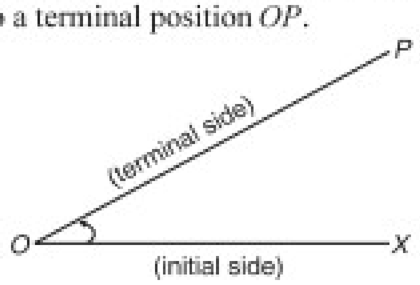
11 TRIGONOMETRY

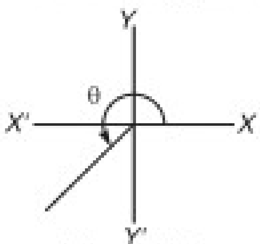
This chapter is basically written for those students who were either not so good in the secondary classes maths or who are not in touch with the following concepts from 4-5 years. Hardly a question is asked in CAT, directly from this chapter, but almost every year there are one or two questions in which we can find the application of the concepts of trigonometry *e.g.*, sometimes in geometry or in height and distance problems. So just refresh the concepts of trigonometry. Therefore CAT aspirants are not required to waste so much of precious time on this chapter. But this chapter is useful for IIFT, NIFT, FMS etc.

Angle : An angle is thought of as traced out by the rotation of a line (called the revolving line) from the initial position OX to a terminal position OP.

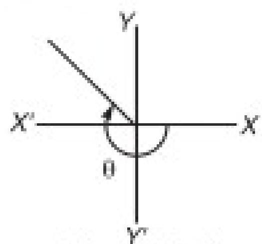


Then O is the vertex, OX is called the initial side and OP is called the terminal side of the angle.

Convention of Sign of Angles : It is customary to regard the angle traced out by a **counter-clockwise** rotation as positive and one traced out by a **clockwise** rotation as negative. The angle is indicated by using an arrow, starting from the initial line and ending at the terminal line.

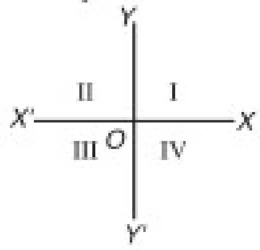


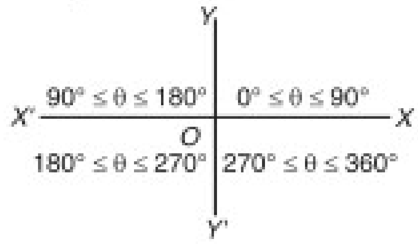
(θ is positive)



(θ is negative)

Quadrants : Let XOX' and YOY' be two mutually perpendicular lines in any plane. These lines divide the plane into four regions called quadrants, numbered counter-clockwise. Thus the quadrants XOY, YOX', $X'OY'$ and $Y'OX$ are respectively called the first quadrant, the second quadrant, the third quadrant and the fourth quadrant.





NOTE θ will be more than 360°, if after making one complete revolution, the revolving line does not stop at the initial position OX but proceeds on further on.

Measures of Angles : There are three different systems of units for measurement in trigonometry, *viz.* (i) Sexagesimal system (ii) Centesimal system (iii) Circular system.

Sexagesimal System : In this system, 1 right angle is divided into 60 equal parts and each part (or division) is called a degree (*i.e.*, 1°, read as 1 degree).

$$1° = 60' \text{ (60 sexagesimal minutes)}$$

$$1' = 60'' \text{ (60 sexagesimal seconds)}$$

This system is called the Common or the English system.

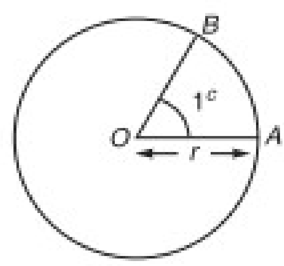
Centesimal System : In this system, 1 right angle is divided into 100 equal parts and each part (or division) is called a grade (or 1^g).

$$90° = 100^g$$

$$1^g = 100' \quad (K' \to K \text{ centesimal minute})$$

$$1' = 100'' \quad (K'' \to K \text{ centesimal second})$$

Circular Measure : The unit of measurement of angles in this system is a radian (or 1^c)



A **radian** is defined as the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle and it is denoted by 1^c.

In the figure $OA = OB = r$, (radius of the circle) and $\angle AOB = 1$ radian or 1^c.

IMPORTANT

1. In all circles, the ratio of the circumference to its diameter is constant.
(Circumference = $\pi \times$ diameter)
π is an incommensurable number, *i.e.*, it cannot be expressed exactly either by a whole number or by a fraction. Its approximate value is 22/7 or 3.1416, correct to four places of decimal only.
2. A radian is a constant angle.
1 radian (1^c) = a constant angle
3. π radian $= 180°$ (or $1° = \dfrac{\pi}{180}$ radian)
4. If an arc of length '*s*' subtends an angle θ radian at the centre of a circle of radius r, then $s = r\theta$. $\left(\theta = \dfrac{\text{arc}}{\text{radius}}\right)$
5. (i) Area of the sector $AOB = \dfrac{1}{2}r^2\theta$
(ii) Area of the sector $AOB = \dfrac{1}{2}rs$
($s \rightarrow$ length of arc AB)

EXAMPLE 1 *Express the following angles in radian measure and centesimal measure :*
(i) 45° (ii) 20° 35′ (iii) 50° 38′ 40′′

SOLUTION (i) $\because$ $180° = \pi$ rad

$\therefore \quad 1° = \dfrac{\pi}{180}$ rad

$\therefore \quad 45° = 45 \times \dfrac{\pi}{180} = \dfrac{\pi}{4}$ rad

Now, $\quad 90° = 100$ grade

$\Rightarrow \quad 45° = 50$ grade

(ii) $20° \, 35' = 20\dfrac{35}{60}$ degree $= \dfrac{247}{12}$ degree

Now, 180 degree = π rad

$\therefore \quad \dfrac{247}{12}$ degree $= \dfrac{247}{12} \times \dfrac{\pi}{180} = \dfrac{247}{2160}\pi$ rad

and $\quad \because \; 90° = 100$ grade

$\therefore \quad \left(\dfrac{247}{12}\right)^{\circ} = \dfrac{247}{12} \times \dfrac{100}{90} = \dfrac{2470}{108}$ grade $= \dfrac{1235}{54}$ grade

(iii) $50° \, 38' \, 40'' = 50° \, 38\dfrac{40}{60}$ min $= 50° \dfrac{116}{3}$ min

$= 50° \dfrac{116}{3 \times 60}$ degree

$= \dfrac{2279}{45}$ degree

Now $\quad 180° = \pi$ rad

$\because \quad \left(\dfrac{2279}{45}\right)^{\circ} = \dfrac{2279}{45} \times \dfrac{\pi}{180} = \dfrac{2279}{8100}\pi$ rad

and $\quad 90° = 100^g$

$\Rightarrow \quad \left(\dfrac{2279}{8100}\right)^{\circ} = \dfrac{2279}{8100} \times \dfrac{100}{90} = \dfrac{2279}{7290}$ grade.

