* $ax^2+bx+c=0.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad b^2-4ac \rightarrow \text{discriminant}$$

$\downarrow$

Δ

Sum of roots $= -b/2a$
Product of roots $= c/a$.

* Nature of roots :-

$a, b, c \rightarrow$ real numbers.

$\Delta<0 \rightarrow$ roots are complex & unequal.
$\Delta=0 \rightarrow$ real & equal
$\Delta>0 \rightarrow$ real & unequal.

*

Condition	Roots	Property
- $\Delta<0$, $\Delta\neq ps$ ~~$\Delta \neq ps$~~	$p+iq, p-iq$	$p\rightarrow$ rational, $q\rightarrow$ irrational
$\Delta<0$, $\Delta=ps$	$p+iq, p-iq$	$p,q\rightarrow$ rational.
$\Delta=0$	$-b/2a, -b/2a$	Rational, Equal.
$\Delta>0$, $\Delta=ps$	$\frac{-b+\sqrt{\Delta}}{2a}, \frac{-b-\sqrt{\Delta}}{2a}$	Rational, unequal.
$\Delta>0$, $\Delta\neq ps$	$\frac{-b+\sqrt{\Delta}}{2a}, \frac{-b-\sqrt{\Delta}}{2a}$	Conjugate surds.

* Sign of Roots :-

Product	Sum	Sign
+	+	both roots +ve.
+	−	both roots −ve
−	+	numerically larger root +ve, other root −ve
+	−	numerically larger root +ve, other root +ve.

* Max or Min value of a quadratic expression:-

ax^2+bx+c

If $a>0$ $\cup \rightarrow$ min value $= \frac{4ac-b^2}{4a}$ $x=\frac{-b}{2a}$

If $a<0$ $\cap \rightarrow$ max value $= \frac{4ac-b^2}{4a}$ $x=\frac{-b}{2a}$.

* Remainder Theorem:-

$p(x) \rightarrow$ divided by $x-a$ Remainder $p(a)$.

Ex Find remainder when x^2+6x+8 is divided by $x+4$.

$(-4)^2+6\times(-4)+8$

$=16+8-24=0$.

Ex Find remainder when x^2+6x+8 is divided by $2x+1$.

$(-\frac{1}{2})^2+6\times(-\frac{1}{2})+8$

$=\frac{1}{4}-\frac{6}{2}+8$

$=21/4$.

Ex * A quadratic polynomial in x leaves remainder as 4 & 7 respectively when divided by $(x+1)$ & $(x-2)$. Also it is exactly divisible by $x-1$. Find the quadratic polynomial.

ax^2+bx+c

$a-b+c=4$

$4a+2b+c=7$.

$a+b+c=0$.

$\Rightarrow a+c=2$.

$2a-c=7$.

$a=3$. $c=-1$ $b=-2$.

$\therefore 3x^2-2x-1$

__/__/____

* Find the common factor of $3x^2-x-10$ & $2x^2-x-6$.

Let the common factor be k.

$$3k^2-k-10 = 2k^2-k-6.$$

$$\Rightarrow k^2 = 4$$

$$\Rightarrow k = \pm 2.$$

$3x^2 - 6x + 5x - 10.$

$(3x+5)(x-2)$

$2x^2 - x - 6 = 0,$

$2x^2 - 4x + 3x - 6,$

$(2x+3)(x-2)$.

Alt:

$$\frac{x+10}{3} = \frac{x+6}{2}.$$

$$\Rightarrow 2x+20 = 3x+18$$

$$\Rightarrow x = 2.$$

$\therefore$ Check whether $x=2$ is satisfying both eqⁿ or not. Then only it is a common factor.

* Relation b/w Roots & Coefficients:-

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0$$

$$S_1 = -a_{n-1}/a_n$$

$$S_2 = \frac{a_{n-2}}{a_n}$$

$$S_m = \sum \alpha_1\alpha_2\dots\alpha_m = (-1)^m \frac{a_{n-m}}{a_n}.$$

Ex * If $3+\sqrt{5}$ is one root of eqⁿ with rational coefficients, then find the other root of eqⁿ.

$3-\sqrt{5}$ Conjugate Surds.

* $\boxed{(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta}$ V.V. Imp.

__/__/____

Ex $3^{x+1} + 3^{2x+1} = 270$. Find x.

$3 \cdot 3^x + 3 \cdot 3^{2x} = 270$. $\quad 3^x \to a$.

$3a + 3a^2 = 270$.

$\Rightarrow a^2 + a - 90 = 0$.

