__/__/____

S $\longrightarrow$ D — Special Permutations

12 similar chocolates — 3 persons

S $\longrightarrow$ S — Combination

12 similar balls — 3 similar boxes

D $\longrightarrow$ S — Std. Combination

12 different letters — 5 similar letterboxes

D $\longrightarrow$ D — Std. Permutation

12 different letters — 5 different letterboxes.

Ex Consider the word "PRECIPITATION". Find the no. of ways in which
→ a selection
→ an arrangement of 4 letters can be made ?.

PRECIPITATION

P → 2   C → 1   A – 1
R → 1   I → 3   O – 1
E → 1   T → 2   N – 1

Selection

4 different + (2 same + 2 same) + (2 same + 1 diff + 1 diff.)
+ (3 same + 1 diff)

$$= {}^{9}C_{4} + {}^{3}C_{2} + {}^{3}C_{1} \times {}^{8}C_{2} + {}^{8}C_{1}.$$

Arrangement

$$\_ \times 4!. + \_ \times \frac{4!}{2! \times 2!} + \_ \times \frac{4!}{2!} + \_ \times \frac{4!}{3!}.$$

__/__/____

Ex 19 In how many ways can 10 students be allotted ranks 1 to 10 such that A is above B?

$^{10}C_2 \times 1 \times 8!$ = (2 ranks choose for A, B, but no arrangement for them).

Alt

$\frac{10!}{2}$. (In half arrangements A will be above B).

$$\boxed{S \rightarrow D}$$

Scenarios :-

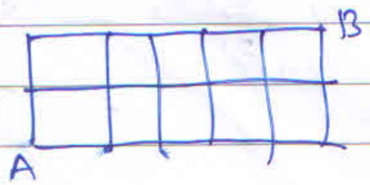
(i) $A+B+C=12$ $A, B, C \geq 0$.
Find no. of solⁿ.

(ii) No. of ways of distributing 12 identical chocolates to 3 persons

(iii) How many no. less than 1000 will have digit sum 12.
$A+B+C=12$
→ additional condition $A, B, C \leq 9$. (not present in (i) & (ii)).

(iv) 3 dices are thrown. What is the probability that sum is 12.
$A+B+C=12$.
→ additional condⁿ → $1 \leq A, B, C, \leq 6$.

(v) $(A+B+C)^{12}$ → how many terms in the expression

(vi) 

Find no. of ways of reaching from A to B.

S → D can be categorized in below formats:-

→ No limit

→ Lower Limit

→ Upper Limit

→ Inequality type

(i) No limit

$$A+B+C=12$$

$$\underbrace{0000+0000+0000}_{14 \text{ terms}}$$

$$\frac{14!}{12!\,2!} = {}^{14}C_2$$

(ii) Lower limit

$$A+B+C=12$$

$$A, B, C \geq 1.$$

First allot 1 to each A, B, C
1 1 1

Now, $A+B+C=9$

$${}^{11}C_2.$$

Ex $A+B+C+D=20$

$A \geq 1,\ B \geq 2,\ C \geq 3,\ D \geq 4.$

Solve $A+B+C+D=10$ no limit

$${}^{13}C_3.$$

(iii) Upper Limit

Convert UL to LL question.

Ex $A+B+C=12$ $A, B, C \le 6$.

(6+1)

Step 1 → Violate the rule by giving 7 or more to one element

$A=7$

$A+B+C=5 \rightarrow {}^{7}C_2$ → represents count of all cases when UL is violated.

Ans → ${}^{14}C_2 - 3 \times {}^{7}C_2$

Ex $A+B+C=12$ $A \le 6, B \le 7, C \le 8$.

${}^{14}C_2 - {}^{7}C_2 - {}^{6}C_2 - {}^{5}C_2$.

Limitation → If UL less than half of total no., means $A+B+C=12$
$A, B, C \le 5$

$\boxed{\text{Leave}}$

(iv) Inequality Type

Ex $A+B+C \ge 12$ (Not determined)

Ex $A+B+C \le 10$.

$A+B+C = 10$ D

Assume another variable.

$A+B+C+D=10$

$\rightarrow {}^{13}C_3$.

