# 10 MENSURATION

It is one of the easiest chapters which contributes almost 6-8% problems in Quantitative Aptitude Section of CAT. Besides there are several other aptitude tests which includes plethora of questions from this topic itself. Therefore it is advised that those students who are not so good in other sections such as algebra or sort of logical questions they must emphasise on this chapter. Even the questions asked from this chapter are not as much complex as in Geometry.

**Definition :** Mensuration is a science of measurement of the lengths of lines, areas of surfaces and volumes of solids.

**Planes :** Planes are two dimensional *i.e.,* these two dimensions are namely length and breadth. These occupy surface.

**Solids :** Solids are three dimensional, namely length, breadth and height. These occupy space.

## CONVERSION OF SOME IMPORTANT UNITS

1 km = 10 hectometre = 100 decametre
= 1000 metre = 10,000 decimetre
= 1,00,000 centimetre = 10,00,000 millimetre

1 hectare = 10,000 square metre

1 are = 100 square metre

1 square hectometre = 100 square decametre

1 square decametre = 100 square metre

1 square metre = 100 square decimetre

1 square decimetre = 100 square centimetre

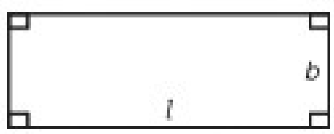
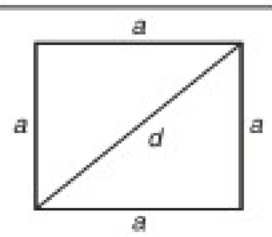
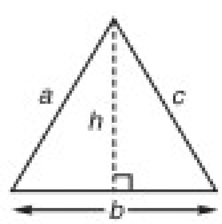
1 square centimetre = 100 square millimetre

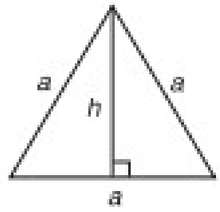
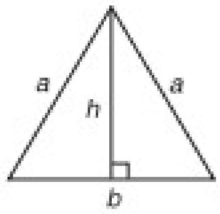
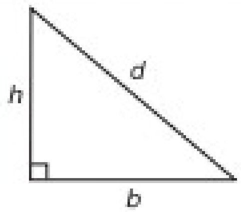
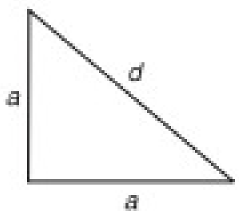
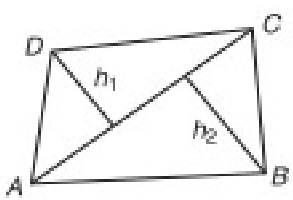
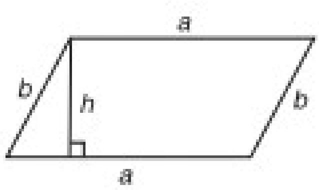
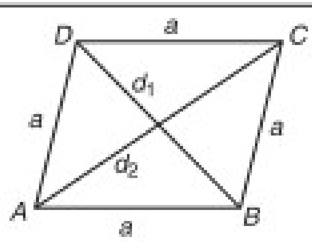
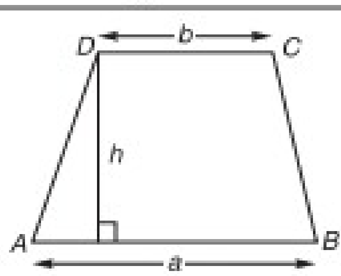
$\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$,

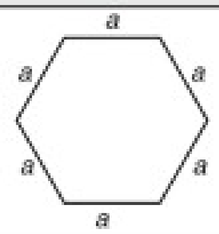
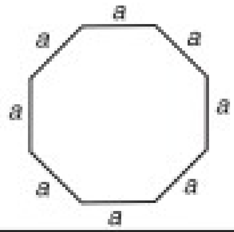
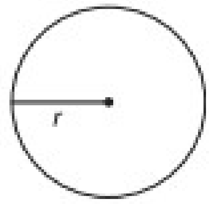
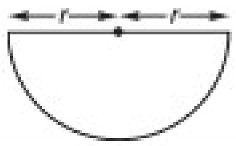
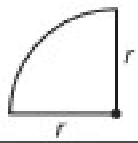
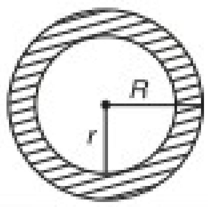
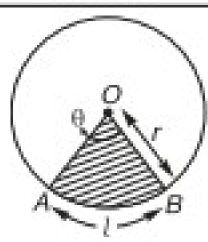
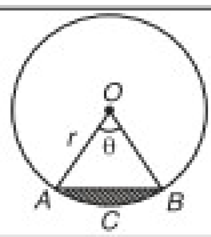
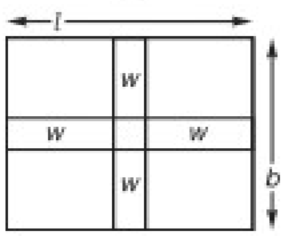
$\sqrt{5} = 2.236$, $\sqrt{6} = 2.45$

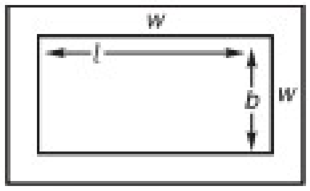
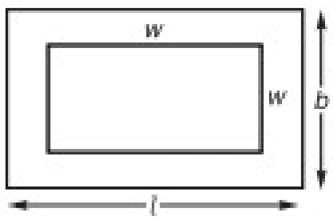
Weight = Volume × density

**2-D Figures (Plane figures)**

| S. No. | Name | Figure | Nomenclature | Area | Perimeter |
|---|---|---|---|---|---|
| 1. | Rectangle |  | $l \to$ length<br>$b \to$ breadth | $l \times b = lb$ | $2l + 2b = 2(l + b)$ |
| 2. | Square |  | $a \to$ side<br>$d \to$ diagonal<br>$d = a\sqrt{2}$ | (i) $a \times a = a^2$<br>(ii) $\frac{d^2}{2}$ | $a + a + a + a = 4a$ |
| 3. | Triangle (Scalene) |  | $a$, $b$ and $c$ are three sides of triangle and $s$ the semiperimeter, where $s = \left(\frac{a+b+c}{2}\right)$<br>$b$ is the base and $h$ is the altitude of triangle | (i) $\frac{1}{2} \times b \times h$<br>(ii) $\sqrt{s(s-a)(s-b)(s-c)}$<br>(Hero's formula) | $a + b + c = 2s$ |

| S. No. | Name | Figure | Nomenclature | Area | Perimeter |
|---|---|---|---|---|---|
| 4. | Equilateral triangle |  | $a \rightarrow$ side<br>$h \rightarrow$ height or altitude<br>$h = \frac{\sqrt{3}}{2}a$ | (i) $\frac{1}{2} \times a \times h$<br>(ii) $\frac{\sqrt{3}}{4}a^2$ | $3a$ |
| 5. | Isosceles triangle |  | $a \rightarrow$ equal sides<br>$b \rightarrow$ base<br>$h \rightarrow$ height or altitude<br>$h = \frac{\sqrt{4a^2 - b^2}}{2}$ | (i) $\frac{1}{2} \times b \times h$<br>(ii) $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$ | $2a + b$ |
| 6. | Right angled triangle |  | $b \rightarrow$ base<br>$h \rightarrow$ altitude/height<br>$d \rightarrow$ diagonal<br>$d = \sqrt{b^2 + h^2}$ | $\frac{1}{2} \times b \times h$ | $b + h + d$ |
| 7. | Isosceles right angled triangle |  | $a \rightarrow$ equal sides<br>$d \rightarrow$ diagonal<br>$d = a\sqrt{2}$ | $\frac{1}{2}a^2$ | $2a + d$ |
| 8. | Quadrilateral |  | $AC$ is the diagonal and $h_1, h_2$ are the altitudes on $AC$ from the vertices $D$ and $B$ respectively | $\frac{1}{2} \times AC \times (h_1 + h_2)$ | $AB + BC + CD + AD$ |
| 9. | Parallelogram |  | $a$ and $b$ are sides adjacent to each other.<br>$h \rightarrow$ distance between the parallel sides | $a \times h$ | $2(a + b)$ |
| 10. | Rhombus |  | $a \rightarrow$ each equal side of rhombus<br>$d_1$ and $d_2$ are the diagonals<br>$d_1 \rightarrow BD$<br>$d_2 \rightarrow AC$ | $\frac{1}{2} \times d_1 \times d_2$ | $4a$ |
| 11. | Trapezium |  | $a$ and $b$ are parallel sides to each other and $h$ is the perpendicular distance between parallel sides | $\left(\frac{a+b}{2}\right) \times h$ | $AB + BC + CD + AD$ |

| S. No. | Name | Figure | Nomenclature | Area | Perimeter |
|---|---|---|---|---|---|
| 12. | Regular hexagon |  | $a \to$ each of the equal side | $\frac{3\sqrt{3}}{2}a^2$ | $6a$ |
| 13. | Regular octagon |  | $a \to$ each of equal side | $2a^2(1+\sqrt{2})$ | $8a$ |
| 14. | Circle |  | $r \to$ radius of the circle<br>$\pi = \frac{22}{7} = 3.1416$(approx) | $\pi r^2$ | $2\pi r$ (called as circumference) |
| 15. | Semicircle |  | $r \to$ radius of the circle | $\frac{1}{2}\pi r^2$ | $\pi r + 2r$ |
| 16. | Quadrant |  | $r \to$ radius | $\frac{1}{4}\pi r^2$ | $\frac{1}{2}\pi r + 2r$ |
| 17. | Ring or circular path (shaded region) |  | $R \to$ outer radius<br>$r \to$ inner radius | $\pi(R^2 - r^2)$ | (outer) $\to 2\pi R$<br>(inner) $\to 2\pi r$ |
| 18. | Sector of a circle |  | $O \to$ centre of the circle<br>$r \to$ radius<br>$l \to$ length of the arc<br>$\theta \to$ angle of the sector<br>$l = 2\pi r\left(\frac{\theta}{360°}\right)$ | (i) $\pi r^2\left(\frac{\theta}{360°}\right)$<br>(ii) $\frac{1}{2} r \times l$ | $l + 2r$ |
| 19. | Segment of a circle |  | $\theta \to$ angle of the sector<br>$r \to$ radius<br>$AB \to$ chord<br>$ACB \to$ arc of the circle | Area of segment $ACB$ (minor segment)<br>$= r^2\left(\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right)$ | $2r\left[\frac{\pi\theta}{360°} + \sin\left(\frac{\theta}{2}\right)\right]$ |
| 20. | Pathways running across the middle of a rectangle |  | $l \to$ length<br>$b \to$ breadth<br>$w \to$ width of the path (road) | $(l + b - w)w$ | $2(l+b) - 4w$<br>$= 2[l + b - 2w]$ |

| S. No. | Name | Figure | Nomenclature | Area | Perimeter |
|---|---|---|---|---|---|
| 21. | Outer pathways |  | $l \to$ length<br>$b \to$ breadth<br>$w \to$ widthness of the path | $(l+b+2w)\,2w$ | (inner) $\to 2\,(l+b)$<br>(outer) $\to 2\,(l+b+4w)$ |
| 22. | Inner path |  | $l \to$ length<br>$b \to$ breadth<br>$w \to$ widthness of the path | $(l+b-2w)\,2w$ | (outer) $\to 2\,(l+b)$<br>(inner) $\to 2\,(l+b-4w)$ |

## RECTANGLES AND SQUARES

1. Area of a rectangle = Length × breadth = $l \times b$
2. Area of a square = $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$

$$= \frac{1}{2}d^2 = a^2$$

3. Diagonal of a rectangle

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2} = \sqrt{l^2 + b^2}$$

4. Diagonal of a square

$$= \sqrt{\text{side}^2 + \text{side}^2} = \text{side}\sqrt{2} = a\sqrt{2}$$

5. Perimeter of a rectangle

$$= 2\,(\text{length} + \text{breadth}) = 2(l+b)$$

6. Perimeter of a square = $4 \times$ side = $4a$
7. Area of four walls of a room = $2\,(l+b) \times h$

**EXAMPLE 1** *The length and breadth of a rectangular room are 15 m and 8 m respectively :*

*(a) Find the perimeter of room.*

*(b) Find the area of the floor of room.*

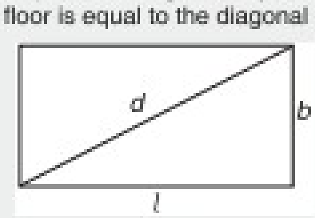
*(c) Find the maximum possible length of the rod that can be put on the floor.*

**SOLUTION** (a) Perimeter = $2(15+8) = 46$ m

(b) Area of floor = $15 \times 8 = 120\ \text{m}^2$

(c) Length of diagonal = $\sqrt{(15)^2 + (8)^2} = 17\,\text{m}$

**NOTE** The maximum possible length of any rod that can be placed on a rectangular floor is equal to the diagonal of the floor of the room.




Here $d > l$

**EXAMPLE 2** *The side of a square shaped garden is 20 m. Find the :*

*(a) area of the garden*

*(b) perimeter (or boundary) of the garden*

*(c) maximum possible distance between any two corners of the garden.*

**SOLUTION** (a) Area = $(\text{side})^2 = (20)^2 = 400\text{m}^2$ (square metre)

(b) Perimeter = $4 \times$ side = $4 \times 20 = 80$m

(c) Diagonal = side $\sqrt{2} = 20 \times \sqrt{2}$

$$= 20 \times 1.414 = 28.28\text{m}$$

**EXAMPLE 3** *One side of a rectangular lawn is 12 m and its diagonal is 13 m. Find the area of the field.*

**SOLUTION** $d = \sqrt{l^2 + b^2} \Rightarrow 13 = \sqrt{12^2 + b^2}$

$\Rightarrow$ $b = 5\,\text{m}$

$\therefore$ Area = $l \times b = 12 \times 5 = 60\ \text{m}^2$

**EXAMPLE 4** *The length of a rectangle is 1 cm more than its breadth. The diagonal is 29 cm. Find the area of the rectangle.*

*(a) 481 cm²* *(b) 841 cm²*

*(c) 420 cm²* *(d) 870 m²*

**SOLUTION** $l = (b+1)$

and $d = \sqrt{l^2 + b^2} = \sqrt{(b+1)^2 + b^2} = 29$

$\Rightarrow$ $b^2 + b = 420$

$\Rightarrow$ $b = 20$

$\therefore$ $l = 21$

$\therefore$ Area = $l \times b = 420\ \text{cm}^2$

**EXAMPLE 5** *The length of a wall is 5/4 times of its height. If the area of the wall be 180 m². What is the sum of the length and height of the wall?*

**SOLUTION** Let the length be $5x$ and height be $4x$

then $l \times h = 180 = 5x \times 4x = 20x^2$

$\Rightarrow$ $x = 3$

$\therefore$ $l + h = 15 + 12 = 27$

**Alternatively :** $1.25\,h^2 = 180$

$\Rightarrow \quad h = 12$ m

$\therefore \quad l = 15$ m

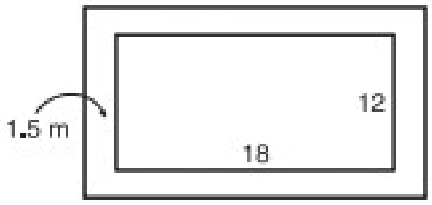
$l + h = 27$ m

**EXAMPLE 6** *A rectangular grassy lawn is 18 m by 12 m. It has a gravel path 1.5 m wide all around it on the outside. What is the area of the path.*

**SOLUTION** Area of path (outside the lawn) $= (l + b + 2w)\,2w$

$= (18 + 12 + 3)\,3$

$= 99 \text{ m}^2$




**Alternatively :**

Area of lawn $= 18 \times 12 = 216 \text{ m}^2$

Total area (lawn + path) $= (18 + 3) \times (12 + 3)$

$= 21 \times 15 = 315 \text{ m}^2$

$\therefore$ Area of path (only) $= 315 - 216 = 99 \text{ m}^2$

**EXAMPLE 7** *Find the cost of carpeting a room 17 m long and 9 m wide with a carpet 60 cm broad at 40 paise per metre.*

**SOLUTION** Area of carpet = Area of room

$l \times 0.6 = 17 \times 9$ (60 cm = 0.6 m)

$\Rightarrow \quad l = 255$ m

$\therefore$ Cost of carpeting the floor = rate × length of carpet

$= 0.4 \times 255 =$ Rs. 102.00

**EXAMPLE 8** *The dimensions of a lawn are in the ratio 4 : 1 and its area is 1/4 hectares. What is the length of the lawn?*

**SOLUTION** $l \times b = 4x \times x = \frac{1}{4} \times 10{,}000$

$\Rightarrow \quad x = 25$

$\Rightarrow$ length $= 4x = 100$ m

**EXAMPLE 9** *A room is 16 m long, 7 m broad and 8 m high. Find the cost of white washing the four walls of room at Rs. 7.5 per m², white washing is not to be done on the doors and windows, which occupy 65 m².*

**SOLUTION** Area of 4 walls of a room $= 2(16 + 7) \times 8 = 368 \text{ m}^2$

Net area of 4 walls $= 368 - 65 = 303 \text{ m}^2$

$\therefore$ Cost of white washing $= 303 \times 7.5 =$ Rs. 2272.5

**EXAMPLE 10** *The ratio between the sides of a room is 5 : 3. The cost of white washing the ceiling of the room at 50 P per square m is Rs. 270 and the cost of papering the walls at 10 P per square metre is Rs. 48. The height of the room is :*

*(a) 6 m (b) 8 m (c) 5 m (d) 10 m*

**SOLUTION** Area of ceiling $= \dfrac{\text{Total cost}}{\text{cost of 1 sq. unit}}$

$= \dfrac{270}{0.5} = 540$ sq. m

Now since $l : b = 5x : 3x$

$\Rightarrow \quad l \times b = 15x^2 = 540 \text{ m}^2$

$\Rightarrow \quad l = 30$ and $b = 18$ m

Now, Area of the 4 walls $= \dfrac{\text{Total cost}}{\text{Cost of 1 sq. unit}} = \dfrac{48}{0.1} = 480 \text{ m}^2$

$\therefore$ Height $= \dfrac{480}{2(30 + 18)} = 5$ m

## INTRODUCTORY EXERCISE-10.1

1. A rectangular field has its length and breadth in the ratio of 16 : 9. If its perimeter is 750 cm, What is its area?
   (a) 7500 cm² (b) 32400 cm²
   (c) 14400 cm² (d) 14000 cm²
2. A rectangular field costs Rs. 110 for levelling at 50 paise per square metre. If the ratio of length : breadth is 11 : 5. Find the length of the field :
   (a) 16 m (b) 21 m
   (c) 22 m (d) none of these
3. A room is half as long again as it is wide. The cost of carpeting it at 62 paise per square metre is Rs. 2916.48. Find the cost of white washing the ceiling at 30 paise per metre :
   (a) Rs. 2211.5 (b) Rs. 1114.2
   (c) Rs. 1411.2 (d) can't be determined
4. The length of a rectangular plot of ground is four times its breadth and its area is 4 hectares. How long will it take to a dog to walk round it at the rate of 3 km/hr?
   (a) 12 min (b) 20 min
   (c) 21 min (d) 18.5 min
5. Find the length of the wire required to fence a square field 6 times having its area 5 hectares and 76 ares :
   (a) 5760 m (b) 6760 m
   (c) 52500 m (d) 11760 m
6. A room is 19 m long and 3.50 m broad. What will be the cost of covering its floor with a carpet of 70 cm wide at 95 paise per metre?
   (a) Rs. 90.25 (b) Rs. 99.25
   (c) Rs. 90.75 (d) none of these
7. Find the cost of paving a courtyard 316.8 m × 65 m with stones measuring 1.3 m × 1.1 m at Rs. 0.5 per stone :
   (a) Rs. 1440 (b) Rs. 7200
   (c) Rs. 72,000 (d) none of these
8. A rectangular garden 63 m long and 54 m broad has a path 3 m wide inside it. Find the cost of paving the path at Rs. 37/2 per square metre :

(a) Rs. 12321 b) Rs. 11100
(c) Rs. 74000 (d) none of these

**9.** If the length of a ractangular field is doubled and its breadth is halved (*i.e.*, reduced by 50%). What is percentage change in its area?
(a) 0% (b) 10%
(c) 25% (d) 33.33%

**10.** A path of uniform width runs all around the inside of rectangular field 116 m by 68 m and occupies 720 sq. m. Find the width of the path :
(a) 1 m (b) 1.5 m
(c) 2 m (d) 4 m

**11.** A drawing room is 7.5 m long 6.5 m broad and 6 m high. Find the length of paper 2.5 dm wide to cover its walls allowing 8 sq. m for doors :
(a) 368 m (b) 640 m
(c) 625 m (d) 888 m

**12.** A square field of 2 sq. km is to be divided into two equal parts by a wall which coincides with a diagonal. Find the length of the wall :
(a) $\sqrt{2}$ km (b) 1 km
(c) 4.2 km (d) 2 km

**13.** There are two square fields. Of the two square fields one contains 1 hectare area while the other is broader by 11 per cent. Find the difference in area expressed in sq. m :
(a) 2321 sq. m (b) 1210 sq. m
(c) 2121 sq. m (d) 7700 sq. m

**14.** The expenses of carpeting a half of the floor were Rs. 759, but if the length had been 6 m less than it was, the expenses would have been Rs. 561. What is the length?
(a) 21 m (b) 23 m
(c) 45 m (d) 27 m

**15.** If a roll of paper 1 km long has area 1/25 hectare, how wide is the paper?
(a) 4 m (b) 40 cm
(c) 40 dm (d) 25 cm

**16.** The diagonal and one side of a rectangular plot are 289 m and 240 m respectively. Find the other side :
(a) 237 m (b) 181 m
(c) 161 m (d) 159 m

**17.** How many tiles 20 cm by 40 cm will be required to pave the floor of a prayer hall of a room 16 m long and 9 m wide :
(a) 18000 (b) 2700
(c) 1800 (d) 14400

**18.** If the area of a square be 22050 sq. cm. Find the length of diagonal :
(a) 201 cm (b) 220 cm
(c) 211 cm (d) 210 cm

**19.** If requires 90 g paint for painting a door 12 cm $\times$ 9 cm, how much paint is required for painting a similar door 4 cm $\times$ 3 cm?
(a) 30 g (b) 27 g
(c) 10 g (d) 45 g

**20.** The area of a rectangular foot ball field is 24200 sq. m. It is half as broad as it is long. What is the approx minimum distance a man will cover if he wishes to go from one corner to the opposite one?
(a) 283 m (b) 246 m
(c) 576 m (d) 289 m

**21.** The area of the four walls of a room is 2640 sq. m and the length is twice the breadth and the height is given as 11 m. What is the area of the ceiling?
(a) 2800 $m^2$ (b) 3200 $m^2$
(c) 320 $m^2$ (d) none of these

**22.** If the perimeter of a square and a rectangle are the same, then the areas $A$ and $B$ enclosed by them would satisfy the inequality :
(a) $A > B$ (b) $A \geq B$
(c) $A < B$ (d) $A \leq B$

**23.** If the perimeter of a rectangle and a square each is equal to 80 cm and the difference of their areas is 100 sq. cm, the sides of the rectangle are :
(a) 25 cm, 15 cm (b) 28 cm, 12 cm
(c) 30 cm, 10 cm (d) 35 cm, 15 cm

**24.** The number of square shaped tin sheets of side 25 cm that can be cut off from a square tin sheet of side 1 m, is :
(a) 4 (b) 40
(c) 16 (d) 400

**25.** The length and breadth of a rectangular field are 120 m and 80 m respectively. Inside the field, a park of 12 m width is made around the field. The area of the park is :
(a) 2358 $m^2$ (b) 7344 $m^2$
(c) 4224 $m^2$ (d) 3224 $m^2$

**26.** A 5m wide lawn is cultivated all along the outside of a rectangular plot measuring 90m $\times$ 40 m. The total area of the lawn is :
(a) 1441 $m^2$ (b) 1400 $m^2$
(c) 2600 $m^2$ (d) 420 $m^2$

**27.** The length of a rectangle is 2 cm more than its breadth. The perimeter is 48 cm. The area of the rectangle (in $cm^2$) is :
(a) 96 (b) 128
(c) 143 (d) 144

**28.** The area of a rectangular field is 52000 $m^2$. This rectangular area has been drawn on a map to the scale 1 cm to 100 m. The length is shown as 3.25 cm on the map. The breadth of the rectangular field is :
(a) 210 m (b) 150 m
(c) 160 m (d) 123 m

**29.** If the length of diagonal $BD$ of a square $ABCD$ is 4.8 cm, the area of the square $ABCD$ is :
(a) 9.6 $cm^2$ (b) 11.52 $cm^2$
(c) 12.52 $cm^2$ (d) 5.76 $cm^2$

**30.** If the side of square is increased by 20%, then how much per cent does its area get increased :
(a) 40% (b) 20%
(c) 44% (d) 24%

**31.** The ratio of the area of a square to that of the square drawn on its diagonal is :
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

**32.** The length and breadth of a square are increased by 60% and 40% respectively. The area of the resulting rectangle exceeds the area of the square by :
(a) 224% (b) 24%
(c) 124% (d) 100%

## TRIANGLES

(1) Area of a triangle $= \frac{1}{2} \times$ base $\times$ height (General formula)

(2) Area of an scalene triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

where $s$ is the semiperimeter of the triangle and $a, b$ and $c$ are the three sides of the triangle, $\left(s = \frac{a+b+c}{2}\right)$

(**Scalene triangle :** A triangle whose all sides are different in length)

(3) Area of a right angle triangle $= \frac{1}{2} \times$ base $\times$ height

(**Right angle triangle :** A triangle in which two sides are perpendicular)

Also, Hypotenuse $= \sqrt{(\text{base})^2 + (\text{height})^2}$

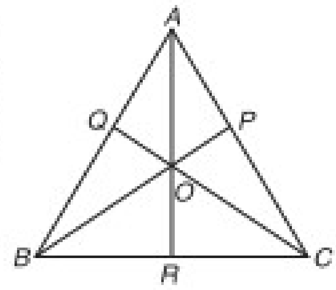
(4) Area of an equilateral triangle

$$= \frac{1}{2} \times \text{side} \times \left(\frac{\sqrt{3}}{2}\text{side}\right) = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

(**Equilateral triangle :** A triangle in which all the three sides are equal also all the three internal angles are equal)

Height of an equilateral triangle $(h) = \frac{\sqrt{3}}{2} \times$ side

**An important property :** In an equilateral $\Delta$ perpendiculars drawn from all the three vertices intersect each other in the ratio of 2 : 1 from the vertex to the base.

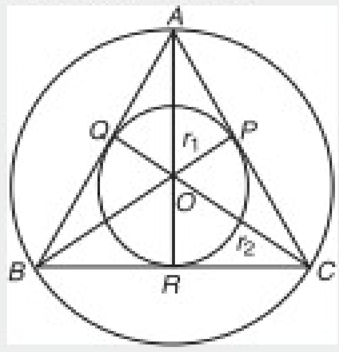



$$\therefore \quad \frac{AO}{OR} = \frac{BO}{OP} = \frac{CO}{OQ} = \frac{2}{1} = \frac{r_2}{r_1}$$

$OP = OQ = OR$, all are the inradii

$OA = OB = OC$, all are the circumradii

**NOTE** Radii means more than one radius.



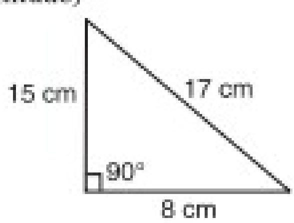

$O$ is the centre of two circles. Circle $PQR$ is called as incircle (touching the sides) and circle $ABC$ is called as circumcircle (touching the vertices)

$$\therefore \quad \text{Inradius} = \frac{\text{side}}{2\sqrt{3}}$$

$$\text{and} \quad \text{Circumradius} = \frac{\text{side}}{\sqrt{3}}$$

**EXAMPLE 1** *The base of a right angled triangle is 8 cm and hypotenuse is 17 cm. Find its area.*

**SOLUTION** Hypotenuse $= \sqrt{(\text{base})^2 + (\text{altitude})^2}$




$(17) = \sqrt{64 + (x)^2}$

$\Rightarrow \quad 289 = 64 + x^2$

$\Rightarrow \quad x^2 = 225$

$\Rightarrow \quad x = 15\text{cm}$

$\therefore \quad \text{Area} = \frac{1}{2} \times \text{base} \times \text{altitude}$

(height is called altitude also)

$= \frac{1}{2} \times 8 \times 15$

$= 60 \text{ cm}^2$

**EXAMPLE 2** *Find the area of an equilateral triangle whose side is $4\sqrt{3}$ cm.*

**SOLUTION** Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

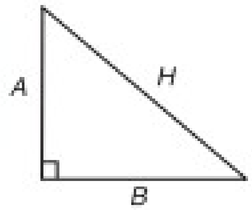
$= \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2$

$= \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3} \text{ cm}^2$

**EXAMPLE 3** *The area of a right angled triangle is 24 cm² and the length of its hypotenuse is 10 cm. The length of the shorter leg is :*
*(a) 5 cm (b) 4 cm (c) 6 cm (d) 3 cm*

**SOLUTION** $H^2 = A^2 + B^2 = 100 \text{ cm}^2$

$\text{Area} = \frac{1}{2} \times A \times B$




$$\Rightarrow \quad \frac{1}{2} A \times B = 24 \text{ cm}^2$$

$$\Rightarrow \quad A \times B = 48 \text{ cm}^2$$

$$\therefore \quad (A+B)^2 = A^2 + B^2 + 2AB$$

$$(A+B)^2 = 196$$

$$\Rightarrow \quad A + B = 14 \qquad \ldots(1)$$

Again $\quad (A-B)^2 = A^2 + B^2 - 2AB$

$$\Rightarrow \quad (A-B)^2 = 4$$

$$\Rightarrow \quad A - B = 2 \qquad \ldots(2)$$

Therefore by solving equation (1) and (2) we get

$A = 8$ and $B = 6$

Therefore the shorter leg is 6 cm.

**Alternatively :** Go through options.

$$\because \quad \frac{1}{2} \times A \times B = 24$$

$$\Rightarrow \quad A \times B = 48$$

Let us assume $B = 6$, then $A = \frac{48}{6} = 8$

Now, $\quad A^2 + B^2 = 100$

$$(8)^2 + (6)^2 = 100$$

$$100 = 100$$

Hence choice (c) is correct.

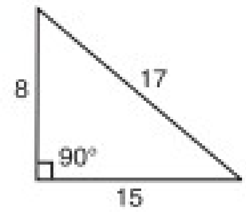
**EXAMPLE 4** *Find the area and perimeter of a triangle whose sides are 17 cm, 8 cm and 15 cm long.*

**SOLUTION** Since $\quad 8^2 + 15^2 = 17^2$

$$64 + 225 = 289$$

$$289 = 289$$

Hence it is a right angled triangle.




$$\therefore \quad \text{Area} = \frac{1}{2} \times b \times h$$

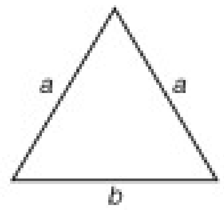
$$= \frac{1}{2} \times 15 \times 8$$

$$= 60 \text{ cm}^2$$

and $\quad$ perimeter $= 8 + 15 + 17 = 40$ cm

**EXAMPLE 5** *Find the area and perimeter of an isosceles triangle whose equal sides are 5 cm each and base is 6 cm.*

**SOLUTION** Area of an isosceles triangle

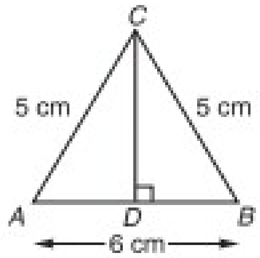



$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{6}{4}\sqrt{4 \times 25 - 36}$$

$$= \frac{3}{2} \times \sqrt{64} = 12 \text{ cm}^2$$

**Alternatively :** We know that the altitude $CD$ bisects the base $AB$ in the isosceles triangle $ABC$.




$$\therefore \quad AD = BD = 3 \text{ cm}$$

Using Pythagorus theorem in $\Delta ADC$ we have,

$$CD^2 = AC^2 - AD^2$$

$$\therefore \quad CD = 4 \text{ cm}$$

$$\therefore \quad \text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$

Also, the perimeter of triangle $= 5 + 5 + 6 = 16$ cm

## INTRODUCTORY EXERCISE-10.2

1. What is the area of the triangle whose sides are 84 m, 80 m and 52 m?
   (a) 1620 sq. m  (b) 2016 sq. m
   (c) 1818 sq. m  (d) none of these
2. Two poles 15 m and 30 m high stand upright in a play ground. If their feet be 36 m apart, find the distance between their tops.
   (a) 41 m  (b) 36 m
   (c) 39 m  (d) none of these
3. The sides of a triangle are 25 m, 39 m and 56 m respectively. Find the perpendicular distance from the vertex opposite to the side 56 m.
   (a) 15 m  (b) 16.5 m
   (c) 18.6 m  (d) 21 m
4. $ABC$ is a triangle and $D, E, F$ are the mid-points of the sides $BC$, $CA$, $AB$ respectively. The ratio of the areas of $\Delta ABC$ and $\Delta DEF$ is :
   (a) 4 : 1  (b) 5 : 1
   (c) 3 : 1  (d) can't be determined
5. The integral base of an isosceles triangle can be whose area is 60 cm$^2$ and the length of one of the equal sides is 13 cm :
   (a) 20 cm  (b) 10 cm
   (c) 16 cm  (d) data insufficient
6. A ladder is resting with one end in contact with the top of a wall of height 60 m and the other end on the ground is at a distance of 11 m from the wall. The length of the ladder is :
   (a) 61 m  (b) 71 m
   (c) 87 m  (d) none of these
7. The base of a triangular field is three times its height. If the cost of cultivating the field is Rs. 36.72 per hectare is Rs. 495.72, find the height and base of the triangular field :
   (a) 480 m, 1120 m  (b) 400 m, 1200 m
   (c) 300 m, 900 m  (d) 250 m, 650 m
8. If every side of a triangle is doubled, then increase in area of the triangle is :
   (a) 200%  (b) 300%
   (c) 400%  (d) none of these

**9.** If the altitude of an equilateral triangle is $2\sqrt{3}$, then its area is :

(a) $4\sqrt{3}$ cm$^2$ (b) $12\sqrt{3}$ cm$^2$

(c) $\frac{8}{\sqrt{3}}$ cm$^2$ (d) none of these

**10.** If the perimeter of an equilateral triangle and a square is same and the area of equilateral triangle is $P$ and the area of square is $Q$, then :

(a) $P < Q$ (b) $P \le Q$

(c) $P > Q$ (d) $P \ge Q$

## PARALLELOGRAM, RHOMBUS AND TRAPEZIUM

1. (a) Area of parallelogram = base ($b$) × height ($h$)

   (b) Area of parallelogram = product of any two adjacent side × sine of the included angle

2. Perimeter = 2 (sum of any two adjacent sides)

3. (a) Area of rhombus $= \frac{1}{2} \times$ (product of diagonals) $= \frac{1}{2} \times d_1 d_2$

   (b) Area of rhombus = product of adjacent sides × sine of the angle included by them

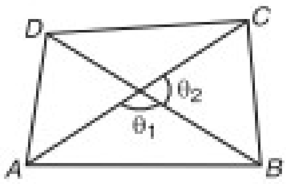
4. Perimeter of rhombus = 4 × side

5. Area of a trapezium $= \frac{1}{2} \times$ sum of parallel sides × height

   height → distance between the two parallel sides

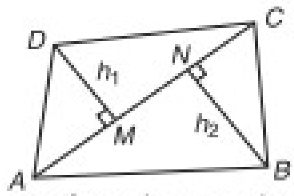
6. Perimeter of trapezium = sum of all the four sides

7. (a) Area of quadrilateral $= \frac{1}{2} \times$ product of diagonal × sine of the angle between them ($AC = d_1$ and $BD = d_2$)




$\therefore$ Area $= \frac{1}{2} d_1 d_2 \sin\theta_1$

$= \frac{1}{2} d_1 d_2 \sin\theta_2$




(b) Area $= \frac{1}{2} \times$ diagonal × sum of the perpendiculars drawn from the opposite vertices on it.

$$= \frac{1}{2} d \times (h_1 + h_2)$$

where, $MD = h_1$ and $BN = h_2$

$AC = d$

## INTRODUCTORY EXERCISE-10.3

**1.** The adjacent sides of a parallelogram are 6 cm and 8 cm and the angle between them is 30°. What is the area of the parallelogram?

(a) 24 cm$^2$ (b) 12 cm$^2$

(c) 40 cm$^2$ (d) $24\sqrt{3}$ cm$^2$

**2.** A parallelogram has sides 30 cm and 20 cm and one of its diagonal is 40 cm long. Then its area is :

(a) $75\sqrt{5}$ cm$^2$ (b) 245 cm$^2$

(c) $150\sqrt{15}$ cm$^2$ (d) 300 cm$^2$

**3.** The distance of a 24 cm long side of a parallelogram from the opposite side is 22 cm. The area of the parallelogram is :

(a) 264 cm$^2$ (b) 246 cm$^2$

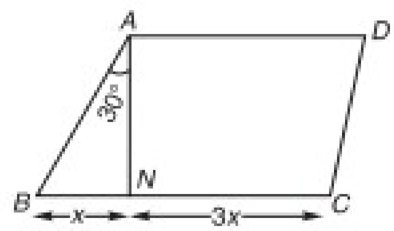
(c) 460 cm$^2$ (d) 528 cm$^2$

**4.** The two adjacent sides of a parallelogram are 25 cm and 40 cm respectively. The altitude drawn on the longer side is 18 cm, then the area of the parallelogram is :

(a) 450 cm$^2$ (b) 720 cm$^2$

(c) 500 cm$^2$ (d) none of these

**5.** In the adjoining figure, the ratio of the areas of the parallelogram $ABCD$ and that of triangle $ABN$ is :




(a) 6 : 1 (b) 5 : 1

(c) 4 : 1 (d) 8 : 1

**6.** If the perimeter of a rhombus is $4p$ and lengths of its diagonals are $a$ and $b$, then its area is :

(a) $\frac{a}{b}$ (b) $\frac{ab}{2}$

(c) $ab/p$ (d) $p(a^2 + b^2)$

**7.** The ratio of the lengths of the diagonal of a rhombus is 2 : 5. Then, the ratio of the area of the rhombus to the square of the shorter diagonal :

(a) 5 : 4 (b) 5 : 2

(c) 2 : 5 (d) none of these

**8.** Area of a rhombus is 256 cm$^2$. One of the diagonal is half of the other diagonal. The sum of the diagonals is :

(a) 38 cm (b) 48 cm

(c) 28 cm (d) 56 cm

**9.** The lengths of two parallel sides of a trapezium are 30 cm and 50 cm and its height is 16 cm. Its area is :
(a) 460 $cm^2$ (b) 750 $cm^2$
(c) 320 $cm^2$ (d) 640 $cm^2$

**10.** *ABCD* is a trapezium in which $AB \parallel CD$ and $AB = 2CD$. If its diagonals intersect each other at *O*, then ratio of areas of triangles *AOB* and *COD* is :
(a) 1 : 4 (b) 1 : 2
(c) 4 : 1 (d) 2 : 1

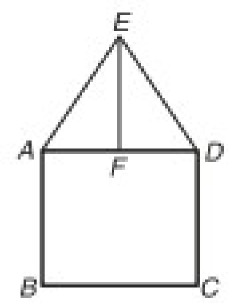
**11.** The area of a trapezium is 441 $cm^2$ and the ratio of parallel sides is 5 : 9. Also the perpendicular distance between them is 21 cm, the longer of parallel sides is :
(a) 36 cm (b) 27 cm
(c) 18 cm (d) 28 cm

**12.** The cross-section of a canal is in the shape of a trapezium and the area of cross-section is 360 $m^2$. If the canal is 12 m wide at the top and 8 m wide at the bottom the depth of the canal is :
(a) 36 m (b) 180 m
(c) 45 m (d) none of these

**13.** The area of a hexagon whose one side is 4 m, is :
(a) $6\sqrt{3}$ $m^2$ (b) $24\sqrt{3}$ $m^2$
(c) $42\sqrt{3}$ $m^2$ (d) 24 $m^2$

**14.** *ABCD* is a quadrilateral $AC = 19$ cm. The lengths of perpendiculars from *B* and *D* on *AC* are 5 cm and 7 cm respectively. Then, the area of *ABCD* (in $cm^2$) is :
(a) 162 (b) 144
(c) 228 (d) 114

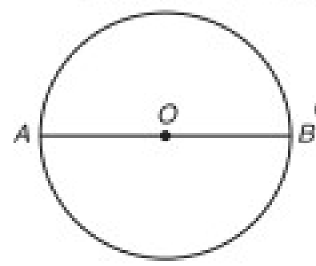
**15.** *ABCD* is a square, $AC = BD = 4\sqrt{2}$ cm, $AE = DE = 2.5$ cm. Find the area of the adjoining figure *ABCDE* :
(a) 19 $cm^2$
(b) 22 $cm^2$
(c) 17 $cm^2$
(d) none of the above




## CIRCLES

1. Area of a circle $= \pi R^2$ ($R \to$ radius of the circle)
2. Circumference of the circle $= 2\pi R$
3. Length of an arc $= 2\pi R\left(\dfrac{\theta}{360°}\right)$
4. Area of a sector $= \pi R^2\left(\dfrac{\theta}{360°}\right) = \dfrac{1}{2}(\text{arc} \times R)$
5. Area of segment $= \pi R^2\left(\dfrac{\theta}{360°}\right) - \dfrac{R^2}{2}\sin\theta$

Diameter $= 2 \times$ radius




$OB = OA \to$ radius
$AB \to$ diameter

**EXAMPLE 1** *The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?*

**SOLUTION** Total distance (travelled) = 11 km = 11000 m

Distance travelled in one revolution

= circumference of the wheel

$= 2 \times \pi \times r = 2 \times \dfrac{22}{7} \times \dfrac{7}{4} = 11$ m

$\therefore$ number of revolutions in 11 km $= \dfrac{11000}{11}$

= 1000 revolution

**EXAMPLE 2** *It takes 13.5 mL to paint the surface of the circular sheet of radius 17 cm. How much paint is required to paint a similar circular sheet with double the radius?*

**SOLUTION** Ratio of radii = 1 : 2

$\therefore$ Ratio of areas = 1 : 4

$$\left(\text{Since } \frac{\text{Area of } C_1}{\text{Area of } C_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}\right)$$

$\therefore$ Quantity of required paint is 4 times
Thus we need $4 \times 13.5 = 54$ mL paint.

