Models for Polytomous Responses

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```
#install.packages("car")
#install.packages("VGAM")
```

For a target with Ordinal Values

Proportional Odds Model

Unless the response variable has numeric values, it is important to ensure that it has been defined as an ordered factor (using ordered()). In the Arthritis data, the response, Improved was setup this way, as we can check by printing some of the values

```
library(vcd)

## Loading required package: grid
```

```
library(grid)
data("Arthritis", package="vcd")
head(Arthritis$Improved, 8)
```

```
## [1] Some None None Marked Marked Marked None Marked
## Levels: None < Some < Marked</pre>
```

```
library(MASS)
arth.polr <- polr(Improved ~ Sex + Treatment + Age,data=Arthritis, Hess=TRUE)
summary(arth.polr)</pre>
```

```
## Call:
## polr(formula = Improved ~ Sex + Treatment + Age, data = Arthritis,
##
       Hess = TRUE)
##
## Coefficients:
##
                        Value Std. Error t value
## SexMale
                    -1.25168
                                 0.54636 - 2.291
## TreatmentTreated 1.74529
                                 0.47589
                                            3.667
## Age
                      0.03816
                                 0.01842
                                            2.072
##
## Intercepts:
##
               Value
                        Std. Error t value
## None | Some
                2.5319 1.0571
                                    2.3952
## Some | Marked 3.4309 1.0912
                                    3.1443
##
## Residual Deviance: 145.4579
## AIC: 155.4579
```

Summary gives us the coefficients, Intercepts labeled by the cutpoint on the ordinal response but no p-values. The car::Anova() method gives the appropriate tests

```
library(car)
```

```
## Loading required package: carData
```

```
Anova(arth.polr)
```

```
## Analysis of Deviance Table (Type II tests)
##
## Response: Improved
##
             LR Chisq Df Pr(>Chisq)
## Sex
               5.6880
                      1 0.0170812 *
## Treatment 14.7095
                      1 0.0001254 ***
## Age
               4.5715
                          0.0325081 *
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

In R, the PO and NPO models can be readily contrasted by fitting them both using vglm() in the VGAM package. This defines the cumulative family of models and allows a parallel option. With parallel=TRUE, this is equivalent to the polr() model, except that the signs of the coefficients are reversed

```
#install.packages("VGAM")
library(VGAM)
```

```
## Loading required package: stats4
## Loading required package: splines
##
## Attaching package: 'VGAM'
## The following object is masked from 'package:car':
##
##
       logit
arth.po <- vglm(Improved ~ Sex + Treatment + Age, data=Arthritis,family = cumulative(</pre>
parallel=TRUE))
arth.po
##
## Call:
## vglm(formula = Improved ~ Sex + Treatment + Age, family = cumulative(parallel = TR
UE),
##
       data = Arthritis)
##
##
## Coefficients:
##
      (Intercept):1
                        (Intercept):2
                                                SexMale TreatmentTreated
         2.53199006
                           3.43098785
                                             1.25167130
                                                             -1.74530408
##
##
                Age
        -0.03816279
##
##
## Degrees of Freedom: 168 Total; 163 Residual
```

The more general NPO model can be fit using parallel=FALSE.

Residual deviance: 145.4579
Log-likelihood: -72.72896

```
arth.npo <- vglm(Improved ~ Sex + Treatment + Age, data=Arthritis, family = cumulativ
e(parallel=FALSE))
arth.npo</pre>
```

```
##
## Call:
## vglm(formula = Improved ~ Sex + Treatment + Age, family = cumulative(parallel = FA
LSE),
##
       data = Arthritis)
##
##
## Coefficients:
##
        (Intercept):1
                            (Intercept):2
                                                    SexMale:1
##
           2.61853875
                               3.43117450
                                                   1.50982704
##
            SexMale:2 TreatmentTreated:1 TreatmentTreated:2
##
           0.86643395
                              -1.83692928
                                                  -1.70401146
##
                Age:1
                                    Age: 2
##
          -0.04086648
                              -0.03729421
##
## Degrees of Freedom: 168 Total; 160 Residual
## Residual deviance: 143.5741
## Log-likelihood: -71.78703
```

