Time, Speed, Distance and Work- Solved Problems

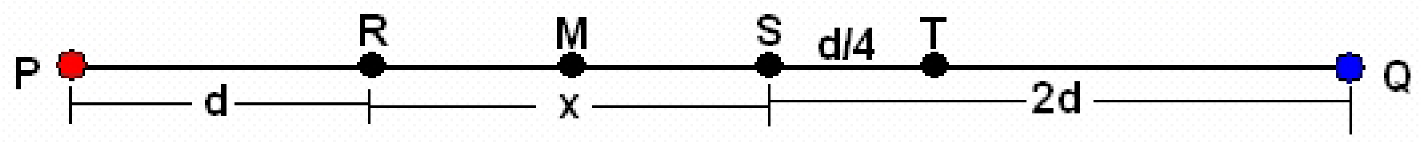
by Total Gadha - Saturday, 6 October 2007, 07:28 AM

I have been suffering from the writerâ€™s block lately and therefore I decided to do what I like to do the most- solving problems. To help all the CAT 2007/008 aspirants on Total Gadha, I decided to hunt TG quant forum for the time, speed and distance problems and solve them. What a treasure I discovered! Some of these problems will baffle brains of even some of the most experienced instructors. I am posting all the problems I solved along with their solutions. All of you please first try to solve the problems on your own and then see the solutions. This way, you would appreciate the solution to many of these problems. Also, notice that I again used only ratios to solve these problems. Still no equations.

Two cars A and B start simultaneously from two points P and Q with certain speeds towards each other. After reaching a point R, speed of A decreases by 1/3. It then meets B at a point S, where SQ = 2PR. If the speed of A had become 1/3 less at the mid point of RS, the cars would have met at T where ST = PR/4. Find RS: PR.

A. 5: 1 B. 6: 1 C. 8: 1 D. 10: 1

Answer: The figure is shown below. M is the midpoint of RS with RS = x. Let the distance PR = d, SQ = 2d and ST = d/4



A starts traveling with $\frac{2}{3}^{rd}$ of his usual speed after point R and then travels a distance of x. If he had traveled with his usual speed, he would have traveled a distance of $\frac{3x}{2}$ in the same time. Since the time of travel of both A and B is the same,

When A travels d + $\frac{3x}{2}$ B travels 2d. Therefore, ratio of speeds = $\frac{d+\frac{3x}{2}}{2d} = \frac{2d+3x}{4d}$.

In the second case, A starts traveling with $\frac{2}{3}^{rd}$ of his usual speed after point M and then travels a distance of $\frac{x}{2}+\frac{d}{4}$. If he had traveled with his usual speed, he would have traveled a distance of $\frac{3}{2}\left(\frac{x}{2}+\frac{d}{4}\right) = \frac{6x+3d}{8}$ in the same time. The total distance traveled = d +

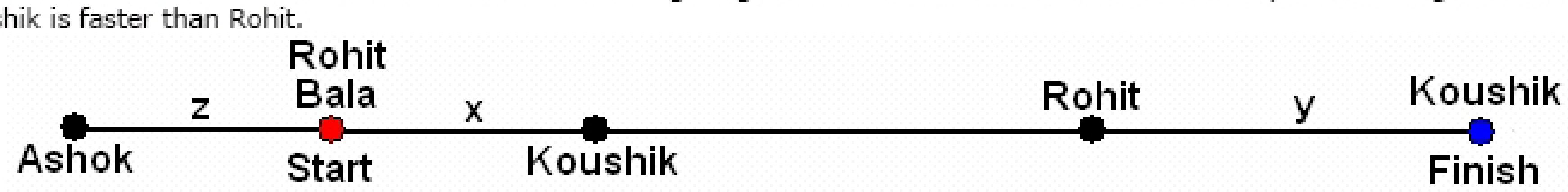
$d+\frac{x}{2}+\frac{6x+3d}{8} = \frac{10x+11d}{8}$ In this time, B travels a distance of $2d-\frac{d}{4}=\frac{7d}{4}$. Therefore, ratio of speeds = $\frac{10x+11d}{14d}$.

