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If A.M. (Arithematic Mean) of AP is known, then

$$S_n = n \times AM$$

Reciprocal of HP is AP.

Ex $|x|<1$, Find $2+4x+6x^2+8x^3+\dots$

$$S = 2+4x+6x^2+8x^3+\dots$$

$$Sx = \quad 2x+4x^2+6x^3+\dots \qquad \text{~~to~~}\, n$$

$$\Rightarrow S(1-x) = 2 + 2x+2x^2+2x^3+\dots$$

$$\Rightarrow S = \cancel{2}\frac{1}{1-x}\left(2+\frac{2x}{1-x}\right)$$

$$= \frac{2}{(1-x)^2}.$$

Ex $S = 1+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\dots$ Find S.

$$S = 1+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\dots$$

$$\frac{S}{5} = \frac{1}{5}+\frac{2}{25}+\frac{4}{125}$$

$$\Rightarrow \frac{4S}{5} = 1+\frac{1}{5}+\frac{2}{25}+\frac{4}{125}+\frac{8}{5^4}$$

$$\frac{4S}{25} = \frac{1}{5}+\frac{1}{25}+\frac{2}{125}+\frac{4}{5^4}$$

$$S = \frac{1}{1-2/5} = 5/3.$$

Ex If $(1^3-t_1)+(2^3-t_2)+(3^3-t_3)+\dots+(n^3-t_n) = \frac{n^2(n-3)}{4}$.
Find t_n.

$$t_n = S_n - S_{n-1}$$

$$1^3+2^3+\dots+n^3-(t_1+t_2+\dots t_n) = \frac{n^2(n-3)}{4}$$

$$\Rightarrow S_n = \frac{n^2(n+1)^2}{4} - \frac{n^2(n-3)}{4} = \frac{n^2}{4}\left(n^2+2n+1-n+3\right)$$

$$= \frac{n^2}{4}\times(n^2+n+4) = \frac{n^4+n^3+4n^2}{4}$$

Then $t_n = S_n - S_{n-1}$

Ex Evaluate $\frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \cdots + \frac{1}{49\times50}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{48} - \frac{1}{49} + \frac{1}{49} - \frac{1}{50}$

$= \frac{1}{2} - \frac{1}{50} = \frac{24}{50} = \frac{12}{25}$

Ex $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \cdots + \sqrt{1+\frac{1}{10^2}+\frac{1}{11^2}}$

$\left(1+\frac{1}{1}-\frac{1}{2}\right) + \left(1+\frac{1}{2}-\frac{1}{3}\right) + \left(1+\frac{1}{3}-\frac{1}{4}\right) + \cdots \left(1+\frac{1}{9}-\frac{1}{10}\right) + \left(1+\frac{1}{10}-\frac{1}{11}\right)$

$= 10 + 1 - \frac{1}{11} = \frac{120}{11}$.

Ex $t_1 = 3$

$t_n = 2t_{n-1} + n$. Find t_{20}.

$t_1 = 3$

$t_2 = 8 \quad = 3\times2^2 - 2 - 2$

$t_3 = 19 \quad = 3\times2^3 - 3 - 2$

$t_4 = 42 \quad = 3\times2^4 - 4 - 2$.

$t_n = 3\times2^n - n - 2$.

$\therefore t_{20} = 3\times2^{20} - 22$.

Ex Find sum of $\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\dots+\frac{1+2+3+\dots+n}{1^3+2^3+\dots+n^3}$

$$t_n = \frac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{2}{n(n+1)} = 2\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$\therefore S_n = 2\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\dots+\frac{1}{n-2}-\frac{1}{n-1}+\frac{1}{n-1}-\frac{1}{n}+\frac{1}{n}-\frac{1}{n+1}\right]$$

$$\cancel{\Rightarrow 2\left[2-\frac{1}{n+1}\right]}$$

$$= 2\left(1-\frac{1}{n+1}\right)$$

$$= \frac{2n}{n+1}.$$

Ex What is the pdt. of first 9 terms of a GP having a total of 13 terms given that 5^{th} term is 2?