The VGAM package defines a coef() method that can print the coefficients in a more readable matrix form giving the category cutpoints

```
coef(arth.po, matrix=TRUE)
```

```
## logitlink(P[Y<=1]) logitlink(P[Y<=2])
## (Intercept) 2.53199006 3.43098785
## SexMale 1.25167130 1.25167130
## TreatmentTreated -1.74530408
## Age -0.03816279 -0.03816279
```

```
coef(arth.npo, matrix=TRUE)
```

```
## logitlink(P[Y<=1]) logitlink(P[Y<=2])
## (Intercept) 2.61853875 3.43117450

## SexMale 1.50982704 0.86643395

## TreatmentTreated -1.83692928 -1.70401146

## Age -0.04086648 -0.03729421
```

In most cases, nested models can be tested using an anova() method, but the VGAM pack- age has not implemented this for "vglm" objects. Instead, it provides an analogous function, Irtest()

```
VGAM::lrtest(arth.npo, arth.po)
```

```
## Likelihood ratio test
##
## Model 1: Improved ~ Sex + Treatment + Age
## Model 2: Improved ~ Sex + Treatment + Age
## #Df LogLik Df Chisq Pr(>Chisq)
## 1 160 -71.787
## 2 163 -72.729 3 1.8838 0.5969
```

The LR test can be also calculated as "manually" shown below using the difference in residual deviance for the two models

```
tab <- cbind(Deviance = c(deviance(arth.npo), deviance(arth.po)),df = c(df.residual(a
rth.npo), df.residual(arth.po)))
tab <- rbind(tab, diff(tab))
rownames(tab) <- c("GenLogit", "PropOdds", "LR test")
tab <- cbind(tab, pvalue=1-pchisq(tab[,1], tab[,2]))
tab</pre>
```

```
## Deviance df pvalue

## GenLogit 143.574067 160 0.8196626

## PropOdds 145.457915 163 0.8343527

## LR test 1.883849 3 0.5968607
```

The vglm() can also fit partial proportional odds models, by specifying a formula giving the terms for which the PO assumption should be taken as TRUE or FALSE. Here we illustrate this using parallel=FALSE ~ Sex,tofitseparateslopesformalesandfemales,butparallellinesforthe otherpredictors. The same model would be fit using parallel = TRUE ~ Treatment + Age.

```
arth.ppo <- vglm(Improved ~ Sex + Treatment + Age, data=Arthritis, family = cumulative
  (parallel=FALSE ~ Sex))
coef(arth.ppo, matrix=TRUE)</pre>
```

```
## logitlink(P[Y<=1]) logitlink(P[Y<=2])
## (Intercept) 2.54245216 3.61556073

## SexMale 1.48333554 0.86736192

## TreatmentTreated -1.77574178 -1.77574178

## Age -0.03962233 -0.03962233
```

Graphical Assesment of Proportional Odds

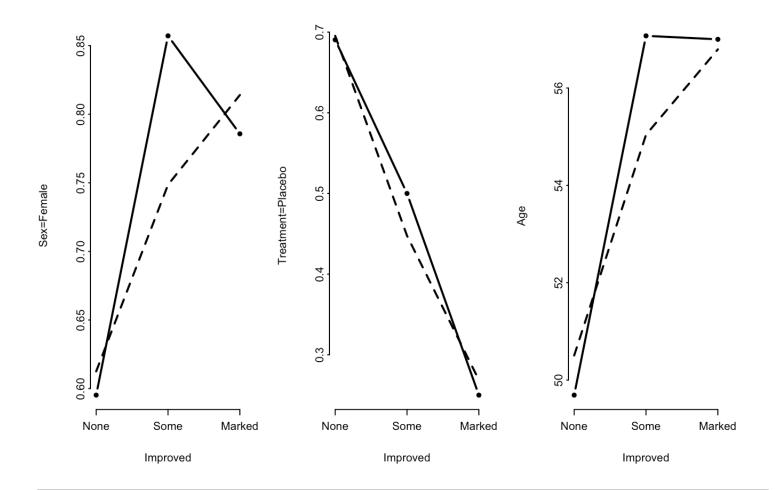
```
#install.packages("polspline",dependencies = TRUE)
#install.packages("rms",dependencies = TRUE)
library("rms")
## Loading required package: Hmisc
## Loading required package: lattice
## Loading required package: survival
## Loading required package: Formula
## Loading required package: ggplot2
##
## Attaching package: 'Hmisc'
  The following objects are masked from 'package:base':
##
##
##
       format.pval, units
## Loading required package: SparseM
##
## Attaching package: 'SparseM'
  The following object is masked from 'package:base':
##
##
       backsolve
##
## Attaching package: 'rms'
## The following objects are masked from 'package:VGAM':
##
##
       calibrate, lrtest
```

```
## The following objects are masked from 'package:car':
##
##
Predict, vif
```

```
arth.po2 <- lrm(Improved ~ Sex + Treatment + Age, data=Arthritis)
arth.po2</pre>
```

```
## Logistic Regression Model
##
##
    lrm(formula = Improved ~ Sex + Treatment + Age, data = Arthritis)
##
                           Model Likelihood
                                                 Discrimination
                                                                    Rank Discrim.
##
                              Ratio Test
                                                     Indexes
                                                                        Indexes
##
##
    Obs
                          LR chi2
                                        24.46
                                                 R2
                                                           0.291
                                                                    C
                                                                             0.750
                    84
##
                    42
                          d.f.
                                                           1.335
                                                                             0.500
     None
                                            3
                                                                    Dxy
                                                 g
##
     Some
                    14
                          Pr(> chi2) <0.0001
                                                 gr
                                                           3.801
                                                                    gamma
                                                                             0.503
                                                           0.280
##
                    28
     Marked
                                                                    tau-a
                                                                             0.309
                                                 gp
    max |deriv| 1e-07
                                                           0.187
##
                                                 Brier
##
                                       Wald Z Pr(>|Z|)
##
                               S.E.
##
   y>=Some
                       -2.5320 1.0570 -2.40
                                             0.0166
##
    y>=Marked
                       -3.4310 1.0911 -3.14
                                              0.0017
    Sex=Male
                       -1.2517 0.5464 -2.29
##
                                              0.0220
##
    Treatment=Treated 1.7453 0.4759
                                        3.67
                                              0.0002
                        0.0382 0.0184 2.07
##
    Age
                                              0.0382
##
```

```
op <- par(mfrow=c(1,3))
plot.xmean.ordinaly(Improved ~ Sex + Treatment + Age, data=Arthritis,lwd=2, pch=16, s
ubn=FALSE)</pre>
```



