Fundamental Theory of Arithmetic :-

Prime No. → With exactly 2 factors (1 & no. itself)
Composite No. → More than 2 factors

1 is neither prime nor composite.

$\boxed{HCF \times LCM = a \times b}$ only for 2 numbers

Ex. Sonia takes 18 min to drive a circular field. Ravi takes 12 min. When will they meet at starting point ?

$$LCM \text{ of } (12,18) = (2^2 \times 3,\ 2 \times 3^2)$$
$$\downarrow$$
$$2^2 \times 3^2 = 36.$$

Fermat's Theorem :-

$$\left.\frac{x^{\mathcal{E}}}{N}\right|_R = 1$$

$\mathcal{E} \to$ Euler's No.

Coprime — There is no factor common other than 1.
(2,7), (1,7) (11,7) etc.

HCF$(x, N) = 1$.

$\mathcal{E}$ of any No. N = $N(1-\frac{1}{a})(1-\frac{1}{b})(1-\frac{1}{c})\ldots$
where a, b, c are prime factors of N.

Ex $7 \to 7(1-\frac{1}{7}) = 6.$

$10 \to 10(1-\frac{1}{2})(1-\frac{1}{5}) = 10 \times \frac{1}{2} \times \frac{4}{5} = 4.$

Note:- For any prime no. n, $\mathcal{E}$ is $(n-1)$.

→ Euler's no. is no. of co-primes less than the numbers. For prime no.s, Euler's no. will be 1 less than the number.

→ If all no. upto that no. are coprime with it, then the no. is a prime no.

$$7 \to 7\left(1-\tfrac{1}{7}\right) \Rightarrow 6.$$

Interpretation

What is the probability of getting a no. multiple of 3? among natural no.s?

$\frac{1}{3}$.

Non-multiple. $\left(1-\frac{1}{3}\right)$.

So, basically $\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ etc. represents probability of getting numbers which is not multiples of a, b & c respectively.

$$E = N\left(1-\tfrac{1}{a}\right)\left(1-\tfrac{1}{b}\right)\cdots$$

Euler's no. is number of non-multiple of prime factors.

Ex Find $\left.\frac{11^{97}}{7}\right|_R$.

$$\cancel{\left(\frac{11}{7}\right)^{97/6}} \to \cancel{(4)}^{1} \qquad \left(\frac{11}{7}\right)^{97/6}$$

note. not 7, but its Euler's no.

$$= 4^1 = 4.$$

__/__/____

Ex $\frac{5^{20}}{7}\Big|_R$

$\left(\frac{5}{7}\right)^2 = 5^2 = 25 - 21 = 4.$

Lecture - 1

→ Basic Remainder Theorem
→ Divisibility Rules
→ Fermat Theorem
→ Eulers No.
→ Digit Sum

Ex There are n books. If taken out in bundles of 9, nothing is left. If taken out in bundles of 11, 2 are left out. Find smallest possible value of n.

$9A = 11B + \cancel{2}$

$A = 10. \quad B = 8$

Ex $\frac{100}{7}\Big|_R$

$\frac{100}{7} = \frac{10}{7} \times \frac{10}{7}$

$= 3 \times 3 = 9 - 7 = 2.$

Final remainder depends upon product of individual remainders.

Ex $\frac{361 \times 363 \times 365}{12}$

$1 \times 3 \times 5 = 15 - 12 = 3.$

Not Factorize ~~Remainder~~ * Denominator, Factorize numerator.

Ex $\left.\dfrac{2^{32}}{15}\right|_R = ?$

$\left.\dfrac{(2^4)^8}{15}\right|_R = 1$ or $\dfrac{(2^4)^8}{\underbrace{2^4-1}_{x}} = (1)^8 = 1.$

Ex $\left.\dfrac{2^{32}}{17}\right|_R = ?$

$\dfrac{(2^4)^8}{17} = (-1)^8 = 1$ or $\dfrac{(2^4)^8}{\underbrace{2^4+1}_{x}} = (-1)^8 = 1.$

Divisibility Rules :—

$\left.\dfrac{10^x}{13}\right|_r = R.$

For base 10.

DR for 9 :-

$\left.\dfrac{10^x}{9}\right|_r = R$

Possible value of R ?

x	R
0	1
1	1
2	1
3	1
⋮	⋮
n	1

For '9', any power of 10, the remainder is "1".

Now, $AB = 10A + B$

$ABC = 10^2A + 10^1B + 10^0C$

$$\left.\frac{ABC}{9}\right|_R = \frac{\overset{+1}{10^2A} + \overset{+1}{10^1B} + \overset{+1}{10^0C}}{9,3}$$

$$= \frac{A+B+C}{9,3} \longrightarrow \text{Sum of digits.}$$

$$\boxed{\text{DR of } 9,3 \rightarrow \langle\text{Sum of digits}\rangle \quad \left.\frac{10^x}{3,9}\right|_R = +1}$$

DR for 11 :-

$$\left.\frac{10^x}{11}\right|_R = R$$

x	R	
0	1	
1	−1	(or 10)
2	1	
3	−1	
⋮	⋮	

So, $ABCD = \dfrac{\overset{-1}{10^3A} + \overset{+1}{10^2B} + \overset{-1}{10^1C} + \overset{1}{10^0D}}{11}$

$\overset{-1}{A} \quad \overset{+1}{B} \quad \overset{-1}{C} \quad \overset{+1}{D}$

$= (B+D) - (A+C)$

Sum of digits at odd places − Sum of digits at even places

(Starting right side)

$+1 \longrightarrow 3, 9$

$+1/-1 \rightarrow 11$.

DR for 7 :- (Look for power of 10 when remainder is +1 or −1)

$$\left.\frac{10^x}{7}\right|_R = R$$

x	R
0	1
1	3
2	2
3	−1 (or 6).

∴ Rule for 7 → difference b/w
sum of triplets at odd places
− sum of triplets at even places.

$$\frac{N}{7} = \frac{ABCPQRXYZ}{7} = \overset{+1}{ABC} \,\Big|\, \overset{-1}{PQR} \,\Big|\, \overset{+1}{XYZ}$$

$$10^6 ABC \,|\, 10^3 PQR \,|\, 10^0 XYZ.$$

Ex $\left.\frac{100\,200\,140\,240}{7}\right|_R = ?$

$$\frac{240+200-140-100}{7} = \left.\frac{200}{7}\right|_R = 4$$

Manual Method :-

$$\begin{array}{cccc} -1 & +2 & +3 & +1 \\ 6 & 2 & 4 & 0 \end{array}$$

$$-6 \quad 4 \quad 12 \quad 0 = \left.\frac{10}{7}\right|_R = 3.$$

$37 \times 27 = 999$

→ So, to formulate divisibility rule for any number,

$\left.\frac{10^x}{B}\right|_r = R$ Look for values of 'x', so that remainder is +1/-1.

DR for 13

$\left.\frac{10^x}{13}\right|_R = R$

x	R
0	1
1	10
2	9
3	-1

So, this case is similar to 7.

Diff b/w sum of triplets at odd places − Sum of triplets at even places.

Ex $\left.\frac{100200140240}{13}\right|_R$

$\frac{240+200-140-100}{13} = \frac{200}{13} = \frac{20\times10}{13} = \underline{\underline{5}}$.

Ex N= 123123123 .... 300 digits

Find $\left.\frac{N}{9}\right|_R$ $\left.\frac{N}{7}\right|_R$ $\left.\frac{N}{11}\right|_R$ $\left.\frac{N}{13}\right|_R$ $\left.\frac{N}{37}\right|_R$.

$\left.\frac{N}{9}\right|_R = \frac{123\times100}{9} = \frac{12300}{9} = \frac{6}{9} = 6$.

$\left.\frac{N}{7}\right|_R = 0$

$\left.\frac{N}{13}\right|_R = 0$.

$\left.\frac{N}{11}\right|_R = \bar{1}\underline{2}\bar{3}\underline{1}\bar{2}\underline{3}$

$= 3+1+2-(2+3+1)$

$= 0$

$\left.\frac{N}{37}\right|_R = \frac{123\times100}{999} = \frac{12300}{999} = \frac{312}{999}$

$= 312$

$\boxed{37\times27=999}$

$\frac{312}{37} = 16$.

Ex $\dfrac{123456789123456789123456789\ldots\ldots 180\text{ Qs}}{11}$

20 groups → +1/−1   Remainder − 0.

For 9, we can take any no.

↓

triplet

4 at a time

5 at a time etc.

Ex $\left.\dfrac{123456789101112\ldots\ldots 40}{9}\right|_{R}$

$\dfrac{\Sigma 40}{9} = \dfrac{40 \times 41}{2 \times 9} = \dfrac{820}{9} = 1.$

DR for 9 → Sum of digits

→ Group as per requirement.

$\left.\dfrac{A^2}{B}\right|_{r} = R$

A → 10   +1 → 9, 3, 37

R → +1/−1   −1/+1 → 11, $\underbrace{7, 13}_{\text{triplet}}$

Rule for 9 and factors of 9 (3) in base 10 is applicable for 8 and its factors in base 9.

Rules of Zero :-

$\left.\frac{10^x}{B}\right|_R = 0.$ $\Rightarrow$ B can be 2, 5, 10
B can be a factor of 10.

For 2, 5

Check for 2^n or 5^n, check divisibility of last n digits.

$5^4, 2^4 \longrightarrow$ Last 4 digits
$5^5, 2^5 \longrightarrow$ Last 5 digits
$5^2, 2^2 \longrightarrow$ Last 2 digits.

Ex $\left.\frac{11^{97}}{7}\right|_R$

$\left(\frac{11}{7}\right)^{97/6} = 4^1 = 4.$

Ex $\left.\frac{5^{20}}{7}\right|_R$

$\left(\frac{5}{7}\right)^{\frac{20}{6}} = 5^2 = 25 - 21 = 4.$

Digit Sum $\rightarrow$ Sum till single digit no. is obtained

Ex Find digit sum of 8888^{8888}.

Divide by 9. $\left(\frac{8888}{9}\right)^{\frac{8888}{6}}$ $9 \times (1 - \frac{1}{3}) = 6.$

$= 5^2 = \frac{25}{9} = 7$

Ans $\rightarrow$ 7

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Ex Suppose, seed(n) = n if n < 10
= seed(s(n)) otherwise.
where s(n) → Sum of digits of(n).
Ex → seed(7) = 7
seed(248) = 2+4+8 = seed(14) = seed(5) = 5 etc.
How many +ve integers n, such that n < 500, will have seed(n) = 9?

No. of multiples of 9 less than 500 = 55.

Digit Sum →
- Sum of digits till single digit is obtained
- Remainder when divided by 9
- AP with d = 9.

Ex How many 6 digit no.s starting with 523 divisible by 7, 8 and 9?

LCM = 7×8×9 = 504.

Ans → 523000 → $\frac{1000}{504}$ → 1 or 2.

$\frac{523000}{504}\Big|_R = 352$

1st no. divisible = 523000 + 504 − 352 = 523152

Ans → 2.

Ex In a shelf if you group of 9, 0 remain and group of 11, then 7 remains. Min. no. of books in shelf.

9A + 0 = 11B + 7

B = 2 Ans → 18.

→ If max. capacity is 1000, then how many no. of possible soln exists?

LCM of 9, 11 → 99. $\frac{1000}{99} \rightarrow \boxed{10/11}$

$\underline{\underline{99k+18}}$.

Ex $\frac{12121212 \ldots \ldots 300 \text{ digits}}{99}\Big| R$

$\frac{12 \times 150}{99} = \frac{1800}{99} = \frac{18}{99} = 18$.

Ex The integers 34041 and 32506, when divided by a 3 digit integer n, leave the same remainder. What it is?

$$\begin{array}{r} 34041 \\ 32506 \\ \hline 1535 \end{array}$$

$\frac{1535}{n} \rightarrow$ how many values of n.

$1535 = 5 \times 307$ Ans → $\underline{307}$

Ex X is a set of 3 digit divisors which gives the same remainder when it divides 2 no.s 12714 & 13914. What is the max. possible no. of element in X?

$$\begin{array}{r} 13914 \\ 12714 \\ \hline 1200 \end{array}$$

$1200 = 2 \times 2 \times 2 \times 2 \times 3 \times 5^2 = 2^4 \times 3 \times 5^2$

$= \underline{\underline{2^4 \times 3 \times 5^2}}$.

$\underline{\underline{\text{Alt.}}}$

$\frac{1200}{n} \rightarrow$ how many value of n

$\frac{1200}{2} \rightarrow$

$\frac{1200}{12} \rightarrow$

[2, 12]

2, 3, 4, 5, 6, 8, 10, 12 — 8

~~2×2×2×2~~

$2^4 \times 3 \times 5$

$2^3 \times 3 \times 5$

$2^3 \times 3 \times 5^2$

$2^2 \times 3 \times 5^2$

$2 \times 3 \times 5^2$

$2^4 \times 5^2$

2

Ans → $\underline{8}$.

