Permutation and Combination
by Total Gadha - Sunday, 17 August 2008, 11:13 AM

Dagny has been telling me smugly that itâ€™s now my turn to write an article. So I decided to write one as soon as possible and pass the buck back to her. We both know that it takes a lot of pain to research for an article and write the text so we both try to avoid it for as long as possible. On the other hand, since we both enjoy writing and like to publish a well-finished chapter, we both undertake the task eagerly once the ball is in our court. Writing these chapters have made me realize how true is the maxim â€œif you want to learn something well, teach it.â€ I can cite a lot of topics which I learnt while teaching in the class or answering studentsâ€™ questions. These years of teaching have made so many topics seem pretty childish to me which were nightmares to me before. But knowing a topic well and teaching it are two different ball games. Many a times, I have found myself struggling in the class to make a student understand something which I have found obvious to understand! The fault is not on the studentâ€™s part. Three or four years ago, I would have been stuck on the same point! Every time I encounter a situation like this, my only measure is to stop at that point, retrace my steps with the students and slowly unravel the difficulty heâ€™s facing. It works most of the times. But sometimes, the student is in so much awe with the topic that his mind gets frozen. I have seen this happening many a times. Topics such as time, speed and distance, Permutation and Combination, geometry etc. inspire so much fear among the students that sometimes simple principles, which they would have otherwise understood do not strike them as simple. I get incessant queries such as â€œSir, time speed distance chhod sakte hain?,â€ â€œSir, permutation combination na karein to chalega?â€ from my students all the time. And the sad part is, that the level of CAT in these topics is very simple. They are doable. In the present chapter, I am trying to present one of these dreaded chapters as I understand it. I hope my students understand it too.

Let us start this chapter with a simple example:

Ravi has his IIM Ahmedabad interview two days later and he has come to a garment store to buy a dress for his interview. In the store, Ravi notices 20 trousers and 30 shirts that he can buy. In how many ways can Ravi buy

1) **1 shirt and 1 trouser?**
2) **only one garment out of these?**

Answer: Let the trouser be $T_1, T_2, T_3, \ldots T_{19}, T_{20}$ and the shirts be $S_1, S_2, S_3, \ldots S_{29}, S_{30}$.

1) One shirt and one trouser- How many pairs of one shirt and one trouser can we make. The pairs will be from (T_1S_1, T_1S_2, T_1S_3, … T_1S_{30}) to ($T_{20}S_1$, $T_{20}S_2$, $T_{20}S_3$, …$T_{20}S_{30}$). For every trouser, Ravi can make 30 pairs. As there are 20 trousers, the number of pairs will be $20 \times 30 = 600$.
2) There are $20 + 30 = 50$ garments in all, out of which we have to choose one garment. Since we can choose any one of the 50 garments, the number of ways is = 50.

Notice that when we were choosing shirt **AND** trouser we multiplied and when were choosing shirt **OR** trouser we added. This principle is known as fundamental principle of counting:

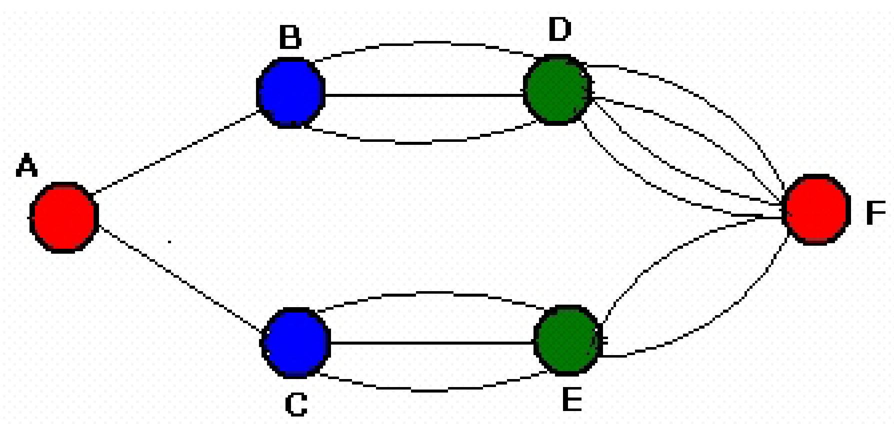
FUNDAMENTAL PRINCIPLE OF COUNTING:

If one operation can be performed in *m* ways, and a second operation can be performed in *n* ways, the number of ways of performing both the operations will be $m \times n$ and the number of ways of performing either one of the operations will be $m + n$.

TIPS
1: In case of **OR** we **add**.
2. In case of **AND** we **multiply**.

Examples

- *The number of routes between few cities A, B, C, D, E, and F are shown in the figure. Find*



1. *In how many ways can you go from city A to city F via city B?*
Answer: We can go from A to B in one way, from B to D in 3 ways, and from D to F in 4 ways. Since we have to go from A to B, **AND** from B to D, **AND** from D to F the number of ways = $1 \times 3 \times 4 = 12$

2. *In how many ways can you go from city A to city F via city C?*
Answer = $1 \times 3 \times 2 = 6$

3. *Find the total number of ways you can go from city A to city F.*
Answer We can go from A to F either via city B **OR** via city C. Hence the number of ways = 12 + 6 = 18

- *In how many ways can you post 10 letters in 4 letterboxes?*

Answer: The first letter can be posted in 4 ways, the second can be posted in 4 ways, the third can be posted in 4 ways… and so on. Since we have to post all the letters (**the case of AND**) the number of ways = $4 \times 4 \times 4 \ldots = 4^{10}$.

- *Four friends go to a city in which there are 10 hotels.*

1. *In how many ways can they stay?*
Answer: the first friend can stay in 10 ways, the second friend can stay in 10 ways, the third friend can stay in 10 ways and the fourth friend can stay in 10 ways. Since **all** the friends are staying (**AND**), the number of ways in which the friends can stay = $10 \times 10 \times 10 \times 10 = 10^4 = 10\,000$

2. *In how many ways can they stay if no two friends stay together?*
Answer: If no two friends stay together, the first friend can stay in 10 ways, the second friend can stay in 9 ways, the third friend can stay in 8 ways and the fourth friend can stay in 7 ways. Hence the number of ways in which no two friends stay together = $10 \times 9 \times 8 \times 7 = 5\,040$

3. *In how many ways can they stay if at least two friends stay together?*
Answer: The total number of ways in which the four friends can stay is 10^4. The number of ways in which no two friends stay together is 5 040. Therefore, the number of ways in which at least any two friends stay together = 10 000 – 5040 = 4 960.

- How many functions are there from a set with 5 elements to one with 7 elements? (Counting Functions).
  Answer: A function corresponds to a choice of one of the 7 elements in the co-domain for each of the 5 elements in the domain. Hence, by the product rule there are $7 \times 7 \times 7 \times 7 \times 7 = 7^5 = 16807$ functions from a set with 5 elements to one with 7 elements.

- How many one-to-one functions are there from a set with 5 elements to one with 7 elements? (Counting Functions).
  Answer: Suppose the elements in the domain are a_1, a_2, a_3, a_4 and a_5. There are 7 ways to choose the value of the function at a1. There are 6 ways to choose value of the function at a2, and so on. Therefore, the number of one-to-one functions is $= 7 \times 6 \times 5 \times 4 \times 3 = 2520$

- How many one-to-one functions are there from a set with 7 elements to one with 5 elements? (Counting Functions).
  Answer: zero.