$\Rightarrow a^2 + 10a - 9a - 90 = 0$.

$\Rightarrow (a-9)(a+10) = 0$. $\quad a = 9 \quad x = 2$.

$\quad -10 \to$ Not possible.

* What can be concluded about roots of $x^3 - 7x + 6 = 0$, on basis of Descarte's rule of signs.

$x^3 - 7x + 6 = 0$.

$\longrightarrow$ 2 sign changes

No. of +ve roots $\to$ 2 or 0.

$f(-x) = -x^3 + 7x + 6$

$\longrightarrow$ 1 sign change.

No. of -ve root $\to$ 1.

* How many real roots does the eq^n have?

$x^4 - 4x^3 + 3x^2 + 2x - 6 = 0$.

$x^4 - 4x^3 + 3x^2 + 2x - 6 = 0$. $\quad \to 3 \qquad$ +ve $\to$ 3, 1.

$+x^4 + 4x^3 + 3x^2 - 2x - 6 \quad \to 1 \qquad$ -ve $\to$ 1.

$f(x) = \underbrace{(x+1)}_{-ve}\ \underbrace{(x-3)}_{+ve}\ \underbrace{(x^2 - 2x + 2)}_{\text{2 complex roots.}}$ $\quad \Rightarrow$ have to solve for this by trial & error.

Q-Type

quadratic eqⁿ in denominator.

Ex Find range of expression $\dfrac{x+2}{x^2+5x+7}$, when x is real.

$$\frac{x+2}{x^2+5x+7} = k.$$

$$\Rightarrow kx^2+(5k-1)x+7k-2=0.$$

$$\Delta \geq 0.$$

$$\Rightarrow (5k-1)^2 - 4\times k\times(7k-2) \geq 0.$$

$$\Rightarrow 25k^2+1-10k-28k^2+8k \geq 0.$$

$$\Rightarrow -3k^2-2k+1 \geq 0.$$

$$\Rightarrow 3k^2+2k-1 \leq 0.$$

$$\Rightarrow 3k^2+3k-k-1 \leq 0.$$

$$\Rightarrow (3k-1)(k+1) \leq 0$$

$+ \quad -1 \quad - \quad 1/3 \quad +$

Range :- $[-1, \tfrac{1}{3}]$

Ex Solve the eqⁿ $8x^4-2x^3-27x^2+6x+9=0$, 2 of the roots being equal but opposite in sign.

$\alpha \quad -\alpha \quad \beta \quad \gamma.$

$$\beta+\gamma = 2/8 = 1/4.$$

$$-\alpha^2+\alpha\beta+\alpha\gamma-\alpha\beta-\alpha\gamma+\beta\gamma = -27/8 \Rightarrow -\alpha^2+\beta\gamma = -27/8$$

Find pair of 3 → 3 → $\alpha\beta\gamma - \alpha^2\beta - \alpha\beta\gamma - \alpha^2\gamma = -6/8.$

2 doesn't simplify.

$$\Rightarrow -\alpha^2(\beta+\gamma) = -6/8$$

$$\Rightarrow \alpha^2 \times \tfrac{1}{4} = 6/8$$

$$\Rightarrow \alpha^2 = \frac{24}{8} = 3 \quad \Rightarrow \alpha = \pm\sqrt{3}.$$

4 → $-\alpha^2\beta\gamma = 9/8 \Rightarrow \beta\gamma = -3/8 \quad \beta+\gamma = 1/4. \quad \therefore \beta = 3/4 \quad \gamma = -1/2.$

Ex The sum of square of root of eq^n $x^3+3x^2+2x-4=0$ is.

$$\alpha^2+\beta^2+\gamma^2$$
$$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$=(-3)^2-2\times 2.$$
$$=9-4=5.$$

Ex Find remainder when x^{999} is divided by x^2-4x+3.

$$x^{999}=(x-3)(x-1)\,q(x)+\underbrace{ax+b}_{\text{Remainds.}}$$

$a+b=1.$

$3a+b=3^{999}.$

$\Rightarrow 2a=3^{999}-1$

$\Rightarrow a=\frac{1}{2}\left(3^{999}-1\right).$

$b=1-\frac{1}{2}\left(3^{999}-1\right)$

$=\frac{3}{2}-\frac{1}{2}\times 3^{999}$

$=\frac{3}{2}\left(1-\frac{3^{998}}{2}\right)$

$\therefore$ Remainder $\rightarrow$ $\frac{1}{2}\left(3^{999}-1\right)x+\frac{3}{2}\left(1-\frac{3^{998}}{2}\right).$