Ex Consider the set $A = (1, 2, 3, \ldots\ldots, 1000)$. How many AP can be formed from the elements of A that start with 1 and end with 1000 and have at least 3 terms?

$$1000 - 1 = 999$$

$$999 = 3\times3\times3\times37$$

$$= 3^3\times37$$

Ans → $4\times2-1 = \cancel{8}\ \underline{7}$

CD can't be 999.

Q-Type

Ex The eq^n $4x - Ay = 8$ has a no. of integral sol^{ns}. If HCF$(A,4)=1$ and the no. of sol^{ns} is (x,y) for $0 < x,y < 500$ is 45, then the no. of possible values of A is.

$4x - Ay = 8$ A, 4 → Coprimes.

x	y	
2	0	✗
2+A	4	
2+2A	8	
2+3A	12	
	16	
⋮	⋮	
2+45A	180	

$2+45A < 500$

$A = 11$

$\frac{500}{45} \sim 11$

Q-Type

Ex For given pair (x,y) of +ve integer, such that $3x - 11y = 1$ and given that x can take max. value of 1000, how many such pairs possible?

$$3x - 11y = 1$$

x (C.D = 11)	y (C.D. = 3)
4	1
15	4
⋮	⋮

Ans → $\frac{1000}{11} \approx 90/91$ $4 + \overset{n-1}{\underline{90}}\times\underline{11} = 994$. Ans → $\underline{91}$

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Ex $\left.\dfrac{123123123\ldots\ldots 300 \text{ digits}}{504}\right|_R = ?$

Q-Type

$504 = 2\times2\times2\times3\times3\times7 = 2^3\times3^2\times7$

$\frac{123\times100}{9} = \frac{312}{9} = 6$

$8A+3 = 7B = 9C+6.$

$8A+3 = 7B$		$8A = 9C+3$		$7B = 9C+6$	
4	5	6	5	6	4
11	13	15	13	15	11
18	21	24	21		
25	29	33	29		
32	37				

~~4k~~ $7K+4 = 6+9K'$

$\Rightarrow 7K = 9K'+2$

8 6

$5K+8 = 6+9K'$

$\Rightarrow 5K+2 = 9K'$

5 3

60 79 $8A+3 = 483$

$123123\ldots 300\text{digits} = 504K + \underline{\underline{483}}.$

Alt. $8A+3 = 9C+6$

$\Rightarrow 8A = 9C+3.$

6 5 $\Rightarrow 72K+51$

$72K+51 = 7K'$

$\Rightarrow \left(\frac{2K+2}{7}\right) = K' \quad \Rightarrow \underline{\underline{K=6.}}$

$72\times6+51 = 483$

Format $\rightarrow 504K+483.$

Ans $\rightarrow \underline{\underline{483.}}$

Ex $\frac{37^{288}}{100}\Big|_R = ?$

$100 \times \frac{1}{2} \times \frac{4}{5}$

$= \underline{40}$.

$\frac{(37)^8}{100}$.

$= \frac{(37^4)^2}{100}$ $7^4 \to$ unit digit 1.

$(\times 1)$

$(41)^{2} = 81$

$= \frac{(37^2 \times 37^2)^2}{100}$

$= \frac{(69 \times 69)^2}{100}$

$= \frac{61^2}{100} = \underline{\underline{21}}$.

Ex Find last 2 digits of 4^{123}.

$\frac{4^{123}}{100}\Big|_R = \frac{2^{246}}{100}$

$= \frac{(2^{10})^{24} \times 2^6}{100}$

$= 76 \times 64$

$= \underline{\underline{64}}$

$2^{10 \times a}$ Last 2 digits

$a \to$ odd $\to 24$

Even $\to 76$

~~76 ×~~ any ~~multiple~~ pa

$76 \times 2^n = \ldots$ last 2 digit of 2^n.

$76 \times 24 = ..24$

$\times 64 = 64$

$\times 28 = 28$

$\times 56 = 56$

Ex $\frac{12^{107}}{37}$

$\left.\frac{12^{36}-1}{37}\right|_R = 0.$ $\left.\frac{12^{36}}{37}\right|_R = +1$

$\left.\frac{12^{35}}{37}\right|_R$ $\cancel{\frac{12}{37}} \times \frac{12^{35}}{37}$ $\frac{12}{37} \times \frac{12^{35}}{37}$

12×3 12

$37K+1 = 12K'$

$\Rightarrow K' = \frac{37K+1}{12}$

$= \left(\frac{K+1}{12}\right) \Rightarrow K = 11$

$\frac{12^{107}}{37} \rightarrow \frac{12^{36}}{37} \times \frac{12^{36}}{37} \times \frac{12^{35}}{37}$

~~999 = 3×3×3×37~~

Ex Find last digit of $7^{3\times4n}+1$, when n belongs to N.

Ans → $1+1=2$.

Ex Sum of 3rd power of 1st 100 natural no.s will have a unit's digit of.

$1^3+2^3+\cdots+100^3$

$= \left(\frac{n\times(n+1)}{2}\right)^2 = \left(\frac{100\times101}{2}\right)^2 = (50\times101)^2$ Ans → 0.

Ex What is the first non-zero integer from the right in $8330^{1957}+8370^{1982}$?

$8330^{1957}+8370^{1982}$

$8330^{1957} + 8370^{1982}$

$\downarrow$

will have more 0's. So no need to considr.

8330^{1957}

$\downarrow$

833^{1957}

3
9
7
1

Ans $\rightarrow \underline{\underline{3}}$.

Ex In how many ways 105 can be written as sum of 2 or more consecutive integers?

Check

$$\frac{n}{2}\left[2a+(n-1)d\right] = 105 \quad (d=1)$$

$$n\left[2a+n-1\right] = 210 = 2\times3\times5\times7.$$

$$\left.\begin{matrix} n>0 \\ a>0 \end{matrix}\right\} \Rightarrow 2a+n-1 > n$$

$$\text{Ans} \rightarrow \frac{2\times2\times2\times2}{2} = 8.$$

Ex How many pairs of natural no.s are there such that difference of there squares is 36?

$$a^2-b^2=36$$

$$\Rightarrow (a+b)(a-b) = 9\times4 \qquad 9=3^2$$

$$K_1\times K_2 = 9 \qquad \text{Ans} \rightarrow \underline{\underline{1}}. \leftarrow \frac{3}{2}$$

~~a+b=3~~
~~a-b=3~~

Ex No. of solⁿ of eqⁿ $m^2=1614+n^2$, where m & n are integers.

$$1614 = 4K+2 \Rightarrow \text{No sol}^n.$$

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Ex How many no.s below 100 can be expressed as a difference of 2 perfect squares in one way?

$$a^2 - b^2 = k$$

$$\Rightarrow (a+b)(a-b) = k.$$

All odd perfect squares 9, 25, 49, 81

Ex When 10^x is divided by 13, remainder is 1. If x is a natural no. less than 100, how many values x can take?

x	R
0	1
1	10
2	9
3	12/−1
4	3
5	4
6	1

Ans $\rightarrow \frac{100}{6} \approx 16.$

Ex $x^6/9$ remainder is 1 where $100 \le x \le 200$ & x is integer. How many sol^n are possible for x?

$9 \times \frac{2}{3} = 6$

$x \rightarrow$ coprime to 9

$198 = 102 + (n-1)3$

$\Rightarrow n = 33$

Ans = $101 - 33 = 68$.

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Ex $AB = 9C+1$, where A, B, C are natural no.s and $100 \le A \le 200$. How many different values can A take?

$AB = 9C+1$

$\Rightarrow$

Lecture-2

Objective:-

→ $1 \times 2 \times 3 \times \dots \times n = n!$

→ $1+2+3+\dots+n = \Sigma n$

→ Product of consecutive numbers starting from anywhere

→ Sum of consecutive numbers

Factorial

$$1 \times 2 \times 3 \times \dots n = n!$$

$\boxed{f(x+1) = (x+1) f(x)}$ → function for factorial

→ Arrangements of n different things at n places.

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Ex If $f(x+1) = (x+1)\,f(x)$ and $f(x+2) = 20\,f(x)$. Find x.

$f(x) = n!$

Then $(n+2)! = 20 \times n!$ $\Rightarrow (n+2)(n+1) = 20$

$\Rightarrow n = 3.$

→ Functions are no. patterns in variables.

Highest power of a $\boxed{\text{prime no.}}$ in a Factorial

Highest power of 5 in 100!

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \\ \hline & 4 \end{array}$$

Ans → $20 + 4 = 24$.

Also, $\frac{100}{5} + \frac{100}{5^2} + \frac{100}{5^3} + \cdots = 20 + 4 + 0 \cdots = 24.$

Ex $100! = 2^{96} \times 5^{23} \times n$

How many zeroes in n?

$$\frac{100}{2} + \frac{100}{2^2} + \frac{100}{2^3} + \frac{100}{2^4} + \frac{100}{2^5} + \frac{100}{2^6} + \frac{100}{2^7} + \cdots = 50 + 25 + 12 + 6 + 3 + 1 + \cdots = 97.$$

$$\frac{100}{5} + \frac{100}{5^2} + \frac{100}{5^3} + \cdots = 24.$$

$\therefore n = 2 \times 5.$ Ans → 1.

Ex Find H.P. (highest power) of 567 that can divide 285! exactly?

$567 = 3 \times 3 \times 3 \times 3 \times 7 = 3^4 \times 7$.

$$\begin{array}{r|l} 3 & 285 \\ 3 & 95 \\ 3 & 31 \\ 3 & 10 \\ 3 & 3 \\ & 1 \end{array}$$

$\underline{95+31+10+3} = 139$

$\frac{139}{4} = 34.35$

$$\begin{array}{r|l} 5 & \cancel{285} \\ 5 & \cancel{57} \\ 5 & \cancel{11} \\ & 2 \end{array}$$

$\cancel{57+11+2} = \cancel{70}$.

$$\begin{array}{r|l} 7 & 285 \\ 7 & 40 \\ & 5 \end{array}$$

$40+5=45$.

Ans $\rightarrow$ $\underline{\underline{34}}$.

No. of zeroes in a product or factorial.

= H.P. of 10 in base 10

= H.P. of 2 & 5.

$\rightarrow$ ~~H.P.~~ No. of zeroes = H.P. of 'n' in base 'n'.

Ex How many natural no.s such that their factorial is ending with 5 zeroes?

$\underline{\underline{5 \cdot 5}}$

$$\begin{array}{r|l} 5 & 20 \\ & 4 \end{array} \rightarrow$$

$$\begin{array}{r|l} 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \quad 6$$

Not possible Ans $\rightarrow$ $\underline{0}$

- In every multiple of 25 $\rightarrow$ count up by 2
  125 $\rightarrow$ count up by 3.

__/__/__

Ex How many natural no.s such that n! ends with exactly 30 zeroes ?

$$\frac{100}{5}+\frac{100}{5^2}+\frac{100}{5^3}=24.$$

$$\frac{115}{5}+\frac{115}{5^2}+\frac{115}{5^3}=23+4=27$$

5 | 115
5 | 23
4

$$\frac{120}{5}+\frac{120}{5^2}+\frac{120}{5^3}=24+4=28.$$

$$\frac{125}{5}+\frac{125}{5^2}+\frac{125}{5^3}+\frac{125}{5^4}=25+5+1=31$$

Ans → $\underline{\underline{0}}$.

Ex $x!$ ends with n zeroes. $(x+1)!$ ends with $n+3$ zeroes. $1 \le x \le 1000$. How many solⁿ are possible for x.

Multiple of 125

$$\frac{1000}{125}=8 \qquad \text{Ans} \rightarrow 8-1=\underline{7}$$

not possible for 625, increase by 5^4.

Ex How many zeroes will be there in 15! (defined in base 10) in base 12 ?

Q-Type

$12 = 2^2 \times 3$

2 | 15
2 | 7
2 | 3
1

11 → 5

3 | 15
3 | 5
2

7

Ans → $\underline{\underline{5}}$.

Ex Find no. of zeroes at the end of $5^6!$?

5 | 5^6
5 | 5^5
5 | 5^4
5 | 5^3
5 | 5^2
5 | 5^1
1

$$1+\dots+5^5=\frac{1\times(5^6-1)}{4}=3906. \qquad \text{Ans} \rightarrow \underline{\underline{3096}}$$

Ex If $n!$ and $4n!$ end with 25 zeroes and 106 zeroes respectively, then which of the following is a factor of n?
(a) 25   (b) 26   (c) 27   (d) 28.

$$\frac{100}{5}+\frac{100}{5^2}+\frac{100}{5^3}=24.$$

$105! - 109! \longrightarrow 25$ zeroes.

$$\frac{400}{5}+\frac{400}{5^2}+\frac{400}{5^3}+\frac{400}{5^4}+\cdots = 80+16+3 = 99.$$

$$\frac{420}{5}+\frac{420}{5^2}+\frac{420}{5^3}+\frac{420}{5^4}+\cdots = 84+16+3 = \underline{103}.$$

$$\frac{425}{5}+\frac{435}{5^2}+\frac{435}{5^3}+\cdots = 87+17+3 = 107$$

$430! - 434! \longrightarrow 106$ zeroes.

