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Ex Which is greater 10^{11} or 11^{10} ?

$$\frac{10^{11}}{11^{10}} = 10 \times \left(\frac{10}{11}\right)^{10} \ldots$$

$$\frac{11^{10}}{10^{11}} = \frac{1}{10} \times \left(\frac{11}{10}\right)^{10}$$

$$< 1.$$

$$\therefore 10^{11} > 11^{10}.$$

$$\boxed{\begin{array}{c} 2 \le \left(1+\frac{1}{x}\right)^{x} \le 2.8 \\ \left(1+\frac{1}{10}\right)^{10}. \end{array}}$$

Ex Solve $\dfrac{x^2-8x+12}{2x^2+7x-4} < 0.$

$2x^2+8x-x-4.$

$$\frac{(x-6)\ (x-2)}{(2x-1)\ (x+4)} < 0.$$

$+\quad -4 \quad -\quad \tfrac{1}{2} \quad +\quad 2 \quad -\quad 6 \quad +$

Ans $\rightarrow$ $\left(-4, \frac{1}{2}\right) \cup (2, 6)$

If $ax+by=k$ where a, b, x, y are all +ve, maximize $x^m y^n$ where m & n are all +ve numbers.

Ex If $5x+2y=15$, find max. value of x^3y^2.

$$\frac{3 \times \frac{5x}{3} + 2 \times y}{5} \geq \sqrt[5]{\left(\frac{5x}{3}\right)^3 \times (y)^2}$$

$$\Rightarrow 3^5 \geq \frac{5^3}{3^3} x^3 y^2 \qquad \Rightarrow x^3y^2 \le \frac{3^5 \times 3^3}{5^3}$$

When expression $ax+by$ is constant, max. value of $x^m y^n$ is realized when $\dfrac{ax}{m} = \dfrac{by}{n}$.

If $x^m y^n = k$, where $x>0$, $y>0$ and m & n are +ve integers, minimise $ax+by$ where $a>0$, $b>0$.

Ex If $x^5y^3 = \cancel{2}5^5 2^3$, then min value of $2x+3y$?

$$\frac{5\times\frac{2x}{5}+3y}{8} \geq \left[\left(\frac{2x}{5}\right)^5 (y)^3\right]^{1/8}$$

$$\Rightarrow 2x+3y \geq 8\times\left[\frac{2^5}{5^5}\times 5^5\times 2^3\right]^{1/8}$$

$$\Rightarrow 2x+3y \geq 8\times 2$$
$$\geq 16.$$

When $x^m y^n$ is constant, min. value of $ax+by$ is realized when $\frac{ax}{m} = \frac{by}{n}$.

Greatest value of $(a-x)^m (b+x)^n$, for any real value of x numerically less than a, b and $m, n \in Z^+$ occurs when $\frac{a-x}{m} = \frac{b+x}{n}$.

Ex Find max. value of $(9-x)^2(-4+x)^3$

$$\frac{9-x}{2} = \frac{-4+x}{3}$$

$$\Rightarrow 27-3x = -8+2x$$
$$\Rightarrow 5x = 35 \Rightarrow x = 7$$

$\therefore$ Max value $= 2^2 \times 3^3 = 108$.

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$\frac{(x+a)(x+b)}{(x+c)}$ Min. value $= a-c+b-c+2\sqrt{(a-c)(b-c)}$

$x = \sqrt{(a-c)(b-c)} - c$

Ex Find min. value of $\frac{(x+3)(x+6)}{(x+2)}$, when $x+2>0$

Min. value $= 3-2+6-2+2\sqrt{1\times 4}$

$= 5+2\times 2$

$= 9.$

Alt.

$\frac{(x+3)(x+6)}{(x+2)} = \frac{(t+1)(t+2)}{t}$

$= \frac{t^2+5t+4}{t}$

$= t+\frac{4}{t}+5$

$\geq 4+5$

$\geq 9.$

Ex If x is a -ve real number, then max. value of $x+\frac{1}{x}$ is.

Ans $\rightarrow -2$.

Ex Find the complete range of values of x for which $(x^2-x+1)^x < 1$

$\underbrace{\left[(x-\frac{1}{2})^2+\frac{3}{4}\right]}_{A}{}^{x}$.

If $A>1$ then A^x can be less than 1 when $x<0$.

