DOUBLEROOT

Cheat Sheet – Trigonometry: Identities

Basic

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

Compound Angles

$$sin(A + B) = sin A cos B + cos A sin B$$

$$sin(A - B) = sin A cos B - cos A sin B$$

$$cos(A + B) = cosA cosB - sinA sinB$$

$$cos(A - B) = cosA cosB + sinA sinB$$

$$tan(A + B) = \frac{tan A + tan B}{1 - tan A tan B}$$

$$tan(A - B) = \frac{tan A - tan B}{1 + tan A tan B}$$

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Sum to Product

$$\sin C + \sin D = 2\sin \frac{C+D}{2}\cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2\sin \frac{C - D}{2}\cos \frac{C + D}{2}$$

$$\cos C + \cos D = 2\cos \frac{C+D}{2}\cos \frac{C-D}{2}$$

$$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$$

Product to Sum

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A - B) - \cos(A + B)$$

Multiple Angles

$$\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$=\cos^2\theta - \sin^2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

$$\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$$

$$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Sub-Multiple Angles

$$\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$$
$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$$

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

Misc

$$\sqrt{1+\sin 2\theta} = |\sin \theta + \cos \theta|$$

$$\sqrt{1-\sin 2\theta} = |\sin \theta - \cos \theta|$$

$$\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$$

$$\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$$

$$\cos \theta + \sin \theta = \sqrt{2} \sin \left(\frac{\pi}{4} + \theta\right) = \sqrt{2} \cos \left(\frac{\pi}{4} - \theta\right)$$

$$\cos \theta - \sin \theta = \sqrt{2} \sin \left(\frac{\pi}{4} - \theta \right) = \sqrt{2} \cos \left(\frac{\pi}{4} + \theta \right)$$

$$\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$