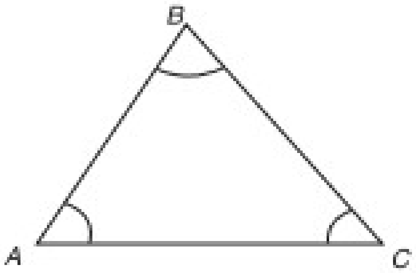
EXAMPLE 2 *One angle of a triangle is 54° and another angle is $\dfrac{\pi}{4}$ radian. Find the third angle in centesimal unit.*

SOLUTION Let $\angle A = 54°$ and $\angle B = \dfrac{\pi}{4}$ rad

$\therefore \quad B = 45°$

Thus, $\quad \angle A + \angle B = 99°$

$\therefore \quad \angle C = 180 - 99 = 81° \quad (\because \angle A + \angle B + \angle C = 180°)$



Now, $\quad$ since $90° = 100$ grade

$\Rightarrow \quad 81° = 81 \times \dfrac{100}{90} = 90$ grade

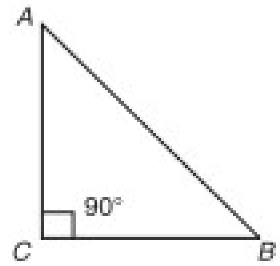
EXAMPLE 3 *The difference between two acute angles of a right-angled triangle is $\dfrac{\pi}{6}$ radians. Find the angle in degrees.*

SOLUTION $\quad \angle A + \angle B = 90°$

and $\quad \angle A - \angle B = \dfrac{\pi}{6} = 30°$

$\therefore \quad \angle A = 60°$ and $\angle B = 30°$

Hence the two acute angles are 60° and 30°.



EXAMPLE 4 *In a circle of diameter 60 m, the length of a chord is 30 m. Find the length of the minor arc on one side of the chord. What is the length of the major arc?*

SOLUTION $\quad 2r = 60$ m $\Rightarrow r = 30$ m

$\therefore \quad OA = OB = AB = 30$ m

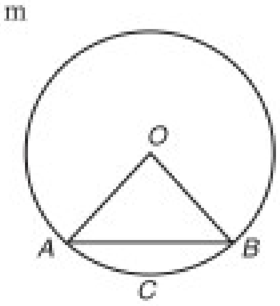
$\therefore$ ΔOAB is an equilateral triangle.

Thus, $\quad \angle AOB = 60° = \dfrac{\pi}{3}$ rad

Now, since $\quad s = \theta \; r = \dfrac{\pi}{3} \times 30$

$= 10 \times 3.142$

$= 31.42$ m (approx.)



Thus the length of minor arc $ACB = 31.42$ m and the length of major arc

$= (2\pi r -$ minor arc$)$

$= 2\pi \times 30 - 31.42 = 157.1$ m (approx.)

EXAMPLE 5 *An arc AB of a circle subtends an angle x radians at the centre O of the circle. Given that the area of the sector AOB is equal to the square of the length of the arc AB, find the value of x.*

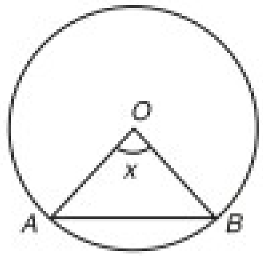
SOLUTION Let arc $AB = s$

Given that area of the sector

$$AOB = (\text{arc } AB)^2$$

$$\therefore \quad \frac{1}{2}r^2x = s^2 \quad \text{and} \quad s = rx \quad (\because \theta = x)$$

$$\therefore \quad \frac{1}{2}r^2x = r^2x^2 \quad \text{or} \quad x = \frac{1}{2}\text{ radian}$$



EXAMPLE 6 *Two arcs of two different circles are of equal lengths. If these arcs subtends angles of 45° and 60° at the centres of the circles. Find the ratio of the radii of the two circles.*

SOLUTION $45° = \frac{\pi}{4}$ rad and $60° = \frac{\pi}{3}$ rad

Let r_1 and r_2 be the radii of the two circles and s be the length of each arc.

$$\therefore \quad s = r_1\frac{\pi}{4} = r_2\frac{\pi}{3} \qquad (\because s = r\theta)$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

Hence the required ratio of radii = 4 : 3

EXAMPLE 7 *The wheel of a railway carriage is 4 ft in diameter and makes 6 revolution per second, how fast is the train going ?* ($\pi = 3.14$)

SOLUTION Radius (r) of the wheel = 2 ft

$\therefore$ Circumference of wheel = $2 \times 3.14 \times 2 = 12.56$ ft

Since the wheel makes 6 revolutions per second,

$\therefore$ Distance traversed in 1 second = $6 \times 12.56 = 75.36$ ft.

Hence the velocity of the train = 75.36 ft/s

$$= \frac{75.36 \times 60 \times 60}{3 \times 1760}$$

$$= 51.38 \text{ mile/h}$$

EXAMPLE 8 *A horse trots uniformly along a circular track of radius 27 m. The angle subtended at the centre of the track by the arc passed over by the horse in 3 seconds is 70°. What distance will the horse pass over in $\frac{1}{2}$ minute.*

SOLUTION $r = 27$ m

$$\theta = 70° = \frac{7\pi}{18}\text{ radian}$$

$$\therefore \quad s = r\theta = 27 \times \frac{7\pi}{18} = 33 \text{ m (approx.)}$$

$\therefore$ Distance passed over by the horse in $\frac{1}{2}$ minute

$$= \frac{33}{3} \times 30 = 330 \text{ m (approx.)}$$

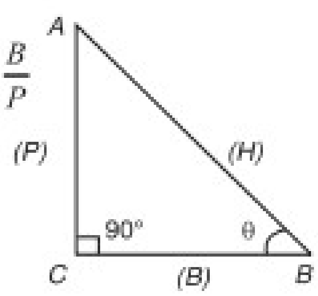
INTRODUCTORY EXERCISE-11.1

1. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle having the same radius. The angle of the sector (approx.) is :
   (a) 65° 27′ 16″ (b) 68° 18′ 19″
   (c) 56° 52′ 18″ (d) none of these
2. The length of a pendulum is 8 m while the pendulum swings through 1.5 rad, find the length of the arc through which the tip of the pendulum passes :
   (a) 8 m (b) 9 m
   (c) 12 m (c) none of these
3. The angles of a triangle are in AP and the greatest angle is 75°. Find all the three angles in degrees :
   (a) 55°, 55°, 70° (b) 45°, 60°, 75°
   (c) 40°, 65°, 75° (d) none of these
4. A circular wire of radius 2.5 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 1.29 m. Find (in degrees) the angle which is subtended at the centre of the hoop :
   (a) 9.67° (b) 7.69°
   (c) 6.97° (d) none of these
5. A circle is drawn on AB as diameter. The centre of the circle is O and the length $AB = 13$ cm, P is a point on the circumference of the circle such that the chord $AP = 12$ cm. Calculate the value of the angles PAB and POB in radians :
   (a) 0.395, 0.789 (b) 0.786, 0.735
   (c) 0.398, 0.689 (d) 0.786, 0.753

TRIGONOMETRICAL RATIOS

$$\sin\theta = \frac{P}{H}, \cos\theta = \frac{B}{H}, \tan\theta = \frac{P}{B}$$

$$\text{cosec}\,\theta = \frac{H}{P}, \sec\theta = \frac{H}{B}, \cot\theta = \frac{B}{P}$$



Remember :

sin θ	cos θ	tan θ	
P	B	P	⇒ Pandit Badri Prasad
H	H	B	⇒ Hari Hari Bol
cosec θ	sec θ	cot θ	

$P \rightarrow$ Perpendicular (AC)
$B \rightarrow$ Base (BC)
$H \rightarrow$ Hypotenuse (AB)

