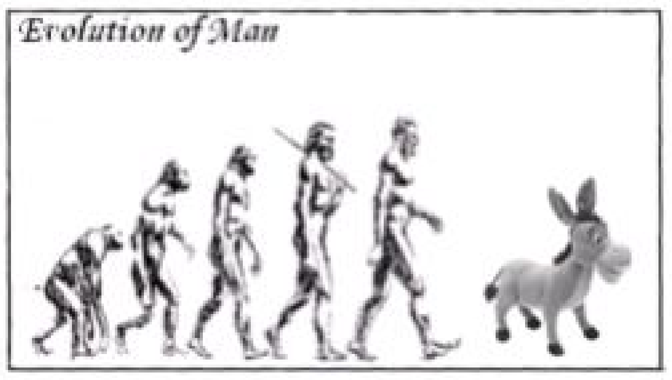
Factorials
by Software Engineer - Wednesday, 17 June 2009, 04:53 PM



Most of the CAT aspirants don't know that the hardest work they have to do for their CAT preparation is developing a healthy psychology. What is a healthy psychology? That you feel good about yourself, your family, your relationships, your academics, your job, your friends, everything. Which means that you will have to work to improve your relationships, do well in your job, perform great in your college etc. Is it important? It is critical. If you want to ask me how critical it is, let me tell you something- when I took CAT in 2004, I had quit my engineering career to prepare for CAT. I knew in my heart that if I didn't clear CAT I would have a hard time going back to my profession. The pressure killed me in the exam, despite the fact that my All-India ranks in the mocks used to be in top 20. I couldn't perform in the paper. Come 2005, I started loving my teaching profession, so much so that I wanted to come back to it even if I cracked CAT. I used to teach for 14 hours a day at time (morning 7:00 to evening 9:00) and still come back home and study because I used to feel happy and refreshed. If you can wake up happy every morning to go to your office and come back happy, it means you love your job. Anyways, coming back to my story, where in 2004 I used to score 16- 18 marks in quant and DI, I started scoring 27- 30. A phenomenal jump, not because I was preparing hard- in fact I was studying one-tenth of what I was studying in 2004- but because I felt great about myself. In 2005, when I sat for the CAT paper, the hours passed away smoothly. No screw-up. Same happened in 2008. So if you ask me if you should quit your job or not pay attention to your college exams to prepare for CAT? My answer is that you have killed your chances before you have even started- *Total Gadha*.

This article comes from an old-timer, Software Engineer- commonly called as SE in our forums. When SE came on TG, Dagny and I thought he was a brat but since brats are rare among sweet gadhas of donkeyland we kind of liked it. Well, we still think he is a brat but he is a lovable brat. And an intelligent one too, which you will find out from this article. Young and fresh TGites, please pay attention to this article. You cannot get better fundas than this. And don't forget to thank SE for his effort!

The Highest Power of Prime Contained in Factorial

The highest power of prime number p in n! = $\left[\frac{n}{p^1}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots+\left[\frac{n}{p^y}\right]$ where y is the largest positive integer such that $p^y \le n \le p^{y+1}$ and [x] denotes the greatest integer less than or equal to x.

Find the highest power of 2 in 50!

The highest power of 2 in 50! = $\left[\frac{50}{2^1}\right]+\left[\frac{50}{2^2}\right]+\left[\frac{50}{2^3}\right]+\left[\frac{50}{2^4}\right]+\left[\frac{50}{2^5}\right]$ = 25 + 12 + 6 + 3 + 1 = 47

Note Here the successive terms (i.e. 25, 12, 6, 3 and 1) can be obtained most easily by repeated division by 2; collecting all quotients and throwing away any remainder.

Quotient $\left(\frac{50}{2}\right)$ = 25 (term 1)

Quotient $\left(\frac{25}{2}\right)$ = 12 (term 2)

Quotient $\left(\frac{12}{2}\right)$ = 6 (term 3)

Quotient $\left(\frac{6}{2}\right)$ = 3 (term 4)

Quotient $\left(\frac{3}{2}\right)$ = 1 (term 5)

If $a_k = \left[\frac{n}{p}\right]$ then $a_k+1 = \left[\frac{a_k}{p}\right]$ where a, n are natural numbers, p is a prime number and [x] denotes the greatest integer less than or equal to x.

Checko!

The highest power of 2 in 50!

= $\left[\frac{50}{2^1}\right]+\left[\frac{50}{2^2}\right]+\left[\frac{50}{2^3}\right]+\left[\frac{50}{2^4}\right]+\left[\frac{50}{2^5}\right]$

= 25 + 12 + 6 + 3 + 1

Here, the successive terms are $a_1=25$, $a_2=12$, $a_3=6$, $a_4=3$ and $a_5=1$.