par(op)

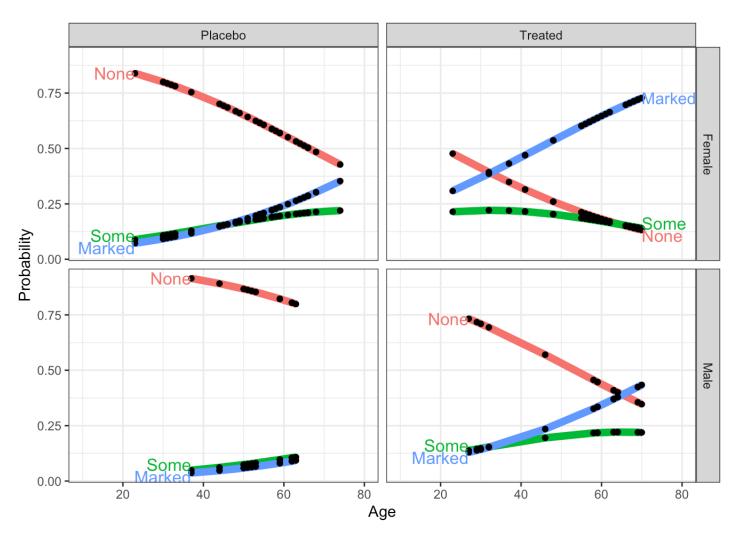
Visualizing results for Proportional Odds Model

```
arth.fitp <- cbind(Arthritis,predict(arth.polr, type="probs"))
head(arth.fitp)</pre>
```

```
##
     ID Treatment
                   Sex Age Improved
                                          None
                                                     Some
                                                             Marked
     57
          Treated Male
                                Some 0.7326185 0.1380586 0.1293229
                                None 0.7174048 0.1444325 0.1381627
    46
          Treated Male
                        29
   3 77
          Treated Male
                        30
                                None 0.7096042 0.1476259 0.1427699
     17
          Treated Male
                        32
                              Marked 0.6936286 0.1540035 0.1523679
                              Marked 0.5702499 0.1950359 0.2347142
  5 36
          Treated Male
                        46
  6 23
                        58
                              Marked 0.4563432 0.2171302 0.3265266
          Treated Male
```

```
##
      Sex Treatment Age Improved Level Probability
## 1 Male
           Treated
                    27
                           Some
                                 None
                                        0.7326185
## 2 Male
           Treated 29
                                        0.7174048
                           None None
## 3 Male
           Treated 30
                           None None
                                        0.7096042
## 4 Male
           Treated 32
                         Marked None
                                        0.6936286
## 5 Male
                         Marked None
                                        0.5702499
           Treated 46
## 6 Male
           Treated 58
                         Marked None
                                        0.4563432
```

```
library(ggplot2)
library(directlabels)
gg <- ggplot(plotdat, aes(x = Age, y = Probability, colour = Level)) +
   geom_line(size=2.5) + theme_bw() + xlim(10,80) +
   geom_point(color="black", size=1.5) +
   facet_grid(Sex ~ Treatment) #,labeller = function(x, y) sprintf("%s = %s", x, y)
direct.label(gg)</pre>
```



