



## Percentage and its applications

by Total Gadha - Wednesday, 4 April 2007, 07:31 PM

Your competence with percentages and its application should form a very important part of your armory of CAT preparation or, for that matter, preparation for any other MBA exam. The concept of percentage will be applied in not only your quant section but also your data interpretation (DI) section. Therefore, master this important concept and make it your habit to calculate percentages mentally.

The word "percentage" literally means "per hundred" or "for every hundred." Therefore, whenever you calculate something as a part of 100, that part is numerically termed as percentage.

In other words, percentage is a ratio whose second term is equal to 100. For example, 1:4 can be written as 25: 100 or 25%, 3: 8 can be written as 37.5: 100 or 37.5%, 3: 2 can be written as 150: 100 or 150%, and so on.

### IMPORTANT CONCEPTS ASSOCIATED WITH PERCENTAGE

#### Basic formula of percentage:

p% of a number N is  $= N \times \frac{p}{100}$ .

#### Example:

1. What is 45% of 500?

Answer: 45% of 500  $= 500 \times \frac{45}{100} = 225$

2. What is 20% of 50% of 60% of 200?

Answer: Required percentage  $\left( \left( \left( 200 \times \frac{60}{100} \right) \times \frac{50}{100} \right) \right) \times \frac{20}{100} = 12$

#### Percentage of:

To calculate what percentage of a is b, use the formula  $\text{Percentage} = \frac{b}{a} \times 100$ . In short, if you ask yourself, "of what are we calculating percentage?" you will get the answer "of a." Therefore, a will form the whole and b will form the part of the whole.

#### Example:

1. What percentage of 240 is 90?

Answer: Percentage  $= \frac{90}{240} \times 100 = 37.5\%$

2. What percentage of 75 is 125?

Answer: Percentage  $= \frac{125}{75} \times 100 = 166.66\%$

#### Percentage increase/decrease:

Percentage increase/decrease when a quantity a increase/decreases to become another quantity b is

$$\text{Percentage Increase / Decrease} = \frac{\text{Increase / Decrease}}{\text{Initial Value}} \times 100 = \begin{cases} \frac{b-a}{a} \times 100 & \text{when } b > a \text{ (increase)} \\ \frac{a-b}{a} \times 100 & \text{when } a < b \text{ (decrease)} \end{cases}$$

$$\text{Therefore new quantity } b = \begin{cases} a \left( 1 + \frac{\text{percentage increase}}{100} \right) \\ a \left( 1 - \frac{\text{percentage decrease}}{100} \right) \end{cases}$$

#### Example:

1. A dealer buys products for Rs80 and hikes the price to Rs125. He sells it to the customer after giving a discount of Rs5. Find his profit percentage.

Profit percentage = Percentage Increase/Decrease in his income

$$= \frac{\text{Profit}}{\text{Cost Price}} \times 100 = \frac{120 - 80}{80} \times 100 = 50\%$$

2. A dealer sells goods priced at Rs180 after giving a discount of 25%. Find his selling price.

$$\text{Selling price after discount} = 180 \left( 1 - \frac{25}{100} \right) = 135$$

#### Percentage less than/greater than:

Have a look at the picture given below:



You can see that Johnny is taller than Vicky. What will your answer be if I ask

- (a) By what percentage is Johnny taller than Vicky?
- (b) By what percentage is Vicky shorter than Johnny?

Answer:

(a) We are looking at Johnny from the viewpoint of Vicky (taller than Vicky), therefore in the denominator we will have Vicky's height. Therefore, the percentage by which Johnny is taller than Vicky is equal

$$\text{to } \frac{\text{Johnny's height} - \text{Vicky's height}}{\text{Vicky's height}} \times 100.$$

(b) We are looking at Vicky from Johnny's point of view (shorter than Johnny), therefore in the denominator we will have Johnny's height. Therefore, the percentage by which Vicky is shorter than Johnny is equal to  $\frac{\text{Johnny's height} - \text{Vicky's height}}{\text{Johnny's height}} \times 100$ .

1. Malaika Arora's income is 25% more than John Abraham's income. By what percentage is John's salary less than Malaika's?

Answer: Let John's income be Rs100 (of course some women will protest that he is worth more than Rs100). Then Malaika's salary = Rs125. Now to find out by what percentage is John's salary less than Malaika's, we look at John's salary from Malaika's side.

Therefore, John's salary is  $\frac{\text{Malaika's salary} - \text{John's salary}}{\text{Malaika's Salary}} \times 100 = \frac{25}{125} \times 100 = 20\%$  less than Malaika's.