**EXAMPLE 3** *A circular road runs round a circular garden. If the difference between the circumference of the outer circle and the inner circle is 44 m. Find the width of the road.*

**SOLUTION** Let *R* and *r* be radii of outer circle and inner circle respectively

$\therefore$ Width of the road $= R - r$

$\therefore$ $2\pi R - 2\pi r = 44$m

$\Rightarrow$ $2\pi(R - r) = 44$ m

$\Rightarrow$ $(R - r) = 7$ m $\left(\because \pi = \dfrac{22}{7}\right)$

**EXAMPLE 4** *The radius of a circle is 5 m. What is the radius of another circle whose area is 25 times that of the first?*

**SOLUTION** Ratio of areas = (ratio of radii)$^2$

$\dfrac{25}{1}$ = (ratio of radii)$^2$

$\Rightarrow$ ratio of radii $= \dfrac{5}{1}$

Therefore radius of another circle is 5 times

Hence the required radius = 25 m

**EXAMPLE 5** *What is the radius of a circle whose area is equal to the sum of the areas of two circles whose radii are 20 cm and 21 cm.*

**SOLUTION**

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$
$$\pi R^2 = \pi (r_1^2 + r_2^2)$$
$$R^2 = (400 + 441)$$
$$R^2 = 841$$
$$\Rightarrow \quad R = 29 \text{ cm}$$

**EXAMPLE 6** *In a circle of radius 28 cm, an arc subtends an angle of 108° at the centre.*

*(a) Find the area of the sector.*

*(b) Find the length of the arc.*

**SOLUTION** (a) Area of the sector $= \pi r^2 \left(\frac{\theta}{360°}\right)$

$$= \frac{22}{7} \times 28 \times 28 \times \frac{108°}{360°}$$
$$= 22 \times 4 \times 28 \times \frac{3}{10}$$
$$= 739.2 \text{ cm}$$

(b) Length of the arc $= 2\pi r \left(\frac{\theta}{360°}\right)$

$$= 2 \times \frac{22}{7} \times 28 \times \frac{108}{360°} = 52.8 \text{ cm}$$

## Introductory Exercise-10.4

**1.** If the circumference of a circle is 704 cm, then its area is :
(a) 49324 $m^2$ (b) 39424 $m^2$
(c) 3672 $cm^2$ (d) 39424 $cm^2$

**2.** If the circumference of a circle is 4.4 m, then the area of the circle (in $m^2$) is :
(a) $49/\pi$ (b) $49\pi$
(c) $4.9\pi$ (d) none of these

**3.** A circular wire of radius 4.2 m is cut and bent in the form of a rectangle whose longer side is 20% more than its shorter side. The longer side of the rectangle is :
(a) 7.2 m (b) 72 cm
(c) 8 m (d) none of these

**4.** The inner circumference of a circular path around a circular lawn is 440 m. What is the radius of the outer circumference of the path, if the path is 14 m wide?
(a) 96 m (b) 84 m
(c) 70 m (d) 88 m

**5.** The sum of the radius and the circumference of a circle is 51 cm. The area of the circle is :
(a) 151 cm (b) 152 cm
(c) 154 cm (d) data insufficient

**6.** The difference between the circumference and the diameter of the circle is 15 m. What is the area of the circle?
(a) 225 $m^2$ (b) 165 $m^2$
(c) 156 $m^2$ (d) none of these

**7.** The radius of a circle is increased by 2 cm from 5 cm to 7 cm. What is the percentage change in area of the circle?
(a) 96% (b) 35%
(c) 70% (d) 74%

**8.** If the circumference of a circle is increased by 20%, then its area will be increased by :
(a) 44% (b) 32%
(c) 40% (d) none of these

**9.** The area of a circular field is 124.74 hectares. The cost of fencing it at the rate of 80 paise per metre is :
(a) Rs. 3168 (b) Rs. 1584
(c) Rs. 1729 (d) none of these

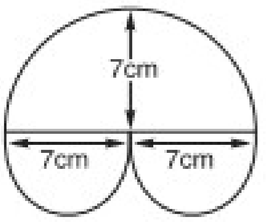
**10.** Eldeco Housing Pvt. Ltd purchased a circular plot of land for Rs. 158400 at the rate of 1400 per sq. metre. The radius of the plot is :
(a) 5 m (b) 6 m
(c) 7 m (d) 14 m

**11.** A figure consists of a square of side '$a$' m with semicircles drawn on the outside of the square. The area (in $m^2$) of the figure so formed will be :
(a) $a^2(\pi + 1)$ (b) $a^2\left(\pi + \frac{1}{4}\right)$
(c) $a^2 + \frac{\pi a^2}{2}$ (d) none of these

**12.** The length of a rope by which a buffalo must be tethered so that she may be able to graze a grassy area of 2464 sq. m is :
(a) 35 m (b) 27 m
(c) 24 m (d) 28 m

**13.** A circle of radius '$a$' is divided into 6 equal sectors. An equilateral triangle is drawn on the chord of each sector to lie outside the circle. Area of the resulting figure is :
(a) $3a^2(\pi + \sqrt{3})$ (b) $3\sqrt{3}\,a^2$
(c) $3(a^2\sqrt{3} + \pi)$ (d) $\frac{3\sqrt{3}\pi a^2}{2}$

**14.** In the following figure, the area in ($cm^2$) is :

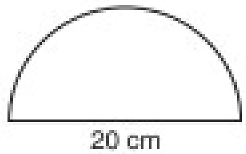



(a) 115.5
(b) 228.5
(c) 154
(d) none of the above

**15.** If a piece of wire 25 cm long is bent into an arc of a circle subtending an angle of 75° at the centre, then the radius of the circle (in cm) is :
(a) $\frac{\pi}{120}$ (b) $\frac{60}{\pi}$
(c) $60\pi$ (d) none of these

**16.** Four horses are tethered at four corners of a square plot of 42 m so that they just cannot reach one another. The area left ungrazed is :
(a) 378 $m^2$ (b) 438 $m^2$
(c) 786 $m^2$ (d) none of these

**17.** The circumference of the following figure is :




(a) $(20 + 10\pi)$ (b) $20\pi$
(c) $10\pi$ (d) $30\pi$

**18.** The area of a minor sector subtending the central angle at the centre 40° is 8.25 cm². What is the area of the remaining part (*i.e.*, major sector) of the circle?
(a) 82.5 cm² (b) 74.25 cm²
(c) 66 cm² (d) none of these

**19.** The area of a sector of a circle of radius 8 cm, formed by an arc of length 5.6 cm, is :
(a) 22.4 cm² (b) 2.24 cm²
(c) 56 cm² (d) none of these

**20.** How long will a man take to go, walking at 13.2 km/h, round a circular garden of 700 m radius?
(a) 12 minute (b) 30 minute
(c) 20 minute (d) none of these

**21.** What is the radius of circular field whose area is equal to the sum of the areas of three smaller circular fields of radii 8 m, 9 m and 12 m respectively?
(a) 17 m (b) 20 m
(c) 21 m (d) 29 m

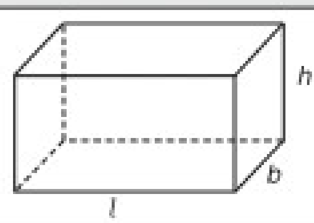
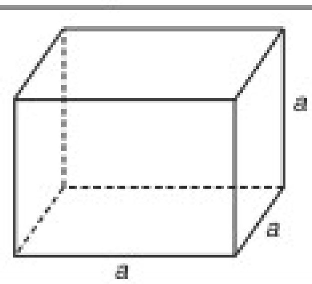
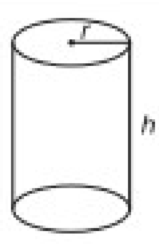
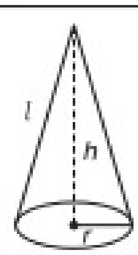
**22.** A rope by which a calf is tied is decreased from 23 m to 12 m. What is the decrease in area to be grazed by it?
(a) 1110 m² (b) 1210 m²
(c) 1120 m² (d) 1221 m²

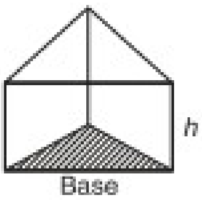
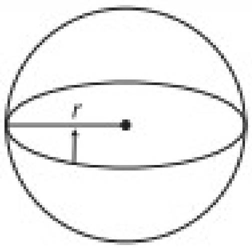
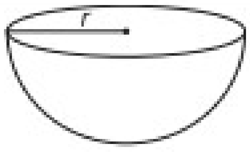
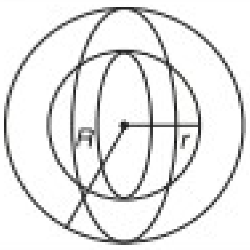
**23.** A wire is bent in the form of a square of side 66 m. It is cut and again bent in the form of a circle. The diameter of this circle is :
(a) 42 m (b) 84 m
(c) 21 m (d) none of these

**24.** A wire is in the form of a circle of radius 42 m is cut and again bent in the form of a square. What is the diagonal of the square?
(a) 66 m (b) $66\sqrt{3}$ m
(c) $66\sqrt{2}$ m (d) none of these

**25.** If the driving wheel of a bicycle makes 560 revolutions in travelling 1.1 km. Find the diameter of the wheel :
(a) 31.5 cm (b) 30.5 cm
(c) 62.5 cm (d) none of these

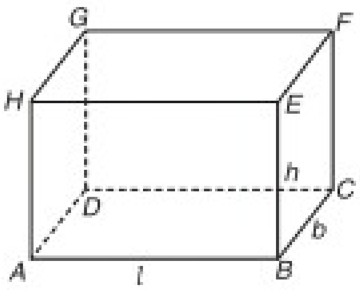
## 3-D FIGURES (SOLIDS)

| S. No. | Name | Figure | Nomenclature | Volume | Curved/Lateral surface area | Total surface area |
|---|---|---|---|---|---|---|
| 1. | Cuboid |   | $l \to$ length<br>$b \to$ breadth<br>$h \to$ height | $lbh$ | $2(l+b)h$ | $2(lb+bh+hl)$ |
| 2. | Cube |   | $a \to$ edge/side | $a^3$ | $4a^2$ | $6a^2$ |
| 3. | Right circular cylinder |   | $r \to$ radius of base<br>$h \to$ height of the cylinder | $\pi r^2 h$ | $2\pi rh$ | $2\pi r(h+r)$ |
| 4. | Right circular cone |   | $r \to$ radius<br>$h \to$ height<br>$l \to$ slant height<br>$l = \sqrt{r^2+h^2}$ | $\frac{1}{3}\pi r^2 h$ | $\pi rl$ | $\pi r(l+r)$ |

| S. No. | Name | Figure | Nomenclature | Volume | Curved/Lateral surface area | Total surface area |
|---|---|---|---|---|---|---|
| 5. | Right triangular prism |  | — | area of base × height | perimeter of base × height | lateral surface area + 2 (area of base) |
| 6. | Right pyramid |  | — | $\frac{1}{3}$ × area of the base × height | $\frac{1}{2}$ × perimeter of the base × slant height | lateral surface area + area of the base |
| 7. | Sphere |  | $r \to$ radius | $\frac{4}{3}\pi r^3$ | — | $4\pi r^2$ |
| 8. | Hemisphere |  | $r \to$ radius | $\frac{2}{3}\pi r^3$ | $2\pi r^2$ | $3\pi r^2$ |
| 9. | Spherical shell |  | $r \to$ inner radius $R \to$ outer radius | $\frac{4}{3}\pi[R^3 - r^3]$ | — | $4\pi[R^2 + r^2]$ |
| 10. | Frustum of a cone |  | — | $\frac{\pi}{3}h(r^2 + Rr + R^2)$ | $\pi(r+R)l$ | lateral surface area + $\pi[R^2 + r^2]$ |

## CUBOID AND CUBE

**Cuboid :** A cuboid has 6 faces, 12 edges, 8 vertices and 4 diagonals.




**Faces :**

*ABCD*, *EFGH*, *ABEH*, *CFGD*, *BCFE*, *ADGH*

**Edges :** *AB*, *BC*, *CD*, *AD*, *EF*, *FG*, *GH*, *EH*, *BE*, *CF*, *DG*, *AH*

**Vertices :** *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*

**Diagonals :** *AF*, *BG*, *CH*, *DE*

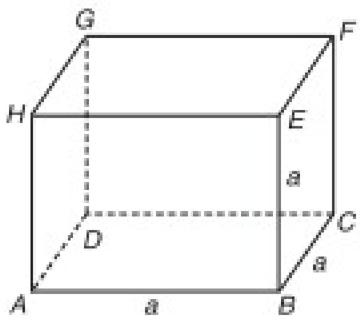
**Formulae :** Volume = $l \times b \times h$

($l \to$ length, $b \to$ breadth, $h \to$ height)

Total surface area = $2(lb + bh + hl)$

Diagonal $(d) = \sqrt{l^2 + b^2 + h^2}$

**Cube :** A cube has 6 equal faces, 12 equal edges, 8 vertices and 4 equal diagonals




**Formulae :**

Volume : $(a)^3$

Total surface area = $6(a)^2$

($a \to$ edge of the cube)

Diagonal $(d) = a\sqrt{3}$

**Euler's Theorem** $\to (V + F) = (E + 2)$; where $V \to$ no. of vertices, $F \to$ no. of faces, $E \to$ no. of edges

**EXAMPLE 1** *The dimensions of a cuboid are 16 cm, 18 cm and 24 cm. Find :*
*(a) volume (b) surface area (c) diagonal*

**SOLUTION** (a) Volume $= l \times b \times h = 16 \times 18 \times 24 = 6912 \text{ cm}^3$

(b) Surface area $= 2(lb + bh + hl)$
$= 2(16 \times 18 + 18 \times 24 + 24 \times 16)$
$= 2208 \text{ cm}^2$

(c) Diagonal $= \sqrt{l^2 + b^2 + h^2}$
$= \sqrt{16^2 + 18^2 + 24^2} = \sqrt{1156} = 34 \text{ cm}$

**EXAMPLE 2** *Edge of a cube is 5 cm. Find :*
*(a) volume (b) surface area (c) diagonal*

**SOLUTION** Volume $= a^3 = (5)^3 = 125 \text{ cm}^3$
Surface area $= 6a^2 = 6 \times (5)^2 = 150 \text{ cm}^3$
Diagonal $= a\sqrt{3} = 5\sqrt{3} = 8.660 - 8.66 \text{ cm}$

**EXAMPLE 3** *Three cubes of volumes, $1 \text{ cm}^3$, $216 \text{ cm}^3$ and $512 \text{ cm}^3$ are melted to form a new cube. What is the diagonal of the new cube?*

**SOLUTION** Volume of new cube $= 1 + 216 + 512 = 729 \text{ cm}^3$
$\therefore$ Edge of new cube $= \sqrt[3]{729} = 9 \text{ cm}$
$\therefore$ Surface area $= 6a^2 = 6 \times (9)^2 = 486 \text{ cm}^2$
$\therefore$ Diagonal of the new cube $= a\sqrt{3} = 9\sqrt{3}$
$= 15.6$ cm (approx)

**EXAMPLE 4** *The surface area of a cube is $864 \text{ cm}^2$. Find its volume.*

**SOLUTION** $6a^2 = 864 \Rightarrow a^2 = 144 \Rightarrow a = 12 \text{ cm}$
$\therefore \quad a^3 = (12)^3 = 1728 \text{ cm}^3$

**EXAMPLE 5** *Find the length of the longest pole that can be placed in a room 30 m long, 24 m broad and 18 m high.*

**SOLUTION** $d = \sqrt{l^2 + b^2 + h^2}$
$d = \sqrt{900 + 576 + 324}$
$d = 30\sqrt{2} \text{ m}$

**EXAMPLE 6** *A brick measures 20 cm × 10 cm × 7.5 cm. How many bricks will be required for a wall 20 m × 2 m × 0.75 m?*

**SOLUTION** Number of bricks $= \dfrac{\text{total volume of a wall}}{\text{volume of one brick}}$

$$= \frac{20 \times 2 \times 0.75 \times 100 \times 100 \times 100}{20 \times 10 \times 7.5}$$

$= 20,000$

## INTRODUCTORY EXERCISE-10.5

1. A cube of metal, each edge of which measures 4 cm, weighs 400 kgs. What is the length of each edge of a cube of the same metal which weighs 3200 kg?
(a) 64 cm (b) 8 cm
(c) 2 cm (d) none of these

2. The length of a tank is thrice that of breadth, which is 256 cm deep and holds 3000 L water. What is the base area of the tank? (1000 L = 1 cubic metre)
(a) $111775 \text{ m}^2$ (b) $1171.875 \text{ m}^2$
(c) $1.171875 \text{ m}^2$ (d) none of these

3. The three co-terminus edges of a rectangular solid are 36 cm, 75 cm and 80 cm respectively. Find the edge of a cube which will be of the same capacity :
(a) 60 cm (b) 52 cm
(c) 46 cm (d) none of these

4. A tank 10 m long and 4 m wide is filled with water. How many litres of water must be drawn off to make the surface sink by 1 m. (1000 L = 1 cubic metre)
(a) 20 kilolitre (b) 40 kilolitre
(c) 50 kilolitre (d) none of these

5. How many cubes each of surface area 24 sq. dm can be made out of a metre cube, without any wastage :
(a) 75 (b) 250
(c) 125 (d) 62

6. Three cubes of metal, whose edges are 3 cm, 4 cm and 5 cm respectively are melted to form a new cube. What is the surface area of the new cube?
(a) $216 \text{ cm}^2$ (b) $56 \text{ cm}^2$
(c) $36 \text{ cm}^2$ (d) none of these

7. A lid of rectangular box of sides 39.5 cm by 9.35 cm is sealed all around with tape such that there is an overlapping of 3.75 cm of the tape. What is the length of the tape used?
(a) 111.54 cm (b) 101.45 cm
(c) 110.45 cm (d) none of these

8. A cistern from inside is 12.5 m long, 8.5 m broad and 4 m high and is open at top. Find the cost of cementing the inside of a cistern at Rs. 24 per sq. m :
(a) Rs. 6582 (b) Rs. 8256
(c) Rs. 7752 (d) Rs. 8752

9. 250 men took a dip in a water tank at a time, which is 80 m × 50 m. What is the rise in the water level if the average displacement of 1 man is $4 \text{ m}^3$?
(a) 22 cm (b) 25 cm
(c) 18 cm (d) 30 cm

10. The edge of a cube is increased by 100%, the surface area of the cube is increased by :
(a) 100% (b) 200%
(c) 300% (d) 400%

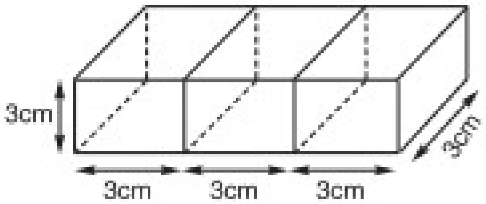
**11.** The edge of a cube is doubled. What will be the new volume?
(a) 2 times (b) 3 times
(c) 4 times (d) 8 times

**12.** The external dimensions of a wooden box closed at both ends are 24 cm, 16 cm and 10 cm respectively and thickness of the wood is 5 mm. If the empty box weighs 7.35 kg, find the weight of 1 cubic cm of wood :
(a) 10 g (b) 12.5 g
(c) 27 g (d) 15 g

**13.** The internal dimensions of a tank are 12 dm, 8 dm and 5 dm. How many cubes each of edge 7 cm can be placed in the tank with faces parallel to the sides of the tank. Find also, how much space is left unoccupied?
(a) 35; 113 $dm^3$ (b) 1313; 31.13 $dm^3$
(c) 1309; 31.013 $dm^3$ (d) 1309; 13.31 $dm^3$

**14.** The length, breadth and height of box are 2 m, 1.5 m and 80 cm respectively. What would be the cost of canvas to cover it up fully, if one square metre of canvas costs Rs. 25.00?
(a) Rs. 260 (b) Rs. 290
(c) Rs. 285 (d) none of these

**15.** Three cubes each of edge 3 cm long are placed together as shown in the adjoining figure. Find the surface area of the cuboid so formed :




(a) 182 sq. cm (b) 162 sq. cm
(c) 126 sq. cm (d) none of these

**16.** A room is 36 m long, 12 m wide and 10 m high. It has 6 windows, each 3 m × 2.5 m; one door 9.5 m × 6 m and one fire chimney 4 m × 4.5 m. Find the expenditure of papering its walls at the rate of 70 paise per metre, if the width of the paper is 1.2 m :
(a) Rs. 490 (b) Rs. 690
(c) Rs. 1000 (d) none of these

**17.** A school hall has the dimensions 30 m, 12 m by 6 m. Find the number of children who can be accommodated, if each child should get 8 $m^3$ of space :
(a) 240 (b) 270
(c) 250 (d) 150

**18.** When each side of a cube is increased by 2 cm, the volume is increased by 1016 $cm^3$. Find the side of the cube. If each side of it is decreased by 2 cm, by how much will the volume decrease?
(a) 12 cm, 729 $cm^3$ (b) 8 cm, 512 $cm^3$
(c) 9 cm, 729 $cm^3$ (d) 12 cm, 728 $cm^3$

**19.** Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes :
(a) 5 : 7 (b) 7 : 9
(c) 9 : 7 (d) none of these

**20.** A hollow square shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 $cm^3$ of iron in the tube. Find its thickness :
(a) 2 cm (b) 0.5 cm
(c) 1 cm (d) can't be determined

**21.** A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of base are 15 cm and 12 cm. Find the rise in water level in the vessel :
(a) 6.85 cm (b) 7 cm
(c) 7.31 cm (d) 7.39 cm

**22.** A rectangular tank 25 cm long and 20 cm wide contains water to a depth of 5 cm. A metal cube of side 10 cm is placed in the tank so that one face of the cube rests on the bottom of the tank. Find how many litres of water must be poured into the tank so as to just cover the cube?
(a) 1 L (b) 1.5 L
(c) 2 L (d) 2.5 L

**23.** A rectangular block has length 10 cm, breadth 8 cm and height 2 cm. From this block, a cubical hole of side 2 cm is drilled out. Find the volume and the surface area of the remaining solid :
(a) 152 $cm^3$, 512 $cm^2$ (b) 125 $cm^3$, 215 $cm^2$
(c) 152 $cm^3$, 240 $cm^2$ (d) 125 $cm^3$, 512 $cm^2$

**24.** A rectangular tank of dimensions 24 m × 12 m × 8 m is dug inside a rectangular field 600 m long and 200 m broad. The earth taken out is evenly spread over the field. By how much will the level of the field rise?
(a) 1.925 cm (b) 0.02 m
(c) 0.2 cm (d) none of these

**25.** How many bricks (number near to next hundred) will be required to build a wall 30 m long, 30 cm thick and 5 m high with a provision of 2 doors, each 4 m × 2.5 m and each brick being 20 cm × 16 cm × 8 cm when one-ninth of the wall is filled with lime?
(a) 13500 bricks (b) 13600 bricks
(c) 20050 bricks (d) 18500 bricks

**26.** A rectangular water reservoir is 15 m by 12 m at the base. Water flows into it through a pipe whose cross-section is 5 cm by 3 cm at the rate of 16 m per second. Find the height to which the water will rise in the reservoir in 25 minutes :
(a) 0.2 m (b) 2 cm
(c) 0.5 m (d) none of these

**27.** Which of the following pairs is not correctly matched :

| Geometrical objects | Number of vertices |
|---|---|
| (A) Tetrahedron | 4 |
| (B) Pyramid with rectangular base | 5 |
| (C) Cube | 6 |
| (D) Triangle | 3 |

(a) (A) (b) (B)
(c) (C) (d) (D)

**28.** If the length of diagonal of a cube is $6\sqrt{3}$ cm, then the length of its edge is :

(a) 2 cm (b) 3 cm
(c) 6 cm (d) $\frac{36}{\sqrt{3}}$ cm

**29.** The length of longest pole that can be placed on the floor of a room is 12 m and the length of longest pole that can be placed in the room is 15 m. The height of the room is :

(a) 3 m (b) 6 m
(c) 9 m (d) none of these

**30.** The sum of length, breadth and depth of a cuboid is 12 cm and its diagonal is $5\sqrt{2}$ cm. Its surface area is :

(a) 152 cm$^2$ (b) 94 cm$^2$
(c) 108 cm$^2$ (d) $60\sqrt{2}$ cm$^2$

**31.** The volume of a wall, 3 times as high as it is broad and 8 times as long as it is high, is 36,864 m$^3$. The height of the wall is :

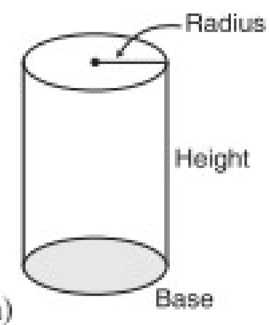
(a) 1.8 m (b) 2.4 m
(c) 4.2 m (d) none of these

**32.** If the areas of 3 adjacent sides of a cuboid are $x, y, z$ respectively, then the volume of the cuboid is :

(a) $xyz$ (b) $\sqrt{xyz}$
(c) $3xyz$ (d) none of these

## CYLINDER AND CONE

**Cylinder :**




Volume = base area × height

$= \pi r^2 h$

Curved surface area

= perimeter × height

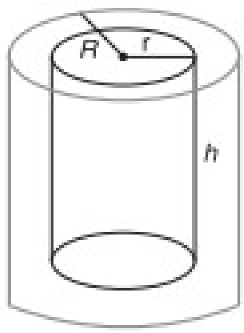
$= 2\pi rh$

Total surface of the cylinder

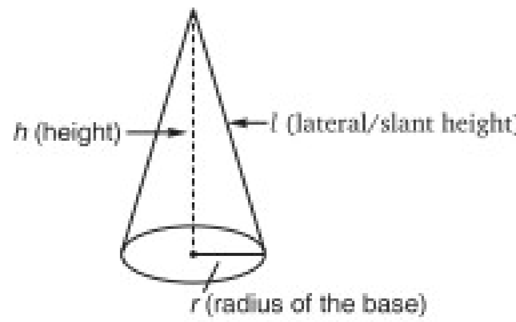
= curved surface area + 2 (base area)

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Volume of a hollow cylinder $= \pi h(R^2 - r^2)$




**Cone :**

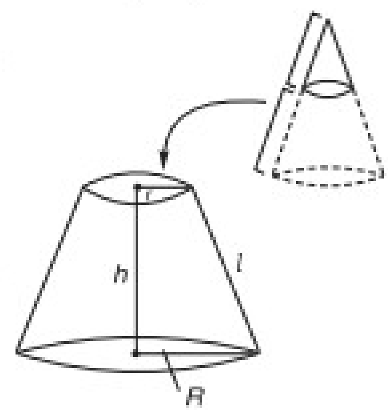



Volume $(V) = \frac{1}{3} \times$ base area × height $= \frac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$

Total surface area $= \pi rl + \pi r^2 = \pi r(l + r)$

**Frustum of a cone :**




Volume of the frustum of a cone

$= \frac{\pi}{3} h(r^2 + Rr + R^2)$

Curved/Lateral surface area of the frustum of cone

$= \pi l(r + R)$

**EXAMPLE 1** *The base radius of a cylinder is 14 cm and its height is 30 cm. Find :*

*(a) volume* *(b) curved surface area*
*(c) total surface area*

**SOLUTION** (a) Volume of cylinder $= \pi r^2 \times h = \frac{22}{7} \times 14 \times 14 \times 30$

$= 18480$ cm$^3$

(b) Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 14 \times 30$

$= 2640$ cm$^2$

(c) Total surface area $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 14(30 + 14)$

$= 2 \times \frac{22}{7} \times 14 \times 44 = 3872$ cm$^2$

**EXAMPLE 2** *How many cubic metres of earth must be dug to make a well 14 m deep and 4 m in diameter?*

**SOLUTION** Earth to be dugout from the well

= volume of the cylindrical well

$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 14$

$= 176$ m$^3$

**EXAMPLE 3** *A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube is 210 cm, find the number of cubic cm of iron in it.*

**SOLUTION** External radius $(R) = 25$ cm

Internal radius $(r) = (25 - 2) = 23$ cm

Volume of iron $= \pi h(R^2 - r^2)$

$$= \frac{22}{7} \times 210 \times (25^2 - 23^2)$$
$$= 63360 \text{ cm}^3$$

**EXAMPLE 4** *A well with 14 m inside diameter is dugout 15 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embarkment. What is the height of the embarkment?*

**SOLUTION** Area of embarkment × height of embarkment

= volume of earth dugout

$$\pi (R^2 - r^2) \times h = \pi \times 7 \times 7 \times 15$$
$$\Rightarrow \quad (28^2 - 7^2) h = 7 \times 7 \times 15$$
$$\Rightarrow \quad (35 \times 21) \times h = 7 \times 7 \times 15$$
$$\Rightarrow \quad h = 1 \text{ m}$$

**EXAMPLE 5** *A cylindrical cistern whose diameter is 21 cm is partly filled with water. If a rectangular block of iron 14 cm in length, 10.5 cm in breadth and 11 cm in thickness is wholly immersed in water, by how many centimetres will the water level rise?*

**SOLUTION** Volume of the block $= 14 \times 10.5 \times 11 \text{ cm}^3$

Radius of the tank $= \frac{21}{2} = 10.5$ cm

Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h$

$$\therefore \quad \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h = 14 \times 10.5 \times 11$$
$$h = \frac{14}{3} = 4\frac{2}{3} \text{ cm}$$

**EXAMPLE 6** *If the radius of cylinder is doubled, but height is reduced by 50%. What is the percentage change in volume?*

**SOLUTION** $\frac{r_1}{r_2} = \frac{r}{2r}$ and $\frac{h_1}{h_2} = \frac{h}{h/2}$

$\therefore$ Actual volume $= \pi r^2 h$

New volume $= \pi (2r)^2 \times \frac{h}{2} = 2\pi r^2 h$

Therefore new volume is the twice of the original volume.

Hence the change in volume $= \frac{2-1}{1} \times 100 = 100\%$

**EXAMPLE 7** *The radius of the base of a right cone is 35 cm and its height is 84 cm. Find :*

*(a) slant height* *(b) curved surface area*
*(c) total surface area* *(d) volume*

**SOLUTION** (a) Slant height $(l) = \sqrt{r^2 + h^2}$

($r \to$ radius of the circular base)

$= \sqrt{35^2 + 84^2}$ ($h \to$ height of the cone)

$= \sqrt{1225 + 7056}$

$= \sqrt{8281} = 91$ cm

(b) Curved surface area $= \pi r l = \frac{22}{7} \times 35 \times 91$

$= 10010 \text{ cm}^2$

(c) Total surface area = lateral surface area + base area

$= \pi r l + \pi r^2 = \pi r (l + r)$

$= \frac{22}{7} \times 35 (91 + 84)$

$= 110 \times 175 = 19250 \text{ cm}^2$

(d) Volume $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84$

$= 107800 \text{ cm}^3$

**EXAMPLE 8** *Find the area of the iron sheet required to prepare a cone 20 cm high with base radius 21 cm.*

**SOLUTION** $r = 21$ cm, $h = 20$ cm

$\therefore \quad l = \sqrt{r^2 + h^2} = 29$ cm

$\therefore$ Area of the sheet = total surface area of the cone

$= \pi r l + \pi r^2$

$= \pi r (l + r) = \frac{22}{7} \times 21 [29 + 21]$

$= 3300 \text{ cm}^2$

**EXAMPLE 9** *A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to make n solid cones of height 1 cm and base radius 1 mm. Find the value of n.*

**SOLUTION** $n = \frac{\text{volume of cylinder}}{\text{volume of one cone}}$

$$= \frac{\pi \times 3 \times 3 \times 5}{\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1} = 13500$$

## INTRODUCTORY EXERCISE-10.6

1. How many cubic metres of water fill a pipe which is 3500 m long and 0.08 m in diameter?
   (a) $17.5 \text{ m}^3$ (b) $17.6 \text{ m}^3$
   (c) $21 \text{ m}^3$ (d) $35 \text{ m}^3$
2. A cube of metal, whose edge is 10 cm, is wholly immersed in water contained in cylindrical tube whose diameter is 20 cm. By how much will the water level rise in the tube?
   (a) 3.3 cm (b) $6\frac{3}{11}$ cm
   (c) $3\frac{2}{11}$ cm (d) none of these

**3.** Find the height of the cylinder whose volume is 511 $m^3$ and the area of the base is 36.5 $m^2$ :

(a) 7 m (b) 10.5 m
(c) 14 m (d) none of these

**4.** The lateral surface area of a cylinder is 1056 $cm^2$ and its height is 16 cm. What is its volume?

(a) 5566 $cm^3$ (b) 4455 $cm^3$
(c) 5544 $cm^3$ (d) none of these

**5.** There is a cubical block of wood of side 2 cm. If the cylinder of the largest possible volume is carved out from it. Find the volume of the remaining wood :

(a) $\frac{7}{12}$ $cm^2$ (b) $\frac{12}{7}$ $cm^3$
(c) $5\frac{5}{7}$ $cm^3$ (d) none of these

**6.** The amount of concrete required to build a cylindrical pillar whose base has a perimeter of 8.8 m and whose curved surface area is 17.6 $m^2$ :

(a) 12.32 $m^3$ (b) 12.23 $m^3$
(c) 9.235 $m^3$ (d) 8.88 $m^3$

**7.** If the diameter of the base of a closed right circular cylinder be equal to its height '$h$', then its whole surface area is :

(a) $\frac{2}{3}\pi h^2$ (b) $\frac{3}{2}\pi h^3$
(c) $\frac{3}{2}\pi h^2$ (d) $\pi h^3$

**8.** A right circular cylindrical tunnel of diameter 4 m and length 10 m is to be constructed from a sheet of iron. The area of the iron sheet required :

(a) $\frac{280}{\pi}$ (b) $40\pi$
(c) $80\pi$ (d) none of these

**9.** The ratio between curved surface area and total surface area is 2 : 3. If the total surface area be 924 $cm^2$, find the volume of the cylinder :

(a) 2156 $cm^3$ (b) 1256 $cm^3$
(c) 1265 $cm^3$ (d) none of these

**10.** If the volume and curved surface area of a cylinder are 269.5 $cm^3$ and 154 $cm^2$ respectively, what is the height of the cylinder?

(a) 6 (b) 3.5
(c) 7 (d) can't be determined

**11.** If the curved surface area of a cylinder is 1320 $cm^2$ and its base radius is 21 cm, then its total surface area is :

(a) 4092 $cm^2$ (b) 2409 $cm^2$
(c) 4920 $cm^2$ (d) none of these

**12.** The ratio between the radius of the base and the height of a cylindrical pillar is 3 : 4. If its volume is 4851 $m^3$, the curved surface area of the pillar is :

(a) 924 $m^2$ (b) 1617 $m^2$
(c) 425 $m^2$ (d) none of these

**13.** If the ratio of total surface area to the curved surface area of a cylinder be 4 : 1, what is the ratio of radius to the height?

(a) 4 : 1 (b) 2 : 3
(c) 3 : 2 (d) 3 : 1

**14.** The circumference of the base of a right cylinder is 33 cm and height is 330 cm. What is the volume of this cylinder?

(a) 28586.25 $cm^3$ (b) 3344 $cm^3$
(c) 4433 $cm^3$ (d) 3456 $cm^3$

**15.** The radius of an iron rod decreased to one-fourth. If its volume remains constant, the length will become :

(a) 2 times (b) 8 times
(c) 4 times (d) 16 times

**16.** The total surface area of the cylinder is 2640 $m^2$ and the sum of height and radius of base of cylinder is 30 m. What is the ratio of height and radius of the cylinder?

(a) 7 : 9 (b) 9 : 7
(c) 8 : 7 (d) 3 : 7

**17.** The radii of two cylinders are in the ratio of 3 : 5 and their heights are in the ratio 4 : 3. The ratio of their volumes is :

(a) 12 : 25 (b) 13 : 25
(c) 4 : 5 (d) 5 : 4

**18.** The heights of two cylinders are in the ratio of 3 : 1. If the volumes of two cylinders be same, the ratio of their respective radii are :

(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$
(c) 1 : 9 (d) none of these

**19.** The ratio of heights of two cylinders is 3 : 2 and the ratio of their radii is 6 : 7. What is the ratio of their curved surface areas?

(a) 9 : 7 (b) 1 : 1
(c) 7 : 9 (d) 7 : 4

**20.** A hollow garden roller 42 cm wide with a girth of 132 cm is made of iron 3 cm thick. The volume of the iron of the roller is :

(a) 15544 $cm^3$ (b) 15444 $cm^3$
(c) 15545 $cm^3$ (d) none of these

**21.** A conical vessel has a capacity of 15 L of milk. Its height is 50 cm and base radius is 25 cm. How much milk can be contained in a vessel in cylindrical form having the same dimensions as that of the cone?