Ex You have some chocolates and you have to distribute those among A, B, C such that total of A, B, C can not exceed 10.

Ex Suppose $8 \le A+B+C \le 10$.

then count

$^{12}C_2 + {}^{11}C_2 + {}^{10}C_2$.

Ex

(V) <u>Type 5:- Special Case :-</u>

<u>Case 1:-</u>

$$A+B+3C = 10.$$

Ex 10 chocolates are getting distributed. C will take if it is in multiple of 3.

$A+B+3C$		
0	$A+B=10$	$^{11}C_1$
3	$=7$	$^{8}C_1$
6	$=4$	$^{5}C_1$
9	$=1$	$^{2}C_1$
		26.

$\boxed{2A+3B+5C = ?}$ workshop.

<u>Case 2:-</u>

Total 35 chocolates to 5 people so that each gets odd no. of chocolates.

$$(2A+1)+(2B+1)+(2C+1)+(2D+1)+(2E+1) = 35$$

$$\Rightarrow A+B+C+D+E = 15$$

Ans $\rightarrow$ $^{19}C_4$.

<u>Case 3</u>

30 chocolate to 7 person so that each get distinct no. of chocolates.

$1+2+3+4+5+6+7 = 28$

For values to be distinct extra 2 can be allotted towards end.

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ & & & & & 1 & 1 \\ & & & & & 2 & 0 \\ & & & & & 0 & 2 \end{array} \Bigg\} \underline{\underline{3}}$$

Ans $\to 3 \times 7!$

Combinations

	$D \to S$	$S \to S$
r items	${}^nC_r$	1
≥ 0	2^n	$n+1$
≥ 1	$2^n - 1$	n

Ex No. of sol^n of $A+B+C+D=10$ such that $A > B$.

$$\left\{ \frac{{}^{13}C_3 - \text{when } (A=B)}{2} \right\}$$

Alt:

$A = B + k \qquad k \geq 1$

$2B + C + D + k = 10.$

$\downarrow \qquad \downarrow k \geq 1$

$2B + C + D + k = 9.$

0	$C+D+K=9$	${}^{11}C_2$
2	$=7$	${}^{9}C_2$
4	$=5$	${}^{7}C_2$
6	$=3$	${}^{5}C_2$
8	$=1$	${}^{3}C_2$

__/__/__

Ex Find no. of ways of arranging 4 similar books in 5 different boxes?

$$A+B+C+D+E = 4. \qquad \underline{\underline{{}^{8}C_4}}.$$

→ If 4 sim books 5 sim. boxes

4, 0, 0, 0, 0
3, 1, 0, 0, 0
2, 1, 1, 0, 0 → 5 ways.
2, 2, 0, 0, 0
1, 1, 1, 1, 0

→ If 4 diff books in 5 sim. boxes.

4, 0, 0, 0, 0 →	${}^{4}C_4$
3, 1, 0, 0, 0	${}^{4}C_3$
2, 1, 1, 0, 0	${}^{4}C_2$
2, 2, 0, 0, 0	$\frac{{}^{4}C_2}{2!}$
1, 1, 1, 1, 0	1

Ex $x+y+z+t = 20 \qquad x, y, z, t, \geq -1$

$+1+1+1+1$

$$\underline{\underline{{}^{27}C_3}}.$$

Ex Group of friends met. Everyone shook hand with each other. Total handshake = 66. On new year, everyone sent greeting card to each other. How many greeting card exchanged?

2 people do = 1 handshake.

2 people send = 2 greeting card

Ans → $\underline{132}$.

Ex 12 points in a plane. Only 4 of them on a straight line. Find diff b/w no. of Δs and no. of st. lines.

$\Delta s = {}^{12}C_3 - {}^{4}C_3$

St. line $= {}^{12}C_2 - {}^{4}C_2 + 1.$

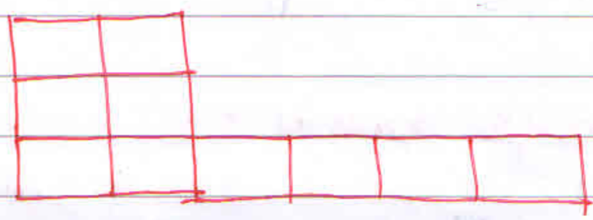
Ex A person has $(2n+1)$ friends. No. of ways he can invite at least $(n+1)$ for dinners is 4096. Find no. of friends.