Thus, $\boxed{\begin{array}{c}\sin\theta\,\text{cosec}\,\theta = 1\\ \cos\theta\sec\theta = 1\\ \tan\theta\cot\theta = 1\end{array}}$ and $\boxed{\begin{array}{c}\tan\theta = \dfrac{\sin\theta}{\cos\theta}\\[2ex] \cot\theta = \dfrac{\cos\theta}{\sin\theta}\end{array}}$

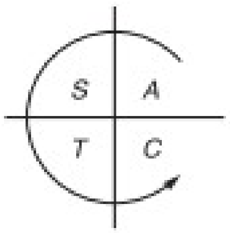
NOTE $(\sin\theta)^{-1}$ is not written as $\sin^{-1}\theta$. Thus $\sin^{-1}\theta \neq (\sin\theta)^{-1}$ etc but $(\sin\theta)^2 = \sin^2\theta$ and $(\sin\theta)^3 = \sin^3\theta$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\text{cosec}^2\theta - \cot^2\theta = 1$$

SIGN OF TRIGONOMETRICAL FUNCTIONS

sin and cosec +ve	All +ve
tan and cot +ve	cos and sec +ve

+ve → (Positive)



$A \to$ All
$S \to$ sin/cosec
$T \to$ tan/cot
$C \to$ cos/sec

Remember : "Add Sugar To Coffee"

Values of T-Ratios :

Angle	$\theta = 0$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 90°$	$\theta = 180°$
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0
cosec	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1	∞
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞	–1
cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	∞

Remember :

$$\text{cosec}\,\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \cot\theta = \frac{1}{\tan\theta}$$

and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

and $\tan\theta = \dfrac{1}{\cot\theta}$ or $\cot\theta = \dfrac{1}{\tan\theta}$

Range of Trigonometric Ratios :

(i) $-1 \le \sin\theta \le 1 \Rightarrow |\sin\theta| \le 1$

(ii) $-1 \le \cos\theta \le 1 \Rightarrow |\cos\theta| \le 1$

(iii) $\text{cosec}\,\theta \le -1$ and $\text{cosec}\,\theta \ge 1 \Rightarrow |\text{cosec}\,\theta| \ge 1$

(iv) $\sec\theta \le -1$ and $\sec\theta \ge 1 \Rightarrow |\sec\theta| \ge 1$

(v) $-\infty < \tan\theta < \infty$ *i.e.*, $\tan\theta$ may assume any value.

INCREASING AND DECREASING FUNCTIONS OF T-RATIOS

	1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
$\sin\theta$	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0
$\cos\theta$	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0	increases from 0 to 1
$\tan\theta$	increases from 0 to ∞	increases from $-\infty$ to 0	increases from 0 to ∞	increases from $-\infty$ to 0

INTRODUCTORY EXERCISE-11.2

Directions for question number 1 to 7: Find the values of the following expressions.

1. $\dfrac{\sec^2\theta - 1}{\tan^2\theta} = ?$

(a) 1 (b) 2
(c) 3 (d) 4

2. $\sin^4\theta + \sin^2\theta\cos^2\theta = ?$

(a) 0 (b) 1
(c) 2 (d) $\sin^2\theta$

3. $\dfrac{\sin\theta\,\text{cosec}\,\theta\tan\theta\cot\theta}{\sin^2\theta + \cos^2\theta} = ?$

(a) 0 (b) 1
(c) –1 (d) 2

4. $\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = ?$

(a) –1 (b) 0
(c) 3 (d) 1

5. $\tan\theta + \cot\theta$ is :

(a) 1 (b) $\tan\theta$
(c) $\text{cosec}\,\theta\cot\theta$ (d) $\sec\theta\,\text{cosec}\,\theta$

6. $\dfrac{3 - 4\sin^2\theta}{\cos^2\theta} + \tan^2\theta$ is :

(a) 1 (b) 2
(c) 3 (d) none of these

7. $\dfrac{\cot A + \tan B}{\cot B + \tan A}$ is :

(a) $\tan A\cot B$ (b) $\cot A\tan B$
(c) 1 (d) none of these

8. If $\sin\theta = \dfrac{21}{29}$, find the value of $\sec\theta + \tan\theta$, if θ lies between 0 and $\pi/2$:

(a) 1 (b) $\pi/2$
(c) 5/2 (d) none of these

9. If A is in the fourth quadrant and $\cos A = \dfrac{5}{13}$, find the value of $\dfrac{13\sin A + 5\sec A}{5\tan A + 6\,\text{cosec}\,A}$:

(a) $-2/37$ (b) $-3/27$
(c) $2/37$ (d) can't be determined

10. Find the value of,

$\frac{4}{3}\cot^2 30° + 3\sin^2 60° - 2\operatorname{cosec}^2 60° - \frac{3}{4}\tan^2 30°$:

(a) 10 / 3 (b) 11 / 3
(c) 4 (d) none of these

11. Find the perimeter of a regular octagon inscribed in a circle of radius 100 cm :

(a) 532.87 cm (b) 612.32 cm
(c) 378.32 cm (d) 875 cm

12. Find the altitude and base of an isosceles triangle whose vertical angle is 65° and whose equal sides are 415 cm :

(a) 350 cm, 646 cm (b) 350 cm, 446 cm
(c) 630 cm, 445 cm (d) none of these

13. If $5\sin^2\theta + 3\cos^2\theta = 4$, find the value of $\sin\theta$ and $\cos\theta$:

(a) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$ (b) $\pm\frac{\sqrt{3}}{2}, \pm\sqrt{2}$
(c) $\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$ (d) none of these

14. Find the value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$:

(a) $2\sec\theta$ (b) $\sec\theta$
(c) $2\operatorname{cosec}\theta$ (d) none of these

15. If $\tan\theta = \frac{a}{b}$, find the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

(a) $\frac{a^2-b^2}{a^2+b^2}$ (b) $\frac{b^2-a^2}{b^2+a^2}$
(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) none of these

✍ ***Direction for question number 16-18 :*** *For the every* $\theta, 0 < \theta \le 90$, *find the values of the following angles.*

16. $\tan\theta + \cot\theta = 2$:

(a) 30° (b) 45°
(c) 60° (d) none of these

17. $2\sin^2\theta + 4\cos^2\theta = 3$:

(a) 30° (b) 60°
(c) 45° (d) none of these

18. $2\sin^2\theta - 3\sin\theta = -1$:

(a) 30° (b) 45°
(c) 90° (d) both (a) and (c)

TRIGONOMETRICAL RATIOS OF NEGATIVE AND ASSOCIATED ANGLES

Angle	$-\theta$	$(90-\theta)$	$(90+\theta)$	$(180-\theta)$	$(180+\theta)$	$(360-\theta)$	$(360+\theta)$
$\sin\theta$	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\sin\theta$	$\sin\theta$
$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$\cos\theta$	$\cos\theta$
$\tan\theta$	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$-\tan\theta$	$\tan\theta$

SUM, DIFFERENCE AND PRODUCT FORMULAE

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
6. $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
7. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
8. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
9. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
10. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
11. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
12. $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
13. $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$
14. $\cot(A-B) = \frac{\cot A\cot B + 1}{\cot B - \cot A}$
15. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
16. $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
17. $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
18. $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

INTRODUCTORY EXERCISE-11.3

1. If $A + B = 45°$, find the value of $\tan A + \tan B + \tan A \tan B$:

(a) -1 (b) $\frac{1}{2}$

(c) $\sqrt{3}$ (d) 1

2. Find the value of $\tan 75°$:

(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2+\sqrt{2}}$

(c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (d) $\frac{2+\sqrt{2}}{\sqrt{3}}$

