* $8x + 15y = 153$ $x > 0,\ y > 0.$ Find x, y.

$$x = \frac{153 - 15y}{8}$$

$153 - 15y \rightarrow$ multiple of 8

$\Downarrow$

y has to be odd.

$$\boxed{y \rightarrow 7 \qquad x = 6} \rightarrow \text{sol}^n$$

* Ramya told Krishna "12 times the date of my birth added to 31 times month of my birth is 376. On which date was Ramya born?

$$12D + 31M = 376 \qquad D \leq 31 \quad M < 12.$$

$$\Rightarrow D = \frac{376 - 31M}{12} \qquad M \text{ has to be even}$$

$$\therefore M = \cancel{24} \quad 21 \quad \therefore \text{ Ans} \rightarrow \underline{21}.$$

other approach

$$\frac{376 - 31M}{12} = \frac{4 - 7M}{12}$$

$$12D + 31M = 376.$$

$$\frac{12D}{12} + \frac{24M}{12} + \frac{7M}{12} = \frac{372}{12} + \frac{4}{12}.$$

$$\frac{7M - 4}{12} = K.$$

$$M = (12k + 4)\frac{1}{7} \qquad 12k + 4 < 84$$

$$K \rightarrow 2 \qquad M \rightarrow 4$$

* Atal & Bharat have a collection of less than 50 stamps together. If Bharat gives a certain no. of stamps, then Atal has 5 times as many as Bharath has instead. If Atal gives the same no. of stamps to Bharath, then Atal will now have 3 times as many as Bharath. The no. of stamps with Atal can be

(i) 18   (ii) 16   (iii) 38   (iv) 15.

$$a + b < 50$$

$$5(b-x) = a+x \quad \Rightarrow \; 5b - a = 6x \quad 6x = 5b - a$$

$$3(b+x) = a - x \quad \Rightarrow \; 3b + a \quad 4\cdot 2x = -a - 3b$$

$$\therefore \; 5b - a = 3a - 9b$$

$$\Rightarrow 14b = 4a$$

$$\Rightarrow 7b = 2a.$$

$a \rightarrow$ multiple of 7

$b \rightarrow$ multiple of 2

	a	b
	7	2

$$10b - 2a = 3a - 9b$$

$$\Rightarrow 19b = 5a$$

$a \rightarrow$ multiple of 19

$b \rightarrow$ multiple of 5.

a	b
19	5
38	10

$$\therefore \; a \text{ can be } 38.$$

* Average weight of a certain group of 'n' men is 75 kg. 3 men whose weights are 80 kg, 76 kg & 74 kg. join the group and one man whose weight is between 90 kg & 100kg leaves the group. Avg. age of group has been come down by 2 kg. If the no. of men initially is a perfect square, then the weight of man who left is

(a) 98 kg   (b) 96 kg   (c) 94 kg   (d) 92 kg.

$$\frac{75n + 80 + 76 + 74 - x}{n + 3 - 1} = 73$$

$$\Rightarrow 75n + 230 - x = 73(n+2). \qquad n \rightarrow \text{perfect square}$$

$$\Rightarrow 2n + 84 = x.$$

$$90 \leq 84 + 2n \leq 100$$

$$\Rightarrow 6 \leq 2n \leq 16$$

$$\Rightarrow 3 \leq n \leq 8 \qquad \therefore n = 4.$$

$$\therefore x = 2 \times 4 + 84 = 92.$$

* In an organization, there are totally 38 employees belonging to 3 different departments – A, B & C with each department having more than 5 employees. The organisation decides to pay a bonus of Rs 11,000, Rs 6000 & Rs 3000 to each of the employees of dept. A, B & C respectively. If total amount paid to the employee as bonus is Rs 3,33,000, then no. of employees in Dept 'C' is

(a) 6   (b) 7   (c) 14   (d) 15.

$$11a + 6b + 3c = 332 \qquad a + b + c = 38$$
$$\Rightarrow 11(38 - b - c) + 6b + 3c = 332 \qquad \Rightarrow b + c = 38 - a$$
$$\Rightarrow 418 - 5b - 8c = 332. \qquad \Rightarrow a = 38 - b - c$$
$$\Rightarrow 5b + 8c = 86.$$
$$\Rightarrow b = \frac{86 - 8c}{5} \qquad \begin{array}{c} C \to 2. \\ C \to 7 \end{array} \Big\| \begin{array}{c} X \quad \times 5 \\ \searrow \; 7 - 2 = 5 \; \checkmark \end{array}$$

$$\therefore C \to 7 \qquad b \to 6 \qquad a = 25.$$

* In a △, all the ∠s are acute. 19 times an ∠ equals 15 times the other. What is least possible ∠ in degrees?

$$\alpha, \beta, \gamma < 90°.$$

Let's assure $19\alpha = 15\beta$.