Ans → $2^9 = 512$.

Ex Every no. of an infinite GP of +ve terms is equal to m time the sum of no.s that follow it. What is r?

$$a = m \times \frac{ar}{1-r}$$

$$\Rightarrow a - ar = mar$$

$$\Rightarrow 1 = r + mr \Rightarrow r = \frac{1}{m+1}.$$

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Ex The sum of first n terms of an AP is P. Sum of first $2n$ terms is Q. If $Q = \frac{4}{3}P$. Find sum of first $3n$ terms.

$$\frac{n}{2}\left[2a+(n-1)d\right] = P$$

$$\frac{2n}{2}\left[2a+(2n-1)d\right] = Q.$$

$$Q = \frac{4P}{3}$$

$$\Rightarrow \frac{2n}{2}\left[2a+(2n-1)d\right] = \frac{4}{3}\times\frac{n}{2}\times\left[2a+(n-1)d\right]$$

$$\Rightarrow \left[4a+(2n-1)3d\right]\times 3 = 8a+(n-1)4d.$$

$\Rightarrow$ ~~12a+~~

Sum of first P terms P_1, next P terms P_2 & 3^{rd} set P_3.

Sum of first n terms, next n terms & last n terms are in AP.

$$P_1 = P$$

$$P_1+P_2 = \frac{4P}{3} \Rightarrow P_2 = \frac{P}{3}.$$

Imp. $\Rightarrow$ || $P_1 \quad P_2 \quad P_3 \rightarrow$ A.P.

$$P+P_3 = \frac{2P}{3} \Rightarrow P_3 = -\frac{P}{3}.$$

$$\therefore \text{Ans} \rightarrow P+\frac{P}{3}-\frac{P}{3} = 0.$$

Ex X is the set of first 1990 natural no.s. A is an AP having at least 3 elements. First element is the least element of X and its last element is the greatest element of X. How many possibilities does A have?

$$1990-1 = 1989.$$

$1989 = 3\times3\times13\times17$ no. of factors $\rightarrow 12$ $\therefore$ Ans

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But Common difference can't be 1989.

$\therefore$ Ans $\rightarrow$ 11.

Ex $a_n = 1$, if $n = 0$.

$a_n = a_{n-1}$ if $n = 3k$ $k \rightarrow Z^+$

$a_n = 3a_{n-1}$ if $n = 3k+1$ Find a_{50}.

$a_n = 2a_{n-1}$ if $n = 3k+2$.

$a_0 = 1$

$a_1 = 3 \times 1 = 3$ $a_{50} = 2^{17} \times 3^{17} = 6^{17}$.

$a_2 = 2 \times 3 = 6$

$a_3 = a_2 = 6. = 2 \times 3$

Ex There are 4 distinct no.s such that first and last term are equal. First 3 are in AP. Last 3 are in G.P. Find r of GP.

$a-d \quad a \quad a+d \quad a-d$

$a(a-d) = (a+d)^2$

$\Rightarrow a^2 - ad = a^2 + d^2 + 2ad$ $r = \dfrac{a+d}{a}$

$\Rightarrow d^2 = -3ad$

$\Rightarrow d = -3a$. $= \dfrac{-2a}{a} = -2$.

Ex Find sum of first 75 terms of $150 \times 2 + 148 \times 4 + 146 \times 6 + \ldots$

$t_n = (152 - 2n) \times 2n$

$= 304n - 4n^2$.

$S_{75} = 304 \times \dfrac{75 \times 76}{2} - 4 \times \dfrac{75 \times 76 \times 151}{6}$.

Ex $50^2-49^2+48^2-47^2+\cdots\quad -3^2+2^2=$

$99\times1+95\times1+\cdots\quad 7\times1+4$ $\qquad 99=7+(n-1)4$

$=4+\frac{24}{2}(99+7)$ $\qquad \Rightarrow n=24$

Ex $S_1=1(30)+2(29)+3(28)+\cdots+30(1)$

$S_2=1(60)+2(59)+3(58)+\cdots+60(1)$

Find $2\frac{S_2}{S_1}$.