Although we now have three response curves in each panel, this plot is relatively easy to un- derstand: (a) In each panel, the probability of no improvement decreases with age, while that for marked improvement increases. (b) It is easy to compare the placebo and treated groups in each row, showing that no improvement decreases, while marked improvement increases with the active treatment. (On the other hand, this layout makes it harder to compare panels vertically for males and females in each condition.) (c) The points show where the observations are located in each panel; so, we can see that the data is quite thin for males given the placebo

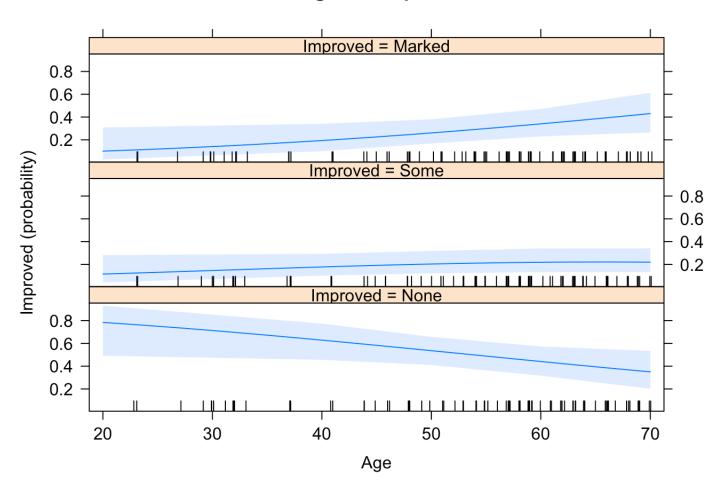
Effect Plots

```
#install.packages("effects")
library(effects)
```

```
## Use the command
## lattice::trellis.par.set(effectsTheme())
## to customize lattice options for effects plots.
## See ?effectsTheme for details.
```

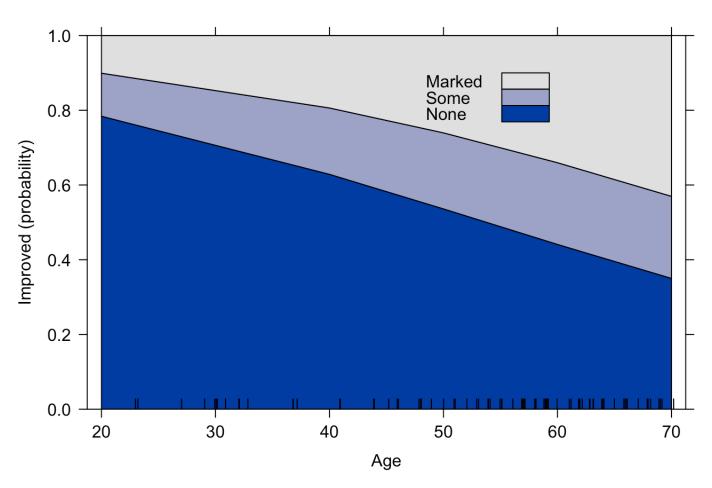
plot(Effect("Age", arth.polr))

Age effect plot

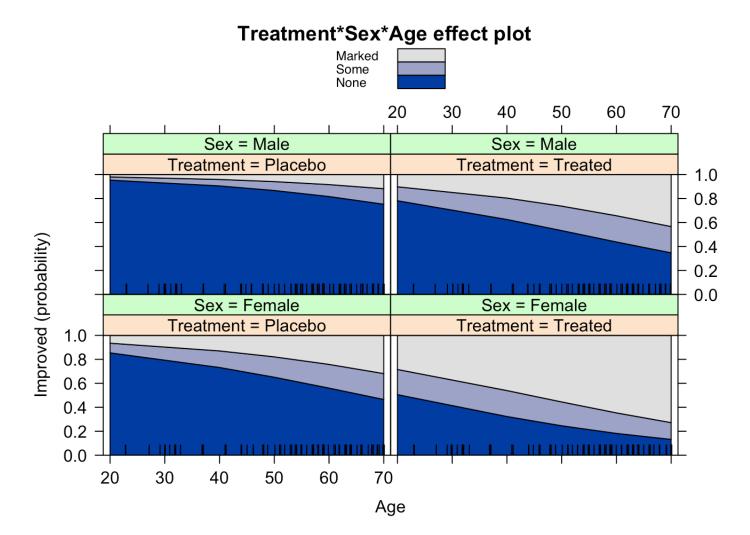


plot(Effect("Age", arth.polr), style='stacked',key.args=list(x=.55, y=.9))