3. Find the value of $\cos 28° \cdot \cos 32° - \sin 28° \cdot \sin 32°$:

(a) 1 (b) 1/2

(c) 1/3 (d) can't be determined

4. If $\cos A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, find the value of $\frac{\tan A + \tan B}{1 - \tan A \tan B}$:

(a) $\frac{63}{16}$ (b) $\frac{36}{16}$

(c) $\frac{61}{36}$ (d) none of these

5. Find the value of

$$\sin^2 (120° - A) + \sin^2 A + \sin^2 (120° + A):$$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{\sqrt{3}}{2}$

6. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lie between 0° and 45°, find the value of $\tan 2\alpha$:

(a) $\frac{56}{63}$ (b) $\frac{56}{33}$

(c) $\frac{25}{33}$ (d) $\frac{65}{63}$

7. Find the maximum and minimum values of $7\cos\theta + 24\sin\theta$:

(a) 25 and -25 (b) 16 and -9

(c) 25 and 0 (d) 36 and 25

8. Find the value of $2\cos 45° \cdot \cos 15°$:

(a) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$ (b) $\frac{(\sqrt{3}-1)}{2}$

(c) $\frac{\sqrt{3}+1}{2}$ (d) none of these

9. Find the value of $\frac{\sin 75° + \sin 15°}{\cos 75° + \cos 15°}$:

(a) 1 (b) 2

(c) $\frac{\sqrt{3}}{2}$ (d) can't be determined

10. Find the value of $\cos 20° + \cos 100° + \cos 140°$:

(a) -1 (b) 0

(c) 1 (d) none of these

11. $\cos 20° \cdot \cos 40° \cdot \cos 60° \cdot \cos 80°$ is:

(a) $\frac{1}{16}$ (b) $\frac{1}{50}$

(c) $\frac{1}{24}$ (d) $\frac{1}{25}$

12. What is the value of;

$$\frac{\sin(90° - \theta)\sec(180° - \theta)\sin(-\theta)}{\sin(180° + \theta)\cot(360° - \theta)\operatorname{cosec}(90° + \theta)}:$$

(a) $\sin\theta$ (b) $\cos\theta$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

13. If $\sin(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$, find the value of $\tan 2\alpha$:

(a) $\frac{16}{36}$ (b) $\frac{63}{16}$

(c) $\frac{36}{16}$ (d) none of these

14. Find the value of $\frac{\cos 2B - \cos 2A}{\sin 2A + \sin 2B}$:

(a) $\tan(A - B)$ (b) $\cos(A - B)$

(c) $\cot(A - B)$ (d) $\tan(A + B)$

15. $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A}$ is:

(a) $\tan 2A$ (b) $\tan 3A$

(c) $\tan 4A$ (d) $\tan 8A$

TRIGONOMETRICAL RATIOS OF MULTIPLE AND SUB-MULTIPLE ANGLES

1. $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

2. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

   $= 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$

3. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

4. $\sin 3A = 3\sin A - 4\sin^3 A$

5. $\cos 3A = 4\cos^3 A - 3\cos A$

6. $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

7. $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right) = \frac{2\tan(A/2)}{1+\tan^2(A/2)}$

8. $\cos A = \cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right)$ etc.

9. $\tan A = \frac{2\tan(A/2)}{1-\tan^2(A/2)}$

INTRODUCTORY EXERCISE-11.4

1. If $\sin\theta = \frac{4}{5}$, find the value of $\sin 2\theta$:

(a) 24/25 (b) 16/25
(c) 9/20 (d) none of these

2. $\frac{\sin 2A}{1+\cos 2A}$ is :

(a) $\tan 2A$ (b) $\cos 2A$
(c) $\tan A$ (d) none of these

3. $\cos^4\theta - \sin^4\theta$ is :

(a) $\cos 2\theta$ (b) $\sin 2\theta$
(c) $\tan 2\theta$ (d) none of these

4. If $\cot x = \frac{a}{b}$, find the value of $a\cos 2x + b\sin 2x$

(a) a (b) b
(c) $\sin a$ (d) a/b

5. Find the value of $\cot 3\theta$:

(a) $\frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta}$ (b) $\frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$
(c) $\frac{3\cot\theta - \cot^3\theta}{3\cot^2\theta - 1}$ (d) none of these

6. Find the value of $\sin\left(22\frac{1}{2}\right)^\circ$:

(a) $\frac{\sqrt{(2-\sqrt{2})}}{2}$ (b) $\frac{\sqrt{(2-\sqrt{3})}}{3}$
(c) $\frac{\sqrt{2-\sqrt{2}}}{4}$ (d) none of these

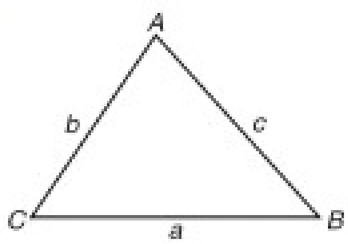
7. If $2\cos\theta = x + \frac{1}{x}$, find the value of $2\cos 3\theta$

(a) $x^3 + \frac{1}{x^3}$ (b) $x^2 + \frac{1}{x^2}$
(c) $x^3 - \frac{1}{x^3}$ (d) $\sqrt{x + \frac{1}{x}}$

8. Find the value of $\tan A + \tan B + \tan C$, if $A + B + C = \pi$:

(a) $\tan A \tan B \tan C$ (b) 1
(c) 0 (d) $\cot A \cot B \cot C$

PROPERTIES OF TRIANGLES



A triangle ABC has three angles and three sides. The three angles are denoted by A, B and C while the three sides are denoted by a, b and c respectively. The area of the triangle is denoted by Δ and its perimeter is denoted by $2s$, so that $a + b + c = 2s$. Thus 's' is the semiperimeter of the triangle and $s = \frac{a+b+c}{2}$. Also we denote the circumradius by R and inradius by r.

1. The law of sines : In any triangle the sides are proportional to the sines of the opposite angles *i.e.*,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. The law of cosines : The square on any side of a triangle is equal to the sum of the squares on the other two sides, minus twice the product of those two sides and the cosine of the included angle *i.e.*,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

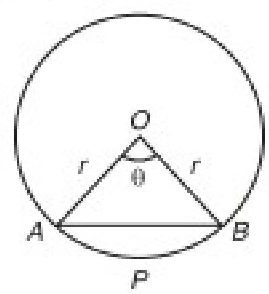
$$c^2 = a^2 + b^2 - 2ab\cos C$$

3. Area of a triangle :

(i) $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$

(ii) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (Hero's formula)

4. The area of a segment of a circle :



Area of the segment APB

= Area of the sector AOB

− Area of ΔAOB

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

5. Circumradius of a circle (R) :

(i) $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

(ii) $R = \frac{abc}{4\Delta}$

6. Inradius of a circle (r)

$$r = \frac{\Delta}{s} \qquad \left(s = \frac{a+b+c}{2}\right)$$

INTRODUCTORY EXERCISE-11.5

1. If $\cos B = \dfrac{\sin A}{2\sin C}$, find the nature of triangle :

(a) isosceles (b) scalene
(c) can't be determined (d) none of these

2. If the angles of a triangle are in the ratio $1:2:3$ and its circumradius is 10 cm, find the sides of the triangle (in cm) :