Now,

$$a_1 = \left[\frac{n}{p}\right] = \left[\frac{50}{2}\right] = 25$$

$$a_2 = \left[\frac{a_1}{p}\right] = \left[\frac{25}{2}\right] = 12$$

$$a_3 = \left[\frac{a_2}{p}\right] = \left[\frac{12}{2}\right] = 6$$

$$a_4 = \left[\frac{a_3}{p}\right] = \left[\frac{6}{2}\right] = 3$$

$$a_5 = \left[\frac{a_4}{p}\right] = \left[\frac{3}{2}\right] = 1$$

Find the highest power of 2 in 50!
Let's solve it using the repeated division process.
25 (Divide 50 by 2; get the quotient and throw away remainder, if there is any)
25 + 12 (Now, the new dividend is 25; the quotient obtained in last iteration of the repeated division process. Divide 25 by 2 and add the quotient thus obtained to the old quotient)
25 + 12 + 6 (Now, divined=12. Divide 12 by 2; add the quotient thus obtained to the old quotients.)
25 + 12 + 6 + 3 (Now, dividend=6, divisor=2 (which is always constant), quotient = 3. Add 3.)
25 + 12 + 6 + 3 + 1 (3 = (1 * 2) +1. Add 1.)
The repeated division process stops when the last quotient is lower than divisor. Here, 1<2.
The highest power of 2 in 50! can be obtained by adding all of these successive quotients.
The highest power of 2 in n! = 25 + 12 + 6 + 3 + 1 = 47

Find the highest power of 7 in 400!
The highest power of 7 in 400! = 57 + 8 + 1 = 66
Explanation
First, divide 400 by 7 and get the quotient; it's 57.
Now, new divined is equal to the quotient obtained in the last step i.e. 57.
Divide 57 by 7 and add the quotient thus obtained to the quotient obtained earlier. 57 + 8
Now, divined=8. Divide 8 by 7 and add the quotient i.e. 1 to the quotients obtained earlier. 57 + 8 + 1
Stop the repeated division process as the last quotient, here it's 1, is lower than divisor, here it's 7.
The highest power of 7 in 400! = 57 + 8 + 1 = 66

What is the greatest power of 5 which can divide 80! exactly? CAT 1991
(1). 16 (2). 20 (3). 19 (4). None of these
16 + 3 = 19

Find the highest power of 30 in 50!
30 = 2 * 3 * 5. Here, 5 is the largest prime factor of 30, therefore, the highest power of 5 in 50! will be less than that of 2 and 3. Therefore, there cannot be more 30s than there are 5s in 50! So we'll find the highest power of 5 in 50!
The highest power of 5 in 50! = 10 + 2 = 12
Hence, the highest power of 30 in 50! = 12

If 146! is divisible by 6^n then find the maximum value of n. posted by **Priyanka Keshri**
(1). 74 (2). 70 (3). 76 (4). 75
All in all, we need to find the highest power of 6 contained in 146!
6 = 2 * 3. As 3 is the largest prime factor of 6, we'll find the highest power of 3 in 146!
The highest power of 3 in 146! = 48 + 16 + 5 + 1 = 70
Hence, n=70.

Find the number of divisors of 15!
To find the number of divisors of 15!, we need to find out the highest power of every prime number, that is less than or equal to 15, in 15!. The prime numbers less than or equal to 15 are 2, 3, 5, 7, 11 and 13.

the highest power of 2 in 15! = 7 + 3 + 1 = 11
the highest power of 3 in 15! = 5 + 1 = 6
the highest power of 5 in 15! = 3
the highest power of 7 in 15! = 2
the highest power of 11 in 15! = 1
the highest power of 13 in 15! = 1

Therefore, $15! = 2^{11} * 3^6 * 5^3 * 7^2 * 11^1 * 13^1$
Hence, the number of divisors of 15! = 12 * 7 * 4 * 3 * 2 * 2 = 4032

The highest power of the number p^a in n! = $\left[\frac{\text{the highest power of p in n!}}{a}\right]$ where p is a prime number, a is a natural number and [x] denotes the greatest integer less than or equal to x.

Find the highest power of 2^3 in 51!
The highest power of 2^3 in 51!
$= \left[\frac{\text{the highest power of 2 in 51!}}{3}\right] = \left[\frac{25+12+6+3+1}{3}\right] = \left[\frac{47}{3}\right] = 15$

Find the highest power of 72 in 100!
72 = $2^3 * 3^2$. Therefore, we need to find the highest power of 2^3 and 3^2 in 72!.
The highest power of 2^3 in 100! = $\left[\frac{50+25+12+6+3+1}{3}\right] = \left[\frac{97}{3}\right] = 32$
The highest power of 3^2 in 100! = $\left[\frac{33+11+3+1}{2}\right] = \left[\frac{48}{2}\right] = 24$
As the highest power of 3^2 is less than that of 2^3, the highest of 72 in 100! is equal to that of 3^2.
Hence, the highest power of 72 in 100! = 24.