$\Rightarrow \frac{10x+11d}{14d} = \frac{2d+3x}{4d} \Rightarrow$ x = 8d. Therefore, RS: PR = 8: 1

Bala and Rohit start at the starting line of a 100m track, while Koushik is given a head start of 'x' m. Ashok starts from point which is 'z' m behind the starting line. Rohit is beaten by Koushik by 'y' m. In a 100m race, Ashok can beat Bala by 'y' m. y > x and z = (100y)/(100-y). If Ashok and Koushik finished the race together, who runs the least distance of the four?

A. Koushik B. Rohit C. Bala D. Cannot be determined.

Answer: The known situation has been summarized in the figure given below. Since Koushik beats Rohit by a distance greater than the lead (y > x), Koushik is faster than Rohit.



In a 100 m race Ashok can beat Bala by y m $\Rightarrow$ When Ashok travels 100 m, Bala travels 100 – y metres.
Therefore, in this race when Ashok travels 100 + z metres, Bala will travel
$\frac{100-y}{100}\times(100+z)$ metres = $\frac{100-y}{100}\times\left(100+\frac{100y}{100-y}\right) = \frac{100-y}{100}\times\left(\frac{10000}{100-y}\right) = 100$ metres. Therefore, Ashok and Bala finish together. And Koushik finishes with them.

Koushik travels 100 – x, Rohit travels 100 – y, Bala travels 100 and Ashok travels 100 + z. The least distance is traveled by Rohit.

"When I take my dog for a walk," said a friend. "He frequently supplies me with some interesting puzzle to solve. One day he waited to see which way I should go, and when I started he raced along to the end of the road, immediately returning to me again racing to the end of the road and again returning. He did this four times in all, at a uniform speed, and then ran at my side the remaining distance, which according to my paces measured 81 metres. I afterwards measured the distance from my door to the end of the road and found it to be 625 metres. I walk at the rate of 4 km per hour.

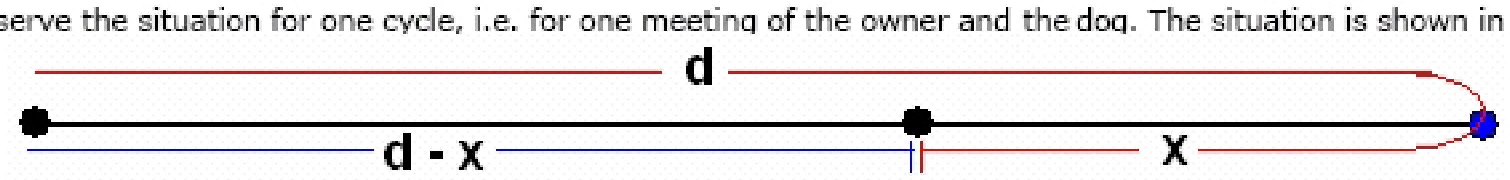
What is the speed of my dog when racing to and fro?

A. 48 kmph B. 4 kmph C. 16 kmph D. 8 kmph

The distance traveled by the dog till the time of the first reunion with me is equal to

A. 1000 m B. 850 m C. 1200 m D. 1250 m

Answer: Let's observe the situation for one cycle, i.e. for one meeting of the owner and the dog. The situation is shown in the image below:



Let the ratio of the speed of the owner to that of his dog be k. The dog travels the whole distance d and comes back a distance x to meet its owner. The owner meanwhile travels a distance equal to d – x. Therefore, the ratio of the speed = $\frac{d+x}{d-x} = k \Rightarrow x = \frac{d(k-1)}{k+1}$

For the second cycle, x would be the total distance. Therefore, the remaining distance after the second cycle $x_1 = \frac{x(k-1)}{k+1} = d\left(\frac{k-1}{k+1}\right)^2$

The remaining distance after the 3rd cycle = $x_2 = d\left(\frac{k-1}{k+1}\right)^3$

The remaining distance after the 4th cycle = $x_3 = d\left(\frac{k-1}{k+1}\right)^4$

Now, d = 625, and the remaining distance after the 4^{th} cycle = 81.