$n! \rightarrow 108!$ $4n! \rightarrow 432!$

Ans → $\underline{27}$

$1+2+3+\cdots+n = \Sigma n = \frac{n(n+1)}{2}$.

→ sum of pages starting from 1
→ double counting page
→ missing page.

Ex If $\Sigma n = 288$. Find missing page.

$\Sigma 24 = \frac{24\times 25}{2} = 300$ Ans → $\underline{12}$.

Types of Questions:-

$\Sigma n \quad \Sigma n^2 \quad \Sigma n^3 \quad \underline{\Sigma(\Sigma n)}$.

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Ex Double counting a page.

$\Sigma n = 1000$. Which page?

$$\Sigma 45 = \frac{45 \times 44}{2} = 45 \times 22 = 990 \qquad \text{Ans} \rightarrow \underline{10}.$$

How to find 45?

Double $\Sigma n = \underline{\underline{2000}}$. Find the nearest square. $(45^2 = 2025)$.

This will give some idea.

Ex A BB CCC DDDD EEEEE .... Find 312^{th} term

1 22 333 4444 55555

$312 \times 2 \rightarrow \underline{624}$.

$\frac{25 \times 26}{2} = 25 \times 13 = 325$

$\frac{24 \times 25}{2} = 300 \qquad \text{Ans} \rightarrow \underline{Y}$.

Ex 1, 2, 3, 4, ... 40 no. are there on screen. The first person removes 2 no.s and replacing it with $a+b-1$. The person is doing it 39 times. What is last no. on the screen?

1 2 3 4 40

↓ 2 3 4 ... 40

↓ 4 4 5 6 ... 40.

Ans $\rightarrow \Sigma 40 - \underline{39}$.

__/__/____

Ex "N" no.s written on board. First person comes and delete 1st no. and write the sum. Process is repeated till only 'N' is left on the screen. Find the sum of sums.

$$1+2+3+\cdots\cdots+n=$$
$$2+3+\cdots\cdots+n=$$
$$3+4+\cdots\cdots+n=$$

$$1+2\times2+3\times3+\cdots\cdots+n\times n=\Sigma n^2.$$

Normal way

$$\Sigma n-1+\Sigma n-(1+2)+\Sigma n-(1+2+3)\cdots$$
$$=n\Sigma n-(\Sigma(\Sigma n))$$
$$=n\times\frac{(n+1)n}{2}-\Sigma\frac{n(n+1)}{2}$$
$$=\frac{n^2}{2}(n+1)-\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}+n\left(\frac{n+1}{2}\right)\right]$$
$$=\frac{n(n+1)}{2}\left[n-\frac{2n+1}{6}+\frac{1}{2}\right]$$
$$=\frac{n(n+1)}{2}\times\frac{6n-2n-1+3}{6}$$
$$=\frac{n(n+1)}{2}\times\frac{4n+2}{6}=\frac{n(n+1)(2n+1)}{6}=\Sigma n^2.$$

Ex

1

1 2

1 2 3

1 2 3 4

⋮

1 2 3 4 . . . . n

Find sum of all.

$$\Sigma(\Sigma n).$$

__/__/____

Ex Books are kept in order.
Row 1 has 1 book.
2 has 3 books
3 has 6 books.

n^{th} has X books.
Total count is given. Find no. of rows.
$\Sigma(\Sigma n)$ = find n

Remember
$\Sigma 1 = 1$
$\Sigma 2 = 3$
$\Sigma 3 = 6$
$\Sigma 4 = 10$
$\Sigma 5 = 15$
$\Sigma 6 = 21$
$\Sigma 7 = 28$
$\Sigma 8 = 36$
$\vdots$

Ex How many ways 100 can be written as a sum of consecutive natural no.s?

$$\frac{n}{2}\left[2a+n-1\right] = 100$$

$$\Rightarrow n(2a+n-1) = 100 = 2\times2\times5^2 = 2^2\times5^2.$$

$$n < 2a+n-1$$

Ans → $\frac{3\times3\cancel{+}1}{2} = \cancel{5} - 1 = \cancel{4}3.$

1, 100 not allowed

distinct.

Alt.

No. of odd factors of 100 $= 2^2\times5^2$
-1 $= 2+1 = 3-1 = 2.$

$$\boxed{\text{No. of ways a no. is written as a sum of consecutive no.} = \text{No. of odd factors} - 1}$$

Ex How many sum of consecutive no. add upto 7!.

$\frac{7!}{3} + \frac{7!}{3^2} = 2.$ $\quad \frac{7!}{7} + \frac{7!}{7^2} = 1$

$\frac{7!}{5} + \frac{7!}{5^2} = 1$

$\therefore 3^2\times5\times7\times x$

Ans → $3\times2\times2 - 1$
$= 11.$

Powers

	Frequency
1 − 1	
2 − 2, 4, 8, 6, 2, 4, 8, 6	4
3 − 3, 9, 7, 1,	4
0, 1, 5, 6	1

→ last digit Qs
→ Remainder Qs where divisibility depends upon unit digit 2, 5, 10.

→ Right most non-zero integer
→ Power cycle in groups of 10s.

No.s	Frequency
2, 3, 7, 8	4
9, 4	2
0, 1, 5, 6	1

Last 2 digits in a^b :-

Odd numbers :- 1, 3, 5, 7, 9.

⇒ $21^{67} = 41$

$31^{42} = 61$

$171^{32} = 41$.

No.s ending in '1'

⇒ No.s ending in 5 → 25/75.

$\Rightarrow$ $3^4, 7^4, 9^2 \longrightarrow$ all will end in 1.

Ex Find las 2 digits of 37^{65}.

$$\underline{37\times(37^4)^{16}} = 37\times(69\times69)^{16}$$
$$= 37\times(61)^{16}$$
$$= \cancel{37\times(61)} \; 37\times 61$$
$$= \underline{57}.$$

Alt.

$37^5 = \underline{57}$

> Any last 2 digits que/ when remainder when divided by zero. Powers can be reduced in multiple of 20

$\Downarrow$

37^{65} last 2 digits = 37^5 last 2 digits.

Ex 33^{65} last 2 digits.

$$33^5 = 33\times 33^4 = 33\times 89^2$$
$$= 33\times 21$$
$$= 93$$

For Even Numbers

$\rightarrow$ $(2^{10})^{even} = \ldots\ \underline{76}$

$(2^{10})^{odd} = \ldots\ \underline{24}$

Remember.

Ex 2^{246} last 2 digits.

$$2^6\times(2^{10})^{24}$$
$$= 64\times 76$$
$$= \underline{64}.$$

$76\times 64 = 64$
$\times 28 = 28$
$\times 12 = 12$
$\times 56 = 56$
$\times 16 = 16.$

Lecture - 3

→ Factors/Multiples
→ Division
→ Constant Remainders
→ No.s — as difference of squares / as sum of squares.

Prime Factors

$N = a^p b^q c^r$ No. of factors $= (p+1)(q+1)(r+1)$

a, b, c → prime factors.

→ There are 3 factors — Prime^2
2 factors — Prime No.

Ex How many natural no. less than 1000 will have exactly 3 factors?

No. of Prime^2 $= 2^2, 3^2, 5^2, 7^2, 11^2, 13^2, 17^2, 19^2, 23^2,$
$29^2, 31^2, \cancel{37^2}$

Ans → 11

Ex No. less than 1000 which will have odd no. of factors.

No. of perfect squares.   Ans → 31.

Ex Assume there are 1000 lights numbered.
1, 2, 3, 4, 5, 6, ... 1000
There are 1000 students.
1^{st} person goes and switch ON all the lights.
2^{nd} person change state for 2, 4, 6, 8 . . .
3^{rd} person change state for 3, 6, 9, 12
⋮
So on upto 1000.

__/__/__

After the complete cycle, how many of them will remain 'ON'.

All no.s having odd no. of factors will remain ON
$\Rightarrow$ perfect square.

Ans $\rightarrow$ 31.

Division/Multiplication Questions:-

Ex $7A+1 = 8B+1$

$7A = 8B$

LCM of (7,8) Ans $\rightarrow$ $56k+1$

Ex $7A+2 = 8B+3$ Special case.

$\cancel{7A = 8B}$

$\rightarrow 5$ $\rightarrow 5$

$56k+51$.

Ex $7A+0 = 8B+3$.

$7A = 8B+3$

$56k+35$

	A	B
	5	4
C.D.	13	11
8	21	18

$\rightarrow$ C.D. $= 7$

$7A+0 = 8B+3$

Ex $9A+2 = 11B+7 = N$. Find no. of sol^n, when $N<1000$.

3 2

$99k+29$ Ans $\rightarrow$ $\frac{1000}{99} \approx 10/11$. Find out.

Ex $3x \mp 11y+1$. Find no. of sol^n, when $N<1000$.

$33k+12$.

Ans $\rightarrow$ $\frac{1000}{33}$.

__/__/____

Ex 2, 5, 8, 11 . . . . 50 terms
3, 5, 7, 9, 11 . . . . 60 terms.

No. of identical terms.

2, 5, 8, 11 . . . . .
→ Interval of 2 ⇒ 25 terms.

3, 5, 7, 9, 11, 13 . . .
Interval of 3 ⇒ 20 terms.

Ans → **20**.

⇒ divide an AP with C.D, then remainder will be constant.

$$\left.\frac{10}{14}\right|_R = \left.\frac{24}{14}\right|_R = \left.\frac{38}{14}\right|_R = \left.\frac{52}{14}\right|_R = 10.$$

$$\left.\frac{10}{13}\right|_R \neq \left.\frac{24}{13}\right|_R \neq \left.\frac{38}{13}\right|_R \;\ldots$$

But will be same for factors of C.D.

$$\left.\frac{10}{7}\right|_R = \left.\frac{24}{7}\right|_R = \left.\frac{38}{7}\right|_R .$$

$\boxed{\text{Constant Remainder Theorem}}$

__/__/____

Ex After division of a no. successively by 3, 4 and 7 the remainder obtained are 2, 1, & 4 respectively. What is the remainder if the no. is divided by 16?

3 4 7 $\qquad$ $84K + 53$ $\rightarrow$ Ans $\rightarrow$ can't be determined,

2 1 4 $\qquad$ as 16 not a factor of 84.

If divided by 12, ans will always be 5.

Ex If x & y are integers, then the eqn $5x + 19y = 64$ has

(a) no soln for $x < 300$ & $y < 0$.

(b) no soln for $x > 250$ & $y > -100$

(c) a soln for $250 < x < 300$

(d) a soln for $-59 < y < -56$

$$5x + 19y = 64$$

$9 \qquad 1$

$-10 \qquad 6$

$9 + 19K \qquad 1 + 5\cancel{K}13$ $\qquad$ $x \uparrow \; y \downarrow$

Ex How many ways 1954 & 1614 can be written as difference of square.

$a^2 - b^2 = 1954 = 4K+2$ $\qquad$ 0 ways.

$1614 = 4K+2$ $\qquad$ 0 ways.

Ex How many ways prime no. be expressed as difference of squares?

1 way. $\qquad$ Except $\rightarrow$ 2

Ex How many 2 digit odd no.s are there with exactly 8 factors?

$8 \to 1\times 8$
2×4
$2\times 2\times 2$

8 factors → a^7, ab^3, abc

$2^7 = 128$ → ✗

3×5^3 ✗

5×3^3 ✗

$3\times 5\times 7$

Ans → $\underline{0}$.

Ex If P is set of all natural no.s with 4 factors such that sum of factors excluding the no. is 31. Find no. of elements in P.

4 factors → a^3, ab.

$4 \to 1\times 4$
2×2

$1+a+a^2 = 31$

$\Rightarrow a+a^2 = 30$

$\Rightarrow a^2+6a-5a-30 = 0$

$\Rightarrow (a-5)(a+6) = 0$

$\Rightarrow \underline{a=5}$

$1+a+\cancel{ab} = 31$

$\Rightarrow \cancel{ab}\ a+b = 30$ // a, b are prime.

$\Rightarrow a = 30-b$

23	7
19	11
17	13

$ab < \underline{30}$

Ans → $125 + 23\times 7 + 19\times 11 + 17\times 13$
$= 716$.

Ex N is a set of all natural numbers less than 500 which can be written as sum of 2 or more consecutive natural no.s. Find the max. no. of elements possible in N.

Only no.s without any prime odd factors can be shown.

Ans → $499 - \cancel{8}9 = \underline{490}$.