$S_1\to n(31-n)=31n-n^2$ $\qquad$ Find & solve.

$S_2\to n(61-n)=61n-n^2$.

Ex Find sum of first 20 terms of

$1^2\times3+2^2\times4+3^2\times5+4^2\times6+\cdots$

$t_n=n^2(n+2)$

$=n^3+2n^2$.

Ex $\frac{1}{3^2-2^2}+\frac{1}{7^2-2^2}+\frac{1}{11^2-2^2}+\cdots+\frac{1}{39^2-2^2}$

$\frac{1}{5\times1}+\frac{1}{9\times5}+\frac{1}{13\times9}+\cdots+\frac{1}{41\times37}$

$=\frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{9}+\frac{1}{9}-\frac{1}{13}+\cdots+\frac{1}{33}-\frac{1}{37}+\frac{1}{37}-\frac{1}{41}\right]$

$=\frac{1}{4}\left(1-\frac{1}{41}\right)$

$=10/41$.

Ex $X=\frac{1}{90\times46}+\frac{1}{89\times47}+\cdots+\frac{1}{46\times90}$

$Y=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots-\frac{1}{90}$. Find X/Y

$$X = \frac{2}{136}\left[\frac{1}{46}+\frac{1}{47}+\dots+\frac{1}{90}\right]$$

$$Y = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots-\frac{1}{90}$$

$$= \left(1+\frac{1}{2}+\dots+\frac{1}{90}\right)-2\times\left(\frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{90}\right)$$

$$= \left(1+\frac{1}{2}+\dots+\frac{1}{90}\right)-\left(1+\frac{1}{2}+\dots\frac{1}{45}\right)$$

$$= \frac{1}{46}+\frac{1}{47}+\dots+\frac{1}{90}$$

$$\therefore \frac{X}{Y} = \frac{1}{68}$$

Ex $\frac{1}{2\cdot5\cdot8}+\frac{1}{5\cdot8\cdot11}+\frac{1}{8\cdot11\cdot14}+\dots$ upto 15 terms

$$\frac{1}{2d^2}\left[\frac{1}{a}-\frac{2}{a+d}+\frac{1}{a+2d}\right]$$

$2+(15-1)\times3$
$=2+42$
$=\underline{\underline{44}}$

$$\therefore \frac{1}{18}\left[\frac{1}{2}-\frac{2}{5}+\frac{1}{8}+\frac{1}{5}-\frac{2}{8}+\frac{1}{11}+\frac{1}{8}-\frac{2}{11}+\frac{1}{14}\right.$$
$$\left.+\dots\dots+\frac{1}{44}-\frac{2}{47}+\frac{1}{50}\right]$$

$$= \frac{1}{18}\left(\frac{1}{2}-\frac{1}{5}-\frac{1}{47}+\frac{1}{50}\right) = \frac{39}{2350}$$

Ex Find sum of $\frac{1}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\dots\frac{n^2}{2^n}$.

$$S = \frac{1}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\frac{4^2}{2^4}$$

$$\frac{S}{2} = \frac{1}{2^2}+\frac{2^2}{2^3}+\frac{3^2}{2^4}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\frac{9}{2^5}$$

$$\frac{S}{4} = \frac{1}{2^2} + \frac{3}{2^3} + \frac{5}{2^4} + \frac{7}{2^5}$$

$$\Rightarrow \frac{S}{2} - \frac{S}{4} = \frac{S}{4} = \frac{1}{2} + \frac{2}{2^2} + \frac{2}{2^3} + \frac{2}{2^4} + \frac{2}{2^5}$$

$$\Rightarrow \frac{S}{4} = \frac{1}{2} + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\right]$$

$$= \frac{1}{2} + \frac{1/2}{1-1/2}\Big]$$

$$\Rightarrow \frac{S}{4} = 3/2 \quad \Rightarrow S = \underline{\underline{6}}.$$

