Time, Speed and Distance Unplugged
by Total Gadha - Monday, 21 May 2007, 07:17 PM

For an MBA aspirant, a problem on time, speed and distance means solving complex situations with the help of many equations. Whether you are a CAT 2007 or 2008 aspirant, time speed and distance will give you many sleepless nights. The chapter has both easy and tough aspects to it. The easy aspect is the formula; there is only one formula- Distance = speed Â´ time- and it is known to every student. The tough aspect of course is the application of the formula; most students apply equations to solve the problems based on this formula. Letâ€™s have a look at a typical time, speed and distance problem:

Two trains start from stations A and B, respectively, towards each other at 12:00 noon. The trains take 7 hours and 13 hours, respectively, to cover the whole trip. What time do the two trains meet?

Well how would you solve it? If you already started writing an equation, hold your pen for a while. Thereâ€™s a better method. The method owes its existence to fundamentals of ratios and proportions. Letâ€™s have a look at the formula once again:

$$\boxed{\text{Distance} = \text{speed} \times \text{time}}$$

There are two special cases that are derived from this equation-

- **DISTANCE CONSTANT**- If the distance for travel is constant then the time taken for travel is inversely proportional to time, i.e.

$$\text{Time} \propto \frac{1}{\text{Speed}} \Rightarrow \frac{T_1}{T_2} = \frac{V_2}{V_1}$$

Therefore, if the speed of travel is doubled, the time of travel is halved. If the speed of travel is $\frac{1}{3}^{rd}$ of the original speed, the time of travel is thrice the original time. If the speed of travel is $\frac{2}{3}^{rd}$ of the original speed, the time of travel is $\frac{3}{2}$ times the original time. Let's put this principle into practice:

Examples:
I leave my home everyday at 8:00 am and reach my office at 9:00 am, not stopping anywhere along the way. One day, I left my home at my normal time and traveled the first half of the distance at $\frac{2}{3}^{rd}$ of my original speed. What should be my speed for the second half so that I reach my office in time?
Answer: If I travel with my normal speed, I take one hour to reach from my home to my office. Therefore, I take 30 minutes to reach halfway. Therefore, I reach the midpoint of my path at 8:30. If I travel with $\frac{2}{3}^{rd}$ of my original speed, the time to reach the midpoint of my path is $\frac{3}{2}$ of my original time. Therefore, I take $\frac{3}{2} \times 30 = 45$ minutes to reach halfway, as shown in the figure.

$$\mathbf{8{:}00}\ \underset{\text{Current speed}}{\overset{\text{Normal speed}}{\rule{5em}{0.4pt}}}\ \overset{\mathbf{8{:}30}}{\underset{\mathbf{8{:}45}}{|\,|}}\ \underset{\text{Required speed}}{\overset{\text{Normal speed}}{\rule{5em}{0.4pt}}}\ \mathbf{9{:}00}$$

I reach midway at 8:45. To reach the office in time, I need to travel the second half of the distance in 15 minutes. With my normal speed, I travel the second half of the distance in 30 minutes. Therefore, to travel the same distance in 15 minutes, I will have to travel with a speed **double** of my normal speed.

In the previous question, I traveled the first half of the distance at $\frac{2}{3}^{rd}$ of my original speed and the second half of the distance at $\frac{3}{2}^{nd}$ of my original speed. At what time will I reach office?

Answer: As discussed in the previous question, I will reach the midpoint of the path at 8:45 with a speed $\frac{2}{3}^{rd}$ of my original speed.

$$\mathbf{8{:}00}\ \underset{V_1}{\overset{\text{Normal speed}}{\rule{5em}{0.4pt}}}\ \overset{\mathbf{8{:}30}}{\underset{\mathbf{8{:}45}}{|\,|}}\ \underset{V_2}{\overset{\text{Normal speed}}{\rule{5em}{0.4pt}}}\ \mathbf{9{:}00}$$

If I travel the second half at $\frac{3}{2}$ of my original speed, my time becomes $\frac{2}{3}^{rd}$ of my original time, i.e. $\frac{2}{3} \times 30 = 20$ minutes. Therefore, the total time that I take is = 45 + 20 = 65 minutes. Therefore, I reach my office 5 minutes late.