① →

___/___/___

Ex $\left.\frac{21^{231}}{25}\right|_R = ?$

$25 \times \frac{4}{5} = 20$

For divisibility with 25, find last 2 digits.

$=$ Find last 2 digits.

$21^{231} = 21^{11}$

$= 21$ Ans $\rightarrow \underline{\underline{21}}$.

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Ex A is set of +ve integers such that when divided by 2, 3, 4, 5, 6 leaves the remainder 1, 2, 3, 4, 5 respectively. How many integers between 0 to 100 belong to set A?

$2-1=1 \quad 3-2=1 \quad 4-3=1 \quad 5-4=1 \quad 6-5=1.$

$60-1=59$ Form $\rightarrow 60K+59$ Ans $\rightarrow \underline{\underline{1}}$.

Lecture-4

Ex N is a +ve integer, which when divided by 16, 17 and 18 leaves a remainder of 6, 7 and 8 respectively. Find the remainder when N^2+5N+6 is divided by 12.

$16-6 = 17-7 = 18-8 = 10.$

$LCM(\underset{2^4}{16}, \underset{17}{17}, \underset{2\times3^2}{18}) = 2^4 \times 3^2 \times 17 = \underline{\underline{2448}}$.

Form $= \frac{2448K + 2438}{12}$. $\frac{2448K+2438}{12} = 2$

$\left.\frac{N^2+5N+6}{12}\right|_R = \frac{(N+2)(N+3)}{12} = 4 \times 5 = \underline{\underline{8}}.$

__/__/____

Ex If $a=1^2, b=2^3, c=3^4, \ldots z=26^{27}$. In the product of all the alphabets, how many zeroes exist in the end?

$5^6 \quad 10^{11} \quad 15^{16} \quad 20^{21} \quad 25^{26}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$6 \quad 11 \quad 16 \quad 21 \quad 52$

$= \underline{\underline{106}}$.

Ex How many integer values of a & b are there such that $4a+7b=3$ while $|a|<100$ & $|b|<100$?

$4a+7b=3$

$\Rightarrow a = \frac{3-7b}{4}$.

a	b
-1	1
-8	15

-99 to 99

Ans $\rightarrow \frac{199}{7} = 28$ or 29

$-99+\underline{28}\times 7$

$= -99+196$

$= \underline{\underline{97}}$.

Ans $\rightarrow \underline{\underline{29}}$

Ex Find unit digit of $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$.

$\left(\frac{(3^{11})^5}{3^{11}}\right) + \frac{2^{144}}{2^{72}}$

$= \underline{\underline{0}}$..

Ex Find the remainder of 5^{2p} divided by 13, given that $p = 1!^2 + 2!^2 + \ldots + 10!^2$

$p\rightarrow 1 \quad \frac{5^2}{13} = 12$

$p\rightarrow 2 \quad \frac{5^{2\times 2!^2}}{13} = \frac{5^8}{13}$.

$p\rightarrow 3 \quad \frac{5^{2\times 3!^2}}{13} = (\ )^0 \quad \frac{2\times 3!^2}{12}$

$= 1$

$$= 12\times\frac{5^8}{13}\times 1\times 1\times 1$$

$$= 12\times 12$$

$$= 144 = \underline{1}.$$

Ex Students in a college decide to form a club. The requirement for being a member of this club is no. of stamps a member owns must be a perfect square. Also, each member must have a no. of stamp which differs by 192 from only one other member. If club has max. no. of members possible, find difference b/w greatest no. of stamps owned by a member and least no. of stamps owned by a member.

$$a^2-b^2 = 192$$

$$K_1\times K_2 = 48 = 2\times2\times2\times2\times3 = 2^4\times3$$

~~Ans~~ → No. of members $= \frac{5\times2}{2} = \underline{5}$

$a+b = 24$
$a-b = 2$

2 13 11

$a+b = 96$
$a-b = 2$ $\Rightarrow 2a = 98 \Rightarrow a = 49 \quad b = 47.$

		192	
49, 47	←	2	96
		4	48
		6	32
16, 8	←	8	24
2, 14	←	12	16

Ans → $49^2 - 2^2$

$= \underline{2397}.$

Ex A, B and C are 3 consecutive odd no.s. If ABC = 531117, find A+B+C

$$531117 = 3\times177039$$
$$= 3\times9\times19671$$
$$= 3^3\times3\times6557$$
$$= 3^4\times79\times83$$
$$= 79\times81\times83$$

Ans → $\underline{243}$

Alt. Find cube root of 531117. Ans. must be around them.

~~Any cube can be found out by~~

$\boxed{\text{Any cube is equal to sum of consecutive odd numbers.}}$

Ex S is a set of all natural nos. having 24 factors and 3 prime factors. Find the digital sum of smallest element in S.

$24 = 2\times12$
$= 2\times3\times4$
$= 2\times2\times6$

$a\,b^2\,c^3$ or $a\,b\,c^5$

$a=5,\ b=3,\ c=2$; $a=3,\ b=5,\ c=2$

$= 5\times3^2\times2^3$; $= 32\times15$

$= 360$; $= \underline{\underline{480}}$

Ans → $\underline{\underline{9}}$

Ex $\left.\dfrac{200^{1000}}{17}\right|_R = ?$

$\left(\dfrac{200}{17}\right)^{\frac{1000}{16}}$

$= \dfrac{(13)^8}{17}$

$\dfrac{13^2}{17}\times\dfrac{13^2}{17}\times\dfrac{13^2}{17}\times\dfrac{13^2}{17} = -1\times-1\times-1\times-1$
$= 1.$

Ex $\left.\dfrac{12^{107}}{37}\right|_R.$

$\left.\dfrac{12^{36}}{37}\right|_R = 1$

$\dfrac{12^{35}}{37}$ $\quad 12\times$ $\quad \left.\dfrac{12^{35}}{37}\right|_R = k.$ $\qquad \left.\dfrac{12\times}{37}\ \dfrac{12^{35}}{37}\right|_R = 1$

$\Rightarrow \left.\dfrac{12\times k}{37}\right|_R = 1$. Solve from options.

__/__/____

Ex $\dfrac{6^{83}+8^{83}}{49}\Big|_R$

$\dfrac{(14)\,(\qquad\qquad)}{49}$

$\dfrac{6^{83}}{49}$

Ex LCM & HCF of 2 no.s are 609 & 21 respectively. Find no. of max. possible ordered pairs of such no.s?

$21a \quad 21b.$ $\quad 21ab = 609$

$\Rightarrow ab = \underline{\underline{29}}.$

Ans → $\underline{\underline{2}}$.

Ex How many non-zero integral values of x, y and z are there, such that $z^2 = x^2 + y^2$ and $z^2 \le 100$.

x	y	z
3	4	5
4	3	5
6	8	10
8	6	10

Ans → 32 Both +ve & -ive

Ex How many natural no.s are there with 24 factors where 2, 3, 5 are only factors.

$24 = 2 \times 2 \times 6 \rightarrow \frac{3!}{2!} = 3$

$2 \times 3 \times 4. \quad \cdot \; 3! = 9.$

Ans → 9.

Lecture - 5

Ex In a number system, the product of 122 and 41 is 5442. The no. 4434 of this system when converted to decimal system become.

$$\begin{array}{r} 122 \\ 41 \\ \hline 122 \\ 488 \\ \hline 5002 \end{array}$$

$b^3 \times 5 + b^2 \times 4 + b \times 4 + 2 = 5442$

but it is 4.

∴ Base → 6.

$6^3 \times 4 + 6^2 \times 4 + 6 \times 3 + 4 = 1030.$

Ex How many 3 digit nos in base 10 are there which can be expressed using 3 digits in base 9 as well as base 11?

10 → 100 ($\rightarrow 10^2$) — 999 $(10^3 - 1)$

9 → $9^2 \rightarrow 81$ — 728 $(9^3 - 1)$

11 → $11^2 \rightarrow 121$ — 1330 $(11^3 - 1)$

Ans → $728 - 121 + 1 = 608$

__/__/____

Ex $\left.\dfrac{402^{12}}{1000}\right|_R$.

$402 \to 2 \times 3 \times 67$

$\dfrac{2^{12} \times 3^{12} \times 67^{12}}{1000}$

$\cdots + {}^{12}C_{10}\, 400^2 \times 2^{10} + {}^{12}C_{11} \times 400 \times 2^{11} + {}^{12}C_{12} \times 2^{12}.$

$\dfrac{12 \times 400 \times 2^{11}}{1000} + \dfrac{2^{12}}{1000}$

$= 400 + 96$

$= \underline{\underline{496}}.$

Ex How many different remainder can result when 100^{th} power of a natural no. is divided by 125?

$125 = 125 \times 4/5 = \underline{\underline{100}}.$

Any no. coprime to 125 → remainder 1 $\left(\dfrac{a}{125}\right)^{\frac{100}{100}} = \left(\dfrac{a^0}{125}\right) = 1.$

Others will be of form $5^a \times$ __ . All these cases will yield remainder 0.

Ans → $\underline{2}$.

Ex $\dfrac{\left[(61)^{7!}\right]^{13345}}{13}$

Ans → $\left(\dfrac{61}{13}\right)^{\frac{7! \times 13345}{12}} = \underline{\underline{1}}.$

Ex No.s 1, 2, 3 .... n are written in sequence. No.s at odd places are struck off and a new sequence formed. The process repeated until a single no. remains. What is the final no. left if n = 527?

		2^2	2^3	2^4
~~1~~	2	4	8	
2	~~2~~			
~~3~~	4	~~4~~		
4	~~6~~	8	8.	
~~5~~	8	~~12~~	16	
6	~~10~~	16		
~~7~~	12			
8	~~14~~			
~~9~~	16			
10				

Ans → **512**.

Ex If n = 10,000! is divisible by p^p, where p is a prime no. What is maximum value of p?

$$\left.\frac{10,000!}{p^p}\right|_R = 0.$$

$10000! \to (100^2)!$

Greatest prime no. less than 100   Ans → **97**

Ex A set P consists of all odd numbers from 1 to 150. What is the highest power of 3 in the product of all elements in P?

3	6	9	12	18	27	30	33	36	39	42
1	1	1	1	1	1	1	1	1	1	1
1	1	2	2	2	3.	3	3	3	3	3

~~150~~
~~$150 = 81 + (n-1)3$~~
~~$69 = (n-1)3$~~
~~$\Rightarrow n = 24$~~

$1\times2 + 2\times3 + 3\times4 + \underline{4\times5}$.

$$\begin{array}{cccc} 3 & 9 & 27 & 81 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 50 & 16 & 5 & 1 \end{array}$$

$50+16+5+1 = \underline{\underline{72}}$.

$$\begin{array}{r|l} 3 & 150 \\ 3 & 50 \\ 3 & 16 \\ 3 & 5 \\ & 1 \end{array}$$

$50+16+5+1$
$= \underline{\underline{72}}$.

Ex $\dfrac{362 \times 363 \times \cdots \times 454}{93!} \Big| R$.

$\dfrac{362 \times 363 \times 364 \times \cdots \times 454}{93!}$

Ex If $n > 47$ & n is a natural no. Then how many of following will exactly divide $n(n^2-1)(n^2-4)$.
(i) 120 (ii) 24 (iii) 48 (iv) 49

$(n-2)(n-1)\,n\,(n+1)(n+2)$

$2^3 \times 3 \times 5$. Ans $\rightarrow \underline{2}$

Ex $\dfrac{11^{10}-11}{100} \Big| R = ?$

$= 1-11 = -10 = \underline{\underline{90}}$.

Ex. $AB = 9C + 1$, where A, B, C are natural no.s and $100 \le A \le 200$. How many different values can A take.

Not possible.

$9B \neq 9C + 1$

$10B = 9C + 1$ ✓

$11B = 9C + 1$ ✓

$12B \neq 9C + 1$

$A, 9$ should be coprimes.

~~Ans~~ $\frac{101}{3} = 33$ ~~Not pos.~~ Ans. $101 - 33 = \underline{\underline{68}}$.

Ex. A 50 digit no. has all 7's. Find the remainder when divided by 74.

$$\frac{7777\ldots \text{ 50 digits}}{74}$$

$$\frac{\underbrace{7777\ldots \times 100 \times 100}_{48\text{ times}} + 77}{74}$$

$= \underline{\underline{3}}$.

Ex. A maxi power no. is defined as a no. which is perfect square, cube, 4th power, 5th power, 6th power, 7th power, 8th power, 9th power & 10th power of distinct natural no.s. Last digit of difference b/w smallest 2 maxi power no. is:

2 3 4 5 6 7 8 9 10

$$3^{2520} - 2^{2520} = \underline{5}.$$

$3^{2520} \downarrow 1$, $2^{2520} \downarrow 6$

Ex How many 3 digit no.s are there which can be expressed as a perfect square, perfect cube and perfect 4th power?