- *How many three-digit numbers are there?*
  Answer: We have to fill three places with three digits. The first digit cannot be 0. Therefore, we can choose the first digit in 9 ways. We can choose the second and third digits in 10 ways each. Therefore, the total number of ways of choosing = Total number of three-digit numbers = $9 \times 10 \times 10 = 900$.

- *How many three-digit numbers are there which have at least one 7 in them?*
  Answer: To find the number of three-digit numbers with at least one 7 in them, we first find the number of three-digit numbers with no 7 in them, and then subtract this number from the total number of three-digit numbers. To find the number of three-digit numbers with no 7 in them we proceed as follows: Now the first digit cannot be 0 or 7. Therefore, we choose the first digit in 8 ways. The rest two digit cannot be 7 and we can choose them in 9 ways. Hence, the number of ways = number of three-digit numbers without the digit 7 = $8 \times 9 \times 9 = 648$. Therefore, total number of numbers with at least one 7 in them $= 900 - 648 = 252$.

NOTE: If there is a condition given, we fulfill the condition first.

- How many three-digit numbers are there which are even and have no repeated digits? (Here we are using all digits 0 through 9).
  Answer: For a number to be even it must end in 0, 2, 4, 6 or 8. There are two cases to consider.

  The number ends in 0; then there are 9 possibilities for the first digit and 8 possibilities for the second since no digit can be repeated. Hence there are $9 \times 8 = 72$ three-digit numbers that end in 0.

  The number does not end in 0; Then there are 4 choices for the last digit (2, 4, 6 or 8). When this digit is specified, then there are only 8 possibilities for the first digit, since the number cannot begin with 0. Finally, there are 8 choices for the second digit and therefore there are $8 \times 8 \times 4 = 256$ numbers that do not end in 0. Accordingly since these two cases are mutually exclusive, the sum rule gives 72 + 256 = 328 even three-digit numbers with no repeated digits.

- There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned either to person 3 or person 4. Every person must be assigned one task. In how many ways can the assignment be done? **(CAT 2006)**

Answer: We first fulfill the condition of assigning task 2. Task 2 can be assigned in 2 ways. Then task 1 can be assigned in 3 ways. The other 4 tasks can be assigned in $4 \times 3 \times 2 \times 1 = 24$ ways. Therefore, the number of ways in which the assignment can be done $= 2 \times 3 \times 24 = 144$.

- How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no repetition is allowed?
  Answer: The number of digits is not specified in this problem so we can form one-digit numbers, two-digit numbers, or three digit numbers, etc. But since no repetitions are allowed and we have only the 7 numbers to work with, the maximum number of digits would have to be 7.

  Applying the product rule, we see that we may form 7 one-digit numbers, $7 \times 6 = 42$ two-digit numbers $7 \times 6 \times 5 = 210$ three digit numbers, $7 \times 6 \times 5 \times 4 = 840$ four digit numbers, $7 \times 6 \times 5 \times 4 \times 3 = 2520$ five digit numbers, $7 \times 6 \times 5 \times 4 \times 3 \times 2 = 5040$ six-digit numbers, and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$ seven-digit numbers. The events of forming one-digit numbers, two digit numbers, three digit numbers, etc., are mutually exclusive events so we apply the sum rule to see that there are 7 + 42 + 210 + 840 + 2520 + 5040 + 5040 = 13699 different numbers we can form according to the problem.

- How many 7-digit numbers are there in binary (base 2)? How many of them are divisible by 8 when converted to base 10?
  Answer: In binary, each place can be filled by two digits, 0 or 1. Also, the first place cannot be filled by a 0. Therefore, the number of 7-digit numbers in binary $= 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.
  If the numbers are divisible by 8 when converted to base 10, the last 3 digits of the numbers will be 0. Therefore, the number of numbers divisible by 8 $= 1 \times 2 \times 2 \times 2 \times 1 \times 1 \times 1 = 8$.

- *How many different words, with or without meaning can you form by arranging the letters of the word **ROCKET**?*
  Answer: We are going to form a six-letter word. The first letter can be filled in 6 ways, the second letter can be filled in 5 ways, the third letter can be filled in 4 ways… and so on. Therefore, the total number of ways of filling up 6 places with the alphabets of ***ROCKET*** $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

- Each user on a computer system has a password, which is six to eight characters long, where each character is an upper case letter or a digit. Each password must contain at least one digit. How many possible passwords are there?
  Answer: Let P be the total number of possible passwords, and let P_6, P_7 and P_8 denote the number of possible passwords of length 6, 7, and 8 respectively. By the sum rule, $P = P_6 + P_7 + P_8$

  We will now find P_6, P_7, and P_8. Finding P_6, P_7 or P_8 directly is difficult. To find P_6 it is easier to find the number of strings of upper case letters and digits that are six characters long, including those with no digits, and subtract from this the number of strings with no digits. By the product rule, the number of strings of six characters is 36^6 and the number of strings with no digits is 26^6.
  Hence, $P_6 = 36^6 - 26^6 = 2, 176, 782, 336 - 308, 915, 776 = 1, 867, 866, 560$.
  Similarly, it can be shown that $P_7 = 36^7 - 26^7 = 78, 364, 164, 096 - 8, 031, 810, 176 = 70, 332, 353, 920$.
  and $P_8 = 36^8 - 26^8 = 2, 821, 109, 907, 456 - 208, 827, 064, 576 = 2, 612, 282, 842, 880$.

  Consequently, $P = P_6 + P_7 + P_8 = 2, 684, 483, 063, 360$.

- In how many ways can the sum of 8 be obtained by rolling two dice if
  1. The dice are distinguishable
     Answer: If the dice are distinguishable, the number of ways we obtain a sum of 8 = (2, 6), (6, 2), (3, 5), (5, 3), (4, 4). As all these ways can occur only separately, number of ways = 5
  2. The dice are indistinguishable
     Answer: If the dice are indistinguishable, the number of ways we obtain a sum of 8 = (2, 6), (3, 5), (4, 4). As all these ways can occur only separately, number of ways = 3

- The number plates in Delhi start with the initials DL and are of the form DL NA NNNN, where A represents an alphabet and N represents a number with the condition that the first N is not equal to zero. Also, the last 4 Ns cannot be all equal to zero. How many maximum number plates can be issued in Delhi?

Answer: The first N can have the value 1 to 9. The alphabet A can be selected from 26 alphabets. The last 4 Ns can be filled in $10 \times 10 \times 10 \times 10 - 1$ ways (the case of all N equal to 0 is removed).
Therefore, maximum number of number plates that can be issued = $9 \times 26 \times 9999 = 2339766$.

- *How many words, with or without meaning, can be made from the letters of the word* ***triangle****,* without repetition, *such that the word begins with* ***t*** *and ends with* ***e****?*
Answer: We can make two-letter word, three-letter words, four-letter words, ... and so on till eight-letter words. Except for two-letter word, all other words will have letters filled between **t** and **e**. Suppose we have to make a n-letter word ($n \le 8$). After putting **t** and **e** at the ends, the remaining $n - 2$ places will have to be filled from the remaining 6 letters. The results are summarized below:
Two-letter word (**te**) = 1
Three-letter words (**t** _ **e**) = 6
Four- letter words (**t** _ _ **e**) = The second place can be filled in 6 ways and the third place can be filled in 5 ways. Therefore, the number of word = $6 \times 5 = 30$.
Five-letter words (**t** _ _ _ **e**) = $6 \times 5 \times 4 = 120$.
Six-letter words (**t** _ _ _ _ **e**) = $6 \times 5 \times 4 \times 3 = 360$.
Seven-letter words (**t** _ _ _ _ _ **e**) = $6 \times 5 \times 4 \times 3 \times 2 = 720$
Eight- letter words (**t** _ _ _ _ _ _ **e**) = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

To find the total number of words we sum these results. Why do we sum? Notice that it is an **OR** case- we will make either a two letter word, **OR** three-letter words, **OR** four-letter words... and so on.