(a) 15 L (b) 30 L
(c) 45 L (d) none of these

**22.** The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

(a) 20 cm (b) 18 cm
(c) 27 cm (d) 15 cm

**23.** A tent is in the form of right circular cone 10.5 m high, the diameter of the base being 13 m. If 8 men are in the tent, find the average number of cubic metres of air space per man :

(a) $32\frac{3}{58}$ (b) 59.75
(c) $36\frac{9}{13}$ (d) $58\frac{3}{32}$

**24.** The radius and height of a right circular cone are in the ratio of 5 : 12. If its volume is $314\frac{2}{7}$ $m^3$, its Slant height is :
(a) 26 m (b) 19.5 m
(c) 13 m (d) none of them

**25.** The volume and height of a right circular cone are 1232 $cm^3$ and 24 cm respectively, the area of its curved surface (in $cm^2$) is :
(a) 1100 (b) 225
(c) 616 (d) 550

**26.** The circumference of the base of a right circular cone is 220 $cm^3$ and its height 84 cm. The curved surface area of the cone is :
(a) 20020 $cm^2$ (b) 2020 $cm^2$
(c) 2200 $cm^2$ (d) 10010 $cm^2$

**27.** How many metres of cloth 10 m wide will be required to make a conical tent with base radius of 14 m and height is 48 m?
(a) 110 m (b) 55 m
(c) 77 m (d) 220 m

**28.** A cone of height 2.8 cm has a lateral surface area 23.10 $m^2$. The radius of the base is :
(a) 3.5 cm (b) 2 cm
(c) 2.1 cm (d) 4 cm

**29.** The radii of two cones are equal and their slant heights are in the ratio 3 : 2. If the curved surface area of the smaller cone is 300 $cm^2$, then the curved surface area of the bigger cone (in $cm^2$) is :
(a) 250 (b) 450
(c) 150 (d) 200

**30.** The ratio of the volume of a right circular cylinder and a right circular cone of the same base and height will be :
(a) 2 : 3 (b) 1 : 3
(c) 3 : 1 (d) 9 : 1

**31.** If the diameter of the base of right circular cone is equal to 8 cm and its slant height is 5 cm, then the area of its axial section is :
(a) 9 $cm^2$ (b) 12 $cm^2$
(c) 20 $cm^2$ (d) 40 $cm^2$

**32.** If the base radius and the height of a right circular cone are increased by 40% then the percentage increase in volume (approx) is :
(a) 175% (b) 120%
(c) 64% (d) 540%

**33.** From a circular sheet of paper of radius 25 cm, a sector area 4% is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is :
(a) 16 : 25 (b) 9 : 25
(c) 7 : 12 (d) 24 : 7

**34.** If the radius of the base is doubled, keeping the height constant, what is the ratio of the volume of the larger cone to the smaller cone?
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 4 : 3

**35.** A largest posible cone is cut out from a cube of volume 1000 $cm^3$. The volume of the cone is :
(a) 280 $cm^3$ (b) 261.9 $cm^3$
(c) 269.1 $cm^3$ (d) 296.1 $cm^3$

**36.** If the height and the radius of a cone are doubled, the volume of the cone becomes :
(a) 2 times (b) 8 times
(c) 16 times (d) 4 times

**37.** A conical tent has 60° angle at the vertex. The ratio of its radius and slant height is :
(a) 3 : 2 (b) 1 : 2
(c) 1 : 3 (d) can't be determined

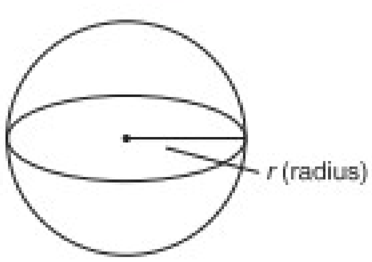
**38.** Water flows at the rate of 5 m per min from a cylindrical pipe 8 mm in diameter. How long will it take to fill up a conical vessel whose radius is 12 cm and depth 35 cm?
(a) 315 s (b) 365 s
(c) 5 min (d) none of these

**39.** A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. It is 6 m deep its capacity is :
(a) 224 $m^3$ (b) 176 $m^3$
(c) 225 $m^3$ (d) none of these

**40.** A conical vessel whose internal radius is 10 cm and height 72 cm is full of water. If this water is poured into a cylindrical vessel with internal radius 30 cm, the height of the water level rises in it is :
(a) $2\frac{2}{3}$ cm (b) $3\frac{2}{3}$ cm
(c) $5\frac{2}{3}$ cm (d) none of these

## SPHERE, PRISM AND PYRAMID

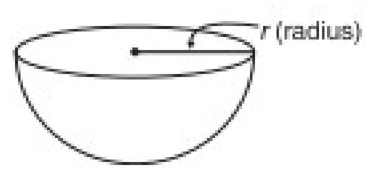
**Sphere :**




Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

**Hemisphere :**




Volume $= \frac{2}{3}\pi r^3$

Curved surface area $= 2\pi r^2$

Total surface area $= 3\pi r^2$ $(2\pi r^2 + \pi r^2)$

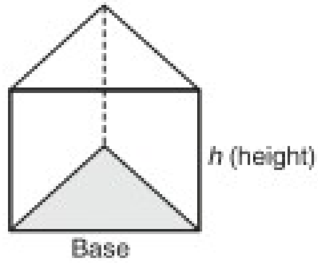
**Spherical shell :**

Volume $= \frac{4}{3}\pi(R^3 - r^3)$

Total surface area $= 4\pi(R^2 + r^2)$

**Prism :**

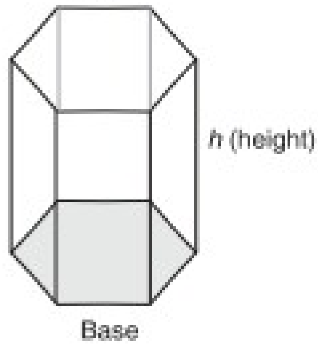
Volume = Base area × height




Lateral surface area

= perimeter of the base

× height



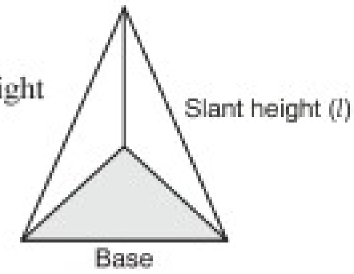

**Pyramid :**

Volume $= \frac{1}{3} \times$ base area × height

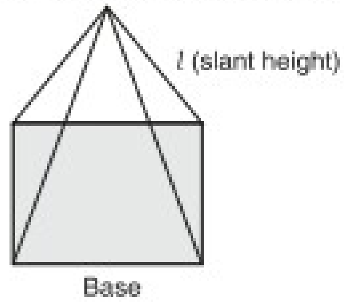
Lateral surface area

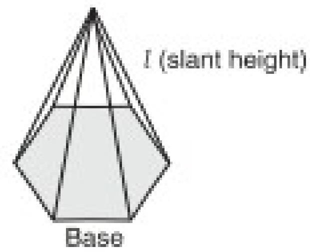
$= \frac{1}{2} \times$ perimeter of the base

× slant height




Total surface area = lateral surface area + base area




**EXAMPLE 1** *Find the volume and surface area of a sphere of radius 3.5 cm.*

**SOLUTION** Volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 = 179.66 \text{ cm}^3$$

Surface area $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5 = 154 \text{ cm}^2$$

**EXAMPLE 2** *Find the volume, curved surface area and total surface area of a hemisphere of diameter 7 cm.*

**SOLUTION** Volume $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 89.833 \text{ cm}^3$$

Curved surface area $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77 \text{ cm}^2$$

Total surface area $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 115.5 \text{ cm}^2$$

**EXAMPLE 3** *How many bullets can be made from a sphere of 8 cm radius. The radius of each bullet must be 0.2 cm.*

**SOLUTION** Number of bullets $= \dfrac{\text{volume of sphere}}{\text{volume of 1 bullet}}$

$$= \frac{\frac{4}{3}\pi \times 8 \times 8 \times 8}{\frac{4}{3}\pi \times 0.2 \times 0.2 \times 0.2} = 64000$$

**EXAMPLE 4** *A sphere has the same curved surface as a cone of height 12 cm and base radius 5 cm. Find the radius to the nearest cm.*

**SOLUTION** $4\pi r^2 = \pi \times 5 \times 13$

$\Rightarrow$ $r^2 = \frac{65}{4}$

$\Rightarrow$ $r = 4$ cm (approx)

## INTRODUCTORY EXERCISE-10.7

1. A spherical ball of lead 6 cm in radius is melted and recast into three spherical balls. The radii of two of these balls are 3 cm and 4 cm. What is the radius of the third sphere?
   (a) 4.5 cm (b) 5 cm
   (c) 6 cm (d) 7 cm
2. The radius of a copper sphere is 12 cm. The sphere is melted and drawn into a long wire of uniform circular cross-section. If the length of the wire is 144 cm, the radius of wire is :
   (a) 1 cm (b) 2 cm
   (c) 4 cm (d) none of these
3. A hemispherical bowl of internal radius 6 cm contains alcohol. This alcohol is to be filled into cylindrical shaped small bottles of diameter 6 cm and height 1 cm. How many bottles will be needed to empty the bowl?
   (a) 36 (b) 27
   (c) 16 (d) 4

**4.** A hemisphere of lead of diameter 14 cm is cast into a right circular cone of height 14 cm. The radius of the base of the cone is :

(a) 7 cm (b) 14 cm
(c) 21 cm (d) none of these

**5.** The volume of a spherical shell whose external and internal diameters are 14 cm and 10 cm respectively :

(a) $42\pi$ cm$^3$ (b) $\frac{872}{3}\pi$ cm$^3$
(c) $118\pi$ cm$^3$ (d) $86\pi$ cm$^3$

**6.** A solid metal ball of diameter 16 cm is melted and cast into smaller balls, each of radius 1 cm. The number of such balls is :

(a) 256 (b) 2048
(c) 512 (d) 4096

**7.** If a hemispherical dome has an inner radius 21 cm then its volume (in m$^3$) is :

(a) 4910 m$^3$ (b) 18354 m$^3$
(c) 19404 m$^3$ (d) none of these

**8.** A sphere of radius 9 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 12 cm. If the sphere is submerged completely, then the surface of the water rises by :

(a) 27.5 cm (b) 27 cm
(c) 12 cm (d) 6.75 cm

**9.** If the height of a cone is half the radius of a sphere then the radius of the base of the cone, which has the same volume as a sphere of radius 7 cm is :

(a) 14 m (b) $\frac{14}{\sqrt{2}}$ cm
(c) $14\sqrt{2}$ cm (d) none of these

**10.** From a solid sphere of radius 15 cm, a right circular cylindrical hole of radius 9 cm whose axis passing through the centre is removed. The total surface area of the remaining solid is :

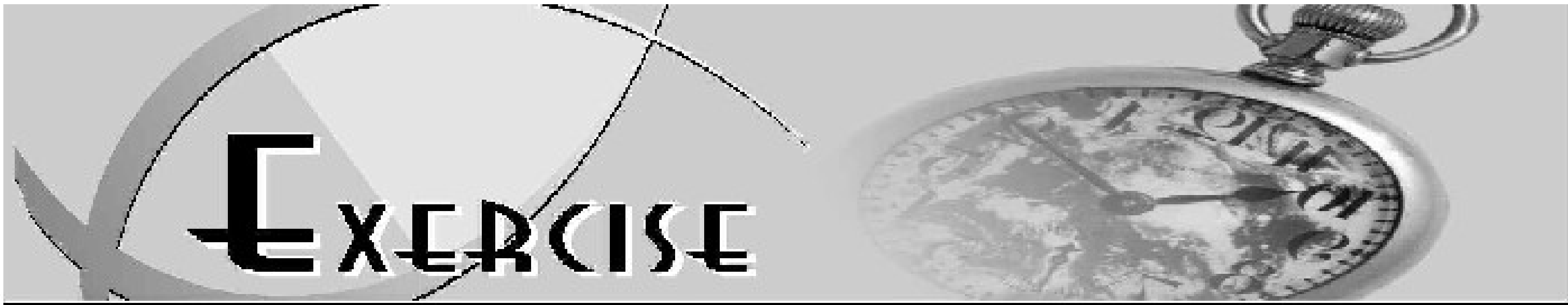
(a) $1188\pi$ cm$^2$ (b) $108\pi$ cm$^2$
(c) $1170\pi$ cm$^2$ (d) $144\pi$ cm$^2$

**11.** The volume of a pyramid of base area 25 cm$^2$ and height 12 cm is :

(a) 200 cm$^3$ (b) 100 cm$^3$
(c) 400 cm$^3$ (d) 800 cm$^3$

**12.** If the base of right rectangular prism remains constant and the measures of the lateral edges are halved, then its volume will be reduced by :

(a) 50% (b) 33.33%
(c) 66.66% (d) none of these




## MISCELLANEOUS

**1.** If the side of an equilateral triangle is $r$, then the area of the triangle varies directly as :

(a) $\sqrt{r}$ (b) $r$
(c) $r^2$ (d) $r^3$

**2.** The length of the minute hand of the clock is 6 cm. The area swept by the minute hand in 30 minutes is :

(a) $\frac{1}{36\pi}$ cm$^2$ (b) $\frac{1}{18\pi}$ cm$^2$
(c) $18\pi$ cm$^2$ (d) $36\pi$ cm$^2$

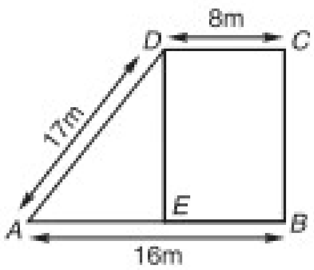
**3.** If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its perimeter is :

(a) 15 cm (b) 42 cm
(c) 60 cm (d) 70 cm

**4.** If the ratio of diagonals of two squares is 3 : 2 then the ratio of the areas of two squares is :

(a) 4 : 5 (b) 6 : 5
(c) 9 : 4 (d) $\sqrt{3} : \sqrt{2}$

**5.** In the given figure, $ABCD$ is a trapezium in which the parallel sides $AB, CD$ are both perpendicular to $BC$. Find the area of the trapezium :




(a) 140 m$^2$
(b) 168 m$^2$
(c) 180 m$^2$
(d) 156.4 m$^2$

**6.** One cubic metre piece of copper is melted and recast into a square cross-section bar, 36 m long. An exact cube is cut off from this bar. If cubic metre of copper cost Rs. 108, then the cost of this cube is :

(a) 50 paisa (b) 75 paisa
(c) one rupee (d) 1.50 rupee

**7.** If '$h$' be the height of a pyramid standing on a base which is an equilateral triangle of side '$a$' units, then the slant height is :

(a) $\sqrt{h^2 + a^2/4}$ (b) $\sqrt{h^2 + a^2/8}$
(c) $\sqrt{h^2 + a^2/3}$ (d) $\sqrt{h^2 + a^2}$

**8.** The area of the square base of a right pyramid is 64 cm$^2$. If the area of each triangle forming the slant surface is 20 cm$^2$, then the volume of the pyramid is :

(a) 64 cm$^3$ (b) $\frac{128}{3}$ cm$^3$
(c) $\frac{64}{3}\sqrt{3}$ cm$^3$ (d) $64\sqrt{2}$ cm$^3$

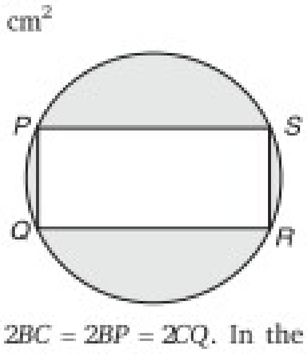
**9.** If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volumes is :

(a) 8 : 25 (b) 8 : 26
(c) 8 : 27 (d) 8 : 28

**10.** The side of a rhombus are 10 cm and one of its diagonal is 16 cm. The area of the rhombus is :

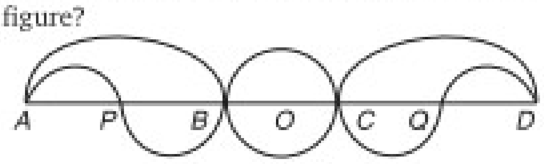
(a) 96 cm$^2$ (b) 95 cm$^2$
(c) 94 cm$^2$ (d) 93 cm$^2$

**11.** In the adjoining figure $PQRS$ is a rectangle $8 \text{ cm} \times 6 \text{ cm}$, inscribed in the circle. The area of the shaded portion will be :




(a) 48 cm$^2$
(b) 42.50 cm$^2$
(c) 32.50 cm$^2$
(d) 30.5 cm$^2$

**12.** In the adjoining figure $AB = CD = 2BC = 2BP = 2CQ$. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?




(a) $4\pi$ (b) $8\pi$
(c) $10\pi$ (d) none of these

**13.** In a shower 10 cm of rain fall the volume of water that falls on 1.5 hectares of ground is :

(a) 1500 m$^3$ (b) 1400 m$^3$
(c) 1200 m$^3$ (d) 1000 m$^3$

**14.** The base of a prism is a right angle triangle and the two sides containing the right angle are 8 cm and 15 cm. If its height is 20 cm, then the volume of the prism is :

(a) 1600 cc (b) 300 cc
(c) 1200 cc (d) 600 cc

**15.** A conical circus tent is to be made of canvas. The height of the tent is 35 m and the radius of the base is 84 m. If $\pi = \frac{22}{7}$, then the canvas required is :

(a) 24000 m$^2$ (b) 24004 m$^2$
(c) 24014 m$^2$ (d) 24024 m$^2$

**16.** The radius of base and the volume of a right circular cone are doubled. The ratio of the length of the larger cone to that of the smaller cone is :

(a) 1 : 4 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

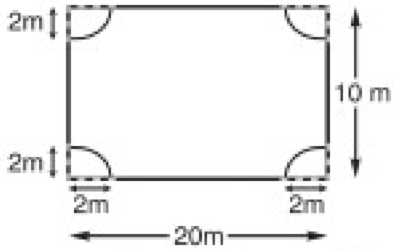
**17.** A cone and a hemisphere have equal base diameter and equal volumes. The ratio of their heights is :

(a) 3 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 3

**18.** A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted to form a solid cylinder of base diameter 8 cm. The height of the cylinder is approximately :

(a) 4.5 cm (b) 4.57 cm
(c) 4.67 cm (d) 4.7 cm

**19.** The perimeter of the figure given below correct to one decimal place is :




(a) 56 m
(b) 56.6 m
(c) 57.2 m
(d) 57.9 m

**20.** The sum of the radii of the two circle is 140 cm and the difference between their circumference is 88 cm. The radius of the larger circle is :
(a) 60 cm (b) 70 cm
(c) 63 cm (d) 77 cm

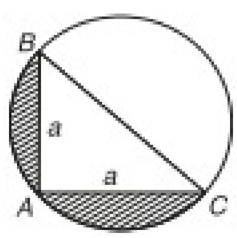
**21.** If the lateral surface of a right circular cone is 2 times its base, then the semi-vertical angle of the cone must be :
(a) $15°$ (b) $30°$
(c) $45°$ (d) $60°$

**22.** There is a pyramid on a base which is a regular hexagon of side $2a$. If every slant edge of this pyramid is of length $\frac{(5a)}{2}$ then the volume of this pyramid must be :
(a) $3a^3$ (b) $3a^3\sqrt{2}$
(c) $3a^3\sqrt{3}$ (d) $6a^3$

**23.** The slant height of a conical tent made of canvas is $\frac{14}{3}$ m. The radius of tent is 2.5 m. The width of the canvas is 1.25 m. If the rate of canvas per metre is Rs. 33, then the total cost of the canvas required for the tent (in Rs.) is :
(a) 726 (b) 950
(c) 960 (d) 968

**24.** A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube (in cm) is :
(a) 23 (b) 24
(c) 25 (d) 26

**25.** If $BC$ passes through centre of the circle, then the area of the shaded region in the given figure is :




(a) $\frac{a^2}{2}(3-\pi)$
(b) $a^2\left(\frac{\pi}{2}-1\right)$
(c) $2a^2(\pi-1)$
(d) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$

**26.** A river 3 m deep and 60 m wide is flowing at the rate of 2.4 km/h. The amount of water running into the sea per minute is :
(a) 6000 $m^3$ (b) 6400 $m^3$
(c) 6800 $m^3$ (d) 7200 $m^3$

**27.** A cone whose height is 15 cm and radius of base is 6 cm is trimmed sufficiently to reduce it to a pyramid whose base is an equilateral triangle. The volume of the portion removed is :
(a) 330 $cm^3$ (b) 328 $cm^3$
(c) 325 $cm^3$ (d) 331 $cm^3$

**28.** If a solid right circular cylinder is made of iron is heated to increase its radius and height by 1% each, then the volume of the solid is increased by :
(a) 1.01% (b) 3.03%
(c) 2.02% (d) 1.2%

**29.** The base of a prism is a regular hexagon. If every edge of the prism measures 1 m, then the volume of the prism is :
(a) $\frac{3\sqrt{2}}{2}$ $m^3$ (b) $\frac{3\sqrt{3}}{2}$ $m^3$
(c) $\frac{6\sqrt{2}}{5}$ $m^3$ (d) $\frac{5\sqrt{3}}{2}$ $m^3$

**30.** If the side of a square is 24 cm, then the circumference of its circumscribed circle (in cm) is :
(a) $24\sqrt{3}\pi$ (b) $24\sqrt{2}\pi$
(c) $12\sqrt{2}\pi$ (d) $24\pi$

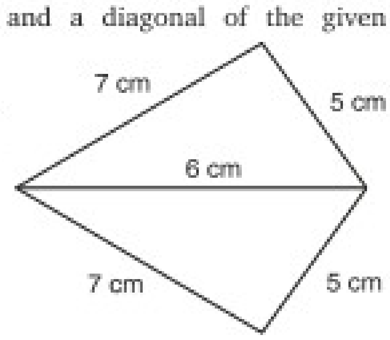
**31.** An isosceles right triangle has area 112.5 $m^2$. The length of its hypotenuse (in cm) is :
(a) 21.213 (b) 21.013
(c) 21.113 (d) 21.313

**32.** Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region, common to both the circles, is :
(a) $\frac{(4\pi-3\sqrt{3})}{12}$ (b) $\frac{(4\pi-6\sqrt{3})}{12}$
(c) $\frac{(4\pi-3\sqrt{3})}{6}$ (d) $\frac{(4\pi-6\sqrt{3})}{6}$

**33.** If the right circular cone is separated into three solids of volumes $V_1$, $V_2$ and $V_3$ by two planes which are parallel to the base and trisects the altitude, then $V_1 : V_2 : V_3$ is :
(a) 1 : 2 : 3 (b) 1 : 4 : 6
(c) 1 : 6 : 9 (d) 1 : 7 : 19

**34.** Water flows at the rate of 10 m per minute from a cylindrical pipe 5 mm in diameter. A conical vessel whose diameter is 40 cm and depth 24 cm is filled. The time taken to fill the conical vessel is :
(a) 50 min (b) 50 min. 12 sec.
(c) 51 min. 12 sec (d) 51 min. 15 sec

**35.** The length of four sides and a diagonal of the given quadrilateral are indicated in the diagram. If $A$ denotes the area and $l$ the length of the other diagonal, then $A$ and $l$ are respectively :




(a) $12\sqrt{6}$, $4\sqrt{6}$
(b) $12\sqrt{6}$, $5\sqrt{6}$
(c) $6\sqrt{6}$, $4\sqrt{6}$
(d) $6\sqrt{6}$, $5\sqrt{6}$

**36.** If a regular square pyramid has a base of side 8 cm and height of 30 cm, then its volume is :
(a) 120 cc (b) 240 cc
(c) 640 cc (d) 900 cc

**37.** A cylinder curcumscribes a sphere. The ratio of their volumes is :
(a) 1 : 2 (b) 3 : 2
(c) 4 : 3 (d) 5 : 6

**38.** In triangle $ABC$, $BC = 8$ cm, $AC = 15$ cm and $AB = 17$ cm. The length of the altitude drawn from $B$ on $AC$ is :
(a) 6 cm (b) 7 cm
(c) 8 cm (d) 10 cm

**39.** The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :
(a) 3 (b) 4
(c) $2\sqrt{3}\pi$ (d) 2

**40.** The area of the largest triangle that can be inscribed in a semicircle of radius $r$ is :

(a) $r^2$ cm$^2$ (b) $\left(\frac{r}{3}\right)^2$ cm$^2$
(c) $r\sqrt{2}$ cm$^2$ (d) $3\sqrt{3r}$ cm$^2$

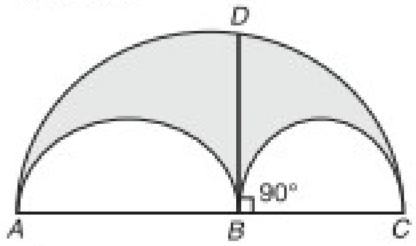
**41.** If a regular hexagon is inscribed in a circle of radius $r$, then its perimeter is :

(a) $6\sqrt{3}r$ (b) $6r$
(c) $3r$ (d) $12r$

**42.** If a regular hexagon circumscribes a circle of radius $r$, then its perimeter is :

(a) $4\sqrt{3}r$ (b) $6\sqrt{3}r$
(c) $6r$ (d) $12\sqrt{3}r$

**43.** In the adjoining figure there are three semicircles in which $BC = 6$ cm and $BD = 6\sqrt{3}$ cm. What is the area of the shaded region (in cm) :

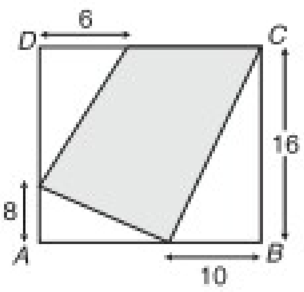



(a) $12\pi$ (b) $9\pi$
(c) $27\pi$ (d) $28\pi$

**44.** Area of a rhombus is 144 cm$^2$ and the ratio of length of two diagonals is 1 : 2. The sum of lengths of its diagonals are :

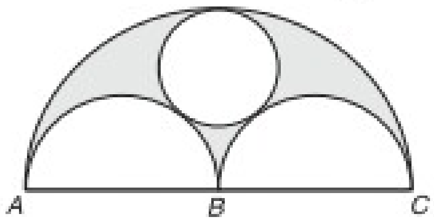
(a) 72 cm (b) 40 cm
(c) 36 cm (d) $18\sqrt{2}$ cm

**45.** Find the area of the shaded region in the given figure of square $ABCD$ :




(a) 128 cm$^2$
(b) 192 cm$^2$
(c) 148 cm$^2$
(d) 168 cm$^2$

**46.** In the following figure $AB = BC$ and $AC = 84$ cm. The radius of the inscribed circle is 14 cm. $B$ is the centre of the largest semi- circle. What is the area of the shaded region?




(a) 335 cm$^2$ (b) 770 cm$^2$
(c) 840 cm$^2$ (d) 650 cm$^2$

**47.** A tank 4 m long and 2.5 m wide and 6 m deep is dug in a field 10 m long and 9 m wide. If the earth dugout is evenly spread over the field, the rise in level of the field will be :

(a) 80 cm (b) 75 cm
(c) 60 cm (d) 30 cm

**48.** An open box is made of wood 2 cm thick. Its internal length is 86 cm, breadth 46 cm and height is 38 cm. The cost of painting the outer surface of the box at Rs. 10 per m$^2$ is :

(a) Rs. 18.5 (b) Rs. 8.65
(c) Rs. 11.65 (d) Rs. 17.50

**49.** A rectangular tin sheet is 22 m long and 8 m broad. It is rolled along its length to form a cylinder by making the opposite edges just to touch each other. The volume of the cylinder (in m$^3$) is :

(a) 385 (b) 204
(c) $280\pi$ (d) 308

**50.** The lateral surface of a cylinder is developed into a square whose diagonal is $2\sqrt{2}$ cm. The area of the base of the cylinder (in cm$^2$) is :

(a) $3\pi$ (b) $1/\pi$
(c) $\pi$ (d) $6\pi$

**51.** If from a circular sheet of paper of radius 15 cm, a sector of 144° is removed and the remaining is used to make a conical surface, then the angle at the vertex will be :

(a) $\sin^{-1}\left(\frac{3}{10}\right)$ (b) $\sin^{-1}\left(\frac{6}{5}\right)$
(c) $2\sin^{-1}\left(\frac{3}{5}\right)$ (d) $2\sin^{-1}\left(\frac{4}{5}\right)$

**52.** A right circular cone of radius 4 cm and slant height 5 cm is carved out from a cylindrical piece of wood of same radius and height 5 cm. The surface area of the remaining wood is :

(a) $84\pi$ (b) $70\pi$
(c) $76\pi$ (d) $50\pi$

**53.** If $h$, $s$, $V$ be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is equal to :

(a) 0 (b) $\pi$
(c) $\frac{V}{sh}$ (d) $\frac{36}{V}$

**54.** If a cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volumes of the upper part and the frustum is :

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 7

**55.** A cone, a hemisphere and a cylinder stand on equal bases of radius $R$ and have equal heights $H$. Their whole surfaces are in the ratio :

(a) $(\sqrt{3}+1) : 3 : 4$ (b) $(\sqrt{2}+1) : 7 : 8$
(c) $(\sqrt{2}+1) : 3 : 4$ (d) none of these

**56.** If a sphere is placed inside a right circular cylinder so as to touch the top, base and the lateral surface of the cylinder. if the radius of the sphere is $R$, the volume of the cylinder is :

(a) $2\pi R^3$ (b) $8\pi R^3$
(c) $\frac{4}{3}\pi R^3$ (d) none of these

**57.** A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of :

(a) $3 : \sqrt{3} : 2$ (b) $3 : 2 : 1$
(c) $1 : 2 : 3$ (d) $2 : 3 : 1$

**58.** The base of a pyramid is a rectangle 40 m long and 20 m wide. The slant height of the pyramid from the mid-point of the shorter side of the base to the apex is 29 m. What is the volume of pyramid?

(a) 5600 m$^3$ (b) 400 m$^3$
(c) 6500 m$^3$ (d) $1753\sqrt{110}$ m$^3$

**59.** A copper wire when bent in the form of a square, encloses an area of 121 m$^2$. If the same wire is bent to form a circle, the area enclosed by it would be :

(a) 122 m$^2$ (b) 112 m$^2$
(c) 154 m$^2$ (d) 308 m$^2$

# LEVEL 1

**1.** The perimeter of a parallelogram with one internal angles 150° is 64 cm. Find the length of its sides when its area is maximum :

(a) 16, 16 (b) 15, 17
(c) 14, 18 (d) can't be determined

**2.** A sphere of 30 cm radius is dropped into a cylindrical vessel of 80 cm diameter, which is partly filled with water, then its level rises by $x$ cm. Find $x$ :

(a) 27.5 cm (b) 22.5 cm
(c) 18.5 cm (d) none of these

**3.** Amit walked 12 m toward east, then he turned to his right and walked 18 m. He then turned to his right and walked 12 m. He again turned to his right and walked 28 m then he again turned to his right and walked 24 m. At what distance is he from the starting point and in which direction?

(a) 23 m north-east (b) 26 m north-east
(c) 26 m west (d) 34 m north-east

**4.** Find the inradius of triangle if its area is 30 $cm^2$ and hypotenuse is 13 cm :

(a) 1 cm (b) 2 cm
(c) 2.5 cm (d) $2\sqrt{2}$ cm

**5.** Which of the following figure will have maximum area if the perimeter of all figures is same?

(a) Square (b) Octagon
(c) Circle (d) Hexagon

**6.** $ABCD$ is a trapezium with $\angle A = 90°$ and $AB$ parallel to $CD$. Then $\angle B$ is :

(a) 90° (b) $90° - \angle C$
(c) $360° - \angle C$ (d) $180° - \angle C$

**7.** A square $ABCD$ has an equilateral triangle drawn on the side $AB$ (interior of the square). The triangle has vertex at $G$. What is the measure of the angle $CGB$ ?

(a) 60° (b) 80°
(c) 75° (d) 90°

**8.** There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?

(a) 32 $cm^2$ (b) $64\pi$ $cm^2$
(c) 36 $cm^2$ (d) 48 $cm^2$

**9.** A square and rhombus have the same base. If the rhombus is inclined at 60°, find the ratio of area of square to the area of the rhombus :

(a) $2\sqrt{3} : 3$ (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : 2$ (d) none of these

**10.** Four isoscelles triangles are cut off from the corners of a square of area 400 $m^2$. Find the area of new smaller square (in $m^2$) :

(a) $200\sqrt{2}$ (b) $\dfrac{200}{\sqrt{2}}$
(c) 200 (d) $100\sqrt{2}$

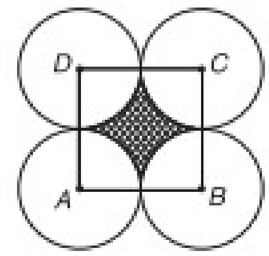
**11.** Altitude and base of a right angle triangle are $(x + 2)$ and $(2x + 3)$ (in cm). If the area of the triangle be 60 $cm^2$, the length of the hypotenuse is :

(a) 21 cm (b) 13 cm
(c) 17 cm (d) 15 cm

**12.** Find the area of a regular octagon with each side '$a$' cm :

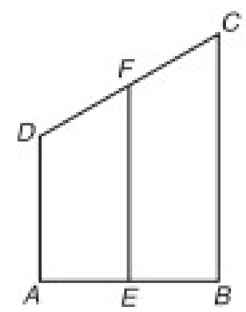
(a) $2a^2 (1 + \sqrt{2})$ (b) $\sqrt{2}a (1 + \pi)$
(c) $a^2 (\sqrt{2} + 2)$ (d) none of these

**13.** $ABCD$ is a square, 4 equal circles are just touching each other whose centres are the vertices $A, B, C, D$ of the square. What is the ratio of the shaded to the unshaded area within square?




(a) $\dfrac{8}{11}$ (b) $\dfrac{3}{11}$
(c) $\dfrac{5}{11}$ (d) $\dfrac{6}{11}$

**14.** $ABCD$ is a trapezium, in which $AD \parallel BC$, $E$ and $F$ are the mid-points of $AB$ and $CD$ respectively, then $EF$ is :




(a) $\dfrac{(AD + BC)}{2}$

(b) $\dfrac{(AB + CD)}{2}$

(c) $\dfrac{DF \times CF}{AE \times BE}$

(d) $\dfrac{AD + EF + BC}{2}$

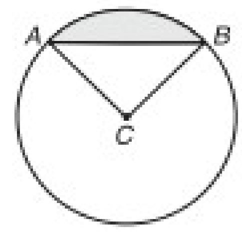
**15.** A right circular cone resting on its base is cut at $\dfrac{4}{5}$th its height along a plane parallel to the circular base. The height of original cone is 75 cm and base diameter is 42 cm. What is the base radius of cut out (top portion) cone?

(a) 4.2 cm (b) 2.8 cm
(c) 3.5 cm (d) 8.4 cm

**16.** $l$, $b$ are the length and breadth of a rectangle respectively. If the perimeter of this rectangle is numerically equal to the area of the rectangle. What is the value of $l - b$ (where $l > b$)?

(a) 1 (b) 2
(c) 3 (d) can't be determined

**17.** In the adjoining figure $ABC$ is an equilateral triangle and $C$ is the centre of the circle, $A$ and $B$ lie on the circle. What is the area of the shaded region, if the diameter of the circle is 28 cm?




(a) $\left(102\dfrac{2}{3} - 49\sqrt{3}\right)$ $cm^2$

(b) $\left(103\dfrac{2}{3} - 98\sqrt{3}\right)$ $cm^2$

(c) $(109 - 38\sqrt{3})$ cm

(d) none of the above

**18.** $l_1, b_1$ and $l_2, b_2$ are the lengths and breadths of the two rectangles respectively, but the areas of the rectangles are same. $l_1$ is increased by 25% and $b_1$ is decreased by 25%. Similarly $l_2$ is decreased by 25% and $b_2$ is increased by 25%. If $A_1$ and $A_2$ is the new areas of the two rectangles respectively, then :

(a) $A_1 > A_2$ (b) $A_1 < A_2$
(c) $A_1 = A_2$ (d) can't be determine

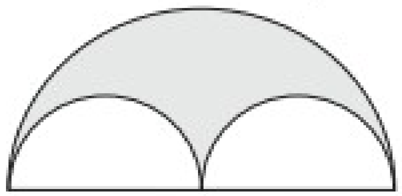
**19.** An acute angle made by a side of parallelogram with other pair of parallel sides is 60°. If the distance between these parallel sides is $6\sqrt{3}$, the other side is :

(a) 12 cm (b) $12\sqrt{3}$ cm
(c) $15\sqrt{3}$ cm (d) none of these

**20.** A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of the sphere. What is the ratio of the height and radius of the cone so formed?

(a) 4 : 3 (b) 2 : 3
(c) 3 : 4 (d) none of these

**21.** In the given figure there are 3 semicircles, the radii of each smaller circle is equal. If the radius of the larger circle be 22 cm, then the area of the shaded region is :



(a) $363\frac{\pi}{4}$ (b) $363\frac{\pi}{3}$
(c) $236.5\ \pi$ (d) $363\pi$

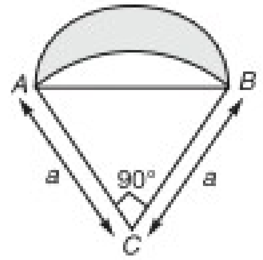
**22.** A rectangular lawn 60 m × 40 m has two roads each 5 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 80 paise per sq. m is :

(a) Rs. 380 (b) Rs. 385
(c) Rs. 400 (d) none of these

**23.** There are two rectangular fields of same area. The length of first rectangular field is $x$% less than the length of the second field and breadth of the first field is $(5x)$% greater than the breadth of the second field. What is the value of $x$?

(a) 15 (b) 25
(c) 50 (d) 80

**24.** In the adjoining figure $ACB$ is a quadrant with radius '$a$'. A semicircle is drawn outside the quadrant taking $AB$ as a diameter. Find the area of shaded region :

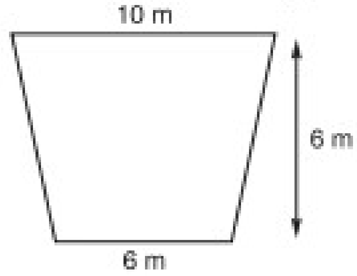



(a) $\frac{1}{4}(\pi - 2a^2)$
(b) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$
(c) $\frac{a^2}{2}$
(d) can't be determined

**25.** Ravi made an error of 5% in excess while measuring the length of rectangle and an error of 8% deficit was made while measuring the breadth. What is the percentage error in the area?

(a) − 3% (b) − 40%
(c) − 3.4% (d) can't be determined

**26.** In the adjoining figure the cross-section of a swimming pool is shown. If the length of the swimming pool is 120 m then the amount of water it can hold is :




(a) 5760 $m^3$
(b) 9600 $m^3$
(c) 7200 $m^3$
(d) none of these

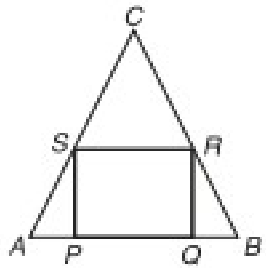
**27.** Around a circular garden a circular road is to be repair which costs Rs. 22176 at the rate of Rs. 1 per sq. m. If the inner radius is 112 m, find the width of the circular road :

(a) 18 m (b) 28 m
(c) 14 m (d) none of these

**28.** An equilateral triangle is cut from its three vertices to form a regular hexagon. What is the percentage of area wasted?

(a) 20% (b) 50%
(c) 33.33% (d) 66.66%

**29.** $ABC$ is an equilateral triangle and $PQRS$ is a square inscribed in the triangle in such a way that $P$ and $Q$ lie on $AB$ and $R$, $S$ lie on $BC$ and $AC$ respectively. What is the value of $RC : RB$ ?

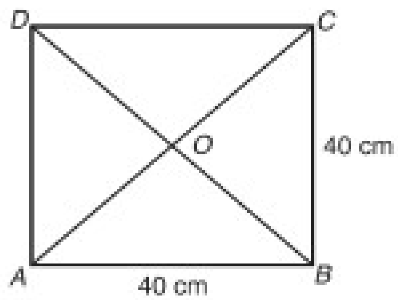



(a) $1 : \sqrt{2}$
(b) $1 : \sqrt{3}$
(c) $\sqrt{3} : 2$
(d) 1 : 2

**30.** The area of a square and circle is same and the perimeter of square and equilateral triangle is same, then the ratio between the area of circle and the area of equilateral triangle is :

(a) $\pi : 3$ (b) $9 : 4\sqrt{3}$
(c) $4 : 9\sqrt{3}$ (d) none of these

**31.** Adjoining figure shows a square $ABCD$ in which $O$ is the point of intersection of diagonals $AC$ and $BD$. Four squares of maximum possible area are formed inside each four triangles $AOB$, $BOC$, $COD$ and $AOD$. What is the total area of these 4 squares?




(a) 400 $cm^2$ (b) 100 $cm^2$
(c) 80 $cm^2$ (d) none of these

**32.** What is the ratio of the area of circumcircle of equilateral triangle to the area of square with the same side length as the equilateral triangle?

(a) $\pi : 3$ (b) $\pi : \sqrt{3}$
(c) $\sqrt{3} : 2$ (d) none of these

**33.** It is required to construct a big rectangular hall that can accommodate 400 people with 25 $m^3$ space for each person. The height of the wall has been fixed at 10 m and the total inner surface area of the walls must be 1300 $m^2$. What is the length and breadth of the hall (in metres)?

