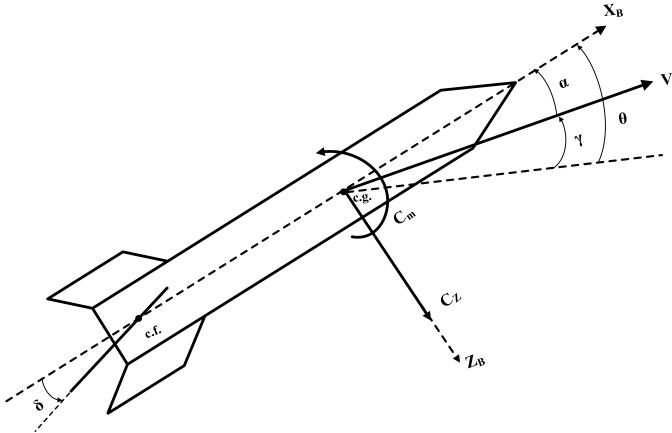


Fig. 1: Missile longitudinal airframe dynamics

TABLE I: Aerodynamic parameters and coefficients [1]

Physical parameters	Value
Mass (m)	13.98 slug
Reference area (S)	0.44 ft ²
Static pressure (P_0)	973.3 lb/ft ²
Speed of sound (a)	1036.4 ft/s
Pitch moment of inertia (I_Y)	182.5 slug - ft ²
Reference length (d)	0.75 ft
Actuator bandwidth (ω_a)	50 rad/s
Actuator damping (ξ)	0.7
$a_n = 19.373$	$a_m = 40.440$
$b_n = -31.023$	$b_m = -64.015$
$c_n = -9.717$	$c_m = 2.922$
$d_n = -1.948$	$d_m = -11.803$
$C_{D0} = 0.300$	$e_m = -1.719$

new set of coordinates. The new system exhibits quasi-linear dynamics due to new control inputs [8]. At its core, Feedback Linearization (FL) aims to transform nonlinear system dynamics into an equivalent framework that behaves linearly. Consider the following non-linear system:

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \quad , \quad x(0) = x_0 \\ y &= h(x)\end{aligned}\quad (7)$$

$$\text{where } f(x) = \begin{bmatrix} \frac{0.7P_0S}{ma} MC_Z \cos \alpha + q \\ \frac{0.7P_0Sd}{I_Y} M^2 C_m \\ \delta_b \\ -2\zeta\omega_a \delta_b - \omega_a^2 \delta_a \end{bmatrix}, g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_a^2 \end{bmatrix}, \text{ and } h(x) = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x = x_1.$$

Here, the state vector is represented by x , the output vector is represented by y and u denotes the input vector. The control law is formulated as follows:

$$u = \alpha(x) + \beta(x)v \quad (8)$$

Equation (8) demonstrates that the input-output dynamics from v to y have been transformed into a linear form. The non-linear model exhibits input-state linearizability and controllability exclusively in regions where the subsequent criteria are met [5]. The following canonical form is obtained after

the transformation of the non-linear missile model through the specific selection [5]:

$$z = T(x), \quad \dot{z} = Az + Bv \quad (9)$$

where z contains the transformed state variables and v is the control effort of the quasi-linearized system,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Matrix A and B represent the Brunovsky canonical form. The control input v enables the original control input u to be constructed for the nonlinear plant. Controllability ensures that, although v can be designed using simple linear control methods, the overall non linear system continues to meet the desired performance specifications.

B. LQR Approach

We often face the challenge of determining the feedback gain without prior knowledge of the system's eigenvalues in case of standard state feedback techniques. This limitation has led researchers to adopt the LQR (LQR) framework. The LQR approach facilitates the computation of the state feedback gain K_{LQR} , by selecting optimal weighting matrices Q and R . Through the LQR methodology, K_{LQR} is derived by minimizing a cost function, which is typically represented as a quadratic performance index, concerning the control effort (v)

$$J = \int_0^\infty z^T Q z + v^T R v dt \quad (10)$$

Here, Q denotes a symmetric, positive semi-definite matrix, and R is a symmetric, positive definite matrix. The optimal state-feedback gain is subsequently determined as follows:

$$K_{LQR} = R^{-1} B^T P \quad (11)$$

where P is a symmetric, positive definite matrix that satisfies the Algebra Riccati Equation (ARE):

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (12)$$

The matrices play a crucial role in determining the behavior of the LQR controller. The Q matrix weights the state variables, influencing how fast they can converge and the R matrix influences the control effort. In the following section, we present a new approach to selecting weight matrices Q and R using meta-heuristic algorithms such as Genetic Algorithms (GA), and Particle Swarm Optimization (PSO).