LCM (2 3 4) → 12

Min → $2^{12} = 4096$ Ans → $\underline{0}$.

Ex Find the first non-zero integer from right in 25!

$\frac{25}{2} + \frac{25}{2^2} + \frac{25}{2^3} + \frac{25}{2^4} + \ldots = 12 + 6 + 3 + 1 = 2^{22}$

$\frac{25}{3} + \frac{25}{3^2} = 8 + 2 = 3^{10}$

$\frac{25}{5} + \frac{25}{5^2} = 6$ $\quad 5^6$

$\frac{25}{7} =$ $\quad 7^3$

$\frac{25}{11}$ $\quad 11^2$

$\frac{25}{13}$ $\quad 13$ $\quad \underline{6}$ $\quad$ Check

$\frac{25}{17}$ $\quad 17$

$\frac{25}{19}$ $\quad 19$

$\frac{25}{23}$ $\quad 23$

__/__/____

Ex Which of following pair of no.s are not twin primes?

(i) 71 & 73 (ii) 101 & 103

(iii) 137 & 139 (iv) 161 & 163

Twin primes are prime no.s differing by 2.

Prime No. → $6K+1$ / $6K-1$

~~Ans → (iv) 163 → not a prime no~~ All no.s are $6K+1/6K-1$ form.

Check individually.

71 → need to check divisibility till $\sqrt{71} \approx 9$

71 – prime

73 – prime

101 → prime (check till 11)

103 → prime "

137 → prime (Check till 12) Ans → (iv).

139 → prime "

161 → Not prime 7×23.

Ex How many no.s are there in $(2ABC)^4$ where 2ABC is a 4 digit no.?

$(2ABC)^2$ = 7 or 8 digit.

$2000^2 \to 40,00,000$

$3000^2 \to 9,000,000$

$(2ABC)^2$ has 7 digits.

$(2ABC)^2 \times (2ABC)^2$ = ~~1~~14 or 13 digits.

~~Ans~~ → $(4000000)^2 = 16,000,000,000,000$

Ans → Only 14 digits.

$\underline{\underline{Ex}}$ The difference b/w a no. and the sum of its digits is always a multiple of :-

ABC $\quad 100A + 10B + C - (A+B+C)$

$= 99A + 9B +$ $\quad$ Ans → $\underline{\underline{9}}$.

$\underline{\underline{Ex}}$ If $LCM(A,B,C) = A \times B \times C$, then $HCF(A,B) =$

Ans → $\underline{\underline{1}}$ (has to be coprimes).

$\underline{\underline{Ex}}$ The $HCF(A,B,C) = 1$. Is the $LCM(A,B,C) = A \times B \times C$?

Not necessarily

Ex → 2 6 7 $\quad HCF = 1$ $\quad$ Ans → Can not say.

$LCM = 42$ $\quad (\underline{\underline{6 \times 7}})$.

$\underline{\underline{Ex}}$ No. of factors of $3^6 \times 6^3$?

Always remember to factorize to prime factors

$3^6 \times 2^3 \times 3^3 = 2^3 \times 3^9 = 4 \times 10 = 40$.

$\underline{\underline{Ex}}$ If no. of factors of a no. is odd, is it a perfect cube?

Can not say

$a^3 \to 4$ $\quad a^6 \to \underline{\underline{7}}$.

$\underline{\underline{Ex}}$ If $K = \dfrac{\text{sum of all factors of } N}{N}$, where N is a perfect no., what is the value of k?

$k = \dfrac{2N}{N} = \underline{\underline{2}}$.

Ex What is LCM $\left(\frac{4}{6}, \frac{6}{9}, \frac{5}{4}\right)$?

First bring to simplest form.

$\frac{2}{3}, \frac{2}{3}, \frac{5}{4}$ $\qquad \frac{10}{1}$

Ex Is HCF [a,b,c,d] = HCF [HCF (a,b), HCF(c,d)]

Yes.

Ex Is LCM [a,b,c,d,e] = LCM [LCM (a,b), LCM(b,c), LCM(c,d), LCM(a,e), LCM(b,c)] ?

Yes.

Ex No. of zeroes at the end of 125! ?

$\frac{125}{5} + \frac{125}{5^2} + \frac{125}{5^3} + \dots = 25+5+1 = \underline{31}$

Ex If 2484x36y is divisible by 36, find $(x-y)_{min}$.

$36 \to \underline{9 \times 4}$.

60	x+y = 0
64	9
68	18

$x \to 1, \quad y \to 8.$ $\qquad$ Ans ⇒ $\underline{-7}$

Ex If y (y≥4) is an even natural no. and $x = y^2 - 2y$, then the largest no. that always divides $(x^2 - 8x)$ is :-

$$x^2 - 8x = (y^2-2y)^2 - 8(y^2-2y)$$
$$= y^4 + 4y^2 - 4y^3 - 8y^2 + 16y$$
$$= y(y^3 - 4y^2 - 4y + 16)$$

$= y[y^2(y-4)-4(y-4)]$

$= y(y-4)(y-2)(y+2)$

$= (y-4)(y-2)\,y\,(y+2)$ $\qquad y \to \underline{\underline{\text{even}}}$

$2^4 \times 2^2 \times 2$ $\qquad 4 \to$ multiple of 2

$= \underline{\underline{2^7}}$ $\qquad 2 \to$ " " 4

$= \underline{\underline{128}}.$ $\qquad 1 \to$ " " 8.

$1 \to$ " " 3.

Ans $\to$ $128 \times 3 = \underline{\underline{384}}.$

Ex If x, y, z are prime no.s and $y = x+2$, $z = y+2$, then no. of possible solⁿ of (x, y, z)

$n \quad (n+2) \quad (n+4)$

Only solⁿ $\to$ $(3, 5, 7)$ $\qquad$ Ans $\to \underline{\underline{1}}.$

Ex An integer m is divisible by 24 but not by 48 while another integer n is divisible by 20 but not by 40. Which of following is definitely an integer?

$m \to 2^3 \times 3 \times x$ $\qquad n \to 2^2 \times 5 \times y$

$= 48a + 24$ $\qquad = 40b + 20.$

(i) $\frac{5m}{16} + \frac{3n}{8}$ $\qquad \frac{120}{16} + \frac{60}{8} = \frac{120}{8}$ 15. $(\checkmark)$

(ii) $\frac{5m}{8} + \frac{3n}{16}$ $\qquad (\times)$

(iii) $\frac{5m}{16} + \frac{3n}{16}$ $\qquad \frac{120}{16} + \frac{120b}{16} + \frac{60}{16}$

$= \frac{136}{16} + \frac{120b}{16}$ $\qquad (\times)$

Ex How many natural no.s below 50 are there in the condition satisfying $(n-1)!$ not a multiple of n?

True when ~~is~~ n is prime.

remember

Ans → $\underline{15} + \underline{1}$ ⟵ For $n=4$, $3!$ not a multiple of 4.
$= 16$ ~~4~~ → special case.

Ex If x & y are even integers, then no. of soln of $x^2-y^2 = 7389746232$ is.
(i) 0 (b) 1 (c) 2 (d) more than 2.

$$7389746232 = 4 \times 1847436558$$
$$= 4 \times 2 \times 923718279$$
$$= 4 \times 2 \times 3 \times 107906093$$

Ans → More than 2.

As x and y are even, they are of the form $\underline{4k}$ or $\underline{4k+2}$.

Then x^2-y^2 can be $16p$
$16p-4$
or $16p+4$.

But the given no. is $16p+8$. Hence, '0' soln.

Ex P is a composite integer, which is not a perfect square.
(i) P has a factor lying between 1 and $\sqrt{P}$.
(ii) P has a factor lying between $\sqrt{P}$ and P.
Find T/F?

Both True.

Any composite no. 'P' has at least 1 factor 'm' b/w 1 and $\sqrt{P}$ and corresponding factor $\left(\frac{P}{m}\right)$ b/w $\sqrt{P}$ & P.

P → not perfect square.

Ex 2 no.s when divided by a divisor leave remainder 248 & 372. The remainder obtained when the sum of the no.s is divided by the same divisor is 68. Find the divisor.

Ans → $248+372-k=68$
$\Rightarrow k=552.$

Ex LCM & HCF of 2 no.s are 2376 and 22 respectively. Find the larger of the 2 no.s if their sum is 682.

$22a+22b=682.$
$\Rightarrow a+b=\underline{\underline{31}}.$

$22a \times 22b = 2376 \times 22.$
$\Rightarrow ab = \frac{2376}{22} = \underline{\underline{108}}.$

∴ a → 27, b → 4.

Ans → $22 \times 27 = \underline{\underline{594}}.$

Ex Frustrated with his consistently low scores in Maths, Pradip tore off a certain no. of consecutive leaves from his book. Each leaf is numbered on either side and is considered 2 pages. If the sum of page no.s which he tore off was 720, then how many pages did Pradip tore?

~~$720 = 2\times2\times2\times2\times3\times3\times5 = 2^4\times3^2\times5$~~

~~Ans → $3\times2-1 = 5$~~

__/__/____

$\frac{n}{2}\left[2a+n-1\right] = 720$

$\Rightarrow n \times (2a+n-1) = 1440.$

$n < 2a+n-1.$

n is even
2a+n-1 odd

2	720.
4	~~180~~ 360
6	240.
8	180
10	144
12	120
16	90
18	80
20	72
24	60
30	48

Ans $\Rightarrow$ $\longleftarrow$ 32 45

Ex How many natural no.s are ~~there~~ factors of exactly one of the no.s of 675 & 1080?

$675 = 3 \times 3 \times 3 \times 5^2 = 3^3 \times 5^2.$

$1080 = 3\times3\times3\times2\times2\times2\times5 = 3^3 \times 2^3 \times 5.$

$HCF = 3^3 \times 5$

$675 \rightarrow 5^2 \rightarrow 4.$

$1080 \rightarrow \cancel{3^3\times5}\ 2\times$ — 8

2^2 8

2^3 8,

Ans $\rightarrow 8\times3+4 = 28.$

Ex Which of following need not be a factor of $n(n^2-1)(n^4-13n^2+36)$, where n is a natural no. > 3.
(i) 112 (ii) 44 (iii) 315 (iv) 140.

$n(n^2-1)(n^2-9)(n^2-4)$

$= \underline{(n-3)(n-2)(n-1)}\ n\ \underline{(n+1)(n+2)(n+3)}$

$2^3 \times 2 \times 3^2 \times 5 \times 7 = 2^4 \times 3^2 \times 5 \times 7$.

$112 = 2\times2\times2\times2\times7 = 2^4 \times 7$ ✓

$44 = 2\times2\times11$ ✗

$315 = 3\times3\times5\times7$ ✓ Ans → 44.

$140 = 2\times2\times5\times7$ ✓

Ex A natural no. P is equal to one or more than pdt. of 4 consecutive +ve integers. Which of following are true?

(i) P is a perfect square. ✓
(ii) P is composite. ✓
(iii) P is odd. ~~✗~~ ✓

Any no. of the form

$(x)(x+1)(x+2)(x+3)+1$ is a perfect square.

$x \rightarrow$ +ve integer.

Ex For a set of particular distinct integral values of a, b, c, d the eqn $(n+a)(n+b)(n+c)(n+d) = 25$ holds. For how many distinct integral values of n does it hold?

$(n+a)\ (n+b)\ (n+c)\ (n+d)$ all are distinct

only one combination possible.

$+1\ \ -1\ \ +5\ \ -5$ (in any sequence, depends on values of a, b, c, d)

Ans → $\underline{1}$. (For given set of a, b, c, d).

__/__/____

Ex $N = 3! + 4! + \dots + 64!$

Which are true?

(i) N is a perfect square.

(ii) N is a perfect cube.

(iii) Both (A) & (B)

(iv) Neither (A) & (B)

Last 2 digits

$= 3! + 4! + 5! + \dots + 9! + \underline{00}$

$= 6 + 24 + 120 + 720 + 40 + 20 + 80 + 00$

$= \underline{\underline{10}}.$

$\therefore$ Not a perfect square. Nor perfect cube.

Ex The sets P_p are defined to be $\{p, p+1, p+2, p+3 \dots, p+6\}$ where $p = 1, 2, 3, \dots 88$. How many sets contain 8 or its multiple?..

Not contain → Starting from 1, 9, 17.

Ans → $1 + \cancel{8}\ 10 \times 8 = \underline{\underline{81}}$. 11

Ans → $88 - 11 = \underline{\underline{77}}$.

Ex If x, y, z and p are natural no.s such that $x^p + y^p = z^p$; which is true?

(a) p is always less than or equal to min. of x, y, z.

(b) p is always, greater than or equal to max. of x, y, z.

(c) $(x, y, z)_{min} < p < (x, y, z)_{max}$

(d) Either A or B

$$x^p + y^p = z^p.$$

No soln for $p > 2$.

Ans → $\underline{\underline{A}}$.