(a) 30, 20 (b) 45, 20
(c) 40, 25 (d) 35, 30

**34.** The perimeter of a rectangle and an equilateral triangle are same. Also, one of the sides of the rectangle is equal to the side of the triangle. The ratio of the areas of the rectangle and the triangle is :
(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$
(c) $2:\sqrt{3}$ (d) $4:\sqrt{3}$

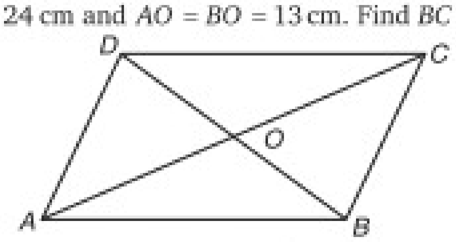
**35.** If $l$, $b$, $p$ be the length, breadth and perimeter of a rectangle and $b$, $l$, $p$ are in GP (in order), then $l/b$ is :
(a) 2 : 1 (b) $(\sqrt{3}-1):1$
(c) $(\sqrt{3}+1):1$ (d) $2:\sqrt{3}$

**36.** A spherical steel ball was silver polished then it was cut into 4 similar pieces. What is ratio of the polished area to the non polished area :
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) can't be determined

**37.** What is the total surface area of the identical cubes of largest possible size that are cut from a cuboid of size 85 cm × 17 cm × 5.1 cm?
(a) 26010 $cm^2$ (b) 21600 $cm^2$
(c) 26100 $cm^2$ (d) none of these

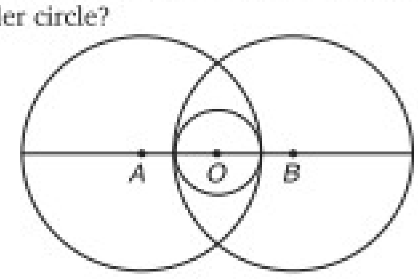
**38.** 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube?
(a) $234\frac{2}{3}\%$ (b) $235\frac{1}{3}\%$
(c) $134\frac{2}{3}\%$ (d) none of these

**39.** In the adjoining figure a parallelogram $ABCD$ is shown. $AB = 24$ cm and $AO = BO = 13$ cm. Find $BC$ :



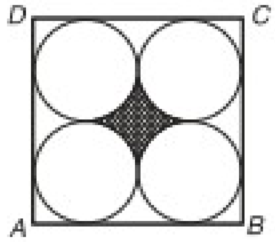

(a) 8 cm (b) 10 cm
(c) 11 cm (d) none of these

**40.** There are two circles intersecting each other. Another smaller circle with centre $O$, is lying between the common region of two larger circles. Centres of the circle (*i.e.*, $A$, $O$ and $B$) are lying on a straight line. $AB = 16$ cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?



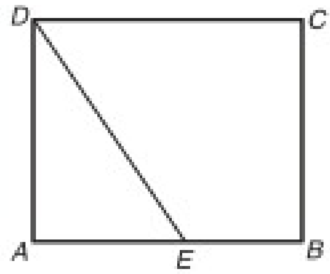

(a) $4\pi$ $cm^2$ (b) $2\pi$ $cm^2$
(c) $\frac{4}{\pi}$ $cm^2$ (d) $\frac{\pi}{4}$ $cm^2$

**41.** $ABCD$ is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?




(a) $(2\pi - 3)$ $cm^2$
(b) $(4 - \pi)$ $cm^2$
(c) $(16 - 4\pi)$ $cm^2$
(d) none of these

**42.** $ABCD$ is a square, $E$ is a point on $AB$ such that $BE = 17$ cm. The area of triangle $ADE$ is 84 $cm^2$. What is the area of square?

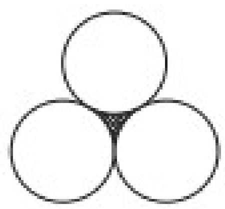



(a) 400 $cm^2$
(b) 625 $cm^2$
(c) 729 $cm^2$
(d) 576 $cm^2$

**43.** If the volume of a sphere, a cube, a tetrahedron and a octahedron be same then which of the following has maximum surface area?
(a) Sphere (b) Cube
(c) Octahedron (d) Tetrahedron

**44.** In a rectangle the ratio of the length is to breadth is same as that of the sum of the length and breadth to the length. If $l$ and $b$ be the length and breadth of the rectangle then which of the following is true?
(i) $\frac{l}{b} = \frac{l^2}{b^2} + 1$ (ii) $\frac{b}{l-b} = \frac{l+b}{l}$
(iii) $lb = (l+b)(l-b)$
(a) only (i) is true
(b) only (ii) is true
(c) only (ii) and (iii) are true
(d) only (i) and (ii) are true

**45.** Three circle of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?



(a) $\left(\frac{2\sqrt{3}-\pi}{2}\right)$ $cm^2$
(b) $\left(\frac{3\sqrt{2}-\pi}{3}\right)$ $cm^2$
(c) $\frac{2\sqrt{3}}{\pi}$ $cm^2$
(d) none of the above

**46.** How many spheres of radius 1.5 cm can be cut out of a wooden cube of edge 9 cm?
(a) 216 (b) 81
(c) 27 (d) can't be determined

**47.** Kaurav and Pandav have a rectangular field of area 20,000 sq. m. They decided to divide it into two equal parts by dividing it with a single straight line. Kaurav wanted to fence their land immediately, so they incurred total expenses for the fencing all the four sides alone at Rs. 2 per metre. What is the minimum cost that Kaurav had to incur?
(a) Rs. 800 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 600

**48.** There is a cone of height 12 cm, out of which a smaller cone (which is the top portion of the original cone) with the same vertex and vertical axis is cut out. What is the ratio of the volume of the larger (actual) cone to the remaining part (frustum) of the cone, if the height of the smaller cone is 9 cm?
(a) 3 : 1 (b) 9 : 1
(c) 64 : 37 (d) 16 : 7

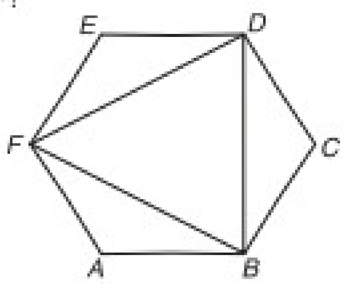
**49.** Radhey can walk along the boundary of a rectangular field and also along the diagonals of the field. His speed is 53 km/h. The length of the field is 45 km. Radhey started from one corner and reached to the diagonally opposite corner in 1 hour. What is the area of the field?
(a) 860 km$^2$ (b) 1260 km$^2$
(c) 1060 km$^2$ (d) can't be determined

**50.** Charles has a right circular cylinder which he inserted completely into a right circular cone of height 30 cm. The vertical angle of the cone is 60° and the diameter of the cylinder is $8\sqrt{3}$ cm. What is the volume of the cone?
(a) $\frac{3000}{7}\pi$ cm$^3$ (b) $3000\pi$ cm$^3$
(c) $4860\pi$ cm$^3$ (d) can't be determind

**51.** There are six faces in a cube. Rajeev fix one cube on each of the faces. The dimensions of all the cubes are same. What is the ratio of total surface area of the newly formed solid to the area of a single cube?
(a) 7 : 1 (b) 6 : 1
(c) 5 : 1 (d) 41 : 9

**52.** If the ratio of diagonals of two cubes is 3 : 2 then the ratio of the surface areas of the two cubes respectively is :
(a) 5 : 4 (b) 9 : 5
(c) 9 : 4 (d) can't be determind

**53.** *ABCDEF* is a regular hexagon of side 6 cm. What is the area of triangle *BDF*?




(a) $32\sqrt{3}$ cm$^2$ (b) $27\sqrt{3}$ cm$^2$
(c) 24 cm$^2$ (d) none of these

***Directions for question number 54 and 55 :*** *King Dashratha of Ayodhya had a rectangular plot of area 9792 m$^2$. He divided it into 4 square shaped plots by fencing parallel fences to the breadth of the rectangular plot. All the four sons got each square shaped plot. However, some area of plot was still left which could not be formed as a square shaped. So, four more square shaped plots were formed by fencing parallel to the longer side of the original plot. The king gave one smaller square shaped plot to each of his wives and one of the smaller square shaped plot retained with himself and then nothing left to divide.*

**54.** What is the ratio of the area of larger square shaped plot to the area of the smaller square shaped plot?
(a) 17 : 1 (b) 25 : 9
(c) 16 : 1 (d) can't be determined

**55.** What are dimensions of the original plot?
(a) 288 m, 34 m (b) 102 m, 96 m
(c) 306 m, 32 m (d) 204 m, 48 m

## LEVEL 2

***Directions for question number 1, 2 and 3 :*** *Each edge of an equilateral triangle is 'a' cm. A cone is formed by joining any two sides of the triangle.*

**1.** What is the radius and slant height of the cone?
(a) $a, \frac{a}{2\pi}$ (b) $\frac{a}{\pi}, \frac{a}{2}$
(c) $\frac{a}{2\pi}, a$ (d) $2a, \frac{a}{\pi}$

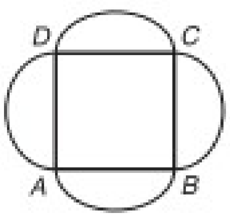
**2.** What is the volume of the cone?
(a) $\frac{a^2}{24\pi^3}\sqrt{4-\pi^2}$ (b) $\frac{a^3}{24\pi^2}\sqrt{4\pi^2-1}$
(c) $\frac{a^3}{8\pi^2}\sqrt{1-4\pi^2}$ (d) $\frac{a}{\sqrt{3}}\pi^2\left(1-\frac{2}{\pi}\right)$

**3.** If the cone is cut along its axis from the middle, the new shape we obtain after opening the paper is :
(a) isosceles triangle (b) equilateral triangle
(c) right angle triangle (d) none of these

**4.** If the sum of the radius and the height of a closed cylinder is 35 cm and the total surface area of the cylinder is 1540 cm$^2$, then the circumference of the base of the cylinder is :
(a) 66 cm (b) 44 cm
(c) 56 cm (d) can't be determined

**5.** An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of base of cone, as well as cylinder is 21 cm. The cylindrical part is 80 cm high and conical part is 16 cm high. Find the weight of the pillar, if 1 cm$^3$ of iron weighs 8.45 g :
(a) 999.39 kg (b) 111 kg
(c) 1001 kg (d) 989 kg

**6.** *ABCD* is a square of side *a* cm. *AB*, *BC*, *CD* and *AD* all are the chords of circles with equal radii each. If the chords subtends an angle of 120° at their respective centres, find the total area of the given figure, where arcs are part of the circles :




(a) $\left[a^2+4\left(\frac{\pi a^2}{9}-\frac{a^2}{3\sqrt{2}}\right)\right]$ (b) $\left[a^2+4\left(\frac{\pi a^2}{9}-\frac{a^2}{4\sqrt{3}}\right)\right]$
(c) $[9a^2-4\pi+3\sqrt{3}a^2]$ (d) none of these

**7.** A rectangle has a perimeter of 26. How many combinations of integral valued length are possible?
(a) 4 (b) 8
(c) 6 (d) 12

**8.** A hollow sphere of outer diameter 24 cm is cut into two equal hemisphere. The total surface area of one of the hemisphere is $1436\frac{2}{7}$ cm$^2$. Each one of the hemisphere is filled with water. What is the volume of water that can be filled in each of the hemisphere?

(a) $3358\frac{2}{3}$ cm$^3$ (b) $3528\frac{2}{3}$ cm$^3$
(c) $2359\frac{2}{3}$ cm$^3$ (d) $9335\frac{2}{3}$ cm$^3$

**9.** A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes each of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in total surface area of the big cube?

(a) 16 cm$^2$, decreases
(b) 48 cm$^2$, decreases
(c) 32 cm$^2$, decreases
(d) remains the same as previously

**10.** A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller spheres over that of the large sphere?

(a) 200% (b) 400%
(c) 500% (d) can't be determined

**11.** A cone is made of a sector with a radius of 14 cm and an angle of 60°. What is total surface area of the cone?

(a) 119.78 cm$^2$ (b) 191.87 cm$^2$
(c) 196.5 cm$^2$ (d) none of these

**12.** Kishan Chand is a very labourious farmer, he erected a fence around his paddy field in a square shape. He used 26 poles in each side, each at a distance of 4 m. What is the area of field?

(a) 1.6 hectare (b) 2.6 hectare
(c) 5.76 hectare (d) 1 hectare

**13.** A rectangular lawn is surrounded by a path of width 2 m on all sides. Now if the length of the lawn is reduced by 2 m the lawn becomes a square lawn and the area of path becomes 13/11 times, what is the length of the original lawn?

(a) 8 m (b) 9 m
(c) 10 m (d) 12 m

✍ ***Directions for question number 14, 15 :*** *A cylinder with height and radius 2 : 1 is filled with soft drink and then it is tilted so as to allow some soft drink to flow off to an extent where the level of soft drink just touches the lowest point of the upper mouth.*

**14.** If the 2.1 L soft drink is retained in the cylinder, what is the capacity of the cylinder?

(a) 3.6 L (b) 4 L
(c) 1.2 L (d) 4.2 L

**15.** If the quantity of soft drink left is poured into a conical flask whose height and base radius are same as that of the cylinder so as to fill the conical flask completely, the quantity of soft drink left in the cylinder as a fraction of its total capacity is :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$
(c) $\frac{1}{9}$ (d) $\frac{1}{10}$

**16.** An elephant of length 4 m is at one corner of a rectangular cage 16 m × 30 m and facing towards the diagonally opposite corner. If the elephant starts moving towards the diagonally opposite corner it takes 15 seconds to reach the opposite corner. Find the speed of the elephant :

(a) 1 m/s (b) 2 m/s
(c) 1.87 m/s (d) can't be determined

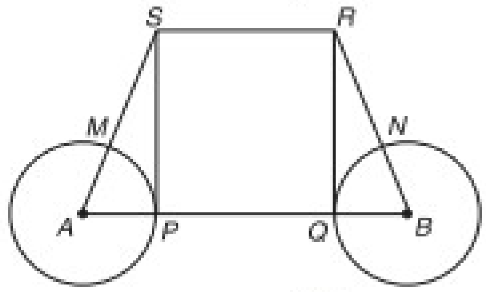
**17.** What is the height of the cone which is formed by joining the two ends of a sector of circle with radius $r$ and angle 60°?

(a) $\frac{\sqrt{35}}{6}r$ (b) $\frac{\sqrt{25}}{6}r$
(c) $\frac{r^2}{\sqrt{3}}$ (d) none of these

**18.** If a cube of maximum possible volume is cut off from a solid sphere of diameter $d$, then the volume of the remaining (waste) material of the sphere would be equal to :

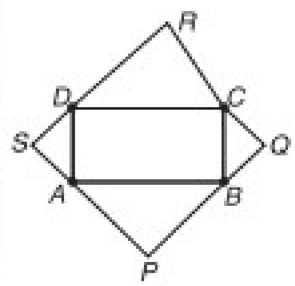
(a) $\frac{d^3}{3}\left(\pi - \frac{d}{2}\right)$ (b) $\frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$
(c) $\frac{d^2}{4}(\sqrt{2} - \pi)$ (d) none of these

**19.** In the adjoining figure $PQRS$ is a square and $MS = RN$ and $A$, $P$, $Q$ and $B$ lie on the same line. Find the ratio of the area of two circles to the area of the square. Given that $AP = MS$ :



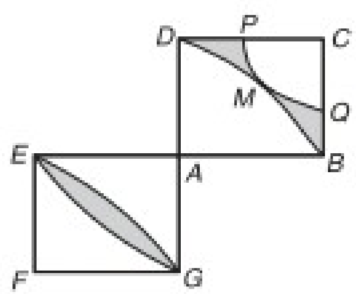

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{3\pi}{2}$ (d) $\frac{6}{\pi}$

**20.** $ABCD$ is a rectangle and there are four equilateral triangles. Area of $\Delta\, ASD$ equals to area of $\Delta\, BQC$ and area of $\Delta\, DRC$ equals to area of $\Delta\, APB$. The perimeter of the rectangle is 12 cm. Also the sum of the areas of the four triangles is $10\sqrt{3}$ cm$^2$ then the total area of the figure thus formed :




(a) $2(4 + 5\sqrt{3})$ cm$^2$ (b) $5(4 + 2\sqrt{3})$ cm$^2$
(c) $42\sqrt{3}$ cm$^2$ (d) none of these

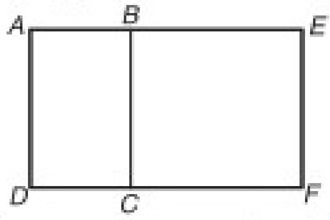
**21.** $ABCD$ and $EFGA$ are the squares of side 4 cm, each. In square $ABCD$, $DMB$ and $PMQ$ are the arcs of circles with centres at $A$ and $C$ respectively. In square $AEFG$, the shaded region is enclosed by two arcs of circles with centres at $A$ and $F$, respectively. What is the ratio of the shaded regions of the squares $ABCD$ and $AEFG$ respectively :



(a) $\dfrac{2+\pi(\sqrt{2}-2)}{(\pi-2)}$ (b) $\dfrac{(\pi-2)}{2(\sqrt{2}+1-\pi)}$

(c) $\dfrac{4}{3}$ (d) none of these

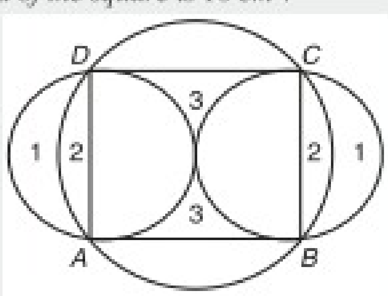
**22.** In the adjoining figure $\dfrac{AB}{BC}=\dfrac{AD}{DF}$, where $EBCF$ is a square. Find the ratio of $\dfrac{AE}{EF}$ :




(a) $\dfrac{(1\pm\sqrt{7})}{3}$ (b) $\dfrac{(1-\sqrt{7})}{2}$

(c) $\dfrac{(1+\sqrt{5})}{2}$ (d) $\dfrac{(1\pm\sqrt{5})}{2}$

✍ ***Direction for question number 23–25 :*** *In the adjoining figure ABCD is a square. A circle ABCD is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square, AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm$^2$.*




**23.** What is the area of region 1?

(a) 2.4 cm$^2$ (b) $\left(2-\dfrac{\pi}{4}\right)$ cm$^2$

(c) 8 cm$^2$ (d) $(4\pi-2)$ cm$^2$

**24.** What is the area of region 2?

(a) $3(\pi-2)$ cm$^2$ (b) $(\pi-3)$ cm$^2$

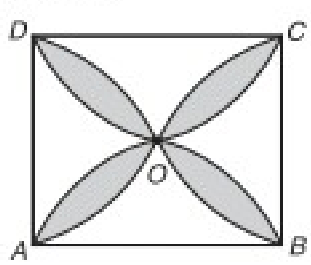
(c) $(2\pi-3)$ cm$^2$ (d) $4(\pi-2)$ cm$^2$

**25.** What is the area of region 3?

(a) $(4-4\pi)$ cm$^2$ (b) $4(4-\pi)$ cm$^2$

(c) $(4\pi-2)$ cm$^2$ (d) $(3\pi+2)$ cm$^2$

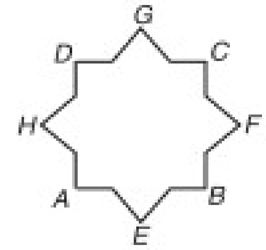
**26.** In the adjoining figure *ABCD* is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides *AB*, *BC*, *CD* and *AD* are the respective diameters of the four semicircles. Each of the side of the square is 8 cm. Find the area of the shaded region :




(a) $32(\pi-2)$ cm$^2$ (b) $16(\pi-2)$ cm$^2$

(c) $(2\pi-8)$ cm$^2$ (d) $\left(\dfrac{3}{4}\pi-4\right)$ cm$^2$

**27.** *ABCD* is a square. Another square *EFGH* with the same area is placed on the square *ABCD* such that the point of intersection of diagonals of square *ABCD* and square *EFGH* coincide and the sides of square *EFGH* are parallel to the diagonals of square *ABCD*. Thus a new figure is formed as shown in the figure. What is the area enclosed by the given figure if each side of the square is 4 cm :




(a) $32(2-\sqrt{2})$ (b) $16\left(\dfrac{3+\sqrt{2}}{2+\sqrt{2}}\right)$

(c) $32\left(\dfrac{2+\sqrt{2}}{3+\sqrt{2}}\right)$ (d) none of these

**28.** A piece of paper is in the form of a right angle triangle in which the ratio of base and perpendicular is 3 : 4 and hypotenuse is 20 cm. What is the volume of the biggest cone that can be formed by taking right angle vertex of the paper as the vertex of the cone?

(a) 45.8 cm$^3$ (b) 56.1 cm$^3$

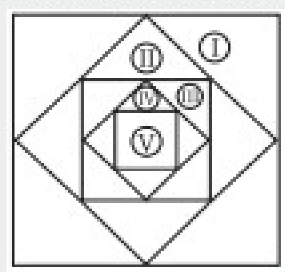
(c) 61.5 cm$^3$ (d) 48 cm$^3$

**29.** In a particular country the value of diamond is directly proportional to the surface area (exposed) of the diamond. Four thieves steel a cubical diamond piece and then divide equally in four parts. What is the maximum percentage increase in the value of diamond after cutting it?

(a) 50% (b) 66.66%

(c) 100% (d) none of these

✍ ***Directions for question number 30 and 31 :*** *In the figure shown square II is formed by joining the mid-points of square I, square III is formed by joining the mid-points of square II*




*and so on. In this way total five squares are drawn. The side of the square I is 'a' cm.*

**30.** What is the perimeter of all the five squares?

(a) $\dfrac{(4\sqrt{2}+1)a}{(\sqrt{2}+1)}$ (b) $\dfrac{(4\sqrt{2}-1)a}{(\sqrt{2}+1)}$

(c) $\dfrac{5}{6}a$ (d) $(7+3\sqrt{2})a$

**31.** What is the total area of all the five squares?

(a) $\dfrac{(4\sqrt{2}-1)a^2}{(4\sqrt{2}-1)}$ (b) $\dfrac{(4\sqrt{2}-1)a^2}{4(\sqrt{2}-1)}$

(c) $\dfrac{31}{16}a^2$ (d) none of these

✍ ***Directions for question number 32-35 :*** *Each edge of a cube is equally divided into n parts, thus there are total $n^3$ smaller cubes. Let,*

*$N_0 \rightarrow$ Number of smaller cubes with no exposed surfaces*

*$N_1 \rightarrow$ Number of smaller cubes with one exposed surfaces*
*$N_2 \rightarrow$ Number of smaller cubes with two exposed surfaces*
*$N_3 \rightarrow$ Number of smaller cubes with three exposed surfaces*

**32.** What is the number of unexposed smaller cubes ($N_0$)?
(a) $(n-2)^3$ (b) $n^3$
(c) $n!$ (d) 8

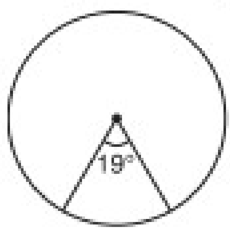
**33.** What is the number of smaller cubes with one exposed surface ($N_1$)?
(a) $4(n-3)^3$ (b) $6(n-2)^2$
(c) $(n-3)^2$ (d) $(n+1)^2$

**34.** What is the value of ($N_2$)?
(a) $8(n-2)^2$ (b) $6(n-2)$
(c) $12(n-2)$ (d) $3(n-3)^2$

**35.** What is the value of $N_3$?
(a) $(n-1)!$ (b) $(n-2)^2$
(c) $\dfrac{n(n+1)}{2}$ (d) 8

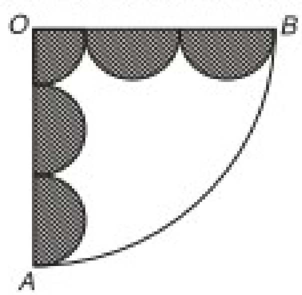
**36.** In a bullet the gun powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 5 cm. The ratio of height of cylinder and cone is 3 : 2. A cylindrical hole is drilled through the metal solid with height two-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gun powder is to be filled up) is one-third the volume of metal solid after drilling?
(a) $\sqrt{\dfrac{88}{5}}$ cm (b) $\sqrt{\dfrac{55}{8}}$ cm
(c) $\dfrac{55}{8}$ cm (d) $33\pi$ cm

**37.** A sector of the circle measures 19° (see the figure). Using only a scale, a compass and a pencil, is it possible to split the circle into 360 equal sectors of 1° central angle?




(a) Yes
(b) No
(c) Yes, only if radius is known
(d) can't be determined

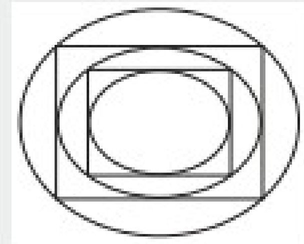
**38.** A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure, after it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant *OAB* is 5 cm and radius of each semicircle is 1 cm) :




(a) 25 : 16 (b) 25 : 9
(c) 20 : 9 (d) none of these

**39.** A cubical cake is cut into several smaller cubes by dividing each edge in 7 equal parts. The cake is cut from the top along the two diagonals forming four prisms. Some of them get cut and rest remained in the cubical shape. A complete cubical (smaller) cake was given to adults and the cut off part of a smaller cake is given to a child (which is not an adult). If all the cakes were given equally each piece to a person, total how many people could get the cake?
(a) 343 (b) 448
(c) 367 (d) 456

***Directions for question number 40-42 :*** *A square is inscribed in a circle then another circle is inscribed in the square. Another square is then inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the figure. The radius of the outer-most circle is R.*

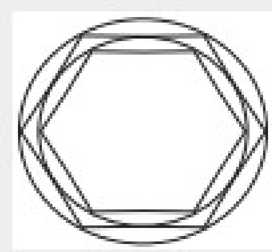


**40.** What is the radius of the inner-most circle?
(a) $\dfrac{R}{2}$ (b) $\dfrac{R}{\sqrt{2}}$
(c) $\sqrt{2}R$ (d) none of these

**41.** What is the sum of areas of all the squares shown in the figure?
(a) $3R^2$ (b) $3\sqrt{2}R^2$
(c) $\dfrac{3}{\sqrt{2}}R^2$ (d) none of these

**42.** What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?
(a) $(2+\sqrt{3})\pi R$ (b) $(3+\sqrt{2})\pi R$
(c) $3\sqrt{3}\pi R$ (d) none of these

***Directions for question number 43-45 :*** *A regular hexagon is inscribed in a circle of radius R. Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.*

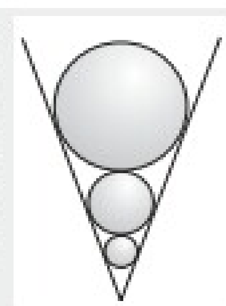


**43.** What is the sum of perimeters of both the hexagons?
(a) $(2+\sqrt{3})R$ (b) $3(2+\sqrt{3})R$
(c) $3(3+\sqrt{2})R$ (d) none of these

**44.** What is the ratio of area of inner circle to the outer circle?
(a) 3 : 4 (b) 9 : 16
(c) 3 : 8 (d) none of these

**45.** If there are some more circles and hexagons inscribed in the similar way as given above, then the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outside) :
(a) $9 : 3\sqrt{2}$ (b) 16 : 9
(c) $8 : 3\sqrt{3}$ (d) none of these

***Directions for question number 46-47 :*** *Five spheres are kept in a cone in such a way that each sphere touch each other and also touch the lateral surface of the cone, this is due to increasing radius of the spheres starting from the vertex of the cone. The radius of the smallest sphere is 16 cm.*



**46.** If the radius of the fifth (*i.e.*, largest) sphere be 81 cm, then find the radius of the third (*i.e.*, middlemost) sphere :

(a) 25 cm (b) $25\sqrt{3}$ cm
(c) 36 cm (d) data insufficient

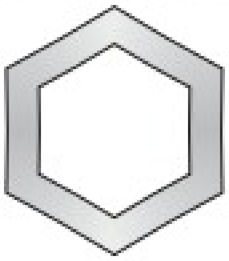
**47.** What is the least distance between the smallest sphere and the vertex of the cone?

(a) 64 cm (b) 80 cm
(c) 28 cm (d) none of these

**48.** Saumya has a pencil box of volume 60 $cm^3$. What can be the maximum length of a pencil that can be accommodated in the box. Given that all the sides are integral (in cm) and different from each other?

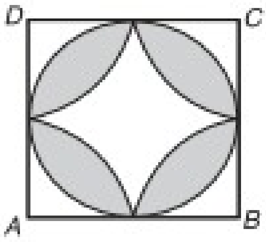
(a) $7\sqrt{2}$ cm (b) $\sqrt{905}$ cm
(c) $\sqrt{170}$ cm (d) $\sqrt{3602}$ cm

**49.** There are two concentric hexagons. Each of the side of both the hexagons are parallel. Each side of an internal regular hexagon is 8 cm. What is the area of the shaded region, if the distance between corresponding parallel sides is $2\sqrt{3}$ cm :



(a) $120\sqrt{3}$ $cm^2$
(b) $148\sqrt{3}$ $cm^2$
(c) 126 $cm^2$
(d) none of the above

**50.** *ABCD* is a square. *A* circle is inscribed in the square. Also taking *A*, *B*, *C*, *D* (the vertices of square) as the centres of four quadrants, drawn inside the circle, which are touching each other on the mid-points of the sides of square. Area of square is 4 $cm^2$. What is the area of the shaded region?




(a) $\left(4-\frac{3\pi}{2}\right)$ $cm^2$ (b) $(2\pi - 4)$ $cm^2$
(c) $(4 - 2\pi)$ $cm^2$ (d) none of these

**51.** In a factory there are two identical solid blocks of iron. When the first block is melted and recast into spheres of equal radii '*r*', then 14 cc of iron was left, but when the second block was melted and recast into sphere each of equal radii '2*r*', then 36 cc of iron was left. The volumes of the solid blocks and all the spheres are in integers. What is the volume (in $cm^3$) of each of the larger spheres of radius '2*r*'?

(a) 176 (b) $12\pi$
(c) 192 (d) data insufficient

**52.** There is a vast grassy farm in which there is a rectangular building of the farm-house whose length and breadth is 50 m and 40 m respectively. A horse is tethered at a corner of the house with a tether of 80 m long. What is the maximum area that the horse can graze?

(a) 5425 $\pi$ (b) 5245 $\pi$
(c) 254 $\pi$ (d) none of these

**53.** A cube of side 6 cm is painted on all its 6 faces with red colour. It is then broken up into 216 smaller identical cubes. What is the ratio of $N_0 : N_1 : N_2$.

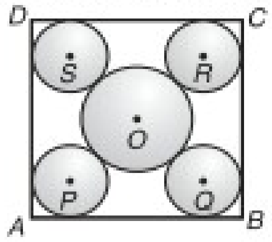
Where, $N_0 \to$ number of smaller cubes with no coloured surface.
$N_1 \to$ number of smaller cubes with 1 red face.
$N_2 \to$ number of smaller cubes with 2 red faces :

(a) 3 : 4 : 6 (b) 3 : 4 : 5
(c) 4 : 6 : 3 (d) can't be determined

**54.** Assume that a mango and its seed, both are spherical, now if the radius of seed is 2/5 of the thickness of the pulp. The seed lies exactly at the centre of the fruit. What per cent of the total volume of the mango is its pulp?

(a) $63\frac{3}{5}\%$ (b) 97.67%
(c) $68\frac{2}{3}\%$ (d) none of these

**55.** In the adjoining diagram *ABCD* is a square with side '*a*' cm. In the diagram the area of the larger circle with centre '*O*' is equal to the sum of the areas of all the rest four circles with equal radii, whose centres are *P*, *Q*, *R* and *S*. What is the ratio between the side of square and radius of a smaller circle?




(a) $(2\sqrt{2}+3)$ (b) $(2+3\sqrt{2})$
(c) $(4+3\sqrt{2})$ (d) can't be determined

**56.** Initially the diameter of a balloon is 28 cm. It can explode when the diameter becomes 5/2 times of the initial diameter. Air is blown at 156 cc/s. It is known that the shape of balloon always remains spherical. In how many seconds the balloon will explode?

(a) 1078 s (b) 1368 s
(c) 1087 s (d) none of these

**57.** The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of the volume of the cone to the volume of the cube?

(a) 3.18 $\pi$ (b) 2.25 $\pi$
(c) 2.35 (d) can't be determined

**58.** Raju has 64 small cubes of 1 $cm^3$. He wants to arrange all of them in a cuboidal shape, such that the surface area will be minimum. What is the diagonal of this larger cuboid?

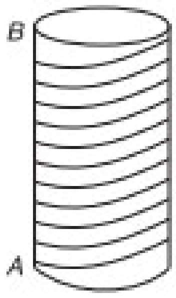
(a) $8\sqrt{2}$ cm (b) $\sqrt{273}$
(c) $4\sqrt{3}$ cm (d) $\sqrt{129}$ cm

**59.** The volume of a cylinder is 48.125 $cm^3$, which is formed by rolling a rectangular paper sheet along the length of the paper. If a cuboidal box (without any lid *i.e.*, open at the top) is made from the same sheet of paper by cutting out the quare of side 0.5 cm from each of the four corners of the paper sheet, then what is the volume of this box?

(a) 20 $cm^3$ (b) 38 $cm^3$
(c) 19 $cm^3$ (d) none of these

✍ ***Directions for question number 60-62 :*** *Consider a cylinder of height h cm and radius* $r = \frac{4}{\pi}$ *cm as shown in the figure. A string of certain length when wound on its cylindrical surface, starting at point A, gives a maximum of n turns.*

**60.** What is the vertical spacing (in cm) between two consecutive turns?




(a) $\frac{h}{n}$

(b) $\frac{\sqrt{h}}{n}$

(c) $\frac{h^2}{n}$

(d) can't be determined

**61.** If there is no spacing between any two courecutive turns and the width of string be $x$ cm, then the required length of the string is :

(a) $\frac{8x}{h}$ cm (b) $\frac{8h}{x}$ cm

(c) $8hx$ cm (d) $2\sqrt{2}\frac{h}{x}$ cm

**62.** If the string is wound on the exterior four walls of a cube of side $a$ cm starting at point $C$ and ending at point $D$ exactly above $C$, making equally spaced 4 turns. The side of the cube is :

(a) $a = \frac{2n}{\sqrt{255}}$ (b) $a = \frac{(n)^2}{16}$

(c) $a = \frac{8n}{\sqrt{257}}$ (d) $a = 2\sqrt{15}n$

**63.** A blacksmith has a rectangular iron sheet 10 ft long. He has to cut out 7 circular discs from this sheet. What is the minimum possible width of the iron sheet if the radius of each disc is 1 ft?

(a) $2\sqrt{3}$ ft (b) $(2+\sqrt{3})$ ft

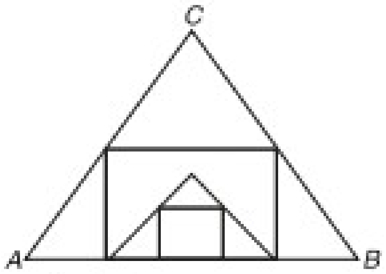
(c) $(3+\sqrt{2})$ ft (d) $(2+2\sqrt{3})$ ft

**64.** The perimeter of a square, a rhombus and a hexagon are same. The area of square, rhombus and hexagon be $s$, $r$, $h$, respectively then which of the following is correct :

(a) $r > s > h$ (b) $s > h > r$

(c) $h > s > r$ (d) data insufficient

**65.** In the adjoining figure $ABC$ is an equilateral triangle inscribing a square of maximum possible area. Again in this square there is an equilateral triangle whose side is same as that of the square. Further the smaller equilateral triangle inscribes a square of maximum possible area. What is the area of the innermost square if the each side of the outermost triangle be 0.01 m?




(a) $(873-504\sqrt{3})$ cm$^2$ (b) $(738-504\sqrt{3})$ cm$^2$

(c) $(873-405\sqrt{2})$ cm$^2$ (d) none of these

**66.** A blacksmith has a rectangular sheet of iron. He has to make a cylindrical vessel both circular ends are closed. When he minimises the wastage of the sheet of iron, then what is the ratio of the wastage to the utilised area of sheet?

(a) $\frac{1}{11}$ (b) $\frac{2}{17}$

(c) $\frac{3}{22}$ (d) none of these

**67.** Barun needs an open box of capacity 864 m$^3$. Actually where he lives, the rates of paints are soaring high so he wants to minimize the surface area of the box keeping the capacity of the box same as required. What is the base area and height of such a box?

(a) 36 m$^2$, 24 m (b) 216 m$^2$, 4 m

(c) 144 m$^2$, 6 m (d) none of these

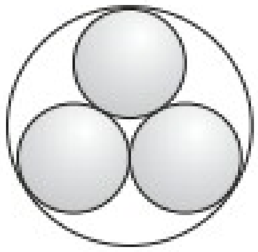
**68.** There are two cylindrical containers of equal capacity and equal dimensions. If the radius of one of the container is increased by 12 ft and the height of another container is increased by 12 ft, then the capacity of both the container is equally increased by $K$ cubic ft. If the actual heights of each of the container be 4 ft, then find the increased volume of each of the container :

(a) 1680 $\pi$ cu ft (b) 2304 $\pi$ cu ft

(c) 1480 $\pi$ cu ft (d) can't be determined

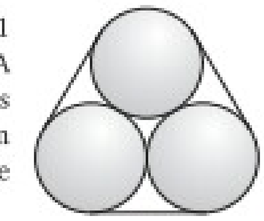
## Test of Your Learning

1. Three equal circles each of radius 1 cm are circumscribed by a larger circle. Find the perimeter of the circumscribing circle :



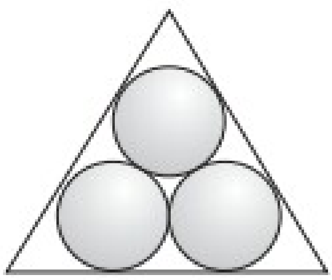
(a) $\frac{\sqrt{3}}{2}(2-\sqrt{3})\pi$ cm
(b) $\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$ cm
(c) $\frac{2}{\sqrt{3}}(2+\sqrt{3})\pi$ cm
(d) none of the above

2. Three circular rings of equal radii of 1 cm each are touching each other. A string runs all around the set of rings very tightly. What is the minimum length of string required to bind all the three rings in the given manner?



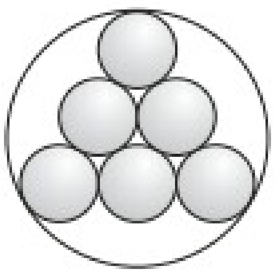
(a) $\frac{6}{\pi}$ cm (b) $2(3+\pi)$ cm
(c) $\frac{\pi}{6}$ cm (d) can't be determined

3. An equilateral triangle circumscribes all the three circles each of radius 1 cm. What is the perimeter of the equilateral triangle?



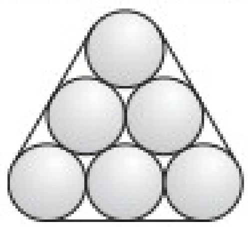
(a) $6(\sqrt{3}+1)$ cm
(b) $\sqrt{3}(8+\sqrt{2})$ cm
(c) $15(\sqrt{3}-1)$ cm
(d) none of the above

4. Six circles each of unit radius are being circumscribed by another larger circle. All the smaller circles touch each other. What is the circumference of the larger circle?



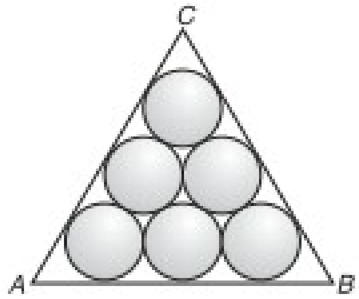
(a) $\left(\frac{\sqrt{3}+4}{\sqrt{2}}\right)\pi$ cm
(b) $4\sqrt{3}\pi$ cm
(c) $2\left(\frac{4+\sqrt{3}}{\sqrt{3}}\right)\pi$ cm
(d) can't be determined

5. There are six circular rings of iron, kept close to each other. A string binds them as tightly as possible. If the radius of each circular iron ring is 1 cm. What is the minimum possible length of string required to bind them?