IV. OPTIMAL SELECTION OF LQR PARAMETERS

A. Genetic Algorithm for LQR tuning

Genetic algorithms are powerful search techniques based on the principles of biological evolution. These algorithms mimic the natural selection process to address complex optimization challenges, including the LQR problem [13]. Fig.2 describes

a generic workflow to obtain optimal solutions through the Genetic Algorithm. The workflow namely follows three steps: Initialization of parameters followed by the core iterative optimization (Selection, Crossover, Mutation), termination check and display of results. The Genetic algorithm requires the following parameters: population size, number of variables and a termination determining criteria (tolerance). Lower bound and upper bound values for the Q and R are required to be specified. In our case, RMSE is chosen as the objective function. Candidate parameters for Q and R matrices are chosen, and the fitness function (in our case, RMSE) is evaluated. The candidates with the best fitness values (minimum value of J_{RMSE}) are selected. The fitness values are evaluated using this expression:

$$Fitness = 1/(J_{RMSE} + e) \quad (13)$$

where e is a small value to avoid division by zero. The selection probability of a candidate i :

$$P(i) = \frac{F(i)}{\sum F(j)} \quad (14)$$

$P(i)$ represents selection probability while $F(i)$ represents the fitness value of i and $\sum F(j)$ is the sum of fitness values for all individuals in the population.

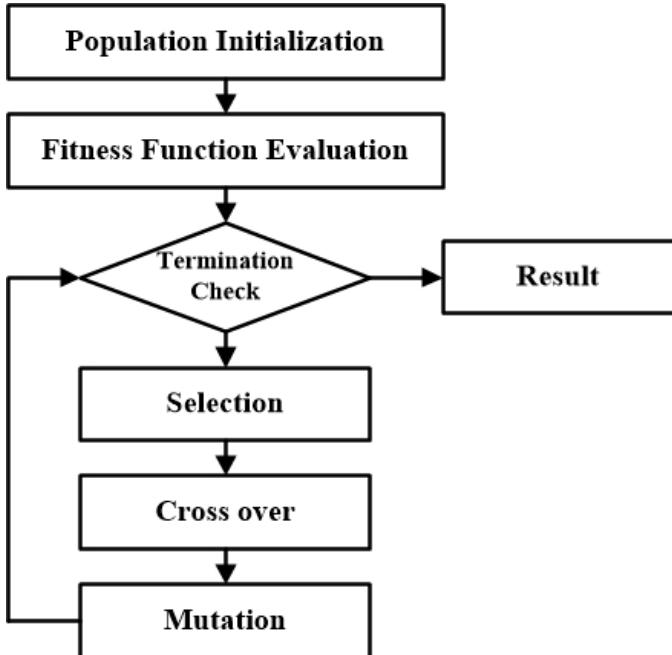


Fig. 2: Genetic Algorithm Workflow

This paper focuses on minimizing the RMSE by optimally tuning the weighting matrices Q and R of the LQR controller. The expression for RMSE, which serves as a critical performance metric, is articulated with respect to the reference and output signals related to the Angle of Attack (AOA):

$$J_{RMSE} = \sqrt{\frac{1}{N+1} \sum_{i=0}^N (reference_i - output_i)^2} \quad (15)$$

Achieving a trade-off between precise state regulation and minimal control effort is crucial. We need to ensure that we meet our performance goals for the system, specifically an overshoot of less than 5% and a settling time of under 1 second [5], while also optimizing our control efforts. The RMSE provides a reliable metric for quantifying system performance by assessing the discrepancies between the actual and reference trajectories.

B. Particle Swarm Optimization for LQR tuning

Particle Swarm Optimization at its core involves a swarm of particles traversing the problem domain in search of the optimal solution [14]. These particles iteratively update their positions based on a combination of individual and collective knowledge. Three factors influence the movement of the particles: their current position, their personal best position (cognitive component), and the global best position discovered by the entire swarm (social component). The expression for particle position update is given by:

$$X_i(k+1) = X(k) + V_i(k+1) \quad (16)$$

The expression for particle velocity update is given by:

$$V_i(k+1) = wV_i(k) + c_1r_1(p_{best_i} - X(k)) + c_2r_2(g_{best_i} - X_i(k)) \quad (17)$$

The flowchart in Fig. 3 outlines the execution process of the PSO used for tuning LQR parameters.

C. Configuration of the Missile Control system

The block diagram illustrated in Fig. 4 represents a closed-loop control system specifically designed for missile guidance. This system employs LQR controller to effectively attain the desired flight characteristics. The LQR control algorithm utilizes a full state feedback mechanism, whereby the controlled states are fed back through the feedback gain, denoted as K_{LQR} . This gain is derived by minimizing a selected objective function through the careful selection of optimal weight matrices Q and R, employing a meta-heuristic approach to ensure desired performance.