Ex Find sum of

$$\frac{1}{1+2^{1/3}+2^{2/3}} + \frac{1}{2^{2/3}+6^{1/3}+3^{2/3}} + \frac{1}{3^{2/3}+12^{1/3}+4^{2/3}} + \cdots \frac{1}{63^{2/3}+4032^{1/3}+64^{2/3}}$$

$$2^{1/3}-1^{1/3}+3^{1/3}-2^{1/3}+4^{1/3}-3^{1/3}+\cdots+63^{1/3}-62^{1/3}+64^{1/3}-63^{1/3}$$

$= 4-1 = 3.$

Ex $t_n = t_{n-1} + 2n - 1 \quad n \geq 2$

$t_1 = 1.$

Find t_{50}.

$t_1 = 1$

$t_2 = 1 + 4 - 1 = 4$

$t_3 = 4 + 2\times3 - 1 = 9$

$t_4 = 9 + 2\times4 - 1 = 16.$

$t_5 = 16 + 2\times5 - 1 = 25$

$t_n = n^2 \quad n \geq 2.$

$\therefore t_{50} = 2500.$

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Ex Find sum of squares of first 12 terms of the AP for which sum of first n terms is $3n^2+6n$.

$t_n = S_n - S_{n-1}$
$= 3n^2+6n - [3(n-1)^2 + 6(n-1)]$
$= 3n^2+6n - [3n^2-6n+3+6n-6]$
$= 3n^2+6n - [3n^2-3]$
$= \underline{\underline{6n+3}}$

$9, \cancel{15}, 21 \dots$

$t_n^2 = 36n^2+9+36n$
$= 9(4n^2+4n+1)$. Find..

Ex 10^{th} term of an AP is ~~minimum~~ 15. If sum of squares of 7^{th}, 10^{th} & 13^{th} term is minimum. Find C.D.

$15-3d \quad 15 \quad 15+3d$

$(15-3d)^2 + (15+3d)^2 + 15^2$.
$= 3\times 15^2 + \underline{\underline{18d^2}}$ Min when $d=0$. $\therefore$ Ans $\to 0$.

Ex 3 terms $a+2b, b+2c, c+2a$ are in A.P.
$16a, 2(b+c), c$ are in G.P.
Then, $\frac{C.D.}{r} = ?$ (i) -2 (b) 4 (c) 8 (d) 0.

$a+2b+c+2a = 2b+4c$
$\Rightarrow 3a+2b+c = 2b+4c$
$\Rightarrow 3a = 3c \Rightarrow a = c$.

$16a \times \overset{c}{a} = [2(b+a)]^2$.
$\Rightarrow 16a^2 = 4\times(a^2+b^2+2ab)$
$\Rightarrow 4a^2 = a^2+b^2+2ab$.
$\Rightarrow 3a^2 = b^2+2ab$. $\Rightarrow b^2+2ab-3a^2 = 0$.
$\Rightarrow b^2+3ab-ab-3a^2$
$\Rightarrow (b-a)(b+3a)$
$\therefore b=a$ or $-3a$.

C.D. $= b+2c-a-2b$
$= \cancel{b+a}\ b+a-2b$
$= a-b$.

$r = \frac{2(b+c)}{16a} = \frac{a+b}{8a}$.

$\frac{C.D.}{r} = \frac{(a-b)\times 8a}{(a+b)} = \cancel{8a}\ 8a \times \frac{a-b}{a+b}$.

Ans $\to 0$ when $a=b$.

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Ex Find sum of $-7, -25, -61, -121, \dots -3361$.

$$S = -(7+25+61+121+\dots+3361)$$

$$S = -7 -25 -61 -121$$

$$S = \quad -7 \quad -25 \quad -61 \qquad \cancel{\ } \dots t_n$$

$$\Rightarrow 0 = -7 -18 -36 -60$$

$$S = -(7+25+61+\dots+3361)$$

$$= -(2^3-1 + 3^3-2+4^3-3+\dots+15^3-14).$$

$\Downarrow$

Solve.