${}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} \dots {}^{2n+1}C_{2n+1} = 4096$

$\Rightarrow 2^{2n} = 4096 = 2^{12}$

$\Rightarrow \underline{\underline{n = 6}}.$ Ans $\rightarrow$ $\underline{\underline{13}}$.

Ex No. of ways of placing letters of the word SUBJECT in the given diagram such that no row is empty.



SUBJECT

Total letters = 7
Total squares = 10.

${}^{10}P_7 - \left(\text{No. of ways top row empty} + \text{No. of ways middle row empty}\right)$

$= {}^{10}P_7 - 2 \times {}^{8}P_7$

$= \frac{10!}{3!} - 2 \times 8! = 8!\,(15 - 2) = 13 \times 8!$

__/__/____

Ex A set of 6 elements is divided into 2 groups. Find no. of ways.

1	5	$^6C_1 = 6$
2	4	$^6C_2 = 15$
3	3	$^6C_3 = 20$

$$\underline{\underline{41}}$$

Ex No. of ways of arranging letters of the word CALENDAR in such a way that exactly 2 letters are present b/w L and D.

A L/D __ __ L/D B → no. of place

A	B
4	0
0	4
2	2
3	1
1	3

→ 5 ways

$$\underset{L/D}{2} \times 5 \times \frac{6!}{2!}$$

$2!$ → because of 2 A's.

Ex How many different signals can be made by waving 5 different coloured flags one along the other when one or more of them can be waved at a time?

$$^5C_5 \times 5! + {}^5C_4 \times 4! + {}^5C_3 \times 3! + {}^5C_2 \times 2! + {}^5C_1 \times 1!$$

Ex In how many ways can one divide 12 books

(i) into 4 equal bundles $\longrightarrow \dfrac{12!}{4!(3!)^4}$

(ii) equally among 4 boys. → $\dfrac{12!}{(3!)^4}$

__Ex__ GYRATION

How many words can be formed such that either G is in the 1st place or T is in the last place?

Start with G, not end with T + Start with G, end with T + Not start with G, end with T

$= 6 \times 6! + 6! + 6 \times 6! = 13 \times 6!$

How many words can be formed which neither start with G nor end with T?

Ans $\rightarrow 8! - 13 \times 6! = \underline{\underline{43 \times 6!}}$

__Alt__

words starting with T + words not starting with T

$= 7! + 6 \times 6 \times 6! = 43 \times 6!$

__Ex__ How many 5 letter words without repetition can be formed using letters of word "CRANE" so that vowels are never together?

X C X C X C X

$3! \times {}^4C_2 \times 2! = 6 \times 6 \times 2 = \underline{\underline{72}}.$

MISTAKE

$4! \times {}^5C_3 \times 3! = 24 \times 10 \times 6 = 1440.$

Ex Find the no. of ways in which
(i) a selection
(ii) an arrangement of 4 letters can be made from letters of the word "DISTILLATIONS".

DISTILLATIONS

D − 1   T − 2   O − 1
I − 3   L − 2   N − 1
S − 2   A − 1

Selection

4 diff + 3 same, 1 diff + 2 same, 2 same + 2 same, 1 diff, 1 diff

$^{8}C_4$ $^{7}C_1$ $^{4}C_2$ $^{4}C_1 \times {}^{7}C_2$.

Arrangement $\times$ $4!$ $\frac{4!}{3!}$ $\frac{4!}{2!\,2!}$ $\frac{4!}{2!}$.

Ex How many 4 digit no.s, that are divisible by 4 can be formed, using the digits 0 to 7, if no digit is to occur more than once in a no.?

04 → 4 with '0'
12
16
20
24
32
36
40
52
56
60
64
72
76

$4 \times {}^{6}C_2 \times 2! + 10 \times 5 \times 5$
$= 4 \times 15 \times 2 + 250$
$= 370$.

