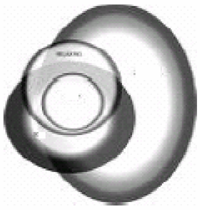
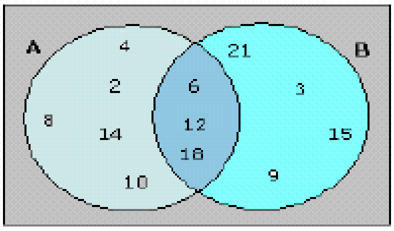
Venn Diagrams- Basics, Problems, Maxima and Minima
by Total Gadha - Tuesday, 18 September 2007, 04:49 PM



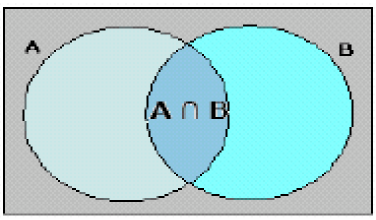
For all the CAT aspirants taking CAT in 2007 or 2008, this chapter should provide some insight into Venn diagrams and methods for solving the problems. This chapter comes on the demand of some high octane TG users who are responsible for my lack of sleep and excessive intake of caffeine last night. I hope this resolves many of their problems in Venn diagrams.

Venn diagrams are pictorial representations used to display mathematical or logical relationships between two or more given sets (groups of things). The drawing consists of two or more circles, each representing a specific group. Each Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Any values that belong to more than one set will be placed in the sections where the circles overlap. A typical venn diagram is shown in the figure below:



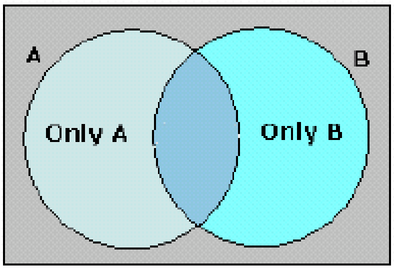
In the figure, set A contains the multiples of 2 which are less than 30 and set B contains multiples of 3 which are less than 25. Therefore, A = {2, 4, 6, 8, 10, 12 â€¦ 26, 28} and B = {3, 6, 9, 12 â€¦ 21, 24}. The various areas in the above diagram depict the following relationships:

ℜ• Intersection (Aâˆ©B)- Denotes the set of elements that are shared by two or more given sets. In the figure
given below, the intersection of the two sets is shown.



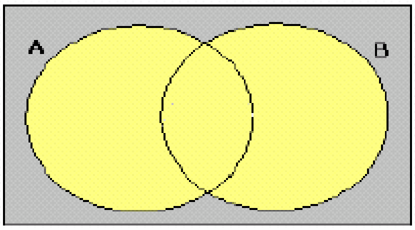
A âˆ© B = {6, 12, 18, 24}

ℜ• â€˜Only Aâ€™ or â€˜Only Bâ€™- The part of set A, or set B, which is not shared by any other set is known as â€˜only A,â€™ or â€˜only B.â€™ In the figure given below, the two parts are shown:



Only A = {2, 4, 8, 10, 14, 16, 20, 22, 26, 28}, only B = {3, 9, 15, 21}

ℜ• Union (AUB)- Denotes all the elements of the given sets taken once.

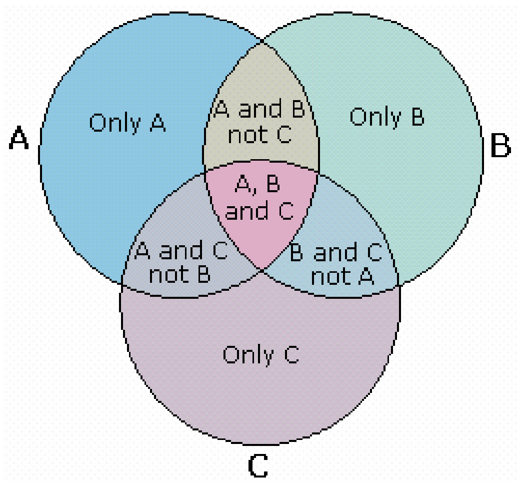


A U B = {2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 26, 28}

It can be seen that

$$\boxed{n(AUB) = n(A) + n(B) - n(A \cap B)}$$

The venn diagram for three sets is shown below:



It can be shown that

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Problem-solving through Venn diagrams:

I use the following method to solve problems through Venn diagrams:

Fill the areas with their respective values. If one or more values are missing, try to assu least number of variables such that the empty areas can be filled.

