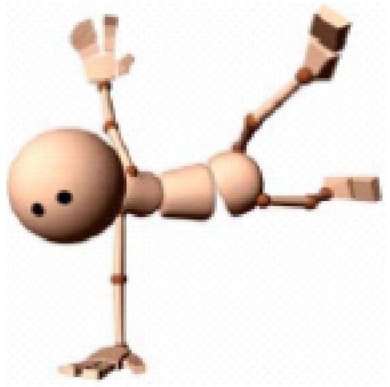
Mensuration- Solids
by Total Gadha - Saturday, 1 March 2008, 11:31 PM

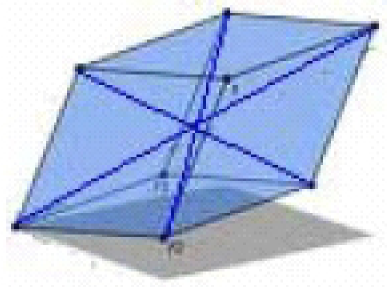


For nearly every CAT 2008 aspirant, the question of â€˜what to do and how to do itâ€™ is the most crucial question of all. Besieged by too many topics and pulled in too many directions, a student is dazed and intimidated by the seemingly mammoth task at hand. The trick to winning a long-drawn-out battle like CAT preparation is to get started first and think later. Catch any end of the rope you can lay your hands on and start pulling. Some helpful and cultivated good habits, such as making a timetable for the next day and practicing all the three sections everyday, will also go a long way in preparing you for the D-day. And if you are one of them who forget what they read a month ago, you can certainly move around in TG forums and keep solving problems all the time to keep you in shape! But the most important thing to do is to find that zeal and excitement for your preparations in place of that CAT fear.

Needless to say, CAT preparations will bring you one of the most memorable times of your life. Even with all that stress, those class exercises, those depressing mocks and the fierce competition, CAT preparation is so much fun.

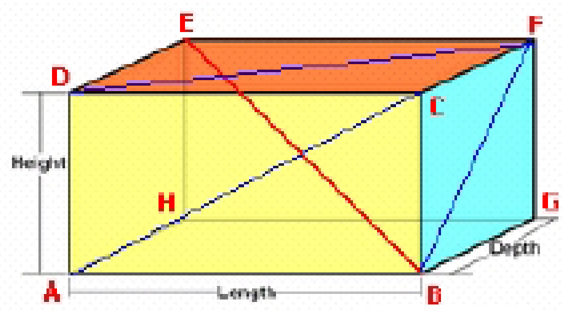
A solid figure, or **solid**, is any portion of space bounded by one or more surfaces, plane or curved. These surfaces are called the **faces** of the solid, and the intersections of adjacent faces are called **edges**. Let's have a look at some common solids and their properties.

Parallelepiped: A parallelepiped is a solid bounded by three pairs of parallel plane faces.



- Each of the six faces of a parallelepiped is a parallelogram.
- Opposite faces are congruent.
- The four diagonals of a parallelepiped are concurrent and bisect one another.

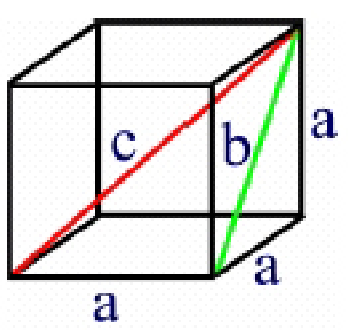
Cuboid: A parallelepiped whose faces are rectangular is called a cuboid. The three dimensions associated with a cuboid are its length, breadth and height (denoted as l, b and h here.)



- The length of the three pairs of face diagonals are $BF = \sqrt{b^2 + h^2}$, $AC = \sqrt{l^2 + h^2}$, and $DF = \sqrt{l^2 + b^2}$.
- The length of the four equal body diagonals $AF = \sqrt{l^2 + b^2 + h^2}$.
- The total surface area of the cuboid = 2(lb + bh + hl)
- Volume of a cuboid = lbh
- The radius of the sphere circumscribing the cuboid = $\frac{\text{Diagonal}}{2} = \frac{\sqrt{l^2 + b^2 + h^2}}{2}$.
- Note that if the dimensions of the cuboid are not equal, there cannot be a sphere which can be inscribed in it, i.e. a sphere which touches all the faces from inside.