Therefore, the total number of ways = $1 + 6 + 30 + 120 + 360 + 720 + 720 = 1957$

- *How many arrangements can be made out of the letters of the word* ***draught****, such that vowels are never separated?*
Answer: Assume that we tie a string around the vowels A and U so that they are together. If we now consider A and U as one letter, we have 6 letters in all. Now we need to arrange these 6 letters in 6 places, as shown below:

T H D G R AU

As seen earlier, the number of arrangements is = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. Now inside the string A and U can be arranged in two ways, i.e. AU and UA. Therefore, the total number of ways = $2 \times 720 = 1440$.

- *A college group of 5 boys and 5 girls goes to cinema hall to watch 'Spiderman- 3.' They get 10 seats in a row, numbered 1 to 10. Every boy wants to sit with a girl therefore the boys decide that no matter what they will not let any two girls sit together. In how many ways can they sit such that the wish of every boy comes true?*
Answer: To make the desired seating arrangement the guys will have to first sit on the alternate seats and the girls will have to sit in between. There are two arrangements possible, as shown below:

B	G	B	G	B	G	B	G	B	G
1	2	3	4	5	6	7	8	9	10

G	B	G	B	G	B	G	B	G	B
1	2	3	4	5	6	7	8	9	10

In both the arrangements, the boys can sit in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways and then the girls can sit in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. Therefore, the number of ways for one seating arrangement = $120 \times 120 = 14400$.

Therefore, the total number of ways considering both the arrangements = $2 \times 14400 = 28800$.

- *During the 'Spiderman- 3' movie, two of the boys fell in love with a girl sitting next to them. Next time when this group of 10 went to watch 'Good Boy Bad Boy' movie, these two boys wanted to sit with their respective girlfriends. The remaining people don't mind where they sit. In how many ways can this group now sit in a row of 10 seats?*
Answer: Again, assume that each of these two couples is tied with a string. Assuming these two couples as two persons we have to arrange seating for 8 persons. This can be done in $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$. Also, the two persons in each couple can sit in two ways BG and GB. Therefore, the total number of ways of seating the group = $2 \times 2 \times 40320 = 161280$.

- During the 'Good Boy Bad Boy' movie, the couples decided that they wanted to sit at the end of the row in order to have some privacy. How many seating arrangements can be made such that the two couples are sitting at either end?

Find the answer for me please

:MUTATION

- In how many ways can you seat n boys in r seats?
Answer: The first seat can be filled in n ways, the second seat can be filled in $n - 1$ ways, the third seat can be filled in $n - 2$ ways, and so on. The r^{th} seat can be filled in $n - r + 1$ ways.
Therefore, total number of ways = $n \times (n - 1) \times (n - 2) \times \ldots (n - r + 1) = \frac{n \times (n-1) \times (n-2) \cdots (n-r+1) \times (n-r) \cdots 3 \times 2 \times 1}{(n-r) \times (n-r-1) \cdots 3 \times 2 \times 1} = \frac{n!}{(n-r)!}$

Therefore, the number of ways of arranging r things out of n dissimilar things is equal to $\frac{n!}{(n-r)!}$ and this known as ^nP_r. Therefore, $^nP_r = \frac{n!}{(n-r)!} \Rightarrow {}^nP_n = n!$ or number of ways of arranging n dissimilar things = n!

- At an election meeting 10 speakers are to address the meeting. The only protocol to be observed is that whenever they speak P.M. will speak before M.P., and M.P. will speak before M.L.A. In how many ways can the meeting be addressed?
Answer: Let's denote P.M. by P, M.P. by Q and M.L.A. by R. Ten speakers can address in 10! Ways including P, Q, R. Now P, Q, R can be arranged amongst themselves in 3! = 6 ways like PQR, PRQ, QRP, QPR, RPQ, RQP. Since we want only PQR order, hence we will take $1/6^{th}$ of the total. Hence the required number is 10!/6.

Order Matters or NOT?

Does the order matter in the State Lotteries?

We need to think if the numbers on a ticket have to be in the same order as the order in which they became the winning numbers. In
other words, let's assume that the winning numbers are rolled out of the machine in the order of: 1,2,3,4,5,6. Do the numbers on our
ticket have to be in this same order to win? Or will any order such as 2, 3, 1, 5, 6, 4 also be a winning ticket?

The answer is that any order of the winning numbers will produce the winning ticket. Therefore, the order does not matter.

An exacta in horse racing is when we correctly guess which horses will finish first and second. If there are six horses in the race, how many different possible outcomes for the exacta are there?

In this problem the order matters. There are 6 horses, and we want to know how many different ways there are to choose 2 horses at a time where order matters. In other words, once we choose two horses, this count as two arrangements. If horse A comes in first and horse B comes in second, it is a different arrangement then if horse B comes in first and horse A comes in second.

ARRANGEMENT OF THINGS WHEN SOME OF THEM ARE SIMILAR

The number of arrangements of four letters a, b, c, and d is 24:

abcd, abdc, acbd, adbc, acdb, adcb, cabd, dabc, bacd, badc, cadb, dacb, cdab, dcab, cbad, dbac, bcad, bdac, cdba, dcba, dbca, cbda, bcda, bdca.

The number of arrangements of four letters a, a, a, b is 4:
aaab, aaba, abaa, baaa.

In the second case, the number of things is still 4. But what happened to 24 arrangements as in the earlier case?

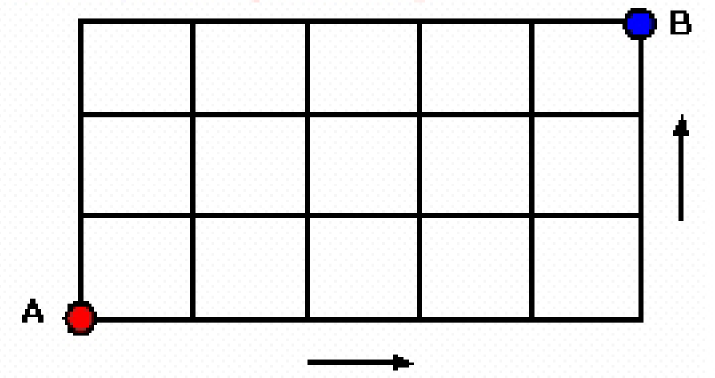
We can see that when the three things, a, c and d, were different, their number of arrangements was 3! = 6. But the moment these three things became similar those 3! arrangements became similar **without** changing the positions of the letters. For example, in the arrangement acbd, we can keep the position of b fixed and arrange a, c and d in 3! ways (acbd, cabd, adbc, dabc, cdba, dcba). But the moment we replace a, c and d by three similar things, these 3! Arrangements become a single arrangement (aaba). Therefore the number of arrangements in this case = $\frac{4!}{3!} = 4$.

We can see that when some of the elements are similar the number of arrangements decrease in number.

The number of ways in which n things may be arranged among themselves, taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest are all different: $\frac{n!}{p!q!r!}$

- In how many ways can the letters in MISSISSIPPI be arranged?
  Answer: The word MISSISSIPPI contains 11 letters of which 4 are S, 4 are I, and 2 are P. If the S's, I's, and P's are **distinguishable** there would be 11! permutations. However, the 4 S's can themselves be arranged 4! different ways as can the 4 I's. The 2 P's have 2! = 2 arrangements. Thus assuming these repeated elements are truly **indistinguishable,** the number of arrangements would be $\frac{11!}{4!4!2!} = 34650$.