Solved Examples:

Of all the users on Totalgadha.com, 80% spend time in CAT Quant-DI forum whereas 60% spend time in CAT verbal forum. If only those users will crack CAT who spend time in both the forums, what percentage of users of TotalGadha

ℜ• will crack CAT?
ℜ• will not crack CAT?

Answer: n(AUB) = n(A) + n(B) – n(Aâˆ©B) ℘ 100% = 80% + 60% – n(Aâˆ©B) ℘ n(Aâˆ©B) = 40%
Therefore, 40% users of TG will crack CAT. And 60% of users (only A + only B) will not crack CAT.

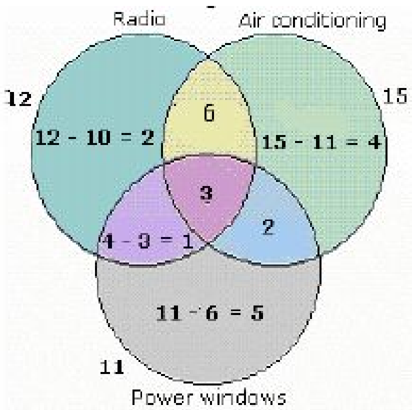
NOTE: See that the surplus (superfluous part) can only be adjusted inside the area denoted for the intersection of the sets, a fact we will use in maxima- minima type of questions.

A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options â€” air conditioning, radio and power windows â€” were already installed. The survey found:

15 had air conditioning
2 had air conditioning and power windows but no radios
12 had radio
6 had air conditioning and radio but no power windows
11 had power windows
4 had radio and power windows
3 had all three options.

What is the number of cars that had none of the options? (CAT 2003)
1. 4 2. 3 3. 1 4. 2

Answer: We make the Venn diagram and start filling the areas as shown:



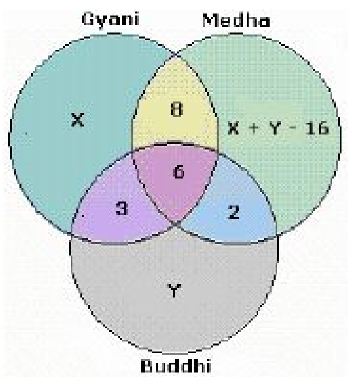
Total Number of cars according to the diagram = 2 + 6 + 3 + 1 + 5 + 2 + 4 = 23.
Therefore, number of cars having none of the given options = 25 – 23 = 2.

New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved. (CAT 2003- Leaked)

What is the number of projects in which Gyani alone is involved?
1. 0
2. 1.
3. 4.
4. cannot be determined

What is the number of projects in which Medha alone is involved?
1. 0
2. 1.
3. 4.
4. cannot be determined

Answer: The Venn diagram for the three consultants is shown below:

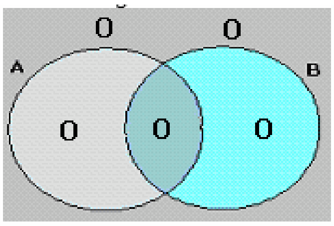


Total Number of projects = 2 ℘ number of projects in which more than one consultant is involved – 1 = 2 ℘ 19 – 1 = 37.
Therefore, X + 8 + 6 + 3 + Y + 2 + X + Y – 16 = 37 ℘ X + Y = 17. The values of X or Y cannot be uniquely determined. Medha alone is involved in X + Y – 16 = 17 – 16 = 1 project.