Ex Find sum of first 15 terms of

$$\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\frac{9}{400}$$

$$S_n = \left(\frac{1}{1^2}-\frac{1}{2^2}\right)+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots+\frac{1}{15^2}-\frac{1}{16^2}$$

$$= 1-\frac{1}{256} = \frac{255}{256}.$$

Ex Find sum of 1^{st} 25 terms

$$S_n = 3+12+25+42+63+88+\dots$$

$$S_n = 3+12+25+42+63+88+\dots$$

$$S_n = \qquad 3+12+25+42+63+\dots$$

$$t_n = 3+9+13+17+21+\dots(n-1)_{term}$$

$$= 3+\cancel{9}\ \frac{n-1}{2}\left[18+(n-2)4\right]$$

$$= 3+\frac{n-1}{2}\times(4n+10) \quad = 3+\frac{n-1}{2}\times(2n+5) =$$

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$$\cancel{= 2n^2 - n + 2}$$

$$= 2n^2 + 3n - 2.$$

Solve

Ex $\frac{1}{1\cdot2\cdot3\cdot4} + \frac{1}{2\cdot3\cdot4\cdot5} + \cdots\cdots + \frac{1}{15\cdot16\cdot17\cdot18} = ?$

$$\frac{1}{6}\left[1 - \frac{3}{2} + \frac{3}{3} - \frac{1}{4} + \frac{1}{2} - \frac{3}{3} + \frac{3}{4} - \frac{1}{5} + \cdots\cdots + \frac{1}{15} - \frac{3}{16} + \frac{3}{17} - \frac{1}{18}\right]$$

$$= \frac{1}{6}\left[1 - \frac{2}{2} + \frac{1}{3} + \frac{1}{16} + \frac{2}{17} - \frac{1}{18}\right]$$

$$= \frac{1}{6}\left[\frac{1}{3} + \frac{1}{16} + \frac{2}{17} - \frac{1}{18}\right]$$

$$= \frac{1}{6} \times \frac{816 + 153 + 288 - 136}{2448}$$

$$= \frac{815}{14688}.$$

Ex $1 + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots\cdots + \frac{1^3+2^3+3^3+\cdots+10^3}{1+3+5+\cdots+19} = ?$

$$t_n = \frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n}{2}\left[2+(n-1)2\right]} = \frac{n^2 \times (n+1)^2}{4} \div n^2 = \frac{(n+1)^2}{4} = \frac{1}{4}(n^2+2n+1).$$

Solve.

Ex Find sum of first 15 terms

$3 + 5 + 9 + 15 + 23 + 33 + \cdots$

$S_n = 3 + 5 + 9 + 15 + 23 + 33 + \cdots$

$S_n = 3 + 5 + 9 + 15 + 23 + \cdots + t_n$

$\Rightarrow t_n = 3 + 2 + 4 + 6 + 8 + 10 + \cdots (n-1)\text{ term}$

$\Rightarrow t_n = 3 + \frac{n-1}{2}\left[4 + (n-2)2\right] = 3 + \frac{n-1}{2} \times 2n = 3 + n^2 - n.$

Ex $t_n = \left(1 + \frac{1}{n^2-1}\right) t_{n-1} \quad n > 1$

$t_1 = 1.$

Find $t_{12} - t_3$.

$t_1 = 1$

$t_2 = 4/3$

$t_3 = \frac{9}{8} \times \frac{4}{3} = \frac{3}{2}.$

$t_n = \frac{2 \times n}{n+1} \times t_1$

$\therefore t_{12} - t_3 = \frac{24}{13} - \frac{6}{4}$

Ex If $T_n = 2T_{n-1} + 1$ for $n \geq 2$. $T_1 = 5$. Find T_{100}.