Ex In how many ways can the crew of a 10 oared boat be arranged, when of the 10 persons available, 2 of whom can row only on the bow side and 3 of whom can row only on stroke side?

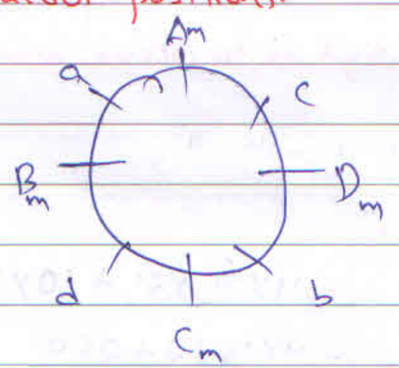
$\underline{2} \qquad \underline{3}$

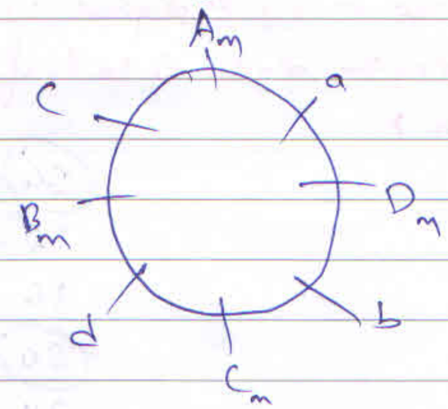
Ans $\rightarrow$ ${}^{5}C_{3} \times (5!)^2$

Ex 12 friends go out for a dinner to a restaurant, where they find 2 circular tables, one with 7 chairs and one with 5 chairs. In how many ways the group settle down themselves for dinner?

${}^{12}C_{7} \times 6! \times 4!$

Ex In how many ways can 4 couples be seated around a circular table such that people of the same gender do not sit in adjacent positions and exactly one of the 4 couples sit in adjacent positions.





Only 2 cases.

$3! \times 2 \times {}^{4}C_{1} = 12 \times 4 = \underline{\underline{48}}$.

Ex In how many ways can 4 prizes each having $1^{st}, 2^{nd}, 3^{rd}$ positions be given to 3 boys, if each boy is eligible to receive one prize of each event?

$$(3!)^4$$

Ex Find sum of all no:s that can be formed by taking all digits at a time from 4,5,6,7,8 without repitition?

$$4! \times (4+5+6+7+8) \times 11111$$
$$= 24 \times 30 \times 11111$$
$$= 7999920$$

Ex There are 15 points in a plane of which 8 of them are on a st. line. Then how many (i) st. lines (ii) Δ^s can be formed?

(i) $^{15}C_2 - {}^{8}C_2 + 1 = 105 - 28 + 1 = \underline{\underline{78}}$.

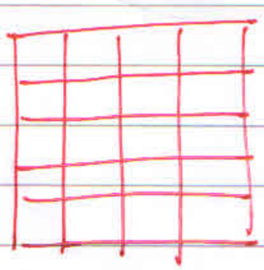
(ii) $^{15}C_3 - {}^{8}C_3 = 455 - 56 = \underline{\underline{399}}$.

Ex In a convex nonagon all the diagonals are drawn. These diagonals intersect each other at p points inside the polygon, q points on the polygon and r points outside the polygon. Find max. value of p.

$$^{9}C_4 = \frac{\cancel{9}^{3} \times 8 \times 7 \times 6}{\cancel{24}} = \underline{\underline{126}}.$$

Ex

B



A

Find no. of ways A to B?

$$VVVVVHHHH = \frac{9!}{5!\,4!}$$
$$= \frac{9 \times 8 \times 7 \times 6}{24} = \underline{\underline{126}}.$$

Ex The no. of terms in expansion of $(a+b+c+d)^{20}$ is

$$a+b+c+d = 20$$

$$^{23}C_3 = \frac{23 \times \cancel{22}^{11} \times \cancel{21}^{7}}{\cancel{6}} = \underline{\underline{1771}}.$$

Ex Let k be an integer such that sum of the digits of k is 4 and $10^5 < k < 10^6$. How many values can 'k' have?

