DOUBLEROOT Cheat Sheet – Straight Line

Equations of a Line

Parallel to X-axis

At a distance a from the X-axis

$$y = a$$

Parallel to Y-axis

At a distance b from the Y-axis

$$x = b$$

Slope-Intercept Form

Slope: m and y-intercept: c

$$y = mx + c$$

Intercept Form

x-intercept: a and y-intercept: b

$$\frac{x}{a} + \frac{y}{b} = 1$$

Point-Slope Form

Slope: m and through a point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Two-Point Form

Passing through the points (x_1, y_1) and (x_2, y_2)

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Normal Form

At a distance p from origin and perpendicular from origin inclined at angle α with the X-axis

$$x\cos\alpha + y\sin\alpha = p$$

Parametric Form

Slope: tanθ and passing through the point (x_1, y_1)

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

(x, y) is a point on the line at a distance r from (x_1, y_1)

General Form

$$ax + by + c = 0$$

Slope: -a/b

Equations representing the same line

$a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ represent the same line if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Angle between lines with slopes m_1 and m_2

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

For parallel lines: $m_1 = m_2$

For perpendicular lines: $m_1m_2=-1$

Point of intersection of non-parallel lines $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

Condition for the lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ & $a_3x+b_3y+c_3=0$ to be concurrent

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Distance of a point (x_1, y_1) from a line $ax+by+c=0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Distance between parallel lines $ax+by+c_1=0$ & $ax+by+c_2=0$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Position of points (x_1, y_1) and (x_2, y_2) w.r.t a line $ax+by+c=0$

Same side: $(ax_1+by_1+c)(ax_2+by_2+c) > 0$

Opposite side: $(ax_1+by_1+c)(ax_2+by_2+c) < 0$

Angle bisectors of $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Family of lines

Equation of a line passing through the point of intersection of $L_1=0$ and $L_2=0$

$$L_1 + \lambda L_2 = 0 \qquad (\lambda \in R)$$

Pair of Straight Lines passing through Origin

Equation

$$ax^2 + 2hxy + by^2 = 0$$

Angle between the Lines

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

Coincident lines: $h^2 = ab$, Perpendicular lines: $a+b=0$

Equation of angle bisectors

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$