- In the figure given below, the lines represent one-way roads allowing travel only northwards or only eastwards. Along how many distinct routes can a car reach point B from point A? **(CAT 2004)**



  Answer: Let an eastward step be denoted by E and a northward step be denoted by N. There are 5 steps eastward and 3 steps northward. Therefore, we are trying to find the total number of ways of arranging EEEEENNN. This is equal to $\frac{8!}{5!3!} = 56$

- How many arrangements of six 0's, five 1's, and four 2's are there in which
  i) the first 0 precedes the first 1?
  ii) the first 0 precedes the first 1, precedes the first 2?
  iii) the first 0 precedes the first 1?

  (i) This is more difficult than it first appears. We have six 0's, five 1's, four 2's = 15 digits. You must consider several non-overlapping configurations. For example, you could have 0 followed by every possible arrangement of the remaining digits, or 20 followed by every possible arrangement of the remaining digits, or 220 followed etc.

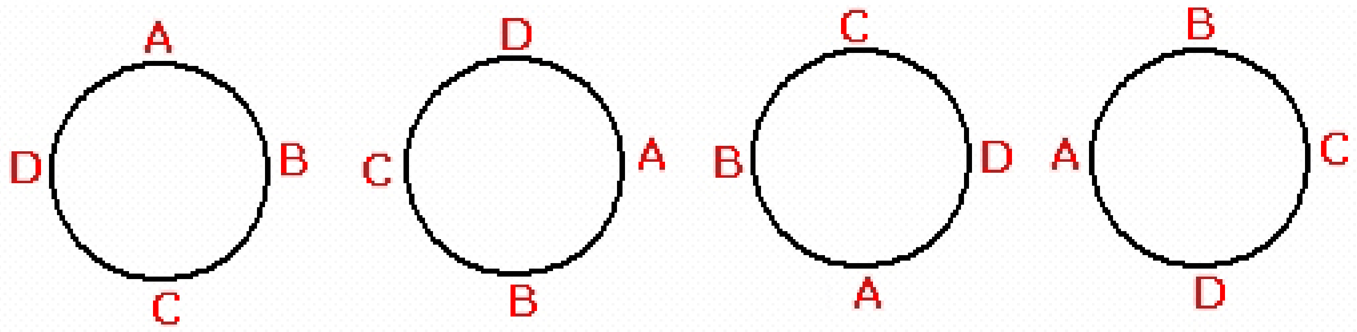
First Positions	Number and type of remaining digits	Number of arrangements
0	Five 0's, five 1's, four 2's	$\frac{14!}{5!5!4!}$
20	Five 0's, five 1's, three 2's	$\frac{13!}{5!5!3!}$
220	Five 0's, five 1's, two 2's	$\frac{12!}{5!5!2!}$
2220	Five 0's, five 1's, one 2	$\frac{11!}{5!5!1!}$
22220	Five 0's, five 1's	$\frac{10!}{5!5!}$

  Total = 343980.

  Solve the other two parts in the same manner.

CIRCULAR ARRANGEMENT

Let's consider that 4 persons A, B, C, and D are sitting around a round table. Shifting A, B, C, D, one position in clockwise direction, we get the following agreements:-



Thus, we use that if 4 persons are sitting at a round table, then they can be shifted four times, but these four arrangements will be the same, because the sequence of A, B, C, D, is same. But if A, B, C, D, are sitting in a row, and they are shifted, then the four linear-arrangement will be different.

A-B-C-D, B-C-D-A, C-D-A-B, D-A-B-C.

We can see that in circular permutation these four arrangements have reduced to one arrangement. Likewise, in circular permutation of n things, a set of n linear arrangements reduce into one arrangement.

Hence the number of permutations in circular arrangement = $\frac{\text{number of permutations in linear arrangement}}{n} = \frac{n!}{n} = (n-1)!$

You can think of circular arrangement in terms of orientation also. The first person can sit in only 1 way since all places are the same. The rest of the persons can now seat with respect to the first person in (n – 1)! ways.

When clock-wise and anti-clock wise arrangements are not different, then observation can be made from both sides, and this will be the same. This happens when we have beads in a necklace or flowers in a garland. Here two permutations will be counted as one. So total permutations will be half and in this case.

Circular permutations for pearls in a necklace etc. = $\frac{(n-1)!}{2}$

Let there be an m-sided table with p chairs on each side and we need to seat mp persons around it. Since all sides are the same, the first person has p option on one side. The rest of the persons can sit in (mp – 1)! ways. Therefore total number of ways = p(mp – 1)!

SELECTION

Suppose we want to take four people, Bhuvan, Mangal, Shyam and Aangad, and arrange them in groups of three at a time where order matters. The number of arrangements will be $^4P_3 = \frac{4!}{(4-3)!} = 24$. The following demonstrates all the possible arrangements:

BMS, BSM, BAS, BSA, BMA, BAM
MBS, MSB, MAS, MSA, MBA, MAB
SBM, SMB, SBA, SAB, SMA, SAM
ABM, AMB, ABS, ASB, AMS, ASM

Now suppose we only wanted to select 3 people out of the four people, Bhuvan, Mangal, Shyam and Aangad. Then what?

Then many of the arrangements are same to us. For example, the arrangements BMS, BSM, MBS, MSB, SBM, SMB are the same to us where selection is concerned because we in effect are selecting the same three people, Bhuvan, Mangal and Shyam!

Therefore, there are only 4 selections of three people- BMS, MSA, SAB, and MAB. What happened to those 24 earlier cases?

We can see that there are 3! = 6 ways of arranging three people. All these 3! ways became one way where selection was concerned.

Therefore, the number of selection of 3 people out of 4 = $\frac{\text{number of arrangements of three people out of four}}{3!} = \frac{4!}{3!(4-3)!}$

COMBINATION

- In how many ways can you select r things out of n different things?
  Answer: As we see saw in the previous example, the number of arrangements of r things out of n different things would be nPr.
  The number of selections would be $\frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

Selection of r things out of n dissimilar things = $\frac{\text{Arrangements of r things out of n dissimilar things}}{r!} = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$. This is known as combination and denoted by ^nC_r. Therefore, $^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

PERMUTATION OR COMBINATION?

In permutation (arrangement) the order matters whereas in combination (selection) the order does not matter.

The **arrangements** of four letters a, b, c, d taken two at a time are twelve in total, namely,
ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc

The **selections** of four letters a, b, c, d taking two letters at a time are six in number: namely,
ab, ac, ad, bc, bd, cd

Understand selection and arrangement in a different manner: Suppose we need to seat r people out of n people. We can first select r people out of n people and then arrange this group of r people in order to seat them. Therefore, selection × r! = arrangement ⇒ $^nC_r \times r! = {}^nP_r$

- A baseball team has 13 members. How many lineups of 9 players are possible?
  Answer: We can first choose 9 players in $^{13}C_9$ ways and then arrange them in 9! Ways. Therefore, the number of ways = $^{13}C_9 \times 9!$
  Or they can straightaway arrange them in $^{13}P_9$ ways.

- A multiple-choice test has 30 questions, each with five choices. How many answer keys are possible?
  Answer: For the first question, 5 answers are possible. For the second question also, 5 answers are possible, and so on. Therefore, total number of possible answer keys = $5 \times 5 \times 5 \times 5 \ldots \times 5 = 5^{30}$.