$\downarrow$

$(-\infty, 0)$

Ans $\rightarrow (-\infty, 0) \cup (\frac{1}{2}, 1) \cup (0, \frac{1}{2}) \cup \{\frac{1}{2}\}$

$\Rightarrow (-\infty, 0) \cup (0, 1)$

Ex $a, b, c > 0 \quad \left(\frac{a^4+b^4}{a^2b^2} + \frac{b^4+c^4}{b^2c^2} + \frac{c^4+a^4}{c^2a^2}\right)_{min} = ?$

$$\frac{a^4+b^4}{2} \geq \sqrt{a^4b^4}$$

$$\Rightarrow \frac{a^4+b^4}{a^2b^2} \geq 2.$$

$\therefore$ Ans $\rightarrow 2+2+2 = 6$.

Ex If $a^2+b^2 = c^2+d^2 = 2$, then which is true?

(a) $ac+bd \leq 2$ (c) $ad+bc \leq 2$

(b) $ab+cd \leq 2$ (d) (A), (B) & (C)

$$\frac{a^2+b^2}{2} \geq ab \quad \Rightarrow ab \leq 1 \qquad \Rightarrow (b) \checkmark$$

$$a^2+b^2+c^2+d^2 = 4.$$

$$\frac{a^2+c^2}{2} \geq ac \qquad \frac{b^2+d^2}{2} \geq bd$$

$$\Rightarrow ac+bd \leq 2 \qquad \Rightarrow (a) \checkmark$$

Similarly (c) $\checkmark$.

$\therefore$ Ans $\Rightarrow$ (A), (B) & (C)

Ex If a, b, c are +ve real numbers, then which is true?

(a) $x^2+y^2+z^2 \geq xy+yz+zx$

$$\frac{x^2+y^2}{2} \geq xy \qquad \frac{y^2+z^2}{2} \geq yz \qquad \frac{x^2+z^2}{2} \geq xz.$$

$\therefore x^2+y^2+z^2 \geq xy+yz+xz$ (T)

(b) $(x+y)(y+z)\ z+x \geqslant 8xyz$

$x+y \geqslant 2\sqrt{xy}$ $\quad y+z \geqslant 2\sqrt{yz}$ $\quad x+z \geqslant 2\sqrt{zx}$.

$\therefore (x+y)(y+z)(z+x) \geqslant 8xyz$ (T)

(c) $\frac{1}{x}+\frac{1}{y} \geqslant \frac{2}{x+y}$

$\cancel{\frac{x+y}{xy}}$ $\quad \frac{x^2+y^2}{2} \geqslant xy$

$\cancel{\geqslant x}$

$\frac{1}{x}+\frac{1}{y} \geqslant 2\frac{1}{\sqrt{xy}}$ $\quad \frac{x+y}{2} \geqslant \sqrt{xy}$

$\therefore \frac{1}{\sqrt{xy}} \geqslant \frac{2}{x+y}$.

$\therefore \frac{1}{x}+\frac{1}{y} > \frac{2}{x+y}$. (T)

(d) $a+b+c \leq \frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}$

$\cancel{\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b} \geqslant 3\times\sqrt[3]{abc}}$

$\frac{a}{b}+\frac{b}{a} \geqslant 2$.

$\frac{ac}{b}+\frac{bc}{a} \geqslant 2c$. Similarly $\frac{b}{c}a+\frac{c}{b}a \geqslant 2a$

$\frac{a}{c}b+\frac{c}{a}b \geqslant 2b$.

$\therefore \frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b} \geqslant a+b+c$. (T)

(e) If $a>b$, $a^b \cdot b^a > a^a b^b$

$\left(\frac{a}{b}\right)^b > \left(\frac{a}{b}\right)^a$ (F)

(f) $a^2+b^2+c^2 > 2(ab+bc+ca)$

$a^2+b^2 \geq 2ab.$

$b^2+c^2 \geq 2bc$

$c^2+a^2 \geq 2ac.$

$\therefore a^2+b^2+c^2 \geq (ab+bc+ca)$ nothing can be said about $2(ab+bc+ca)$

(F)

Ex Find the bounds of expression $\frac{(a+b+c)^2}{ab+bc+ca}$.

a, b, c are sides of a Δ.