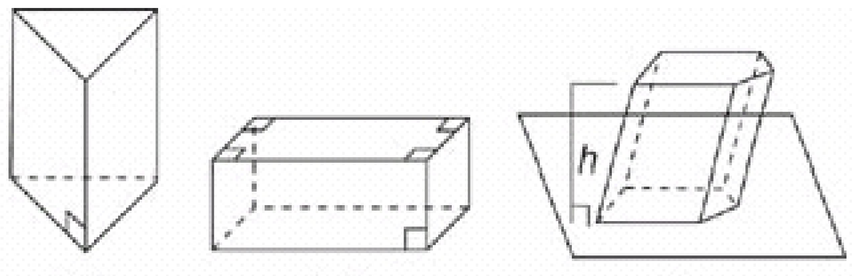
Euler's Formula: the number of faces (*F*), vertices (*V*), and edges (*E*) of a solid bound by plane faces are related by the formula *F* + *V* = *E* + 2 gives here 6 + 8 = 12 + 2.

Cube: A cube is a parallelepiped all of whose faces are squares.



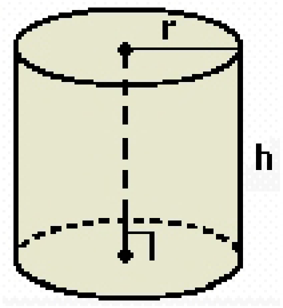
- Total surface area of the cube = $6a^2$
- Volume of the cube = a^3
- Length of the face diagonal b = $\sqrt{2}a$
- Length of the body diagonal c = $\sqrt{3}a$
- Radius of the circumscribed sphere = $\frac{\sqrt{3}a}{2}$
- Radius of the inscribed sphere = $\frac{a}{2}$
- Radius of the sphere tangent to edges = $\frac{a}{\sqrt{2}}$

Prism: A prism is a solid bounded by plane faces, of which two, called the ends, are congruent figures in parallel planes and the others, called side-faces are parallelograms. The ends of a prism may be triangles, quadrilaterals, or polygons of any number of sides.



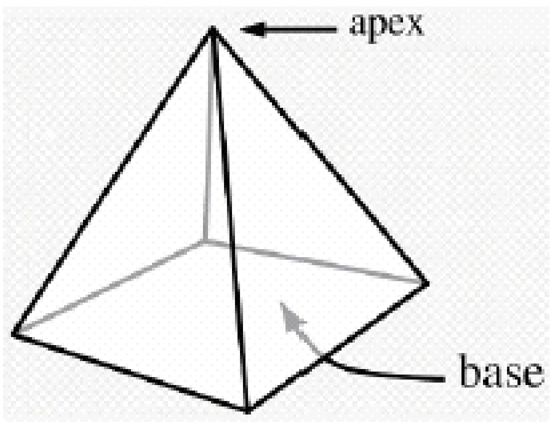
- The side- edges of every prism are all parallel and equal.
- A prism is said to be right, if the side-edges are perpendicular to the ends: In this case the side faces are rectangles. Cuboids and cubes are examples.
- Curved surface area of a right prism = perimeter of the base × height
- Total surface area of a right prism = perimeter of the base × height + 2 × area of the base
- Volume of a right prism = area of the base × height

Right Circular Cylinder: A right circular cylinder is a right prism whose base is a circle. In the figure given below, the cylinder has a base of radius r and a height of length h.



