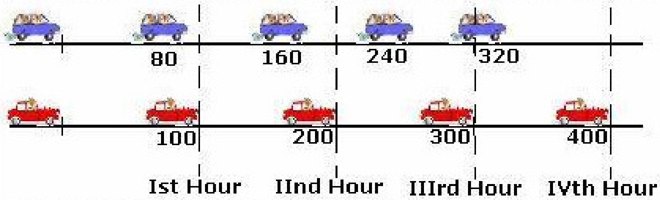
Relative Speed, Clocks and Circular Motion
by Total Gadha - Monday, 18 June 2007, 02:02 AM

The concept of relative speed is not a new or extra concept that CAT 2007 or 2008 aspirants need to learn. It is still the basic time, speec and distance formula applied to distance between two moving bodies. In a simple case of the distance formula, a body traveling with a sp of 50 km/h is reducing the gap between its starting point and the finish point by 50 km every hour. In the relative speed case of the dista formula two moving bodies, traveling at a relative speed of 50 km/h towards/away from each other, are reducing/increasing the gap between them by 50 km every hour.

Let me explain this through an example:

In the figure given below, two cars start from the same point with speeds of 100 km/h and 80 km/h, respectively, in the same direction.



We can see that every hour the faster car covers 20 km more than the distance traveled by the slower car. Therefore, the distance between the slower car and the faster car increases by 20 km every hour. The gap between the cars is 20 km in one hour, 40 km in two hours, 60 km in three hours, and so on. Note that **the gap between the cars does not depend on their absolute speeds but the difference between their speeds.** The gap would still be increasing by 20 km every hour if the speeds of the cars were 90 km/h and 70 km/h. After four hours, the gap between the cars is 80 km.
If at this moment, we press the rewind button, the cars will start going backwards and the faster car will now be **chasing** the slower car. The initial distance between the faster car and the slower car is 80 km. The faster car will be reducing the gap between the cars by 20 km every hour. Therefore, to reduce the gap of 80 km, it will take 4 hours. From this analysis we deduce that

$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{80}{20} = 4$$

Notice that the time taken for the faster car to catch up with the slower car from an initial distance of 80 km will be the same as when their speeds are 90 km/h and 70 km/h, 120 km/h and 100 km/h, or 160 km/h and 140 km/h.

In the figure given below, the two cars, traveling with 80 km/h and 100 km/h, are moving towards each other with an initial distance of 360 km between them. In one hour, the distance traveled by the first car is 80 km and the distance traveled by the second car is 100 km. Therefore, every hour, the two reduce the gap by 180 km.

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$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{360}{180} = 2$$

Notice that the time taken for the two cars to meet from an initial distance of 360 km will be the same as when their speeds are 90 km/h and 90 km/h, 120 km/h and 60 km/h, or 160 km/h and 20 km/h.

Notice that the distance reduced per unit time is nothing but relative speed- the sum or difference of the speeds depending on the relative direction of travel of the two bodies. Therefore,

$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Relative speed}}$$

A thief ran out of a police station at 4:00 pm with a speed of 8 km/h. At 5:00 pm, a policeman ran after the thief with a speed of 10 km/h. At what time will the policeman catch the thief?
Answer: From 4:00 pm to 5:00 pm, the thief will cover a distance of 8 km. When the policeman starts running after the thief, he reduces the gap between them by 2 km per hour. Therefore, $\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{8}{2} = 4$ h

Two boats, traveling at 5 and 10 km per hour, head directly towards each other. They begin at a distance of 20 km from each other. How far apart are they (in km) one minute before they collide?
Answer: As the boats are traveling towards each other, they are reducing the gap by 15 km every hour or 1/4 km every minute. Therefore, the gap before collision will be 1/4 km.

CLOCKS

The concept of relative speed can be applied to solve the problem of clocks. The two hands of a clock are nothing but two runners running around a circular track.

While solving the problem of clocks, we remember the following things:

- The distance is measured in degrees. There are 30 degrees between any two consecutive hour positions. At 4:00 o clock, the distance between the minute hand and the hour hand is 120°. At 7 o clock, the distance between the minute hand and hour hand is 210°, and so on.
- The speed is measured in degrees per minute. The minute hand rotates one full circle in one hour, or it travels 360° in 60 minutes. Therefore, the speed of the minute hand is 6°/min. The hour hand travels 30° (from one hour position to the next) in one hour. Therefore, the speed of the hour hand is 1/2°/min.
- The relative speed of the minute hand with respect to the hour hand is 6°/min – 1/2°/min = 11/2°/min. Therefore, the minute hand chases the hour hand around the circle with a relative speed of 11/2°/min

At what time between 4:00 pm and 5:00 pm, will the minute hand and the hour hand meet?
Answer: At 4:00 pm, the distance between the minute hand and the hour hand is 120°, as shown below:

Therefore, the minute hand is 120° behind at 4:00 pm and then it starts closing the gap by 11/2° every minute. Therefore, the time taken to catch up with hour hand completely = $\frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{120}{\frac{11}{2}} = \frac{240}{11} = 21\frac{9}{11}$ minutes. Therefore, at $21\frac{9}{11}$ minutes past 4:00, the two hands will be together.

Chinmanbhai starts a trip when the hands of the clock are together between 8 am and 9 am. He arrives at his destination between 2 pm and 3 pm when the hands are exactly 180° apart. How long did the trip take?