$$\underline{100000}$$

$$a+b+c+d+e+f = \underline{\underline{4}}.$$

$a \geq 1$

$$a+b+c+d+e+f = \underline{\underline{3}}.$$

$$\text{Ans} \rightarrow {}^{8}C_5 = \frac{8 \times 7 \times 6}{6} = \underline{\underline{56}}.$$

Ex 7 boxes numbered 1 to 7 are arranged in a row. Each is to be filled by either a black or blue coloured balls such that no two adjacent boxes contain blue coloured balls. In how many ways can the boxes be filled with the balls?

blue → A   black → B

7A	0B	×
6A	1B	×
5A	2B	×
4A	3B	1
3A	4B	$^5C_3 = 10$
2A	5B	$^6C_2 = 15$
1A	6B	$^7C_1 = 7$
0A	7B	1
		$\underline{\underline{34}}$.

__/__/__

Ex An advertisement board is to be designed with 6 vertical stripes using some or all of the colors: red, green, blue, black and orange. In how many ways can the board be designed such that no 2 adjacent stripes have same colour?

$$5 \times 4 \times 4 \times 4 \times 4 \times 4$$
$$= \underline{\underline{5120}}.$$

Ex How many different words can be formed using all the letters of the word "COMBINATION" such that the vowels as well as consonants appear in alphabetical order?

COMBINATION

$$^{11}C_5 = \frac{11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8} \times 7}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} = \underline{\underline{462}}.$$

Ex In how many ways 7 identical ~~bo~~ balls be placed ~~to~~ into 4 boxes P, Q, R, S such that 2 boxes P, Q have at least 1 ball each?

$$P+Q+R+S = 7 \qquad P, Q \geq 1$$

$$P+Q+R+S = 5$$

$$^{8}C_3 = \frac{8 \times 7 \times 6}{6} = \underline{\underline{56}}.$$

Byju's.

Ex In how many ways 12 similar chocolates be distributed among 3 persons where each person can get a max. of 6 chocolates?

$$a+b+c = 12 \qquad \underline{\underline{a, b, c \leq 6}}$$

$^{14}C_2 - {^7C_2} \times 3$ $a+b+c=\underline{\underline{5}}$

$= 91 - 3 \times 21 = \underline{\underline{28}}$.

Ex The no. of ways that 7 persons can address a meeting so that 3 persons A, B and C from them A will speak before B and B before C ~~will~~ –

$^7C_3 \times 4!$

$= \frac{7!}{4! \times 3!} \times 4! = \frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = \underline{\underline{840}}$

Alt. $\frac{7!}{3!} \rightarrow \frac{\text{all}}{6} \rightarrow$ no. of arrangements possible b/w A, B, C.

Ex How many 3 digit no. can be formed using 1, 2 .... 9 such that hundred's place greater that ten's place greater than unit's place?

$$\frac{9 \times 8 \times 7}{6} = 3 \times 4 \times 7 = \underline{\underline{84}}.$$

Ex There are 5 math books, 4 Science books and 3 Literature books. How many different collections a person can make selecting at least one of each kind when all books are different? And when the books in a category are same?

Diff $\rightarrow (2^5-1) \times (2^4-1) \times (2^3-1)$
$= 31 \times 15 \times 7 = 3255$???? $2^{12}-1$

Books in a category are same $\rightarrow 5 \times 4 \times 3$
$= \underline{\underline{60}}$.

__/__/____

Ex In an exam. of 9 papers, a candidate has to pass in more papers than the no. of papers in which he fails, in order to be successful. No. of ways he can be unsuccessful is:

$$^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$$

$$= \frac{2^9}{2} = 256$$

Ex The sides BC, CA and AB of a Δ have 3, 4, 5 interior points respectively on them. The no. of Δs can be constructed using these points as vertices are:-

3 on 3 sides → $3 \times 4 \times 5 = \underline{\underline{60}}$.

2 on 1 side + 1 on other → $^3C_2 \times {}^9C_1 + {}^4C_2 \times {}^8C_1 + {}^5C_2 \times {}^7C_1$

$= 3\times9 + 6\times8 + 10\times7$

$= 27+48+70$

$= 145$

Ans → $145+60 = \underline{\underline{205}}$.