Ex Find remainder when x^5 is divided by x^3-4x.

$$x^5=x(x+2)(x-2)\,p(x)+\underbrace{ax^2+bx+c}_{R}$$

$c=0.$

$4a+2b+c=32.$ $\quad\Rightarrow\quad 2a+b=16$

$4a-2b+c=-32.$ $\quad\quad 2a-b=-16$

$\therefore\ a=0.$

$b=16.$

$\therefore$ Remaindr $\rightarrow 16x.$

__/__/__

Ex Solve. $3^{2x+1} - 39(3^{x+2}) + 8748 = 0.$

$$3\times 3^{2x} - 39\times 9\times 3^{x} + 8748 = 0.$$

$$3y^2 - 39\times 9\times y + 8748 = 0.$$

Ex If the eqns $x^2+7ax+10=0$ & $x^2+3bx-85=0$, where a & b are integers, have a common root, then value of b can be
(a) 4 (b) 2 (c) -2 (d) None of these.

$$7ax+10 = 3bx-85$$

$$\Rightarrow (3b-7a)x = 95.$$

$$\Rightarrow x = \frac{95}{3b-7a}.$$

$x^2+7ax+10=0$ $\qquad\qquad$ $x^2+3bx-85=0.$

1 10 $\qquad\qquad$ 1 85

± 2 ± 5 $\qquad\qquad$ ± 5 $\pm 17.$

A. $3b = \pm 12$

$\Rightarrow b = \pm 4.$

Ex Let m & n be the roots of $x^2-(a+2)x+a+1=0$. What is the minimum value of m^2+n^2?

$$m^2+n^2 = (m+n)^2 - 2mn$$

$$= (a+2)^2 - 2(a+1)$$

$$= a^2+4a+4-2a-2$$

$$= a^2+2a+2 \qquad = (a+1)^2+1$$

Ans $\rightarrow$ 1

Ex If difference between roots of quadratic eqn $x^2-ax+2a=0$, which is real, is less than $2\sqrt{3}$, find the range of a.

$$\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$\Rightarrow \sqrt{a^2 - 4\times 2a} < 2\sqrt{5}$

$\Rightarrow a^2 - 8a < 20$

$\Rightarrow (a-10)(a+2) < 0$ $\quad + \;\; -2 \;\; - \;\; 10 \;\; +$

~~Ans~~ $(-2, 10)$

Then $\Delta \geq 0$.

$a^2 - 8a \geq 0 \qquad a(a-8) \geq 0$ $\quad + \;\; 0 \;\; - \;\; 8 \;\; +$

Ans $\rightarrow (-2, 0] \cup [8, 10)$.

Ex In an quadratic eqⁿ $ax^2+bx+c=0$, $a=1$, $b=c$ and both roots are integers. Difference b/w roots is :-

x^2+bx+b.

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{b^2 - 4b}$

$\Delta \geq 0$

$b^2 - 4b \geq 0$ $\quad + \;\; 0 \;\; - \;\; 4 \;\; +$

$\alpha + \beta + \alpha\beta = -b + b = 0$.

$\Rightarrow \beta = \dfrac{-\alpha}{\alpha+1}$ $\qquad \alpha = -2 \quad \beta = -2$

$\beta = \alpha = 0 \quad \beta = 0$.

$\therefore$ Difference $= \boxed{0}$.

Ex Find value of $\cfrac{1}{1+\cfrac{1}{-4+\cfrac{1}{1+\cfrac{1}{-4+\dots}}}}$

$x = \cfrac{1}{1+\cfrac{1}{-4+x}} \Rightarrow x = \dfrac{x-4}{x-3}$

$\Rightarrow x(x-3) = x-4.$

$\Rightarrow x^2-4x+4=0. \quad \Rightarrow x = 2.$

Ex Giridhar & Harish noted down a quadratic eqn from the blackboard. Giridhar made an error in noting down coefficient of x and he got 5 and -3 as roots, while Harish made an error in noting down the constant term and got -5 and 7 as the roots. What are the roots of actual equation?

$$ax^2+bx+c=0.$$

Pdt of roots :- $\alpha\beta = -15$

$\alpha+\beta = 2.$

Ans $\rightarrow$ $-3, 5$.

Ex The hypotenuse of a right $\angle \Delta$ is 2 more than twice of one of the other sides while the 3rd side is 13 more than half of hypotenuse. Find length of median to hypotenuse.