Therefore, $625\left(\frac{k-1}{k+1}\right)^4 = 81 \Rightarrow k = 4$. Therefore, the ratio of the speeds = 4: 1

$\Rightarrow$ The speed of the dog = 4 × 4 = 16 kmph

The total distance traveled by the dog in the first cycle = d + x = $d + \frac{d(k-1)}{k+1} = \frac{2dk}{k+1} = \frac{2 \times 625 \times 4}{5} = 1000$ m

Mr. Sharma starts from his house to office 30 min late then his usual time. So, he increases his speed by 25% over his usual speed but still reaches 15 min late. Another day he starts 15 min later then his usual time and increases his speed by 50% comparatively to the previous day speed and reaches 20 min before the office time. What is the distance between Mr. Sharma's house and office?

A. 10Km B. 15 Km C. 5 Km D. 20Km E. 25 Km

Answer: Let me Sharma take time t to reach his office after getting 30 min late. This time will be his normal time he takes daily to reach his office. Since he increases his speed by 25% ($\frac{1}{4}$) his traveling speed becomes $\frac{5}{4}^{th}$ of his usual speed. Therefore, the time taken becomes $\frac{4}{5}t$.

Therefore, time saved = $t - \frac{4}{5}t = \frac{t}{5} = 15 \text{ min} \Rightarrow t = 75$ min

The second information is redundant as it will give the same result of usual time taken = 75 min. I don't think distance can be calculated. Does anyone have the answer?

Angle between hour and minute hands is exactly 1 degree. The time is an integral number n of minutes after noon (0< n <720).Total possible values of n are

A. 3 B. 2 C. 1 D. 4

Answer: The relative speed of the minute hand is $\frac{11}{2}$ degrees/minute. Therefore, in one minute, the minute hand gets ahead of the hour hand by $\frac{11}{2}$ degrees. The minute hand takes $\frac{2}{11}$ minutes to gain/cover one degree on the hour hand.

To meet the hour hand again, the minute hand takes $\frac{360}{\frac{11}{2}} = \frac{720}{11}$ minutes. $\frac{2}{11}$ minutes before meeting, the minute hand would be 1 degree behind. Therefore, time taken = $\frac{720}{11} - \frac{2}{11} = \frac{718}{11}$. After meeting minute hand would take $\frac{2}{11}$ minutes to get ahead by 1 degree. Therefore, time taken = $\frac{720}{11} + \frac{2}{11} = \frac{722}{11}$

Therefore after every $\frac{720k}{11} - \frac{2}{11}$ minutes, the minute hand is one degree behind the hour hand and after every $\frac{720k}{11} + \frac{2}{11}$ minutes, the minute hand is one degree ahead of the hour hand. We need to find integral values for these times. For the first condition, we get k = 7 and n = 458 and for the second condition, we get k = 4 and n = 262. Therefore, we have two integral values of n below 720.

Without stoppage a train travels at average speed of 75 km/h and with stoppage it covers the same distance at 60 km/h. How many minutes per hour does the train stop?

Answer: Let the speed of the train be 75 km/h and the distance to be traveled be also 75 km. Therefore, in the first case, the train takes 1 h to cover the distance. In the second case, the average speed of the train = 60 km/h.

Therefore, the train travels 60 km in one hour. To travel 60 km, the train takes $\frac{60}{75} \times 60 = 48$ min. Therefore, the train stos for 60 – 48 = 12 min every hour.

In a race of 600m, Ajay beats Vijay by 60m and in a race of 500 m Vijay beats Anjay by 25m. By how many metres will Ajay beat Anjay in a 400m race?

Answer:

When Ajay travels 600m, Vijay travels 540m. Therefore, when Ajay travels 400m, Vijay travels $\frac{540}{600} \times 400 = 360$m.

When Vijay travels 500m, Anjay travels 475m. Therefore, when Vijay travels 360m, Anjay travels $\frac{475}{500} \times 360 = 342$m. Therefore, Ajay beats Anjay by 400 – 342 = 58m.

R walked down a descending escalator and took 40 steps to reach the bottom. S started simultaneously from the bottom, taking 2 steps for every 1 step taken by R. Time taken by R to reach the bottom from the top is same as time taken by S to reach the top from the bottom.

How many steps more than R did S take before they crossed each other?
If R were to walk at the speed of S, what percentage of the initial time would he be able to save?