__/__/____

Ex $\left[\frac{15}{6^2-1}+\frac{1}{8^2-1}+\frac{1}{10^2-1}+\cdots\cdots+\frac{1}{16^2-1}\right]$

$$\frac{14}{6^2-1}+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}\cdots\cdots+\frac{1}{13}-\frac{1}{15}+\frac{1}{15}-\frac{1}{17}\right)$$

$$=\frac{2}{5}+\frac{1}{2}\times\left(\frac{1}{5}-\frac{1}{17}\right)$$

$$=\frac{2}{5}+\frac{1}{2}\times\frac{12}{85}$$

$$=\frac{2}{5}+\frac{6}{85}=\frac{40}{85}=\frac{8}{17}.$$

Ex If p is a no. such that $25\le p\le 49$ and $q=\frac{p^2+3\sqrt{p}\,[p+9]+81}{p+6\sqrt{p}+9}$, then q satisfies

(a) $18\le q<36$ (c) $19\le q<38$

(b) $20\le q<45$ (d) $23\le q<29$.

$\sqrt{p}=x$

$$\frac{x^4+3x\,(x^2+9)+81}{(x+3)^2}$$

$$=\frac{x^4+3x^3+27x+81}{(x+3)^2}$$

$$=\frac{x^3(x+3)+27(x+3)}{(x+3)^2}$$

$$=\frac{(x+3)\,(x+3)\,(x^2+9-3x)}{(x+3)^2}$$

$$=x^2+9-3x.$$

$\left(x-3/2\right)^2+\frac{27}{4}$. ~~Ans~~- $x=\sqrt{p}=5$ 19.

$x=\sqrt{p}=7$ 37

Ans→ $19\le q<38$.

Ex Pdt. of all factors of a number N is equal to 10^{th} power of the no. Which of following can't be no. N?

(a) 90 (b) 200 (c) 210 (d) 126.

$N^{d/2} = N^{10}$

$\Rightarrow d = 20$. a^{19} ab^{9} abc^{4} $a^{3}b^{4}$.

$2\times2\times5$.
$20 \rightarrow 2\times10$
4×5

$90 = 2\times3\times3\times5 = 2\times3^2\times5$ ✓

$200 = 2\times2\times2\times5^2 = 2^3\times5^2$ ✓

$210 = 3\times2\times5\times7 = 2\times3\times5\times7$ ✗

$126 = 2\times3\times3\times7 = 2\times3^2\times7$ ✓

Ans → c.

Ex If pdt. of all the factors of a number is equal to the square of the no. and sum of all the factors of the no. other than the no. itself is 21, find no. of possible values for the no.

(a) 0 (b) 1 (c) 2 (d) >2.

$N^{d/2} = N^2 \Rightarrow d = 4$ $4 = 1\times4$, 2×2.

a^3 ab.

$1+a+a^2 = 21$
$\Rightarrow a^2+a-20 = 0$
$\Rightarrow (a+5)(a-4) = 0$
$\Rightarrow a = 4$. not prime.

$1+a+b = 21$
$\Rightarrow a+b = 20$
prime

20	
a	b
3	17
7	13

∴ Ans → 2

Ex Using the digits 9, 8, 7, 6 and 5 exactly once, two numbers N_1 and N_2 are formed such that their product is max. possible no. Find sum of N_1 and N_2.

Pdt. will be max. when one is a 3 digit no. and other is a 2 digit no.

987	65	64155
876	95	83220
765	98	74,970
965	87	83,995
975	86	83,850
985	76	74,860
986	75	73,950
976	85	82,960
875	96	84000 ⟶

Ans → $875 + 96 = \underline{971}$.

Ex Let p, q, r are 3 natural no.s such that $p+q+r = 6t+7$, where t is a +ve integer. Given t, which is true?

(i) $p^2+q^2+r^2\big|_{min} = 12t^2-28t+17$
(ii) $p^2+q^2+r^2\big|_{max} = 12t^2-28t+17$
(iii) $p^2+q^2+r^2\big|_{min} = 12t^2+28t+17$
(iv) $p^2+q^2+r^2\big|_{max} = 12t^2+28t+17$

$(p+q+r)^2 = 36t^2 + 49 + 84t.$

$\Rightarrow p^2+q^2+r^2 = 36t^2 + 84t + 49 - 2(pq+pr+qr).$

Min value when, p $(2t+2)^2 + (2t+2)^2 + (2t+3)^2$

$= 12t^2 + 17 + 28t.$

Ans → (iii).

Max value $= 1^2 + 1^2 + (6t+5)^2$

$= 36t^2 + 60t + 27.$

__/__/____

Ex N is a perfect square having at least 3 digits. Its last 2 digits are equal and not equal to 0.

(i) Last digit of N must be

0 ✗
11 ✗
44
99 ✗
66 ✗
55 ✗

Note

If last digit odd & perfect square, 10th digit has to be even

If 6 and p.s., 10th digit must be odd.

Ans → 4

(ii) How many 5 digit values can N assume?

_ _ _ 44.

$\sqrt{4}$ is of form $10k \pm 2$. Since only $(10k+2)^2$ and $(10k+8)^2$ ends in 4.

N is $100k^2 + 4 \pm 40k$.

Case 1

$100k^2 + 40k + 4$.

$40k$ must end in 40.

$k \to 1, 6, 11$ $k = 5a+1$

$\sqrt{N} = 10(5a+1) + 2 = 50a + 12$.

Case 2

$100k^2 - 40k + 4$.

$k \to 4, 9,$ $5b+4$.

$\sqrt{N} = 10(5b+4) - 2$

$= 50b - 38$

$= 50(b+1) - 12$.

__/__/____

So, $\sqrt{N}$ is of form $50L \pm 12$.

6 digit

Ans → 112, 138, 162, ~~200~~, 188, 212, 238, 262, 288, 312, ~~338~~

Ans → $\underline{\underline{9}}$.

Ex There are 3 consecutive natural nos. The middle no. was cubed, the least was squared and the greatest was raised to its first power. Sum of results was tripled and then its square root was found. The value obtained was observed to be equal to the sum of the original nos. The least of the original no. is L. Which of the following can be concluded ?

(a) $L \geq 16$ (b) $12 \leq L \leq 15$

(c) $8 \leq L \leq 11$ (d) $1 \leq L \leq 4$.

$$k \quad k+1 \quad k+2$$

$$\sqrt{(k^2 + (k+1)^3 + k + 2)3} = 3k + 3$$

$$\Rightarrow \sqrt{3(k^3 + 3k^2 + 3k + 1 + k^2 + k + 2)} = 3k + 3$$

$$\Rightarrow \sqrt{3(k^3 + 4k^2 + 4k + 3)} = 3k + 3$$

$$\Rightarrow 3(k^3 + 4k^2 + 4k + 3) = 9(k^2 + 2k + 1)$$

$$\Rightarrow k^3 + k^2 - 2k = 0.$$

$$\Rightarrow k(k^2 + k - 2) = 0.$$

$$\Rightarrow (k+2)(k-1)k = 0. \qquad k = 0 \text{ or } 1 \text{ or } -2.$$

$k \to 1$

$\therefore$ Ans → $1 \leq L \leq 4$.

__/__/____

Ex Sum of 4 consecutive 2 digit no.s is S. When S is divided by 10, the quotient is Q and the remainder is 0. Q has an odd no. of factors. How many possible combinations exist for the no.s?

k k+1 k+2 k+3

Q → perfect square of prime no.   Q → $\underline{\underline{4, 9, 25,}}$ ~~3~~ 49 → 490 ✗ $\underline{\underline{>400}}$

40 → 0

90 → $n \times (2a + n - 1) = 180$

$\Rightarrow 2a + 3 = 45$

$\Rightarrow a = \underline{\underline{21}}$.

250 → $n \times (2a + n - 1) =$ ~~250~~ 500

$\Rightarrow 2a + 3 = 125$

$\Rightarrow a = \underline{\underline{61}}$.

Ans → $\underline{\underline{2}}$.

Ex Raju has a certain no. of chocolates (< 1000) of chocolates with him. If he distributes them equally among a group of 12 or 15 or 18 children, he would be left with 1 chocolate in each case. If he distributes the chocolates equally among 19 children he would be left with no chocolates. How many chocolates does Raju have.

$12A + 1 = 15B + 1 = 18C + 1$

$\downarrow$ $2^2 \times 3$ $\downarrow$ 3×5 $\downarrow$ 2×3^2.

$2^2 \times 3^2 \times 5 = \underline{\underline{180}}$

$180K + 1 = 19K'$ Ans → $\underline{\underline{361}}$.

$\downarrow$ 2. $\downarrow$ 19

__/__/____

Ex The square of a 2 digit no. is equal to that of a 3 digit no. The tens digit and the hundreds digit of the 3 digit no. are equal. The units digit of 2 two digit and the 3 digit no. are equal. If the 3 digit no. exceeds 250, find its unit digit.

ab   ccb.

b → 1 → 11, 21, 31

0 → not possible. 000

5 → 15, 25, 35

6 → 16, 26, 36

Ans → $21^2 = 441$ (1)

Ex Raju was given a problem of adding a certain no. of consecutive natural no.s starting from 1. By mistake he added a natural no. twice. He obtained the sum as 825. Find the no. that was added twice.

$\sqrt{825} = \cancel{28}\ \cancel{29 \to 841}$ $1650 \to n^2 = 40.$

$\cancel{\frac{29 \times 30}{2}} = \frac{41 \times 40}{2} = 820$

Ans → 5.

Ex Find the no. of digits in the smallest no. containing all 5s which is divisible by 7.

$\frac{5}{7} \to 5$ $\frac{555}{7}\Big|_R = 0$

$\frac{55}{7}\Big|_R \neq 0$

:

:

$\frac{555555}{7}\Big|_R = 0.$

Alt:

7 follows tripled rule.

555 | 555   555 − 555 = 0.

__/__/____

Ex Find the no. of triplets of prime no.s in ascending order such that the difference b/w each pair of successive no.s is 2.

Ans – 1 $\quad$ 3, 5, 7.

Ex In an attempt to find HCF of 2 no.s using the division method, the quotients were 1, 3, 2, 6 in order. The final divisor was 16. Find the 2 no.s.

→ ~~16) 96 (~~ 16

720 | 928, 720 | 1

208 | 208×3+96, 208×3 | 3

96 | 192+16, 192 | 2

16 | 96, 96 | 6

0

Ans → 720, 928..

Ex When 5, 8, 12 divide a multiple of 13 the remainders are 3, 6, 10 respectively. Find the least such no.

$$\underbrace{5A+3}_{2} = \underbrace{8B+6}_{2} = \underbrace{12C+10}_{2} = 13K.$$

$120 - 2 = \underline{118}$ $\qquad$ $120K + 118 = 13K'$

↓

4

598

Ans → 598.

Ex The remainder obtained when $N^5 - N$ is divided by 6, where $N \in Z^+$ and $N > 1$.

$N(N^4-1) = \underline{N(N+1)(N-1)}(N^2+1)$ Ans → $\underline{0}$.

Ex The expression $n^3 - 6n^2 + 5n$ where $n \in Z^+$ & $n > 5$ is always divisible by which?

(i) 5   (b) 7   (c) 9   (d) 6

$n^3 - 6n^2 + 5n$

$= \cancel{n(n-3)(n-2)}\ n(n^2 - 6n + 5)$

$= \underbrace{n(n-1)}_{2}(n-5)$ $n \to 3k$ — chill

$n \to 3k+1 \Rightarrow n-1 = 3k$

$n \to 3k+2 \Rightarrow n-5 = \underline{3k-3}$.

Ans → $2 \times 3 = \underline{6}$.

Ex If $x \in Z^+$ such that $4x^4 + 3x^3 + 2x^2 + x + 24$ is perfectly divisible by x, then how many values of x are possible?

$24 \to 2^3 \times 3 \quad = 4 \times 2 = \underline{8}$.

Ex When a no. is divided by any divisor from 2 to 7, it always leaves a remainder 1 less than the divisor. Which of following can't be the no?

(a) 419   (b) 839   (c) 2099   (d) 1249

LCM (2, 3, 4, 5, 6, 7) $12 \times 5 \times 7 = \underline{420}$

$420k + \underline{419}$.

Ans → (d) ~~(a)~~ $\underline{1249}$

__/__/__

Ex A number N when divided by a divisor D leaves a remainder of 19. When 3N is divided by D, the remainder is 14. What is D.?

$$DK+19 \rightarrow 3DK+57 = DK'+\underline{14}$$

$\therefore D = \cancel{57-14}\ \underline{43}$.

Ex If pdt. of all factors of a no. is equal to square of the no. and sum of all the factors of the no. other than the no. itself is 13, find the sum of all values possible for the no.?

$$N^{d/2} = N^2$$

$\Rightarrow d = \underline{\underline{4}}$..

a^3 ab.