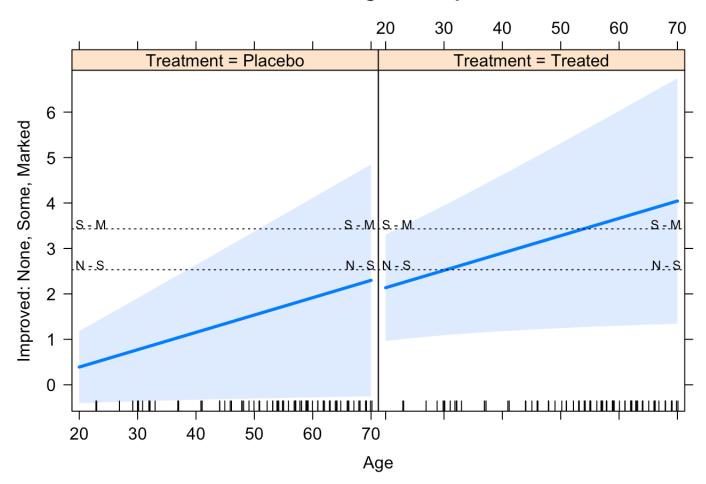
plot(Effect(c("Treatment", "Sex", "Age"), arth.polr),style="stacked", key.arg=list(x=
.8, y=.9))



Latent variable effect plot for the effects of Treatment and Age in the Arthritis data

plot(Effect(c("Treatment", "Age"), arth.polr, latent=TRUE), lwd=3)

Treatment*Age effect plot



EXAMPLE 8.1: Women's labor force participation

```
library(car)
rm(Womenlf)
data("Womenlf", package="car")
#head(Womenlf)
Womenlf <- Womenlf
```

In this example, it makes sense to consider a first dichotomy (working) between women who are not working, vs. those who are (full time or part time). A second dichotomy (fulltime) contrasts full time work vs. part time

work, among those women who are working at least part time. These two binary variables are created in the data frame using the recode() function from the car package

```
#Create dichotomies// using ifelse. Recode is another option which was not working in
this case
library(dplyr)
##
## Attaching package: 'dplyr'
   The following objects are masked from 'package:Hmisc':
##
##
##
       src, summarize
   The following object is masked from 'package:car':
##
##
##
       recode
   The following object is masked from 'package:MASS':
##
##
##
       select
## The following objects are masked from 'package:stats':
##
##
       filter, lag
   The following objects are masked from 'package:base':
##
##
##
       intersect, setdiff, setequal, union
working = ifelse(Womenlf$partic=='not.work', "no", "yes")
fulltime = ifelse(Womenlf$partic=='fulltime', "yes",
                  ifelse(Womenlf$partic=='parttime', "no", NA)
Womenlf = cbind(Womenlf, working, fulltime)
#Womenlf = cbind(Womenlf,fulltime)
head(Womenlf)
```

```
##
       partic hincome children region working fulltime
## 1 not.work
                   15
                       present Ontario
                                                     <NA>
## 2 not.work
                   13
                       present Ontario
                                                     <NA>
                                             no
## 3 not.work
                       present Ontario
                   45
                                                     <NA>
                                             no
## 4 not.work
                   23
                       present Ontario
                                                     <NA>
                                             no
## 5 not.work
                   19
                       present Ontario
                                                     <NA>
                                             no
## 6 not.work
                    7
                       present Ontario
                                             no
                                                     <NA>
```

```
with(Womenlf, table(partic, working))
```

```
## working
## partic no yes
## fulltime 0 66
## not.work 155 0
## parttime 0 42
```

```
with(Womenlf, table(partic, fulltime, useNA="ifany"))
```

```
##
              fulltime
## partic
                no yes <NA>
     fulltime
##
                  0
                     66
                            0
##
     not.work
                  0
                      0
                         155
##
     parttime
                42
                      0
                            0
```

