Simple and Compound Interest
by Total Gadha - Sunday, 15 April 2007, 01:00 PM

Every MBA aspirant who understands percentages and the principles of arithmetic and geometric progressions will be able to handle problems of simple interest (SI) and compound interest (CI) easily. In CAT, XAT or other MBA exams, the problems on simple and compound interest are asked as a part of percentages only. Let me start this lesson with a simple problem:

Tommy and his dad planted two plants on the same day. The first plant is 10 ft high and grows by 10 ft every day. The second plant is 1 ft high and doubles its height everyday. On which day will the 1 ft high plant overtake the 10 ft high plant?

Answer: Letâ€™s make a simple table to answer this question. We note down the heights of the plants on each day, as shown below:

	1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day	7th Day	8th Day
1st Plant	10 ft	20 ft	30 ft	40 ft	50 ft	60 ft	70 ft	80 ft
2nd Plant	1 ft	2 ft	4 ft	8 ft	16 ft	32 ft	64 ft	128 ft

We can see from the table that the 2nd plant outgrows the 1st plant on the 8th day (colored orange in the table). Now, answer the following questions:

1. Which plant has a fixed increase in height everyday? How much is this increase?
2. Which plant is growing at a faster rate?
3. What is the percentage increase in the height of 2nd plant everyday?
4. What will be the difference in heights of the plants on the 10th day?

You can see that the 1st plant has a fixed increase of 10 ft in its height everyday. If you observe harder, this increase of 10 ft is nothing but 100 % of its original height. Therefore, **the 1st plant is increasing with a fixed percentage of its original value**. In other words, the first plant is growing with a simple interest (SI) rate.

In simple interest (SI) the increase happens by a constant amount every time. This constant amount is a fixed percentage of the original value

You can also see that the 2nd plant is increasing at a faster rate and **increases by 100% on its present value** everyday. In other words, the second plant is increasing at a compound interest (CI) rate.

In compound interest (SI) the increase happens by a constant percentage every time. This constant percentage happens on the present value.

Therefore, **both the plants are increasing by the same percentage, first on the initial value and the second on the current value. Hence, the first is in simple interest and the second is in the compound interest.**

Note that the heights of the 1st plant form an arithmetic progression with a common difference of 10 ft and the heights of the second plant form a geometric progression with a common ratio of 2.

Therefore, to calculate the heights on the 10th day we use the formula for the Nth terms of the arithmetic and geometric progressions.

For the 1st plant

$T_n = a + (n - 1)d$ ⇒ $T_{10} = 10 + 9 \times 10 = 100$

For the 2nd plant

$T_n = ar^{n-1}$ ⇒ $T_{10} = 1 \times 2^9 = 512$.

Therefore difference in heights = 512 − 100 = 412 ft

The common difference of the arithmetic progression in the case of simple interest is nothing but a fixed percentage on the principle P i.e. $SI = \frac{Pr}{100}$. Therefore the terms of the arithmetic progression, or the amounts every year, are $P, P+\frac{Pr}{100}, P+\frac{2Pr}{100}, P+\frac{3Pr}{100}\ldots, P+\frac{nPr}{100}+\ldots$

Simple interest after t years = $\frac{Prt}{100}$

The common ratio of the geometric progression in the compound interest case is nothing but the fixed percentage growth of $(1+\frac{r}{100})$. Therefore, the terms of the geometric progression, or the amounts every year are $P, P(1+\frac{r}{100}), P(1+\frac{r}{100})^2, \ldots P(1+\frac{r}{100})^n, \ldots$

Compound interest after t years = $P(1+\frac{r}{100})^t - P$

Now let's start with the same amount and see the results of their growths in simple and compound interests:

This time, we invest the two amounts of Rs100 each, both at the annual rate of 10%, one at simple interest and the other at compound interest. Letâ€™s see their growth:

	Initial Quantity	1st Year	2nd Year	3rd Year	4th Year
Simple Growth	Rs100	Rs110	Rs120	Rs130	Rs140
Yearly Interest	0	Rs10	Rs10	Rs10	Rs10
Compounded Growth	Rs100	Rs110	Rs121	Rs133.1	Rs146.41
Yearly Interest	0	Rs10	Rs11	Rs12.1	Rs13.31

Notice a few things:

1. The simple interest in every year is constant.
2. The simple interest and the compound interest are the same at the end of the 1st year.

3. The compound interest is increasing every year and it is increasing by a factor equal to the ratio of the geometric series, i.e. the percentage growth $(1+\frac{r}{100}) = (1+\frac{10}{100}) = 1.1$. Therefore, interest in 2nd year = interest in the 1st year multiplied by percentage growth = 10 × 1.1 = 11, interest in the 3rd year = interest in the 2nd year multiplied by percentage growth = 11 × 1.1 = 12.1... and so on. Therefore, **compound interest is the interest on interest**.