Find the number of zeroes present at the end of 310!
The number of zeroes present at the end of 310! will be equal to the number of times 10 divides 310! completely. Therefore, we need to find the highest power of 10 contained in 310!.
10 = 2 * 5. As 5 is the largest prime factor of 10, we'll find the highest power of 5 contained in 310!

The highest power of 5 in 310! = 62 + 12 + 2 = 76
Hence, there are 76 zeroes at the end of 310!

Find the number of zeroes at the end of 100! CAT 1993
(1). 26 (2). 25 (3). 24 (4). 23
20 + 4 = 24.

How many zeroes are present at the end of 24! + 25!? posted by **Total Gadha**
The total number of zeroes at the end of 24! = 4
The total number of zeroes at the end of 25! = 5 + 1 = 6
Hence, the expression 24! + 25! has four trailing zeroes.

The number 2006! is written in base 22. How many zeroes are there at the end?
(1). 450 (2). 500 (3). 199 (4). 200 CAT Number System Quiz
A number written in base 10 can be converted to any base b by first dividing the number by b and then dividing the successive quotient by b. The remainders thus obtained, written in reverse order, give the equivalent number in base b.

Let's convert $(56)_{10}$ to base 7.

$$\begin{array}{r|ll} 7 & 56 & \\ \cline{2-2} & 8 & 0 \\ \cline{2-2} & 1 & 1 \\ & & 1 \end{array}$$

$(56)_{10} = (110)_7$

There is exactly one zero at the end of the number converted to base 7 because the number 56 is divisible once by 7 i.e. the highest power of 7 contained in the number 56 in exactly equal to one. 56 = $2^3 * 7^1$.

Similarly, if we convert 56 to base 2 then there will be exactly three zeroes at the end of converted number in base 2 because the highest power of 2 contained in 56 is exactly equal to three i.e. the number 56 is divisible thrice by 2.

Here, the number of zeroes present at the end of 2006! in base 22 will be equal to the number of times 22 divides 2006! completely. Therefore, we need to find the highest power of 22 contained in 2006!
22 = 2 * 11. As 11 is the largest prime factor of 22, we'll find the highest power of 11 contained in 2006!

The highest power of 11 in 2006! = 182 + 16 + 1 = 199
Hence, there are 199 zeroes at the end of 2006! in base 22.

30! when expressed in base 12 ends with k zeroes. Find k. posted by **Praneeth Koralla**
12 = $2^2 * 3$.
The highest power of 2^2 in 30! = $\left[\frac{15+7+3+1}{2}\right] = \left[\frac{26}{2}\right] = 13$
The highest power of 3 in 30! = 10 + 3 + 1 = 14
As the highest power of 2^2 is less than that of 3, the highest power of 12 in 100! is equal to that of 2^2
Hence, k=13.

For a given z(n), the smallest value of a natural number n such that there are z(n) zeroes present at the end of n! can be calculated approximately as $n = 4 * z(n)$ where z(n) denotes the number of zeroes present at the end of n! in base 10.

Let, z(n) be the number of zeroes present at the end of n! in base 10. Therefore, the highest power of 10 contained in n! is z(n).
10 = 2 * 5. As 5 is the largest prime factor of 10, the highest power of 5 contained in n! is z(n).
Therefore,

$$z(n) = \left[\frac{n}{5^1}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\left[\frac{n}{5^4}\right]+\ldots+\left[\frac{n}{5^y}\right]$$

$$z(n) = \frac{1}{5}\left[n+\left[\frac{n}{5^1}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\ldots+\left[\frac{n}{5^{(y-1)}}\right]\right] \quad \text{(approximately)}$$

$$z(n)*5 = n+\left[\frac{n}{5^1}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\ldots+\left[\frac{n}{5^{(y-1)}}\right] \quad \text{(approximately)}$$

$$z(n) * 5 - n = z(n) \quad \text{(approximately) [put } \left[\frac{n}{5^1}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\ldots+\left[\frac{n}{5^{(y-1)}}\right] = z(n)]$$

$$n = 4 * z(n) \quad \text{(approximately)}$$

Find the least value of n such that n! has exactly 2394 zeroes. posted by **Fundoo Bond**
Here, z(n) = 2394. n = 4 * 2394 = 9576 (approximately)

Let's count the number of zeroes present at the end of 9576!.
The number of zeroes present at the end of 9576! = 1915 + 383 + 76 + 15 + 3 = 2392

To get more zero(es) we'll have to find the least multiple of five greater than 9576 and it is 9580. How many more zeros we'll get at the end of 9580! depends on whether 9580 is multiple of 5, 25, 125, 625, ...
The number of zeroes present at the end of 9580! = 1916 + 383 + 76 + 15 + 3 = 2393

Still there is one less zero than the total number of required zeroes. Next multiple of five is 9585.
The number of zeroes present at the end of 9585! = 1917 + 383 + 76 + 15 + 3 = 2394

Hence, n=9585.