(a) 8, 16, 24 (b) 18, 12, 6
(c) 10, 20, $20\sqrt{3}$ (d) 10, 20, $10\sqrt{3}$

3. If $a = 1$, $b = \sqrt{3}$ and $A = 30°$, find the value of the angle B:

(a) 60° (b) 30°
(c) 120° (d) either (a) or (c)

4. Find B and C of a triangle ABC, if $b = 2$ cm, $c = 1$ cm and $A = 60°$:

(a) 90° and 45° (b) 90° and 30°
(c) 60° and 30° (d) none of these

5. In a triangle ABC, $a = 1$ cm, $b = \sqrt{3}$ cm and $C = \dfrac{\pi}{6}$, find the third side of the triangle :

(a) 1 cm (b) 1.5 cm
(c) 2 cm (d) $3\sqrt{3}$ cm

6. If the angles of a triangle are in the ratio $1:2:3$, find the ratio between corresponding sides :

(a) $3:2:1$ (b) $1:\sqrt{3}:2$
(c) $6:3:2$ (d) $1:\sqrt{2}:\sqrt{3}$

7. If $a = 2b$ and $A = 3B$, find the angles A, B, C of ΔABC :

(a) 90°, 30°, 60° (b) 30°, 60°, 90°
(c) 45°, 45°, 90° (d) data insufficient

8. If in any triangle $a = 13$ cm, $b = 14$ cm and $c = 15$ cm, find the inradius of the circle :

(a) 2 cm (b) 6.5 cm
(c) 4 cm (d) 7 cm

9. In a right angled triangle $a^2 + b^2 + c^2$ is :

(a) R^2 (b) $4R^2$
(c) $8R^2$ (d) none of these

10. Abhinav and Brijesh started walking simultaneously @ 3 km/h and 4 km/h from the same point 'C' on two different paths which diverge from each other at an angle of 120°. They walked for 6 hours and they stopped at A and B respectively. What is the least possible distance between Abhinav and Brijesh when they are at A and B respectively.

(a) $36\sqrt{37}$ (b) $6\sqrt{37}$ km
(c) $12\sqrt{37}$ (d) none of these

HEIGHT AND DISTANCE

Angle of Elevation : $\angle OPM$ is called as the angle of elevation.

Suppose a person P is looking an object O which is at a higher level than P, then the angle $\theta\,(\angle OPM)$ is the angle of elevation for that person.

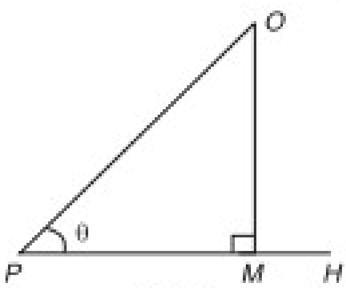


Fig. (i)

Angle of Depression : $\angle OPM$ in fig (ii) is called as angle of depression.

Suppose a person P is looking downward an object O, which is at a lower level than P, then the angle $\theta\,(\angle OPM)$ is called as the angle of depression.

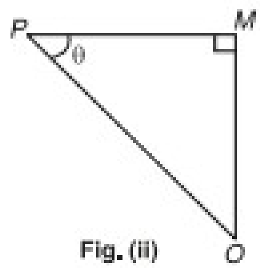


Fig. (ii)

NOTE It should be noted that the angle of elevation of one position as seen from the other is equal to the angle of depression of the latter as seen from the former.

EXAMPLE 1 *From the top of a tower 180 m high, it was observed that the angle of depression of the bottom of a cat sitting on the ground was 30°. Find the distance of the cat from the foot of the tower.*

SOLUTION Let PT be the tower and cat is sitting at C

then $\angle MTC = \angle TCP$

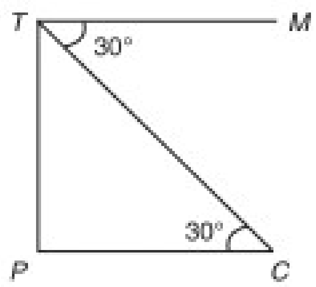
Now in ΔCPT, $\angle TCP = 30°$

$\because$ $\tan 30° = \dfrac{TP}{PC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{180}{PC}$

$\Rightarrow$ $PC = 180\sqrt{3}$ m

$= 311.76$ m



Hence cat is 311.76 m away from the foot of the tower.

NOTE For a particular case, the angle of depression is equal to the angle of elevation.

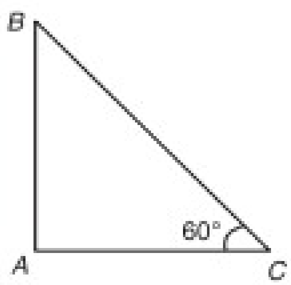
EXAMPLE 2 *The angle of elevation of the top of a tower at a distance of 100 m from its foot on a horizontal plane is found to be 60°. Find the height of the tower.*

SOLUTION Let AB be the tower

then $\tan 60° = \dfrac{AB}{AC}$

$\sqrt{3} = \dfrac{AB}{100}$

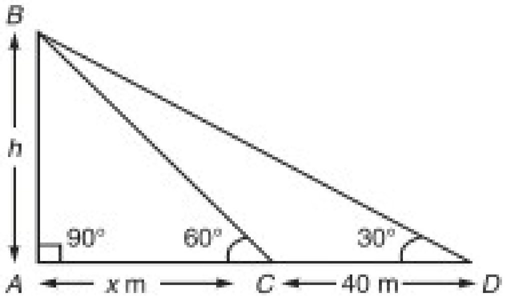
$AB = 100\sqrt{3}$ m $= 173.2$ m



Thus the height of the tower is 173.2 m.

EXAMPLE 3 *A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retreats 40 m from the bank he finds the angle to be 30°. Find the height of the tree and the breadth of the river.*

SOLUTION Let height of the tree be AB and width of the river be AC.



In ΔABC, $\tan 60° = \dfrac{h}{x}$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad \ldots(1)$$

and in ΔABD, $\dfrac{AB}{AD} = \tan 30°$

$$\frac{h}{(x+40)} = \frac{1}{\sqrt{3}} \quad \ldots(2)$$

From equation (1) and (2), we get

$$\frac{x\sqrt{3}}{(x+40)} = \frac{1}{\sqrt{3}}$$

$$x = 20 \text{ m}$$

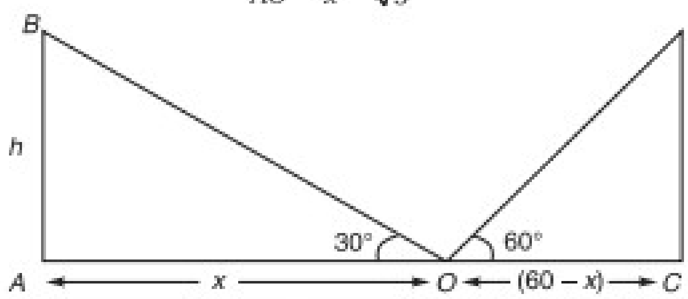
$\therefore \quad h = x\sqrt{3} = 20\sqrt{3} = 34.64$ m (approx.)

Thus the height of the tree is 34.46 m and breadth of the river is 20 m.