$a \quad b \quad c \rightarrow$ hypotenuse

$c = 2+2a.$

$b = 13+ c/2.$

$a^2+b^2 = c^2.$

$\Rightarrow \left(\frac{c-2}{2}\right)^2 + \left(13+c/2\right)^2 = c^2.$

$\Rightarrow \frac{\cancel{c^2}-4c+4}{4} + \quad c = 34.$

Median to hypotenuse is half of hypotenuse.

Median $= c/2 = \underline{\underline{17}}.$

Ex. Let $g(y) = py^2 + qy + r$, where p, q, r are constants, $p \neq 0$ and $3g(8) = -4g(2)$. The roots of $g(y) = 0$ are 6 and m.

(i) Find m.

$$3 \times (64p + 8q + r) = -4\,(4p + 2q + r)$$
$$\Rightarrow 192p + 24q + 3r = -16p - 8q - 4r.$$
$$\Rightarrow 208p + 32q + 7r = 0.$$

$$36p + 6q + r = 0. \Rightarrow 144p + 24q + 4r = 0.$$

$$64p + 8q + 3r = 0. \quad 192p + 24q + 9r = 0.$$
$$48p + 5r = 0. \quad \Rightarrow 48p = -5r$$
$$\Rightarrow \frac{r}{p} = \frac{-48}{5}$$

$$6 + m = \frac{-q}{p} \qquad 6m = \frac{r}{p}$$

$$\therefore 6 \times m = \frac{-48}{5}$$

$$\Rightarrow m = -\frac{8}{5} = -1.6.$$

(ii) Find g(1).

$p + q + r \Rightarrow$ can't be determind as we don't have values for unique p, q & r.

Ex. How many common roots do $x^3 - 7x^2 + 6x + 1 = 0$ & $x^3 - 6x^2 + x + 7 = 0$ have?

$$7x^2 - 6x - 1 = 6x^2 - x - 7.$$
$$\Rightarrow x^2 - 5x + 6 = 0.$$
$$\Rightarrow x = 3, 2. \qquad 3^3 - 7 \times 9 + 6 \times 3 + 1 \neq 0.$$
$$2^3 - 7 \times 4 + 6 \times 2 + 1 \neq 0.$$

As both 3 & 2 are not satisfying the eqns, so no soln possible.

__Ex__ ~~The roots~~ A student by trial discovers one -ve and 2 +ve roots of $x^5-9x^4+13x^3+57x^2-86x-120=0$. How many non-real roots does the eq^n have ?

$x^5-9x^4+13x^3+57x^2-86x-120=0.$ → +ve roots → 3.

$- \quad - \quad - \quad + \quad + \quad -$ → -ve roots → 2.

∴ Non-real roots → 0.

__Ex__ The remainder of $x^{83}+x^{65}+x^{47}+x^{29}+x^{20}+x^{11}+x^5$ is divided by x^5-x^3 is.

$f(x) = (x^5-x^3)\,q(x) + \underline{ax^4+bx^3+cx^2+dx+e}$

$f(0)=0 \Rightarrow \cancel{a+b+c+d}+e=0$

$f(1)=7 \Rightarrow a+b+c+d+\cancel{e}=7 \Rightarrow a+b+c+d=7.$

$f(-1)=-5 \Rightarrow a-b+c-d=-5$

∴ $a+c=1$ if $a=1$, $c=0$

$b+d=6$ Check options.

__Alt.__

$x^5-x^3 = x^3(x^2-1)$

$f(x) = \underbrace{(x^{80}+x^{62}+x^{44}+x^{26}+x^{17}+x^8+x^2)}_{g(x)}\,x^3.$

$g(x) = (x^2-1)\,q(x) + \underline{\underline{ax+b}}$

$a+b=7.$ ∴ $a=1,\ b=6.$

$-a+b=5$

Ans → $\underline{\underline{x^3(x+6)}}$.

Q-type

Ex $x+\frac{2}{x}=\sqrt{6}$, find $x^6+x^{12}+x^{18}$.

$x+\frac{2}{x}=\sqrt{6}$.

$x^3+\frac{8}{x^3}+3\times x^2\times\frac{2}{x}+3\times x\times\frac{4}{x^2}=6\sqrt{6}$.

$\Rightarrow x^3+\frac{8}{x^3}+6x+\frac{12}{x}=6\sqrt{6}$.

$\Rightarrow x^3+\frac{8}{x^3}+6\left(x+\frac{2}{x}\right)=6\sqrt{6}$

$\Rightarrow x^3+\frac{8}{x^3}+6\sqrt{6}=6\sqrt{6}$.