Find the smallest positive integer n such that n! is a multiple of 10^{2009}. posted by **Kamal Lohia**
It means that find the smallest positive integer n such that n! has at least 2009 zeroes at the end.
n = 4 * 2009 = 8036 (approximately)

The number of zeroes present at the end of 8036! = 1607 + 321 + 64 + 12 + 2 = 2006

The number of zeroes present at the end of 8040! = 1608 + 321
As we ended up with 321, it'll give the same successive quotients obtained during the calculation of 8036! Therefore we don't need to calculate the same thing again. Therefore, 8040! has 2007 trailing zeroes.

The next multiple of five is 8045. The number of zeroes present at the end of 8045! = 1609 + 321
Therefore, 8045! has 2008 trailing zeroes.

The next multiple of five is 8050. The number of zeroes present at the end of 8050! = 1610 + 322 + 64
Therefore, 8050! has 2010 trailing zeroes.

Hence, 8050! is the least multiple of 10^{2009}.

n! has x number of zeroes at the end and (n + 1)! has x + 3 zeroes at the end. Find the number of possible values of n if n is a three digit number.
The highest power of 5 in n! is x. How many more zeroes we'll get at the end of (n+1)! depends on whether (n+1) is a multiple of 5^0, 5^1, 5^2, ... and so on. Here (n+1)! has x+3 zeroes at the end. Therefore the highest power of 5 in the number (n+1) is exactly equal to 3.

The possible values of the number (n+1) where the highest power of 5 is exactly equal to 3 are
$5^3 * 1 = 125$
$5^3 * 2 = 250$
$5^3 * 3 = 375$
$5^3 * 4 = 500$
$5^3 * 6 = 750$
$5^3 * 7 = 875$
$5^3 * 8 = 1000$
The highest power of 5 in $5^3 * 5$ is 4 which is why we ignored it. Hence, the possible values of n = 7.

Note n is a multiple of 5^3 and the highest power of 5 in the number n is exactly equal to 3; both are really different things. The former includes 625 but the later does not.

For a given z(n), the smallest value of a natural number n such that there are z(n) zeroes present at the end of n! in base b can be calculated approximately as $n = (p-1) * z(n)$ where z(n) denotes the number of zeroes present at the end of n! in base b and p is the largest prime factor of base b.
Here, $\text{base } b = 2^{e_2} * 3^{e_3} * 5^{e_5} * \ldots$ where e_i denotes the highest power of prime p_i contained in base b.
The exponent e_i can be either 0 or 1 i.e. $e_i \in \{0, 1\}$

Let, z(n) be the number of zeroes present at the end of n! in base b. Therefore, the highest power of b contained in n! is z(n). The base $b = 2^{e_2} * 3^{e_3} * 5^{e_5} * \ldots$ where e_i denotes the power of prime p_i contained in base b. Here, the exponent e_i can be either 0 or 1 i.e. $e_i \in \{0, 1\}$ Let p be the largest prime factor of base b. Therefore, the highest power of p contained in n! is z(n).

$$z(n) = \left[\frac{n}{p^1}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \ldots + \left[\frac{n}{p^y}\right]$$
$$z(n) = \frac{1}{p}\left[n + \left[\frac{n}{p^1}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^{(y-1)}}\right]\right] \quad \text{(approximately)}$$
$$z(n) * p = n + \left[\frac{n}{p^1}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^{(y-1)}}\right] \quad \text{(approximately)}$$
$$z(n) * p - n = z(n) \quad \text{(approximately)}$$
$$n = (p-1) * z(n) \quad \text{(approximately)}$$

n! has 100 trailing zeroes in base 35. Find the minimum value of n.
Here, b = 35 = 5 * 7. As 7 is the largest prime factor of 35 p=7. z(n)=100.
n = (p-1) * z(n) = 6 * 100 = 600 (approximately)

Let's count the number of zeroes present at the end of 600! in base 35.
The number of zeroes present at the end of 600! in base 35 = 85 + 12 + 1 = 98

To get more zero(es) we'll have to find the least multiple of 7 greater than 600. 600 = 85 * 7 + 5. Therefore, the next multiple of 7 is 600 + (7 – 5) = 602. How many more zeros we'll get at the end of 602! depends on whether 602 is multiple of 7^1, 7^2, 7^3, ...
The number of zeroes present at the end of 602! in base 35 = 86 + 12 + ... = 99

The next multiple of 7 is 602 + 7 = 609.
The number of zeroes present at the end of 609! in base 35 = 87 + 12 + ... = 100

Hence, n=609 .