EXAMPLE 4 *Two towers of the same height stand on either side of a road 60 m wide. At a point on the road between the towers, the elevations of the towers are 60° and 30°. Find the height of the towers and the position of the point.*

SOLUTION In ΔAOB

$$\frac{AB}{AO} = \frac{h}{x} = \frac{1}{\sqrt{3}}$$



and in ΔCOD, $\dfrac{CD}{OC} = \dfrac{h}{(60-x)} = \sqrt{3} \quad \ldots(1)$

from equation (1) and (2)

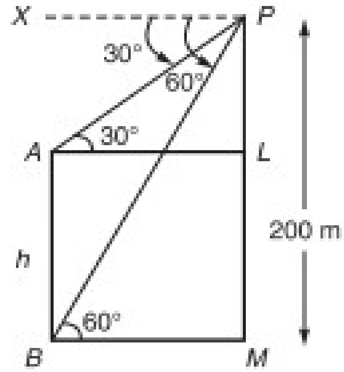
$$\frac{x}{\sqrt{3}(60-x)} = \sqrt{3} \Rightarrow x = 45 \text{ m}$$

$$\therefore \quad h = \frac{45}{\sqrt{3}} = 15\sqrt{3} = 15 \times 1.732 = 25.98 \text{ m}$$

Hence, the height of the towers is 25.98m and the distance of the point 'O' from A is 45 m and C is 15 m.

EXAMPLE 5 *From the top of a cliff, 200 m high, the angle of depression of the top and bottom of a tower are observed to be 30° and 60°, find the height of the tower.*

SOLUTION Let the height of the tower be h m, then



In ΔPBM, $\tan 60° = \dfrac{PM}{BM}$

$$\sqrt{3} = \frac{200}{BM}$$

$BM = \dfrac{200}{\sqrt{3}}$ m $= AL$ (also)

Now, in ΔALP,

$$\tan 30° = \frac{PL}{AL}$$

$$\frac{1}{\sqrt{3}} = \frac{PL}{(200/\sqrt{3})}$$

$$\Rightarrow \quad PL = \frac{200}{3}$$

$$\therefore \quad LM = PM - PL = 200 - \left(\frac{200}{3}\right)$$

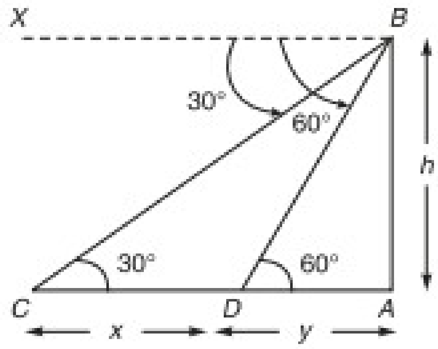
$$LM = \frac{400}{3}$$

But $LM = AB = h$

$\therefore$ Height of the tower $(h) = \dfrac{400}{3}$ m

EXAMPLE 6 *A man on the top of a rock rising on a seashore observes a boat coming towards it. If it takes 10 minutes for the angle of depression to change form 30° to 60°, how soon the boat reach the shore ?*

SOLUTION Let AB be the rock of height 'h' metres.



Let C and D be the two positions of boat such that

$$\angle ACB = \angle XBC = 30°$$

and $\angle ADB = \angle XBD = 60°$

Let $CD = x$ m and $AD = y$ m

Now, in ΔABD, $\tan 60° = \dfrac{h}{y}$

$$\sqrt{3} = \frac{h}{y} \quad \ldots(1)$$

and in ΔABC, $\tan 30° = \dfrac{h}{x+y} = \dfrac{1}{\sqrt{3}} \quad \ldots(2)$

from equation (1) and (2), we get

$$\frac{\sqrt{3}y}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x = 2y \Rightarrow y = \frac{x}{2}$$

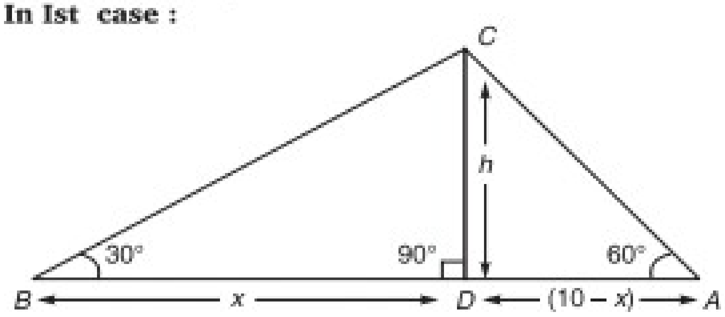
Since the boat takes 10 minutes to cover x m, hence it will take $\dfrac{10}{2} = 5$ minutes to cover $y = \dfrac{x}{2}$ metres. Thus the required time

$= 5$ minutes.

EXAMPLE 7 *The angles of elevation of an aeroplane from two places 10 km apart are found to be 60° and 30° respectively. Find the height of the aeroplane.*

SOLUTION Let A and B be the two places such that $AB = 10$ kms and let C be the position of the aeroplane at a height of h metres above AB. Let CD be perpendicular to AB.

In Ist case :



In ΔBCD, $$\frac{h}{x} = \frac{1}{\sqrt{3}} \quad \ldots(1)$$

and $$\frac{h}{10 - x} = \sqrt{3} \quad \ldots(2)$$

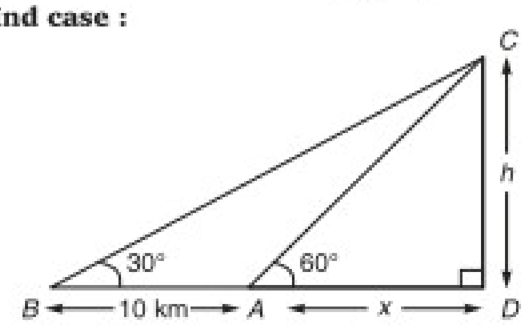
from equation (1) and (2)

$$\frac{x}{\sqrt{3}\,(10 - x)} = \sqrt{3} \Rightarrow x = 7.5 \text{ km}$$

$$\therefore \quad h = \frac{7.5}{\sqrt{3}} = \frac{7.5}{3} \times \sqrt{3} = 2.5 \times \sqrt{3}$$

$$= 4.33 \text{ km (approx.)}$$

In IInd case :



In ΔACD, $$\frac{h}{x} = \sqrt{3} \quad \ldots(1)$$

In ΔBCD, $$\frac{h}{10 + x} = \frac{1}{\sqrt{3}} \quad \ldots(2)$$

Therefore from equation (1) and (2)

$$h = 5\sqrt{3} \text{ km} \Rightarrow h = 8.66 \text{ km}$$

INTRODUCTORY EXERCISE-11.6

1. Two towers of the same height stand on opposite sides of a road 100 m wide. At a point on the road between the towers, the elevations of the towers are 30° and 45°. Find the height of the towers and the position of the point from the nearest tower :
(a) 36.6 m and 63.4 m (b) 63.3 m and 63.4 m
(c) 66.3 m and 63.4 m (d) 36.6 m and 86.4 m

2. The elevation of a tower at a station A due north of it is 45° and at a station B due west of A is 30°. If $AB = 40$ m, find the height of the tower :
(a) 26.26 m (b) 28.28 m
(c) 38.5 m (d) none of these

3. If the shadow of a tower is 30 m when the sun's altitude is 30° what is the length of the shadow when the sun's altitude is 60° ?
(a) $10\sqrt{3}$ m (b) 20 m
(c) 10 m (d) 12 m

4. A helicopter is in a stationary position at a certain height over the lake. At a point 200 m above the surface of the lake, the angle of elevation of the helicopter is 45°. At the same time, the angle of depression of its reflection in the lake is 75°. Calculate the height of the helicopter from the surface of the lake :
(a) $\frac{200}{\sqrt{3}}$ m (b) $200\sqrt{3}$ m
(c) 200 m (d) 150 m