$\Rightarrow x^3+\frac{8}{x^3}=0$.

$\Rightarrow x^6=(-8)$.

$\therefore$ Ans $\rightarrow -8+64-512$

$=-456$.

Ex If the roots of quadratic eqⁿ $cx^2-bx+4=0$ are in the ratio $c:4$ and $c\neq b-4$, then $c=$

$\frac{\alpha}{\beta}=\frac{c}{4}$ $\alpha=kc$ $\beta=k\times4$.

$4ck^2=\frac{4}{c} \Rightarrow c=\pm\frac{1}{k}$.

$k(4+c)=\frac{b}{c}$

$\Rightarrow 4+c=\frac{b}{c}\times\frac{1}{k}=\frac{b}{c}\times\pm c$

$\Rightarrow 4+c=\pm b$

$\Rightarrow c=\pm b-4$. Ans $\rightarrow -b-4$.

Ex If sum of roots of $x^2 - 5(x^{4\log_x k})x + k = 0$ is 80, then what is/are the possible value of k ?

$x^2 - 5k^4 x + k = 0$.

$\Rightarrow 5k^4 = 80$

$\Rightarrow k = \pm 2$.

Ex Mr Girish has written a number A, such that $B = A - 1$, where B is the product of 4 consecutive +ve integers. Which of the following statements is/are true regarding A ?.

(i) It is even
(ii) It is prime
(iii) It is odd.
(iv) It is a perfect square.

$B \rightarrow n(n+1)(n+2)(n+3)$ $\qquad A = B+1$

B has to even $\Rightarrow$ A has to be odd.

(i) $\rightarrow$ ~~defin~~ ✗

(iii) $\rightarrow$ ✓

$B \rightarrow (n-1)\, n\, (n+1)(n+2)$

$= (n^2-1) \times n\, (n+2)$

$= (n^3 - n^2)(n+2)$

$= n^4 + 2n^3 - n^2 - 2n$

$A = (n-1)\, n\, (n+1)(n+2) + 1$ $\qquad n^2+n-1 \rightarrow t$

$= (n^2+n-2)(n^2+n) + 1$

$= (t-1)(t+1) + 1$

$= t^2$ ..

$\therefore$ (ii) ✗

(iv) ✓

$\_\_/\_\_/\_\_$

Ex How many real roots are possible for the eq^n $\frac{A}{x}+\frac{B}{x+2}=4$, where A & B are 2 +ve integers.

$$xA+2A+B=(x^2+2x)4$$

$$\Rightarrow 4x^2+(8-A)x-2A-B=0.$$

$$\Delta=(8-A)^2+4\times4\times(2A+B)>0.$$

$\therefore$ Ans $\rightarrow$ 2 real roots.

Ex The eq^n $14x^2+px+1=0$ has 2 roots, one root being $\frac{5}{14}$ more than the other root. If $p>0$, find p.

$$\alpha-\beta=\frac{5}{14}.$$

$$\cancel{(\alpha-\beta)^2=\frac{25}{256}}$$

$$\cancel{\Rightarrow\left(\frac{p}{14}\right)^2-4\times\frac{1}{14}=\frac{25}{256}}$$

$$\alpha+\beta=\sqrt{(\alpha-\beta)^2+4\alpha\beta}$$

$$=\sqrt{\left(\frac{5}{14}\right)^2+4\times\frac{1}{14}}$$

$$=\sqrt{\frac{25}{196}+\frac{4}{14}}$$

$$=\sqrt{\frac{81}{196}}=\pm\frac{9}{14}.$$

$\therefore p=\pm9$ as $p>0$ $p=9$.

Ex If the quadratic eq^n $ax^2+bx+c=0$ & $bx^2+cx+a=0$, where a,b,c are distinct, have one common root, then the common root is ?.

$$\frac{-bx-c}{a} = \frac{-cx-a}{b}$$

$\Rightarrow b^2x + bc = \cancel{c^2ax}\ cax + a^2.$

$\Rightarrow x = \dfrac{a^2-bc}{b^2-ca}.$

not helping.

$ax^2+bx+c=0 \quad \ldots (1)$

$bx^2+cx+a=0 \quad \ldots (2)$

$$(cb-a^2)^2 = (ac-b^2)(ab-c^2)$$

$\Rightarrow \cancel{c^2b^2} + a^4 - 2a^2bc = a^2bc - ac^3 - ab^3 + \cancel{b^2c^2}$

$\Rightarrow a^3 - 2abc = abc - b^3 - c^3$

$\Rightarrow a^3+b^3+c^3 = 3abc \qquad \Rightarrow a+b+c=0.$

By introspection, $x=1$.