$\alpha \to$ multiple of 15
$\beta \to$ multiple of 19.

α	β	γ	
15	19	>90	X
30	38	>90	X
45	57	78	
60	76	(44)	
	95		X

Ans → 44°

※ How many 3 digit no.s leave a remainder of 4, when divided by 21 and a remainder of 8 when divided by 13

$$21k_1 + 4 = 13k_2 + 8.$$

$21k - 4 \rightarrow$ multiple of 13.

$k \rightarrow 7$ $\boxed{143}$

∴ Form :- $LCM(21,13) + 143 + 8$

$= 273 + 151$ $\boxed{x > 1}$

$x \rightarrow 4$ ~~970~~ ∴ Ans → 4.

※ Airtite bolts & Nuts manufacturer of precision bolts have a unit which produce 15mm bolts. Turner, a worker in that unit is supposed to pack less than 252 bolts into boxes. Turner finds that if he can pack 5 bolts less per box, he can pack 10 boxes more per day. How many bolts does he pack per day given that he packs atleast 16 boxes per day.

no. of boxes ⟼ n
no. of bolts in a box → b.

$$nb = (n+10)(b-5) < 252.$$

$\Rightarrow nb = nb - 5n + 10b - 50.$

$\Rightarrow 5n - 10b = -50.$

$\Rightarrow 10b - 5n = 50.$

$\Rightarrow 2b - n = 10.$

$\Rightarrow 2b = \dfrac{10 + n}{2}.$ $n \rightarrow$ even Integer.

~~5×16~~ ∴ ~~17×22.~~

$\Rightarrow n = 2b - 10$ $n \geqslant 16$ ∴ $b \geqslant 13.$

n	b	
(16	13)	208
18	14	252 X

∴ Ans → 208.

* Sanghamitra goes to a stationary shop to buy some scale, pencils & crayons. She decided to buy twice as many crayons as pencils and at least one pencil more than scales. Also, she finds that pencils are thrice as expensive as scales and scales are at half the price of crayons. If each scale cost Re 1 & Sanghamitra spends a total of Rs 47, how many items did she purchase in all?

$$S \quad p \quad c \qquad no.\rightarrow \quad S \qquad p \qquad c$$
$$c = 2p \qquad\qquad\qquad\qquad S \qquad p \qquad 2p$$
$$p \geqslant S+1 \qquad cost\rightarrow \quad 1 \cdot \quad 3 \cdot \quad 2 \cdot$$

$$S + 3p + 4p = 47.$$
$$\Rightarrow S + 7p = 47 \qquad \Rightarrow S = 47 - 7p$$

	S	p	c/2p.
✗	40	1	
✗	33	2	
✗	26	3	
✗	19	4	
✗	12	5	
Ans → (23) ←	5	6	⑫ 12

* Ramesh went to a bank with a cheque. He had no money with him. He handed over the cheque to one of the bank tellers. The bank teller was in a confused state of mind and cashed the cheque. He gave Ramesh as many Rupees as those of paise he had to give and as many paise as those of Rupees he had to give. Ramesh took the amount and left the bank. He then bought a chocolate for 75 paise from a shop nearby. He then observed that he was left with an amount which was the excess of amount on the cheque over the amount he received from the teller. Find the sum of no. of rupees and paise on the cheque.

Cheque $\rightarrow$ r p

$100p + r - 75 = 100r + p - (100p + r)$.

$\Rightarrow 200p + 2r - 75 = 100r + p$.

$\Rightarrow 199p - 75 = 98r$.

$\Rightarrow \cancel{p} = \cancel{\dfrac{98r + 75}{9}}$

$\Rightarrow r = \dfrac{199p - 75}{98}$

$\qquad = 2p + \dfrac{3p}{98} - \dfrac{75}{98}$.