5. A vertical tower stands on a horizontal plane and surmounted by a flagstaff of height 'h'. At a point on the plane, the angle of elevation of bottom of the flagstaff is α and that of the top of the flagstaff is β. Find the height of the tower :
(a) $h \tan \alpha$ (b) $\frac{h \tan \alpha}{\tan \alpha - \tan \beta}$
(c) $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$ (d) $\frac{\tan \alpha + \tan \beta}{h \tan \alpha}$

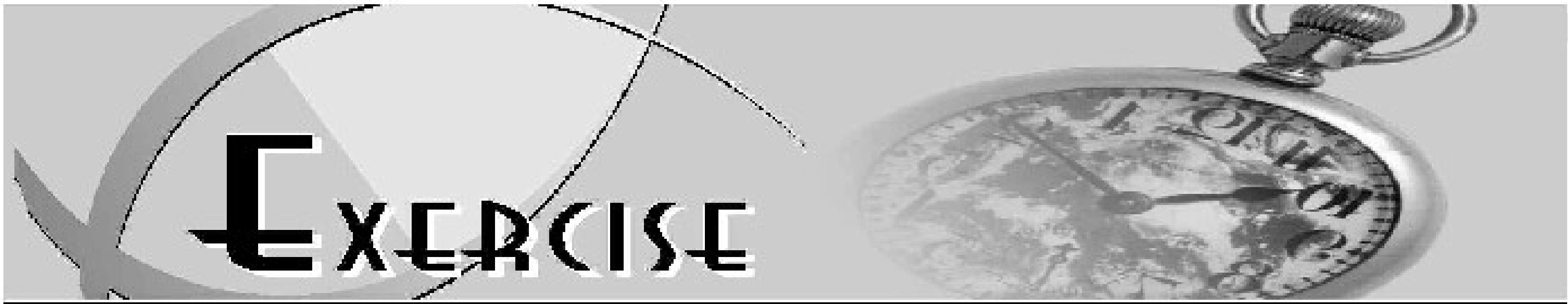
6. A round balloon of radius 'r' subtends an angle α at the eye of the observer, while the angle of elevation of its centre is β. Find the height of the centre of balloon.
(a) $r \operatorname{cosec}\left(\frac{\alpha}{2}\right) \sin \beta$ (b) $r \sin \alpha \operatorname{cosec} \beta$
(c) $\frac{r}{2} \sin \beta$ (d) $r \sec\left(\frac{\alpha}{2}\right)$

7. At the foot of a mountain the elevation of its summit is 45°, after ascending 1000 m towards the mountain up a stop of 30° inclination, the elevation is found to be 60°. Find the height of the mountain :
(a) 1.3 km (b) 1.366 km
(c) 2.72 km (d) none of these

8. An aeroplane when 3000 m high passes vertically above another aeroplane at an instant when their angles of elevation at the same observing point are 60° and 45° respectively. How many metres higher is the one than the other?
(a) 1248 m (b) 1188 m
(c) 1752 m (d) 1268 m

9. The angle of elevation of a cloud from a height h above the level of water in a lake is α and the angle of depression of its image in the lake is β. Find the height of the cloud above the surface of the lake :
(a) $\frac{h \sin(\beta - \alpha)}{\sin(\alpha + \beta)}$ (b) $h \sin \alpha$
(c) $\frac{h \sin(\alpha + \beta)}{\sin(\beta - \alpha)}$ (d) none of these

10. A palm tree 90 ft high, is broken by the wind and its upper part meet the ground at an angle of 30°. Find the distance of the point where the top of the tree meets the, ground , from its root :
(a) 43.69 ft (b) 51.96 ft
(c) 60 ft (d) 30 ft



LEVEL 1

1. If $0 < \theta < 90°$, then $(\sin\theta + \cos\theta)$ is :
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) greater than 2

2. The value of x satisfying the equation $\sin x + \dfrac{1}{\sin x} = \dfrac{7}{2\sqrt{3}}$ is :
(a) 10° (b) 30°
(c) 45° (d) 60°

3. If $\sin\theta - \cos\theta = 0$ and $0 < \theta \le \pi/2$, then θ is equal to :
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{6}$ (d) 0

4. Given that θ is acute and then $\sin\theta = \dfrac{3}{5}$. Let x, y be positive real numbers such that $3(x - y) = 1$, then one set of solutions for x and y expressed in terms of θ is given by :
(a) $x = \sec\theta$, $y = \operatorname{cosec}\theta$ (b) $x = \cot\theta$, $y = \tan\theta$
(c) $x = \text{csoec}\,\theta$, $y = \cot\theta$ (d) $x = \sec\theta$, $y = \tan\theta$

5. Which one of the following pairs is correctly matched ?

If	then
(a) $x = \dfrac{1+\sin 60° - \cos 60°}{1+\sin 60° + \cos 60°}$	$x = \tan 60°$
(b) $x = \dfrac{1+\sin 90° - \cos 90°}{1+\sin 90° - \cos 90°}$	$x = \tan 30°$
(c) $x = \dfrac{2\tan 30°}{1-\tan^2 30°}$	$x = \tan 60°$
(d) $x = \dfrac{1-\tan^2 30°}{1+\tan^2 30°}$	$x = \cos 60°$

6. If $x\tan 45° \,.\, \cos 60° = \sin 60° \cot 60°$, then x is equal to
(a) 1 (b) 1/2
(c) $\sqrt{3}$ (d) $1/\sqrt{2}$

7. If θ lies in the second quadrant, then $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}$ is equal to :
(a) $-2\sec\theta$ (b) $2\sec\theta$
(c) $2\operatorname{cosec}\theta$ (d) $2\tan\theta$

8. $\sin^6 A + \cos^6 A$ is equal to :
(a) $1 - 3\sin^2 A\cos^2 A$ (b) $1 - 3\sin A\cos A$
(c) $1 + 3\sin^2 A\cos^2 A$ (d) 1

9. If $\sec x = P$, $\operatorname{cosec} x = Q$, then :
(a) $P^2 + Q^2 = PQ$ (b) $P^2 + Q^2 = P^2Q^2$
(c) $P^2 - Q^2 = P^2Q^2$ (d) $P^2 + Q^2 = -P^2Q^2$

10. $\sin^2 A\cos^2 B - \cos^2 A\sin^2 B$ simplifies to :
(a) $\sin^2 A + \sin^2 B$ (b) $\cos^2 A + \cos^2 B$
(c) $\sin^2 A - \sin^2 B$ (d) $\sin^2 A - \cos^2 B$

11. If $\sin 2x = n\sin 2y$, then the value of $\dfrac{\tan(x+y)}{\tan(x-y)}$ is :
(a) $\dfrac{n+1}{n-1}$ (b) $\dfrac{n-1}{n+1}$
(c) $\dfrac{1-n}{n+1}$ (d) $\dfrac{1+n}{1-n}$

12. The least value of $2\sin^2\theta + 3\cos^2\theta$ is :
(a) 1 (b) 2
(c) 3 (d) 5

13. The value of $\tan(180 + \theta)\cdot\tan(90 - \theta)$ is :
(a) 1 (b) -1
(c) 0 (d) none of these

14. $\log\tan 1° + \log\tan 2° + \ldots. + \log\tan 89°$ is :
(a) 1 (b) $1/\sqrt{2}$
(c) 0 (d) -1

