

Basic

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

Compound Angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Sum to Product

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\sin C - \sin D = 2 \sin \frac{C - D}{2} \cos \frac{C + D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}$$

Product to Sum

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

Multiple Angles

$$\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$= \cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

Sub-Multiple Angles

$$\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\begin{aligned} \cos \theta &= 2 \cos^2 \frac{\theta}{2} - 1 = 1 - 2 \sin^2 \frac{\theta}{2} \\ &= \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \end{aligned}$$

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

Misc

$$\sqrt{1 + \sin 2\theta} = |\sin \theta + \cos \theta|$$

$$\sqrt{1 - \sin 2\theta} = |\sin \theta - \cos \theta|$$

$$\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)$$

$$\cos^2 A - \sin^2 B = \cos(A + B) \cos(A - B)$$

$$\cos \theta + \sin \theta = \sqrt{2} \sin \left(\frac{\pi}{4} + \theta \right) = \sqrt{2} \cos \left(\frac{\pi}{4} - \theta \right)$$

$$\cos \theta - \sin \theta = \sqrt{2} \sin \left(\frac{\pi}{4} - \theta \right) = \sqrt{2} \cos \left(\frac{\pi}{4} + \theta \right)$$

$$\tan \left(\frac{\pi}{4} + \theta \right) = \frac{1 + \tan \theta}{1 - \tan \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$$

$$\tan \left(\frac{\pi}{4} - \theta \right) = \frac{1 - \tan \theta}{1 + \tan \theta} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$$