What is the smallest n such that n! ends in exactly 91 zeroes in base 91? posted by **Software Engineer**
Here, b = 91 = 7 * 13. As 13 is the largest prime factor of 91 p=13. z(n)=91.
n = (p-1) * z(n) = 12 * 91 = 1092 (approximately)

Let's count the number of zeroes present at the end of 1092! in base 91.
The number of zeroes present at the end of 1092! in base 91 = 84 + 6 = 90

To get more zero(es) we'll have to find the least multiple of 7 greater than 600. 600 = 85 * 7 + 5. Therefore, the next multiple of 7 is 600 + (7 – 5) = 602. How many more zeros we'll get at the end of 602! depends on whether 602 is multiple of 7^1, 7^2, 7^3, ...
The number of zeroes present at the end of 602! in base 35 = 86 + 12 + ... = 99

The next multiple of 7 is 602 + 7 = 609.
The number of zeroes present at the end of 609! in base 35 = 87 + 12 + ... = 100

Hence, n=609 .

What is the smallest n such that n! ends in exactly 91 zeroes in base 91? posted by **Software Engineer**
Here, b = 91 = 7 * 13. As 13 is the largest prime factor of 91 p=13. z(n)=91.
n = (p-1) * z(n) = 12 * 91 = 1092 (approximately)

Let's count the number of zeroes present at the end of 1092! in base 91.
The number of zeroes present at the end of 1092! in base 91 = 84 + 6 = 90

To get more zero(es) we'll have to find the least multiple of 13 greater than 1092. 1092 = 84 * 13 + 0. Therefore, the next multiple of 13 is 1092 + (13 – 0) = 1105
The number of zeroes present at the end of 1105! in base 91 = 85 + 6 = 91

Hence, n=1105 .

How many numbers less than or equal to 9000 are multiple of 49?
183. (You know why?)

The highest power of 2 contained in 20! is 10 + 5 + 2 + 1 = 33. What does it mean?
The highest power of 2 in 20! = $\left[\frac{20}{2^1}\right]+\left[\frac{20}{2^2}\right]+\left[\frac{20}{2^3}\right]+\left[\frac{20}{2^4}\right]$ = 10 + 5 + 2 + 1 = 33.

Meaning 01
The first quotient is 10; it means that there are 10 numbers between 1 and 20 (including both) which are multiple of 2^1. They are 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20.

The second quotient is 5; it means that there are 5 numbers between 1 and 20 (including both) which are multiple of 2^2. They are 4, 8, 12, 16 and 20.

The third quotient is 2; it means that there are 2 numbers between 1 and 20 (including both) which are multiple of 2^3. They are 8 and 16.

The fourth quotient is 1; it means that there is a single number between 1 and 20 (including both) which is multiple of 2^4. It's 16.

As 10 numbers contain at least one power of 2 in their prime factorization; the remaining (20 – 10) = 10 numbers don't contain any power of 2 i.e. such numbers are not multiple of 2.

Meaning 02
$n – q_1$ = 20 – 10 = 10. It means that there are 10 numbers between 1 and 20 (including both) in which the highest power of 2 is exactly equal to zero. The numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17 and 19.

$q_1 – q_2$ = 10 – 5 = 5. It means that there are 5 numbers between 1 and 20 (including both) in which the highest power of 2 is exactly equal to one. The numbers are 2, 6, 10, 14 and 18.

$q_2 – q_3$ = 5 – 2 = 3. It means that there are 3 numbers between 1 and 20 (including both) in which the highest power of 2 is exactly equal to two. The numbers are 4, 12 and 20.

$q_3 – q_4$ = 2 – 1 = 1. It means that there is one number between 1 and 20 (including both) in which the highest power of 2 is exactly equal to three. The number is 8.

q_4=1. It means that there is one number between 1 and 20 (including both) in which the highest power of 2 is exactly equal to four. The number is 16.

How many numbers less than or equal to 1000 are divisible by 125?
$125 = 5^3$. The highest power of 5 contained in 1000! = 200 + 40 + 8 + …
The third quotient denotes the total number of numbers less than or equal to 1000 and divisible by 5^3. As we got the answer in third iteration, we stopped the division process there.
Hence, there are 8 such numbers.

How many numbers less than or equal to 10000 are divisible by 121?
$121 = 11^2$. The highest power of 11 contained in 10000! = 909 + 82 + …
Hence, there are 82 such numbers.

How many numbers less than or equal to 9000 are multiple of 49?
183. (You know why.)

Find the number of possible values of n = 125 * m if n is less than or equal to 1000 and m is not divisible by 5.
$n = 5^3 * m$ where m is not divisible by 5 i.e. the highest power of 5 in the number n is exactly equal to three.

The highest power of 5 contained in 1000! = 200 + 40 + 8 + 1

1000 – 200 = 800.
There are 800 numbers less than or equal to 1000 in which the highest power of 5 is exactly equal to zero.
200 – 40 = 160.
There are 160 numbers less than or equal to 1000 in which the highest power of 5 is exactly equal to one.
40 – 8 = 32.
There are 32 numbers less than or equal to 1000 in which the highest power of 5 is exactly equal to two.
8 – 1 = 7.
There are 7 numbers less than or equal to 1000 in which the highest power of 5 is exactly equal to three.

Hence, the number of possible values of n is 7.