### Conversion of fractions into percentages:

Knowing conversion of common fractions into percentages helps you convert many fractions into percentage immediately. For example, knowing that  $\frac{1}{8} = 12.5\%$  will help you convert fractions like

$\frac{3}{8}$  or  $\frac{5}{8}$  into percentages immediately.

In DI, if I need to convert fraction like  $\frac{3132}{8214}$  into percentage, I will immediately realize that I can write it as

approximately equal to  $\frac{3000}{8000} = \frac{3}{8} = 37.5\%$ . The actual value of the fraction is 38.1%. Realize how close I

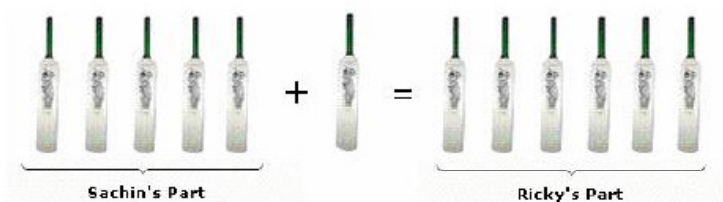
can get without moving a muscle and saving precious time. Given below are the fractions converted into percentage.

Fraction	Percentage	Fraction	Percentage	Fraction	Percentage
$\frac{1}{2}$	50%	$\frac{1}{10}$	10%	$\frac{1}{18}$	5.55%
$\frac{1}{3}$	33.33%	$\frac{1}{11}$	9.09%	$\frac{1}{19}$	5.26%
$\frac{1}{4}$	25%	$\frac{1}{12}$	8.33%	$\frac{1}{20}$	5%
$\frac{1}{5}$	20%	$\frac{1}{13}$	7.69%	$\frac{1}{21}$	4.76%
$\frac{1}{6}$	16.66%	$\frac{1}{14}$	7.14%	$\frac{1}{22}$	4.54%
$\frac{1}{7}$	14.28%	$\frac{1}{15}$	6.66%	$\frac{1}{23}$	4.34%
$\frac{1}{8}$	12.5%	$\frac{1}{16}$	6.25%	$\frac{1}{24}$	4.16%
$\frac{1}{9}$	11.11%	$\frac{1}{17}$	5.88%	$\frac{1}{25}$	4%

1. The salary of Ricky Ponting is 20% more than that of Sachin Tendulkar. By what percentage is Sachin's salary less than that of Ricky's?

Answer: it is similar to the earlier question on incomes. But let's solve it in a different way.

$20\% = \frac{1}{5}$ , so if Sachin's salary is divided into 5 parts, Ricky's salary will one more part added to Sachin's salary, i.e.



Therefore, if Sachin's salary is 5 parts, Ricky's salary is 6 parts. Now if Ricky's salary is 6 parts,

Sachin's salary is one part less. Therefore, for Ricky, Sachin's salary is  $\frac{1}{6}$  less or 16.66% less.

2. TCS sells its products 25% cheaper than those of Infosys and 25% dearer than those of HCL. By what percentage are HCL products cheaper than those of Infosys?

Answer:  $25\% = \frac{1}{4}$ . Therefore, if HCL's price is made of 4 parts, TCS' price is made of one part more, i.e. 5 parts. Also, If Infosys's price is made of 4 parts, TCS's price is made of one part less, i.e. 3 parts. To equate the parts of TCS in both cases, we take LCM of 3 and 5, i.e. 15.

If HCL's part is made of 12 parts, TCS's price is made of 15 parts.

If Infosys' price is made of 20 parts, TCS' part is made of 15 parts.

Therefore, HCL's price is 8 parts less in Infosys' 20 parts. 8 parts out of 20 is 40%. Therefore, HCL's products are priced 40% lower than those of Infosys.

3. A ball drops from a height of 4802 m. Thereafter, it bounces every time to a height which is 14.28% less than its previous height. What height will the ball reach on its 4<sup>th</sup> bounce?

Answer: I wonder how many of you will notice that  $14.28\% = \frac{1}{7}$ . Therefore, the ball is rising up

to a height which is  $\frac{1}{7}$ <sup>th</sup> less than the previous height. Or, the ball is rising up to a height which is  $\frac{6}{7}$  of the previous height. Therefore on its 4<sup>th</sup> bounce the ball will reach a height of  $4802 \times \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} = 2592\text{m}$

#### Multiplication factors for percentage increase/decrease:

We saw in percentage increase/ decrease that new quantity  $b = \begin{cases} a(1 + \frac{\text{percentage increase}}{100}) \\ a(1 - \frac{\text{percentage decrease}}{100}) \end{cases}$

For example, for a 20% increase the new quantity = old quantity  $(1 + \frac{20}{100}) = \text{old quantity} \times 1.2$ .