We proceed to fit two separate binary logistic regression models for the derived dichotomous variables. For the working dichotomy, we get the following results:

```
mod.working <- glm(working ~ hincome + children, family=binomial,data=Womenlf)
summary(mod.working)</pre>
```

```
##
## Call:
## glm(formula = working ~ hincome + children, family = binomial,
##
       data = Womenlf)
##
## Deviance Residuals:
##
       Min
                 10
                    Median
                                   3Q
                                           Max
## -1.6767 -0.8652 -0.7768 0.9292
                                        1.9970
##
## Coefficients:
##
                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                  1.33583
                               0.38376
                                         3.481
                                                 0.0005 ***
## hincome
                   -0.04231
                               0.01978 - 2.139
                                                 0.0324 *
## childrenpresent -1.57565
                               0.29226 - 5.391
                                               7e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 356.15 on 262
                                      degrees of freedom
## Residual deviance: 319.73 on 260 degrees of freedom
## AIC: 325.73
##
## Number of Fisher Scoring iterations: 4
```

```
mod.fulltime <- glm(fulltime ~ hincome + children, family=binomial,data=Womenlf)
summary(mod.fulltime)</pre>
```

```
##
## Call:
## glm(formula = fulltime ~ hincome + children, family = binomial,
##
       data = Womenlf)
##
## Deviance Residuals:
##
       Min
                 10
                      Median
                                   30
                                            Max
                      0.3949
##
  -2.4047 -0.8678
                                         1.7641
                               0.6213
##
## Coefficients:
##
                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                    3.47777
                               0.76711
                                          4.534 5.80e-06 ***
## hincome
                   -0.10727
                               0.03915 -2.740 0.00615 **
## childrenpresent -2.65146
                               0.54108 -4.900 9.57e-07 ***
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
##
       Null deviance: 144.34 on 107
                                       degrees of freedom
## Residual deviance: 104.49 on 105
                                       degrees of freedom
     (155 observations deleted due to missingness)
## AIC: 110.49
##
## Number of Fisher Scoring iterations: 5
```

Although these were fit separately, we can view this as a combined model for the three-level response, with the following coefficients

```
cbind(working=coef(mod.working), fulltime=coef(mod.fulltime))
```

```
## working fulltime

## (Intercept) 1.33582979 3.4777735

## hincome -0.04230843 -0.1072679

## childrenpresent -1.57564843 -2.6514557
```

For both dichotomies, increasing income of the husband and the presence of young children decrease the log odds of a greater level of work. However, for those women who are working the effects of husband's income and and children are greater on the choice between full time and part time work than they are for all women on the choice between working and not workin

```
LRtest <- function(model) c(LRchisq=(model$null.deviance - model$deviance),df=(model$
df.null - model$df.residual))
tab <- rbind(working=LRtest(mod.working),fulltime=LRtest(mod.fulltime))
tab <- rbind(tab, All = colSums(tab))
tab <- cbind(tab, pvalue = 1- pchisq(tab[,1], tab[,2]))
tab</pre>
```

```
## LRchisq df pvalue

## working 36.41835 2 1.235536e-08

## fulltime 39.84682 2 2.225215e-09

## All 76.26518 4 1.110223e-15
```

Anova(mod.working)

```
## Analysis of Deviance Table (Type II tests)
##
## Response: working
## LR Chisq Df Pr(>Chisq)
## hincome   4.8264  1   0.02803 *
## children   31.3229  1  2.185e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

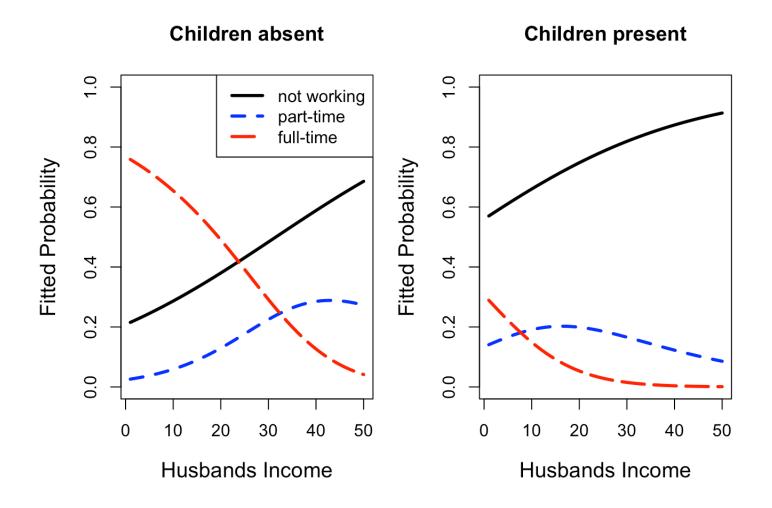
Anova(mod.fulltime)