The quotient obtained by dividing n by 121 and the number 11 are co-prime to each other. Find the number of possible values of n if n is less than or equal to 10000.
It means that the highest power of 11 contained in the number n is exactly equal to two.
The highest power of 11 contained in 10000! = 909 + 82 + 7
82 – 7 = 75.
Hence, the number of possible values of n is 75.

Funda#1

The product of all of the numbers from 1 to 10 not divisible by 5 ends in 6.
1 * 2 * 3 * 4 * 6 * 7 * 8 * 9 = x6.

The product of all of the numbers from 11 to 20 not divisible by 5 ends in 6.
11 * 12 * 13 * 14 * 16 * 17 * 18 * 19 = x6.

The product of all of the numbers from 21 to 30 not divisible by 5 ends in 6.
21 * 22 * 23 * 24 * 26 * 27 * 28 * 29 = x6.
………………………………………………………………………………………………

and so on.

Funda#2

The unit digit of 6^n is always 6 where n is natural number.

Funda#3

In base 10, the rightmost non zero digit of n! is 2, 4, 6 or 8 where n>1.

Let $n! = 2^{e_2} * 3^{e_3} * 5^{e_5} * 7^{e_7} * 11^{e_{11}} * \ldots$
In base 10, as 5 is the largest prime factor of 10 the highest power of 5 contained in n! is always less than that of 2 i.e. $e_2 > e_5$. Therefore, the highest power of 10 in n! is e_5.

If we divide n! by 10^{e_5+1} we get the rightmost non zero digit of n! as remainder. It means that if we remove all the factors of 5 and an equal number of factors of 2, the remaining factors multiplied together modulo 10 will be the rightmost non zero digit of n!

After removing all the factors of 5 and an equal number of factors of 2 the product of remaining factors is (n!)` = $2^{e_2-e_5} * 3^{e_3} * 7^{e_7} * 11^{e_{11}} * \ldots$ As $e_2 > e_5$; (e2 - e5) > 0.

The unit digit of (n!)` is the rightmost non zero digit of n! As (n!)` contains some factors of 2, the right most non zero digit of n! must be even.

In the value of the number 30! all the zeroes at the end are erased. Then, the unit digit of the number that is left is (1). 2 (2). 4 (3). 6 (4). 8 **CAT Number System Quiz**

30! = 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10
* 11 * 12 * 13 * 14 * 15 * 16 * 17 * 18 * 19 * 20
* 21 * 22 * 23 * 24 * 25 * 26 * 27 * 28 * 29 * 30

Let's remove all factors of 5 from 30!

(30!)` = 1 * 2 * 3 * 4 * **1** * 6 * 7 * 8 * 9 * **2**
* 11 * 12 * 13 * 14 * **3** * 16 * 17 * 18 * 19 * **4**
* 21 * 22 * 23 * 24 * **1** * 26 * 27 * 28 * 29 * **6**

The highest power of 5 in 30! = 6 + 1 = 7.

Black Numbers The highest power of 5 in each of these numbers is exactly equal to zero. The total number of black numbers is (30 – 6) = 24.
Green Numbers Initially, the highest power of 5 in each of these numbers was exactly equal to one. The total number of green numbers is (6 – 1) = 5.
Red Numbers Initially, the highest power of 5 in each of these numbers was exactly equal to two. The total number of red numbers is (1) = 1.
(Suppose there are yellow numbers. Initially the highest power of 5 in each of these numbers would be exactly equal to three. But 30 is less than 125 which is why there are no yellow numbers)

Let's arrange the numbers in an appropriate manner so we can apply Funda#1 here.

(30!)` = 1 * 2 * 3 * 4 * 6 * 7 * 8 * 9 (Complete)
* 11 * 12 * 13 * 14 * 16 * 17 * 18 * 19 (Complete)
* 21 * 22 * 23 * 24 * 26 * 27 * 28 * 29 (Complete)
* **1** * **2** * **3** * **4** * **6** (Incomplete)
* **1** (Incomplete)

Definition. If a row contains exactly 8 numbers then it's a complete row; otherwise it's an incomplete row.

Let's calculate the unit digit of (30!)`
(30!)` = (6 * 6 * 6) * (1*2*3*4*6) * (1) [Accord to Funda#1 the product of each complete row ends in 6]
= 6^3 * 4 * 1
= 6 * 4 * 1 [Accord to Funda#2 6^3 ends in 6.]
= 4

Therefore, the unit digit of (30!)` is 4. So far we have divided the 30! by $5^7$ i.e. we removed all the factors of 5 from 30! The quotient of the division is (30!)` and it's unit digit is 4.

In order to get the right most non zero digit of 30!, we'll have to divide (30!)` by 2^7 because the highest power of 5 in 30! is 7.
(30!)` ÷ 2^7 = R where R is the rightmost non zero digit of 30!
x4 = x8 * xR (2^7 ends in 8.)
Which single digit number when multiplied with 8 produces 4 as the unit digit of the product?
R is 3 or 8. But accord to Funda#3 R is always even (2, 4, 6 or 8).

