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Name: Shreejan Pandit
Level: Bachelor
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Q.1

Ans. Given,

$F(x) = 3x + \sin x + e^{x}$

Here,

x	0	1
$f(x)$	-1	1.1232

1st iteration:

Here $f(0) = -1 < 0$ and $f(1) = 1.1232 > 0$

∴ Now Roots lies between 0 and 1

$x_0 = \frac{0+1}{2} = 0.5$

$f(x_0) = f(0.5)$

$= 3 * 0.5 + \sin(0.5) - e^{-0.5}$

$= 0.3307 > 0$

Now like iteration 1st we solve it in table shown.

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n	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	f(c)	update
1	0	-1	1	1.1232	0.5	0.3307	b = c
2	0	-1	0.5	0.3307	0.25	-0.2866	a = c
3	0.25	-0.2866	0.5	0.3307	0.375	0.0363	b = c
4	0.25	-0.2866	0.375	0.0363	0.3125	-0.1219	a = c
5	0.3125	-0.1219	0.375	0.0363	0.3438	-0.042	a = c
6	0.3438	-0.042	0.375	0.0363	0.3594	-0.0026	a = c
7	0.3594	-0.0026	0.375	0.0363	0.3672	0.0169	b = c
8	0.3594	-0.0026	0.3672	0.0169	0.3633	0.0071	b = c
9	0.3594	-0.0026	0.3633	0.0071	0.3613	0.0023	b = c
10	0.3594	-0.0026	0.3613	0.0023	0.3604	-0.0002	a = c
11	0.3604	-0.0002	0.3613	0.0023	0.3608	0.001	b = c
12	0.3604	-0.0002	0.3608	0.001	0.3606	0.0004	b = c
13	0.3604	-0.0002	0.3606	0.0004	0.3605	0.0001	b = c
14	0.3604	-0.0002	0.3605	0.0001	0.3604	0	a = c

$\therefore$ Approximate root of the equation $f(x) = 3x + \sin(x) - e^{-x}$ using Bisection method is 0.3604 #

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⇒ Using False Position method

Soln

Given: $f(x) = 3x + \sin(x) - e^x$

x	0	1
$f(x)$	-1	1.1232

Formula:

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

1st iteration:

Here $f(0) = -1 < 0$ and $f(1) = 1.1232 > 0$

∴ Now root lies betn x_0 and $x_1 = 1$

$$x_2 = 0 - (-1) \cdot \frac{1-0}{1.1232 - (-1)}$$

$\therefore x_2 = 0.471$

$f(x_2) = f(0.471) = 3 * 0.471 + \sin(0.471) - e^{0.471}$

$= 0.2652 > 0$

2nd iteration :-

Here $f(0) = -1 < 0$ and $f(0.471) = 0.2652 > 0$

∴ Now, Root lies betn $x_0 = 0$ and $x_1 = 0.471$

$$x_3 = 0 - (-1) \cdot \frac{0.471 - 0}{0.2652 - (-1)}$$

$x_3 = 0.3723$

$f(x_3) = f(0.3723) = 3 * 0.3723 + \sin(0.3723) - e^{0.3723}$

$= 0.0295 > 0$

3rd iteration:

Here $f(0) = -1 < 0$ and $f(0.3723) = 0.0295 > 0$

$\therefore$ Now, Root lies betn $x_0 = 0$ & $x_1 = 0.3723$

$$x_4 = 0 - (-1) \cdot \frac{0.3723 - 0}{0.0295 - (-1)}$$

$x_4 = 0.3616$

$f(x_4) = f(0.3616) = 3 * 0.3616 + \sin(0.3616) - e^{0.3616}$

$= 0.0029 > 0$

4th iteration:

Here $f(0) = -1 < 0$ and $f(0.3616) = 0.0029 > 0$

$\therefore$ Root lies betn $x_0 = 0$ and $x_1 = 0.3616$

$$x_5 = 0 - (-1) \frac{0.3616 - 0}{0.0029 - (-1)}$$

$x_5 = 0.3605$

$f(x_3) = f(0.3605) = 3 \times 0.3605 + \sin(0.3605) - e^{0.3605}$

$= 0.0003 > 0$

$\therefore$ Approximate root of eq^n $f(x) = 3x + \sin(x) - e^x$ using False Position method is 0.3605.

