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$$\boxed{x > 0}$$

$$\rightarrow \text{If } a < b \Rightarrow \frac{a}{b} < 1$$

$$\frac{a+x}{b+x} > \frac{a}{b}$$

$$\rightarrow \text{If } a > b \Rightarrow \frac{a}{b} > 1$$

$$\frac{a+x}{b+x} < \frac{a}{b}$$

$$\rightarrow \text{If } a = b, \text{ then } \frac{a+x}{b+x} = \frac{a}{b}$$

Ex Divide 784 into 4 parts such that 4 times the 1<sup>st</sup> part, 3 times the 2<sup>nd</sup> part, twice the 3<sup>rd</sup> part are each equal to 12 times the 4<sup>th</sup> part.

$$a \quad b \quad c \quad d$$

$$4a = 3b = 2c = 12d$$

$$\therefore a + \frac{4a}{3} + 2a + \frac{4a}{12} = 784. \quad \text{Solve.}$$

Ex Divide 4200 into 4 parts such that a fourth of first part, a sixth of 2<sup>nd</sup> part, an eighth of 3<sup>rd</sup> part and a 10<sup>th</sup> of 4<sup>th</sup> part are all equal.

$$\frac{a}{4} = \frac{b}{6} = \frac{c}{8} = \frac{d}{10} = k$$

$$\therefore 4k + 6k + 8k + 10k = 4200 \quad \text{Solve.}$$

Ex If  $x$  varies directly with  $y$ , check whether  $(x+y)^2$  varies directly with  $(x-y)^2$ .

$$x = ky$$

$$\frac{(x+y)^2}{(x-y)^2} = \frac{x^2+y^2+2xy}{x^2+y^2-2xy} = \frac{k^2y^2+y^2+2ky^2}{k^2y^2+y^2-2ky^2} = \frac{y^2}{y^2} \cdot \frac{k^2+2k+1}{k^2-2k+1} = K$$

Hence,  $(x+y)^2$  varies directly with  $(x-y)^2$ .

Ex The expenses per month of Ravi's car are partly constant and partly vary with the no. of km he travels in that month. When he travels 100 km in a month, total expense come to Rs 3200. If he travels 150 km, it is Rs 3800. Find the total expense, if he travels 250 km in a month.

$$3200 = F + 100V$$

$$3800 = F + 150V$$

$$\Rightarrow 50V = 600 \Rightarrow V = 12.$$

$$\therefore F = 2000$$

$$\text{Ans} \rightarrow 2000 + 250 \times 12 = 5000.$$

Ex There are 2 numbers m and n. Which of the following must be added to the numbers m and n so that their ratio becomes  $x:y$ ?

$$\frac{m+k}{n+k} = \frac{x}{y}$$

$$\Rightarrow my + ky = nx + kx$$

$$\Rightarrow k = \frac{my - nx}{x - y}$$

Ex Ratio of Earnings to Expenditure of A is 5:3 and that of B is 7:6. If Savings of A is twice that of B, then what could be the ratio of total earnings of A and B together to the total expenditure of A & B together?

	Earning	Expen	Saving
A	5x	3x	2x
B	7y	6y	y



Then,  $2x = 2y$   
 $\Rightarrow x = y$ .

Ans  $\rightarrow \frac{12x}{9x} = \frac{4}{3}$ .

Ex If  $a/b$  is an improper fraction satisfying the eq<sup>n</sup>  
 $16a^2 - 26ab + 9b^2 = 0$ , find value of  $a:b$ .

Improper fraction  $\Rightarrow a \geq b$ .

$$16\frac{a^2}{b^2} - 26\frac{a}{b} + 9 = 0.$$

$$\Rightarrow 16t^2 - 26t + 9 = 0.$$

$$\Rightarrow 16t^2 - 8t - 18t + 9 = 0.$$

$$\Rightarrow 8t(2t-1) - 9(2t-1) = 0.$$

$$\Rightarrow (8t-9)(2t-1) = 0 \quad \Rightarrow t = \frac{1}{2} \text{ or } \frac{9}{8}.$$

As  $a/b$  is an improper fraction, ans  $\rightarrow \frac{9}{8}$ .

Ex There are 5 vessels, with equal capacities, each containing some milk. The quantities of milk in the 5 vessels are 4:5:6:7:8. The total quantity of milk in the 5 vessels is equal to 75% of total capacity of 5 vessels. How many vessels are at least 64% full of milk?

Capacity  $\rightarrow V$

$$\frac{4x}{V} \quad \frac{5x}{V} \quad \frac{6x}{V} \quad \frac{7x}{V} \quad \frac{8x}{V}$$

$$\Rightarrow \frac{30x}{5V} = \frac{3}{4} \Rightarrow \frac{x}{V} = \frac{3}{4} \times \frac{5}{30} = \frac{5}{40} = \frac{1}{8} = 12.5\%$$

Ans  $\rightarrow \boxed{3}$  6:7:8

Ex - A number is divided into 5 parts. Twice the 1<sup>st</sup> part, thrice the 2<sup>nd</sup> part, 4 times the 4<sup>th</sup> part are equal. Twice the 2<sup>nd</sup> part, 5 times the 3<sup>rd</sup> part and 6 times last part are equal. If all are integers, which is always true?