(a) $2(6+3\sqrt{3}+\pi)$ cm
(b) $6(2+\sqrt{3})\pi$ cm
(c) $2(6+\pi)$ cm
(d) none of the above

6. An equilateral triangle circumscribes all the six circles, each with radius 1 cm. What is the perimeter of the equilateral triangle?




(a) $6(2+\sqrt{3})$ cm
(b) $3(\sqrt{3}+2)$ cm
(c) $12(\sqrt{3}+4)$ cm
(d) none of the above

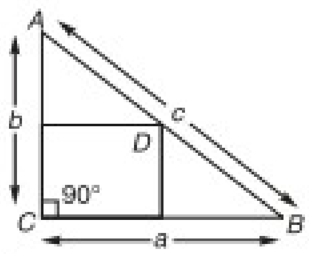
7. A cube of maximum possible volume is cut from the sphere of diameter $3\sqrt{3}$ cm. What is the ratio of volume of the sphere to that of cube?

(a) $\frac{4\sqrt{3}}{\pi}$ (b) $\frac{\sqrt{3}}{2}\pi$
(c) $\frac{4}{\sqrt{3}}\pi$ (d) none of these

8. A cube of maximum possible volume is cut from the solid right circular cylinder. What is the ratio of volume of cube to that of cylinder if the edge of a cube is equal to the height of the cylinder?

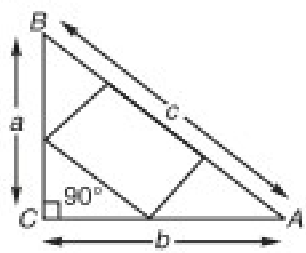
(a) $\frac{11}{7}$ (b) $\sqrt{2}\frac{\pi}{7}$
(c) $\frac{7}{11}$ (d) none of these

9. In a right angle triangle *ABC*, what is the maximum possible area of a square that can be inscribed when one of its vertices coincide with the vertex of right angle of the triangle?



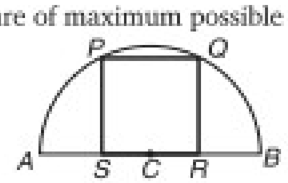

(a) $\frac{a}{b}$ (b) $\frac{ab}{a+b}$
(c) $\frac{a+b}{ab}$ (d) $\left(\frac{ab}{a+b}\right)^2$

10. In the adjoining figure a square of maximum possible area is circumscribed by the right angle triangle *ABC* in such a way that one of its side just lies on the hypotenuse of the triangle. What is the area of the square?




(a) $\left(\frac{abc}{a^2+b^2+ab}\right)^2$ (b) $\frac{a^2+b^2+c^2}{abc}$
(c) $\frac{abc}{a^2+b^2+c^2}$ (d) none of these

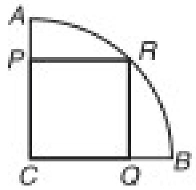
11. In the adjoining figure *PQRS* is a square of maximum possible area which is circumscribed by the semicircle. Points *R* and *S* lie on the diameter *AB*. What is the area of the square if the radius of the circle is '*r*'?



(a) $\frac{\sqrt{3}}{4}r^2$ (b) $\frac{4}{5}r^2$

(c) $\frac{3}{5}r^2$ (d) $\frac{\sqrt{5}}{4}r^2$

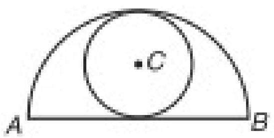
**12.** In the adjoining figure a quadrant (of circle) inscribes a square of maximum possible area. If the radius of the circle be '$r$' then what is the area of the square?




(a) $\frac{r^2}{2}$ (b) $\frac{3r^2}{5}$

(c) $\frac{r^2}{\sqrt{3}}$ (d) $2\sqrt{6}r$

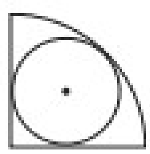
**13.** In the adjoining figure, $AB$ is the diameter of a semicircle which inscribes a circle of maximum possible area. If the radius of the larger circle (*i. e.*, semicircle) is $r$, the area of the inscribed circle is :




(a) $\frac{5r^2}{\pi}$ (b) $\frac{2\pi}{\sqrt{3}}r^2$

(c) $\frac{\pi r^2}{4}$ (d) none of these

**14.** In a quadrant (of a circle) a circle of maximum possible area is given. If the radius of the circumscribing quadrant be $r$, then what is the area of the inscribed circle?



(a) $(2+3\sqrt{2})r^2$ (b) $\frac{\pi r^2}{(3+2\sqrt{2})}$

(c) $\left(\frac{3+2\sqrt{2}}{r^2}\right)\pi$ (d) none of these

**15.** A cylindrical chocobar has its radius $r$ unit and height '$h$' unit. If we wish to increase the volume by same unit either by increasing its radius alone or its height alone, then how many unit we have to increase the radius or height?

(a) $\frac{r^2+2r}{h}$ (b) $\frac{r^2-2rh}{h}$

(c) $\frac{2r^2-rh}{h^2}$ (d) $\frac{\pi r^2}{2h}$

**16.** A 12 cm long wire is bent to form a triangle with one of its angle as 60°. Find the sides of the triangle (in cm) when its area is largest?

(a) 3, 4, 5 (b) 2, 4, 6

(c) 4, 4, 4 (d) 2.66, 3.66, 4.66

**17.** Let $S_1, S_2, \ldots, S_n$ be the squares such that for each $n > 1$, the length of a side of $S_n$ equals the length of the diagonal of $S_{(n+1)}$. If the length of a circle of $S_1$ is 10 cm, then for which of the following values of $n$ the area of $S_n$ is less than 1 square cm?

(a) 4 (b) 8

(c) 6 (d) 7

**18.** Area of a regular hexagon and a regular octagon is same. Which one of the two has larger perimeter?

(a) Hexagon (b) Octagon

(c) can't be determined (d) none of these

# Answers

## INTRODUCTORY EXERCISE-10.1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (b) | **2.** (c) | **3.** (c) | **4.** (b) | **5.** (a) | **6.** (a) | **7.** (b) | **8.** (a) | **9.** (a) | **10.** (c) |
| **11.** (b) | **12.** (d) | **13.** (a) | **14.** (b) | **15.** (b) | **16.** (c) | **17.** (c) | **18.** (d) | **19.** (c) | **20.** (b) |
| **21.** (b) | **22.** (a) | **23.** (c) | **24.** (c) | **25.** (c) | **26.** (b) | **27.** (c) | **28.** (c) | **29.** (b) | **30.** (c) |
| **31.** (b) | **32.** (c) | | | | | | | | |

## INTRODUCTORY EXERCISE-10.2

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (b) | **2.** (c) | **3.** (a) | **4.** (a) | **5.** (d) | **6.** (a) | **7.** (c) | **8.** (b) | **9.** (a) | **10.** (a) |

## INTRODUCTORY EXERCISE-10.3

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (a) | **2.** (c) | **3.** (d) | **4.** (b) | **5.** (d) | **6.** (b) | **7.** (a) | **8.** (b) | **9.** (d) | **10.** (c) |
| **11.** (b) | **12.** (a) | **13.** (b) | **14.** (d) | **15.** (a) | | | | | |

## INTRODUCTORY EXERCISE-10.4

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (d) | **2.** (d) | **3.** (a) | **4.** (b) | **5.** (c) | **6.** (d) | **7.** (a) | **8.** (a) | **9.** (a) | **10.** (b) |
| **11.** (c) | **12.** (d) | **13.** (b) | **14.** (a) | **15.** (b) | **16.** (a) | **17.** (a) | **18.** (c) | **19.** (a) | **20.** (c) |
| **21.** (a) | **22.** (b) | **23.** (b) | **24.** (c) | **25.** (c) | | | | | |

## INTRODUCTORY EXERCISE-10.5

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (b) | **2.** (c) | **3.** (a) | **4.** (b) | **5.** (c) | **6.** (a) | **7.** (b) | **8.** (a) | **9.** (b) | **10.** (c) |
| **11.** (d) | **12.** (a) | **13.** (c) | **14.** (b) | **15.** (c) | **16.** (a) | **17.** (b) | **18.** (d) | **19.** (b) | **20.** (c) |
| **21.** (d) | **22.** (b) | **23.** (c) | **24.** (a) | **25.** (b) | **26.** (a) | **27.** (c) | **28.** (c) | **29.** (c) | **30.** (b) |
| **31.** (b) | **32.** (b) | | | | | | | | |

## INTRODUCTORY EXERCISE-10.6

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (b) | **2.** (c) | **3.** (c) | **4.** (c) | **5.** (b) | **6.** (a) | **7.** (c) | **8.** (b) | **9.** (a) | **10.** (c) |
| **11.** (a) | **12.** (a) | **13.** (d) | **14.** (a) | **15.** (d) | **16.** (c) | **17.** (a) | **18.** (b) | **19.** (a) | **20.** (b) |
| **21.** (c) | **22.** (a) | **23.** (d) | **24.** (c) | **25.** (d) | **26.** (d) | **27.** (d) | **28.** (c) | **29.** (b) | **30.** (c) |
| **31.** (b) | **32.** (a) | **33.** (d) | **34.** (c) | **35.** (b) | **36.** (b) | **37.** (b) | **38.** (a) | **39.** (b) | **40.** (a) |

## INTRODUCTORY EXERCISE-10.7

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (b) | **2.** (c) | **3.** (c) | **4.** (a) | **5.** (b) | **6.** (c) | **7.** (c) | **8.** (d) | **9.** (c) | **10.** (c) |
| **11.** (b) | **12.** (a) | | | | | | | | |

## MISCELLANEOUS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (c) | **2.** (c) | **3.** (c) | **4.** (c) | **5.** (c) | **6.** (a) | **7.** (c) | **8.** (a) | **9.** (c) | **10.** (a) |
| **11.** (d) | **12.** (c) | **13.** (a) | **14.** (c) | **15.** (d) | **16.** (b) | **17.** (b) | **18.** (c) | **19.** (b) | **20.** (d) |
| **21.** (b) | **22.** (c) | **23.** (d) | **24.** (c) | **25.** (d) | **26.** (d) | **27.** (d) | **28.** (b) | **29.** (b) | **30.** (b) |
| **31.** (a) | **32.** (c) | **33.** (d) | **34.** (c) | **35.** (a) | **36.** (c) | **37.** (b) | **38.** (c) | **39.** (d) | **40.** (a) |
| **41.** (b) | **42.** (a) | **43.** (c) | **44.** (c) | **45.** (a) | **46.** (b) | **47.** (b) | **48.** (c) | **49.** (d) | **50.** (b) |
| **51.** (c) | **52.** (c) | **53.** (a) | **54.** (d) | **55.** (c) | **56.** (a) | **57.** (b) | **58.** (a) | **59.** (c) | |

## LEVEL-1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (a) | **2.** (b) | **3.** (b) | **4.** (b) | **5.** (c) | **6.** (d) | **7.** (c) | **8.** (b) | **9.** (a) | **10.** (c) |
| **11.** (c) | **12.** (a) | **13.** (b) | **14.** (a) | **15.** (a) | **16.** (c) | **17.** (a) | **18.** (c) | **19.** (a) | **20.** (d) |
| **21.** (b) | **22.** (a) | **23.** (d) | **24.** (c) | **25.** (c) | **26.** (a) | **27.** (b) | **28.** (c) | **29.** (c) | **30.** (b) |
| **31.** (d) | **32.** (a) | **33.** (c) | **34.** (c) | **35.** (c) | **36.** (a) | **37.** (a) | **38.** (a) | **39.** (b) | **40.** (a) |
| **41.** (b) | **42.** (d) | **43.** (d) | **44.** (c) | **45.** (a) | **46.** (c) | **47.** (a) | **48.** (c) | **49.** (b) | **50.** (b) |
| **51.** (c) | **52.** (c) | **53.** (b) | **54.** (c) | **55.** (d) | | | | | |

## LEVEL-2

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (c) | **2.** (b) | **3.** (c) | **4.** (b) | **5.** (a) | **6.** (b) | **7.** (c) | **8.** (a) | **9.** (d) | **10.** (b) |
| **11.** (a) | **12.** (d) | **13.** (c) | **14.** (d) | **15.** (b) | **16.** (b) | **17.** (a) | **18.** (b) | **19.** (b) | **20.** (a) |
| **21.** (a) | **22.** (c) | **23.** (c) | **24.** (d) | **25.** (b) | **26.** (a) | **27.** (a) | **28.** (b) | **29.** (c) | **30.** (d) |
| **31.** (c) | **32.** (a) | **33.** (b) | **34.** (c) | **35.** (d) | **36.** (b) | **37.** (a) | **38.** (a) | **39.** (b) | **40.** (a) |
| **41.** (a) | **42.** (d) | **43.** (b) | **44.** (a) | **45.** (c) | **46.** (c) | **47.** (a) | **48.** (b) | **49.** (a) | **50.** (b) |
| **51.** (a) | **52.** (a) | **53.** (c) | **54.** (b) | **55.** (b) | **56.** (a) | **57.** (b) | **58.** (c) | **59.** (a) | **60.** (a) |
| **61.** (b) | **62.** (c) | **63.** (b) | **64.** (c) | **65.** (a) | **66.** (a) | **67.** (c) | **68.** (b) | | |

## TEST OF YOUR LEARNING

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (c) | **2.** (b) | **3.** (a) | **4.** (c) | **5.** (c) | **6.** (a) | **7.** (b) | **8.** (c) | **9.** (d) | **10.** (a) |
| **11.** (b) | **12.** (a) | **13.** (c) | **14.** (b) | **15.** (c) | **16.** (c) | **17.** (b) | **18.** (a) | | |

# Hints & Solutions

## Introductory Exercise 10.1

**1.** $2(16x + 9x) = 750$; $l = 16x$ and $b = 9x$

**2.** $\text{Area} = \dfrac{110}{0.5} = 220$ sq. m

and $11x \times 5x = 220$

**3.** Length : breadth = 3 : 2
Also, area of floor = area of roof
and 30 is almost 50% of 62. So you need not to solve just look out for the appropriate option thus 1411.2 is almost 50% of 2916.48 and rest of the options are not satisfactory.

**4.** 1 hectares = 10000 $m^2$, find length and breadth, then perimeter

Perimeter = 1000 m = 1 km

$\therefore \quad \text{Time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{1}{3}$ hrs = 20 min

**5.** 5 hectares and 76 ares = 57600 $m^2$, find perimeter then multiply it by 6.

**6.** $l \times 0.7 = 19 \times 3.5 \Rightarrow l = 95$ m (70 cm = 0.7 m)

Also $(95)^2 = 9025$

So $95 \times 0.95 = 90.25$

**7.** $\text{Number of stones} = \dfrac{\text{Area of courtyard}}{\text{Area of one stone}} = 14400$

Cost = Rate × number of stones
$= (0.5 \times 14400)$

**8.** $\text{Area} = (63 + 54 - 6) \times 6$ $[\because \text{Area} = (l + b - 2w)\,2w]$

$\text{Area} = (111 \times 6)$

and $\text{Rate} = \dfrac{37}{2} = \dfrac{37 \times 3}{2 \times 3} = \dfrac{111}{6}$

$\therefore \quad \text{Cost} = (111 \times 6) \times \dfrac{111}{6}$

$= 111 \times 111 = 12321$

**9.** Original area $= l \times b$

New area $= 2l \times \dfrac{b}{2} = l \times b$

Hence, no change.

**10.** $(116 + 68 - 2w)\,2w = 720$
Solve through quadratic equation.
**Alternatively:** Go through options and put $w = 2$ from option (c) you will find that both sides are equal. So the presumed choice is correct.

**11.** Net area = Total area of 4 walls – 8$m^2$

and Area of 4 walls $= 2(l + b) \times h$

**12.** $d^2 = 2 \times \text{area}$ $\left(\because \text{Area} = \dfrac{d^2}{2}\right)$

$d^2 = 2 \times 2 \Rightarrow d = 2$ km

**13.** $100 \times 100 = 10000$ $m^2$ = 1 hectare
$111 \times 111 = 12321$ $m^2$

$\therefore$ Difference in area = 2321 $m^2$

**14.** $\text{Rate} = \dfrac{759 - 561}{6}$ = Rs. 33 per metre

**15.** $l \times b = \dfrac{1}{25} \times 10000 = 1000 \times b$

$\Rightarrow \quad b = 0.4$ m

**16.** $\text{Side} = \sqrt{(289)^2 - (240)^2}$

**17.** $\dfrac{1600 \times 900}{40 \times 20} = 1800$

**18.** $\text{Area} = \dfrac{d^2}{2}$

**19.** $\dfrac{1}{3} \times \dfrac{1}{3} = \dfrac{1}{9}$

$\therefore \quad 90 \times \dfrac{1}{9} = 10$ g (It depends upon area)

**20.** $l : b = 2 : 1$

$2x^2 = 24200$, find diagonal.

$\sqrt{(110)^2 + (220)^2} = 110\sqrt{1^2 + 2^2}$

$= 110 \times \sqrt{5}$

$= 110 \times 2.236 = 246$ m

**21.** $2(2x + x) \times 11 = 2640$, find $2x \times x = ?$

**22.** Consider some appropriate values.

*e.g.,*


$4a = 16$ cm $\qquad 2(l + b) = 16$ cm

but $a^2 = 16$ cm$^2$ and $l \times b = 12$ cm$^2$

$\Rightarrow \quad a^2 > l \times b$

Remember, when $a + b$ is constant, then the maximum value of $a \times b$ is at when $a = b$.
So, area of square (*i.e.*, $l \times b = a \times a$) is always greater than area of rectangle with same perimeter.
**Alternatively:** Let each side of a square be $a$ and length and breadth of the reactangle be $l$ and $b$, then

$4a = 2(l + b) \Rightarrow a = \left(\dfrac{l + b}{2}\right)$

$\therefore$ Area of the rectangle $= l \times b$

and Area of the square $= a^2 = \dfrac{1}{4}(l + b)^2$

But since we know that for any (different) certain values

AM > GM

$\therefore \quad \dfrac{l + b}{2} > \sqrt{l \times b}$

$\Rightarrow \quad \left(\dfrac{l + b}{2}\right)^2 > l \times b$

$\Rightarrow$ Area of square > Area of rectangle

**23.** Best way is to go through options and verify the result.

**Alternatively:** $2(l+b) = 4a = 80$

$\Rightarrow \quad l + b = 40$ and $a = 20 \Rightarrow a^2 = 400$

Also $\quad a^2 - lb = 100$

$\Rightarrow \quad 400 - lb = 100$

$\Rightarrow \quad lb = 300$

Now $\quad (l-b)^2 = (l+b)^2 - 4lb$

$(l-b)^2 = 1600 - 1200$

$\Rightarrow \quad l - b = 20$

(Solving $l + b = 40$ and $l - b = 20$)

$\therefore \quad l = 30$ and $b = 10$

**24.** $\dfrac{100 \times 100}{25 \times 25} = 16 \quad (1 \text{ m} = 100 \text{ cm})$

**25.** $(120 + 80 - 24) \times 24 = 176 \times 24 = 4224$

**26.** $(90 + 40 + 10) \times 10 = 1400$

**27.** $2[(x+2) + x] = 48 \Rightarrow x = 11$ cm

$\therefore \quad (x + 2) = 13$ cm

**28.** $\dfrac{52000}{325} = 160$

3.25 cm means 325 m, as per scale.

**29.** Area $= \dfrac{d^2}{2}$; $d \to$ diagonal

**30.** $1.2 \times 1.2 = 1.44 \Rightarrow$ 44% increase

**31.** $a^2 : (a\sqrt{2})^2 = 1 : 2$

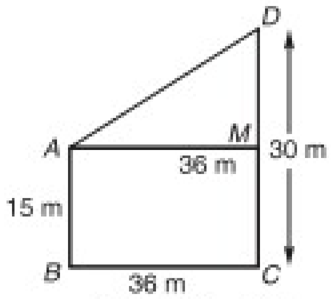
**32.** $1.6 \times 1.4 = 2.24 \Rightarrow$ increase $= 1.24 \Rightarrow$ 124%

## Introductory Exercise 10.2

**1. Use Hero's formula:**

Area of scalene triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

**2.**




$AM = BC = 36$ m

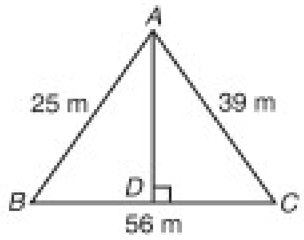
$MD = CD - MC = 30 - 15 \quad (\because MC = AB = 15 \text{ m})$

$\therefore \quad MD = 15$ m

Now using Pythagorus theorem

$AD^2 = AM^2 + MD^2$, find $AD$

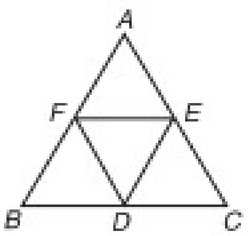
**3.** Find the area, using Hero's formula, then




Area $= \dfrac{1}{2} \times b \times h$

$= \dfrac{1}{2} \times 56 \times AD$

**4.** All the triangles are similar to each other.




$\therefore \quad \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{4}{1}$

(Since in $\Delta ABC$ there are 4 similar triangles having same area as $\Delta DEF$.)

**5.** $60 = \dfrac{b}{4}\sqrt{4a^2 - b^2}$

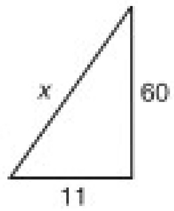
$\Rightarrow \quad 60 = \dfrac{b}{4}\sqrt{676 - b^2}$

$\Rightarrow \quad 57600 = b^2(676 - b^2)$

By solving the above equation we can get the value of $b$.

**Alternatively:** You can go through option.

**6.** Use pythagorus theorem




$x^2 = (60)^2 + (11)^2$

[Pythagorus theorem : (Hypotenuse)$^2$ = (base)$^2$ + (height)$^2$]

**7.** Area of field $= \dfrac{495.72}{36.72}$

$= 13.5$ hectare $= 135000 \text{ m}^2$

Now, if $\quad h = x, \quad b = 3x$

$\therefore \quad \dfrac{1}{2} \times b \times h = 135000$

**8.** Area of triangle $= \dfrac{1}{2} \times b \times h$

Let initially area of triangle $= 1 \times 1 = 1$ unit

Now, the area of triangle $= 2 \times 2 = 4$ unit

Increase in area $= \dfrac{4-1}{1} \times 100 = 300\%$

(For your convenience assume any value of $b$ and $h$.)

**9.** Height of an equilateral triangle $= \dfrac{\sqrt{3}}{2} \times$ side

$\therefore \quad 2\sqrt{3} = \dfrac{\sqrt{3}}{2} \times$ side

$\Rightarrow \quad$ Side = 4 cm

$\therefore \quad$ Area of an equilateral triangle $= \dfrac{\sqrt{3}}{4} \times (\text{side})^2$

$= \dfrac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3} \text{ cm}^2$

**10.** Let the each side of a square be $3x$ and each side of an equilateral triangle be $4x$

then $\quad$ perimeter of square $= 4 \times 3x = 12x$

and $\quad$ perimeter of equilateral triangle $= 3 \times 4x = 12x$

Now $\quad$ area of square $= (3x)^2 = 9x^2$

Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} \times (4x)^2 = 4\sqrt{3}x^2$

Since $\quad 9x^2 > 4\sqrt{3}x^2 \Rightarrow 9 > 4\sqrt{3}$

Hence area of square is greater than area of equilateral triangle, when the perimeter of each is same.

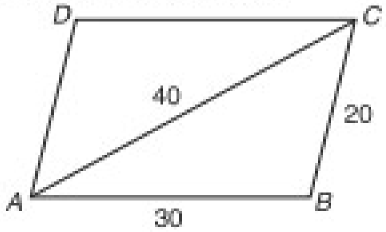
**Alternatively:** Consider any suitable values and verify.

## Introductory Exercise 10.3

**1.** Area $= 6 \times 8 \sin 30°$

$$= 6 \times 8 \times \frac{1}{2} = 24 \text{ cm}^2 \qquad \left(\sin 30° = \frac{1}{2}\right)$$

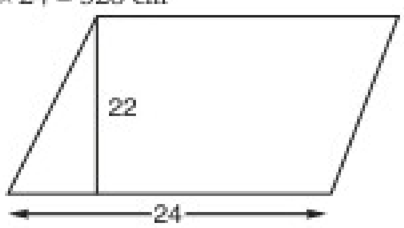
**2.** To find the area of parallelogram, first find the area of $\Delta ABC$ by Hero's formula then double it.



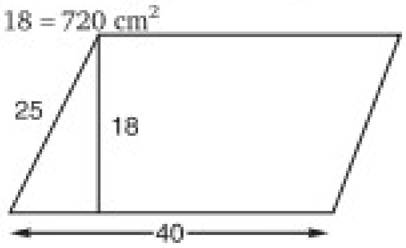

Since, area of $ABCD = 2 \times$ Area of $\Delta ABC$

$$= 2 \times \text{Area of } \Delta ACD$$

**3.** Area $= 22 \times 24 = 528 \text{ cm}^2$




**4.** Area $= 40 \times 18 = 720 \text{ cm}^2$




**5.** $\dfrac{\text{Area of parallelogram } ABCD}{\text{Area of triangle } ABN} = \dfrac{BC \times AN}{\frac{1}{2} \times BN \times AN} = \dfrac{2 \times 4x}{x} = \dfrac{8}{1}$

**6.** Area of rhombus $= \frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times a \times b = \frac{ab}{2}$$

**7.** Area of rhombus $= \frac{1}{2} \times 2x \times 5x = \frac{10x^2}{2} = 5x^2$

and square of the shorter diagonal $= (2x)^2 = 4x^2$

$\therefore \qquad \dfrac{5x^2}{4x^2} = \dfrac{5}{4}$

**8.** $\qquad \frac{1}{2} \times x \times 2x = 256$

$\Rightarrow \qquad x^2 = 256$

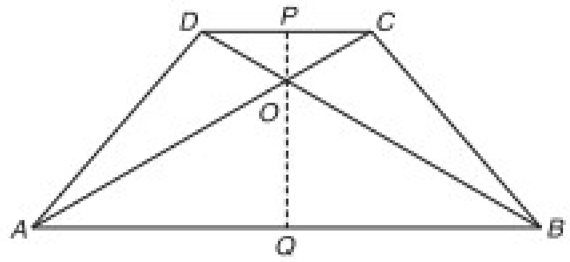
$\Rightarrow \qquad x = 16$

$\therefore \qquad 2x = 32$

Hence, $\qquad x + 2x = 16 + 32 = 48$

**9.** Area $= \frac{1}{2} \times (30 + 50) \times 16 = 640 \text{ cm}^2$

**10.** $\dfrac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \dfrac{\frac{1}{2} \times AB \times OQ}{\frac{1}{2} \times CD \times PO}$




$$= \frac{AB \times OQ}{CD \times PO}$$

$$= \frac{2CD \times 2OP}{CD \times OP} = \frac{4}{1} \qquad \left(\because \; AB = 2CD \text{ and } OQ = 2PO\right)$$

This is due to the similarity of triangles $AOB$ and $COD$.

**11.** Area of trapezium = 441

$$\frac{1}{2}(5x + 9x) \times 21 = 441$$

$\Rightarrow \qquad 14x = 42 \Rightarrow x = 3$

$\therefore \qquad 9x = 27$ cm

**12.** $\qquad \frac{1}{2} \times (12 + 8) \times h = 360$

$\Rightarrow \qquad h = 36$ m

**13.** $6 \times \dfrac{\sqrt{3}}{4} \times (\text{Side})^2 = \dfrac{3\sqrt{3}}{2}(\text{Side})^2$

$$= \frac{3\sqrt{3}}{2} \times 4 \times 4 = 24\sqrt{3} \text{ m}^2$$

**14.** Area of quadrilateral $= \frac{1}{2} \times 19 \times (5 + 7) = 114 \text{ cm}^2$

**15.** $AB = BC = CD = AD = 4$ cm $\qquad \left(\because \text{Side} = \dfrac{\text{Diagonal}}{\sqrt{2}}\right)$

and $EF = 1.5$ cm $\qquad$ (By Pythagorus theorem)

$\therefore$ Area of $ABCDE$ = Area of $ABCD$ + Area of $AED$

$$= (4)^2 + \frac{1}{2} \times 4 \times 1.5 = 19 \text{ cm}^2$$

## Introductory Exercise 10.4

**1.** $\qquad 2\pi r = 704 \Rightarrow r = 112$

$\therefore \qquad \pi r^2 = \frac{22}{7} \times 112 \times 112 = 39424 \text{ cm}^2$

**2.** $\qquad 2\pi r = 4.4 \Rightarrow r = \frac{7}{10}$ m

$\therefore \qquad \pi r^2 = 0.49\pi \text{ m}^2$

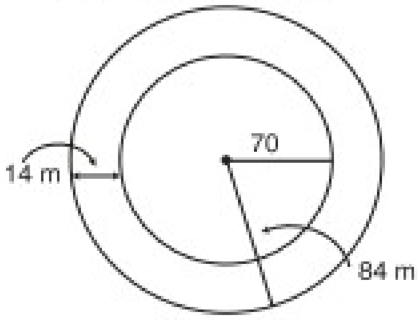
**3.** $\qquad 2\pi r = 2 \times \frac{22}{7} \times 4.2$

$$= 26.4 \text{ metre}$$

$$26.4 = 2(6x + 5x)$$

$\Rightarrow \qquad 6x = 7.2$ m

**4.** $2\pi r = 440 \Rightarrow r = 70$ m




$\therefore \quad R = 70 + 14 = 84$ m

**5.** $r + 2\pi r = 51 \Rightarrow r\left(1 + \frac{44}{7}\right) = 51 \Rightarrow r = 7,$

Find area.

**6.** $(2\pi r - 2r) = 15 \Rightarrow 2r\left(\frac{22}{7} - 1\right) = 15 \Rightarrow r = \frac{7}{2}$

$\therefore \quad \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5 \text{ m}^2$

**7.** $\frac{\text{New area}}{\text{Original area}} = \frac{\pi \times 7 \times 7}{\pi \times 5 \times 5} = \frac{49}{25}$

Change in area $= \frac{24}{25} \times 100 = 96\%$

**8.** $\frac{\text{New circumference}}{\text{Original circumference}} = \frac{6}{5} = \frac{\text{New radius}}{\text{Original radius}}$

$\therefore \quad \frac{\text{New area}}{\text{Original area}} = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$

$\therefore \quad$ Change in area $= \frac{11}{25} \times 100 = 44\%$

**Alternatively:** Change $= (1.2 \times 1.2) - (1 \times 1) = 0.44$

$\therefore \quad$ Change $= 44\%$

**9.** $\pi r^2 = 124.74$ hectare

$\pi r^2 = 1247400 \text{ m}^2$

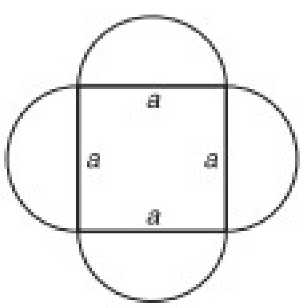
$\Rightarrow \quad r = 630$ m

$\therefore \quad 2\pi r = 3960$

$\therefore \quad$ Cost $= 3960 \times 0.8 = 3168$

**10.** $\pi r^2 = \frac{158400}{1400} \Rightarrow r^2 = 36 \Rightarrow r = 6$ m

**11.**




Area of square $= a^2$

Area of circular parts $= 4 \times \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{2}$

$\therefore \quad$ Total area $= a^2 + \frac{\pi a^2}{2} = a^2\left(1 + \frac{\pi}{2}\right)$

**12.** $\pi r^2 = 2464 \Rightarrow r = 28$ m

**13.** Basically there are 12 equilateral triangles each of side '$a$'.

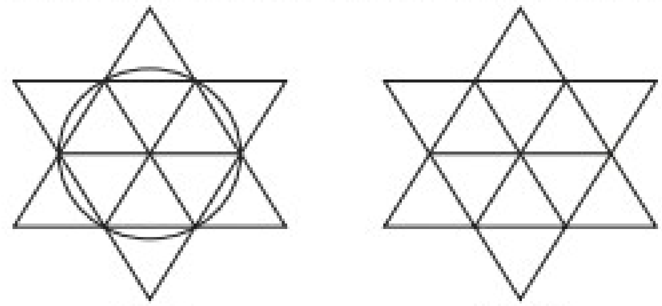


**Fig. (i)** **Fig. (ii)**

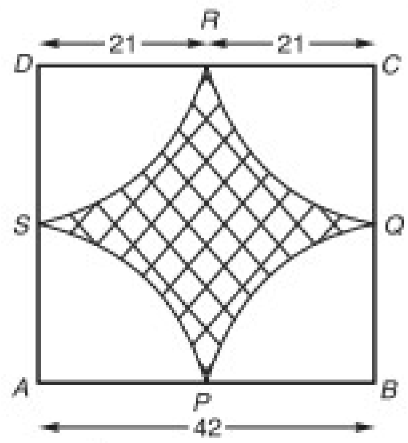
$\therefore \quad 12 \times \frac{\sqrt{3}}{4} \times (a)^2 = 3\sqrt{3}a^2$

**14.** $\frac{1}{2} \times \pi \times (7)^2 + 2\left[\frac{1}{2} \times \pi \times \left(\frac{7}{2}\right)^2\right]$

Area of larger semicircle + 2 (area of smaller semicircle)

**15.** $2\pi \times R \times \frac{75}{360} = 25 \Rightarrow R = \frac{60}{\pi}$

**16.** Ungrazed area = Area of square − 4 (area of quadrants)




$= (42)^2 - 4 \times \frac{1}{4} \times \pi (21)^2 = (21)^2 [4 - \pi] = 378$

**17.** $20 + \frac{1}{2} \times [2\pi \times 10] = 20 + 10\pi$

**18.** $\frac{40}{360} = \frac{1}{9} \quad \therefore \quad \frac{(360 - 40)}{360} = \frac{8}{9}$

$\therefore$ Area of major sector $= 8 \times 8.25 = 66 \text{ cm}^2$

**19.** Area of a sector $= \frac{1}{2} \times \text{arc} \times \text{radius} = \frac{1}{2} \times 8 \times 5.6 = 22.4 \text{ cm}^2$

**20.** $2\pi r = 2 \times \frac{22}{7} \times 700 = 4400$ m $= 4.4$ km

Time $= \frac{\text{Distance}}{\text{Speed}} = \frac{4.4}{13.2} = \frac{1}{3}$ h = 20 minute

**21.** $\pi R^2 = \pi [r_1^2 + r_2^2 + r_3^2]$, find $R$.

**22.** $\pi [23^2 - 12^2] = \frac{22}{7} \times [529 - 144] = 1210$

**23.** $4a = 4 \times 66 = 264 = 2\pi r \Rightarrow r = 42$

$\therefore \quad d = 2r = 84$ m

**24.** $r = 42 \quad \therefore 2\pi r = 264 = 4a \Rightarrow a = 66$

$\therefore \quad d = a\sqrt{2} = 66\sqrt{2}$

**25.** Circumference $= \frac{1100}{560} = \frac{110}{56} = 2\pi r$

$\therefore \quad 2r = \frac{110}{56} \times \frac{7}{22} = \frac{5}{8}$ m $= 62.5$ cm

# Introductory Exercise 10.5

1. Volume of original cube $= (4)^3 = 64 \text{ cm}^3$
and its weight = 400 kg
Since weight of the larger cube is 8 times the weight of smaller cube. Hence, the volume of new cube will be 8 times the volume of smaller cube.
Hence volume of required cube $= 8 \times 64 = (8)^3$
$\therefore$ Edge of this cube = 8 cm

2. Volume of the tank $= 3 \text{ m}^3$
$\therefore$ Base area × height $= 3 \text{ m}^3$
$\Rightarrow$ Base area $= \dfrac{3}{2.56} = 1.171875 \text{ m}^2$
[$\because$ Volume of cuboid $= (l \times b) \times h =$ (base area) × height]

3. Volume of cube = Volume of cuboid
$a^3 = lbh$
$\Rightarrow a^3 = 36 \times 75 \times 80 = 216000$
$\Rightarrow a = 60$ cm

4. Base area × height = Volume
$10 \times 4 \times 1 = 40 \text{ m}^3$
But $1 \text{ m}^3 = 1000$ litre = 1 kilolitre
$\therefore 40 \text{ m}^3 = 40{,}000$ litre = 40 kilolitre

5. Volume of smaller (required) cube $= 8 \text{ (dm)}^3 = 0.008 \text{ m}^3$
$\therefore$ Number of required cubes $= \dfrac{\text{Volume of larger cube}}{\text{Volume of each smaller cube}}$
$= \dfrac{1}{0.008} = 125$

6. Volume of new cube $= (3)^3 + (4)^3 + (5)^3 = 216 \text{ cm}^3$
$\therefore$ Each edge of the new cube = 6 cm
and hence, surface area $= 6 \times (a)^2 = 216 \text{ cm}^2$

7. Total length of tape $= 2(l + b) + 3.75$
$= 2(39.5 + 9.35) + 3.75$
$= 101.45$ cm

8. Area of surface to be cemented $= 2 \times (l + b) \times h + (l \times b)$
*i.e.,* area of four walls + area of floor
$= 2 \times (21) \times 4 + (106.25)$
$= 274.25 \text{ m}^2$
$\therefore$ Cost of cementing $= 24 \times 274.25 =$ Rs. 6582

9. Total volume of water displaced by 250 men
$= 250 \times 4 = 1000 \text{ m}^3$
$\therefore$ Rise in water level $(h) = \dfrac{\text{Volume}}{\text{Base area}}$
$= \dfrac{1000}{80 \times 50} = 25$ cm

10. Let each edge of smaller cube = 1 m
$\therefore$ Each edge of larger cube = 2 m
and Surface area of smaller cube $= 6 \times (1)^2 = 6 \text{ m}^2$
$\therefore$ Surface area of larger cube $= 6 \times (2)^2 = 24 \text{ m}^2$
$\therefore$ % increase in surface area $= \dfrac{24 - 6}{6} \times 100 = 300\%$

**NOTE** It can be determined by using variables *e.g.*, $x$ (edge of cube) instead of solving by assuming some numerals.

**Alternatively :** $\dfrac{S_2}{S_1} = \left(\dfrac{e_2}{e_1}\right)^2 \Rightarrow \dfrac{S_2}{S_1} = \dfrac{4}{1}$
$\therefore$ Percentage increase in surface area $= \dfrac{4 - 1}{1} \times 100 = 300\%$
where $S$ = surface area, $e$ = edge of cube.

11. $\dfrac{V_2}{V_1} = \left(\dfrac{e_2}{e_1}\right)^3$ {$\because V = (e)^3$} {$e$ = edge of cube}
$\therefore \dfrac{V_2}{V_1} = \left(\dfrac{2}{1}\right)^3 = \dfrac{8}{1}$
$\therefore V_2 = 8V_1$

12. External volume of the box $= 24 \times 16 \times 10 = 3840 \text{ cm}^3$
Thickness of the wood = 5 mm = 0.5 cm
$\therefore$ Internal length of box $= 24 - 2 \times 0.5 = 23$ cm
Internal breadth of box $= 16 - 2 \times 0.5 = 15$ cm
Internal height of box $= 10 - 2 \times 0.5 = 9$ cm
$\therefore$ Internal volume of the box $= 23 \times 15 \times 9 = 3105 \text{ cm}^3$
$\therefore$ Volume of the wood $= 3840 - 3105 = 735 \text{ cm}^3$
Now, total weight of wood = Volume × weight of $1 \text{ cm}^3$ wood
$7350 = 735 \times$ weight of $1 \text{ cm}^3$ wood
$\therefore$ Weight of $1 \text{ cm}^3$ wood = 10 gm

13. Length of tank = 120 cm
But since, $\dfrac{120}{7} = 17\dfrac{1}{7}$, hence 17 cubes can be placed along length and breadth of tank = 80 cm
But since, $\dfrac{80}{7} = 11\dfrac{3}{7}$, hence 11 cubes can be placed along breadth and height of tank = 50 cm.
But since, $\dfrac{50}{7} = 7\dfrac{1}{7}$, hence only 7 cubes can be placed along height of the tank.
$\therefore$ Total volume occupied by these cubes $= (7)^3 \times 17 \times 11 \times 7$
$= 448987 \text{ cm}^3$
Total volume of the tank $= 120 \times 80 \times 50 = 480000 \text{ cm}^3$
$\therefore$ Area of unoccupied space $= 480000 - 448987$
$= 31013 \text{ cm}^3 = 31.013 \text{ dm}^3$

14. Surface area of the cuboid $= 2(lb + bh + hl) = 11.6 \text{ m}^3$
$\therefore$ Cost of canvas $= 11.6 \times 25 =$ Rs. 290

15. Required surface area $= 2(9 \times 3 + 3 \times 3 + 3 \times 9) = 126 \text{ cm}^2$

16. Area of 4 walls $= 2(36 + 12) \times 10 = 960 \text{ m}^2$
Total area of (windows + door + chimney) $= 120 \text{ m}^2$
$\therefore$ Net area for papering $= 960 - 120 = 840 \text{ m}^2$
$\therefore$ Length of required paper $= \dfrac{840}{1.2} = 700$ m
Hence, cost of papering $= 700 \times 0.7 =$ Rs. 490

17. Number of children × Space required for one child
= Volume of room
$\therefore$ Number of children $= \dfrac{30 \times 12 \times 6}{8} = 270$

**18.** $(x+2)^3 - x^3 = 1016$

$\Rightarrow$ $x = 12$ cm

and $x^3 - (x-2)^3 = (12)^2 - (10)^2 = 728$

**19.** You can see the figure shown in the question number 15. Now let us consider that surface area of each face of the cube 1 cm$^3$.