SOME SOLVED EXAMPLES:

1. In how many years will an amount invested in simple interest at the rate of 5% get doubled?

Answer: we know that yearly amounts in a simple interest scheme are in arithmetic progression with common difference of Pr/100. In this case, the amounts will be in arithmetic progression with a common difference of 0.05P.

The series is P, 1.05P, 1.1P, 1.15P, â€¦ (P + n Ã— 0.05P), â€¦and so on. Let the amount be double of the original amount after n years.
Therefore, P + n Ã— 0.05P = 2P ⇒ n = 20 years.

Alternative Answer: The amount is growing by 5% every year or 1/20th every year. Since 1/20th of the sum is added every year, the whole sum will be added in 20 years.

2. A is going to pay Rs836 to B 9 months later at 6% annual simple interest. B is going to pay A Rs783 15 months later at 7% annual simple interest. If they decide to settle the debts right now, who will pay what amount to whom?

   Answer: Let the amount A needs to pay B right now be P_1. P_1 at 6% annual simple interest is going to become Rs836 in 9 months.
   $\rightarrow P_1 + \frac{P_1 \times 6 \times 9}{100 \times 12} = 836 \rightarrow P_1 = Rs800.$
   Similarly, let the amount B needs to pay A right now be P_2. P_2 at 7% annual simple interest is going to become Rs783 in 15 months.
   $\rightarrow P_2 + \frac{P_2 \times 7 \times 15}{100 \times 12} = 783 \rightarrow P_2 = Rs720.$
   Therefore, A needs pay Rs800 to B right now and B needs to pay Rs720 right now. Therefore, A needs to pay Rs80 to B.

3. Two equal sums of money were invested at an annual rate of 10%, one sum at simple interest and the other at compound interest. If the difference between the interests after 2 years was Rs200, what were the sums invested?

   Answer: This question can be solved by taking both the amounts equal to Rs100 at 10% and then using the unitary method.
   At Rs100 and at the rate of 10%, the amount at simple interest after two years is Rs120 and the amount at compound interest after 2 years is 121. The difference between the amounts, or the difference between the interests is 1 rupee.

   The difference in the interests is 1 rupee when the amounts invested are Rs100
   → difference between the interests is Rs200 when the amounts invested are 200 × 100
   = Rs20 000.

4. A sum of money was invested in bank at compound interest. The interest in the second year was Rs550 and the interest in the 3rd years was Rs605. Find the rate of interest and the amount invested.

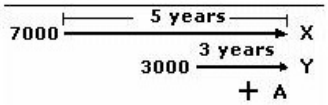
   Answer: we know that compound interest is nothing but interest on interest. Therefore interest in the 3rd year would be interest added on the interest in the 2nd year, i.e., the interest in the 3rd year would be percentage growth on the interest in the 2nd year. Let the rate of interest be r.
   $\rightarrow 550(1+\frac{r}{100}) = 605 \rightarrow r = 10\%$
   Now for rate of interest equal to 10%, here is the table when the amounts invested is Rs100.

	Initial Quantity	1st Year	2nd Year	3rd Year	4th Year
Compounded Growth	Rs100	Rs110	Rs121	Rs133.1	Rs146.41
Yearly Interest	0	Rs10	Rs11	Rs12.1	Rs13.31

   Now we can solve it by unitary method.
   At 10%, the interest is Rs11 in the second year when the amount invested is Rs100.
   → at 10% the interest will be Rs550 in the second year when the amount invested is $\frac{100}{11} \times 550 = Rs5000$.

5. Rajan borrowed Rs7 000 from a bank at 5% simple annual rate of interest. The amount is to be paid after 5 years. Rajan pays Rs4 000 after 2 years. How much amount should he pay at the end of 5 years to pay off his debt completely?

   Answer: The above can be understood by the diagram given below:



   If Rs7000 was lying with the bank itself the bank would have earned an interest for 5 years. The total amount X with bank after 5 years would have been $7000 + \frac{7000 \times 5 \times 5}{100} = Rs8750$.
   Therefore, the total sum Rajan has to pay is Rs8750. Now when he gives Rs4000 after 2 years, the bank earns interest on this amount for next 3 years. Therefore, the total interest earned
   $= \frac{4000 \times 5 \times 3}{100} = Rs600$. Therefore, total amount paid to the bank till the end of 5 years
   = Rs4000 + Rs600 = Rs4600.
   The amount left to be paid = Rs8750 – Rs4600 = Rs4150.

6. An amount is invested in bank at compound interest. The amounts after 1st and 3rd years are Rs1200 and Rs1587, respectively. What is the rate of interest?

   Answer: Let the amount invested be A and the rate of interest be r.
   → Therefore $A(1+\frac{r}{100}) = 1200$ and $A(1+\frac{r}{100})^3 = 1587$.

Dividing the 2nd equation by the 1st we get $(1+\frac{r}{100})^2 = \frac{1587}{1200} \rightarrow (1+\frac{r}{100})^2 = \frac{529}{400} \rightarrow$

$1+\frac{r}{100} = \frac{23}{20} \rightarrow r = 15\%$