$\Rightarrow$ Using Secant method

Given: $f(x) = 3x + \sin(x) - e^x$

Here,

x	0	1
$f(x)$	-1	1.1232

Formula:

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

1st iteration:

$x_0 = 0$ and $x_1 = 1$

$f(x_0) = f(0) = -1$ and $f(x_1) = 1.1232$

$$\therefore x_2 = 0 - (-1) \cdot \frac{1 - 0}{1.1232 - (-1)}$$

$\therefore x_2 = 0.471$

$\therefore f(x_2) = f(0.471) = 3 \times 0.471 + \sin(0.471) - e^{0.471} = 0.2652$

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2^{nd} iteration:

$x_1 = 1$ and $x_2 = 0.471$

$f(x_1) = f(1) = 1.1232$ and $f(x_2) = f(0.471)$

$= 0.2652$

$\therefore x_3 = 1 - 1.1232 * \dfrac{0.471 - 1}{0.2652 - 1.1232}$

$x_3 = 0.3075$

$\therefore f(x_3) = f(0.3075) = 3 * 0.3075 + \sin(0.3075) - e^{0.3075}$

$= -0.1348$

3^{rd} iteration:

$x_2 = 0.471$ and $x_3 = 0.3075$

$f(x_2) = f(0.471) = 0.2652$ and $f(x_3) = f(0.3075)$

$= 0.1348$

$\cancel{\therefore x_4 = x_2}$

$x_4 = 0.471 - 0.2652 \cdot \dfrac{0.3075 - 0.471}{-0.1348 - 0.2652}$

$x_4 = 0.3626$

$\therefore f(x_4) = f(0.3626) = 3 * 0.3626 + \sin(0.3626) - e^{0.3626}$

$= 0.0055$

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4^{th} iteration:

$x_3 = 0.3075$ and $x_4 = 0.3626$

$f(x_3) = f(0.3075) = -0.1348$ and $f(x_4) = f(0.3626)$
$= 0.0055$

$\therefore x_5 = 0.3075 - (-0.1348) \cdot \frac{0.3626 - 0.3075}{0.0055 - (-0.1348)}$

$\therefore x_5 = 0.3605$

$\therefore f(x_5) = f(0.3605) = 3 * 0.3605 + \sin(0.3605) - e^{0.3605}$
$= 0.0001$

$\therefore$ Approximate root of eq^n $f(x) = 3x + \sin(x) - e^x$ using secant method is 0.3605.

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Ans Q5
Using Trapezoidal

Given:

$$\int_0^1 \frac{dx}{1+x^2}, \quad n=4$$

$a = 0$ $\quad F(0) = 1$

$b = 1$ $\quad f(1) = 1/2$

$$h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = \cancel{0.5}\ 0.25$$

we know,

$$\int_a^b F(x) = \frac{h}{2}\left[f(a)+f(b)\right] + h\sum_{i=1}^{n-1} f(a+ih)$$

$$= \frac{0.25}{2}\left[F(0)+f(1)\right] + 0.25\sum_{i=1}^{4-1} f(0+0.25i)$$

$$= \frac{0.25}{2}\left[1+1/2\right] + 0.25\sum_{i=1}^{3} F(0.25i)$$

$$= \frac{3}{16} + 0.25 \cdot \sum_{i=1}^{3} F(0.25i)$$

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$$= \frac{3}{16} + 0.25 * \left[f(0.25*1) + f(0.25*2) + f(0.25*3) \right]$$

$$= \frac{3}{16} + 0.25 * \left[f(0.25) + \cancel{0} f(0.5) + f(0.75) \right]$$

$$= \frac{3}{16} + 0.25 \left[\frac{16}{17} + \frac{4}{5} + \frac{16}{25} \right]$$

$$= \frac{3}{16} + \cancel{0.25}\ \frac{253}{425}$$

$$= 0.7827941176$$

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Using Simpson's method:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Given,

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

$n = 4$

$a = 0$

$b = 1$

$$h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$

$$\therefore \int_0^1 \frac{1}{1+x^2}\,dx = \frac{\frac{1}{4}}{3}\left[f(0) + 4f\left(\frac{0+1}{2}\right) + f(1)\right]$$

$$= \frac{1}{12}\left[1 + 4 \times \frac{4}{5} + \frac{1}{2}\right]$$

$$= \frac{\left(1 + \frac{16}{5} + 0.5\right)}{12}$$

$=$

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$$\int_a^b f(x)\,dx \simeq \frac{h}{3}\Big[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)$$
$$+2f(x_3)+\cdots+4f(x_{n-3})+2f(x_{n-2})$$
$$+4f(x_{n-1})+f(x_n)\Big]$$