Answer:
Not many of you realize that escalator problems are nothing but "upstream/downstream" problems with the river replaced with the escalator.

Let the speed of R be v, speed of S be 2v and speed of escalator = u
R and S are taking the same time to cover the same distance
$\Rightarrow$ v + u (downstream) = 2v - u (upstream) $\Rightarrow$ v = 2u

Therefore, let the escalator come out at the speed of 1 stair/minute. R and S cover stairs at the rate of 2 stairs/minute and 4 stairs/minute, respectively.

When R covers 40 stairs, escalator gives out 20 stairs $\Rightarrow$ total stairs = 40 + 20 = 60.

When S goes up by 4 stairs, escalator brings him down by 1 stairs. Therefore, his upstream speed is 3 stairs/minutes. They meet midway as their upstream and downstream speeds are equal. Therefore, both have covered 30 stairs.

To cover 30 stairs R covers 20 stairs and 10 stairs are given out by the escalator. S covers twice of R, i.e. 40 stairs. **Therefore, S covers 20 stairs more than R.**

Right now, R takes (60 stairs)/(3 stairs/minute) = 20 minutes. When he climbs 4 stairs/minute (Speed of S), his downstream speed will be 5 stairs/minute and time taken = 60/5 = 12 minutes.

Percentage of time saved = (8/20) × 100 = 40%

If a person increases his usual speed by 15km/h, he reaches his destination one hour earlier than his usual time. If he decreases his speed by 10km/h, he will be late by one hour. The distance traveled by him is equal to
A. 420 km B. 360 km C. 480 km D. 300 km

Answer: **1st METHOD:**
I will use the funda of arithmetic and harmonic progression here. Read about the funda in Total Gadha's Quant lessons. Let the normal time taken for travel be t. Therefore, the times taken in two cases are t – 1 and t +1. As the time taken are in arithmetic progression (t – 1, t, t + 1), the speeds (v + 15, v, v – 10) will be in harmonic progression for a fixed distance. Therefore, v would be harmonic mean of v + 15 and v – 10.

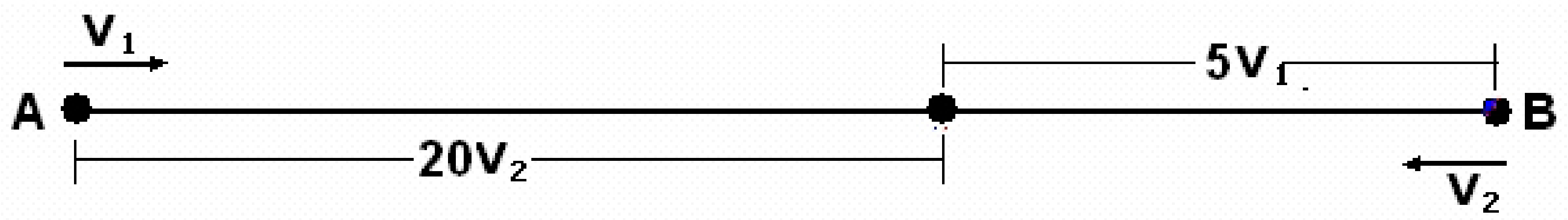
$\Rightarrow v = \frac{2(v+15)(v-10)}{2v+5} \Rightarrow 2v^2 + 5v = 2v^2 + 10v - 300 \Rightarrow v = 60km/h$. Now, 75 × (t – 1) = 50 × (t + 1) ⇒ t = 5h.

Therefore, distance = 60 × 5 = 300km.

2nd METHOD:
Let the usual speed be v and the normal time taken be t. Distance is constant in each case.
(v + 15) × (t – 1) = vt ⇒ 15t – v = 15 --- (1)
(v – 10) × (t + 1) = vt ⇒ v – 10t = 10 --- (1)
Solving (1) and (2) we get v = 50 and t = 5. Therefore, distance = 300km.

Two trains start at the same time from two stations A and B towards each other. They arrive at B and A respectively in 5 hours and 20 hours after they have passed each other. If the speed of the train that started from A is 56 kmph, then the speed of the second train is equal to
A. 26kmph B. 28kmph C. 24kmph D. 30kmph

Answer: The situation is shown in the image given below. Let the speed of trains from A and B be V_1 and V_2, respectively. After assign each other they cover a distance of $5V_1$ and $20V_2$ to reach B and A, respectively, as shown.