Answer: At 8:00 am, the distance between the minute hand and the hour hand is 240°. It takes the minute hand $\frac{240}{\frac{11}{2}} = \frac{480}{11} = 43\frac{7}{11}$ min to catch up with the hour hand. Therefore, at $43\frac{7}{11}$ minutes past 8:00, the two hands are together. At 2:00 pm, the distance between minute hand and hour hand is 60°. Therefore, first the minute hand has to cover this gap of 60° and then get ahead by further 180° to make an angle of 180° with the hour hand. Therefore, the total relative distance it needs to travel = 240° with respect to the hour hand. Therefore, time taken = $\frac{240}{\frac{11}{2}} = \frac{480}{11} = 43\frac{7}{11}$ min. Therefore, at $43\frac{7}{11}$ minutes past 2:00, the two hands are 180° apart. Therefore, the total time taken for the trip = time taken from $8:43\frac{7}{11}$ to $2:43\frac{7}{11}$ = 6 hours.

What is the time interval between two successive meets of the minute hand and the hour hand?

Answer: Once the minute hand and the hour hand are together, the minute hand starts increasing the gap between the hour hand and itself by 11/2° every minute. Therefore, when it has increased the gap by 360°, it again meets the hour hand. The time taken to increase the gap by 360° = $\frac{360}{\frac{11}{2}} = \frac{720}{11} = 65\frac{5}{11}$ minutes. Therefore, the minute hand and the hour hand meet every $65\frac{5}{11}$ minutes.

TG left for Dagny's house between 5:00 pm and 6:00 pm and when he returned at some time between 8:00 pm and 9:00 pm, he noticed that the minute hand and hour hand had interchanged their positions. What time did TG leave?

Answer: The positions are shown in the figure below:

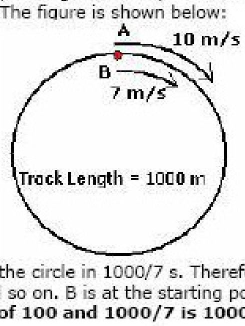
Let the angle between the minute hand and the hour hand be θ initially. To bring the hour hand from between 5 and 6 to between 8 and 9, the minute hand travels two complete circles and the angle 360 − θ. Therefore, the total distance traveled by the minute hand in degrees is 360 + 360 + 360 − θ = 1080 − θ. As the speed of the minute hand is 6 degrees per minute, the time taken for travel = $\frac{1080-\theta}{6}$. In this time, the hour hand travels only θ. As the speed of the hour hand is 1/2 degrees per minute, the time taken for travel is 2θ.

$\Rightarrow \frac{1080-\theta}{6} = 2\theta \Rightarrow \theta = \frac{1080}{13}$. Therefore, the minute hand is 1080/13 degrees ahead of the hour hand between 5:00 and 6:00. At 5:00, the distance between the minute hand and the hour hand is 150°. Therefore, to get ahead by 1080/13 degrees, the minute hand will have to travel a relative distance of 150 + 1080/13 degrees = 3030/13 degrees. The time taken to get ahead = $\frac{\frac{3030}{13}}{\frac{11}{2}} = \frac{6060}{143} = 42\frac{54}{143}$ minutes.

Therefore, TG left at $5:42\frac{54}{143}$

CIRCULAR MOTION

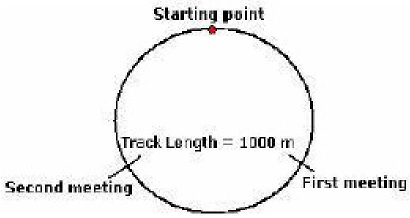
Let's examine two runners running on a circular track, of length 1000 m, in the same direction. The runners, A and B, start from the same point with speeds of 10 m/s and 7 m/s, respectively. The figure is shown below:



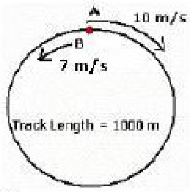
A completes the circle in 1000/10 s and B completes the circle in 1000/7 s. Therefore, A is at the starting point every 1000/10 = 100 s, i.e. A is at the starting point after 100 s, 200 s, 300 s... and so on. B is at the starting point after every 1000/7 s, i.e. B is at the starting point after 1000/7 s, 2000/7 s, 3000/7 s... and so on. **The LCM of 100 and 1000/7 is 1000. Therefore, both A and B are together at the starting point again after 1000 s.**

The moment A and B start from the starting point, A starts getting ahead of B. Every second, A increases the gap between B and him by 3 m. Therefore, the gap between A and B after is 3 m after 1 s, 6 m after 2s, 9 m after 3 s, and so on. When A increases the gap between B and him by 1000 m, he will catch up with B again. **Therefore, time taken for A to catch up with B again is = $\frac{1000}{3}$ s.**

In $\frac{1000}{3}$ s, A travels a distance of $10 \times \frac{1000}{3} = \frac{10000}{3}$ m and B travels a distance of $7 \times \frac{1000}{3} = \frac{7000}{3}$ m. Now, $\frac{1000}{3} = 1000 \times 3\frac{1}{3}$ and $\frac{7000}{3} = 1000 \times 2\frac{1}{3}$. Therefore, A takes $3\frac{1}{3}$ rounds and B takes $2\frac{1}{3}$ rounds (notice that A takes **exactly** one round more than B to catch B again, which is to be expected). Now, A keeps catching B after every 1000/3 s. **Therefore, A and B first meet after 1000/3 s at the first meeting point (shown in the figure), after 2000/3 s at the second meeting point (shown in the figure) and after 3000/3 = 1000 s at the starting point.** We have verified this result already, i.e. A and B meet at the starting point after 1000 s.



If A and B start running in the opposite directions from the starting point, they will reduce the gap between them by 17 m every second. Therefore, time taken for them to meet = 1000/17 s. Thus, A and B will meet every1000/17s. Note that, they would still meeting at the starting point every 1000 s.



Please feel free to ask any question that you have in this lesson.