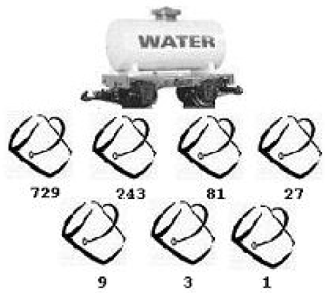
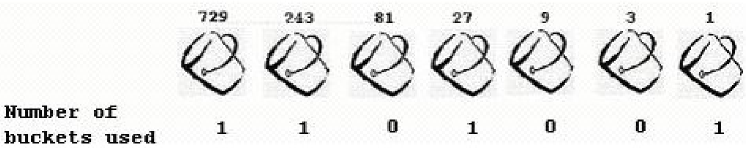
Base system
by Total Gadha - Tuesday, 20 February 2007, 11:59 PM



Suppose you have a 1 000 L tank to be filled with water. The buckets that are available to you all have sizes that are powers of 3, i.e. 1, 3, 9, 27, 81, 243, and 729 L. Which buckets do you use to fill the tank in the minimum possible time?

You will certainly tell me that the first bucket you will use is of 729 L. That will leave 271 L of the tank still empty. The next few buckets you will use will 243 L, 27 L and 1 L. The use of buckets can be shown as below



We can say that

1 000 = 729 + 243 + 27 + 1

= 1 Ã— 3^6 + 1 Ã— 3^5 + 0 Ã— 3^4 + 1 Ã— 3^3 + 0 Ã— 3^2 + 0 Ã— 3^1 + 0 Ã— 3^0.

The number 1 000 has been written in increasing powers of 3. Therefore, 3 is known as the â€˜baseâ€™ in which we are expressing 1 000.

For example, The number 7368 can be written as 8 + 6 Ã— 10 + 3 Ã— $(10)^2$ + 7 Ã— $(10)^3$.

The number 10 is called the 'base' in which this number was written.

Let a number abcde be written in base p, where a, b, c, d and e are single digits less than p. T he value of the number abcde = e + d Ã— p + c Ã— p^2 + b Ã— p^3 + a Ã— p^4

For example, i f the number 7368 is written in base 9,

The value of $(7368)_9$ = 8 + 6 Ã— 9 + 3 Ã— 9^2 + 7 Ã— 9^3 = 5408 (this value is in base 10)

There are two kinds of operations associated with conversion of bases:

1.7A conversion from any base to base ten

The number $(pqrstu)_b$ is converted to base 10 by finding the value of the number. i.e. $(pqrstu)_b$ = u + tb + sb^2 + rb^3 + qb^4 + pb^5.

Example

38. Convert $(21344)_5$ to base 10.

Answer: $(21344)_5$ = 4 + 4 Ã— 5 + 3 Ã— 25 + 1 Ã— 125 + 2 Ã— 625 = 1474

1.7b conversion from base 10 to any base

A number written in base 10 can be converted to any base 'b' by first dividing the number by 'b', and then successively dividing the quotients by 'b'. The remainders, written in reverse order, give the equivalent number in base 'b'.

Example

39. Write the number 25 in base 4.

4	25	
	6	1
	1	2
	0	1

Writing the remainders in reverse order the number 25 in base 10 is the number 121 in base 4.

1.7c Addition, subtraction and multiplication in bases:

Example

40. Add the numbers $(4235)_7$ and $(2354)_7$

Answers: The numbers are written as

$$\begin{array}{cccc} 4 & 2 & 3 & 5 \\ 2 & 3 & 5 & 4 \end{array}$$

The addition of 5 and 4 (at the units place) is 9, which being more than 7

would be written as 9 = 7 Ã— 1 + 2. The Quotient is 1 and written is 2.

The Remainder is placed at the units place of the answer and the Quotient

gets carried over to the ten's place. We obtain

	+1	+1	
4	2	3	5
2	3	5	4
6	6	2	2

At the tens place: 3 + 5 + 1 (carry) = 9

Similar procedure is to be followed when multiply numbers in the same base

Example

41. Multiply $(43)_8$ Ã— $(67)_8$

Answer:

7 Ã— 3 = 21 = 8 Ã— 2 + 5

7 Ã— 4 + 2 = 30 = 8 Ã— 3 + 6

6 Ã— 3 = 18 = 8 Ã— 2 + 2

6 Ã— 4 + 2 = 26 = 8 Ã— 3 + 2

$(4\quad 3)_8$
$(6\quad 7)_8$
 365
322
3605

For subtraction the procedure is same for any ordinary subtraction in base 10 except for the fact that whenever we need to carry to the right we carry the value equal to the base.

EXAMPLE

42. Subtract 45026 from 51231 in base 7.

Answer:

	5	1	2	3	1
—	4	5	0	2	6
		3	2	0	2

In the units column since 1 is smaller than 6, we carry the value equal to the base from the number on the left. Since the base is 7 we carry 7. Now, 1 + 7 = 8

and 8 â€" 6 = 2. Hence we write 2 in the units column. We proceed the same way in the rest of the columns.

1.7D IMPORTANT RULES ABOUT BASES

Rule 1. A number in base N when written in base 10 is divisible by N â€" 1 when the sum of the digits of the number in base N is divisible by N â€" 1.

EXAMPLE

43. The number 35A246772 is in base 9. This number when written in base 10 is divisible by 8. Find the value of digit A.

Answer: The number written in base 10 will be divisible by 8 when the sum of the digits in base 9 is divisible by 8.

Sum of digits = 3 + 5 + A + 2 + 4 + 6 + 7 + 7 + 2 = 36 + A. The sum will be divisible by 8 when A = 4.

Rule 2. When the digits of a k-digits number written in base N are rearranged in any order to form a new k-digits number, the difference of the two numbers, when written in base 10, is divisible by N â€" 1.

EXAMPLE

44. A four-digit number N_1 is written in base 13. A new four-digit number N_2 is formed by rearranging the digits of N_1 in any order. Then the difference N_1 â€" N_2 when calculated in base 10 is divisible by

(a) 9 (b) 10 (c) 12 (d) 13

Answer: c