- In how many ways can four persons be seated out of 5 boys and 3 girls on four different seats?
  Answer: This question is not a simple formulae based permutation or combination question. We'll have to break the problem in cases
  Case1: four boys- number of ways of selection = $^5C_4 = 5$
  Case2: Three boys and one girl- number of ways of selection = $^5C_3 \times {}^3C_1 = 30$
  Case3: Two boys and two girls- number of ways of selection = $^5C_2 \times {}^3C_2 = 30$
  Case4: One boy and three girls- number of ways of selection = $^5C_1 \times {}^3C_3 = 5$

Now for every case we'll do the selection first and then arrange them in 4! ways. For each case the multiplication law of the **principal of counting** will apply since we're both selecting **and** arranging. For finding the total number of ways we'll **add** up all the cases since only **one** of the cases can happen (either-or principal)
Hence the total number of ways = (5 + 30 + 30 + 5) x 4! = 1680.

- If there is a party and every person shakes hands with each other once, and there are 45 handshakes, how many people are there at the party?
  Answer: Let there be n persons in the party. A handshake takes place every time we chose 2 persons out of these n people.
  Therefore, total number of handshakes = total number of ways of choosing two people out of n people = ${}^{n}C_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$
  $\Rightarrow \frac{n(n-1)}{2} = 45 \Rightarrow n = 10$. Therefore, there are 10 people in the party.

- How many diagonals does an n-sided regular polygon have?
  Answer: A diagonal is formed by a line joining two vertices of a polygon. Hence, the number of lines joining two vertices of a polygon = number of ways of selecting two vertices out of n vertices = ${}^{n}C_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$. Out of these $\frac{n(n-1)}{2}$ lines, n of them are forming the sides of the polygons. Therefore, the number of diagonals = $\frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

- How many rectangles are there in 8 × 8 chessboard?
  Answer: To make a rectangle, we need two vertical lines and two horizontal lines. In an 8 × 8 chessboard, there are 9 vertical lines and 9 horizontal lines. Therefore, the number of rectangles = ${}^{9}C_2 \times {}^{9}C_2 = 1296$.

- There are 3 states and 3 students representing each state. In how many ways can 5 students be chosen such that at least one student is chosen from each state?
  Answer: Since at least one student from every state has to be chosen, the ways of selection of 5 students are two:
  (a) Three students from one state and one student from the other two: The state from which three students are chosen can be selected in ${}^{3}C_1$ way. One student each from the remaining two states can be chosen in ${}^{3}C_1$ way. Therefore the total number of ways = ${}^{3}C_1 \times {}^{3}C_1 \times {}^{3}C_1 = 27$ ways

  (b) Two students from one state, two from other, and one from the remaining: Two states can be chosen in ${}^{3}C_2$ ways. From each of these states two students can be chosen in ${}^{3}C_2$ ways. From the remaining state one student can be chosen in ${}^{3}C_1$ way. Therefore, the total number of ways = ${}^{3}C_2 \times {}^{3}C_2 \times {}^{3}C_2 \times {}^{3}C_1 = 81$

  Hence the total number of ways = 27 + 81 = 108

  Wrong Approach!

  An average student approaches the problem in the following manner:

  First, one student from each state can be chosen in ${}^{3}C_1 \times {}^{3}C_1 \times {}^{3}C_1$ ways. Then, the remaining two students can be chosen out of the left 6 students in ${}^{6}C_2$ ways.

  So the total number of ways = ${}^{3}C_1 \times {}^{3}C_1 \times {}^{3}C_1 \times {}^{6}C_2 = 405$

  As can be seen, the answer is higher than the actual answer. The reason is that many cases are being repeated. To illustrate the point let's call the selection of first three students **stage1** and selection of remaining two students **stage2**. So we consider two cases- in one student A is chosen from state 1 and in the other student B is chosen in stage1 from state A.

	CASE1	CASE2
Stage1:	Student A is chosen Among the three students	Student B is chosen Among the three students
Stage2:	Student B is chosen among the two students	Student A is chosen among the two students
Result:	Student A and B are chosen	Student A and B are chosen

  As we can see the two cases give the same result but we're calculating them differently hence the bigger answer.

- How many words can be formed by taking 4 letters at a time out of the letters of the word **MATHEMATICS?**
  Answer: We can choose 4 letters from the 11 listed as under:

  All four different: We have 8 different type of letters and out of these 4 can be arranged in ${}^{8}P_4 = 8!/4! = 1680$ ways
  Two different and two alike: We have 3 pairs of like letters out of which one pair can be selected in 3 ways. Now we have to choose two out of the remaining 7 different types of letters which can be done in ${}^{7}C_2 = \frac{7!}{5!2!} = 21$ ways.

  Hence the total number of groups, of 4 letters in which 2 are different and 2 are alike is 3 x 21 = 63 groups. Each such group has 4 letters out of which 2 are alike and they can be arranged amongst themselves in $\frac{4!}{2!} = 12$ ways.

  Hence the total number of words is 63 x 12 = 756
  Two alike of one kind and two alike of other kind: Out of 3 pairs of like letters we can choose 2 pairs in ${}^{3}C_2 = 3$ ways. These four letters out of which 2 are alike of one kind and 2 alike of other kind, can be arranged in $\frac{4!}{2!2!} = 6$ ways

  Hence the total number of words of this type is 3 x 6 = 18

  Therefore the number of 4 letter words is 1680 + 756 + 18 = 2454

  This problem not only combines selection with arrangement but also is instructive for students who are used to applying formulae because it requires them to break the problem in cases that they have to deal with separately and later on combine all the results.

- In how many different ways can the six faces of a cube be painted with six different colors?
  Answer: The difficulty with this problem is that because the cube is a symmetric solid from all the sides, two arrangements that might seem different will become the same just by turning over the cube or rotating it. So, in order to find different arrangement we first fix one color to one face and find different arrangements of rest of the colors with respect to that face and color. Suppose we choose a face of the cube and a color on it. The face just opposite to it can have 5 different colors and the rest of the four faces on the sides can be considered to be in a circular arrangement. The number of different ways of arranging the 4 colors on the side faces is $(4 - 1)! = 3!$ There total number of different arrangements = 5 x 3! = 30

TOTAL NUMBER OF WAYS OF SELECTION BY TAKING SOME OR ALL OF N DISSIMILAR THINGS

If we are given n things for selection without given the condition about how many things we should select, we can select none of them, one of them, two of them, three of them, ..., or n of them. Therefore, total number of ways of selection = ${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots + {}^{n}C_n = 2^n$.

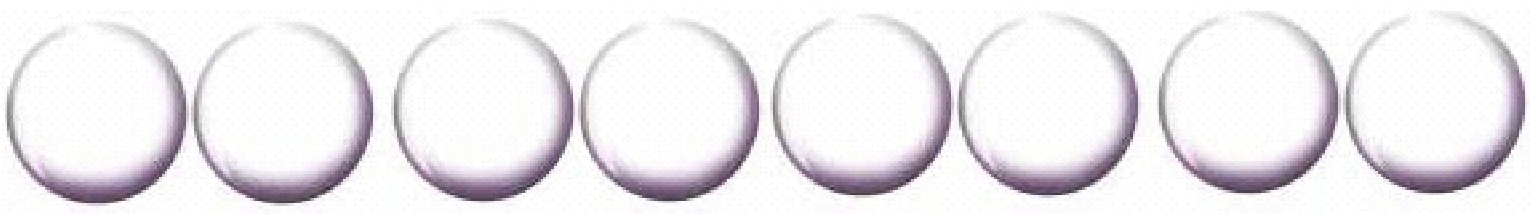
We can also understand it this way: For each of the n things, we have 2 options, either to select it or not to select it. Therefore, the number of selections = $2 \times 2 \times 2 \times \ldots \times 2 = 2^n$.