When they meet, they have taken the same time for travel. In other words, the train from A has taken the same time to cover $20V_2$ as the train from B has taken to cover $5V_1$.

$\Rightarrow \frac{20V_2}{V_1} = \frac{5V_1}{V_2} \Rightarrow \frac{V_1}{V_2} = 2 \Rightarrow V_2 = \frac{V_1}{2} = 28\text{kmph}$

If a person increases his usual speed by 20%, he reaches his office 15 minutes early. By how many minutes will he be late to his office, if he reduces his usual speed by 20%
A. 10 B. 20 C. 15.5 D. 22.5

Answer: If the person increases his speed by 20% ($\frac{1}{5}$), his speed becomes $\frac{6}{5}$ times his original speed and therefore, his time for travel becomes $\frac{5}{6}$ times his original time. Therefore, $\frac{1}{6}^{th}$ time is saved. This is equal to 15 min. therefore, original time taken = 15 × 6 = 90 min.

If the person decreases his speed by 20% ($\frac{1}{5}$), his speed becomes $\frac{4}{5}$ times his original speed and therefore, his time for travel becomes $\frac{5}{4}$ times his original time = $\frac{5}{4} \times 90 = 112.5\text{min}$. Therefore he will be late by 112.5 – 90 = 22.5 min.

In a kilometer race, if A gives B a head start of 40m, then A wins by 19 seconds. If A gives B a head start of 30 seconds, then B wins by 40m. The times taken by each of them to run a kilometer race (in seconds) are
A. 105, 125 B. 125, 145 C. 135, 150 D. 125, 150

I found out a great way to solve this:

Notice the first case. When A gives a lead of 40 m, he wins by 19 s.

Let's see the first case from the perspective of the finish point. When A reaches the finish point B is 19 s behind. If at this moment, I press the rewind button, both A and B will start going toward the start point with B having a lead of 19 s. And then A will beat B by 40 m. The situation can be shown below:

Therefore, when A gives a lead of 19 s to B, he beats B by 40 m. Therefore when A travels 1000 m, B travels 960 – 19v m, where v is B's speed. The speed ratio = $\frac{960-19v}{1000}$

The second case can be shown as below:

When A travels 960 m, B travels 1000 – 30v m. The speed ratio = $\frac{1000-30v}{960}$

$\frac{1000-30v}{960} = \frac{960-19v}{1000} \Rightarrow$ v = 40/6 ⇒ time taken = 150 s

While two trains are crossing each other, a person sitting in the slower train observes that the faster train crossed him in 24 seconds. If the speeds of the two trains are 80km/hr and 65km/hr, what is the length of the faster train?
A. 50m B. 150m C. 100m D. 450m

Answer: Relative speed of the faster train with respect to the slower train = 80 – 65 = 15 km/h = $\frac{25}{6}m/s$.

Time taken = Time taken $= \frac{\text{Relative distance}}{\text{Relative speed}} = \frac{\text{Length of the faster train}}{\text{Relative speed}} \Rightarrow \text{Length of the faster train} = 24 \times \frac{25}{6} = 100m$

Two clocks are set to show the correct time at 7.00pm on a day. One clock loses two minutes in an hour and the other clock gains 3 minutes in a hour. Exactly, after how many days, will both the watches show the correct time?

A. 10 B. 15 C. 30 D. 60

Answer: Every clock will show the correct time when it is late or fast by hours which are multiples of 12. The first clock loses 2 min in 1 hour. Therefore, to lose 12 h it will take $\frac{12 \times 60}{2} = 360 \text{ h} = 15 \text{ days}$. The second clock gains 3 minutes in 1 h. therefore, to gain 12 h it will take $\frac{12 \times 60}{3} = 240 \text{ h} = 10 \text{ days}$. Therefore, both the clocks will show correct time in 30 days (LCM of 15 and 10).

A person left point A for point B . Two hours later, another person left A for B and arrived at B at the same time as the first person .Had both started simultaneously from A and B traveling towards each other , they would have met in 80 minutes .How much time did it take the faster person to travel from A to B .