- (i) First part is a multiple of 72.
- (ii) 2<sup>nd</sup> part is divisible by 4<sup>th</sup> part.
- (iii) 1<sup>st</sup> part is a factor of last part.
- (iv) Pdt. of 1<sup>st</sup> and 4<sup>th</sup> part is divisible by 30.

$$2a = 3b = 4d$$

$$2b = 5c = 6e$$

$$\therefore 4a = 6b = 8d = 15c = 18e.$$

$$\Rightarrow 2^2a = 2 \times 3b = 2^3d = 3 \times 5c = 2 \times 3^2 \times e.$$

$$a \rightarrow 2 \times 3^2 \times 5 \times x$$

$$b \rightarrow 2^2 \times 3 \times 5 \times x$$

$$d \rightarrow 3^2 \times 5 \times x$$

$$c \rightarrow 2^3 \times 3 \times x$$

$$e \rightarrow 2^2 \times 5 \times x.$$

- (i) X
- (ii) X
- (iii) X
- (iv) ✓.

Ex If  $\frac{p+q}{r} = \frac{q+r}{p} = \frac{p+r}{q} = k$ , then find k.

If  $p = q = r$ , then  $k = 2$ .

$$\frac{p+q}{r} = \frac{p+q-q-r}{r-p} = -1.$$

$\therefore$  Ans  $\rightarrow k = 2$  or  $-1$ .



Ex Mean proportional b/w 2 numbers is 12. 3rd proportional of same nos is 96. Find the greater of 2 nos.

$$\frac{a}{12} = \frac{12}{96} \Rightarrow \cancel{a} = \frac{144}{\cancel{96}}$$

$$ab = 144.$$

$$\frac{a}{b} = \frac{b}{96} \Rightarrow b^2 = 96a$$

$$\Rightarrow b^2 = 96 \times \frac{144}{b}$$

$$\Rightarrow b^3 = 12 \times 12 \times 12 \times 2^3$$

$$\Rightarrow b = 12 \times 2 = 24 \quad a = 6.$$

Ans  $\rightarrow 24$ .

Ex If  $a-b$  varies directly with  $a+b$ ,  $a^2-b^2$  will vary directly with

(i)  $a^2+b^2$

(ii)  $ab$

(iii)  $a^2+b^2+3ab$ .

$$a-b = k(a+b)$$

$$\Rightarrow a - ka = kb + b$$

$$\Rightarrow \frac{a}{b} = \frac{k+1}{1-k} = K.$$

$$\text{Then, (i) } \frac{a^2+b^2}{a^2-b^2} = \frac{K^2b^2+b^2}{K^2b^2-b^2} = \frac{K^2+1}{K^2-1} \quad \text{Yc}$$

$$\text{(ii) } \frac{a^2-b^2}{ab} = \frac{K^2b^2-b^2}{Kb \times b} = \frac{K^2-1}{K} \quad \text{Yc}$$

$$\text{(iii) } \frac{a^2+b^2+3ab}{a^2-b^2} = \frac{K^2+1+3K}{K^2-1}$$

Ex Time taken by a group of workers to complete a work varies directly with amount of work to be done by them when the no. of workers is constant & inversely as the no. of workers in the group when amount of work is constant. If 8 workers take  $\frac{1}{2}$  a day to plough 2 acres of field, find the time taken by 16 workers to plough 8 acres of field.

$$\frac{W_1}{N_1} = \frac{W_2}{N_2} \quad \frac{2}{8} \Rightarrow \frac{1}{2}$$

$$\Rightarrow \frac{8}{16} \Rightarrow \frac{1}{2}$$

$$t = k \frac{W}{N} \Rightarrow \frac{1}{2} = k \times \frac{2}{8} \Rightarrow k = 2.$$

$$\text{Then, } 2 \times \frac{8}{16} = 1 \text{ day.}$$

Ex Value of a diamond varies directly with square of its weight. A diamond broke into 3 pieces whose weight were in ratio 32:24:9. The loss caused due to breakage was Rs 25.44 Lakhs. Find initial value of diamond.

$$V = kW^2$$

$$K[(65x)^2 - (32x)^2 - (24x)^2 - (9x)^2] = 25.44 \times 10^5$$

$$\Rightarrow Kx^2 = \frac{25.44 \times 10^5}{4225 - 32^2 - 576 - 81} = 1000$$

$$\begin{aligned} \text{Initial value} &= K 4225 x^2 \\ &= 42.25 \text{ lakh.} \end{aligned}$$



