

* $\frac{7}{x+y} + \frac{6}{x-y} = 7$ $\frac{4}{x-y} - \frac{14}{x+y} = 2$. Find x and y ?

In these type of questions, put values of $\frac{1}{x+y}$ & $\frac{1}{x-y}$.

$$\frac{1}{x+y} \rightarrow a \quad \frac{1}{x-y} \rightarrow b.$$

$$\begin{array}{l} 7a + 6b = 7 \\ 4b - 14a = 2 \end{array} \quad \parallel \quad \text{Relatively simple to solve.}$$

* Indeterminate system of equations:-

$$\begin{array}{l} x+y+z=8 \\ 2x-y+3z=13 \end{array} \quad \begin{array}{l} 2 \text{ eq's, } 3 \text{ variables.} \end{array}$$

* * Q-Type Special case.

$$\begin{array}{l} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \end{array} \quad \text{If } \frac{a_1}{a_2} = \frac{c_1}{c_2}, \text{ then } \bullet y \text{ can be uniquely determined.}$$

$$\begin{array}{l} \text{Ex:- } 2x + 3y + 4z = 46 \quad \times 3 \\ \quad \quad 3x + y + 6z = 62 \quad \times 2 \end{array} \Rightarrow \text{then subtract \& solve.}$$

* * Q-Type

$$2x + 3y + 2z = 54 \dots\dots (1)$$

$$5x + 6y + 4z = 118 \dots\dots (2)$$

Find $3x + 3y + 2z$!

Here, there are 2 eq's & 3 variables. So, not possible to compute value of x, y & z distinctly.

But $(2) - (1)$ gives value of required combination.

* Independent Equations:-

$$I_1 + KI_2 = I_3 \quad K \text{ is any constant.}$$

→ If there are 3 eq^s & 3 variables, but the 3 eq^s are not independent → eq^s can't be solved.

Ex I_1 $3x + 5y + 7z = 12$

I_2 $x - 3y + 9z = 16$

I_3 $9x + 8y + 31z = 54$.

Are they independent?

$$3a + b = 9$$

$$9a + 3b = 27$$

$$5a - 3b = 8$$

$$14a = 35$$

$$\Rightarrow a = \frac{5}{2}$$

$$\therefore b = 9 - \frac{15}{2} = +\frac{3}{2}$$

Then, check. $7a + 9b = 7 \times \frac{5}{2} + 9 \times \frac{3}{2}$

$$= \frac{62}{2} = 31 = \text{coefficient of } z \text{ in } I_3.$$

Then, $I_1 \times \frac{5}{2} + I_2 \times \frac{3}{2} = I_3$.

* Inconsistent Equations:-

$$\begin{array}{l} x + 2y = 4 \\ 2x + 4y = 10 \end{array} \quad \parallel \text{ inconsistent.}$$

* In an Examination, $\frac{3}{5}$ of students who appeared failed by 10 marks and $\frac{1}{5}$ of the students got 10 marks above the pass mark. Students who gave the exam scored 62 marks on average. Find the pass mark.
Each of remaining students got 20 marks above pass mark.
Pass mark → p

$$\frac{3}{5}(p-10) + \frac{1}{5}(p+10) + \frac{1}{5}(p+20) = 62.$$

$$\Rightarrow 3p - 30 + p + 10 + p + 20 = 310.$$

$$\Rightarrow p = 62.$$

- * Venkat takes 2 hours more than Vatsa to cover a distance of 600 km. Instead if Venkat doubles his speed, he would reach the destination 4 hours before Vatsa. Find Vatsa's speed.

Venkat $\rightarrow a$ Vatsa $\rightarrow b$.

$$x \leftarrow \left\{ \frac{600}{a} = y \left\{ \frac{600}{b} + 2. \right. \right.$$

$$\frac{600}{2a} + 4 = \frac{600}{b}.$$

$$x = y + 2$$

$$\frac{x}{2} + 4 = y$$

$$\Rightarrow x - 2 = \frac{x}{2} + 4.$$

$$\Rightarrow x = 6 \times 2 = 12. \quad \therefore y = 10.$$

$$\therefore \frac{600}{b} = 10 \Rightarrow b = 60 \text{ km/hr.}$$

- * A man had enough money to buy 16 apples or 10 mangoes. If the man buys 4 apples & 5 mangoes and is left with Rs 20. Then what is the difference between price of an apple and mango?

$$16a = 10m \Rightarrow 8a = 5m \quad m > a.$$

$$4a + 5m + 20 = 16a = 10m$$

$$\Rightarrow 4a + 5m + 20 = 16a$$

$$\Rightarrow 12a + 20 = 16a \Rightarrow a = 5 \quad \therefore m = 8.$$

$$\text{Ans} \rightarrow 8 - 5 = 3.$$

* $3x + 2y + 4z = 43$
 $5x + 3y + 6z = 66.$ Find x ?