```
## Analysis of Deviance Table (Type II tests)
##
## Response: fulltime
## LR Chisq Df Pr(>Chisq)
## hincome 8.981 1 0.002728 **
## children 32.136 1 1.437e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
      hincome children p.working p.fulltime l.working l.fulltime
## 18
           18
                 absent
                            0.640
                                       0.8245
                                                   0.574
                                                              1.547
## 29
           29
                 absent
                            0.527
                                       0.5907
                                                   0.109
                                                              0.367
           34
## 34
                 absent
                            0.474
                                       0.4578
                                                 -0.103
                                                             -0.169
## 37
                                       0.3796
           37
                 absent
                            0.443
                                                 -0.230
                                                             -0.491
## 96
           46 present
                            0.101
                                       0.0162
                                                 -2.186
                                                             -4.108
```

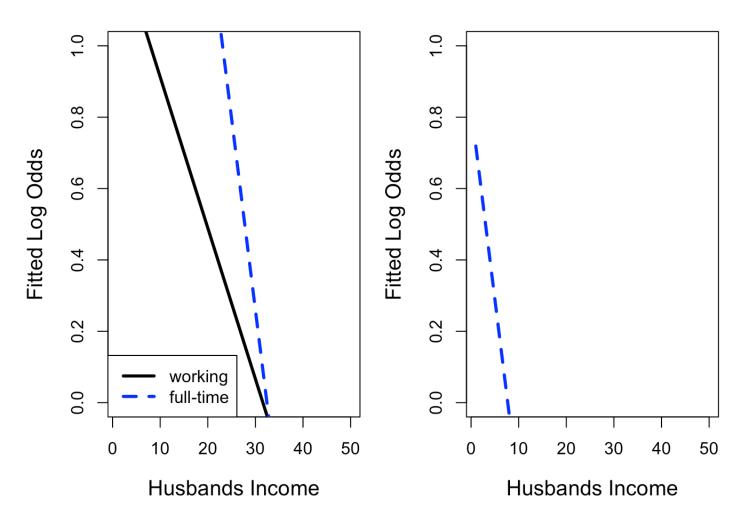
```
fit <- within(fit, { full <- p.working * p.fulltime
part <- p.working * (1 - p.fulltime)
not <- 1 - p.working
})</pre>
```

```
op <- par(mfrow=c(1,2), mar=c(5,4,4,1)+.1)
Hinc <- 1:max(fit$hincome)</pre>
for ( kids in c("absent", "present") )
{
  dat <- subset(fit,children==kids)</pre>
  plot(range(Hinc),c(0,1),type="n",cex.lab=1.25,
       xlab="Husbands Income", ylab="Fitted Probability",
       main=paste("Children", kids))
  lines(Hinc, dat$not, lwd=3, col="black", lty=1)
  lines(Hinc, dat$part, lwd=3, col="blue", lty=2)
  lines(Hinc, dat$full, lwd=3, col="red", lty=5)
  if(kids=="absent")
    legend("topright", lty=c(1,2,5), lwd=3, col=c("black", "blue", "red"),
           legend=c('not working', 'part-time', 'full-time'))
  }
}
```



par(op)

We can see how that the decision not to work outside the home increases strongly with husband's income, and is higher when there are children present. As well, among working women, the decision to work full time as opposed to part time decreases strongly with husband's income, and is less likely with young children. Similarly, we plot the fitted logits for the two dichotomies in I.working and I.fulltime as shown below:



These Graphs didn't come out as expected

Generalized Logit Model

Response: Women working full time, part time or not working Variables: Husbands Income (Continuous), Children Present vs. Absent (Categorical)

```
levels(Womenlf$partic)
```

```
## [1] "fulltime" "not.work" "parttime"
```

```
# choose not working as baseline category
Womenlf$partic <- relevel(Womenlf$partic, ref="not.work")</pre>
```

We fit the main effects model for husband's income and children as follows. As we did with polr() (Section 8.1), specifying Hess=TRUE saves the Hessian and facilitates calculation of standard errors and hypothesis tests

```
library(nnet)
wlf.multinom <- multinom(partic ~ hincome + children,data=Womenlf, Hess=TRUE)</pre>
```

```
## # weights: 12 (6 variable)
## initial value 288.935032
## iter 10 value 211.454772
## final value 211.440963
## converged
```

The summary() method for "multinom" objects doesn't calculate test statistics for the esti- mated coefficients by default. The option Wald=TRUE produces Wald z-test statistics, calculated as $z = \beta/SE(\beta)$