Ex. Expenses for yoga classes in a colony are partly constant & partly varying with no. of members. If there are 50 members, then each member has to bear Rs 220 per month and if there are 10 more members, then share of each of members come down by Rs 15 per month. How many members would be there if the share of each member is Rs 160?

$$220 \times 50 = F + 50 \times V$$

$$205 \times 60 = F + 60 \times V$$

$$\Rightarrow 10V = 12300 - 11000 = 1300$$

$$\Rightarrow V = 130$$

$$\therefore F = 11000 - 50 \times 130 = 4500$$

$$\text{Then, } 160 \times n = 4500 + n \times 130$$

$$\Rightarrow 30n = 4500$$

$$\Rightarrow n = 150$$

Ex. Spice airline has a free luggage allowance of  $f$  kg. If any passenger carries excess luggage, it is charged at a constant rate per kg. The total luggage charge paid by 2 passengers Mohan and Sohan is Rs 1050. If each of Mohan & Sohan had carried twice the luggage he actually did, their luggage charge would have been Rs 2400 and Rs 900 respectively. Find Mohan's luggage charge.

$$(m-f)c + (s-f)c = 1050$$

$$(2m-f)c = 2400$$

$$(2s-f)c = 900$$

$$\Rightarrow (2m+2s-2f)c = 3300$$

$$\Rightarrow (m+s-f)c = 1650$$

$$\Rightarrow (m+s-2f)c = 1050$$

$$\Rightarrow cf = 600$$

$$\Rightarrow 2mc = 3000 \Rightarrow mc = 1500$$

$$\text{Ans} \Rightarrow mc - fc = 900$$

Ex The consumption of diesel per hour of a bus varies ~~with~~ directly as the square of its speed. When the bus is travelling at 40 kmph its consumption is 2 litre per hour. If each litre costs Rs 40 and other expenses per hour cost Rs 40, then what would be the minimum expenditure required to cover a distance of 400 km?

$$C = KV^2$$

$$\Rightarrow 1 = k \times 1600 \Rightarrow k = \frac{1}{1600}$$

Optimize.  $\rightarrow$  time

$$\left( \frac{400}{V} \right) \times \frac{1}{1600} \times V^2 \times 40 + \frac{400}{V} \times 40 \quad \parallel \text{minimize}$$

$$= 10V + \frac{16000}{V}$$

differentiate  $10 - \frac{16000}{V^2} = 0$

$$\Rightarrow V^2 = 1600$$

$$\Rightarrow V = 40$$

double differentiate  $\frac{+16000 \times 2}{V^3} \rightarrow +ve$ . Hence, minima.

$$\therefore \text{Min. Expenditure} = 10 \times 40 + \frac{16000}{40}$$

$$= 800$$

Ex If  $a \neq c$ , and  $\frac{a^2+b^2}{a+b} = \frac{b^2+c^2}{b+c} = k$ . Find  $k$ .

$$\frac{a^2+b^2}{a+b} = \frac{a^2+b^2-b^2-c^2}{a+b-b-c} = \frac{a^2-c^2}{a-c} = a+c = k$$



Ex  $p, q, r, s, t$  are 5 integers satisfying  $p=3q=4r$  and  $2q=5s=12t$ . Which of following pairs contain a number that can never be an integer?

(i)  $(2p/15, r/t)$

(ii)  $(p/t, 4r/t)$

(iii)  $(p/4, rs/180)$

(iv)  $(p/8, s/r)$

$$p=3q=4r \quad 2q=5s=12t$$

$$2p=6q=8r=15s=36t$$

$$\Rightarrow 2p = 2 \times 3q = 2^3 r = 3 \times 5s = 3^2 \times 2^2 t$$

$$p \rightarrow 2^2 \times 3^2 \times 5 \times r$$

$$q \rightarrow 2^3 \times 3 \times 5 \times r$$

$$r \rightarrow 3^2 \times 5 \times r$$

$$s \rightarrow 2^3 \times 3 \times r$$

$$t \rightarrow 2 \times 5 \times r$$

(i) ✓

(ii) ✓

(iii) ✓

(iv) ✗  $\rightarrow \frac{s}{r} = \frac{2^3 \times 3}{2^2 \times 3 \times 5}$

Ex Expense of a hostel are partly fixed and partly varying directly with no. of occupants. If no. of occupants is 60, each of occupant has to bear Rs 1200 per month and if there are 30 more occupants, then share of each of the occupants comes down by Rs 200 per occupant. How many occupant should be there if share of each occupant is Rs 900?

$$1200 \times 60 = F + 60 \times V$$

$$1000 \times 90 = F + 90 \times V$$

$$\Rightarrow 30 \times V = 18000 \Rightarrow V = 600$$

$$\therefore F = 36000$$

Then,  $900 \times n = 36000 + n \times 600$

$$\Rightarrow 9n = 360 + 6n$$

$$\Rightarrow 3n = 360$$

$$\Rightarrow n = 120$$