The number of selections when it is necessary to select at least one of the n things = $2^n - 1$

The number of selections when it is necessary to select at least one of the n things = $2^n - 1$

- A man has 7 friends. In how many ways can he invite one or more of them to dinner?
  Answer: Since the man has to select at least one of the 7 friends, the number of possible selections = $2^7 - 1 = 127$

TOTAL NUMBER OF SELECTIONS WHEN N THINGS ARE SIMILAR



Suppose you are given 8 identical billiards balls. You may select zero, one, or more of them. In how many ways can you make the selection? Suppose you have to select two balls. Does it matter which two you select? Wouldn't all ways of selecting two balls be same? Yes they will be. Therefore, the number of ways of selecting

- 0 ball = 1
- 1 ball = 1
- 2 balls = 1
- 3 balls = 1
- …
- 8 balls = 1

Therefore, the total number of selections possible = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9

Therefore, the total number of selections of n similar things = n + 1

Therefore, total number of selections possible by taking some or all out of p + q + r + … things, where p are alike of one kind, q alike of second kind, r alike of third kind, and so on = (p + 1)(q + 1)(r + 1) …

Therefore, total number of selections possible by taking some or all out of p + q + r + … things, where p are alike of one kind, q alike of second kind, r alike of third kind, and so on, AND m things are dissimilar = $2^m(p + 1)(q + 1)(r + 1)$ …

- A fruit basket consists of 4 identical bananas, 3 identical mangos, and 5 identical strawberries. How many selections are possible?
  Answer: The number of possible selections = 5 × 4 × 6 = 120.

- How many selections are possible if at least one mango has to be taken and the basket also contains 3 different guavas?
  Answer: Since at least one mango has to be taken, the number of possible selections of mangos = 3. Therefore, total number of possible selections = $5 \times 3 \times 6 \times 2^3 = 720$.

NUMBER OF WAYS OF GROUPING DISSIMILAR THINGS

How would you divide seven different objects in two groups of four and three? Simple. You select four things out of those seven and three will be left behind. Or you can select three things out of those seven and four will be left behind. The number of ways you can accomplish this is ${}^7C_4$ or ${}^7C_3$. Now, how would you divide these seven different objects in three groups of one, two and four? Again, you can begin by choosing four things out of these seven, then two things out of the remaining three. The number of ways you can do this is ${}^7C_4 \times {}^3C_2$. Therefore, to divide things into groups, you keep on selecting groups, except for the last group which will be automatically formed.

So how would you divide seven different things into groups of three, three and one?

Your answer would be to take our three things first, then three things next, i.e. ${}^7C_3 \times {}^4C_3 = 140$, and your answer would be **WRONG**!

Why is your answer wrong? Is there anything wrong with the method? No. Only that you need to take some extra precautions when you are making equal groups. There are slight changes to be observed when you divide some number of things into equal groups.

Let's start small. According to the method above, in how many ways can you divide 4 different things (say a, b, c and d) into two groups having two things each? Your answer would be to select two things out of the four and two would be left behind, i.e. ${}^4C_2 = 6$. But are there really 6 ways?

Given below are shown the number of ways we can divide four things, a, b, c and d, into two groups of two:

a b c d

a b	**c d**
a c	**b d**
a d	**b c**

You can keep trying if you want to, but there are only 3 ways of dividing the four things into two groups of two.

Where did the rest of 3 ways calculated through ${}^4C_2 = 6$ disappear?

The answer is that they got merged. When you selected two things out of the four, the things selected were **ab**, **ac**, **ad**, **bc**, **bd**, and **cd**. But the last three groups are already formed when you select the first three groups, i.e. when you select **ab**, you automatically get **cd**. When you select **ac**, you automatically get **bd**, and so on.

Let's do it again. According to the method above, in how many ways can you divide 6 different things (say a, b, c, d, e and f) into three groups having two things each? Your answer would be to select two things out of those six and then select two things from the remaining four so that two would be left behind, i.e. ${}^6C_2 \times {}^4C_2 = 90$. But are there really 90 ways?

The table below shows the number of ways you can divide six different things into three groups of two each:

ab, cd, ef	ac, bf, de	ae, bd, cf
ab, ce, df	ad, bc, ef	ae, bf, cd
ab, cf, de	ad, be, cf	af, bc, de
ac, bd, ef	ad, bf, ce	af, bd, ce
ac, be, df	ae, bc, df	af, be, cd

As you can see, there are only 15 ways in place of 90. What is going on?

The problem is that for every grouping, the formula calculates 3! different ways. For example, for grouping **ab cd** and **ef**, the formula calculates 6 different ways- (**ab cd, ef**), (**ab, ef, cd**), (**cd, ab, ef**), (**cd, ef, ab**), (**ef, ab, cd**) and (**ef, cd, ab**). In essence this is only one way of grouping and **NOT** six. Therefore, to find the total number of ways we divide the result obtained through the formula by 3!

FORMULA:

If **na** different things are divided into **n** groups of **a** things each, the number of ways of grouping

$$= \frac{{}^{na}C_a \times {}^{na-a}C_a \times {}^{na-2a}C_a \times \ldots {}^{2a}C_a}{n!} = \frac{(na)!}{n!(a!)^n}$$

The number of ways of dividing 2m objects into two groups of m objects each = $\frac{(2m)!}{2!(m!)^2}$

The number of ways of dividing 3m things into three groups of m objects each = $\frac{(3m)!}{3!(m!)^3}$

The number of ways of dividing 4m things into four groups of m objects each = $\frac{(4m)!}{4!(m!)^4}$

NOTE: If the groups contain unequal number of things, the method we discussed in the beginning is valid.

Example: How would you divide 5 distinct objects into groups of 2, 2 and 1?
Answer: The single object can be chosen in ${}^5C_1$ = 5 ways. The rest of the 4 objects can be divided into two equal groups in 3 ways.
Therefore, the number of ways = 5 × 3 = 15.
Note that the answer by our first method will be ${}^5C_1 \times {}^4C_2$ = 30 ways. This answer is incorrect.

NUMBER OF WAYS OF GROUPING SIMILAR THINGS

In how many ways can you divide five similar objects (say a, a, a, a, a) into three different groups? When we make groups of similar objects, the only way we can differentiate two different ways of grouping is by differentiating between groups when they have different size (number of objects). Therefore, in case of similar objects, the number of different ways we can group them is the number of different sizes of groups that we can make. Let's see how many groupings of different sizes we can make for 5 similar objects.

Grouping	Number of ways
0 0 5 (0 0 aaaaa)	1
0 1 4 (0 a aaaa)	1
0 2 3 (0 aa aaa)	1
1 1 3 (a a aaa)	1
1 2 2 (a aa aa)	1
Total	**5**

DISTRIBUTION

After you have made groups of some objects, you might want to distribute these groups in various places. For example, after you made groups of some toffees, you might want to distribute these groups among some children. Or, after dividing some number of balls into groups, you might want to distribute these groups into boxes. Just as the objects that we group can be similar or dissimilar, so can the places that we assign these groups to be similar or dissimilar. While distributing groups, we need to keep one rule in mind:

If the places for distribution are dissimilar, the arrangement of groups count, otherwise it doesn't.

Now let's solve an all-encompassing example on what we have learnt.

Question: In how many ways can you put 5 balls in 3 boxes if
I. the boxes are similar and the balls are similar.
II. the boxes are different but the balls are similar.
III. the boxes are similar but the balls are different.
IV. the boxes are different and the balls are different.