$\therefore$ Total surface area of the cuboid = 14 cm$^2$

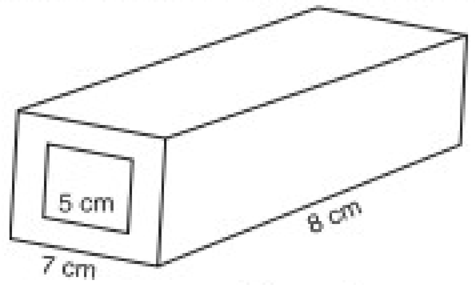
and Total surface area of the 3 cubes = 18 cm$^2$

Hence, required ratio = 14 : 18 = 7 : 9

**NOTE** In the arrangement of cuboid there are only 14 faces visible to us.

**20.** Iron used in the tube

= Difference in external and internal volumes of the tube




$\therefore$ $192 = 8x^2 - 8(5)^2$

$\Rightarrow$ $x = 7$ cm

Hence, the thickness of the tube $= \dfrac{7-5}{2} = 1$ cm

**21.** Base area of vessel × rise in water level = Volume of cube

$15 \times 12 \times h = 11 \times 11 \times 11$

$\Rightarrow$ $h = 7.39$ cm

**22.** (Initial volume of water + required volume of water + volume of cube)

= Base area of vessel × 10

$\therefore$ $25 \times 20 \times 5 +$ required volume of water$+ 1000$

$= 25 \times 20 \times 10$

$\Rightarrow$ Required volume of water = 1500 cm$^3$ = 1.5 litre

**23.** Net volume $= (10 \times 8 \times 2) - (2 \times 2 \times 2) = 152$ cm$^3$

Net surface area $= 2(10 \times 8 + 8 \times 2 + 2 \times 10) + 4(2 \times 2) - 2(2 \times 2)$

$= 240$ cm$^2$

**24.** Available area for spreading the earth

$= (600 \times 200) - (24 \times 12)$

$= 119712$ m$^2$

Volume of the earth = 119712 × rise in level

$24 \times 12 \times 8 = 119712 \times h$

$\Rightarrow$ $h = \dfrac{2304}{119712}$

$= 0.01924$ m $= 1.92$ cm

**25.** Net volume of the wall

= Total volume − Volume taken away due to doors

$= (30 \times 0.3 \times 5) - 2(4 \times 2.5 \times 0.3) = 39$ m$^3$

Volume of the bricks $= 39 \times \dfrac{8}{9}$

(Since $\dfrac{1}{9}$ part is lime in the wall)

$\therefore$ Number of bricks $= \dfrac{39 \times 8}{9 \times 0.2 \times 0.16 \times 0.08}$

$= 13541.66 \approx 13600$

**26.** Volume of water which flow in 25 minutes

$= 25 \times 60 \times 0.05 \times 0.03 \times 16 = 36$ m$^3$

$\therefore$ Rise in water level $= \dfrac{36}{15 \times 12} = \dfrac{1}{5}$ m $= 0.2$ m

**27.** A tetrahedron is a pyramid with triangular base hence it has total $(3+1) = 4$ vertices.

(See the figure shown after exercise 10.6)

**28.** $a\sqrt{3} = 6\sqrt{3} \Rightarrow a = 6$ cm

**29.** $\sqrt{l^2 + b^2} = 12 \Rightarrow l^2 + b^2 = 144$

and $\sqrt{l^2 + b^2 + h^2} = 15$

$\Rightarrow$ $l^2 + b^2 + h^2 = 225$

$\Rightarrow$ $h^2 = 81 \Rightarrow h = 9$ m

**30.** $l + b + h = 12$ cm, $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{2}$

$\Rightarrow$ $l^2 + b^2 + h^2 = 50$

Now, $(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

$\Rightarrow$ $144 = 50 + 2(lb + bh + hl)$

$\Rightarrow$ $2(lb + bh + hl) = 94$ cm$^2$

**31.** $h : b = 3 : 1$ and $l : h = 8 : 1$

$\Rightarrow$ $l : h : b = 24 : 3 : 1$

$\therefore$ $24x \times 3x \times x = 36.864$

$\Rightarrow$ $x^3 = 0.512 \Rightarrow x = 0.8$

$\therefore$ $h = 3x = 2.4$ m

**32.** $lb = x, \quad bh = y, \quad hl = z$

$\therefore$ $lb \times bh \times hl = xyz$

$\Rightarrow$ $(lbh)^2 = xyz \Rightarrow lbh = \sqrt{xyz}$

## Introductory Exercise 10.6

**1.** Required volume of water $= \dfrac{22}{7} \times (0.04)^2 \times 3500$

$= 17.6$ m$^3$

**2.** $h = \dfrac{10 \times 10 \times 10}{\pi \times 10 \times 10} = 3\dfrac{2}{11}$ cm

**3.** $h = \dfrac{511}{36.5} = 14$ m

**4.** $2\pi rh = 1056$ cm$^2$

$\Rightarrow$ $r = \dfrac{33}{\pi}$ cm

$\therefore$ $\pi r^2 h = \pi \times \left(\dfrac{33}{\pi}\right)^2 \times 16$

$\Rightarrow$ Volume = 5544 cm$^3$

**5.** Diameter of the cylinder = edge of the cube = 2 cm

Height of the cylinder = edge of the cube

$\therefore$ Required volume $= (2)^3 - \pi \times (1)^2 \times 2$

$$= 8 - \left(\frac{44}{7}\right) = \frac{12}{7} \text{ cm}^3$$

**6.** $$h = \frac{2\pi rh}{2\pi r} = \frac{17.6}{8.8} = 2 \text{ m}$$

and $2\pi r = 8.8 \Rightarrow r = 1.4$ m

$$\therefore \quad \pi r^2 h = \frac{22}{7} \times (1.4)^2 \times 2 = 12.32 \text{ m}^3$$

**7.** $$2r = h \Rightarrow r = \frac{h}{2}$$

$$\therefore \quad 2\pi r (h + r) = 2\pi \times \frac{h}{2}\left(h + \frac{h}{2}\right) = \frac{3}{2}\pi h^2$$

**8.** $2\pi rh = 2 \times \pi \times 2 \times 10 = 40\pi \text{ m}^2$

**9.** $\dfrac{2\pi rh}{2\pi r (h + r)} = \dfrac{2}{3} \Rightarrow \dfrac{h}{h + r} = \dfrac{2}{3} \Rightarrow \dfrac{h}{r} = \dfrac{2}{1}$

$\therefore \quad 2\pi r (h + r) = 924$

$$\therefore \quad 2\pi rh = 924 \times \frac{2}{3} = 616 \text{ cm}^2$$

$\Rightarrow \quad 2 \times \pi \times x \times 2x = 616$

$\Rightarrow \quad x = 7 \quad \therefore r = 7$ cm and $h = 14$ cm

$$\therefore \quad \pi r^2 h = \frac{22}{7} \times (7)^2 \times 14 = 2156 \text{ cm}^3$$

**10.** $$\frac{\pi \times r \times r \times h}{2 \times \pi \times r \times h} = \frac{269.5}{154}$$

$\Rightarrow \quad r = 3.5$ cm

Now, $2\pi \times 3.5 \times h = 154$

$\Rightarrow \quad h = 7$ cm

**11.** $2\pi rh = 1320 \Rightarrow h = 10$ cm

$$\therefore \quad 2\pi r (h + r) = 2 \times \frac{22}{7} \times 21 \times 31 = 4092 \text{ cm}^2$$

**12.** $\dfrac{r}{h} = \dfrac{3x}{4x}, \quad \pi r^2 h = 4851 \Rightarrow x = 3.5$

$\therefore \quad r = 10.5$ m and $h = 14$ m

$$\therefore \quad 2\pi rh = 2 \times \frac{22}{7} \times 10.5 \times 14 = 924 \text{ m}^2$$

**13.** $$\frac{2\pi r (h + r)}{2\pi rh} = \frac{4}{1} \Rightarrow \frac{h + r}{h} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{3}{1}$$

**Alternatively :** Go through options.

**14.** $$2\pi r = 33 \Rightarrow r = \frac{21}{4}$$

$$\therefore \quad \text{Volume} = \pi r^2 h = \frac{22}{7} \times \left(\frac{21}{4}\right)^2 \times 330$$

$$= 28586.25 \text{ cm}^3$$

**15.** $\dfrac{r_1}{r_2} = \dfrac{4x}{x}$, but $V_1 = V_2$

$\therefore \quad \pi (4x)^2 \times h_1 = \pi (x)^2 h_2$

$\Rightarrow \quad h_2 = 16 h_1$

**16.** $2\pi r (h + r) = 2640$

$\Rightarrow \quad 2\pi r (30) = 2640$

$\Rightarrow \quad r = 14$ m

$\Rightarrow \quad h = 16$ m $\quad (\because r + h = 30 \text{ m})$

$\therefore \quad h : r = 8 : 7$

**17.** $$\frac{V_1}{V_2} = \frac{\pi (r_1)^2 h_1}{\pi (r_2)^2 h_2} = \frac{3 \times 3 \times 4}{5 \times 5 \times 3} = \frac{12}{25}$$

**18.** $$\frac{V_1}{V_2} = \frac{1}{1}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{1} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 \times \frac{3}{1} = \frac{1}{1}$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{3} \Rightarrow \frac{r_1}{r_2} = \frac{1}{\sqrt{3}}$$

**19.** $$\frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{3}{2} \times \frac{6}{7} = \frac{9}{7}$$

**20.** $h = 42$ cm, $2\pi R = 132$ cm

$\Rightarrow \quad R = 21$ cm

$\therefore \quad r = 21 - 3 = 18$ cm

$\therefore$ Required volume $= \pi (R^2 - r^2) h$

$$= \frac{22}{7} [(21)^2 - (18)^2] \times 42$$

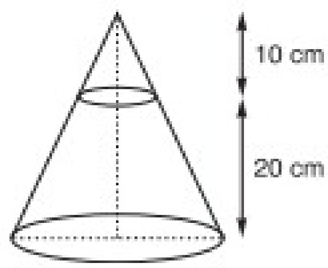
$$= 15444 \text{ cm}^3$$

**21.** Since radius and height of the cylinder are same as that of cone. Therefore cylinder can contain $15 \times 3 = 45$ litre of milk.

**HINT** Volume of cone $= \dfrac{1}{3}\pi r^2 h$ and volume of cylinder $= \pi r^2 h$.

**22.** Best way is to go through option.

**Alternatively :** $\dfrac{V_1}{V_2} = \dfrac{1}{27}$




$$\frac{(r_1)^2 h_1}{(r_2)^2 h_2} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{x_1^3}{x_2^3} = \frac{(1)^3}{(3)^3} \Rightarrow \frac{x_1}{x_2} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{1}{3}$$

$$\therefore \quad h_1 = \frac{h_2}{3} = \frac{30}{3} = 10 \text{ cm}$$

Hence, cone is cut off at $(30 - 10 =)$ 20 cm above the base.

**HINT** For more clarification of the concept seek help from geometry. (Similarity of triangles)

**23.** Volume of conical tent $= \dfrac{1}{3} \times \dfrac{22}{7} \times \dfrac{13}{2} \times \dfrac{13}{2} \times 10.5$

$\therefore$ Average space per man $= \dfrac{\text{Volume of tent}}{8} = 58\dfrac{3}{32}$

**24.** $\dfrac{1}{3} \times \pi \times (5x)^2 \times (12x) = 314\dfrac{2}{7} = \dfrac{2200}{7}$

$\Rightarrow \quad x = 1$

$\therefore \quad r = 5$ and $h = 12$

$\therefore \quad l = 13$ m

**25.** $\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232$

$\Rightarrow \quad r = 7$ cm

$\therefore \quad \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2 \quad (\because l = \sqrt{r^2 + h^2})$

**26.** $2\pi r = 220 \Rightarrow r = 35$ cm

$\therefore \quad l = \sqrt{r^2 + h^2} = 91$ cm

$\therefore \quad \pi rl = \frac{22}{7} \times 35 \times 91 = 10010 \text{ cm}^2$

**27.** Area of cloth $= \pi rl = \frac{22}{7} \times 14 \times 50 = 2200 \text{ m}^2$

$\therefore$ Length of cloth $= \frac{2200}{10} = 220$ m

**28.** $\pi rl = 23.10$

$\Rightarrow \quad rl = 2.1 \times 3.5$

Now, we know that $h = 2.8$. So we can assume the value of $r$ from the given option.

at $\quad r = 2.1, \quad l = 3.5 \quad (\because l = \sqrt{r^2 + h^2})$

**Alternatively :** $r^2 (r^2 + h^2) = (2.1 \times 3.5)^2$

$\Rightarrow \quad r^4 + (2.8)^2 r^2 = (7.35)^2$

$\Rightarrow \quad r^4 + 7.84r^2 = 54.0225$

$\Rightarrow \quad k^2 + 7.84k - 54.0225, \quad k = r^2$

Now, solve the above quadratic equation, if you can else substitute the value of $r$ from the given choices.

**29.** $\frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{3}{2} \quad (\because r_1 = r_2)$

$\therefore \quad \frac{A_1}{A_2} = \frac{3}{2} \Rightarrow \frac{A_1}{300} = \frac{3}{2}$

$\therefore \quad A_1 = 450 \text{ cm}^2$

**30.** $\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\left(\frac{1}{3}\right)\pi r^2 h} = \frac{1}{\left(\frac{1}{3}\right)} = \frac{3}{1}$

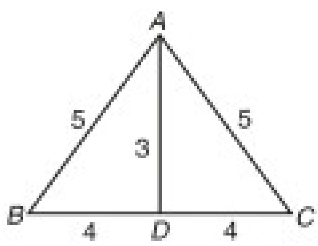
**31.** By Pythagorus theorem

$AD = 3$ cm

$\therefore$ Area of axial section

$= \frac{1}{2} \times 8 \times 3$

$= 12 \text{ cm}^2$




**32.** $\frac{V_2}{V_1} = \frac{(1.4)^3}{(1)^3} = \frac{2.744}{1}$

$\therefore$ % increase in volume $= \left[\frac{2.744 - 1}{1}\right] \times 100 = 174.4\%$

**33.** Area of circular sheet $= 625\pi$

Since length of arc and area of sector are directly proportional to the central angle.

Therefore, length of remaining arc $= \frac{96}{100} \times 2 \times \pi \times 25 = 48\pi$

But the remaining arc is equal to the circumference of the base of circular cone.

$\therefore \quad 2\pi R = 48\pi \Rightarrow R = 24$ cm

Now, since the slant height of cone is equal to the radius of the original circular sheet.

Hence, $\quad l = 25$ cm

$h = 7$ cm $\quad (\because l = \sqrt{r^2 + h^2})$

$\therefore \quad \frac{\text{Radius}}{\text{Height}} = \frac{24}{7}$

**34.** $\frac{V_2}{V_1} = \frac{(r_2)^2 h_2}{(r_1)^2 h_1} = \frac{(2r)^2}{(r)^2} \quad (\because h_1 = h_2)$

$\Rightarrow \quad \frac{V_2}{V_1} = \frac{4}{1}$

**35.** Each edge of cube= 10 cm

Radius of base of cone = 5 cm

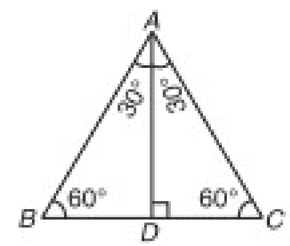
Height of cone = 10 cm

$\therefore$ Volume of cone $= \frac{1}{3} \times \frac{22}{7} \times 25 \times 10 = 261.9 \text{ cm}^3$

**36.** $\frac{V_2}{V_1} = \frac{r_1^2 \times h_1}{r_2^2 \times h_2} = \frac{2 \times 2 \times 2}{1 \times 1 \times 1} = \frac{8}{1}$

**37.** $\frac{BD}{AB} = \cos 60°$

$\frac{BD}{AB} = \frac{1}{2}$




$BD = CD$, are the radii of the base and $AB = AC$ are the slant heights of the cone. $A$ is the vertex and $BC$ is the base.

**38.** Volume of cone $= \frac{1}{3} \pi \times 144 \times 35$

Volume of water flowing per second $= \pi \times (0.8)^2 \times \frac{500}{60}$

$\therefore \quad$ Required time $= \frac{\left(\frac{\pi}{3}\right) \times 144 \times 35}{\pi \times 0.64 \times \left(\frac{500}{60}\right)}$

$= 315$ seconds

**39.** Use the given formula :

Volume $= \frac{\pi}{3} h (r^2 + Rr + R^2)$

where $\pi = \frac{22}{7}, \quad h = 6, \quad R = 4 \quad$ and $\quad r = 2$

**40.** $\frac{1}{3} \pi \times (10)^2 \times 72 = \pi \times (30)^2 \times h$

$\Rightarrow \quad h = \frac{8}{3} = 2\frac{2}{3}$ cm

## Introductory Exercise 10.7

**1.** $$\frac{4}{3}\pi(r_1^3+r_2^3+r_3^3)=\frac{4}{3}\pi(6)^3$$

$\Rightarrow$ $$27+64+r_3^3=216$$

$\Rightarrow$ $$r_3^3=125$$

$\Rightarrow$ $$r_3=5\text{ cm}$$

**2.** $\pi r^2\times 144=\frac{4}{3}\pi\times(12)^3$ ($\because$ Volume remains constant)

$\Rightarrow$ $r=4$ cm

**3.** Number of bottles $\times$ Volume of each bottle

= Volume of hemisphere

$$n\times\pi\times(3)^2\times 1=\frac{2}{3}\pi\times(6)^3$$

$\Rightarrow$ $$n=16$$

**4.** $\frac{1}{3}\pi\times(r)^2\times 14=\frac{2}{3}\pi\times(7)^3$

**5.** Volume of the spherical shell $=\frac{4}{3}\pi(R^3-r^3)$

$$=\frac{4}{3}\pi(7^3-5^3)$$

$$=\frac{872}{3}\pi$$

**6.** Since volume is constant

$\therefore$ $$n\times\frac{4}{3}\pi(1)^3=\frac{4}{3}\pi\times(8)^3$$

$\Rightarrow$ $$n=512$$

**7.** Volume of hemisphere $=\frac{2}{3}\pi r^3$

$$=\frac{2}{3}\times\frac{22}{7}\times(21)^3=19404\text{ m}^3$$

**8.** Change in height (or level) of water $=\dfrac{\text{Volume of sphere}}{\text{Base area of cylinder}}$

$$=\frac{\frac{4}{3}\pi\times(9)^3}{\pi\times(12)^2}=\frac{27}{4}\text{ cm}$$

**9.** Volume of cone = Volume of sphere

$$\frac{1}{3}\pi r^2\times\frac{7}{2}=\frac{4}{3}\pi(7)^3$$

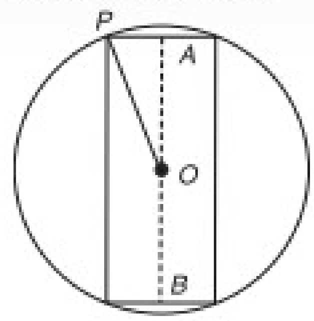
$\Rightarrow$ $$r=14\sqrt{2}\text{ cm}$$

**10.** **Note:** The exact value of the required surface area cannot be found using the general formula. For the exact value of total surface area we have to solve it with the help of calculus. In the following solution we are calculating lumpsum value of total surface area.

Now, $$OA=\sqrt{(OP)^2-(AP)^2}$$

$$=\sqrt{(15)^2-(9)^2}=12\text{ cm}$$

$\therefore$ $$AB=2\times OA=24\text{ cm}$$




$\therefore$ Height of cylinder = 24 cm

$\therefore$ Required area = Surface area of sphere

$-$ 2 (area of each end of cylinder)

+ curved surface area of cylinder

$$=900\pi-2(81\pi)+432\pi=1170\pi\text{ cm}^2$$

**11.** Volume of pyramid $=\frac{1}{3}\times$ base area $\times$ height

$$=\frac{1}{3}\times 25\times 12=100\text{ cm}^3$$

**12.** Volume of prism = Base area $\times$ height

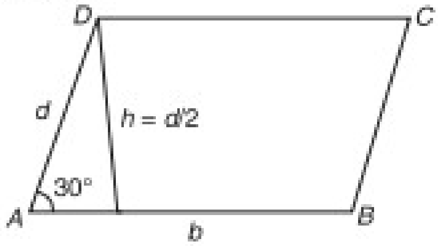
Since base area is constant and height is being halved therefore volume will also be halved. Hence, its volume will be reduced by 50%.

## LEVEL 1

**1.** With the given perimeter the area of parallelogram will be maximum when it will be a rhombus.

Hence (a) is correct.

**Alternatively :**




$$\text{Perimeter} = 2d + 2b = 4h + 2b = 64 \text{ cm}$$

$$\Rightarrow \quad b = \frac{64 - 4h}{2}$$

$$\therefore \quad \text{Area} = b \times h = \frac{64h - 4h^2}{2} = f(A)$$

$\therefore f'(A) = \frac{1}{2}(64 - 8h)$ (Refer the differentiation)

For the maximum area $= f'(A) = 0$

$$\therefore \quad \frac{1}{2}(64 - 8h) = 0$$

$$\Rightarrow \quad h = 8 \text{ cm} \quad \text{and} \quad d = 16 \text{ cm}$$

$$\therefore \quad b = 16 \text{ cm}$$

**2.** Volume of water displaced = volume of sphere

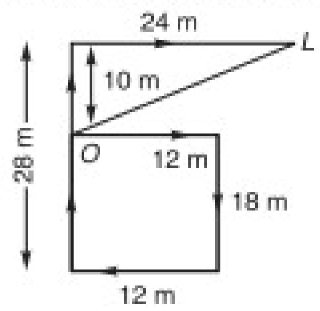
$$\pi \times (40)^2 \times h = \frac{4}{3}\pi \times (30)^3$$

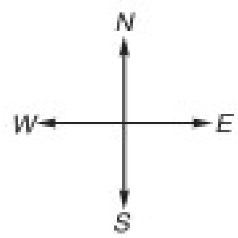
$$\Rightarrow \quad h = \frac{90}{4} = 22.5 \text{ cm}$$

Thus, the level of water rises by 22.5 cm.

**NOTE** The volume of water will be calculated by considering it in the cylindrical shape since the water takes the shape of vessel in which it is filled.

**3.** $O$ is the starting point and $L$ is the end point.



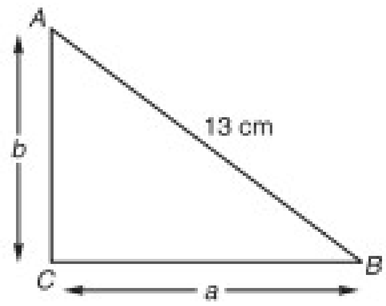

$$(OL)^2 = (24)^2 + (10)^2$$

$$OL^2 = 676$$

$$\Rightarrow \quad OL = 26 \text{ m}$$

**4.** $$a^2 + b^2 = 13^2 = 169$$

and $$\frac{a \times b}{2} = 30$$




Now $\quad (a + b)^2 = a^2 + b^2 + 2ab$

$$(a + b)^2 = 169 + 120$$

$$\Rightarrow \quad a + b = 17 \quad \ldots\text{(i)}$$

Again $\quad (a - b)^2 = (a + b)^2 - 4ab$

$$(a - b)^2 = 289 - 240$$

$$\Rightarrow \quad (a - b) = 7 \quad \ldots\text{(ii)}$$

$\therefore$ From Eq. (i) and (ii), we get

$$a = 12 \text{ cm} \quad \text{and} \quad b = 5 \text{ cm}$$

Now, $\quad \text{inradius} = \frac{(\text{Altitude} + \text{Base} - \text{Hypotenuse})}{2}$

$$= \frac{(12 + 5 - 13)}{2} = 2 \text{ cm}$$

**Alternatively:** $\quad \text{Inradius } (r) = \frac{\text{Area}}{\text{Semiperimeter}}$

$$= \frac{30}{15} = 2 \text{ cm}$$

$$\left(\text{Semiperimeter} = \frac{(12 + 5 + 13)}{2} = 15\right)$$

**NOTE** If you are well aware with Pythagorian triplets then it is very easy to find the sides (*i.e.*, altitude and base) of the right angle triangle.

**5. General phenomenon:** A convex polygon in which there is maximum number of sides, it has the greatest enclosed area when the perimeter of the polygon is constant. Remember that in a circle there are infinite sides of minimum possible length. So, the area of circle will be maximum.

**Alternatively:** Perimeters of hexagon and circumference of circle are same *i. e.*,

$$6a = 2\pi r \quad \Rightarrow \quad 21a = 22r$$

where $a$ is the side of hexagon and $r$ is the radius of circle.

then $\quad \text{Area of circle} = \pi r^2$

and $\quad \text{Area of hexagon} = 6 \times \frac{\sqrt{3}}{4} \times a^2$

$$= \frac{3\sqrt{3}}{4} \times \left(\frac{22r}{21}\right)^2$$

$$= \frac{3\sqrt{3}}{4} \times \frac{22}{7 \times 3} \times \frac{22}{7 \times 3} \times r^2$$

$$= \frac{\sqrt{3}}{4} \times \frac{22}{21} \pi r^2 = 0.45\pi r^2$$

Therefore, area of circle $\pi r^2$ is greater than area of hexagon. Similarly you can compare other figures.

**Alternatively:** Consider some suitable values. Let us assume perimeter of square and hexagon is 24 cm, then each side of square = 6 cm.

and Area = 36 cm$^2$

Similarly, each side of hexagon = 4 cm

and area of hexagon $= \frac{3\sqrt{3}}{2} \times (4)^2 = 24\sqrt{3}$ cm$^2$

Thus, we can say that for a particular (or constant) value of perimeter, area of hexagon is greater than that of square.

Similarly each side of octagon = 3 cm $\left(= \frac{24}{8}\right)$

$\therefore$ Area of octagon $= 2a^2 (1 + \sqrt{2})$

$= 2 \times 9 (1 + \sqrt{2})$

$\approx 43.5$ cm$^2$

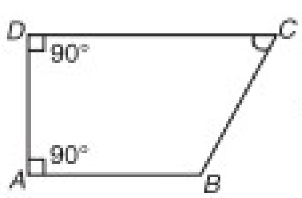
Thus, the area of octagon is greater then the area of hexagon.

Similarly, $2\pi r = 24 \Rightarrow r = \frac{12}{\pi}$ cm

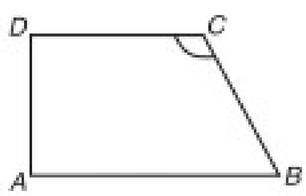
$\therefore$ Area of circle $= \pi \times r^2$

$= \pi \times \frac{12}{\pi} \times \frac{12}{\pi} = 45.81$ cm$^2$

Thus, we can say that ultimately the area of the circle is greatest.

**6.** 


or




Since sum of all the angles of a quadrilateral is 360°.

Therefore, $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle C + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle C$

**7.** $\angle ABC = 60°$

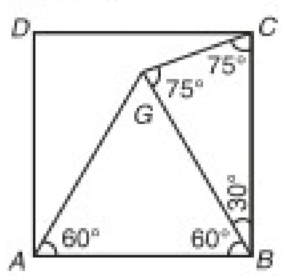
$\therefore \angle GBC = 90° - 60° = 30°$

Again $\angle BGC + \angle BCG = 180° - 30°$

$= 150°$

Now, since $\angle BGC = \angle BCG$

$\therefore \angle BGC = 75°$




**NOTE** In a triangle when two sides are equal, then the two angles opposite to these sides are also equal.

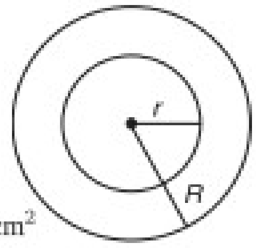
**8.** $\frac{r}{R} = \frac{3x}{4x}$

$\because 4x - 3x = 2$

$\Rightarrow x = 2$

$\therefore$ Outer radius = 8 cm

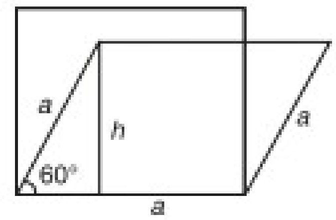
$\therefore$ Area of outer circle $= \pi \times (8)^2 = 64\pi$ cm$^2$




**9.** $\frac{h}{a} = \sin 60°$

$\frac{h}{a} = \frac{\sqrt{3}}{2}$

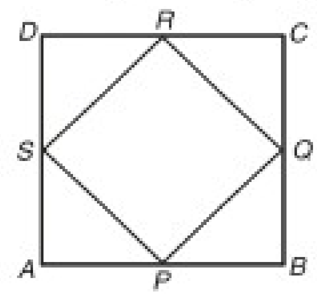
$\Rightarrow h = \frac{a\sqrt{3}}{2}$




$\therefore$ Area of rhombus $= a \times h = \frac{a \times \sqrt{3}a}{2}$

and Area of square $= a^2$

$\therefore$ Required ratio $= \frac{a^2}{\sqrt{3}a^2} \times 2 = \frac{2}{\sqrt{3}} \times \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}$

**10.** 


$ABCD$ is a square,

$AB = BC = CD = AD = 20$ cm

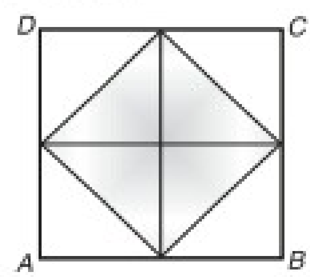
and $P$, $Q$, $R$ and $S$ are the mid-points.

$\therefore PS = SR = RQ = PQ = 10\sqrt{2}$ cm ($\because AP = BP$ etc = 10 cm)

$\therefore$ Area of square $PQRS = (10\sqrt{2})^2 = 200$ cm$^2$

**Alternatively:** We can see that there are total 8 equal parts (or triangles) out of 8 we have taken only 4 parts *i.e.*, 50% area has been cut out.

So, the remaining area = 50% of 400 = 200 cm$^2$.



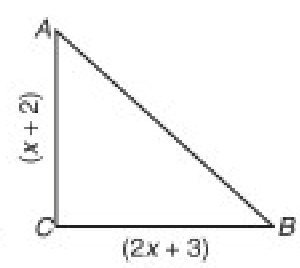

**11.** Area of right angle triangle

$= \frac{(x+2) \times (2x+3)}{2} = 60$

$\Rightarrow 2x^2 + 7x + 6 = 120$

$\Rightarrow 2x^2 + 7x - 114 = 0$

Solving the above quadratic equation, we get




$x = 6$

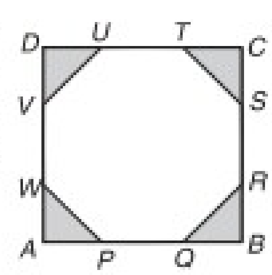
$\therefore x + 2 = 8$ cm

and $2x + 3 = 15$ cm

$\therefore$ Hypotenuse $AB = \sqrt{(8)^2 + (15)^2} = 17$ cm

(Using Pythagorus theorem)

**12.** Let us assume $PQRSTUVW$ is a regular octagon with each side $a$ unit. Again, if we produce all the sides of octagon in both sides we get a square. Since each of the side of octagon is '$a$' then




$UV = WP = QR = ST = a$

$\therefore \quad DU = DV \text{ (etc.)} = \dfrac{a}{\sqrt{2}}$

$\therefore \quad DU + UT + TC = \dfrac{a}{\sqrt{2}} + a + \dfrac{a}{\sqrt{2}}$

$= a(1+\sqrt{2})$

$\therefore \quad$ Area of square $= a^2(1+\sqrt{2})^2$

$= a^2(3+2\sqrt{2})$ sq unit

Again, area of each shaded portion (*i. e.*, an isosceles right angle triangle)

$$= \frac{1}{2}\times\frac{a}{\sqrt{2}}\times\frac{a}{\sqrt{2}} = \frac{a^2}{4}$$

$\therefore$ Total area of all the shaded region $= 4\times\dfrac{a^2}{4} = a^2$

$\therefore$ Area of octagon = Area of square $ABCD$

$-$ Area of total shaded region

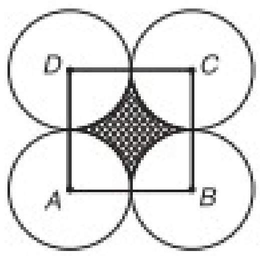
$= a^2(3+2\sqrt{2}) - a^2$

$= 2a^2(1+\sqrt{2})$ sq. unit

**13.** Let the each side of the square be 2 cm, then

area of square = 4 cm$^2$ and

area of 4 quadrants of the four circles (*i. e.*, unshaded part inside the square)




$= 4\times\dfrac{1}{4}\pi\times(1)^2 = \pi$ cm$^2$

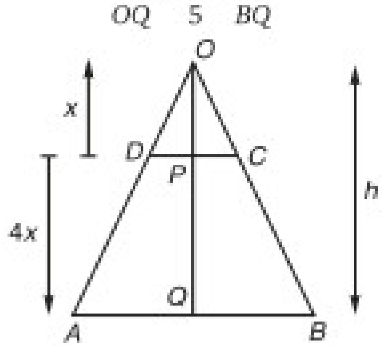
$\therefore \quad$ Area of shaded region $= (4-\pi)$ cm$^2$

Therefore, the required ratio $= \dfrac{4-\pi}{\pi} = \dfrac{6/7}{22/7} = \dfrac{3}{11}$

**14.** From the concept of mid point theorem, it is the average of the length of the parallel sides.

**15.** From the concept of similarity of triangles

$$\frac{OP}{OQ} = \frac{1}{5} = \frac{PC}{BQ}$$




Since, the ratio in radii of the two cones is 1 : 5. Therefore, the radius of smaller cone $ODC$ is $\dfrac{21}{5} = 4.2$ cm.

**Alternatively:** Solve it in the proper way, which is actually a very tedious process. Still you have to apply the concept of similarity of triangles, which you will study in Geometry.

**16.** $\quad l\times b = 2(l+b)$

$\Rightarrow \quad l = \dfrac{2b}{(b-2)} = \dfrac{2(b-2)+4}{b-2} = 2 + \dfrac{4}{(b-2)}$

Since, $l$ is an integer, so 4 must be divisible by $(b-2)$. Thus, $b$ can be 4 or 6 or 3.

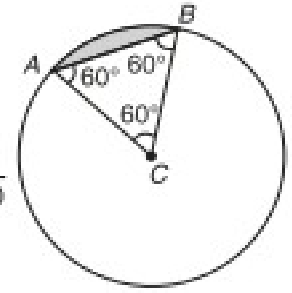
Therefore, if $b = 4$, $l = 4$, it will be a square. If $b = 6$, $l = 3$ and if $b = 3$, $l = 6$.

Hence, $\quad l = 6$ and $b = 3$

$\therefore \quad l - b = 3$

**17.** Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}\times(14)^2$

$= 49\sqrt{3}$ cm$^2$




Area of sector $ABC = \pi\times(14)^2\times\dfrac{60}{360}$

$= 102\dfrac{2}{3}$ cm$^2$

$\therefore$ Area of the shaded region $= \left(102\dfrac{2}{3} - 49\sqrt{3}\right)$ cm$^2$

**18.** $\quad l_1\times b_1 = l_2\times b_2$

New area of first rectangle $= \dfrac{5}{4}l_1\times\dfrac{3}{4}b_1 = \dfrac{15}{16}l_1b_1$

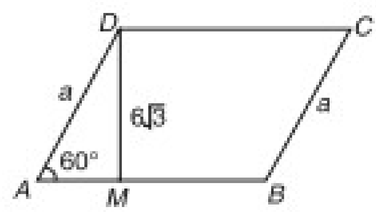
New area of the second rectangle $= \dfrac{3}{4}l_2\times\dfrac{5}{4}b_2 = \dfrac{15}{16}l_2b_2$

Hence, areas of both the new rectangles are same.

**19.** $\quad \dfrac{DM}{AD} = \sin 60°$

$\dfrac{6\sqrt{3}}{AD} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad AD = 12$ cm




**20.** $\quad \dfrac{4}{3}\pi r^3 = \dfrac{1}{3}\pi r^2 h$

(Since radii of sphere and cone are same)

$\Rightarrow \quad 4r = h$

$\therefore \quad \dfrac{h}{r} = \dfrac{4}{1} \Rightarrow h : r = 4 : 1$

**21.** Area of the shaded region $= \left[\dfrac{1}{2}\pi(22)^2 - 2\left(\dfrac{1}{2}\pi\times(11)^2\right)\right]$

$= \dfrac{1}{2}\pi\times(11)^2[4-2] = 121\pi$ cm$^2$

**22.** Area of path $= (l+b-w)w$

$= (60+40-5)5 = 475$ m$^2$

Cost = Area × rate

$= 475\times 0.8 =$ Rs. 380

**23.** Go through options. Let the length and breadth of the second rectangle is $l$ and $b$ respectively, then the area of second field $= l\times b$.

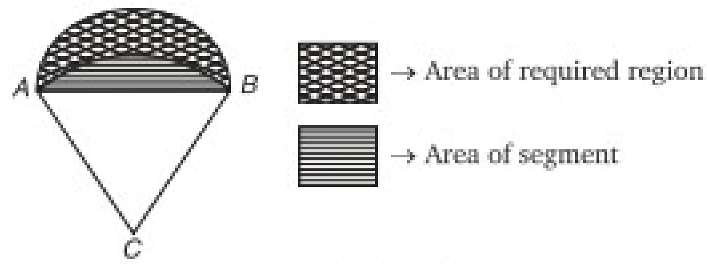
Now consider option (d) *i. e.*, $x = 80$, then

area of first field $= \dfrac{1}{5}l\times 5b = lb$

Hence, area of first field is same as that of second field, hence the presumed option is correct.

**24.** $\quad$ Area of quadrant $= \dfrac{1}{4}\pi a^2$

Area of triangle $ACB = \dfrac{a^2}{2}$




$\therefore$ Area of segment $= \dfrac{\pi a^2}{4} - \dfrac{a^2}{2} = \dfrac{a^2}{4}[\pi - 2]$

Area of semi-circle $= \dfrac{1}{2}\pi\left(\dfrac{a\sqrt{2}}{2}\right)^2 = \dfrac{\pi a^2}{4}$

$\therefore$ Area of required region $= \dfrac{\pi a^2}{4} - \dfrac{a^2}{4}[\pi - 2]$

$= \dfrac{a^2}{2}$ sq. unit

**25.** Wrongly calculated area $= 1.05 \times 0.92 = 0.966 = 96.6\%$

$\therefore$ % error $= 100 - 96.6 = 3.4\%$

**Alternatively:** Actual area $= l \times b$

Wrongly calculated area $= 1.05l \times 0.92b = 0.966lb$

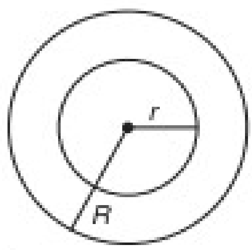
Deficit in area $= lb - 0.966lb = 0.034lb$

% error in area $= \dfrac{0.034lb}{lb} \times 100 = 3.4\%$

**26.** Volume of water = Area of cross-section × Length of pool

$= \dfrac{(10+6)}{2} \times 6 \times 120 = 5760 \text{ m}^3$

**27.** Area of path $= \dfrac{22176}{1} = 22176 \text{ m}^2$




$\pi(R^2 - r^2) = 22176$

$(R^2 - r^2) = \dfrac{22176}{22} \times 7$

$R^2 - (112)^2 = 7056$

$\Rightarrow$ $R^2 = 19600 \Rightarrow R = 140$ m

$\therefore$ Width of the path $= 140 - 112 = 28$ m

**28.** From the figure itself it is clear that there are total 9 equilateral (congruent) triangles. Out of 9 triangles, 3 triangles are cut out.