$\frac{a^2+b^2+c^2}{ab+bc+ca}+2$ Min $\to 1+2=3.$

$a-b < c$

$a^2+b^2-2ab < c^2.$

$\Rightarrow a^2+b^2-c^2 < 2ab.$

Similarly, $b^2+c^2-a^2 < 2bc$

$c^2+a^2-b^2 < 2ac.$

$\therefore a^2+b^2+c^2 < 2(ab+bc+ca)$

$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} < 2.$

$\therefore$ Ans $\to [3,4)$

Ex $a, b, c > 0.$

$\frac{(5a^2+a+5)(7b^2+b+7)(9c^2+c+9)}{19abc}$ can't be

(a) 155 (b) 165 (c) 180 (d) 175.

$$\frac{5a^2+a+5}{a} = 5a+\frac{5}{a}+1$$

$$\geq 11$$

$$\therefore \quad \geq \frac{11\times 15\times 19}{19}$$

$$\Rightarrow \geq 165$$

Ans $\rightarrow$ $\boxed{155}$

Ex $a,b,c,d>0$.
Find min. value of
$$\frac{a^2(bc+bd+cd)+b^2(ac+cd+ad)+c^2(ab+bd+ad)+d^2(bc+ac+ab)}{abcd}.$$

$$\frac{a^2(bc+bd+cd)}{abcd}$$

$$= \frac{a}{d}+\frac{a}{c}+\frac{a}{b}+\frac{b}{a}+\frac{b}{c}+\frac{b}{d}+\frac{c}{a}+\frac{c}{b}+\frac{c}{d}+\frac{d}{a}+\frac{d}{b}+\frac{d}{c}.$$

$$\therefore \quad \frac{\quad}{12} \geq \sqrt[12]{\quad}$$

$$\Rightarrow \quad \geq 12.$$

Ex If x,y,z are +ve real no.s, then the value of $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$ can be

(a) 2.5 (b) 4 (c) 3.5 (d) More than one of above.

$$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy} \geq 3\times 1$$
$$\geq 3.$$

Ans $\rightarrow$ (D)

Ex a,b,c are +ve real no.s and $a^2+b^2+c^2=4$.
Find $(ab+bc+ca)_{max}$.

$$\frac{a^2+b^2+c^2}{3} \geq (abc)^{3/2} \qquad \therefore abc \leq \sqrt{(4/3)}$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$
$$= 4-2(ab+bc+ca).$$

$a^2+b^2+c^2 \geq ab+bc+ca.$

$\therefore (ab+bc+ca)_{max} = 4.$

Ex If x, y, z are +ve real no.s and $x+y+z=12$, then $\frac{3}{x}+\frac{3}{y}+\frac{3}{z}$ can't be

(a) 12 (b) 2 (c) 3 (d) 6.

$\frac{x+y+z}{3} \geq (xyz)^{1/3}.$

$\Rightarrow xyz \leq 4^3$
$\leq 64.$

$xyz^{1/3} \leq 4$
$\Rightarrow \left(\frac{1}{xyz}\right)^{1/3} \geq 1/4$

$\frac{3}{x}+\frac{3}{y}+\frac{3}{z} \geq 3 \times \left(\frac{27}{xyz}\right)^{1/3}.$

$\geq 3 \times 3 \times \left(\frac{1}{xyz}\right)^{1/3}$

$\geq 9 \times \cancel{4}\, 1/4$

$\geq 2.25.$

Ans $\rightarrow$ 2

Ex If a, b, c are +ve real numbers and $abc=27$, then min. value of $\frac{a^6+b^6}{a^4-b^2a^2+b^4}+\frac{b^6+c^6}{b^4-b^2c^2+c^4}+\frac{c^6+a^6}{c^4-c^2a^2+a^4}$.

$2(a^2+b^2+c^2)$

$\frac{a^2+b^2+c^2}{3} \geq (abc)^{2/3}$

$a^2+b^2+c^2 \geq 9 \times 3$

Ans $\rightarrow$ 54.

Ex Is $X > Y$?

(i) $X - Y > X$

(ii) $Y - X > Y$

Using (i) $X - Y > X \Rightarrow -Y > 0 \Rightarrow Y < 0$.

Using (ii) $Y - X > Y \Rightarrow -X > 0 \Rightarrow X < 0$.

Ans $\rightarrow$ Nothing can be said.

Ex Is $X > Y$?

(i) $X + Y > X$

(ii) $X - Y > 2Y$

Using (i) $X + Y > X \Rightarrow Y > 0$.

Using (ii) $X - Y > 2Y \Rightarrow X > 3Y$.

Hence, $X > Y$.