MATHEMATICAL FORMULAE

Algebra

1. $(a+b)^2 = a^2 + 2ab + b^2$; $a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2$; $a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$; $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$; $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$
10. $a^n = a.a.a \ldots n$ times
11. $a^m.a^n = a^{m+n}$
12. $$\begin{aligned} \frac{a^m}{a^n} &= a^{m-n} \text{ if } m > n \\ &= 1 \qquad \text{if } m = n \\ &= \frac{1}{a^{n-m}} \text{ if } m < n; a \in R, a \neq 0 \end{aligned}$$
13. $(a^m)^n = a^{mn} = (a^n)^m$
14. $(ab)^n = a^n.b^n$
15. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
16. $a^0 = 1$ where $a \in R, a \neq 0$
17. $a^{-n} = \frac{1}{a^n}, a^n = \frac{1}{a^{-n}}$
18. $a^{p/q} = \sqrt[q]{a^p}$
19. If $a^m = a^n$ and $a \neq \pm 1, a \neq 0$ then $m = n$
20. If $a^n = b^n$ where $n \neq 0$, then $a = \pm b$
21. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = \sqrt{y}$, then $a = 0$ and $x = y$
22. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = b + \sqrt{y}$ then $a = b$ and $x = y$
23. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a mn = \log_a m + \log_a n$
24. If a, m, n are positive real numbers, $a \neq 1$, then $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
25. If a and m are positive real numbers, $a \neq 1$ then $\log_a m^n = n \log_a m$
26. If a, b and k are positive real numbers, $b \neq 1, k \neq 1$, then $\log_b a = \frac{\log_k a}{\log_k b}$
27. $\log_b a = \frac{1}{\log_a b}$ where a, b are positive real numbers, $a \neq 1, b \neq 1$
28. if a, m, n are positive real numbers, $a \neq 1$ and if $\log_a m = \log_a n$, then $m = n$

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29. if $a+ib=0$ where $i=\sqrt{-1}$, then $a=b=0$
30. if $a+ib=x+iy$, where $i=\sqrt{-1}$, then $a=x$ and $b=y$
31. The roots of the quadratic equation $ax^2+bx+c=0;\ a\neq 0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

    The solution set of the equation is $\left\{\dfrac{-b+\sqrt{\Delta}}{2a}, \dfrac{-b-\sqrt{\Delta}}{2a}\right\}$

    where $\Delta=$ discriminant $=b^2-4ac$
32. The roots are real and distinct if $\Delta>0$.
33. The roots are real and coincident if $\Delta=0$.
34. The roots are non-real if $\Delta<0$.
35. If α and β are the roots of the equation $ax^2+bx+c=0, a\neq 0$ then
    i) $\alpha+\beta=\dfrac{-b}{a}=-\dfrac{\text{coeff. of } x}{\text{coeff. of } x^2}$
    ii) $\alpha\cdot\beta=\dfrac{c}{a}=\dfrac{\text{constant term}}{\text{coeff. of } x^2}$
36. The quadratic equation whose roots are α and β is $(x-\alpha)(x-\beta)=0$

    i.e. $x^2-(\alpha+\beta)x+\alpha\beta=0$

    i.e. $x^2-Sx+P=0$ where $S=$Sum of the roots and $P=$Product of the roots.
37. For an arithmetic progression (A.P.) whose first term is (a) and the common difference is (d).
    i) n^{th} term$=t_n=a+(n-1)d$
    ii) The sum of the first (n) terms $=S_n=\dfrac{n}{2}(a+l)=\dfrac{n}{2}\{2a+(n-1)d\}$

       where $l=$last term$=a+(n-1)d$.
38. For a geometric progression (G.P.) whose first term is (a) and common ratio is (γ),
    i) n^{th} term$=t_n=a\gamma^{n-1}$.
    ii) The sum of the first (n) terms:

$$\boxed{\begin{aligned} S_n &= \frac{a(1-\gamma^n)}{1-\gamma} && \text{if} \gamma<1 \\ &= \frac{a(\gamma^n-1)}{\gamma-1} && \text{if } \gamma>1 \\ &= na && \text{if } \gamma=1 \end{aligned}}.$$

39. For any sequence $\{t_n\}, S_n-S_{n-1}=t_n$ where $S_n=$Sum of the first (n) terms.
40. $\sum\limits_{\gamma=1}^{n}\gamma=1+2+3+\cdots+n=\dfrac{n}{2}(n+1)$.
41. $\sum\limits_{\gamma=1}^{n}\gamma^2=1^2+2^2+3^2+\cdots+n^2=\dfrac{n}{6}(n+1)(2n+1)$.

42. $\sum_{\gamma=1}^{n} \gamma^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2.$
43. $n! = (1).(2).(3).\ldots.(n-1).n.$
44. $n! = n(n-1)! = n(n-1)(n-2)! = \ldots\ldots$
45. $0! = 1.$
46. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n, n > 1.$