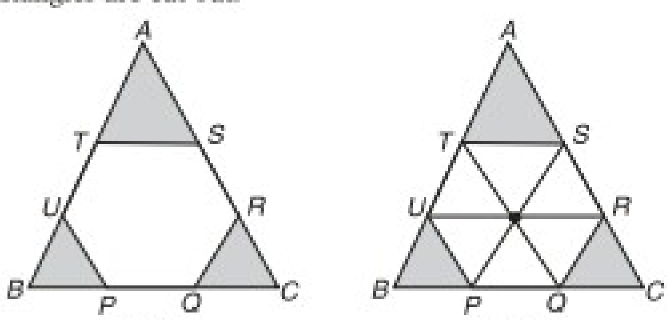
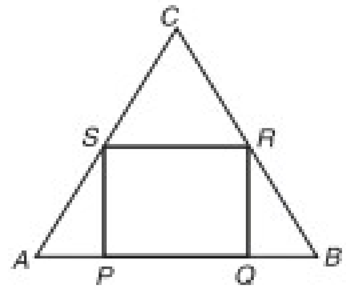



Fig. (i) Fig. (ii)

It means $\dfrac{1}{3}$ (*i. e.*, 33.33%) area has been removed.

**29.** $\dfrac{SP}{AS} = \sin 60° = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $SP = \dfrac{\sqrt{3}}{2} AS$




Also, $SP = SR = CS = CR$

(since, $SR \parallel AB \therefore \angle CSR = \angle SRC = \angle RCS = 60°$)

$\therefore$ $CS = \dfrac{\sqrt{3}}{2} AS$

$\therefore$ $\dfrac{CS}{AS} = \dfrac{\sqrt{3}}{2} = \dfrac{CR}{RB}$

**30.** Let the each side of a square be $a$ then its area will be $a^2$. Therefore, area of circle will also be $a^2$.

Again since the perimeter of square and equilateral triangle is same then, the each side of equilateral triangle is $\dfrac{4a}{3}$.

$\therefore$ Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}\left(\dfrac{4a}{3}\right)^2$

$= \dfrac{4\sqrt{3}}{9} a^2$

$\therefore$ Required ratio of area $= \dfrac{a^2 \times 9}{4\sqrt{3} \times a^2} = 9 : 4\sqrt{3}$

**31.** $ABCD$ is a square $O$ is the point of intersection of diagonals. $P$, $Q$, $R$ and $S$ are the mid-points on the sides $AB$, $BC$, $CD$, $DA$ respectively.

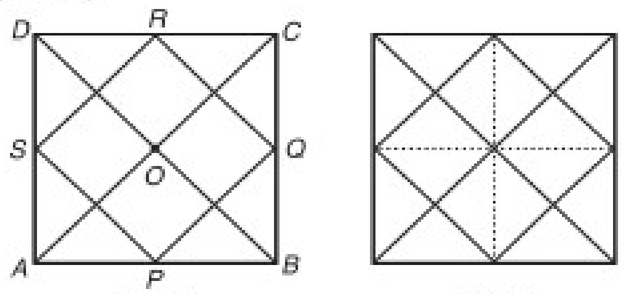



Fig. (i) Fig. (ii)

In the above figure you can see that there are total 16 congruent isosceles right angled triangles.

In figure (iii) you can see that $OMPN$ is a square of maximum possible area which is made up of 2 isosceles right angled triangles $OMP$ and $ONP$. Thus, there are 4 smaller squares around $O$ and thus the total area of these 4 squares is half of the larger square. Hence, the required area = 800 cm$^2$.

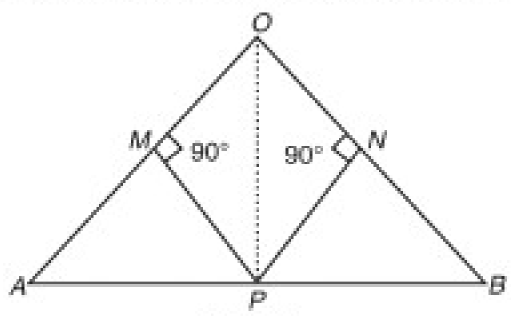


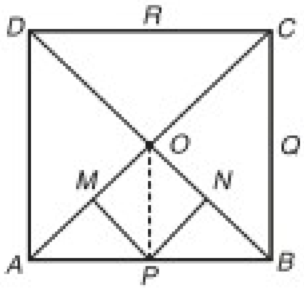

Fig. (iii)

**Alternatively:** $ABCD$ is a square

$AB = 40$ cm

$OA = OB = 20\sqrt{2}$ cm

and $OP = 20$ cm



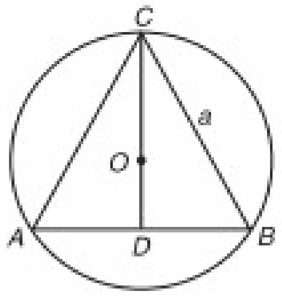
Square $OMPN$ has maximum area inside the triangle and $OP$ is the diagonal of square $OMPN$.

$\therefore$ Area of $OMPN = \frac{(20)^2}{2} = 200$ cm$^2$

Since, there are total 4 similar squares in each of the 4 triangles $AOB$, $BOC$, $COD$, $DOA$ inside the larger square. So, the total required area of 4 smaller squares

$= 4 \times 200 = 800$ cm$^2$

**32.** Let the each side of equilateral triangle be '$a$' then circumradius of the circle $= \frac{a}{\sqrt{3}}$



$\therefore$ Area of circum circle

$$= \pi r^2 = \pi \times \left(\frac{a}{\sqrt{3}}\right)^2 = \frac{\pi}{3} a^2$$

Area of square whose side is equal to that of equilateral triangle $= a^2$.

$\therefore$ Required ratio $= \dfrac{\frac{\pi}{3} a^2}{a^2} = \frac{\pi}{3}$

**NOTE** A circumcircle always passes through the vertices of the inscribed figure (say triangle).

**33.** Best way is to go through option. Given that height of room $= 10$ m.

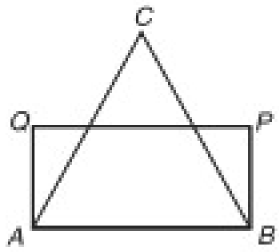
Volume of room $= 25 \times 400 = 10000$ m$^3$

and Surface area of walls $= 2h\,(l + b) = 1300$ m$^2$

Now, consider option (c) and verify it.

where $l = 40$ and $b = 25$

**34.** Let the each side of equilateral triangle be $a$ then its perimeter $= 3a$



Again, $2\,(l + b) = 3a$

$\Rightarrow$ $l + b = \frac{3}{2} a$

$\Rightarrow$ $a + b = \frac{3a}{2}$

$AB = l = a$ (for the rectangle)

$\Rightarrow$ $b = \frac{a}{2}$

$\therefore$ Area of rectangle $= a \times \frac{a}{2} = \frac{a^2}{2}$

and Area of triangle $= \frac{\sqrt{3}}{4} \times a^2$

$\therefore$ Required ratio $= \dfrac{a^2/2}{\sqrt{3}a^2/4} = \frac{2}{\sqrt{3}}$

**35.** Since, $b$, $l$ and $2\,(l + b)$ are in GP, therefore

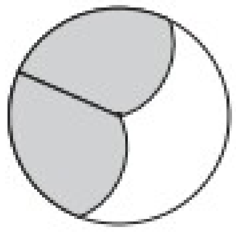
$$\frac{l}{b} = \frac{2(l + b)}{l}$$

Suppose $\frac{l}{b} = x$,

then $x = 2\left(1 + \frac{1}{x}\right) \Rightarrow x^2 - 2x - 2 = 0$

$\Rightarrow$ $x = \sqrt{3} + 1 = \frac{l}{b}$

**36.** Non-polished area $= 4\pi\,(r)^2$



Polished area $= 4 \times \left(2 \times \frac{\pi r^2}{2}\right)$

$= 4\pi r^2$

In the adjoining one of the four parts of the sphere is shown (To understand it properly, take an apple and cut it in the four parts one across horizontal and another cut make vertical to it then you will notice that in a piece there are 2 semicircles.) Therefore, required ratio = 1 : 1.

**37.** Number of cubes $= \frac{85 \times 17 \times 5.1}{1.7 \times 1.7 \times 1.7}$

$= 1500$ (1.7 is the HCF of 85, 17 and 5.1)

Area of each cube $= 6 \times (1.7)^2$

$\therefore$ Area of all the 1500 cubes $= 1500 \times 6 \times (1.7)^2$

$= 26010$ cm$^2$

**38.** Area of large cube $= 6 \times (5)^2 = 150$ (unit)$^2$

Area of cuboid $= 2\,(1 \times 1 + 1 \times 125 + 125 \times 1)$

$= 502$ (unit)$^2$

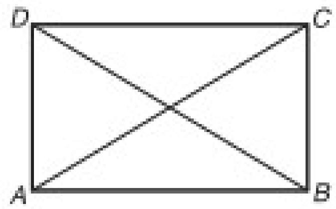
Therefore, increase in surface area $= \frac{(502 - 150)}{150} \times 100$

$= 234\frac{2}{3}\%$

**39.** $AO = BO = 13$ cm

$\Rightarrow$ $AC = BD = 26$ cm ($\because O$ is the point of bisector)

Now, since the diagonals are equal, it means the given figure is actually a rectangle.



$\therefore$ $BC^2 = AC^2 - AB^2$

$BC^2 = (26)^2 - (24)^2$

$BC^2 = 100$

$\Rightarrow$ $BC = 10$ cm

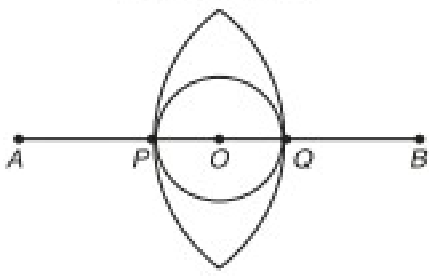
**40.** $AB = 16$ cm

$AQ = 10$ cm

and $AO = \frac{AB}{2} = 8$ cm

$\therefore$ $OQ = AQ - AO$

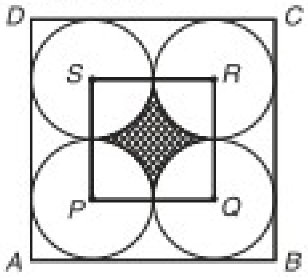
$= 10 - 8 = 2$ cm



$OQ\ (= OP)$ is the radius of smaller enclosed circle between two arcs.

$\therefore$ Area of circle with centre $O$ is $\pi \times (2)^2 = 4\pi$ cm$^2$.

**41.** Area of the shaded region

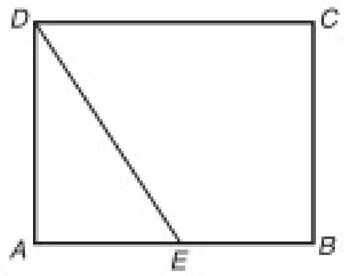



$= $ (Area of square $PQRS - 4 \times$ area of each quadrant of circles)

$$= \left[(2)^2 - 4 \times \frac{1}{4}\pi \times (1)^2\right]$$

$$= (4 - \pi) \text{ cm}^2$$

**42.** Let each side of the square be $a$, then




$$AE = AB - BE = (a - 17)$$

$\therefore$ Area of triangle $= \frac{1}{2} \times AD \times AE$

$$= \frac{1}{2} a \times (a - 17) = 84$$

$\Rightarrow$ $a^2 - 17a - 168 = 0$

$\Rightarrow$ $a = 24$ cm

$\therefore$ Area of square $= (24)^2 = 576$ cm$^2$

**43.** The solid with the least number of sides will have maximum surface area. So, tetrahedron will have maximum surface area. Notice that in a sphere there are infinite number of sides with least possible length. So, the surface area of sphere will be least.

**44.** $$\frac{l}{b} = \frac{(l+b)}{l}$$

$\Rightarrow$ $l^2 = b\,(l + b) = lb + b^2$

$\Rightarrow$ $l^2 - b^2 = lb$ ...(A)

$\Rightarrow$ $(l + b)(l - b) = lb$ ...(B)

and $\frac{(l+b)}{l} = \frac{b}{(l-b)}$ ...(C)

Therefore, statements (ii) and (iii) are true from equation (A)

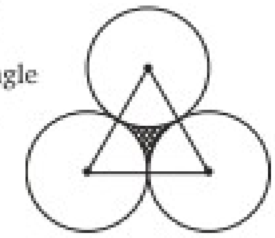
$$\frac{l^2}{b^2} = \frac{bl + b^2}{b^2}$$

$\Rightarrow$ $\frac{l^2}{b^2} = 1 + \frac{l}{b}$

$\Rightarrow$ $\frac{l}{b} = \frac{l^2}{b^2} - 1$

Hence, statement (i) is wrong.

**45.** Area of triangle $= \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3}$ cm$^2$



Area of 3 circles enclosed by the triangle

$$= 3 \times \pi \times (1)^2 \times \frac{60}{360} = \frac{\pi}{2} \text{ cm}^2$$

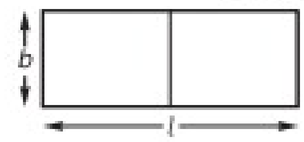
$\therefore$ Area of shaded region

$$= \sqrt{3} - \frac{\pi}{2} = \frac{(2\sqrt{3} - \pi)}{2} \text{ cm}^2$$

**46.** Diameter of cube = 3 cm

$\therefore$ Number of cubes $= \frac{9 \times 9 \times 9}{3 \times 3 \times 3} = 27$ cubes

**47.**




$$l \times b = 20000$$

$\Rightarrow$ $\frac{l}{2} \times b = 10000$, Area of Kaurav's land

For the given area, a square gives the minimum perimeter.

$\therefore$ $\frac{l}{2} = b$

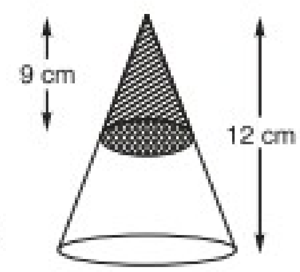
$\Rightarrow$ $b \times b = 10000$

$\Rightarrow$ $b = 100$ m

$\therefore$ Perimeter of Kaurav's land $= 4 \times 100 = 400$ m

$\therefore$ Expenses $= 2 \times 400 =$ Rs. 800

**48.** Ratio of height




$$\frac{h_1}{h_2} = \frac{9}{12} = \frac{3}{4}$$

$\therefore$ Ratio of volumes $= \frac{(3)^3}{(4)^3} = \frac{27}{64}$

Hence, the volume of smaller cone

$$= 27x$$

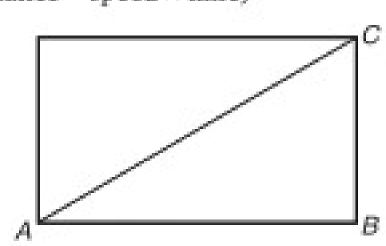
and the volume of larger (actual) cone $= 64x$

$\therefore$ the volume of frustum $= 64x - 27x = 37x$

$\therefore$ Required ratio $= \frac{64x}{37x} = 64 : 37$

**49.** $AB = 45$ km, $AC = 53$ km

(Since distance = speed $\times$ time)




$\therefore$ $BC = \sqrt{AC^2 - AB^2}$

$$BC = \sqrt{(53)^3 - (45)^2}$$

or $BC = 28$ km

$\therefore$ Area of field $= AB \times BC$

$$= 45 \times 28 = 1260 \text{ km}^2$$

**50.**

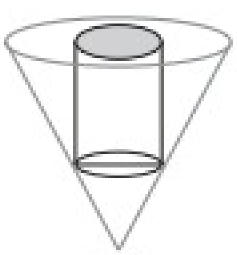


Fig. (i)

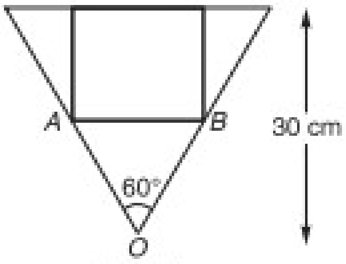



Fig. (ii)

$\angle AOB = 60°$

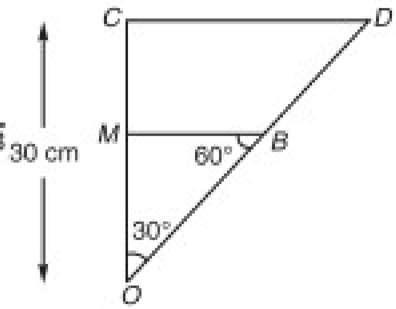
$AB = 8\sqrt{3}$ cm

$OC = 30$ cm

$\frac{OM}{MB} = \tan 60° = \sqrt{3}$

$OM = \sqrt{3}MB$

$OM = 12$ cm




Now, $\frac{OM}{MB} = \frac{OC}{CD}$

(Triangles $OMB$ and $OCD$ are similar)

$\therefore \quad CD = 10\sqrt{3}$ cm (which is the radius of cone)

$\therefore \quad$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (10\sqrt{3})^2 \times 30$

$= 3000\pi$ cm$^3$

**51.** Surface area of one cube $= 6a^2$

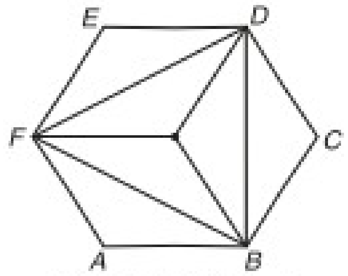
when 6 cubes are fixed on the 6 faces of a cube then only 5 faces of a cube are visible of each cube. Since, the central cube is completely covered. So, the only 6 cubes are visible each with 5 faces. Hence, the total surface area of this solid $= 5a^2 \times 6$.

$\therefore \quad$ Required ratio $= \frac{5a^2 \times 6}{6a^2} = \frac{5}{1} \Rightarrow 5:1$

**52.** Ratio of areas = (Ratio of diagonals)$^2$

$= \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

**53.** You should know that




$\frac{\text{Area of } \Delta BDF}{\text{Area of hexagon}} = \frac{1}{2}$

Actually there is a perfect symmetricity.

$\therefore \quad$ Area of hexagon $= \frac{3\sqrt{3}}{2} \times (6)^2 = 54\sqrt{3}$ cm$^2$

$\therefore \quad$ Area of $\Delta BDF = 27\sqrt{3}$ cm$^2$

**Alternatively:** Subtract the areas of three triangles $DEF$, $BAF$, $BCD$ from the area of hexagon.

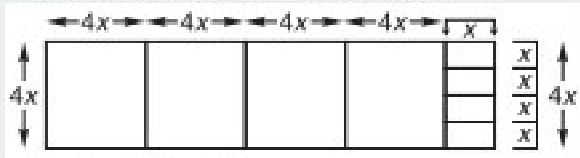
Area of $\Delta DEF = \frac{1}{2} \times 6 \times 6 \sin 120° = 9\sqrt{3}$ cm$^2$

$\therefore$ Area of all the three triangles $= 27\sqrt{3}$ cm

Therefore, required area of $\Delta BDF = 54\sqrt{3} - 27\sqrt{3}$

$= 27\sqrt{3}$ cm$^2$

***Solutions for question number 54 and 55:***




*Total length of plot* $= 17x$

*and total breadth of the plot* $= 4x$

$\therefore \quad$ *Area of plot* $= 17x \times 4x = 9792$

$\Rightarrow \quad x = 12$

$\therefore \quad l = 17 \times 12 = 204$ *m*

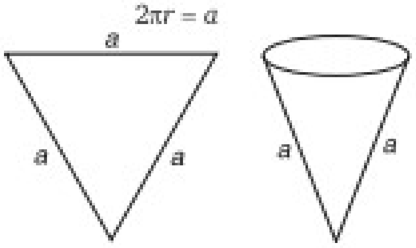
*and* $\quad b = 4 \times 12 = 48$ *m*

**54.** $\frac{4x \times 4x}{x \times x} = \frac{16}{1} \Rightarrow 16:1$

**55.** $l = 204$ m and $b = 48$ m

## LEVEL 2

**1.** $2\pi r = a$




Also, slant height $(l) = a \quad \therefore \quad r = \frac{a}{2\pi}$

**2.** $\because \quad l^2 = h^2 + r^2$

$\Rightarrow \quad h^2 = l^2 - r^2 = a^2 - \left(\frac{a}{2\pi}\right)^2$

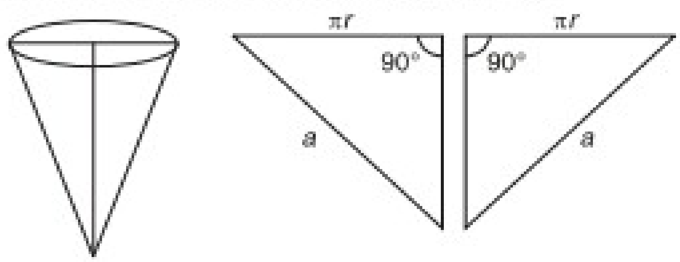
$h^2 = a^2\left[\frac{4\pi^2 - 1}{4\pi^2}\right]$

$\therefore \quad h = \frac{a}{2\pi}\sqrt{4\pi^2 - 1}$

$\therefore \quad$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \frac{a^2}{4\pi^2} \times \frac{a}{2\pi}\sqrt{4\pi^2 - 1}$

$= \frac{a^3}{24\pi^2}\sqrt{4\pi^2 - 1}$

**3.** It will be in the form of a right angled triangle.

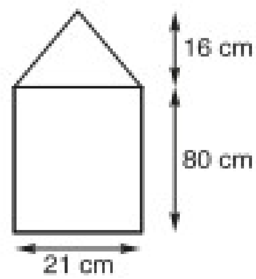


**4.** $$2\pi r\,(r+h) = 1540 \text{ cm}^2$$

and $$(r+h) = 35 \text{ cm}$$

$\therefore$ $$2\pi r = \frac{1540}{35} = 44 \text{ cm}$$

**5.** Total volume $= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$




$$= \pi r^2 \left[h_1 + \frac{h_2}{3}\right]$$

$$= \frac{22}{7} \times (21)^2 \left[80 + \frac{16}{3}\right]$$

$$= \frac{22}{7} \times 441 \times \frac{256}{3}$$

$$\text{Weight} = \frac{22}{7} \times 441 \times \frac{256}{3} \times \frac{8.45}{1000} = 999.39 \text{ kg}$$

**6.** *ABCD* is a square, each side of square is '$a$'.

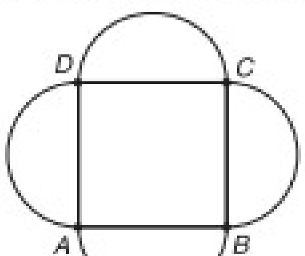



Fig. (i)

In figure (ii),

$$\angle DOC = 120°$$

and $$\angle ODC = \angle OCD = 30°$$

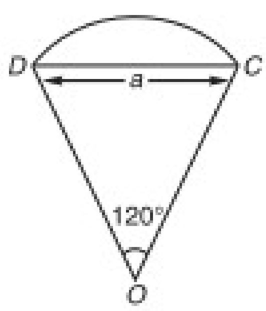



Fig. (ii)

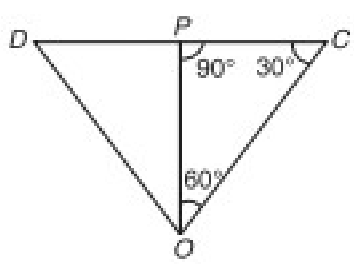



Fig. (iii)

In figure (iii),

$$\frac{PC}{OC} = \sin 60°$$

$$\frac{a/2}{OC} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $OC = \frac{a}{\sqrt{3}}$ $\Rightarrow$ radius of the arc '$CD$'.

$\therefore$ Area of triangle $OCD = \frac{1}{2} \times CD \times OP$

$$= \frac{1}{2} \times a \times \frac{a}{2\sqrt{3}} = \frac{a^2}{4\sqrt{3}}$$

$$\left(\because \frac{OP}{PC} = \tan 30° \text{ and } \tan 30° = \frac{1}{\sqrt{3}}\right)$$

and area of sector $COD$ (figure ii)

$$= \pi r^2 \frac{120}{360}$$

$$= \pi \times \left(\frac{a}{\sqrt{3}}\right)^2 \times \frac{1}{3} = \frac{\pi a^2}{9}$$

$\therefore$ Area of segment = (Area of sector − Area of triangle)

$$= \frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}$$

Total area of all the four segments $= 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)$

and the total area of whole figure $= a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)$

**7.** $$2(l+b) = 26 \;\Rightarrow\; l + b = 13$$

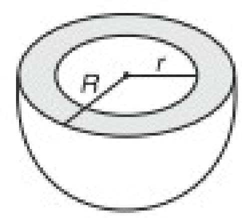
$$12 + 1 = 13$$
$$11 + 2 = 13$$
$$10 + 3 = 13$$
$$9 + 4 = 13$$
$$8 + 5 = 13$$
$$7 + 6 = 13$$

Since, $l > b$, therefore, there are only 6 integral values of the length *viz.*, 7, 8, 9, 10, 11 and 12.

**8.** Total surface area $= 2\pi R^2 + 2\pi r^2 + (\pi R^2 - \pi r^2)$

$$= 3\pi R^2 + \pi r^2$$

$$= \pi\,(3R^2 + r^2)$$




$\Rightarrow$ $$1436\frac{2}{7} = \pi\,[3 \times (12)^2 + r^2]$$

$\Rightarrow$ $$\frac{10054}{7} \times \frac{1}{\pi} = 432 + r^2$$

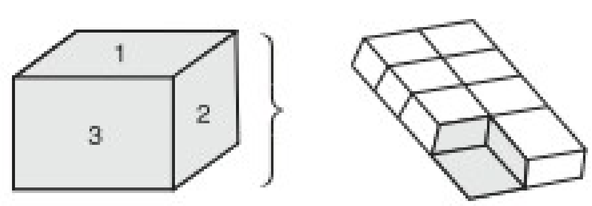
$\Rightarrow$ $$r = 5 \text{ cm}$$

$\therefore$ Internal volume of hemisphere $= \frac{2}{3}\pi\,(R^3 - r^3)$

$$= \frac{2}{3}\pi\,[(12)^3 - (5)^3]$$

$$= 3358\frac{2}{3} \text{ cm}^3$$

**9.** Since, there are 3 faces which are visible in a corner cube. When the cube of a corner is removed then the 3 faces of other cubes will be visible from outside. So, there will not be any change in the surface area of this solid figure.




**10.** Number of spheres $= \dfrac{\frac{4}{3}\pi\left(\frac{15}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{3}{2}\right)^3} = 125$ spheres

Surface area of a large sphere $= 4\pi \times \left(\frac{15}{2}\right)^2$

and surface area of a small sphere $= 4\pi\left(\frac{3}{2}\right)^2$

and total surface area of all the smaller spheres

$$= 125 \times 4\pi \left(\frac{3}{2}\right)^2$$

$$\% \text{ change in area} = \left[\frac{500\pi\left(\frac{3}{2}\right)^2 - 4\pi\left(\frac{15}{2}\right)^2}{4\pi\left(\frac{15}{2}\right)^2} \times 100\right] = 400\%$$

**Alternatively:** Surface area of larger sphere $= 25x$

Surface area of smaller sphere $= x$

$\because$ the ratio of radii is 5 : 1.

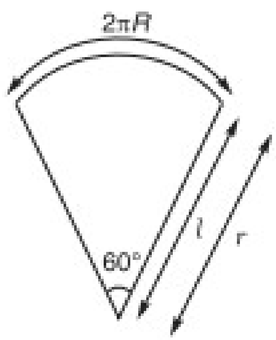
Therefore, ratio in surface areas = 25 : 1

Now, since there are 125 smaller spheres

$\therefore$ Total surface area of smaller spheres $= 125x$

$$\therefore \quad \% \text{ change in area} = \frac{125x - 25x}{25x} \times 100 = 400\%$$

**11.** Let the radius of cone be $R$ and radius of sector $= r$,




$\Rightarrow l = r$

then the slant height of cone $(l) = r$

and $\quad 2\pi R = 2\pi r \times \dfrac{60}{360}$

$$\Rightarrow \quad R = \frac{r}{6} = \frac{14}{6} = \frac{7}{3} \text{ cm}$$

$$\therefore \quad \text{Total surface area} = \pi r (l + r)$$

$$= \frac{22}{7} \times \frac{7}{3}\left(14 + \frac{7}{3}\right)$$

$$= 119.78 \text{ cm}^2$$

**12.** Between 26 poles, total length is $(26 - 1) \times 4 = 100$ m

It means the length of each side of a square field is 100 m.

$\therefore$ Area of field $= (100)^2 = 10{,}000 \text{ m}^2 = 1$ hectare

**13.** It is clear that length of the lawn is 2 m more than the breadth of lawn.

To solve this problem quickly, go through options. Let us take option (c).

$$l = 10 \text{ m} \quad \Rightarrow \quad b = 8 \text{ m}$$

$$\text{Area of path} = (l + b + 2w)\, 2w$$

$$= (10 + 8 + 4)\, 4 = 88 \text{ m}^2$$

and $\quad$ area of lawn $= 10 \times 8 = 80 \text{ m}^2$

Reduced area of lawn $= 8 \times 8 = 64 \text{ m}^2$

$\therefore \quad$ New area of path $= 88 + (80 - 64) = 104 \text{ m}^2$

$$\therefore \quad \text{Ratio of areas of path} = \frac{104}{88} = \frac{13}{11}$$

Hence, option (c) is correct.

**Alternatively:** Let the breadth of the lawn be '$b$' then the length will be $(b + 2)$.

$$\therefore \quad \text{Area of path} = (l + 4)(b + 4) - lb$$

$$= (b + 6)(b + 4) - (b + 2)\, b$$

$$= 8b + 24$$

and the new area of path $= (l + 4)(b + 4) - b \times b$

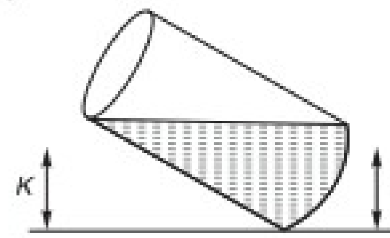
$$= (b + 6)(b + 4) - b^2$$

$$= 10b + 24$$

$$\because \quad \frac{(10b + 24)}{(8b + 24)} = \frac{13}{11}$$

$$\Rightarrow \quad b = 8 \text{ m} \quad \therefore l = 10 \text{ m}$$

**14.** From the figure you can see that just half of the liquid has been flown off and half the liquid is remained in the cylindrical jar.




Thus it is clear that the capacity (or volume) of the cylinder

$$= 2 \times 2.1 = 4.2 \text{ L}$$

**15.** When the height and base of the cone are same as that of cylinder, then the voume of cone is $\frac{1}{3}$ that of the cylinder.

Thus the capacity of cone $= \dfrac{1}{3} \times 4.2 = 1.4\, l$

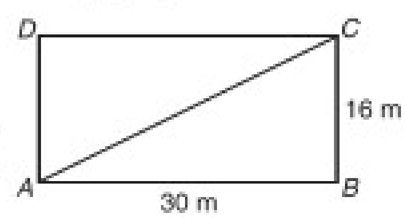
Thus the remaining volume $= 2.1 - 1.4 = 0.7\, l$

$$\therefore \quad \text{the required ratio} = \frac{0.7}{4.2} = \frac{1}{6}$$

**16.** $AC = \sqrt{(30)^2 + (16)^2}$

$AC = 34$ m

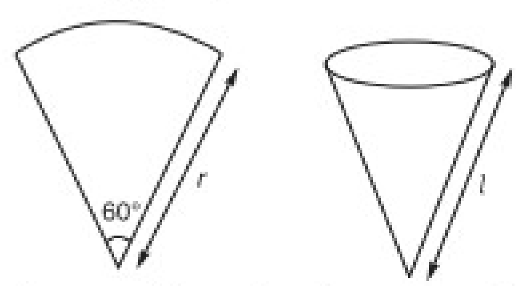
but since elephant is itself 4 m long. So he has to travel only




$$(34 - 4) = 30 \text{ m.}$$

$$\therefore \text{the speed of elephant} = \frac{30}{15} = 2 \text{ m/s}$$

**17.** Arc of sector $= \dfrac{2\pi r\, 60}{360} = \dfrac{2\pi r}{6}$




This arc of sector will be equal to the perimeter of cone. Let the radius of cone be $R$,

then $\quad 2\pi R = \dfrac{2\pi r}{6} \quad \Rightarrow \quad R = \dfrac{r}{6}$

Further the radius of sector will be equal to the slant height of cone

$\therefore \quad l = r$

Now $\quad$ since $= l^2 = h^2 + R^2$

$\Rightarrow \quad h = \sqrt{l^2 - R^2}$

$$h = \sqrt{r^2 - \left(\frac{r}{6}\right)^2}$$

$$h = \frac{\sqrt{35}}{6} r$$

**18.** The diagonal of cube will be equal to the diameter of sphere.

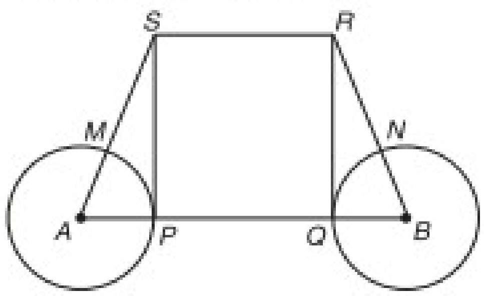
$\therefore \quad$ Volume of sphere $= \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}$

and $\quad$ each side of cube $= a = \frac{d}{\sqrt{3}}$

$\therefore \quad$ Volume of cube $= a^3 = \frac{d^3}{3\sqrt{3}}$

$\therefore$ Remaining volume $= \frac{\pi d^3}{6} - \frac{d^3}{3\sqrt{3}} = \frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$

**19.** Let $AP = x$ then $AM = x$ and $MS = x$

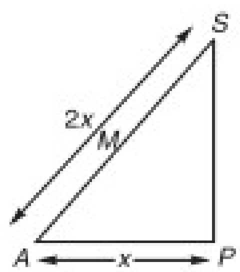



$\therefore \quad AS = AM + MS$

$\quad AS = 2x$

$\therefore \quad PS = \sqrt{AS^2 - AP^2}$

$\quad PS = \sqrt{3}x$



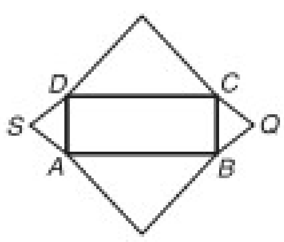

$\therefore$ Area of square $PQRS = (\sqrt{3}x)^2 = 3x^2$

Area of circle $= \pi r^2 = \pi \times x^2 = \pi x^2$

Area of both the circles $= 2\pi x^2$

$\therefore \quad$ Required ratio $= \frac{2\pi x^2}{3x^2} = \frac{2\pi}{3}$

**20.** Let the length of rectangle be '$l$' and breadth be '$b$', then




$$2(l + b) = 12$$

$\Rightarrow \quad l + b = 6$ cm

and area of larger equilateral triangle $= \frac{\sqrt{3}}{4} l^2$

Similarly area of smaller equilateral triangle $= \frac{\sqrt{3}}{4} b^2$

$\therefore$ Total area of all the 4 triangles $= 2 \times \frac{\sqrt{3}}{4}(l^2 + b^2) = 10\sqrt{3}$

$\Rightarrow \quad l^2 + b^2 = 20$

$\therefore \quad (l + b)^2 = l^2 + b^2 + 2lb$

$\Rightarrow \quad 36 = 20 + 2lb$

$\Rightarrow \quad lb = 8$

$\therefore \quad (l - b)^2 = l^2 + b^2 - 2lb = 20 - 16$

$\quad (l - b)^2 = 4$

$\Rightarrow \quad l - b = 2$

$\because \quad l + b = 6$ and $l - b = 2$

$\therefore \quad l = 4$ and $b = 2$

$\therefore \quad$ area of rectangle $= 4 \times 2 = 8$ cm$^2$

$\therefore \quad$ Total area of the figure $= 8 + 10\sqrt{3}$

$\quad = 2(4 + 5\sqrt{3})$ cm$^2$

**21.** Area of each square $= 16$ cm$^2$

Area of quadrant $ADMB = \frac{1}{4}\pi \times (4)^2 = 4\pi$

and radius of smaller quadrant

$$CPMQ = CM = AC - MA$$

$$= 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$$

$\therefore \quad$ Area of smaller quadrant $= \frac{1}{4}\pi[4(\sqrt{2} - 1)]^2$

$$= 4\pi(3 - 2\sqrt{2})$$

Area of shaded region inside the square $ABCD$

$$= 16 - [4\pi + 4\pi(3 - 2\sqrt{2})]$$

$$= 16 - [4\pi(1 + 3 - 2\sqrt{2})]$$

$$= 16 - [4\pi(4 - 2\sqrt{2})]$$

$$= 8[2 - 2\pi + \sqrt{2}\pi]$$

Now, area of quadrants $= AEG + EFG = 2AEG$

$$= 2 \times \frac{1}{4}\pi(4)^2 = 8\pi$$

$\therefore$ Area of shaded region inside the square $EAGF$

$$= 8\pi - 16 = 8(\pi - 2)$$

$\therefore \quad$ Required ratio $= \frac{8(2 - 2\pi + \sqrt{2}\pi)}{8(\pi - 2)}$

$$= \frac{[2 + \pi(\sqrt{2} - 2)]}{(\pi - 2)}$$

**22.** Given that $\quad \frac{AB}{BC} = \frac{AD}{DF}$

Also $\quad BE = BC$

Let $\quad AD = 1$ and $AE = x$

$\therefore \quad \frac{AE}{EF} = \frac{AE}{AD} = \frac{AE}{BC} = x$

$\therefore \quad \frac{AE}{EF} = \frac{AD}{AB} \quad \left(\begin{array}{l}\because \; AD = BC = BE \\ \text{and } AB = AE - BE\end{array}\right)$

$$\frac{x}{1} = \frac{1}{x - 1}$$

$\Rightarrow \quad x^2 - x - 1 = 0$

$\Rightarrow \quad x = \frac{(1 \pm \sqrt{5})}{2}$

$\therefore \quad x = \frac{(1 + \sqrt{5})}{2}$

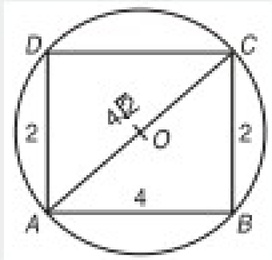
Since ratio of two sides can never be negative.