Concept of Maxima and Minima:

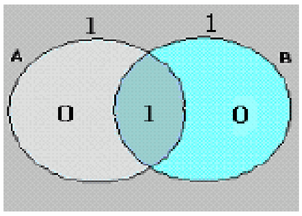
1. When the total number of elements is fixed

Letâ€™s have a look at the Venn diagram of two sets again:



Imagine that in the beginning, the number of elements in all the areas is zero, as shown above. All the sets are empty right now.

Letâ€™s see what happens if I insert one element inside Aâˆ©B:



We can see that adding 1 element to Aâˆ©B increases the number of elements in both A and B by 1. The total number of elements in all areas combined is 1 only (0 + 1 + 0) but if you add the number of elements in A and B (A + B), the addition will come up to 2. Therefore, adding 1 element to Aâˆ©B gives an extra 1 element. Hence, for every surplus of 1 element we can add 1 element to Aâˆ©B.

Letâ€™s see the Venn diagram for 3 sets:

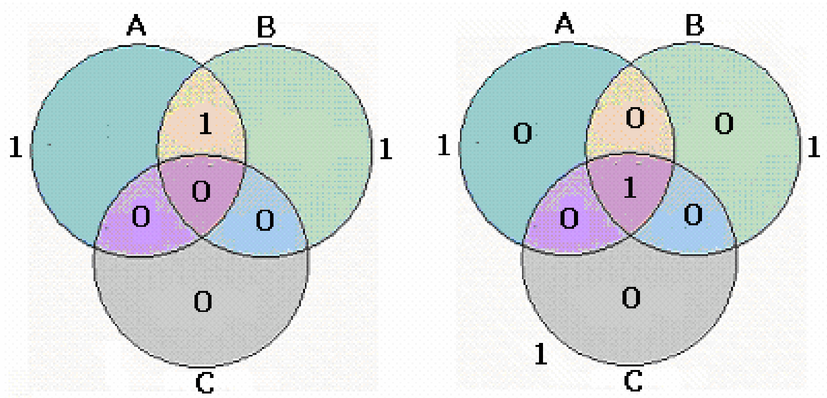


Diagram 1 Diagram 2

In diagram 1, we have added 1 element to intersection of only two sets (A and B but not C). We can see that A and B both increase by 1 and therefore we get a surplus of 1 element.

In diagram 2, we have added 1 element to intersection of all the three sets (A and B and C). We can see that A, B and C all three increase by 1 element each and therefore we get a surplus of 2 elements.

Therefore, in case of three sets, we can accommodate the surplus by

ℜ• adding elements to intersection of only two sets in which case a surplus of 1 element can be accommodated by increase of 1 element in the intersection of only two sets.

ℜ• adding elements to intersection of three sets in which case a surplus of 2 elements can be accommodated by increase of 1 element in the intersection of three sets.

How is this related to maxima and minima?

Letâ€™s see:

According to a survey, at least 70% of people like apples, at least 75% like bananas and at least 80% like cherries. What is the minimum percentage of people who like all three?

Answer: Letâ€™s first calculate the surplus:

percentage of people who like apples + percentage of people who like bananas + percentage of people who like cherries = 70% + 75% + 80% = 225% ℘ a surplus of 125%.

Now this surplus can be accommodated by adding elements to either intersection of only two sets or to intersection of only three sets. As the intersection of only two sets can accommodate only a surplus of 100%, the surplus of 25% will still be left. This surplus of 25% can be accommodated by adding elements to intersection of three sets. For that we have to take 25% out of the intersection of only two sets and add it to intersection of three sets. Therefore, the minimum percentage of people who like all three = 25%

The question can be solved mathematically also. Let the elements added to intersection of only two sets and intersection of three sets be x and y, respectively. These elements will have to cover the surplus.

—>x + 2y = 125%, where x + y ℜ≤ 100%. For minimum value of y, we need maximum value of x.
—> x = 75%, y = 25%.