$h = \frac{b-a}{n}$

we have $a=0$, $b=1$, $n=4$

$\therefore h = \frac{1-0}{4} = \frac{1}{4}$

Divide the interval $[0,1]$ into $n=4$ sub intervals of length $\Delta x = \frac{1}{4}$ with the following end points:

$a=0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1=b$

Now, just evaluate the function at these end points

$f(x_0) = f(0) = 1$

$4f(x_1) = 4f\left(\frac{1}{4}\right) = \frac{64}{17} \simeq 3.7647058823529$

$2f(x_2) = 2f\left(\frac{1}{2}\right) = 8/5 = 1.6$

$4f(x_3) = 4f\left(\frac{3}{4}\right) = \frac{64}{25} = 2.56$

$f(x_4) = f(1) = \frac{1}{2} = 0.5$

Finally, just sum up the above values and multiply by $\frac{h}{3} = \frac{1}{12}$

$\frac{1}{12}$ ~~(1+3.07~~

$\frac{1}{12}(1 + 3.7647058823529 4 + 1.6 + 2.56 + 0.5)$

$= 0.78539215686 2745$

Actual value of $\int_0^1 \frac{dx}{1+x^2}$

$= [\tan^{-1}(x)]_0^1$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \pi/4$

$= 0.7853981 6$

∴ We see that the value given by simpson's rule is much closer to actual value than that given by trapezoidal rule. If we take 5 decimal places the value by simpson's rule is exactly correct.
Hence, Simpson's rule is better.

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Q 4

Ans Second order RK method

→ The essential formula to compute $y(n+1)$ is

$y_1 = y_0 + 1/2 \, (k_1 + k_2)$

where

$k_1 = h f_0 = h_0 + (x_0, y_0)$

$k_2 = h f(x_0 + h, y_0 + k)$

$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$

→ The method is 2nd method meaning that the local trancullation error is on the of (h^3), while the total accumulated error is order $O(h^4)$ #

4th order RK method

→ The essential formula

$k_1 = h f(x_0, y_0)$

$k_2 = h f(x_0 + h/2, y_0 + k_1/2)$

$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$

$k_4 = h f(x_0 + h, y_0 + k_3)$

and $k = 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4)$

and

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= y_i + k$

→ The method is 4th order method, meaning that local ~~function~~ trunclation error on the order of $O(h^5)$ while the total accumulated error in order $O(h^4)$.

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Given,

$y(0.1) = 2$ at $x = 0.1$

$\frac{dy}{dx} = 3e^{x} + 2y = f(x, y)$

$y(0) = 0$ at $x = 0$

$h = 0.1$

now for 2^{n} order method

$k_1 = h f_0 = h f(x_0, y_0)$

$= 0.1 * (3e^{x_0} + 2y_0)$

$= 0.1 * (3e^{1} + 2*0)$

$= 0.1 * 3$

$= 0.3$

$K_2 = h f(x_0 + h, y_0 + k_1)$

$= 0.1 * (3e^{(x_0+h)} + 2(y_0 + k_1))$

$= 0.1 * (3e^{(0.1)} + 2(0.3))$

$= 0.3915$

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now, $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$

$y_1 = y_0 + \frac{1}{2}(0.3 + 0.3915)$

$= 0 + 0.3452$

$= 0.3457$

Put $x = 0.1$

$k_1 = h f(x_1, y_1)$

$= 0.1 * (3e^{0.1} + 2(0.3457))$

$= 0.40069$

$k_2 = h f(x_1 + h, y_1 + k_1)$

$= 0.1 * (3^{(0.1+0.1)} + 2(0.3457 + 0.40069))$

$= 0.51596$

$k_1 + k_2 = 0.91665$

$y_2 = y_1 + \frac{1}{2} k$

$= 0.3457 + \frac{1}{2} * 0.91665$

$= 0.3453 + \frac{0.91665}{2}$

$= 0.80395$

$\therefore y(0.1) = 2.179$ at 2^n order RK method

now, for 4th order

k_1 = same as 2^{nd} so,

$= hf(x_0, y_0)$

$= 0.3$

$h_2 = hf(x_0 + h/2, y_0 + k/2)$

$= 0.1 * (3 * e^{x_0 + h/2} + 2(y_0 + k_1/2))$

$= 0.1 * (3e^{0.1/2} + 2(0.3/2))$

$= 0.34538$

$k_3 = hf(x_0 + h/2, y_0 + k2/2)$

$= 0.1[3e^{0.1/2} + 2(\frac{0.34538}{2})]$

$= 0.34991$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1(3e^{0.1} + 2[0.34991])$