I go to my office everyday with the same speed. One day I traveled with a speed that was $\frac{3}{4}^{th}$ of my normal speed and I reached my office 15 minutes late. What time do I normally take to reach my office?

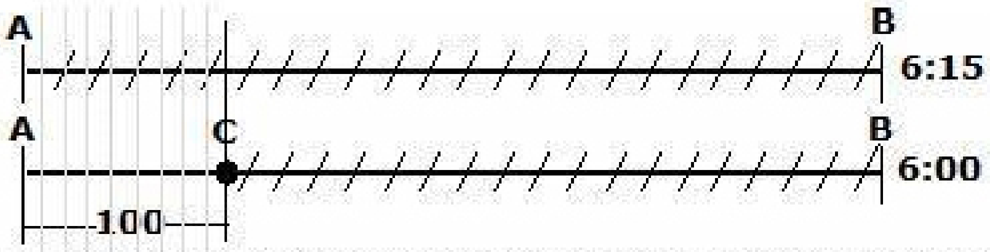
Answer: As distance is constant time taken will be inversely proportional to the speed. Since the speed becomes $\frac{3}{4}^{th}$ of the original speed, the time taken becomes $\frac{4}{3}^{rd}$ of the original time. Let the normal time taken everyday be t. Therefore, at $\frac{3}{4}^{th}$ of the normal speed, the time taken is $\frac{4t}{3}$.

Extra time taken = $\frac{4t}{3} - t = \frac{t}{3} = 15 \text{ min} \Rightarrow t = 45 \text{ min}$.

A train goes from station A to station B everyday. One day, it traveled with $\frac{4}{5}^{th}$ of its original speed because of engine trouble at A and reached station B at 6:15 pm. If the trouble had occurred after the train had traveled 100 km from A, the train would have reached B at 6:00 pm. What is the normal speed of the train?

Answer: The situation is summarized in the picture below. When the train travels the complete distance with speed $\frac{4}{5}^{th}$ of its original speed it reaches B at 6:15. When it travels the distance CB (where C is 100 km from A) with the speed $\frac{4}{5}^{th}$ of its original speed, it reaches B at 6:00

6:00.



The situation is similar to the previous question. From the figure we can see that the difference in arrival time in the two scenarios is occurring because of the difference in two speeds for the distance AC. Because the train is traveling the distance CB with the same speed in both the cases, this distance is not contributing to the change in arrival time.

For distance AC, the train is traveling with the normal speed in the second case and $\frac{4}{5}^{th}$ of the normal speed in the first case. Let the time to travel the distance AC with the normal speed be t. Therefore, in the first case, the time taken will be $\frac{5t}{4}$.

The extra time taken = $\frac{5t}{4} - t = \frac{t}{4}$ = 15 minutes (the change in arrival time is 15 min) ⇒ t = 60 minutes.
Therefore, time taken by the train to travel 100 km = 1 h ⇒ the speed of the train = 100 km/h

If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr, he will arrive at the same place at 11 a.m. At what speed must he cycle to get there at noon? **(CAT 2004)**
1. 11 km/h 2. 12 km/h 3. 13 km/h 4. 14 km/h

Answer: The distance is constant in this case. Let the time taken for travel with a speed of 10 km/h be t. Now the speed of 15 km/h is $\frac{3}{2}$ times the speed of 10 km/h. Therefore, time taken with speed of 15 km/h will be $\frac{2t}{3}$.

Therefore, extra time taken = $t - \frac{2t}{3} = \frac{t}{3}$ =2 h ⇒ t = 6 h. Therefore, the distance = 10 × 6 = 60 km. To reach at noon, the man will take 5 h. Therefore, the speed = 60/5 = 12 km/h

Ram and Shyam run a race between points **A** and **B**, 5 km apart. Ram starts at 9 a.m. from **A** at a speed of 5 km/hr, reaches **B**, and returns to **A** at the same speed. Shyam starts at 9:45 a.m. from **A** at a speed of 10 km/hr, reaches **B** and comes back to **A** at the same speed. (**CAT 2005)**
At what time does Shyam overtake Ram?
1. 10:20 a.m. 2. 10:30 a.m. 3. 10:40 a.m. 4. 10:50 a.m.

Answer: Both Ram and Shyam are reaching B from A and then returning. When Shyam overtakes Ram, the distances traveled by both of them are equal. Therefore, the times taken are inversely proportional to their speeds. Let Ram take t minutes for travel Shyam overtakes him. As Shyam started 45 minutes after Ram he takes t – 45 minutes.