V. RESULTS AND DISCUSSIONS

The results obtained from simulations are discussed in this section. In particular, we simulated the feedback-driven autopilot system, which consists of the nonlinear missile model (7) with Mach Number set to 2.5. The control approach presented in Sections III and IV are used to execute the simulation. It is shown in Fig. 4. The intended design aims to achieve a precise tracking of a specified angle of attack (AOA) for the missile, with a convergence time of 1 second, while maintaining optimal tracking performance quantified by RMSE, ISE, and IAE. The proposed controller design aims to minimize the controller effort by searching for the optimum weighting parameters of an LQR controller using meta-heuristic algorithms. We arrive at optimal values by minimizing the RMSE between desired and reference values of the Angle of Attack.

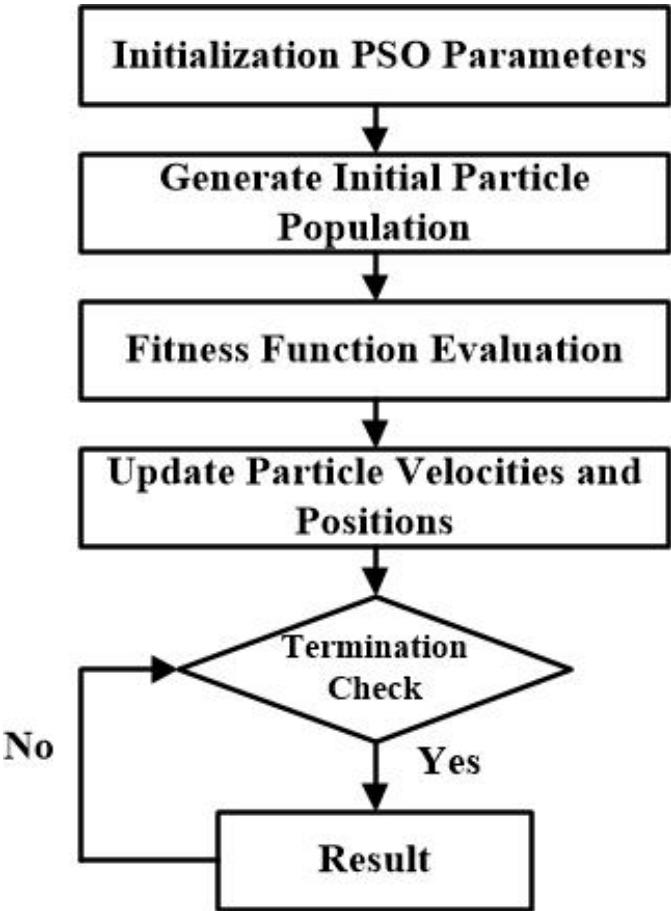


Fig. 3: Particle Swarm Optimization Workflow

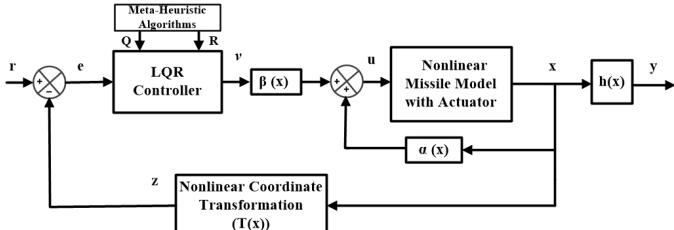


Fig. 4: Configuration of the proposed LQR Controller using FL and meta-heuristic algorithms

After executing the genetic algorithm for LQR tuning, the optimized weights Q and R are given as follows:

$$Q = \text{diag}([119384 \ 3388 \ 43 \ 0.8]), \quad R = 0.01$$

$$\text{and } K_{LQR} = [3058 \ 1326 \ 243.9 \ 23.47]$$

Using Particle Swarm Optimization as follows:

$$Q = \text{diag}([120000 \ 4000 \ 50 \ 0.1]), \quad R = 0.01$$

$$\text{and } K_{LQR} = [3464 \ 1517 \ 274.6 \ 25.48]$$

Leveraging both analytical methods and expert insight, the optimized weighting matrices Q and R have been strategically determined as follows [5]:

$$Q = \text{diag}([100000 \ 2000 \ 30 \ 0.1]), \quad R = 0.01$$

$$\text{and } K_{LQR} = [3162 \ 1282 \ 228 \ 22]$$

The simulation results of the nonlinear missile model with FL and LQR using the meta-heuristic algorithms and analytical/iterative approach [5] to a desired AOA given in the form of a step signal are shown in Fig. 5.