**Alternatively:** Since ratio of two side can never be negative therefore only option (c) is correct.

***Solutions for question number 23 to 25:***

$AB = 4$

$\therefore \quad AO = AC = \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$



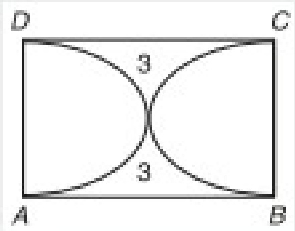

$\therefore$ *Area of circle*

$ABCD = \pi \times (2 \times \sqrt{2})^2 = 8\pi$

*Area of region 2 (only left part)*

$$= \frac{\text{Area of circle} - \text{Area of square}}{4}$$

$$= \frac{8\pi - 16}{4} = (2\pi - 4)$$

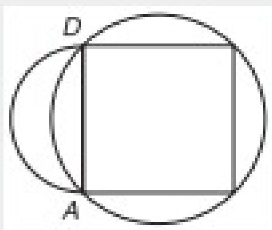



*Area of region 3 = Area of square*

*− 2 (Area of semicircle)*

$$= 16 - 2\left(\frac{1}{2} \times \pi \times 4\right)$$

$$= 16 - 4\pi = 4\,(4 - \pi)\, cm^2$$




*Area of region 1*

*= Area of semicircle AD*

*− Area of region 2*

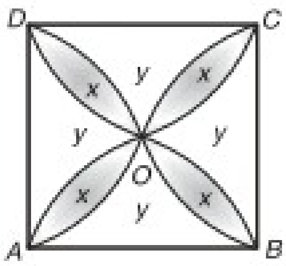
$$= \frac{1}{2}\pi \times (2)^2 - (2\pi - 4) = 4\ cm^2$$

**23.** Total area of region $1 = 2 \times 4 = 8$ cm$^2$

**24.** Total area of region $2 = 2 \times (2\pi - 4) = 4\,(\pi - 2)$ cm$^2$

**25.** Total area of region $3 = 4\,(4 - \pi)$ cm$^2$

**26.** Total area of square = 64 cm$^2$




$\therefore \quad 4\,(x + y) = 64$

$\Rightarrow \quad x + y = 16 \qquad \ldots$(i)

Again in a semicircle

$AOB = x + y + x = \dfrac{1}{2}\pi \times (4)^2$

$2x + y = 8\pi \qquad \ldots$(ii)

From Eq. (i) and (ii), we get

$$x = 8\pi - 16$$

Total area of shaded region $= 4\,(8\pi - 16)$

$= 32\,(\pi - 2)$ cm$^2$

**Alternatively:** Area of square − 2 (Area of semicircles) = $2y$

$(64 - 16\pi) = 2y$

$\therefore \quad 4y = (128 - 32\pi)$

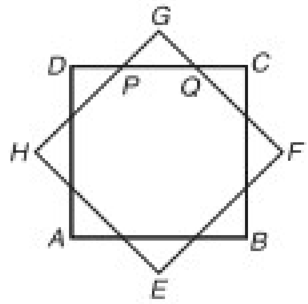
$\therefore$ Required area of shaded region $(4x)$ = (Area of square − $4y$)

$= 64 - (128 - 32\pi)$

$= 32\pi - 64$

$= 32\,(\pi - 2)$ cm$^2$

**27.** You can see in the figure that the sides of one square is parallel to the diagonals of the other square.




Let $DP = a$, then

$\left\{\begin{array}{l} \because \quad DP = QC \\ \text{and } DP = PG = GQ = QC \\ \angle G = 90^\circ \\ \therefore \quad PQ = \sqrt{2}a \end{array}\right\}$

$DC = DP + PQ + QC$

$= a + a\sqrt{2} + a$

$DC = a\,(2 + \sqrt{2})$

$\therefore$ Area of $\Delta PGQ = \dfrac{1}{2} \times a \times a = \dfrac{a^2}{2}$

$\therefore$ Area of all the triangles outside the square $ABCD$

$$= 4 \times \frac{a^2}{2} = 2a^2$$

But $\qquad DC = a\,(2 + \sqrt{2}) = 4$ cm

$\Rightarrow \qquad a = \dfrac{4}{(2 + \sqrt{2})}$

$\therefore \qquad 2\,(a)^2 = 2 \times \left(\dfrac{4}{2 + \sqrt{2}}\right)^2$

$$= \frac{16}{(3 + 2\sqrt{2})} \times \frac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})}$$

$$= 16\,(3 - 2\sqrt{2})$$

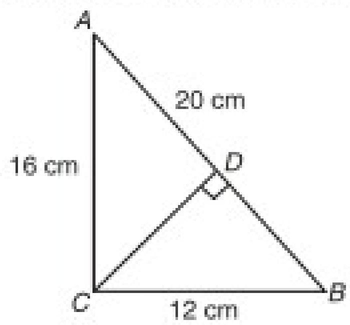
and $\quad$ Area of square = 16 cm

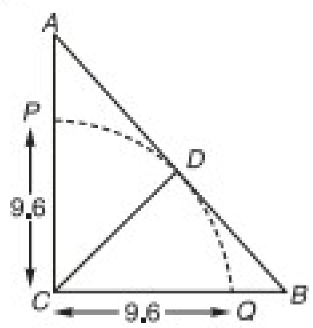
$\therefore \quad$ Total area of the figure $= 16 + 16\,(3 - 2\sqrt{2})$

$= 16\,(4 - 2\sqrt{2})$

$= 32\,(2 - \sqrt{2})$ cm$^2$

**28.** When $l = CD$, then the volume of cone will be maximum, where $l$ is the slant height of the cone and the largest possible angle at the vertex of cone is 90°.




$$CD = \frac{12 \times 16}{20} = 9.6 \text{ cm},$$

which is the radius of the sector.

Therefore, arc of the sector $= 2\pi \times 9.6 \times \dfrac{90}{360} = 4.8\pi$

Let the radius of the cone be $r$, then

$2\pi r$ = arc of the sector

$2\pi r = 4.8\pi$

$r = 2.4$

$\therefore$ Height of the cone $(h) = \sqrt{l^2 - r^2}$

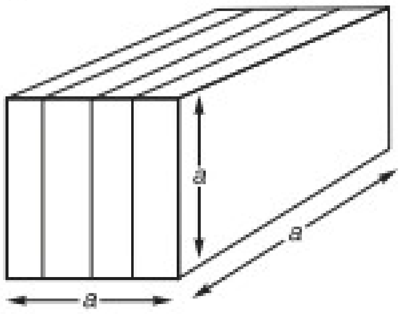
$$= \sqrt{(9.6)^2 - (2.4)^2} = 2.4\sqrt{15} \text{ cm}$$

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 2.4\sqrt{15}$$

$$= 56.1 \text{ cm}^3$$

**29.** To increase the value (or price of diamond) they should cut (divide) the diamond in such a way that the surface area will be maximum.




Thus, when four parts are parallel to each other.

In this way total surface area

$$= 6a^2 + 2a^2 + 2a^2 + 2a^2 = 12a^2$$

Actual surface area of cubical diamond $= 6a^2$

Therefore, percentage increase in area

$$= \frac{12a^2 - 6a^2}{6a^2} \times 100 = 100\%$$

Remember that for the given volume, minimum surface area is possessed by a cube. So to maximize the area we have to increase the maximum possible difference between the edges of cuboid.

**30.** Side of square I $= a$

Side of square II $= \frac{a}{\sqrt{2}}$

Side of square III $= \frac{a}{2}$

Side of square IV $= \frac{a}{2\sqrt{2}}$

Side of square V $= \frac{a}{4}$

Therefore, sum of perimeters of all the squares

$$= 4\left(a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \frac{a}{2\sqrt{2}} + \frac{a}{4}\right)$$

$$= 4a\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \frac{1}{4}\right)$$

$$= 4a\left(\frac{4 + 2\sqrt{2} + 2 + \sqrt{2} + 1}{4}\right)$$

$$= a(7 + 3\sqrt{2})$$

**31.** Total area of the five squares

$$= a^2 + \left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2\sqrt{2}}\right)^2 + \left(\frac{a}{4}\right)^2$$

$$= a^2\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right]$$

$$= a^2\left[\frac{16 + 8 + 4 + 2 + 1}{16}\right]$$

$$= a^2 \times \frac{31}{16} = \frac{31a^2}{16}$$

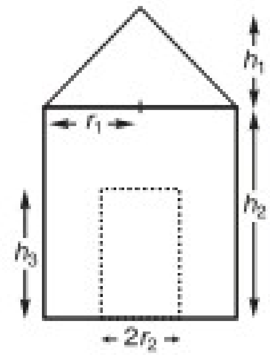
**32.** $(n-2)^3$

**33.** $6(n-2)^2$

**34.** $12(n-2)$

**35.** These are the 8 cubes at the corners, which is always fix.

**36.** Volume of the whole body




$$V_1 = \frac{1}{3}\pi r_1^2 h_1 + \pi r_1^2 h_2$$

but $\quad \frac{h_1}{h_2} = \frac{2}{3}$

$\therefore \quad V_1 = \pi r_1^2 \frac{11h_1}{6}$

and $\quad h_3 = \frac{2}{3}(h_1 + h_2) = \frac{5h_1}{3}$

Hence, volume of the hole $(V_2) = \pi r_2^2 h_3$

$$= \frac{5}{3}\pi r_2^2 h_1$$

But it is given that $V_2 = \frac{V_1 - V_2}{3}$

$\therefore \quad V_1 = 4V_2$

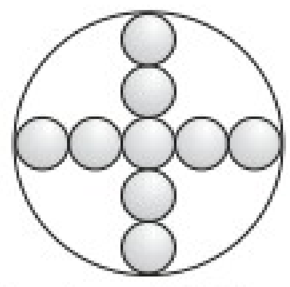
$\Rightarrow \quad 4 \times \frac{5}{3}\pi r_2^2 h_1 = \pi r_1^2 \times \frac{11}{6} h_1$

$\Rightarrow \quad r_2 = \sqrt{\frac{55}{8}}$ cm

**37.** $19 \times 19 = 361$

Thus, we make equal 19 measurements each of 19°, then we get $(361 - 360) = 1°$ angle at the centre. Thus, moving continuously in the similar fashion, we can get all the 360° angle *i.e.*, 360 equal sectors of 1°.

**38.** When we open the paper after cutting it, we will find it as shown in the following figure.



Radius of the larger circle = 5 cm

$\therefore$ Area of larger circle $= 25\pi$

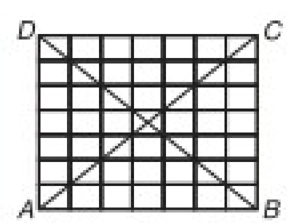
and the radius of each smaller circle is 1 cm.

Therefore, total area of all the 9 circles $= 9 \times \pi \times (1)^2 = 9\pi$

$\therefore$ Remaining area $= (25 - 9)\pi = 16\pi$

Hence, the required ratio = 25 : 16

**39.** In the above layer we can see that total 13 cubes get a cut. So, in 7 layers total $13 \times 7 = 91$ cubes will get a cut and the remaining $(7^3 - 91) = 252$ cubes are without any cut.




Total number of pieces which are not a cube

$$= 12 \times 2 \times 7 + 4 \times 7 = 196$$

(Since 84 cubes are diagonally cut into two parts and 7 cubes which are in the centre are divided into 4 parts.)
Thus, total 196 children will get one-one piece and 252 adults get one-one piece.
Thus total 252 + 196 = 448 people can get a piece of cake.

**NOTE** It is clear that everyone get equal number of pieces but not according to the volume of pieces.

✍ ***Solutions for questions number 40 to 42:*** *Diameter* ($2R$) *of the outermost circle is equal to the diagonal of larger square.*

*Hence, the side of square* $= \frac{2R}{\sqrt{2}} = \sqrt{2}R$

*Again the side of larger square is equal to the diameter of middle most circle.*

*Hence, the radius of mid-circle is* $\frac{R}{\sqrt{2}}$.

*Once again the diameter of the mid-circle is equal to the diagonal of smaller square. Hence, side of the smaller square* $= R$. *Similarly the diameter of innermost circle is equal to the side of the smaller square. Hence, radius of the innermost circle* $= \frac{R}{2}$.

**40.** $\frac{R}{2}$

**41.** Area of larger square $= (\sqrt{2}R)^2 = 2R^2$

and area of smaller square $= R^2$

$\therefore$ Total area of both squares $= 3R^2$

**42.** Sum of all the circumferences $= 2\pi\left(R + \frac{R}{\sqrt{2}} + \frac{R}{2}\right)$

$$= 2\pi R\left(\frac{2 + \sqrt{2} + 1}{2}\right)$$

$$= (3 + \sqrt{2})\pi R$$

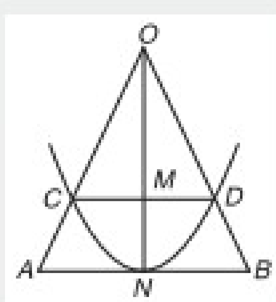
Sum of perimeters of all the squares $= 4(\sqrt{2}R + R)$

$$= 4R(\sqrt{2} + 1)$$

$\therefore$ Required ratio $= \frac{(3 + \sqrt{2})\pi R}{(\sqrt{2} + 1)4R} = \frac{(3 + \sqrt{2})\pi}{(\sqrt{2} + 1)4}$

**Alternatively:** Since circumference and perimeter both has $R$. So $R$ will be cancelled in the ratio of circumference to the perimeter. Thus, neither of the choices $a$, $b$ and $c$ are admissible. Hence, the only correct choice is (d).

✍ ***Solutions for questions number 43 and 44:*** *Each side of outer (larger) hexagon is equal to the radius of circle which is R.*




*Now,* $OC = ON = OD$

*radii of the inner (smaller) circle*

*But* $\frac{ON}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $ON = \frac{\sqrt{3}}{2}OA = \frac{\sqrt{3}}{2}R$, *radius of the inner circle*

*and this is also equal to the side of the inner hexagon.*

**43.** Sum of perimeters of both the hexagons $= 6R + 6 \times \frac{\sqrt{3}}{2}R$

$$= 6R\left(1 + \frac{\sqrt{3}}{2}\right) = 3(2 + \sqrt{3})R$$

**44.** $\frac{\text{Area of inner circle}}{\text{Area of outer circle}} = \frac{\pi}{\pi}\frac{\left[\left(\frac{\sqrt{3}}{2}\right)R\right]^2}{(R)^2} = \frac{3}{4}$

**45.** Radius of the first hexagon $= R$

Radius of the second hexagon $= \frac{\sqrt{3}}{2}R$

Radius of the third hexagon $= \frac{3}{4}R$

Radius of the fourth hexagon $= \frac{3\sqrt{3}}{8}R$

$\therefore$ Required ratio $= \frac{R}{\frac{3\sqrt{3}}{8}R} = \frac{8}{3\sqrt{3}}$

**46.** From the concept of similarity of triangles. All the five quadrilaterals viz., $AOA'$, $BOB'$, $COC'$, $DOD'$ and $EOE'$ are similar.

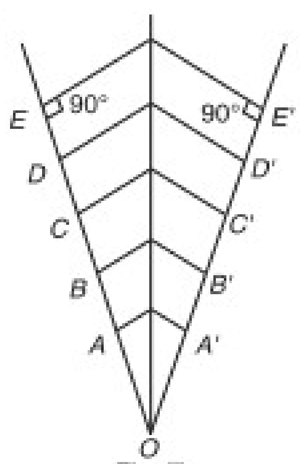


Fig. (i)

From the figure (ii)

$$\frac{r_2 - r_1}{r_2 + r_1} = \frac{r_3 - r_2}{r_3 + r_2} = \frac{r_4 - r_3}{r_4 + r_3} = \frac{r_5 - r_4}{r_5 + r_4} = K$$

$$\Rightarrow \frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3} = \frac{r_5}{r_4} = K$$

(By componendo and dividendo)

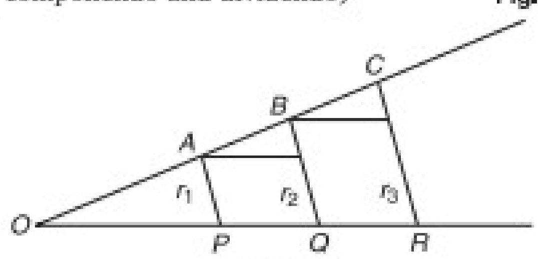


Fig. (ii)

It means all the radii are in GP

Therefore, $\frac{r_5}{r_1} = (K)^4 = \frac{81}{16} = \left(\frac{3}{2}\right)^4$

$\Rightarrow$ $K = \frac{3}{2}$ $\therefore$ $r_3 = r_1(K)^2$

**46.** $r_3 = r_1 \times \frac{9}{4} = \frac{9r_1}{4} = \frac{9}{4} \times 16 = 36$ cm

**47.** $\because$ $r_1 = 16$, $r_2 = 24$, $r_3 = 36$, ... etc.

$\therefore$ $\frac{OP}{AP} = \frac{OQ}{BQ}$

$$\frac{h + r_1}{r_1} = \frac{h + 2r_1 + r_2}{r_2}$$

$$\frac{h + 16}{16} = \frac{h + 56}{24} \Rightarrow h = 64 \text{ cm}$$

**48.**
$$\begin{aligned} 60 &= 1\times1\times60\\ &= 1\times2\times30\\ &= 1\times3\times20\\ &= 1\times4\times15\\ &= 1\times5\times12\\ &= 1\times6\times10\\ &= 2\times2\times15\\ &= 2\times3\times10\\ &\dots\ \dots\ \dots\\ &\dots\ \dots\ \dots\\ &= 3\times4\times5 \end{aligned}$$

Out of the given different combinations the first combination $(1\times1\times60)$ gives maximum length of diagonal of cuboid, but in this case two of the edges are same. So, the second combination gives the proper value *i.e.*, which gives the maximum length of diagonal whose all sides are different. Hence, the length of such a pencil is equal to the diagonal of cuboid

$$= \sqrt{1^2+2^2+30^2} = \sqrt{905}$$

**49.**

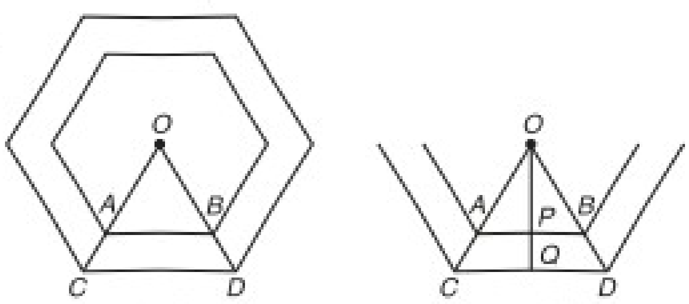



Fig. (i) Fig. (ii)

In figure (ii)

$$OP = \frac{\sqrt{3}}{2}OA = 4\sqrt{3}\text{ cm}$$

Again $\quad \dfrac{OP}{OQ} = \dfrac{OA}{OC}$

$$\frac{4\sqrt{3}}{6\sqrt{3}} = \frac{8}{OC} \qquad (OQ = OP + PQ = 4\sqrt{3} + 2\sqrt{3})$$

$\Rightarrow \quad OC = 12$ cm

$\therefore$ Each side of the outer hexagon is 12 cm.

$\therefore$ Required area = (Area of outer hexagon − Area of inner hexagon)

$$= \frac{3\sqrt{3}}{2}[12^2 - 8^2] = 120\sqrt{3}\text{ cm}^2$$

**50.** Area of region $x$ = Area of square − Area of inscribed circle $= (4-\pi)$

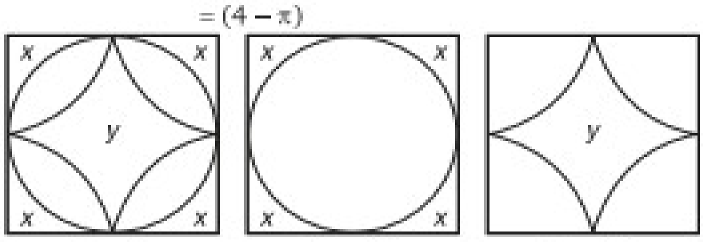



Fig. (i) Fig. (ii) Fig. (iii)

Area of region $y$ = Area of square − 4 (area of quadrant)

$$= 4 - 4\left(\frac{1}{4}\pi\times(1)^2\right) = (4-\pi)$$

$\therefore$ Required area (of shaded region)

= Area of square − [Area of region $x$ + Area of region $y$]

$= 4 - [4-\pi+4-\pi] = 2\pi - 4$

**51.** Let the volume of solid block be $V$ and radius of the spheres formed from the first block be $r_1$, then the volume of each sphere be $V_1$.

Similarly, let the radius of each sphere obtained from second block be $r_2\ (=2r_1)$, then the volume of each sphere be

$$V_2 = (8V_1)$$

$\therefore \quad V = kV_1 + 14 \quad \dots$(i)

and $\quad V = lV_2 + 36$

or $\quad V = 8lV_1 + 36 \quad \dots$(ii)

From equation (i) and (ii)

$$kV_1 + 14 = 8lV_1 + 36$$

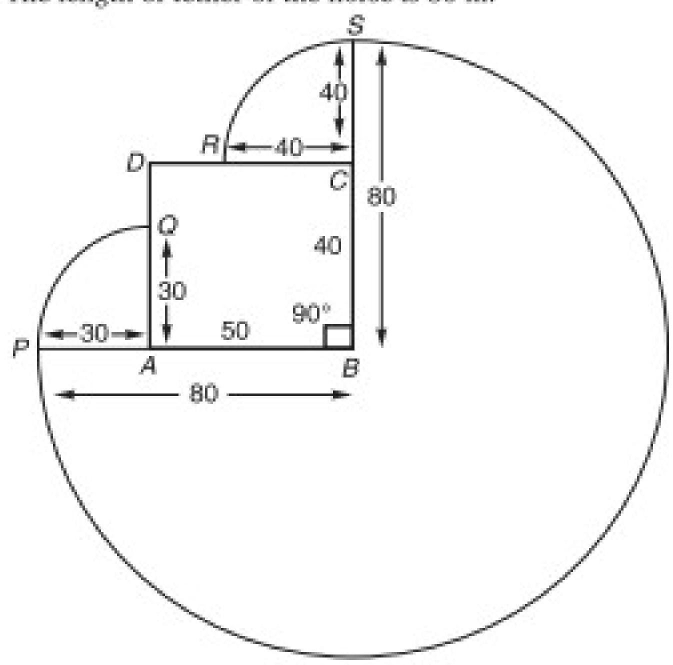
$\Rightarrow \quad V_1(k - 8l) = 22$

The possible value of $V_1 = 22, 11, 2$ or 1

But $V_1$ can never be equal to or less than 14 (since remainder is always less than divisior) So, the only possible value of $V_1 = 22$

$\therefore \quad V_2 = 8\times V_1 = 176\text{ cm}^3$

**52.** The length of tether of the horse is 80 m.




Area grazed by horse

$$= \left[\pi\times(80)^2\times\frac{270}{360} + \pi\times(30)^2\times\frac{90}{360} + \pi\times(40)^2\times\frac{90}{360}\right]$$

$$= \pi\left(6400\times\frac{3}{4} + 900\times\frac{1}{4} + 1600\times\frac{1}{4}\right) = \pi\left[\frac{21700}{4}\right]$$

$$= 5425\pi\text{ m}^2$$

**HINT** When horse is tethered at $B$, then he can move freely 270° *i.e.*, from $P$ to $S$ with the full length of his tether as the radius of the arc $PS$. Again when horse reaches $P$ and tries to move further in the right direction *i.e.*, towards $D$, his tether gets fixed at $A$ and now only 30 m tether is free which works as a radius of the quadrant $PAQ$. Similarly when horse reaches $S$ and tries to move left further in the direction of $D$, due to wall $BC$ his tether gets fixed at $C$ and now only 40 m tether is free to move further. At this moment point $C$ behaves like a fixed point of tether (as centre of quadrant RSC).

**NOTE** You can tether horse at any of the four corners $A$, $B$, $C$ and $D$ area grazed by horse remains same.

**53.** Here each side is broken up into 6 parts *i. e.*, $n = 6$

Now, $N_0 = (n-2)^3 = (4)^3 = 64$

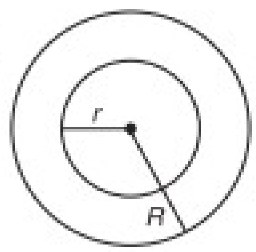
$N_1 = 6(n-2)^2 = 6 \times (4)^2 = 96$

$N_2 = 12(n-2) = 12(4) = 48$

$\therefore$ $N_0 : N_1 : N_2 = 64 : 96 : 98$

$= 4 : 6 : 3$

**54.** Let the radius of seed be $r$ and radius of the whole fruit (pulp + seed) be $R$, then thickness of the pulp $= (R - r)$




Volume of mango fruit $= \frac{4}{3}\pi R^3$

and Volume of pulp $= \frac{4}{3}\pi(R^3 - r^3)$

but $= \frac{4}{3}\pi\left[R^3 - \left(\frac{2}{7}R\right)^3\right]$

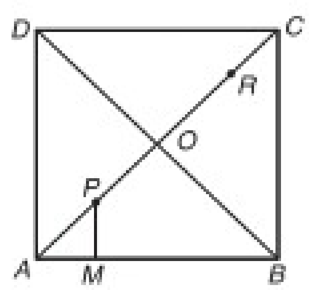
$\left[\because \frac{r}{R-r} = \frac{2}{5} \Rightarrow r = \frac{2}{7}R\right]$

$\therefore$ Percentage of volume of pulp to the total volume of fruit

$$= \frac{\frac{4}{3}\pi R^3\left[1 - \frac{8}{343}\right]}{\frac{4}{3}\pi R^3}$$

$$= \frac{335}{343} \times 100 = 97.66\%$$

**55.** Let the radius of each smaller circle is $r$ and radius of the larger circle is $R$, then




$\pi R^2 = 4\pi r^2 \Rightarrow R = 2r$

$OR = OP = R + r = 3r$

Also $PM = r$

($PM$ is the perpendicular on $AB$)

$\therefore$ $AP = \sqrt{2}r$

$\therefore$ $AO = AP + PO$

$= r\sqrt{2} + 3r = r(3 + \sqrt{2})$

$\therefore AC = 2AO = 2r(3 + \sqrt{2})$, which is the diagonal of square

$\therefore$ Required ratio $= \frac{2r(3+\sqrt{2})}{\sqrt{2}r} = (2 + 3\sqrt{2})$

**56.** Initial radius = 14 cm

Radius at a time when the balloon explodes = 35 cm

Change in volume $= \frac{4}{3}\pi[(35)^3 - (14)^3]$

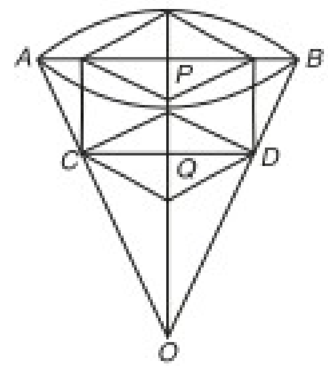
$= \frac{4}{3}\pi(7)^3[125 - 8]$

$= \frac{4}{3}\pi \times 343 \times 117$

Required time to explode $= \frac{\frac{4}{3}\pi \times 343 \times 117}{156}$

$= 1078$ s

**57.** Let the each side of cube be $a$, then




$CD = \sqrt{2}a$

$\therefore$ $CQ = \frac{a}{\sqrt{2}}$

Let the radius of cone be $r$ and height be $h$, then

$r = h\sqrt{2}$

$\therefore$ In $\Delta APO$ and $\Delta CQO$ (Similar triangles)

$$\frac{AP}{PO} = \frac{CQ}{OQ} = \frac{r}{h} = \frac{\frac{a}{\sqrt{2}}}{(h-a)}$$

$\Rightarrow$ $\frac{\frac{a}{\sqrt{2}}}{(h-a)} = \sqrt{2}$

$\Rightarrow$ $a = 2(h-a)$

$\Rightarrow$ $h = \frac{3a}{2}$

$\therefore$ $r = \frac{3a}{2} \times \sqrt{2}$

and $h = \frac{3a}{2}$

$\therefore$ Volume of cone $= \frac{1}{3}\pi \times \left(\frac{3a\sqrt{2}}{2}\right)^2 \times \frac{3a}{2}$

$= \frac{9}{4}a^3\pi$

and Volume of cube $= a^3$

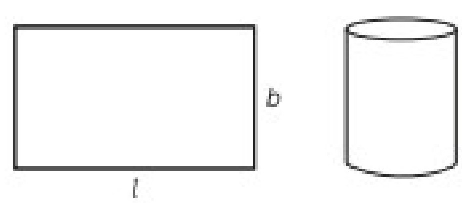
$\therefore$ Required ratio $= \frac{\frac{9}{4}\pi a^3}{a^3} = \frac{9}{4}\pi = 2.25\pi$

**58.** For the given volume, cube has minimum possible length of diagonal.

Therefore each side of cube = 4 cm

and its diagonal $= 4\sqrt{3}$ cm.

**59.** $l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$




where $r$ is the base radius of cylinder and $l$ is the length of paper and $h = b$, where $h$ is the height of cylinder and $b$ is the breadth of the paper.

$\therefore$ Volume of cylinder $= \pi r^2 h = \pi \times \left(\frac{l}{2\pi}\right)^2 \times b$

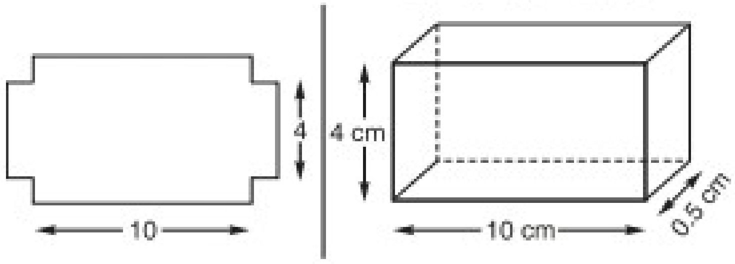
$\Rightarrow$ $\frac{\pi \times l^2 b}{4\pi^2} = 48.125 = \frac{385}{8}$

$\Rightarrow$ $l^2 b = 11 \times 11 \times 5$

$\Rightarrow$ $l = 11$ and $b = 5$ $(\because l > b)$

$\therefore$ Volume of the box $= l \times b \times 4$

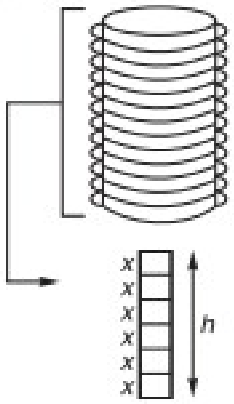
$= 10 \times 4 \times 0.5 = 20 \text{ cm}^3$



**60.** Vertical spacing between any two turns $= \dfrac{\text{Height of cone}}{\text{Number of turns}}$

$= \dfrac{h}{n}$

**61.** Number of turns $= \dfrac{h}{x}$



Length of string in each turn

$= 2\pi r = 2\pi \times \dfrac{4}{\pi} = 8$ cm

$\therefore$ Total length of string in all the $n$ turns

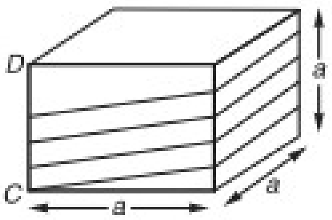
$= \dfrac{h}{x} \times 8 = \dfrac{8h}{x}$ cm

**62.** Total length of string $= 8n$ cm

Since, total length of string

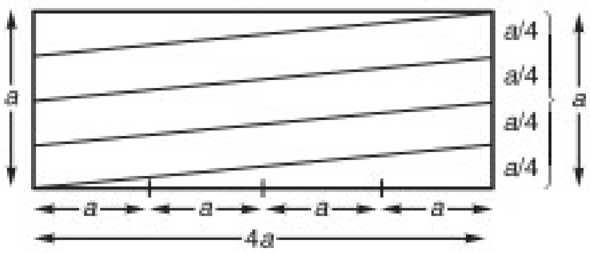
$=$ number of turns $\times$ perimeter of cylinder

$= 8 \times n = 8n$ cm



Length of string required for 1 turn (or round) $= \dfrac{8n}{4} = 2n$

but $\quad 2n = \sqrt{\left(\dfrac{a}{4}\right)^2 + (4a)^2}$



$\Rightarrow \quad a = \dfrac{8n}{\sqrt{257}}$

where $a$ is the side of cube.

**63.** From the sheet of 10 ft long, maximum $\dfrac{10}{2} = 5$ circular discs can be cut along the length of the iron sheet.

$CM = \sqrt{AC^2 - AM^2} = \sqrt{4x^2 - x^2}$

$CM = x\sqrt{3} = \sqrt{3}$ ft (Since $x = 1$ ft)

$\therefore$ Width of the sheet $= AK + MC + CT$

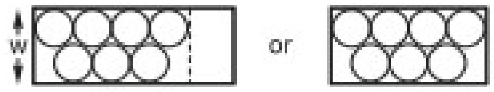


Fig (i) Fig (ii)

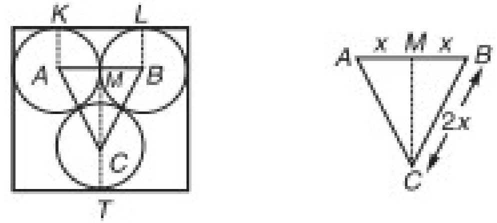


Fig. (iii) Fig. (iv)

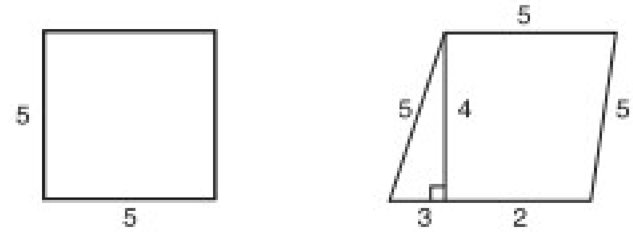
$= 1 + \sqrt{3} + 1$

$= (2 + \sqrt{3})$ ft

**64.** Recall that for given perimeter the polygon of minimum number of sides has minimum area and the polygon of maximum number of sides has maximum area. So, the correct relation is $h > s > r$.

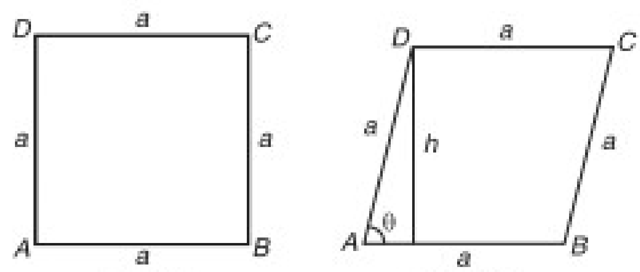
Thus, hexagon (6 sides) has maximum area.

Now, between square and rhombus, square has greater area than rhombus. For easier understanding consider some values.



Area $= 25 \text{ cm}^2$ Area $=$ base $\times$ height $= 5 \times 4 = 20 \text{ cm}^2$

**Alternatively:**



Square Rhombus

**In rhombus *ABCD*:** Area of rhombus $= a \times h$

$= a \times a \sin\theta \qquad \left(\dfrac{h}{a} = \sin\theta\right)$

$= a^2 \sin\theta$

As you know the maximum value of $\sin\theta$ is 1 at $\theta = 90°$ but at $\theta = 90°$ rhombus will become a square. So except $\theta = 90°$ for all the rest values the area of rhombus will be less than the area of square.

**65.**

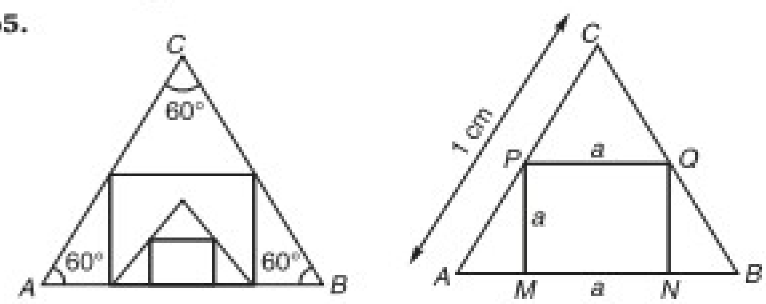


Fig. (i) Fig. (ii)

$PCQ$ is also an equilateral triangle

$\therefore \quad PC = PQ = PM = a$

$\therefore \quad \frac{a}{PA} = \frac{\sqrt{3}}{2}$

$\therefore \quad PA = \frac{2a}{\sqrt{3}}$

$\therefore \quad AC = AP + PC = \frac{2a}{\sqrt{3}} + a = 1 \text{ cm}$

$\Rightarrow \quad a = \frac{\sqrt{3}}{(2+\sqrt{3})} = \sqrt{3}\,(2-\sqrt{3})$

Now, in figure (iii)

$PM = MT = a$

Let the each side of square $RSYX$ be $K$, then $RT = K$ also (since $RTS$ is an equilateral triangle)

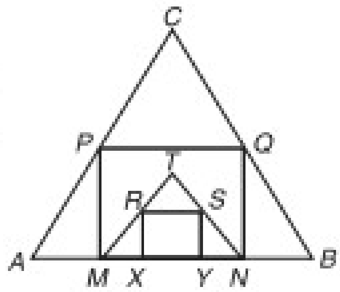


Fig. (iii)

$\therefore \quad \frac{K}{RM} = \frac{\sqrt{3}}{2}$

$\therefore \quad RM = \frac{2K}{\sqrt{3}}$

$\therefore \quad MT = RT + RM = K + \frac{2K}{\sqrt{3}}$

$MT = \frac{(\sqrt{3}+2)}{\sqrt{3}} K$

but $\quad MT = a$

$\therefore \quad a = \left(\frac{\sqrt{3}+2}{\sqrt{3}}\right) K$

$\therefore \quad K = \frac{\sqrt{3}a}{(\sqrt{3}+2)}$

But $\quad a = \sqrt{3}\,(2-\sqrt{3})$

$\therefore \quad K = \frac{\sqrt{3}}{(\sqrt{3}+2)}[\sqrt{3}\,(2-\sqrt{3})]$

$K = \frac{3\,(2-\sqrt{3})}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

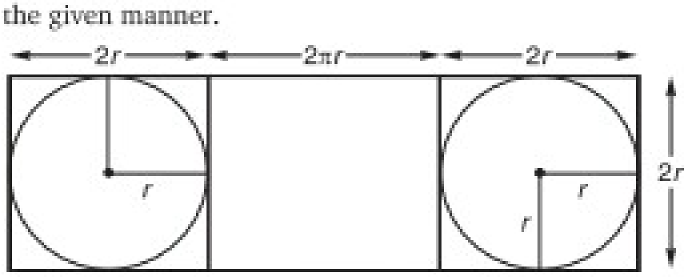
$\Rightarrow \quad K = \frac{3\,(2-\sqrt{3})^2}{1} = 3\,(7-4\sqrt{3})$

$\therefore$ Area of square $RSYX = K^2 = [3\,(7-4\sqrt{3})]^2$

$K^2 = 9\,(49 + 48 - 56\sqrt{3})$

$K^2 = (873 - 504\sqrt{3}) \text{ cm}^2$

**66.** For the minimum wastage of sheet he has to cut the sheet in the given manner.




Total area of sheet required

$(2\pi r + 4r) \times 2r = 4r^2\,(\pi + 2)$

Area of sheet utilised $= (2\pi r \times 2r) + 2\,(\pi r^2) = 6\pi r^2$

Area of wastage sheet $= 4r^2\,(\pi + 2) - 6\pi r^2$

$= 8r^2 - 2\pi r^2$

$\therefore \quad$ Required ratio $= \frac{8r^2 - 2\pi r^2}{6\pi r^2}$

$= \frac{2r^2\,(4-\pi)}{6r^2\pi} = \frac{1}{11}$

**67. Short cut:** Very quickly check the options. If all the options have values.

**Alternatively:** The required capacity of box is 864 $m^3$. Let the length of the base be $l$ and height of the box is '$h$', then

$864 = l^2 h \quad \Rightarrow \quad h = \frac{864}{l^2}$

Now, surface area of the box $A = l^2 + 4lh$

$= l^2 + \frac{4l \times 864}{l^2}$

$= l^2 + \frac{3456}{l}$

Differentiating w.r.t. to $l$, we get

$\frac{dA}{dl} = 2l - \frac{3456}{l^2}$

For the minimum area $= \frac{dA}{dl} = 0$

$\therefore \quad 2l = \frac{3456}{l^2}$

$\Rightarrow \quad l^3 = 1728$

$\Rightarrow \quad l = 12$

$\therefore \quad$ Base area $= (12)^2 = 144$

and $\quad$ Height $= \frac{864}{l^2} = 6$ m

**68.** Let the initial radius be $r$ and volume be $V$, then

$V = \pi r^2 \times 4$

**Ist case:** $V_1 = \pi\,(r+12)^2 \times 4$

**IInd case:** $V_2 = \pi r^2 \times (4 + 12)$

But $\quad V_1 = V + K$

and $\quad V_2 = V + K$

$\therefore \quad V_1 = V_2$

$\Rightarrow \quad \pi\,(r+12)^2 \times 4 = \pi r^2\,(16)$

$\Rightarrow \quad r = 12$ ft

$\therefore$ Increased volume $= V_1 = V_2$

$= \pi \times (24)^2 \times 4$

$= 2304\pi$ cubic ft