$1+a+a^2 = 13$

$\Rightarrow a^2+a-12=0$.

$\Rightarrow (a+4)(a-3)=0$.

$\Rightarrow a = \underline{\underline{3}}$

$1+a+b=13$

$\Rightarrow a+b=12$

prime

a	b
5	7

Ans $\rightarrow$ $27+35 = \underline{\underline{62}}$.

Ex Which of following is smallest 5 digit no. when divided by 8, 11 and 24 leaves a remainder of 5 in each case?

(i) 10301   (ii) 10125

(iii) 10061   (iv) 10037

$LCM(8,11,24) = 264$

$264K+\underline{\underline{5}}$ $\left.\frac{10000}{264}\right|_R = \underline{\underline{37}}$.

$264 \times 38 = 10032$.

Ans $\rightarrow$ $10032+5 = \underline{\underline{10037}}$.

__/__/____

Ex The digits of a 4 digit no. are rearranged to form a new No. Difference of this No. and original no. will always be divisible by

abcd　　　　bcad

$1000a + 100b + 10c + d - 1000b - 100c - 10a - d$

$= 990a - 900b - 90c$

Ans → $\underline{\underline{9}}$.

Ex The set G of no.s is $\left\{\frac{3}{64}, \frac{3}{32}, \frac{3}{16}, \ldots\ 3072\right\}$. H is a subset of G such that pdt. of no 2 elements is 144. Find max. no. of elements in set H.

$\frac{144}{9} = 16$

$\frac{3}{64}$　$\frac{3}{2^5}$　. . . .　3×2　3×2^2　3×2^3

Ans → $\underline{\underline{9}}$.

Ex How many no.s are coprime to 73 b/w the nos 31 and 47, including these 2 ?

prime no.

$47 - 31 + 1 = \underline{17}$.

Ex N = 0.abcabc.... where N is a recurring decimal and at most two of a,b,c are zero. Then which of following no. necessarily result in an integer, when multiplied by N?
(a) 1000　(b) 2997　(c) 3333　(d) 9999

$N = \frac{abc}{999}$　　Ans → $\underline{\underline{(b)}}$.

__/__/__

Ex Let a, b, c be distinct integers, that are odd and +ve. Which of following can be concluded?

(a) a^3b^2c is odd.

(b) $(a-b)^2c^3$ is even.

(c) $(a+b+c)^2(a-b)$ is even.

(d) All of above.

Ex 7 integers b/w 5 and 25 have a product of 61108047. Find the sum of these integers.

All integers must be odd.

No - 5.

7 9 11 13 17 19 21 23   Find out which is not a factor

Ans → $7 \times 9 \times 11 \times 13 \times 17 \times 19 \times 21$

$= 7+9+11+13+17+19+21$

$= \underline{\underline{97}}$.

Ex N is the LCM of +ve integers from 2 to 16. How many digits does N have?

$2^4 \times 3^2 \times 5 \times 7 \times 11 \times 13 = 720720$

Ans → $\underline{\underline{6}}$.

Ex Let n be the no. of no.s divisible by 8 out of all 4 digit no.s that can be formed with the digits 4, 5, 6, 7 and 8, no digit being repeated in the no.s. What is value of n?

— 456
— 568
— 576
— 584 $\qquad 9\times 2 = \underline{\underline{18}}$.
— 648
— 768
— 784
— 856
— 864

Ex The product of all factors of a number N is equal to the no. raised to the power of 4. If N is less than 100, find no. of possible values of N.

$$N^{d/2} = N^4 \Rightarrow \underline{\underline{d=8}}. \qquad 8 = 1\times 8$$
$$= 2\times 4$$
$$= 2\times 2\times 2.$$

$\underline{\underline{a^7}}$ ab^3 abc

$\downarrow$ 0

$ab^3 \downarrow$: 2×3^3, 3×2^3, 5×2^3, 7×2^3, 11×2^3

abc: $2\times 3\times 5$, $2\times 5\times 7$, ~~$2\times 5\times 7$~~, ~~$2\times 5\times$~~, $2\times 3\times 7$, $2\times 3\times 11$, $2\times 3\times 13$

Ans $\rightarrow \underline{\underline{10}}$.

Ex Let $y \in Z^+$ and $x = y^3 - y^2$. If $y \le 5$, the greatest no. by which $x^3 - 4x^2$ is always divisible by

$$(y^3-y^2)^3 - 4(y^3-y^2)^2$$
$$\Rightarrow = y^6(y^3-3y^2+3y-1) - 4y^4(y^2-2y+1)$$
$$= y^9 - 3y^8 + 3y^7 - y^6 - 4y^6 + 8y^5 - 4y^4.$$
$$= y^4(y^5 - 3y^4 + 3y^3 - 5y^2 + 8y - 4).$$

~~$y \times 1$~~ ~~∞~~. $\quad x^3$

y	x	x^3-4x^2
1	0	0
2	4	0
3	18	$18^3-4\times18^2 = 18^2\times14 = 2^3\times3^2\times7$
4	48	$48^3-4\times48^2 = 48^2\times44 = 2^5\times3\times11$
5	100	$100^3-4\times100^2 = 100^2\times96 = 2^9\times5^2\times3$.

$$2^3\times3 = \cancel{18}\ \underline{\underline{24}}$$

Ex How many integer pairs are there which satisfy the condition that the sum of the integers is equal to the pdt. of integers?

a, b

$a+b = ab$

$\Rightarrow \cancel{ab}\ ab - a = b$.

$\Rightarrow a = \dfrac{b}{b-1}$ $\qquad a=2,\ b=2$.

$a=0,\ b=0$.

Ans $\rightarrow \underline{\underline{2}}$.

Ex Let A be the set of prime no.s less than 50. We multiply all the elements of A to obtain a no. B. With how many consecutive zeroes will B end?

$2\times5 = 10$. Ans $\rightarrow \underline{1}$.

Ex Find the latest +ve odd integer which has same no. of factors as 540.

$540 = 2\times2\times3\times3\times3\times5 = 2^2\times3^3\times5$.

$3\times4\times2 = \underline{\underline{24}}. \rightarrow 2\times12$ $\qquad 3\times5^{11}$

3×8 $\qquad 3^3\times5^8$

4×6 $\qquad 3^3\times5^6$ → find min.

$4\times2\times3$ $\qquad 3^1\times5^3\times7^2$ $\qquad 3^3\times5^4\times7^2$

Ans $\rightarrow 3^2\times5\times7\times11$

$2\times2\times6$ $\qquad 3^5\times5\times7$

__/__/____

Ex A teacher wrote 10 digit no. on the board and asked her students to subtract the sum of digits of the no. from the number. Raju performed the subtraction but accidentally erased one of the digits in the result. Remaining digits were 1, 2, 3, 3, 6, 6, 8, 8, 9 (not necessarily in that order). Which digit did Raju erase?

$1+2+3+3+6+6+8+8+9+k = 9k'$

$\Rightarrow 46+k = 9k'$

$\Rightarrow k' = \underline{\underline{8}}$.

Ex What is no. of factors of N?
(i) There are 5 prime factors of N.
(ii) There are 120 factors of 2N.

$120 = \underline{\underline{2\times2\times2\times3\times5}}$

2 must be here

$2^{2}x^1y^1z^1a^4$
$2^{3}x^1y^1z^1a^2$ || 2 cases

Not possible to find.

Ex A no. is divided into 2 parts such that the HCF of those 2 parts is 18. What is the value of the No?
(a) Diff. b/w 2 parts is 522.
(b) Prod. of 2 parts is 10044.

$18a - 18b = 522 \Rightarrow a-b = \underline{\underline{29}}$. many sol^n

$18a \times 18b = 10044 \Rightarrow ab = \frac{558}{18} = 31$ $\underline{\underline{1 \text{ sol}^n}}$

$\underline{\underline{1\times31}}$

∴ Statement 2 is sufficient.

Ex Is the 4 digit no. abcd a perfect square?
(i) $a=1$, $c=3$, $d=6$.
(ii) $a=2$, $c=6$, $d=5$.

(i) → 1b36 → can't say
(ii) → 2b65 → No.

Ex A, B, C are natural no.s. Is $A+2B+8C$ divisible by 4?
(i) B & C are even no.
(ii) A is an odd no.?

(i) → depends on A
(ii) No. (odd no.)

Ex x and y are 2 +ve integers. Find $x+y$.
(i) $x^3+y^2=778$.
(ii) x & y are 2 digit no, each of which can be expressed as diff. of 2 perfect squares.

(i) $x^3+y^2=778$ $\quad 9^3+7^2 = \cancel{\ } \quad 9+7=16$.

(ii) x, y
Any no. which is a multiple of 4 or which is odd can be expressed as a diff of 2 perfect square.
→ not sufficient.

Ex If x and y are 2 natural nos and x/y is in its simplest form, is x/y a recurring decimal?
(i) x is 128.
(ii) y has exactly 2 distinct prime factors.

$128 = 2^7$ $\quad$ 2 is not a factor of y. as $\frac{x}{y}$ is simplest form.

__/__/____

$\frac{x}{y}$ is non-terminating & recurring.

Ex In a 4 digit no., sum of digits is a multiple of 9. How many zeroes does the no. have?

(i) The no. is a p.s. with 5 as the digit in units place.

(ii) The " " " p.c. with 1 as the " " " ".

(i) $\underline{a}\,\underline{b}\,2\,5$ $45^2 = 2025$ || Can't say.

$\cancel{7}5^2 = \overset{56}{\cancel{42}}25$

(ii) _ _ _ 1 → cube of 11, 21, 31

$11^3 \rightarrow 1331$ ✗

$21^3 \rightarrow$ ~~9~~ 9261 || ⟶

∴ Statement 2 alone is sufficient.

Ex A 4 digit no. has all its digits being distinct natural nos. Avg. of its digits is X. Find X.

(i) X+1 is a perfect cube.

(ii) X+2 is a perfect square.

(i) X+1 → 8 ~~27~~ ∴ $x = \underline{7}$. (i) is sufficient.

(ii) $x+2 \rightarrow 4$ or 9 ∴ $x = 7$. (ii) is sufficient.

~~x=2~~ ↓ not possible (not distinct

__/__/____

Periods

49 → 49, 01 ; 49, 01	Period 2.
51 - 51, 01 ; 51, 01	
99 - 99, 01 ; 99, 01	

07 - 07, 49, 03, 01	Period 4.
43 - 43, 49, 07, 01	
57 - 57, 49, 93, 01	
93 - 93, 49, 57, 01	

21 - 21, 41, 61, 81, 01 || Period 5

$$\boxed{\text{Last 2 digits of any power / Remainder when any No. is divided by 100} \rightarrow \left.\frac{a^b}{100}\right|_R \Downarrow \text{Last 2 digits always follow a cycle of 20 or any factor of 20.}}$$

Ex Find remainder when 123,123 .... upto 300 digits is divided by 37.

$37 \rightarrow 999.$

$$\frac{123,123, \ldots 300 \text{ digits}}{999} = \frac{123 \times 100}{999} = \left.\frac{12300}{999}\right|_R = \frac{312}{999} = \underline{\underline{312}}.$$

$$\left.\frac{312}{37}\right|_R = \underline{\underline{16}}.$$

__/__/____

Ex Is the 5 digit no. abc46 a p.s.?

No, If last digit 6, tens digit must be odd.

1x2y75 – No, If last digit is 5, tens digit must 2.

~~A6B~~ – ~~169~~ ~~Can't say~~

AB56 – 34^2 – 1156 ✓ Can't say.

1256 ✗

B36 – No.

Ex If the 3 digit No. A6B is a p.s., is A odd?

169, 361, 961

Ans → Yes.

Ex If X9Y is a p.s., then X+Y is

196 X+Y → 7. Ans → 7

No other no.

Ex If AB9 a p.s., is A odd?

169
289 || Can't say.