The error metrics are documented in Table II:

TABLE II: Error metrics

Errors	Optimization Techniques		
	GA	PSO	Analytical
RMSE	$3.287e^{-5}$	$2.651e^{-5}$	$3.302e^{-5}$
IAE	0.1234	0.2122	0.1172
ISE	0.02309	0.03707	0.02213

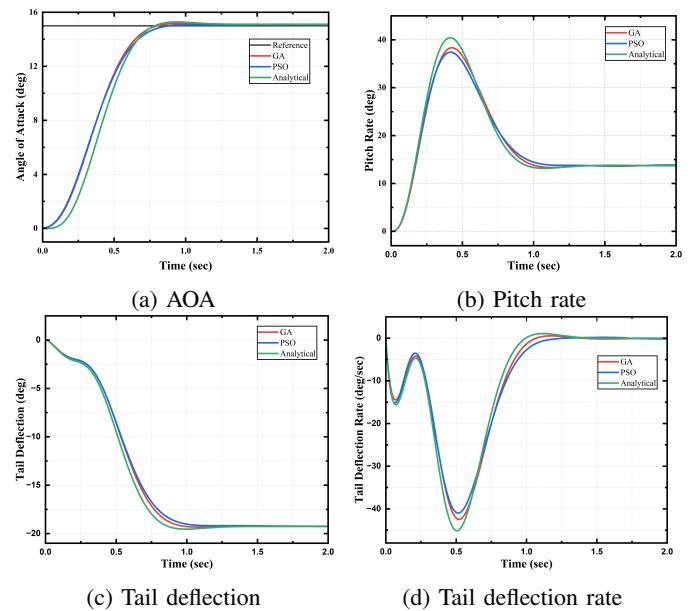


Fig. 5: Control performance of feedback-linearized missile with GA/PSO-tuned-LQR: (a) AOA; (b) Pitch rate; (c) Tail deflection; (d) Tail deflection rate

The RMSE value obtained after optimization using GA and PSO is lower than the value obtained by iterative/analytical procedure used to tune LQR controllers [5]. The performance of the LQR is expressed in terms of time-domain specifications, is listed in Table III

The RMSE value of the angle of attack is obtained to be $2.65e^{-5}$ and $3.287e^{-5}$ after executing the Particle Swarm Algorithm and Genetic Algorithm, respectively. The GA and PSO have been utilized to determine the optimal Q and R matrices for the LQR problem. PSO-based LQR tuning has successfully obtained a reduced value of RMSE compared to the GA. GA-LQR demonstrated a lower settling time compared to PSO-LQR but a higher overshoot percentage. The

TABLE III: Comparative analysis of the time-domain specifications for the three tuning techniques

Time Domain Pa-rameters	GA	PSO	Analytical
Settling Time (sec)	0.6421	0.688	0.7167
Rise Time (sec)	0.287	0.306	0.4077
Overshoot (%)	0.3287	0.0014	1.5507
Peak Time (sec)	0.84	1.88	0.8770
Peak (rad)	0.264	0.263	0.8770
Settling Min (rad)	0.238	0.239	0.2375
Settling Max (rad)	0.264	0.263	0.2678

performance of the LQR controller tuned using an analytical technique shown in [5] is given for comparison purposes. GA and PSO-based LQR successfully manage to reduce the overshoot, settling time and peak time. The LQR can be further tuned by modifying the initialization parameters.

VI. CONCLUSION

In this paper, we have proposed a comprehensive approach for designing a high-performance longitudinal autopilot controller for cruise missiles employing the Linear Quadratic Regulator, enhanced through feedback linearization and further optimized using bio-inspired optimization techniques. The nonlinear missile dynamics were transformed into a quasi-linear framework by leveraging the feedback linearization technique, enabling the deployment of linear state feedback control techniques. Recognizing the critical influence of the weighting matrices Q and R in shaping control performance, we introduced two meta-heuristic optimization algorithms; Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) to automate and refine their selection. The controller's performance was rigorously evaluated using time-domain specifications and error metrics such as RMSE, IAE and ISE. Simulation results demonstrated that both GA and PSO-based LQR controllers significantly outperformed traditional analytical tuning approaches. Notably, PSO achieved lowest RMSE, while GA offered faster settling times, thereby showcasing the trade-offs available through different optimization strategies. The work establishes a robust framework for tuning LQR controllers in nonlinear systems and reinforces the viability of soft computing techniques for control design under stringent performance constraints. The proposed methodology not only improves tracking accuracy and transient response but also ensures adaptability across dynamic operating conditions. Future work could involve extending this methodology to multi-input multi-output (MIMO) systems, incorporating real-time constraints, or hybridizing optimization strategies for enhanced convergence and robustness. This study thus provides a scalable and intelligent control design paradigm that can be generalized beyond missile systems to broader aerospace and robotic applications.

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