Ex Greatest possible no. of points of intersectⁿ of 8 st. lines and 4 circles is:-

8 st. lines + 4 circles + St lines & circles

$= {}^8C_2 + \underset{\times 2}{{}^4C_2} + 8\times2\times4$

$= 28 + \underset{\times2}{6} + \underset{\times4}{16} = 28+12+16\times4 = \underline{\underline{104}}$

Ex No. of non-negative integral solⁿ of $A+B+C \leq 10$

$A+B+C+D = 10$ $\qquad$ $^{13}C_3$

$= \frac{13\times12\times11}{6} = \underline{\underline{286}}$

Ex Rajesh went to the market to buy 18 fruits in all. If there were Mangoes, bananas, apples and oranges for sale then how many ways could Rajan buys at least 1 fruit of each kind?

$$A+B+C+D=18 \qquad A,B,C,D \geq 1$$

$$\underline{\underline{{}^{17}C_3}}.$$

<u>Ex</u> In how many ways you can divide 20 chocolates among 4 people A, B, C and D such that A is getting at least 2, B at least 2, C at least 3 and D at least 4?

$$A+B+C+D=20$$

$$2 \quad 2 \quad 3 \quad 4$$

$$A+B+C+D=9 \qquad {}^{12}C_3 = \frac{12\times11\times10}{6} = \underline{\underline{220}}$$

<u>Ex</u> A person has 4 coins each of different denomination. What is the no. of different sums of money the person can form (using one or more coins at a time)?

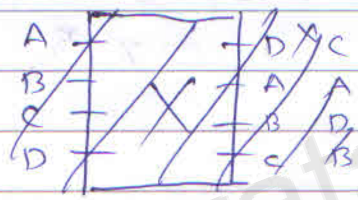
$$ {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = \underline{\underline{15}}$$

<u>Ex</u> No. of ways of factorizing 91,000 into 2 factors, m and n, such that m>1, n>1 and gcd (m,n) = 1.

$$91000 = 1000\times13\times7$$
$$= 2\times2\times2\times5\times5\times5\times7\times13$$
$$= \underbrace{2^3\times5^3\times7\times13}_{4}$$

$$2^{n-1}-1 = 2^{4-1}-1 = 2^3-1 = \underline{\underline{7}}.$$

Ex 4 boys A, B, C, D and their respective girlfriends E, F, G, H are to be seated around a rectangular table with 4 chairs on 2 sides each such that no couples sit exactly facing each other. The boys wanted to be seated on same side, how many seating arrangements are possible?



Derangement → $4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$

$= 12-4+1 = 9$

Ans → $4! \times 9 = 216$.

Ex There are 4 balls to be put in 5 boxes where each box can accomodate any no. of balls. In how many ways one can do this if

→ Balls are similar and boxes are different

$A+B+C+D+E = 4$

$^8C_4 = \frac{8\times7\times6\times5}{4\times3\times2} = 70$.

→ Balls are different, boxes are similar

4, 0, 0, 0, 0		1
3, 1, 0, 0, 0	$^4C_3$	= 4
2, 2, 0, 0, 0	$\frac{^4C_2}{2!}$	= 3
2, 1, 1, 0, 0	$^4C_2 \times \frac{^2C_1}{2}$	= 6
1, 1, 1, 1, 0	1	= 1
		15

divide by 2 bcoz

AB	CD
AC	BD
AD	BC
BC	AD
BD	
AC	

duplicate

→ Both boxes and balls similar

4, 0, 0, 0, 0	1
3, 1, 0, 0, 0	1
2, 2, 0, 0, 0	1
2, 1, 1, 0, 0	1
1, 1, 1, 1, 0	1

5

→ Both boxes and balls are different

~~Sets~~

Check with Abhishek

$4,0,0,0,0$ $\quad 1 \times {}^5C_1 = 5$

$3,1,0,0,0$ $\quad {}^4C_3 = 4 \times {}^5C_1 \times {}^4C_1 = 80$

$2,2,0,0,0$ $\quad {}^4C_2 = 6 \times {}^5C_1 \times {}^4C_1 = 120$

$2,1,1,0,0$ $\quad {}^4C_2 \times {}^2C_1 = 12 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 = 720$

$1,1,1,1,0$ $\quad 1 \times {}^5C_4 = 5$

930

Ans → $5 \times 5 \times 5 \times 5$

Ex Between 2 junctions stations X and Y there are 12 intermediate stations. The no. of ways in which a train can be made to stop at 4 of these stations so that no 2 of these halting stations are consecutive is.