Hence, the rightmost non zero digit of 30! is 8.

Find the right most non zero digit of 34!
The highest power of 5 in 34! = 6 + 1 = 7.

Let's remove all factors of 5 from 34! and arrange the numbers in an appropriate manner so we can apply Funda#1.

(34!)` =	1 *	2 *	3 *	4 *	6 *	7 *	8 *	9	(Complete)
	* 11	* 12	* 13	* 14	* 16	* 17	* 18	* 19	(Complete)
	* 21	* 22	* 23	* 24	* 26	* 27	* 28	* 29	(Complete)
	* 31	* 32	* 33	* 34					(Incomplete)
	* **1**	* **2**	* **3**	* **4**	* **6**				(Incomplete)
	* **1**								(Incomplete)

Black Numbers The highest power of 5 in each of these numbers is exactly equal to zero. The total number of black numbers is (34 – 6) = 28.
Green Numbers Initially, the highest power of 5 in each of these numbers was exactly equal to one. The total number of green numbers is (6 – 1) = 5.
Red Numbers Initially, the highest power of 5 in each of these numbers was exactly equal to two. The total number of red numbers is (1) = 1.

Let's calculate the unit digit of (34!)`
(34!)` = (6 * 6 * 6 * 31 * 32 * 33 * 34) * (1 * 2 * 3 * 4 * 6) * (1) [Each complete row ends in 6.]
= (6^3 * 1 * 2 * 3 * 4) * (1 * 2 * 3 * 4 * 6) * (1) [We are interested in unit digit.]
= (6 * 1 * 2 * 3 * 4) * (1 * 2 * 3 * 4 * 6) * (1) [Funda#2]
= 4 * 4 * 1
= 6

Now, (34!)` ÷ 2^7 = xR
2^7 ends with 8 where R is a non zero single-digit even number.
x6 = x8 * xR (2^7 ends in 8.)
Which single digit number when multiplied with 8 produces 6 as the unit digit of the product?
R is 2 or 7. Apply Funda#3. R=2.

Hence, the rightmost non zero digit of 34! is 2.

Note Accord to Funda#1 EACH complete row ends in 6.

1 *	2 *	3 *	4 *	6 *	7 *	8 *	9	(Complete)
* 11	* 12	* 13	* 14	* 16	* 17	* 18	* 19	(Complete)
* 21	* 22	* 23	* 24	* 26	* 27	* 28	* 29	(Complete)

= 6 * 6 * 6
= 6^3 = x6 (Funda#2)
Therefore, accord to Funda#1 and Funda#2 ALL complete rows also end in 6.

Find the right most non zero digit of 34! (Quickly)
The highest power of 5 in 34! = 6 + 1 = 7.

(34-6) * (6-1) * 1
(28) * (5) * 1
(8*3 + 4) * (0 + 5) * (0 + 1) [express each term in (8*m + r) format]

Question. Why 8*m + r format? Actually we can find the total number of complete and incomplete rows by expressing each term in (8*m + r) format.
8*m denotes that there are m complete rows in that particular group.
The total number of incomplete row is equal to zero or one. If r>0 then there is one incomplete row and if r=0 there is no incomplete row.
Therefore, there are three complete rows in first group and the other two groups don't contain any complete row. Each of the three groups has exactly one incomplete row.

As ALL complete rows end in 6 we can directly put 6 in place of 8*m if m>0. If m=0 the put nothing.

If m>0; write the natural numbers in increasing order starting with (10 * m + 1); skip any multiple of five during this process. The total number of such numbers is equal to r.
If m=0; write the natural numbers in increasing order starting with 1; skip any multiple of five during this process. The total number of such numbers is equal to r.

(6 * 31 * 32 * 33 * 34) * (1 * 2 * 3 * 4 * 6) * (1)

As we're interested in unit digit we can skip the above step
(6 * 1 * 2 * 3 * 4) * (1 * 2 * 3 * 4 * 6) * (1)

Therefore, if m>0; write the natural numbers in increasing order starting with 1; skip any multiple of five during this process. The total number of such numbers is equal to r.

(4) * (4) * 1
6
(34!)` = x6.
2^7 ends with 8.
x6 = x8 * R. Hence, R=2.

For an incomplete row; the product (x1 * x2 * x3 * x4) ends in 4.
the product (x1 * x2 * x3 * x4 * x6) ends in 4.
the product (x1 * x2 * x3 * x4 * x6 * x7 * x8) ends in 4.

the product (x1 * x2 * x3 * x4 * x6 * x7) ends in 8.

The remaining possibilities are (x1), (x1 * x2) and (x1 * x2 * x3).
(x1) = 1
(x1 * x2) = 2
(x1 * x2 * x3) = 6

Find the right most non zero digit of 34! (CAT Hall)
6 + 1 = 7.