$9x + 6y + 12z = 129$
 $10x + 6z + 12z = 132 \Rightarrow x = 3.$

* $3x + 5y + 2z = 68 \quad I_1$
 $6x + 7y + 4z = 121 \quad I_2$ Find z .
 $9x + 15y + 6z = 204 \quad I_3$

$I_3 - I_1 - I_2$
 $\Rightarrow 3y = 15 \Rightarrow y = 5$

$3x + 2z = 43$
 $6x + 4z = 86$
 $9x + 6z = 129$ $\parallel \parallel$ z can't be found.

* $3x + 5y + 7z = 49 \quad I_1$ Find $x + y + z$.
 $5x + 8y + 11z = 78 \quad I_2$

$2I_2 - 3I_1$
 $2x + y + z = 78 \times 2 - 49 \times 3 = 156 - 147 = 9.$

* $4x + 6y + 7z = p$
 $5x + 7y + 9z = q.$
 $2x + 4y + 3z = r.$

Find relation between p, q & r .

Relation exists when they are not independent eqⁿ.

$4a + 5b = 2$ $6a + 7b = 4$

$$12a + 15b = 6$$

$$12a + 14b = 8$$

$$\Rightarrow b = -2. \quad \therefore a = 3$$

$$\text{Check: } -7a + 9b = 7 \times 3 + 9 \times 2 = 3. \quad (\checkmark).$$

$$\therefore 3p - 2q = 8.$$

* 90 is divided into 3 parts such that sum of first two parts exceeded sum of 2nd & 3rd part by 18. If smallest part is 18, find the greatest part.

$$a \quad b \quad 90 - a - b.$$

$$a + b = 90 - a + 18 = 108 - a.$$

$$\Rightarrow 2a + b = 108.$$

$$\text{If } a = 18 \quad b = 72 \quad c = 0. \quad \times$$

$$a = 18 \quad b = 45 \quad c = 27.$$

$$90 - a - b = 18 \Rightarrow a + b = 72$$

$$2a + b = 108$$

$$a = 36$$

$$b = 36$$

$$c = 18.$$

no greatest part.

$$\therefore \text{Ans} \rightarrow 45.$$

* In the year 1980, the age, in years, of a person was one-eightyninth of his year of birth. What was the age of person in 2012?

$$\text{Year of birth} \rightarrow x$$

$$1980 - x = \frac{x}{89} \quad \Rightarrow x = 1958.$$

$$\therefore \text{Age in 2012} = 54.$$

* The difference between a 3 digit number & the no. formed by reversing its digits is 396. Difference of hundred & unit digit is one less than sum of unit & tens digit. Also, hundred digit is twice the units digit. Find the no.

abc

$$(100a + 10b + c) - (100c + 10b + a) = 396$$

$$\Rightarrow 99a - 99c = 396$$

$$\Rightarrow a = c + 4$$

$$a - c + 1 = b + c$$

$$\Rightarrow a = b + 2c - 1$$

$$a = 2c$$

$$\therefore 2c = b + 2c - 1$$

$$\Rightarrow b = 1$$

$$\therefore 2c = c + 4$$

$$\Rightarrow c = 4 \quad \Rightarrow a = 8$$

* In a 4 digit no., sum of digits in units & tens place is equal to sum of digits in hundred & thousand place. The sum of digits in tens & hundreds place is twice the sum of other 2 digits. If sum of digits is more than 20, then the digit in unit place can be
(i) 5 (ii) 6 (iii) 7 (iv) 8.

$abcd$

$$c + d = a + b$$

$$b + c = 2(a + d) \quad \therefore (b + c)_{\max} = 18$$

$$a + d \leq 9$$

$$a + b - d = 2a + 2d - b$$

$$\Rightarrow 2b = a + 3d \dots (i)$$

$$a+b+c+d > 20$$

$$(a+b) = (c+d) > 10.$$

Then, from (i), $2b = a+3d$

$$3b = a+b+3d$$

$$3b-3d > 10.$$

$$b-d > \frac{10}{3} \geq 4$$

$\Rightarrow d \leq 5$, else b will be > 9 .

Case

Ans $\rightarrow 5$

$$d=5 \quad b=9 \quad a=3 \quad c=7 \quad 3975$$

$$d=4 \quad b=8 \quad a=4 \quad c=8 \quad 4884.$$

$$d=3 \quad b=7 \quad a=5 \quad c=9 \quad \cancel{5795} \quad 5793$$

$$d=2 \quad b=6 \quad a=6 \quad c=10 \quad X.$$

* 7 children standing in a line, not all of whom have same no. of cakes. If first child distributes his cakes among the remaining 6 children, such that he doubles their respective no. of cakes, then he will be left with 4 cakes. Instead, if 2nd child takes away 2 cakes from each of remaining 6 children, then he will be left with 3 less than the no. of cakes the first child initially had. What is total no. of cakes with the 3rd child, 4th child, ..., 7th child?

$$\underline{a} \quad \underline{b} \quad \underline{x}$$

$$a-b-x=4.$$

$$b+12 = a-3. \Rightarrow a = b+15.$$

$$\therefore b+15-b-x=4.$$

$$\Rightarrow x=11.$$

* $7x + 5y + 6z = 45 \dots (i)$

~~12x + 10y~~

$8x + 10y + 12z = 78 \dots (ii)$

$4x + 5y + 6z = 39$

Find $3x + 5y + 6z$.