$X \ \underline{A} \ \underset{I}{\_} \ \underline{B} \ \_ \ \underset{II}{\_} \ \underline{C} \ \_ \ \underset{I}{\_} \ \underline{D} \ \underset{I}{\_} \ \underline{E} \ \_ \ Y$

$A+B+C+D+E = 8.$ ~~A,B,C,D,E≥1~~

$A, E \geq 0 \quad B, C, D \geq 1$

$A+B+C+D+E = 5$

$${}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} = 126.$$

Ex If 5 letters are placed in 5 addressed envelopes, no. of ways placing exactly 3 wrong is?

$${}^5C_3 \times 3! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right]$$

$$= 10 \times (3-1) = 20.$$

___/___/___

Ex If $x = (a-1)(b-2)(c-3)(d-4)(e-5)$, where a, b, c, d and e are distinct natural numbers less than 6. If 'x' is non-zero integer, then the no. of sets of possible ~~sets of~~ values a, b, c, d and e are:-

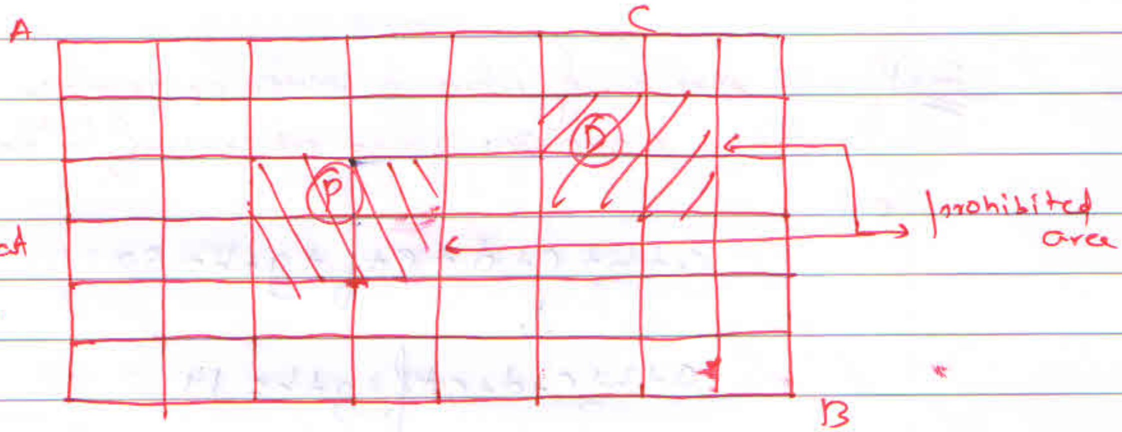
$a, b, c, d, e < 5$.

Derangement $\rightarrow 5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]$

$= 60-20+5-1$

$= \underline{\underline{44}}$

Ex



Streets are at right angles.

$\rightarrow$ Anu riders her bicycle from home at A to office at B, taking the shortest path. Then, no. of paths possible.

CV 814

All — Via P — Via D

$\frac{14!}{6!\,8!} - \frac{6!}{3!\,3!} \cancel{\frac{8!}{6!\,2!}} \times \frac{8!}{5!\,3!} - \frac{8!}{6!\,2!} \times \frac{6!}{4!\,2!}$

$= \frac{\cancel{14}\times13\times\cancel{12}\times11\times\cancel{10}\times\cancel{9}3}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}} - \cancel{\frac{6\times5\times4}{6}} - \cancel{\frac{8\times7}{2}}$

$= \cancel{91\times33-20\times28}$

$= 91\times33 - \frac{6\times5\times4}{6}\times\frac{8\times7\times6}{6} - \frac{8\times7}{2}\times\frac{6\times5}{2}$

$= 91\times33 - 20\times56 - 28\times15 =$

__/__/__

→ Arun rides his bicycle from home at A to club at C, via B taking the shortest path. Then the no. of ~~sh~~ possible shortest path he can take is.