Therefore, to find the final quantity after a 20% increase, we can directly multiply the old quantity by a factor of 1.2 and get the new quantity. Similarly, for a 20% decrease, we can multiply the old quantity by 0.8 and get the new quantity. The factors to be multiplied for various percentage increase/decrease are given below:

Percentage Increase	New quantity = Old quantity multiplied by a factor of	Percentage Decrease	New quantity = Old quantity multiplied by a factor of
5%	1.05	5%	0.95
10%	1.1	10%	0.9
15%	1.15	15%	0.85
20%	1.2	20%	0.8
25%	1.25	25%	0.75
30%	1.3	30%	0.7
40%	1.4	40%	0.6
50%	1.5	50%	0.5
60%	1.6	60%	0.4

The biggest advantage of using the factors is that for subsequent percentage increase/decrease, we just keep on multiplying the corresponding factors and get the final quantity.

#### Example:

- The performance bonus of a salesman increases by 10% in the first year, by 20% in the second year, and by 30% in the third year. What is the overall percentage increase in his bonus in 3 years?

Answers: Let the bonus at the start of the first year be Rs100.

Therefore, to find the final bonus we just multiply by factors.

Final bonus =  $100 \times 1.1 \times 1.2 \times 1.3 = 171.6$ .

Therefore, overall percentage increase = 71.6%

**NOTE:** note that taking initial value of 100 makes the problem simpler; whatever increase we get is directly equal to the percentage increase.

- An amount was first reduced by 10% and then further reduced by 20% and Rs10 800 were left. What was the original amount?

Answer: Let the original amount be A.

Therefore  $A \times 0.9$  (factor for 10% decrease)  $\times 0.8$  (factor for 20% decrease) = 10 800

Or  $A = \text{Rs}15\ 000$ .

#### R • SOME SOLVED EXAMPLES

- If the price of petrol increases by 10%, by what percentage must the consumption be decreased to keep the expenditure constant?

Answer: Let the original price be 100Rs/litre and the original consumption be 100 litres.

Therefore, expenditure =  $100 \times 100 = \text{Rs}10\ 000$ .

New price = Rs110. Since expenditure remains constant

$110 \times \text{new consumption} = 10\ 000 \rightarrow \text{new consumption} = \frac{10000}{110} = 90.90$  litres. Therefore,

decrease in consumption = 9.1%.

- The population of a town is 9 000. If males in the town increase by 6% and females increase by 8%, the population will become 9640. What is the number of females in the town?

Answer: Let the number of females be x.

Therefore  $(9\ 000 - x) \times 1.06 + x \times 1.08 = 9640 \rightarrow x = 5\ 000$ .

- In an examination, a candidate who scores 20% of the total marks fails by 40 marks whereas a candidate who scores 40% of the total marks passes by 120 marks. Then, the total marks possible in the examination are

Answer: Let the passing marks be x. Therefore, one candidate is scoring  $x - 40$  whereas the other is scoring  $x + 120$ . The difference of marks between them is 160. This 160 marks difference is equal to 20% of the marks (40% - 20%).

20% of marks is equal to 160 marks

$\rightarrow$  100% of marks is equal to 800 marks.

Therefore, 800 is the maximum score possible.

4. What quantity of water must be added to a 9 L solution of 50% milk to make it 30% milk?

Answer: The volume of the original solution is equal to 9 L. Therefore, volume of milk in the original solution is equal to 4.5 L. Now this volume of milk remains constant in the new solution.

In the new solution, 4.5 L of milk = 30%. Therefore, total volume =  $\frac{4.5}{30} \times 100 \text{ L} = 15 \text{ L}$ . Therefore,

water added = new volume - old volume = 6 L

5. A watermelon contains 85% water. 30 g of water evaporates from the watermelon and now it contains 80% water. What was the original weight of the watermelon?

Answer: This question is similar to the previous one except that the water is getting reduced in place of getting added. Let the original weight of the watermelon be  $x$ . Therefore, the solid material contained in the watermelon is equal to 15% of  $x = 0.15x$ . When 30 g of water evaporates, weight of watermelon becomes  $x - 30$ , of which  $0.15x$  is the solid material. Now this

solid material is 20% by weight  $\rightarrow \frac{0.15x}{x - 30} \times 100 = 20 \Rightarrow x = 120 \text{ g}$ .