- Curved surface area of the cylinder = $2\pi rh$
- Total surface area of the cylinder = $2\pi rh + \pi r^2$
- Volume of the cylinder = $\pi r^2 h$

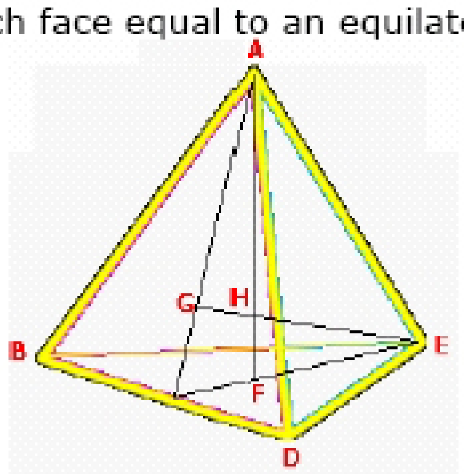
Pyramid: A pyramid is a solid bounded by plane faces, of which one, called the base, is any rectilinear figure, and the rest are triangles having a common vertex at some point not in the plane of the base. The slant height of a pyramid is the height of its triangular faces. The height of a pyramid is the length of the perpendicular dropped from the vertex to the base.



In a pyramid with n sided regular polygon as its base,

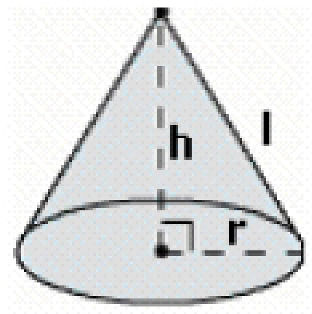
- Total number of vertices = n + 1
- Curved surface area of the pyramid = $\frac{\text{Perimeter}}{2} \times \text{slant height}$.
- Total surface area of the pyramid = $\frac{\text{Perimeter}}{2} \times \text{slant height} + \text{area of the base}$.
- Volume of the pyramid = $\frac{\text{Base area}}{3} \times \text{height}$

Tetrahedron: A tetrahedron is a pyramid which has four congruent equilateral triangles as it four faces. The figure below shows a tetrahedron with each face equal to an equilateral triangle of side a.



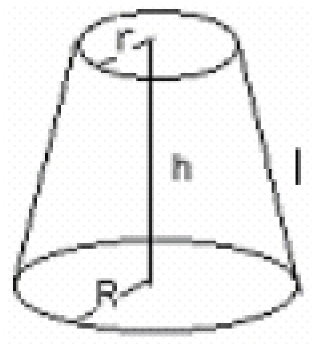
- Total number of vertices = 4
- The four lines which join the vertices of a tetrahedron to the centroids of the opposite faces meet at a point which divides them in the ratio 3: 1. In the figure. AH: HF = 3: 1.
- Curved surface area of the tetrahedron = $\frac{3\sqrt{3}a^2}{4}$.
- Total surface area of the tetrahedron = $\sqrt{3}a^2$.
- Height of the tetrahedron = $\frac{\sqrt{6}a}{3}$
- Volume of the tetrahedron = $\frac{\sqrt{2}a^3}{12}$

Right Circular Cone: a right circular cone is a pyramid whose base is a circle. In the figure given below, the right circular cone has a base of radius r and a height of length h.



- Slant height $l = \sqrt{h^2 + r^2}$
- Curved surface area of the cone = πrl
- Total surface area of the cone = $\pi rl + \pi r^2$
- Volume of the cone = $\frac{\pi r^2 h}{3}$

Frustum of a Cone: When a right circular cone is cut by a plane parallel to the base, the remaining portio known as the frustum.

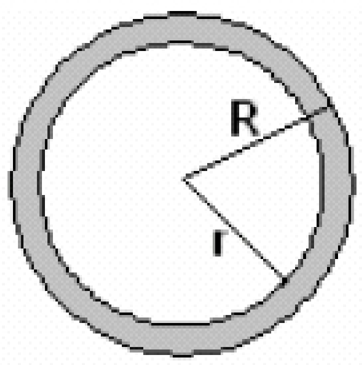


- Slant height $l = \sqrt{h^2 + (R - r)^2}$.
- Curved surface area of the frustum = $\pi(r + R)l$
- Total surface area = $\pi(r + R)l + \pi(r^2 + R^2)$
- Volume of the frustum = $\frac{\pi h(r^2 + R^2 + Rr)}{3}$

Sphere: A sphere is a set of all points in space which are at a fixed distance from a given point. The fixed point is called the centre of the sphere, and the fixed distance is the radius of the sphere.