$\qquad = 2p + \dfrac{3p - 75}{98}$.

$3p - 75 \rightarrow$ multiple of 98.

$p < 100 \qquad p \rightarrow 25, \;\underset{\times}{12}3. \qquad 123 - 25 = 98$

$p \rightarrow 25$.
$r \rightarrow 50$. $\qquad \therefore$ Ans $\rightarrow 50 + 25 = 75$.

※ How many ordered pairs of +ve integers (x, y) satisfy the eq^n $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{12}$?

$\dfrac{x + y}{xy} = \dfrac{1}{12}$.

$\Rightarrow 12x + 12y = xy$.

$\Rightarrow 12x - xy + 12y = 0$.

$\Rightarrow \cancel{x(12 - x) + yr}$

$\Rightarrow xy - 12x - 12y = 0$.

$\Rightarrow x(y - 12) - 12(y - 12) = -144$.

$\Rightarrow (x - 12)(y - 12) = 144$

$$144 \to 2 \times 2 \times 2 \times 2 \times 3^2 = 2^4 \times 3^2$$

Ordered pairs $\to$ $5 \times 3 = \underline{15}$

1, 144   9, 16
2, 72   12, 12
3, 48   16, 9
4, 36
6, 24
8, 18

* How many ordered pairs of integers (x, y) where $y > 0$ satisfy the eq^n $\frac{5}{x} + \frac{1}{y} = \frac{1}{15}$?

$$\frac{5}{x} + \frac{1}{y} = \frac{1}{15}$$

$$\Rightarrow \frac{5y + x}{xy} = \frac{1}{15}$$

$$\Rightarrow xy = 75y + 15x$$

$$\Rightarrow x(y-15) - 75(y-15) = 0$$

$$\Rightarrow (x-75)(y-15) = 0 \qquad 75 \times 15 = 3 \times 5^2 \times 3 \times 5 = 3^2 \times 5^3.$$

$$y > 0.$$

$$\Rightarrow y - 15 > \underline{-15}$$

+ve $\to$ $4 \times 3 = 12$. $\qquad\qquad x - 75 = 75$

± 1	
± 3	
± 5	4
± 9	
± 15	

Ans$\to$ $12 + 4 = 16$.

* How many ordered pair of integers satisfy the eq^n $\frac{3}{x} - \frac{11}{y} = \frac{1}{4}$?

$$(3y - 11x) \, 4 = xy$$

$$\Rightarrow xy + 44x - 12y = 0.$$

$$\Rightarrow x(y+44) - 12(y+44) = -12 \times 44.$$

$$\Rightarrow (x-12)(y+44) = -2 \times 2 \times 3 \times 2 \times 2 \times 11$$

$$= -2^4 \times 3 \times 11$$

+ve → 5 × 2 × 2 = 20
−ve →   20

But x−12 = 12   y+44 = 44. ⇒ not possible.

∴ Ans → 20 + 20 − 1 = 39.

* How many +ve integral values of (x, y) satisfy the eqⁿ $x^2 - y^2 = 357$?

$x^2 - y^2 = 357.$
$\Rightarrow (x+y)(x-y) = 3 \times 119 = 3 \times 7 \times 17.$
$\underline{x > y}$

			x+y	x−y
Total →	$2 \times 2 \times 2 = 8.$		3×7×17	1
	x>y → 4.		17×7	3
			17×3	7
Ans → 4.			7×3	17.

* How many integral values of (x, y) satisfy the eqⁿ $x^2 - y^2 = 1452$?

$x^2 - y^2 = 1452$
$\Rightarrow (x+y)(x-y) = 1452.$

If $x+y \to$ ~~odd~~ Even ⇒ x−y → Even.

1452 → has to multiple of 4.

$(x+y)(x-y) = 4 \times \underline{363}.$

$363 \to 3 \times 11^2.$

x+y	x−y	x	y
1	363	182	−181
3	121	62	−59
11	33	22	−11
33	11	22	11
121	3	62	59
363	1	182	181
−1	−363	−182	−181

Ans → $\underline{2 \times 3} \times \underline{2} = \underline{12}$
      both +ve −ve.

