Greatest Integer Function and its Applications
by Total Gadha - Monday, 26 February 2007, 03:27 AM

The greatest integer function, denoted by [x], gives the greatest integer less than or equal to the given number x.

To put it simply, if the given number is an integer, then the greatest integer gives the number itself, otherwise it gives the first integer towards the left of the number of x on the number line.

For example,

[1.4] = 1

[4]= 4

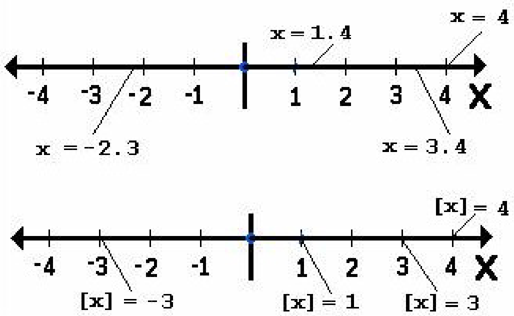
[3.4] = 3

[- 2.3] = - 3

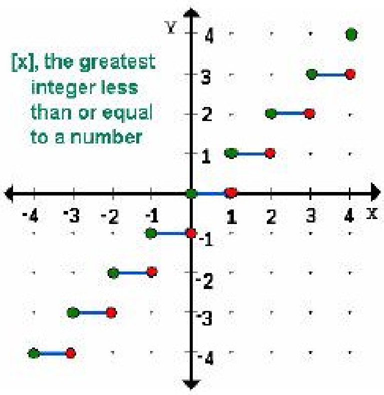
[- 5.6] = - 6, and so on.

NOTE: We can see that [1.4] = 1 + 0.4 or x = [x] + {x}, where {x} is the fractional part of x. For x = - 2.3, [x] = - 3 and {x} = 0.7

In the following figure, we can see that the greatest integer function gives the number itself (when the given number is an integer) or the first integer to the left of the number on the number line.



The graph of greatest integer function is given below. Note that the **red** dot indicates that integer value on the number line is not included while the **green** dot indicates that the integer value is included.



Examples:

- What is the value of $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\cdots+[\sqrt{49}]+[\sqrt{50}]$ where [x] denotes the greatest integer function?
  Answer: It can be seen that
  $[\sqrt{1}] = 1, [\sqrt{2}] = [1.41] = 1, [\sqrt{3}] = [1.73] = 1, [\sqrt{4}] = 2$ and so on..

  Therefore, from $[\sqrt{1}]$ to $[\sqrt{3}]$, the value will be 1, from $[\sqrt{4}]$ to $[\sqrt{8}]$ the value will be 2, from $[\sqrt{9}]$ to $[\sqrt{15}]$ the value will be 3 and so on..
  Therefore, the total value = 3 × 1 + 5 × 2 + 7 × 3 + ... + 13 × 6 + 2 × 7
  = 217.

- If $[\sqrt{x}] = 5$ and $[\sqrt{y}] = 6$, where x and y are natural numbers, what can be the greatest possible value of x + y?
  Answer: It is clear that $[\sqrt{25}] = 5, [\sqrt{26}] = 5, [\sqrt{27}] = 5$ and so on. The highest value of x that we can take is 35, since $[\sqrt{35}] = 5$ but $[\sqrt{36}] = 6$.
  Similarly, the highest value of y we can take is 48, since
  $[\sqrt{48}] = 6$ but $[\sqrt{49}] = 7$
  Therefore, the greatest value of x + y = 35 + 48 = 83.

- What is the value of x for which $x[x] = 28$?
Answer: If the value of x is 5, $x[x] = 25$, and if the value of x is 6 $x[x]=36$.
Therefore, the value of x lies between 5 and 6.
If x lies between 5 and 6, $[x] = 5$.
$\Rightarrow x = \frac{28}{[x]} = \frac{28}{5} = 5.6$

- Let x be number and [x] and {x} denote the greatest integer less than x and fractional part of x. If $[x]^3 + \{x\}^2 = -63.64$, then the value of x is
Answer: Since the value of $[x]^3 + \{x\}^2$ is negative, the value of [x] is negative. The cube lying near -63.64 is -64. Therefore, $[x]^3 = -64$ or $[x] = -4$. Therefore, $\{x\}^2 = 0.36$ or $\{x\} = 0.6$
Hence, $x = [x] + \{x\} = -4 + 0.6 = -3.4$