(28) * (5) * 1
(24 + 4) * (5)
(6 * 2 * 3 * 4) * (2 * 3 * 4 * 6)
4 * 4
6

x6 = x8 * R.
R=2.

Find the right most non zero digit of 15!
The highest power of 5 in 15! = 3
(15-3) * (3)
(12) * (3)
(8 + 4) * (0 + 3)
(6 * 1 * 2 * 3 * 4) * (1 * 2 * 3)
(4) * (6)
4
x4 = 8 * R. Hence, R=8.

Funda#4

The product of all of the numbers from 1 to 10 not divisible by 5 ends with 76.
1 * 2 * 3 * 4 * 6 * 7 * 8 * 9 = x76.

The product of all of the numbers from 11 to 20 not divisible by 5 ends with 76.
11 * 12 * 13 * 14 * 16 * 17 * 18 * 19 = x76.

The product of all of the numbers from 21 to 30 not divisible by 5 ends in 76.
21 * 22 * 23 * 24 * 26 * 27 * 28 * 29 = x76.

....

and so on.

Funda#5

The last two digits of 76^n are always 76 where n is natural number.

Funda#6

In base 10, the number formed by the last two non zero digits of n! is always divisible by 4 where n>3.

For an incomplete row, the product (x1 * x2 * x3 * x4) ends with 24.

Find the last two non zero digits of 11!
The highest power of 5 in 11! = 2

(11-2) * (2)
(9) * (2)
(8 + 1) * (0 + 2)
(76 * 11) * (1 * 2)
36 * 2
72
Therefore, (11!)` = x72

2^2 ends in 4.

x72 = 4 * xab
Let's first find b. xx2 = 4 * xxb Therefore, b=8.
x72 = 4 * xa8
3 + 4a = x7 (8*4 = 32. Carry=3. 4*a = 4a. Therefore, (3 + 4a))
4a = x4
a is 1 or 6.
Apply Funda#6. As 18 is not divisible by 4, ab=68.

Hence, the last two non zero digit of 11! are 68.

Find the last two non zero digits of 25!
The highest power of 5 in 25! = 5 + 1

(25-5) * (5-1) * (1)
(20) * (4) * (1)
(16 + 4) * (0 + 4) * (1)
(76 * 21 * 22 * 23 * 24) * (1 * 2 * 3 * 4) * (1)
(76 * 24) * (24) * (1)
24 * 24 * 1
76
Therefore, (25!)` = x76

2^6 ends in 64.
x76 = 64 * xab
xx6 = x4 * xxb Therefore, b=4.
x76 = 64 * xa4
(1 + 4a) + (4) = x7 (4*4 = 16. Carry=1. 4*a=4. Therefore, (1 + 4a). 6*4 = 24. Therefore, (4))
4a = x2. a is 3 or 8.

As 34 is not divisible by 4 the last two non zero digit of 25! are 84.

Find the last two non zero digits of 36! - 24! posted by **Total Gadha**
The total number of zeroes present at the end of 36! = 7 + 1 = 8
The total number of zeroes present at the end of 24! = 4

36! - 24! = x 000 000 – x ab0 000
Therefore, we need to find the last two non zero digits of 24!
(20) * (4)
(76 * 21 * 22 * 23 * 24) * (1 * 2 * 3 * 4)
(76 * 24) * (24)
76

x76 = x16 * ab
xx6 = xx6 * b. b=6.
x76 = x16 * a6
(3 + 6a) + (6) = x7   (6*6 = 36. Carry=3. 6*a=6a. Therefore, (3 + 6a). 1*6 = 6. Therefore, (6))
6a + 9 = x7
6a = (x-1)8 (Don't 7 – 9 = -2. Do x7 – 9 = (x-1)8.)
a = 3 or 8. Therefore the last two non zero digits of 24! are 36.

Hence, the required digits = 00 – 36 = 64.

How many zeroes are present at the end of 25! + 26! + 27! + 28! + 30!? CAT Number System Quiz
Let's find last two non zero digits of 25!
5 + 1= 6

(20) * (4)
(76 * 21 * 22 * 23 * 24) * (1 * 2 * 3 *4)
(76 * 24) * (24)
76

x76 = x64 * ab
xx6 = xx4 * b Therefore, b=4
x76 = x64 * a4
(1 + 4a) + (4) = x7   (4*4 = 16. Carry=1. 4*a=4a. Therefore, (1 + 4a). 6*4 = 24. Therefore, (4))
4a = x2
a=8
Therefore, the last two non zero digits of 25! are 84 and it has 6 trailing zeroes.

25! * (1 + 26 + 26 * 27 + 26 * 27 * 28 + 26 * 27 * 28 * 30)
25! * (1 + 26 + x02 + x56 + x80)
25! * x65
(x84 000 000) * (x65)
x0 000 000
Hence, the given expression has 7 zeroes at the end.

Compiled By:-

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