* 2 friends Bharathi & Pranathi had a certain no. of 5 rupee coins. If Pranathi gives a certain no. of coins to Bharathi, then Bharathi will have Rs 50 more than Pranathi. Instead, if Bharathi gives the same no. of coins to Pranathi, then Pranathi will have Rs ~~50~~ 30 more than Bharathi. Find the total no. of coins the two can have.

(i) 22   (ii) 21   (iii) 23   (iv) 19.

$\longrightarrow$ no. $\rightarrow b$ $\not{p}$

$$5(p-x)+50 = (b+x)5.$$ $$(b-x)5+30 = (p+x)5$$

$$\Rightarrow p-x+10 = b+x.$$ $$\Rightarrow b-x+6 = p+x$$

$$\Rightarrow p = b+2x-10.$$ $$\Rightarrow b+6-p = 2x.$$

$$\therefore p-b+10 = b+6-p.$$

$$\Rightarrow 2p = 2b-4.$$

$$\Rightarrow \underline{\underline{p+2 = b.}}$$ $$T = p+b$$

$$= p+p+2 = 2p+2$$

even.

Ans $\rightarrow$ 22.

* How many ordered pairs of +ve integers (x,y) satisfy the eqⁿ $\frac{1}{x}+\frac{1}{y} = \frac{1}{18}$?.

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{18}$$

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{18}$$

$$\Rightarrow xy - 18x - 18y = 0.$$

$$\Rightarrow x(y-18) - 18(y-18) = 18 \times 18.$$

$$\Rightarrow (x-18)(y-18) = 2^2 \times 3^4.$$

Ans $\rightarrow$ $3 \times 5 = 15.$

* How many ordered pairs of integers (x,y), where $x>0$ satisfy the eq^n $\frac{1}{x} + \frac{7}{y} = \frac{1}{21}$?

$$\frac{1}{x} + \frac{7}{y} = \frac{1}{21}$$

$$\Rightarrow xy - 147x - 21y = 0$$
$$\Rightarrow x(y-147) - 21(y-147) = 21 \times 147$$
$$\Rightarrow (x-21)(y-147) = 3 \times 7 \times 3 \times 7^2$$
$$= 3^2 \times 7^3.$$

+ve →	$3 \times 4 = 12.$		$x - 21 = -21$	not possible.
-ve →	$\begin{matrix} -1 \\ -3 \\ -7 \\ -9 \end{matrix}$	$\begin{matrix} 5 \times 48 \\ \\ 4 \end{matrix}$	$y - 147 = -147$	

Ans → √2+8   Ans → $12 + 4 = 16$.

* How many ordered pairs of integers satisfy eq^n $\frac{13}{x} - \frac{7}{y} = \frac{1}{4}$.

$$\frac{13}{x} - \frac{7}{y} = \frac{1}{4}.$$

$$\Rightarrow (13y - 7x)4 = xy$$
$$\Rightarrow xy + 28x - 52y = 0.$$
$$\Rightarrow x(y+28) - 52(y+28) = -52 \times 28.$$
$$\Rightarrow (x-52)(y+28) = -2 \times 2 \times 13 \times 2 \times 2 \times 7$$
$$= -2^4 \times 7 \times 13.$$

+ve →	$5 \times 2 \times 2. = 20$		$x - 52 = -52$	not
-ve →	1	20	$y + 28 = 28.$	possible.

Ans → $20 + 20 - 1 = 39$.

* How many integral values of (x,y) satisfy the eq^n $x^2 - y^2 = 625$?

$$x^2 - y^2 = 627$$
$$\Rightarrow (x+y)(x-y) = 3 \times 209$$
$$= 3 \times 11 \times 19.$$

+ve $\rightarrow$ $2 \times 2 \times 2 = 8$ Ans $\rightarrow$ $8 + 8 = 16.$

−ve $= 8$

* How many integral values of (x,y) satisfy the eqn.
$x^2 - y^2 = 220$?

$$x^2 - y^2 = 220.$$
$$(x+y)(x-y) = 220.$$

$(x+y), (x-y) \rightarrow$ Even.

$$(x+y)(x-y) = 4 \times 55 = 4 \times 5 \times 11$$

Ans $\rightarrow$ $2 \times 2 \times 2 = 8.$