$3x = 6 \Rightarrow x = 2$

$\therefore 5y + 6z = 45 - 7 \times 2 = 31$

$\therefore 3x + 5y + 6z = 3 \times 2 + 31 = 37$

* $6x + 7y + 8z = 7500$

$3x + 4y + 5z = 4350$

Find x ?

$\frac{7}{4} \neq \frac{8}{5}$

x can't be determined uniquely.

* The amount with Aarthi is 80 more than that with Bhargavi. Chandni has Rs 50 less than the amount with Bhargavi. Divya has Rs 120 more than the sum of amounts with Bhargavi & Chandni. Total money is Rs 500 & in form of denominations of ₹ 10 & ₹ 20 only. What is least no. of ₹ 10 they can have?

a b c d

$a = b + 80$

$c = b - 50$

$d = b + c + 120$

$a + b + c + d = 500$

$d = b + c + 120$

$d = b + 120 + b - 50$

$\Rightarrow d = 2b + 70$

$b = a - 80$

$2b = 2a - 160$

$a + d + d - 120 = 500$

$a + 2d = 620$

$2a - d = 90$

$4a - 2d = 180$

$2a - 160 = d - 70$

$5a = 800$

$\Rightarrow a = 160$

$d = 230$

$b = 80$

$c = 30$

$\therefore \text{Ans} \rightarrow 0 + 1 + 1 = 2$

- * Sanju & Manju went to a bakery shop. Sanju ate 4 puffs, 3 burgers & 2 cakes & used all money she had. Manju ate 3 puffs, 6 burgers & 4 cakes & paid one-fourth more than what Sanju paid. What fraction of Sanju's money was spent on the puff she ate?

$$4p + 3b + 2c = s \quad 8p + 6b + 4c = 2s.$$

$$3p + 6b + 4c = m = \frac{5s}{4}$$

$$\Rightarrow 5p = 2s - \frac{5s}{4} = \frac{3s}{4}$$

$$\Rightarrow 4p = \frac{3s}{4} \times \frac{4}{5} = \frac{3s}{5}$$

$$\text{Ans} \rightarrow \frac{4p}{s} = \frac{3}{5}$$

- * Present age of a woman is 6 times that of her ^{only} daughter Anu. Her husband's age is 2 years more than 7 times Anu's present age. The average present age of Anu, her parents & only brother (youngest of all 4 members) is 15 years. By how many years is Anu elder to her brother?

$$w = 6d$$

$$d > b.$$

$$h = 2 + 7d$$

$$6d + 2 + 7d + d + b = 15 \times 4 = 60.$$

$$\Rightarrow 14d + b = 58$$

$$\therefore d = 4, b = 2.$$

$$\text{Ans} \rightarrow 2.$$

* A shopkeeper sold a certain number of toys all at a certain price. The no. of toys that he sold is a 3 digit no. in which the tens digit and units digits are the same and are non-zero, and price of each toy is a 2 digit number when expressed in rupees. By mistake he reversed the digits of both, the no. of items sold and the price of each item. In doing so, he found that his stock account at the end of day showed ₹92 items more than what it actually was.

(i) What could be actual no. of toys sold?

(a) 911 (b) 119 (c) 199 (d) 991

(ii) If the faulty calculations show a total sale of ₹5117, what was the actual selling price of each toy?

(a) 43 (b) 37 (c) 45 (d) 34

(iii) What are actual sales?

(a) 39,173

(b) 30,974

(c) 4,046

(d) 4,064.

(i) No. of toys \rightarrow baa

$a > 0$

$x < z$

Faulty Stock account showed \Rightarrow Faulty less sales more items

a & c are probable options

Less sales in faulty calculation

So, Ans \rightarrow A.

baa

$$\begin{aligned} 100x + 10y + z \\ - (100z + 10y + x) \\ = 792 \end{aligned}$$

$$\Rightarrow 99x - 99z = 792$$

$$\Rightarrow x = z + 8$$

(ii) $\frac{119 \times x}{\text{Faulty}} = \frac{\text{₹5117}}{\text{Faulty}} \Rightarrow x = 43.$ Actual $\rightarrow 34$

(iii) Actual sale $\rightarrow 911 \times 34 = 30,974$