ALL − Via C

Ex If n is ~~a~~ the no. of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, find n?

$$a+b+c+d+e+f+g+h=30 \qquad a,\dots,h \geq 2.$$

$$\downarrow$$

$$a+b+c+d+e+f+g+h=14$$

$$^{21}C_7$$

Ex All possible 2 factor products are formed from the numbers 1,2,3,4,..... 200. The no. of factors out of the total obtained which are a multiple of 5 is ___.

$$200 = 5+(n-1)5 \Rightarrow n=40$$

$$^{40}C_1 \times {}^{199}C_1 =$$

Ex Find no. of integral sol^n of to $|x|+|y|+|z| = 10$.

Non-zero

$x+y+z=10$ $\quad$ $x+y+z=7$

${}^9C_2 = \underline{\underline{36}}$ $\quad$ Ans $\rightarrow 36 \times 8$

$= \underline{\underline{288}}$.

1 zero ~~Non-zero~~

$\overset{\text{zero}}{|x|} + \overset{\text{non-zero}}{\overline{|y|+|z|}} = 10$.

$\Rightarrow |y|+|z| = 10$

$\Rightarrow y+z = \underline{\underline{8}}$. $\quad$ ${}^9C_1 = 9 \rightarrow$ Ans $\rightarrow \underline{9 \times 4 \times \overset{\text{Any can be zero}}{3}}$

$= \underline{\underline{108}}$.

2 zero

$\underline{2} \times 3$

Ans $\rightarrow 288 + 108 + 6 = \cancel{380}$ $\underline{\underline{402}}$.

Ex How many nos less than 1000 will have their sum of digits not more than 7?

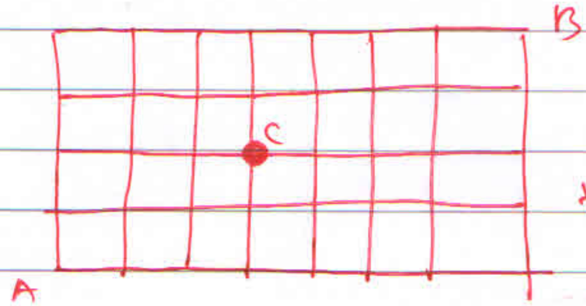
$a+b+c \le 7$

$a+b+c+\underline{\underline{d}} = 7$. $\quad$ ${}^{10}C_3$

$= \dfrac{10 \times 9 \times 8}{6} = \underline{\underline{120}}$.

__/__/____

Ex



A person can move only upwards or towards right. In how many ways can be travel from A to B via C?

$A \to C \quad \cancel{\text{VVH}}\ VVHHH = \frac{5!}{2!\,3!} = 10$

$C \to B \quad VVHHHH = \frac{6!}{4!\,2!} = 15$

Ans $\to$ $10 \times 15 = \underline{\underline{150}}$

Ex How many ways 10 identical chocolates can be distributed to Anil, Bipin and Chirag when Anil takes chocolate only if the no. of chocolates is in multiple of 3?

$A + B + C = 10$

0	$B+C=10$	${}^{11}C_1 =$	11
3	$=7$	${}^{8}C_1 =$	8
6	$=4$	${}^{5}C_1 =$	5
9	$=1$	${}^{2}C_1 =$	2
			$\underline{\underline{26}}$

Ex No. of integer solⁿ for eqⁿ $x+y+z+t = 20$, where $x,y,z,t \geq -1$

$x + y + z + t = 20$

$+1 \quad +1 \quad +1 \quad +1$

24

${}^{27}C_3$

Ex How many ways 30 identical chocolates can be distributed to 4 persons such that all are getting odd no. of chocolates?

$$(2A+1)+(2B+1)+(2C+1)+(2D+1)=30$$

$$\Rightarrow A+B+C+D=13$$

$$^{16}C_3 = \frac{16\times \cancel{15}^{5}\times \cancel{14}^{7}}{\cancel{6}} = \underline{\underline{560}}$$