- Surface area of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

Spherical Shell: A hollow shell with inner and outer radii of r and R, respectively.

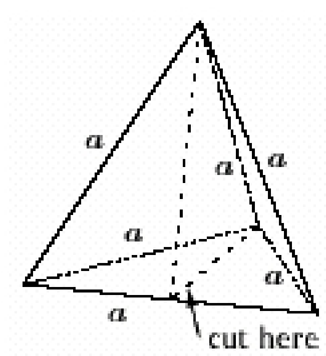


- Volume of the shell = $\frac{4}{3}\pi(R^3 - r^3)$

Given below are some teaser questions for you to try your hands on. Good luck!

1.. An hourglass is formed from two identical cones. Initially, the upper cone is filled with sand and the low one is empty. The sand flows at a constant rate from the upper cone to the lower cone. It takes exactly one hour to empty the upper cone. How long does it take for the depth of the sand in the lower cone to be half the depth of sand in the upper cone? (Assume that the sand stays level in both cones at all times)

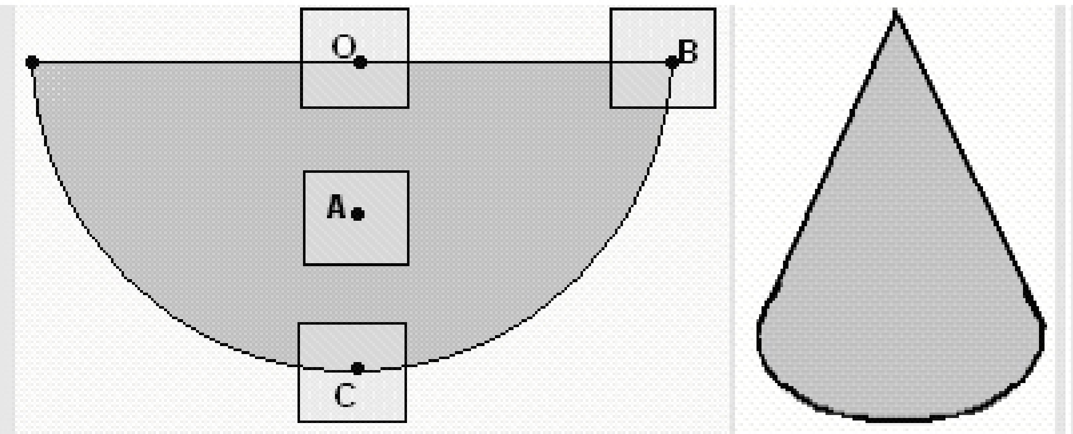
2 . The 6 edges of a regular tetrahedron are of length a. The tetrahedron is sliced along one of its edges to form two identical solids. Find the area of the slice.



3 . A sphere is inscribed in a cone whose radius and height are 12 and 16 units, respectively. Then, the volume of the sphere is

(a) 216π (b) 256π (c) 288π (d) 312π

A semi circular strip of paper, with radius 10 cm, has O as its centre, and A and C as the midpoints of OC and the semicircular part, respectively. The strip is rolled to form a perfect right circular cone without overlap of any two surfaces.



4 . Which of the four points shown forms the apex of the cone?

(a) A (b) B (c) C (d) O (e) None of these

5 . What is the volume of the cone?