Ex $\frac{3^{40}}{11}\Big|_R$

$\left(\frac{3}{11}\right)^{40/10} = 3^0 = 1.$

Ex $\frac{4831 \times 4833 \times 4835}{24}\Big|_R = \frac{7 \times 9 \times 11}{24}$

$= \frac{15 \times 11}{24} = 21.$

__/__/____

Ex N is the no. formed by writing all +ve integers from 90 to 135 one after another. Find the remainder when N divided by 9?

$$\frac{9091\ldots135}{9} \qquad 135-90+1=46.$$

$$\left.\frac{\frac{46}{2}\left[90+135\right]}{9}\right|_R = \underline{\underline{0}}.$$

Ex $$\left.\frac{17^3+19^3+21^3+23^3}{80}\right|_R =$$

$$\frac{40\left(17^2+81^2-17\times81\right)+40\left(21^2+19^2-21\times19\right)}{80}$$

$$= \frac{40\times\overbrace{(\text{odd}+\text{odd})}^{\text{Even}}}{80} = \underline{\underline{0}}.$$

Ex $$\left.\frac{2^{156}}{105}\right|_R$$

$$105\left(1-\frac{1}{5}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{7}\right)$$

$$= 105\times\frac{4}{5}\times\frac{2}{\cancel{3}}\times\frac{\cancel{6}^{\,2}}{7} = \cancel{30}\ \underline{\underline{48}}.$$

$$\underline{\underline{(2)^{12}}}.$$

$$\left.\frac{2^{156}}{3}\right|_R=1 \quad \left.\frac{2^{156}}{5}\right|_R=1 \quad \left.\frac{2^{156}}{7}\right|_R=1.$$

$$2^{156} = \underline{\underline{105}}k+1 \qquad \text{Ans} \rightarrow \underline{\underline{1}}.$$

Ex $\frac{10!+111}{143}\Big|_R =$

$\frac{10!+111}{143} = \frac{10(9!+11)}{143}$

$\frac{10!+111}{11} = \frac{10!+1+110}{11} \rightarrow$ divisible

$\frac{10!+111}{13}$

$12!+1 = 13k.$

$\Rightarrow \frac{12!}{13}\Big|_R = -1$

$\Rightarrow \frac{132}{13} \times \frac{10!}{13}\Big|_R = -1$

$\Rightarrow \frac{10!}{13}\Big|_R = 6.$ $\quad \frac{111}{13}\Big|_R = 7.$ $\quad 6+7=13$

$\therefore$ Ans $\rightarrow 0.$

Ex $\frac{110!}{107^2}\Big|_R$

$\cancel{108 \times 109 \times}$ $\quad \frac{110 \times 108 \times 109 \times 106!}{107}$

$= 107 \times 3 \times 2 \times 1 \times -1$

$= -642.$

Ans $\rightarrow -642 + 107^2.$

Ex $\frac{84^{79}}{100}\Big|_R$

$\frac{84^9}{100} = \frac{{}^9C_8 \times 80 \times 4^8 + {}^9C_9 \times 4^9}{100}$

$= 720 \times 2^{16} + 2^{18}$

$= 20 + 44 = 64.$

__/__/____

Ex Find last 2 digits of 78^{87}.

$78^{7} = \underline{\underline{12}}$.

Y use calculator.

Ex Find remainder when 27^{27} is divided by 100.

$27^{7} = \underline{\underline{03}}$.

Ex $\left.\dfrac{13571357\ldots\ldots \text{upto } 1000 \text{ digits}}{101}\right|_R =$

$= \cancel{\dfrac{(57-13)\times 250}{101} = \dfrac{4400}{101}}$.

$= \dfrac{57\times 250 - 13\times 250}{101}$

$= \dfrac{44\times 250}{101} = \left.\dfrac{11000}{101}\right|_R = \underline{\underline{92}}$.

Ex $\left.\dfrac{123456123456\ldots\ldots\ldots \text{upto } 600 \text{ digits}}{999}\right|_R$

$\dfrac{1|23|456|000}{999} = \underline{\underline{957}}$.

Ex $\dfrac{59^{(73)^{51!}}}{37}$.

$\left(\dfrac{59}{37}\right)^{\frac{73\times 51!}{36}} = \underline{1} \cdot \left(\dfrac{59}{37}\right)^{1\times 12}$

$= 22^{12}$

$= \cancel{\dfrac{22^2}{37} \times \ldots \dfrac{22^2}{37}}$

$= \cancel{\dfrac{3}{}\ \dfrac{3}{37} = \dfrac{729}{37}}$.

$$\frac{59 \times 59^{72 \times 51}}{37}$$

$= \underline{\underline{22}}$.

$\underline{\underline{Ex}}$ $\left.\frac{6^{65^{46}}}{43}\right|_R =$

$\rightarrow \frac{6^{65 \times 46}}{43}$

$= \frac{6^{65 \times 42}}{43} \times \frac{6^{65 \times 4}}{43}$

$= 1 \times \frac{6^{260}}{43}$

$= 1 \times \frac{6^{257}}{43} \times \frac{6^{3}}{43}$

$= 1 \times 1 \times \frac{6^3}{43} \times \frac{6^3}{43}$

$$\frac{6^{65 \times 46}}{43}$$

$\left.\frac{6^{3K}}{43}\right|_R = 1$ $\qquad \frac{6^{3K+2}}{43} = 1 \times 36 = \underline{\underline{36}}$.

$$\frac{6^{65^{56}}}{43} \qquad \left.\frac{6^{3K}}{43}\right|_R = 1.$$

$\left.\frac{6^{3K+1}}{43}\right|_R = \underline{\underline{6}}$.

Ex $\left.\dfrac{20! + 20^{23}}{23}\right|_R =$

$\left.\dfrac{22!}{23}\right|_R = 1.$

$\dfrac{22\times 21}{23} \times \dfrac{20!}{23} = -1 / \underline{\underline{22}}$

$\Rightarrow \left.\dfrac{20!}{23}\right|_R = \underline{\underline{11}}$

$\dfrac{20^{22}\times 20}{23} = 1\times 20 = \underline{\underline{20}}$ $\therefore 20+11 = 31-23 = \underline{\underline{8}}.$

Ex $\left.\dfrac{10^{69}+786}{11}\right|_R$

$\cancel{\dfrac{10^{65}\times 100\times 100}{11}}$

$= \cancel{\left.\dfrac{10^{70}}{11}\right|_R} \cancel{= 1.}$ $= 10+5 = \underline{\underline{4}}.$

$\cancel{\dfrac{10}{11}\times\dfrac{10^{69}}{11}}$

Ex $\dfrac{10^{69}+786}{7}$

$\underline{\underline{3^3}} + 2$

$6+2 = \underline{\underline{1}}.$

Ex P is a +ve integer ≤ 100. Units digit of $3^P + P^3 = 0$. How many values does P have?

$3^P \rightarrow$ odd

$P^3 \rightarrow$ odd $\Rightarrow P \rightarrow$ odd

$$\begin{array}{ll} & \underline{3} \\ 3 & 1-1 \\ 9 & 3-7 \\ 7 & 5-5 \times \\ 1 & 7-3 \\ & 9-9. \end{array}$$

If $p \to 1, \underline{11}, \underline{21}, 31, 41, \ldots 91$

$\to 4k+2 \to$ 0 soln

$p \to 3, 13, 23, \ldots \underline{93}$

$4k+1 \to 13 \quad 33 \quad 53 \quad 73 \quad 93$

$p \to 7, 17, 27, \ldots 97$

$4k+3 \to 7, 27, 47, 67, 87.$

$p \to 9, 19, \ldots 99.$

$4k \to$ not possible.

Ans $\to \underline{\underline{10}}$.

Ex $Y = 40k+2.$

Find $\left.\dfrac{8^Y + 8Y}{5}\right|_R$

$\left.\dfrac{8^Y}{5}\right|_R = 3^2 = \underline{\underline{4}} \quad \dfrac{8Y}{5} = 1 \qquad \therefore 4+1 = 5 \quad$ Ans $\to \underline{\underline{0}}$

Ex $X = 40k+1$

$\left.\dfrac{2^X - 2X}{5}\right|_R =$

$\dfrac{2^X}{5} = 2^1 = 2 \qquad \dfrac{2X}{5} = -2 \qquad 2-2=0. \qquad$ Ans $\to \underline{\underline{0}}$.

Ex Find sum of coefficients of $(7+4x)^{90}$.

11^{90}.

Ex $\frac{48^{50}+50^{50}}{49}\Big|_R$

48^2+1

$=1+1=\underline{\underline{2}}$.

Ex $\frac{33333^{44444}+44444^{33333}}{7}\Big|_R$

6^2+1

$=1+1=\underline{\underline{2}}$.

Ex $\frac{169\times 144^{25}}{13^4}$

$169\times\left(\frac{144^{25}}{13^2}\right)$

$=169\times\left(\frac{12^{50}}{13^2}\right)$

$=169\times\frac{(13-1)^{50}}{13^2}$

$=169\times\left[\frac{{}^{50}C_{48}\times 13^{2}-{}^{50}C_{49}\times 13+{}^{50}C_{50}\times}{13^2}\right]$

$=169\times\left(\frac{-650+1}{169}\right)$

$=169\times\left(\frac{-649}{169}\right)$

$=169\times(-142)=169\times 27=4563$.

__/__/____

Ex If N is a natural No, remainder when $7^{6N+3}+2^{8N+4}-5$ is divided by 359.

$$\frac{7^{6N+3}+2^{8N+4}-5}{359}.$$

$$=\frac{7^{3(2N+1)}+2^{4(2N+1)}-5}{359}$$

$$=\frac{343^{2N+1}+16^{2N+1}-5}{359}$$

$$=\frac{359\times\_\_\_\_ \;-5}{359}$$

$$=354$$

Ex $\left.\frac{12^{13^{14}}}{145}\right|_R$ $\qquad 145=29\times5$

$$\frac{12^{182}}{145}.=\frac{144^{91}}{145}.$$

$$=\frac{{}^{91}C_{89}\times145^2+{}^{91}C_{90}\times145\mp{}^{91}C_{91}\times}{145}$$

$=$

Ex $\dfrac{3737^{77 \times 777!}}{73}\Big|_R$

$\underline{\underline{1}}$.

Ex $\dfrac{987987987\ldots\ldots 123 \text{ digits}}{1001}\Big|_R$

$\underline{\underline{987}}$.

Ex $\dfrac{2^{502} \times 5^{502}}{251}$

$= 4 \times 25 = \underline{\underline{100}}$.

Ex $\dfrac{13^{520}}{2249}\Big|_R$

$\dfrac{13^{520}}{13 \times 173}$

$= 13 \times \dfrac{13^{519}}{173}$

$= 13 \times (13^3)$

$= 13 \times 121 = \underline{\underline{1573}}$

Ex $\dfrac{7^{2000}}{50}\Big|_R$

$50 \times \frac{1}{2} \times \frac{4}{5}$

$= \underline{\underline{20}}$.

7^0

$= \underline{\underline{1}}$.

__/__/____

<u>Ex</u> $\left.\dfrac{787^{2009}}{25}\right|_R$

$\cancel{25\times(1-\frac{1}{5}) = 20}$.

$\dfrac{{}^{2009}C_{2009}\times 12^{2009}}{25}$

$= \dfrac{12^9}{25} = \dfrac{12^3}{25}\times\dfrac{12^3}{25}\times\dfrac{12^3}{25}$

$= \dfrac{--52}{25} = \underline{\underline{2}}$.

<u>Ex</u> $\left.\dfrac{874^{947}}{125}\right|_R$

$\dfrac{{}^{947}C_{947}(-1)^{947}}{125}$

$= \underline{124}$.

<u>Ex</u> $\left.\dfrac{19^{385}}{97}\right|_R = 19^1 = \underline{\underline{19}}$.

<u>Ex</u> $\left.\dfrac{18^{325}}{654}\right|_R$

$18\times\dfrac{18^{324}}{654}$

$= \cancel{6\times 9 \times 2\times}\ 6\times 3\times\dfrac{18^{324}}{\underline{\underline{109}}}$.

$= \cancel{6}\ \underline{\underline{18}}$.

Ex $\left.\dfrac{5^{9000}}{23}\right|_R$

$= 5^2 = 25-23 = 2.$

Ex $\left.\dfrac{17!}{17^2}\right|_R$

$17 \times \dfrac{16!}{17}$

$= 17\times 16 = \cancel{\ } \, 272.$

Ex $\left.\dfrac{35!}{37}\right|_R$

$\left.\dfrac{36!}{37}\right|_R = 36.$

$\Rightarrow \left.\dfrac{35!}{37}\times\dfrac{36}{37}\right|_R = 36.$

$\Rightarrow \left.\dfrac{35!}{37}\right|_R = 1$

Ex $\left.\dfrac{195!}{304}\right|_R =$

$\dfrac{195!}{197}$ $\qquad \dfrac{196!}{197} = 196$

$\Rightarrow \dfrac{196}{197}\times\dfrac{195!}{197} = 196$

$\Rightarrow \left.\dfrac{195!}{197}\right|_R = 1.$

as 195! is even

odd

$\dfrac{197K+1}{304}.$

$\therefore$ Ans $\rightarrow$ 198.

Ex $\dfrac{555555555555 + 5^{192}}{16}$ | R

$16 \times \frac{1}{2} = 8.$

$3 + 1 = 4.$

Ex $\dfrac{47^{100}}{100}$

$47^{20} = 01.$

Ex ~~If~~ x is an integer and y is one odd +ve integer.
If $3^y + x^2 = 2^4 5^4$, how many values does x have.

$3^y + x^2 = 2^4 \times 5^4 = 10,000.$

$x^2 \to$ odd

$y \to$ odd

3^y x^2

3

~~9~~

7

~~1~~

To bring Zeroes, x^2 must end with 3/7.

~~[illegible]~~. Not possible

Ans → 0.