In a college, where every student follows at least one of the three activities- drama, sports, or arts- 65% follow drama, 86% follow sports, and 57% follow arts. What can be the maximum and minimum percentage of students who follow

ℜ• all three activities

ℜ• exactly two activities

Answer: Let us again see the surplus:

Percentage of students who follow drama + Percentage of students who follow sports + Percentage of students who follow arts = 65% + 86% + 57% = 208% ℘□ surplus = 108%. This surplus can be accommodated through adding elements either to intersection of only two sets or to intersection of only three sets. As the intersection of only two sets can accommodate only a surplus of 100%, the surplus of 8% will still be left. This surplus of 8% can be accommodated by adding elements to intersection of three sets. For that we have to take 8% out of the intersection of only two sets and add it to intersection of three sets. Therefore, the minimum percentage of people who like all three = 8%. In this case the percentage of students who follow exactly two activities will be maximum = 92%.

The surplus of 108% can also be accommodated through adding elements to only intersection of three sets. As adding 1 element to intersection of three sets give a surplus of 2 sets, adding 54% to intersection of three sets will give a surplus of 108%. Therefore, the maximum value of students who follow all three activities is 54%. In this case the percentage of students who follow exactly two activities will be minimum = 0%.

We can also solve it mathematically ℘□ x + 2y = 108%, where x + y ℜ≤ 100%. The maximum value of x will give minimum value of y, whereas minimum value of x will give maximum value of y.

2. When the total number of elements is NOT fixed

In this case we assign the variables to every area of the Venn diagram and form the conditions keeping two things in mind:

ℜ• try to express the areas in the Venn diagram through least number of variables.

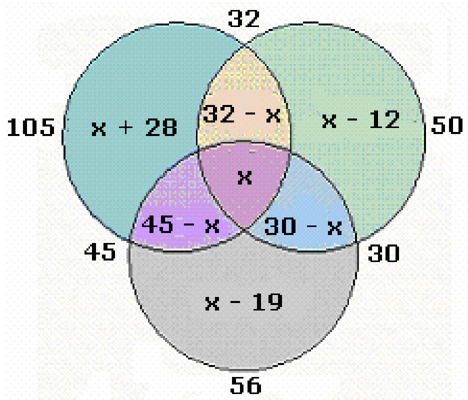
ℜ• all the numbers will be zero or positive. No number can be negative.

Out of 210 interviews of IIM- Ahmedabad, 105 CAT crackers were offered tea by the interview panel, 50 were offered biscuits, and 56 were offered toffees. 32 CAT crackers were offered tea and biscuits, 30 were offered biscuits and toffees, and 45 were offered toffees and tea. What is the

ℜ• maximum and minimum number of CAT crackers who were offered all three snacks?

ℜ• maximum and minimum number of CAT crackers who were offered at least one snack?

Answer: Letâ€™s make the Venn diagram for this question. Since we want to assume least number of variables, we can see that assuming a variable for the number of students who were offered all three snacks will help us express all the other areas. Let the number of students who were offered all three snacks = x.



In the above diagram, we have expressed all the areas in terms of x. To decide maximum value of x, we note that 32 – x, 45 – x and 30 – x will be zero or positive. Therefore, the maximum value of x will be 30. (30 is the lowest among 30, 32 and 45). To decide minimum value of x, we note than x – 19 and x â€“ 12 will be zero or positive. Therefore, x cannot be less than 19 (19 is the higher number between 19 and 12).

Therefore, maximum and minimum number of CAT crackers who were offered all three snacks = 30 and 19.

The number of CAT crackers who were offered at least one snack = Total number of CAT crackers in the Venn diagram = x + 28 + 32 – x + x + 45 – x + x – 19 + 30 – x + x – 12 = 104 + x.

As the maximum and minimum values of x are 30 and 19, respectively, the maximum and minimum value of 104 + x will be 134 and 123, respectively.

Maximum and minimum number of CAT crackers who were offered at least one snack = 134 and 123.