Ex Find min. value $\dfrac{2x+5}{2x^2+6x+7}$, where x is real.

$$\frac{2x+5}{2x^2+6x+7} = k.$$

$\Rightarrow 2kx^2 + (6k-2)x + (7k-5) = 0.$

$\Delta \geq 0.$

$\Rightarrow (6k-2)^2 - 4\times 2k \times (7k-5) \geq 0.$

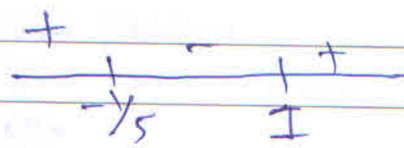
$\Rightarrow 36k^2 + 4 - 24k - 56k^2 + 40k \geq 0.$

$\Rightarrow 20k^2 - 16k - 4 \leq 0.$

$\Rightarrow 5k^2 - 4k - 1 \leq 0.$

$\Rightarrow 5k^2 - 5k + k - 1 \leq 0.$

$\Rightarrow (5k+1)(k-1) \leq 0.$



Ans $\rightarrow -1/5$.

Q-Type **Ex** Find the real roots of eq^n $(x+5)(x+6)(x+7)(x+8) = 840$.

$$\underline{(x+5)}\,\overline{(x+5)\,(x+7)}\,\underline{(x+8)} = 840.$$

$$(x^2+13x+40)(x^2+13x+42) = 840.$$

$$\Rightarrow t(t+2) = \underline{\underline{840}} \qquad 2\times2\times2\times3\times5\times7$$

$t \to 28 \qquad t+2 \to 30$

$t \to -30 \qquad t+2 \to -28.$

$x^2+13x+40 = 28.$ $\qquad$ $x^2+13x+40 = -30$

$\Rightarrow x^2+13x+12 = 0.$ $\qquad$ $\Rightarrow x^2+13x+70 = 0.$

$\Rightarrow (x+1)(x+12) = 0.$ $\qquad$ $\downarrow$ no real roots.

Ans $\to -1, -12.$

Ex How many real roots does the eq^n $x^4+2x^3-10x^2-2x+1=0$ have ?

$x^4+2x^3-10x^2-2x+1=0.$ $\qquad$ $+2 \to$ max +ve root

$+ \;\; - \;\; - \;\; + \;\; +$ $\qquad$ $2 \to$ max −ve root.

nothing can be ascertained. We have to solve the eq^n.

$$x^4+2x^3-10x^2-2x+1=0.$$

$$\Rightarrow x^2+2x-10-\frac{2}{x}+\frac{1}{x^2}=0.$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)+2\left(x-\frac{1}{x}\right)-10=0.$$

$$\Rightarrow t^2+2+2t-10=0.$$

$\Rightarrow t = -4$ or 2. Then solve & check.

__/__/__

Ex Let $f(x) = a_{12}x^{12} + a_{10}x^{10} + a_8x^8 + a_6x^6 + \dots + a_2x^2 + a_0$
The coefficients a_{12}, a_{10}, a_2, a_0 are real. There are 3 sign changes of $f(x)=0$ & $f(x)=0$ has 4 non-real roots. Which of the following is true?

(a) $a_{10}=0$ (b) $a_{10}>0$ (c) $a_{10}<0$ (d) $a_0=0$.

4 non-real roots $\Rightarrow$ 8 real roots
no difference b/w $f(x)$ & $f(-x)$, as all powers are even.

~~∴ 4 +ve real root~~ . ~~4 -ve root.~~

12	10	8	6	4	2	0
+	−	+	−	+	+	+
+	+	−	+	−	+	+

3 sign changes in $f(x)$. 3 sign changes in $f(-x)$.

∴ 6 real root + 4 non-real $\Rightarrow$ Min. 2 roots are 0.

$\Downarrow$

$\boxed{a_0 = 0.}$

Ex The remainders of a polynomial $f(x)$ in x are 10 & 15 respectively when $f(x)$ is divided by $(x-3)$ & $(x-4)$. Find the remainder when $f(x)$ is divided by $(x-3)(x-4)$.

$$f(x) = (x-3)(x-4)\,q(x) + ax+b.$$

$3a+b=10$ $\quad 4a+b=15$ $\quad \Rightarrow a=5,\ b=-5$

∴ Ans $\to$ $5x-5$.