15. If we convert $\sin(-566°)$ to same trigonometrical ratio of a positive angle lying between 0° and 45°, then we get :
(a) cos 26° (b) $-$ cos 26°
(c) sin 26° (d) $-$ sin 26°

16. From the mast head of ship, the angle of depression of a boat is 60°. If the mast head is 150 m, then the distance of the boat from the ship is :
(a) 86.6 m (b) 68.6 m
(c) 66.8 m (d) none of these

17. A portion of a 30 m long tree is broken by tornado and the top struck up the ground making an angle 30° with ground level. The height of the point where the tree is broken is equal to :
(a) $30/\sqrt{3}$ m (b) 10 m
(c) $30\sqrt{3}$ m (d) 60 m

18. Two posts are 25 m and 15 m high and the line joining their tips makes an angle of 45° with horizontal. The distance between these posts is :
(a) 5 m (b) $10/\sqrt{2}$ m
(c) 10 m (d) $10\sqrt{2}$ m

19. The angle of elevation of the top of a tower at a point G on the ground is 30°. On walking 20 m towards the tower the angle of elevation becomes 60°. The height of the tower is equal to :
(a) $10/\sqrt{3}$ m (b) $20\sqrt{3}$ m
(c) $20/\sqrt{3}$ m (d) $10\sqrt{3}$ m

20. If $x = \sec\theta + \tan\theta$, $y = \sec\theta - \tan\theta$, then the relation between x and y is :

(a) $x^2 + y^2 = 0$ (b) $x^2 = y^2$
(c) $x^2 = y$ (d) $xy = 1$

21. The value of θ for which $\sqrt{3}\cos\theta + \sin\theta = 1$ is :

(a) 0 (b) $\pi/3$
(c) $\pi/6$ (d) $\pi/2$

22. If $\tan\theta = \frac{4}{3}$, then the value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is :

(a) 1 (b) 2
(c) 3 (d) 4

23. If the arcs of the same length in two circles subtend angles of 60° and 90° at the centre, then the ratio of their radii is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) 2

24. In the third quadrant, the values of $\sin\theta$ and $\cos\theta$ are :

(a) positive and negative respectively
(b) negative and positive respectively
(c) both positive
(d) both negative

25. The value of $\frac{\cot 40°}{\tan 50°} - \frac{1}{2}\frac{\cos 35°}{\sin 55°}$ is :

(a) 1 (b) -1
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

26. The value of $\theta (0 \le \theta \le \pi/2)$ satisfying the equation $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

27. If $\cos\theta = \frac{4}{5}$ and $0 < \theta < 90°$, then the value of $\frac{3\cos\theta + 2\operatorname{cosec}\theta}{4\sin\theta - \cot\theta}$ is :

(a) $-\frac{43}{2}$ (b) $-\frac{41}{2}$
(c) $\frac{43}{8}$ (d) $-\frac{43}{6}$

28. Maximum value of $(\cos\theta - \sin\theta)$ is :

(a) $\sqrt{2}$ (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

29. The value of $\sin 105°$ is :

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$

30. If $\tan\theta = t$, then $\sin 2\theta$ is equal to :

(a) $\frac{1}{1+t^2}$ (b) $\frac{2t}{1+t^2}$
(c) $\frac{t^2}{1+t}$ (d) $\frac{1+t^2}{1+t}$

31. If $\tan\theta = \sqrt{2}$, then the value of θ is :

(a) less than $\frac{\pi}{4}$ (b) equal to $\frac{\pi}{4}$
(c) between $\frac{\pi}{4}$ and $\frac{\pi}{3}$ (d) greater than $\frac{\pi}{3}$

32. If $\tan\theta = 2-\sqrt{3}$, then $\tan(90-\theta)$ is equal to :

(a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
(c) $3+\sqrt{2}$ (d) $3-\sqrt{2}$

33. If from point 100 m above the ground the angles of depression of two objects due south on the ground are 60° and 45°, then the distance between the objects is :

(a) $\frac{50(3-\sqrt{3})}{3}$ m (b) $\frac{50(3+\sqrt{3})}{3}$ m
(c) $\frac{100(3+\sqrt{3})}{3}$ m (d) $\frac{100(3-\sqrt{3})}{3}$ m

34. If the length of shadow of a vertical pole on the horizontal ground is $\sqrt{3}$ times of its height, then the angle of elevation of sun is :

(a) 15° (b) 30°
(c) 45° (d) 60°

35. A kite is flown with a thread of 250 m length. If the thread is assumed to be stretched and makes an angle of 60° with the horizontal, then the height of the kite above the ground is (approx.) :

(a) 216.25 m (b) 215. 25 m
(c) 212.25 m (d) 210.25 m

Answers

INTRODUCTORY EXERCISE- 11.1

1 (a)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)					

INTRODUCTORY EXERCISE- 11.2

1 (a)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (a)
11. (b)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (d)		

INTRODUCTORY EXERCISE- 11.3

1 (d)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (c)					

INTRODUCTORY EXERCISE- 11.4

1 (a)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (a)		

INTRODUCTORY EXERCISE- 11.5

1 (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (b)

INTRODUCTORY EXERCISE- 11.6

1 (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (b)

LEVEL-1

1 (c)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (d)
21. (d)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (b)
31. (c)	**32.** (a)	**33.** (d)	**34.** (b)	**35.** (a)					

Hints & Solutions

LEVEL 1

1. Let $$z = \sin\theta + \cos\theta$$

$\Rightarrow$ $$z^2 = 1 + \sin 2\theta$$

$\because$ $0 < \theta < 90°$ so $\sin 2\theta < 1$, so that $z^2 < 2$,

Thus $z < \sqrt{2}$ *i.e.*, z is greater than 1.

2. Go through option.

3. $$\sin\theta - \cos\theta = 0$$

$\Rightarrow$ $$\sin\theta = \cos\theta$$

$\Rightarrow$ $$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

4. Go through option.

HINT $\sin\theta = \frac{3}{5}$ and $\text{cosec}\,\theta = \frac{5}{3}$

7. $$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \frac{(1-\sin\theta)+(1+\sin\theta)}{\cos\theta}$$

$$= 2\sec\theta$$

8. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

Let $a = \sin^2\theta$, $b = \cos^2\theta$, so that

$$a + b = \sin^2\theta + \cos^2\theta = 1$$

$\Rightarrow$ $$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$$

9. $$\cos x = \frac{1}{P} \quad \text{and} \quad \sin x = \frac{1}{Q}$$

$$1 = \cos^2 x + \sin^2 x = \frac{1}{P^2} + \frac{1}{Q^2}$$

$\Rightarrow$ $$P^2 + Q^2 = P^2Q^2$$

10. $\sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B = \sin^2 A - \sin^2 B$

11. $$\frac{n}{1} = \frac{\sin 2x}{\sin 2y}$$

$\Rightarrow$ $$\frac{n+1}{n-1} = \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\sin(x-y)}$$

$$= \frac{\tan(x+y)}{\tan(x-y)}$$

12. The value is least when $\theta = 90°$.

14. $\log\tan 1° + \log\tan 89° = \log\tan 1° + \log\tan(90 - 1)$

$$= \log\tan 1° + \log\cot 1°$$

$$= \log\tan 1° \cdot \cot 1° = \log 1 = 0$$

Similarly, $\log\tan 2° + \log\tan 88° = 0$

Also, $\log\tan 45° = \log 1 = 0$

Thus, the value of the expression is 0.

15. $\sin(-566°) = -\sin(566°)$

$$= -\sin(90 \times 6 + 26) = \sin 26°.$$