$T_1 = 5$

$T_2 = 11$

$T_3 = 23.$

$T_n = 3 \times 2^n - 1$

$T_{100} = 3 \times 2^{100} - 1.$

Ex $\frac{\sqrt{2}}{2+\sqrt{2}} + \frac{\sqrt{3}}{3+\sqrt{6}} + \cdots + \frac{\sqrt{16}}{\sqrt{16}+\sqrt{240}}$

$\frac{\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{2}(2-\sqrt{2})}{4-2} = \frac{2-\sqrt{2}}{\sqrt{2}} = \sqrt{2}-1.$

$\frac{\sqrt{3}(3-\sqrt{6})}{3} = \sqrt{3}-\sqrt{2}.$

$\therefore \sqrt{2}-1+\sqrt{3}-\sqrt{2}+\cdots+\sqrt{16}-\sqrt{15}$

$= 4-1 = 3.$

A G.P. has 9 terms. 5^{th} term is 2.

Ex ~~Sum of 9 terms of a GP is 255~~. Find product of all terms.

$$GM = \cancel{\quad} \cancel{\frac{a(r^9-1)}{r-1}} = \cancel{255} \quad (a^9 r^{36})^{1/9}$$
$$= ar^4 = 5^{th} \text{ term}$$

$$Pdt. = G.M.^9$$

$$\boxed{(GM \text{ of } GP)^n = Pdt \text{ of terms of } GP}$$

Ex Ratio of sum of first 11 terms of 2 APs with +ve C.D. is equal to ratio of sum of first 21 terms. What is the ratio of 3^{rd} terms of 2 APs?

(i) 4^{th} term of 2 APs are 1 & 2 respectively

(ii) 6^{th} term of 2^{nd} AP is 0.

$$\frac{2a_1+10d_1}{2a_2+10d_2} = \frac{2a_1+20d_1}{2a_2+20d_2}$$

$$\Rightarrow \frac{a_1+5d_1}{a_2+5d_2} = \frac{a_1+10d_1}{a_2+10d_2}$$

$$\Rightarrow \cancel{a_1a_2} + 10a_1d_2 + 5a_2d_1 + \cancel{50d_1d_2} = \cancel{a_1a_2} + 10a_2d_1 + 5a_1d_2 + \cancel{50d_1d_2}$$

$$\Rightarrow 5a_1d_2 = 5a_2d_1$$

$$\Rightarrow a_1d_2 = a_2d_1 \quad \Rightarrow \frac{a_1}{a_2} = \frac{d_1}{d_2} = k.$$

$$\frac{a_1+3d_1}{a_2+3d_2} = 1/2$$

$$\text{Using (i)} \Rightarrow \frac{ka_2+3kd_2}{a_2+3d_2} = 1/2. \quad \Rightarrow k = 1/2.$$

$$\therefore a_1 = 2a_2$$
$$d_1 = 2d_2$$

$$\Rightarrow 2ka_2 + 6kd_2 = a_2 + 3d_2 \quad \Rightarrow (2k-1)a_2 = (3-6k)d_2.$$

$$\Rightarrow a_2 = -3d_2. \quad \Rightarrow a_1 = -3d_1.$$

$$\text{Then, } \frac{a_1+2d_1}{a_2+2d_2}$$

$$= \frac{Ka_2+2Kd_2}{a_2+2d_2}$$

$$= k = \frac{1}{2}. \qquad \text{So ratio is either } \frac{1}{2} \text{ or } \frac{0}{0}.$$

Using (ii) $a_2+5d_2=0.$

$$\Rightarrow a_2 = -5d_2.$$

As 6^{th} term is 0 & C.D.>0, 3^{rd} term can't be 0.

Both statements needed.

Ex

$$X = 1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots+\frac{1}{80^2}$$

$$Y = 1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{160^2}.$$

Find $\frac{1}{2^2}+\frac{1}{6^2}+\frac{1}{10^2}+\cdots+\frac{1}{318^2}.$

$$\frac{1}{2^2}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots+\frac{1}{159^2}\right)$$

$$= \frac{1}{4}\left[\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\frac{1}{160^2}\right)-\left(\cancel{1}\,\frac{1}{2^2}+\frac{1}{4^2}+\cdots+\frac{1}{160^2}\right]\right.$$

$$= \frac{1}{4}\left[Y-\frac{1}{4}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\frac{1}{80^2}\right]\right]$$

$$= \frac{1}{4}\left(Y-\frac{X}{4}\right) = \frac{4Y-X}{16}.$$