```
summary(wlf.multinom, Wald=TRUE)
```

```
## Call:
## multinom(formula = partic ~ hincome + children, data = Womenlf,
##
       Hess = TRUE)
##
## Coefficients:
##
            (Intercept)
                             hincome childrenpresent
## fulltime
              1.982842 -0.097232073
                                          -2.55860537
              -1.432321 0.006893838
                                           0.02145558
  parttime
##
## Std. Errors:
##
            (Intercept)
                           hincome childrenpresent
## fulltime
            0.4841789 0.02809599
                                          0.3621999
##
  parttime
              0.5924627 0.02345484
                                          0.4690352
##
## Value/SE (Wald statistics):
##
            (Intercept)
                           hincome childrenpresent
               4.095266 -3.4607098
                                        -7.06407045
## fulltime
## parttime
              -2.417573 0.2939197
                                         0.04574407
##
## Residual Deviance: 422.8819
## ATC: 434.8819
```

Notice that the coefficients, their standard errors and the Wald test z values are printed in separate tables. The first line in each table pertains to the logit comparing full time work with the not working reference level; the second line compares part time work against not working

For those who like p-values for significance tests, you can calculate these from the results re- turned by the summary() method in the Wald.ratios component, using the standard normal asymptotic approximation

```
stats <- summary(wlf.multinom, Wald=TRUE)
z <- stats$Wald.ratios
p <- 2 * (1 - pnorm(abs(z)))
zapsmall(p)</pre>
```

```
## (Intercept) hincome childrenpresent
## fulltime 0.0000422 0.0005388 0.0000000
## parttime 0.0156244 0.7688193 0.9635142
```

```
wlf.multinom2 <- multinom(partic ~ hincome * children, data=Womenlf, Hess=TRUE)
```

```
## # weights: 15 (8 variable)

## initial value 288.935032

## iter 10 value 210.797079

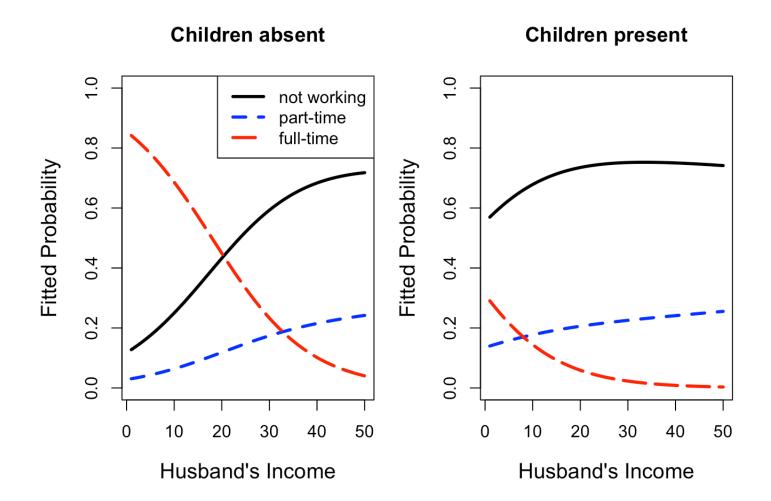
## final value 210.714841

## converged
```

```
Anova(wlf.multinom2)
```

Full model plots of the fitted values can be plotted as shown earlier in Example 8.1: obtain the fitted values over a grid of the predictors and plot these

Plotting these fitted values gives the plot shown in Figure 8.12.



The effects package has special methods for "multinom" models. It treats the response levels in the order given by levels(), so before plotting we use ordered() to arrange levels in their nat- ural order. The update() method provides a simple way to get a new fitted model; in the call, the modelformula. ~

.meanstofitthesamemodelasbefore,i.e.,partic ~ hincome + children

```
## # weights: 12 (6 variable)
## initial value 288.935032
## iter 10 value 211.454772
## final value 211.440963
## converged
```

Asillustratedearlier, you can use plot (all Effects (model), ...) to plot all the high- order terms in the model, either with separate curves for each response level (style="lines") or as cumulative filled polygons (style="stacked"). Here, we simply plot the effects for the combinations of husband's income and children in stacked style, giving a

plot (Figure 8.13) that is analogous to the full-model plot shown in Figure 8.12.

```
plot(Effect(c("hincome", "children"), wlf.multinom),
    style="stacked", key.args=list(x=.05, y=.9))
```

hincome*children effect plot