$= 0.41535$

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$k = 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= 1/6 (0.3 + 2*0.34538 + 2*0.34991 + 0.41535)$

$= \frac{2.10593}{6}$

$= 0.3509$

$y_1 = y_0 + k$

$= 0 + k$

$= 0.35098$

now at $x = 0.1$

$k_1 = h f(x_1, y_1)$

$= 0.1 * (3e^{0.1} + 2(0.35098))$

$= 0.40174$

$k_2 = hf(x_1 + h/2, y_1 + k/2)$

$= 0.1 * (3e^{0.1 + 0.1/2} + 2[0.35098 + 0.45]$

$= 0.4646$

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$y_1(0.1) = y_1 + k$
$= 0.35098 + 0.46304$
$= 0.81402$

$\therefore y(0.15) = 0.81402$ at 4^{th}
$y(0.15) = 0.80395$ at 2^{nd}

The difference $= |0.81402 - 0.80395|$
$= 0.01007$

in percentage $= 1.007\%$

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Q3

	x_0	x_1	x_2	x_3
x	300	304	305	307
$\log x$	2.4771	2.4829	2.4843	2.4871

For $\log_{10} 310$ i.e $x = 310$ [using lagrange's formula]

Here,
from the above value,

$$L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$= \frac{(310-304)(310-305)(310-307)}{(300-304)(300-305)(300-307)}$$

$$= -0.642$$

$$L_1 = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$= \frac{(310-300)(310-305)(310-307)}{(304-300)(304-305)(304-307)}$$

$$= 12.5$$

$$L_2 = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_2-x_1)(x_2-x_3)}$$

$$= \frac{(310-300)(310-304)(310-307)}{(305-300)(305-304)(305-307)}$$

$$= -18$$

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$$L_3 = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(310-300)(310-304)(310-305)}{(307-300)(307-304)(307-305)}$$

$$= 50/7$$
$$= 7.1429$$

Applying lagrange's formula we have

$$\log_{10} 310 = \cancel{\log} L_0 y_0 + L_1 y_1 + L_2 y_2 + L_3 y_3$$

$$= (-0.642 * 2.4771) + (12.5 * 2.4829) + (-18 * 2.4843) + (7.1429 * 2.4871)$$

$$= 2.4937$$

⇒ Using Newton's divide difference formula.

x	$y = f(x)$	Δy $[x_0, x_1]$	$\Delta^2 y$ $[x_0\ x_1\ x_2]$	$\Delta^3 y$ $[x_0\ x_1\ x_2\ x_3]$
$x_0 = 300$	2.4771	$\frac{2.4829-2.471}{304-300} = 0.00145$	$\frac{0.0014-0.00145}{305-300} = -1*10^{-5}$	$-1*10^{-5}$
$x_1 = 304$	2.4829	$\frac{2.4843-2.4829}{305-304} = 0.0014$	0	
$x_2 = 305$	2.4843	$\frac{2.4871-2.4843}{307-305} = 0.0014$		
$x_3 = 307$	2.4871			

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Taking $x = 310$, in Newton's divide difference formula, we obtain
$f(310) = \log_{10} 310 =$

$\Rightarrow y_0 + (x-x_0)(x_0, x_1) + (x-x_0)(x-x_1)[x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2)[x_0, x_1, x_2, x_3]$

$\Rightarrow 2.4771 + (310-300)\, 0.00145 + (310-300)(310-304) * (-1 * 10^{-5}) + (310-300)(310-304)(310-305) * (-1 * 10^{-5})$

$\Rightarrow 2.488$

Using calculator ~~log10~~ $\log_{10} 310 = 2.4917$

Here, the value of $\log_{10} 310$ is closer to the real value while using lagrange's formula than using Newton's divide difference formula.