Answer: Sums up everything, doesn't it? Well, here we go:

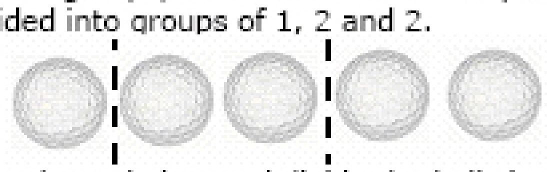
Case I: the boxes are similar and the balls are similar

Groupings	Number of ways for groupings	Number of ways of distribution	Total number of ways
0 0 5 (0 0 bbbbb)	1	1	1
0 1 4 (0 b bbbb)	1	1	1
0 2 3 (0 bb bbb)	1	1	1
1 1 3 (b b bbb)	1	1	1
1 2 2 (b bb bb)	1	1	1
		Total Number of ways =	**5**

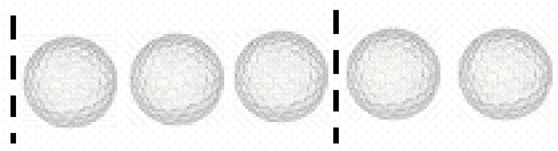
Case II: the boxes are different but the balls are similar

Groupings	Number of ways for groupings	Number of ways of distribution	Total number of ways
0 0 5 (0 0 bbbbb)	1	3	3
0 1 4 (0 b bbbb)	1	3! = 6	6
0 2 3 (0 bb bbb)	1	3! = 6	6
1 1 3 (b b bbb)	1	3	3
1 2 2 (b bb bb)	1	3	3
		Total Number of ways =	**21**

The above case can also be solved through **partition method**, used when the objects are similar but the places of distribution are different. To divide five identical balls into three groups, we need to insert two partitions between them, as shown in the figure. In the figure shown below, the balls have been divided into groups of 1, 2 and 2.



Or we can insert the partitions in the manner shown below and divide the balls into groups of 0, 3 and 2.

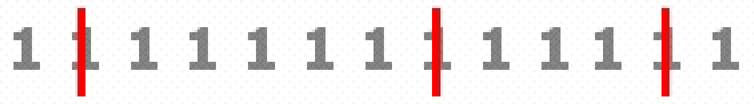


To insert two partitions between the 5 similar balls, we assume the partitions to be 2 extra balls and then we pick out two balls out of the 7 balls. The two balls that we pick out become two partitions, as shown below:

Therefore, the number of ways of dividing 5 similar balls into 3 groups = ${}^{7}C_{2}$ = 21.

Partition method can also be applied to find the whole number solutions of the equation a + b + c + d = 10. The problem basically boils down to partitioning 10 ones into four groups, i.e. inserting three partitions between 10 ones, as shown below:

Again, to insert three partitions, we insert three more ones and pick 3 out of 13 ones, as shown below:



The number of solutions of the equation = ${}^{13}C_{3}$.

Case III: the boxes are similar but the balls are different

Groupings	Number of ways for groupings	Number of ways of distribution	Total number of ways
0 0 5 (0 0 $b_1b_2b_3b_4b_5$)	1	1	1
0 1 4 (0 b_1 $b_2b_3b_4b_5$)	${}^{5}C_{1}$ = 5	1	5
0 2 3 (0 b_1b_2 $b_3b_4b_5$)	${}^{5}C_{2}$ = 10	1	10
1 1 3 (b_1 b_2 $b_3b_4b_5$)	${}^{5}C_{3}$ = 10	1	10
1 2 2 (b_1 b_2b_3 b_4b_5)	${}^{5}C_{1}$ × 3 = 15	1	15
		Total Number of ways =	**41**

Case IV: the boxes are different and the balls are different

Groupings	Number of ways for groupings	Number of ways of distribution	Total number of ways
0 0 5 (0 0 $b_1b_2b_3b_4b_5$)	1	3	3
0 1 4 (0 b_1 $b_2b_3b_4b_5$)	${}^{5}C_{1}$ = 5	3! = 6	5 × 6 = 30
0 2 3 (0 b_1b_2 $b_3b_4b_5$)	${}^{5}C_{2}$ = 10	3! = 6	10 × 6 = 60
1 1 3 (b_1 b_2 $b_3b_4b_5$)	${}^{5}C_{3}$ = 10	3! = 6	10 × 6 = 60
1 2 2 (b_1 b_2b_3 b_4b_5)	${}^{5}C_{1}$ × 3 = 15	3! = 6	15 × 6 = 90
		Total Number of ways =	**243**

DERANGEMENT

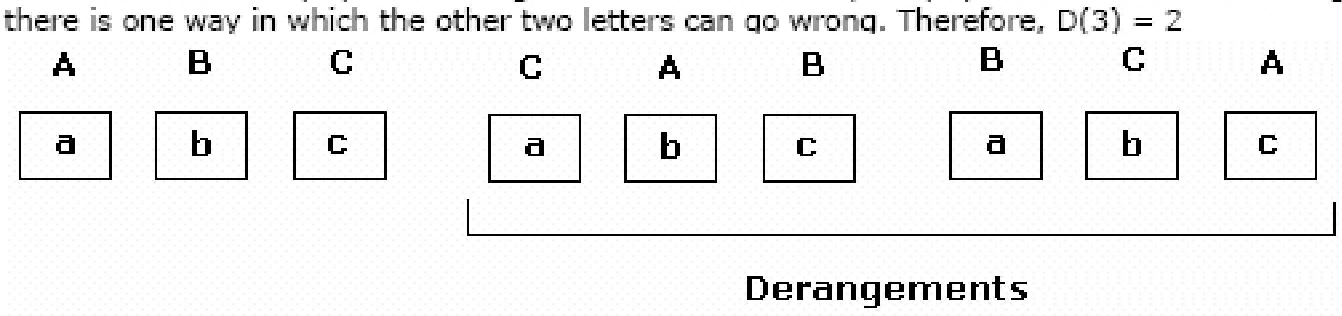
Seven couples go to a dance party. In how many ways can we make dancing couples so that no two spouses are together?

Five stamps of different countries are to be pasted on five different envelopes of the same country. In how many ways can the stamps be pasted so that no stamp is pasted on its designated envelope?

The above questions come under a specific topic of arrangement knows as derangement. A **derangement** of a set of numbers is a permutation in which no number appears in its natural order. So, for example, {5,4,1,2,3} is a derangement of {1,2,3,4,5}.