Answer: Let the time taken by the second person be t minutes. Therefore, the time taken by the first person is t + 120. Let the total distance be d. The second person travels d in t minutes. Therefore he will travel $\frac{80d}{t}$ in 80 minutes. Similarly, the first person will travel $\frac{80d}{t+120}$ in 80 mins.

$\frac{80d}{t} + \frac{80d}{t+120} = d \Rightarrow t = 120 \text{ mins} = 2 \text{ h}.$

In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and slowest runner reach the same point at the end of 5th minute, for the first time after the start of the race. All the runners have the same starting pt. and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of slowest runner, what is the time taken by the fastest runner to finish the race?

A. 20 min B. 15 min C. 10 min D. 5 min

Answer: As the speed of fastest runner is twice the speed of the slowest runner, the fastest runner travels twice the distance as that traveled by the slowest runner in the same time. Also, to catch the slowest runner again, the fastest runner will have to travel one round extra than the distance traveled by the slowest runner. Therefore, it can be reasoned out that the slowest runner is taking one round whereas the fastest runner is taking two rounds in 5 min. therefore, to take 4 rounds, the fastest runner will take 10 min. (**Please work out the logic. It's fun**)

A man builds $\frac{1}{8}^{\text{th}}$ of a wall everyday. Out of the length of the wall built per day, 20% falls off each day(including last day's work).In how many days he can complete the wall?

Answer: The wall would never be completed. The moment the length of the wall reaches 62.5%, 20% of the wall = 12.5% would be demolished. Therefore, the new added length would be equal to the demolished length.

A man starts a piece of work. Starting from 2^{nd} day onwards, everyday a new man joins. With every man joining the work that each man can do per day doubles. The work is completed in 5 days. On which day they would have completed the work if the work each one can do per day remained constant.

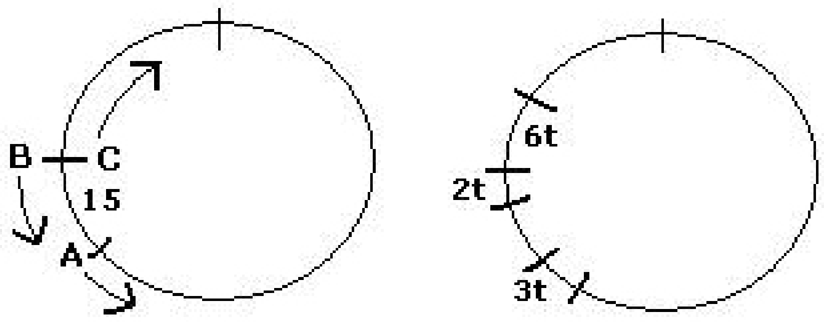
Answer: Let the initial units of work that a man can do on the first day be w. Therefore, the units of work that each man can do on 2^{nd}, 3^{rd}, 4^{th} and 5^{th} day would be 2w, 4w, 8w and 16w, respectively.
Therefore, total units of work done = $w + 2 \times 2w + 3 \times 4w + 4 \times 8w + 5 \times 16w = 129w$.

If the work each man could remained constant the units of work done in n days = $w(1 + 2 + 3 + \ldots + n) = \frac{n(n+1)w}{2}$

$\frac{n(n+1)w}{2} = 129w \Rightarrow n \approx 16\text{days}.$

A and B are running in a circular track in direction opposite to which C is running, in fact running at twice and thrice the speed of A and B, respectively, and on the same track. They start running from same point. It is known that A's average speed is 3m/s and track is 120 meter in length. When will B, after start, find himself equidistant between A and C for first time?

Answer:
See the figure below:



The speeds of A, B, and C are 3, 2 and 6. C will meet A first and then meet B after 120/(2 + 6) = 15 s. In 15 s A will travel 45 m and B will travel 30 m, as shown in the first figure.
Let after next t seconds, B is between A and C. The distance between B and C is 2t + 6t. The distance between A and B is 15 - 2t + 3t = 15 + t.
2t + 6t = 15 + t --> t = 15/7 s.
Total time = 15 + 15/7 = 120/7 s