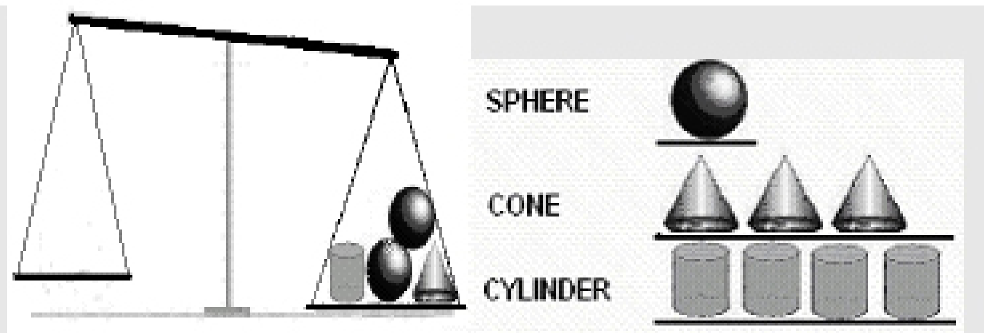
(a) $\frac{250\pi}{3}$

A. $\frac{125\pi}{\sqrt{3}}$

B. $\frac{1000\pi}{\sqrt{3}}$

C. $\frac{1000\pi}{3}$

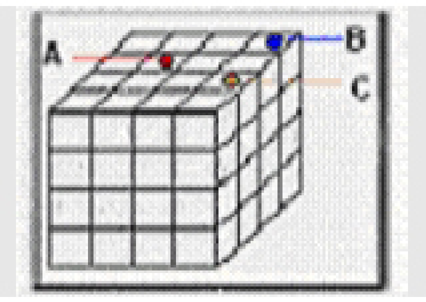
There are three types of solids, spheres, cones and cylinders. The diameter and heights of all the objects are equal. All the solids are made of the same material. Four of the objects, a cylinder, a cone, and two spheres are kept on one pan of a beam balance, as shown in the figure. To balance the beam, 8 solids of the same dimensions and material, a sphere, three cones and four cylinders, are available.



6 . In how many ways can you balance the beam?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

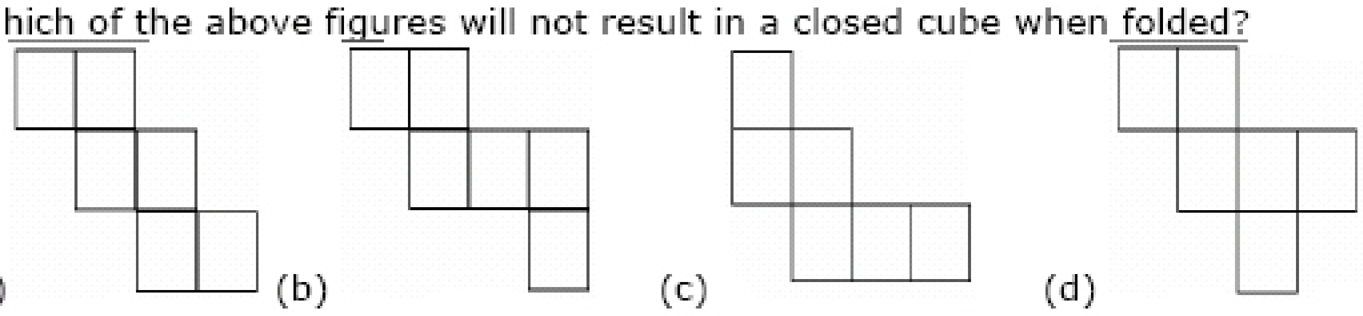
A large 4 cm × 4 cm × 4 cm cube is made up of 64 unit cubes, as shown in the figure. Three unit cubes, A, B, and C (also shown) are marked on the larger cube. The three cubes are taken out all three at a time.



7 . What is the area of the remaining solid?

(a) 100 (b) 101 (c) 102 (d) 104 (e) 105

8. Which of the above figures will not result in a closed cube when folded?



(a) (b) (c) (d)

9. One day sanjeev planned to make lemon tea and used a portion of the spherical lemon as shown in the figure. Find out the volume of the remaining lemon. Radius of the lemon is 6cm.