How to solve such questions? We can proceed iteratively:

Let the derangement of n elements be represented as D(n).
We can see that D(1) = 0, D(2) = 1.
Let's calculate D(3).
The figure below shows three letters A, B, and C designated for three envelopes a, b, and c. Now letter A can go wrong in 2 ways and for both these ways, there is one way in which the other two letters can go wrong. Therefore, D(3) = 2



Let's calculate D(4).
Now D(4) = total number of ways of posting the letters – all the letters in the right envelopes – only 1 letter in the wrong envelope – 2 letters in the wrong envelope and two letters in the right envelopes – 3 letters in the wrong envelope and 1 letter in the right envelope
⇒ D(4) = 24 – 1 – 0 – ${}^{4}C_{2}$ × D(2) – ${}^{4}C_{3}$ × D(3) = 24 – 1 – 6 – 8 = 9

Let's calculate D(5).
D(5) = 5! – 1 – 0 – ${}^{5}C_{2}$ × D(2) – ${}^{5}C_{3}$ × D(3) – ${}^{5}C_{4}$ × D(4) = 120 – 1 – 10 – 20 – 45 = 44

And so on…

In general derangement of n things D(n) = $= n!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots+\frac{(-1)^n}{n!}\right)$ for $n \geq 2$

Problems

How many subsets of {1, 2, 3, 4, 5, 6, 7} contain 6 as its largest number?
(a) 32 (b) 31 (c) 64 (d) 128

A palindromic number reads the same forward and backward (12121 is a palindromic number). How many 6 digits palindromic numbers are there?
(a) 810 (b) 900 (c) 729 (b) 1000

A bag has five balls in it, each with a number written on it. One ball has number two on it, one ball has number five on it, one ball has number seven and two of the balls have number eight on them. The balls are taken out of the bag without replacement and placed in the order they are drawn, from left to right, to create a five-digit number. How many of these numbers are not prime?
(a) 21 (b) 0 (c) 47 (d) 60

In U.P. auto license plates are issued with the following pattern: each plate has three letters followed by three digits (0 through 9). The letters can be in any combination. However, each digit must be equal to or greater than the one preceding it. (For example, 132 is not a legal number combination because the last 1 is less than the digit before it.) How many different license plates can the state issue?
(a) $26^3 \times 9^3$ (b) $26^3 \times 90$ (c) $26^3 \times 120$ (d) $26^3 \times 10^3$

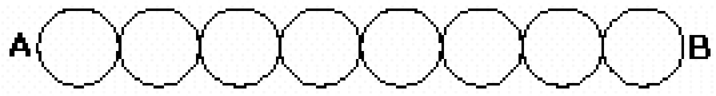
Think about all the five digit numbers that can be formed using the digits 1, 3, 5, 7, and 9. If these numbers are arranged in increasing order, the 500^{th} number is:
(a) 17599 (b) 11799 (c) 17999 (d) 19131

If $(a + 2b + 3c - 5d + 4e - f - 2g - 7h)^2$ is expanded and simplified, the number of terms in the answer is:
(a) 36 (b) 45 (c) 81 (d) 72

Using three colors red, blue and green, in how many **different** ways can we paint the edges of a regular pentagon such that no two edges meeting at a common vertex are of the same color?
(a) 1 (b) 2 (c) 3 (d) 4

3 students are to be chosen to make a team for a quiz contest from a class of 12 students, such that the team includes at least one boy and at least one girl. If exactly 160 different teams can be made out of these 12 students with the given condition, then which of the following can be the difference between the number of boys and the number of girls in the class?
(a) 2 (b) 3 (c) 4 (d) 5

A cube is to be painted with two colors, each color being used for three faces of the cube. How many different cubes painted in this way are possible?
(a) 1 (b) 2 (c) 3 (d) 6



8 circles of diameter 1 cm are kept in a row, each circle touching its neighbours. How many paths of length 4π are possible to go from A (0, 0) to B (8, 0) if you're not allowed to traverse back?
(a) 128 (b) 256 (c) 240 (d) 300

How many odd four-digit numbers are there in which all the digits are different?
(a) 4500 (b) 2240 (c) 2880 (d) 3600

How many odd three-digit numbers are there where the tens digit is greater than the units digit and the hundreds digit is greater than the tens digit?
(a) 225 (b) 45 (c) 50 (d) 230

In how many ways can 4 persons be chosen from a row of 10 persons such that no two persons chosen are sitting next to each other?
(a) 210 (b) 35 (c) 72 (d) 36

The letters of the word **MATHEMATICS** are arranged in all the different ways possible to give 11 letter words, which may or may not make sense. When these 'words' are arranged in alphabetical order, what is the tenth letter of the 50^{th} word?
(a) I (b) T (c) S (d) M

Indian and Japanese teams are the only two teams participating in a marathon. Indian team has 4 runners and Japanese team has 3 runners. In how many ways can the runners win if we're only interested in the country they are representing?
(a) 35 (b) 7!/128 (d) 4!3! (d) 128

A table tennis match continues till one of the players wins three matches. Obviously, the lowest number of matches that can be played is three and the highest number of matches that can be played is five. How many different sequences of matches are possible if a win/loss for one player is considered different than that for the other player?
(a) 10 (b) 20 (c) 12 (d) 6

A three digit number A satisfies the conditions that there is no digit used twice in A, the number always contains the digit 5 in it, and is less than 800. How many values for A are possible?
(a) 240 (b) 168 (c) 184 (d)

How many four digit numbers can be formed from 1, 2, 3, and 4, without repetition, such that 1 is not at the tens place, 2 is not in thousands place, 3 is not in hundreds place, and 4 is not in the units place.
(a) 8 (b) 11 (c) 13 (d) 9

If $(a + 2b - 3c)^{15}$ is expanded and all the like terms are collected, how many terms will be there in all?
(a) 136 (b) 455 (c) 105 (d) 210

There are 9 permutations for four letters and their respective four envelopes such that no letter goes in the correct envelope. How many permutations are there for 7 letters and their respective 7 envelopes such that only 3 letters go in the correct envelopes?
(a) 189 (b) 252 (c) 315 (d) 378

How many squares are there in a 8 x 8 square?

The letters "CFOSU" are arranged in dictionary order. What is the rank of the word "FOCUS" in this order?

In how many ways can the letters in UNUSUAL be arranged? how many have all three U's together? How many of the arrangements in part have no consecutive U's?

In how many ways can the 26 letters in the alphabet be arranged so that the letter b is always to the left of e?

A student must answer 8 out of 10 questions in an exam. How many choices does the student have? How many choices if the first three questions have to be answered? How many choices if at least 4 of the first five questions must be answered?

In a row of 20 seats, in how many ways can 3 blocks of consecutive seats with 5 seats in each block be selected?

There are 50 juniors and 50 seniors. Each class has 25 men and 25 women. In how many ways can an 8 person committee be chosen so that it includes 4 women and 3 juniors?

In how many ways can 2n people be divided into n pairs?

(a) subject to no constraints.
(b) subject to constraint 1.
(c) subject to constraint 2.
(d) subject to both constraints.

I have two checkout registers, and twenty customers. What formula will find how many different ways I can arrange them? Order does matter.

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If each side of a cube is painted red or yellow or blue, how many distinct colour patterns are possible?

How many combinations of pizza can be made with 6 different toppings? Assuming that double toppings are not permitted,

There are 8 identical apples, and they are to be given out to 4 children. How many different ways of distributing the apples are there if each child must receive at least one apple?

How many 6-digit numbers are divisible by 4 if we allow no repeated digits?

How many 3 digit numbers have the digital sum of nine?

How many phone numbers can be made under the following conditions:
(First digit cannot be 0 or 1 because you'll get the operator or long distance.)
a) The first two digits are 3 followed by 6
b) The third digit is even
c) The fourth digit is greater than 5
d) The fifth and seventh digits are odd
e) The sixth digit is 2

If combinations of letters be formed by taking 5 letters at a time out of the letters of the word "**METAPHYSICS**", in how many of them will letter T occur.

How many sequences a_1, a_2, a_3, a_4, a_5 satisfying $a_1 < a_2 < a_3 < a_4 < a_5$ can be formed if a_i must be chosen from the set {1, 2, 3, 4, 5, 6, 7, 8, 9}?

A cube is to be painted with two colors, each color being used for three faces of the cube. How many different cubes painted in this way are possible?

Using three colors red, blue and green, in how many **different** ways can we paint the edges of a regular pentagon such that no two edges meeting at a common vertex are of the same color?

The faces of a regular tetrahedron are to be painted with four different colors. How many different ways are possible?