$\Rightarrow \frac{t}{t-45} = \frac{V_{Shyam}}{V_{Ram}} = 2 \Rightarrow t = 90\,\text{min}$. Therefore, Shyam overtakes Ram at 10:30 am.

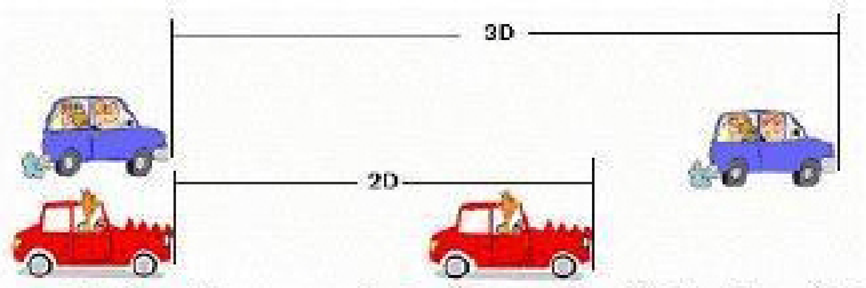
Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour, respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours Arun did Kiranmala start? **(CAT 2006)**
Answer: The ratio of speeds of Arun and Barun is 3: 4. Therefore, to travel the same distance, the times taken will be in the ratio 4: 3. Let Arun take time t. Therefore, Barun takes t – 2 as he started 2 hours after Arun.
1. 3 2. 3.5 3. 4 4. 4.5 5. 5

$\frac{t}{t-2} = \frac{4}{3} \Rightarrow t = 8$. Therefore Arun takes 8 hours. Now Kiranmala will take half that time as her speed is twice of Arun's speed. Therefore, she will take 4 hours for the same distance. Therefore, she should start 4 hours after Arun.

- **TIME CONSTANT**- If the time for travel is constant, then the distance traveled is directly proportional to the speed. i.e.

$$\text{Distance} \propto \text{Speed} \Rightarrow \frac{D_1}{D_2} = \frac{V_1}{V_2}$$

Therefore, if the speed of travel is doubled, the distance traveled in the same time will double. If the speed of travel is halved, the distance traveled in the same time will be halved… and so on.



In the figure above, two cars start from the same point with speeds of 40 km/h and 60 km/h. The ratio of their speed is 2: 3. Therefore, after any time interval t, the distance traveled by them would be in the ratio 2: 3, as shown in the figure.

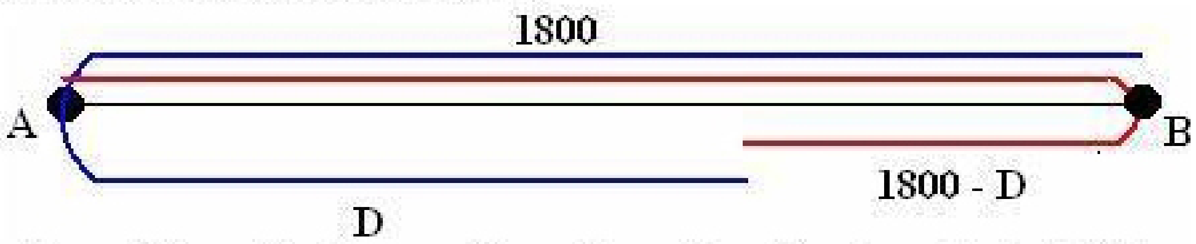
In the figure above, the two cars start simultaneously from two points, A and B, towards each other. The ratio of their speed is 2: 3. Therefore, after any time interval t, the distance traveled by them would be in the ratio 2: 3, as shown in the figure.

Let's put this principle into practise:

Examples:

Two cars simultaneously start traveling towards each other at 40 km/h and 50 km/h from two points A and B, respectively. The distance AB is 1800 km. At what distance from A do the cars meet for the first time?
Answer: Since the cars are taking equal time for travel, the distances traveled by them will be in the ratio 4: 5. The total distance is 1 800 km. Therefore, the distance traveled by the car from A will be equal to $\frac{4}{4+5} \times 1800 = 800$. Therefore, the cars will meet at a distance of 800 km from A.

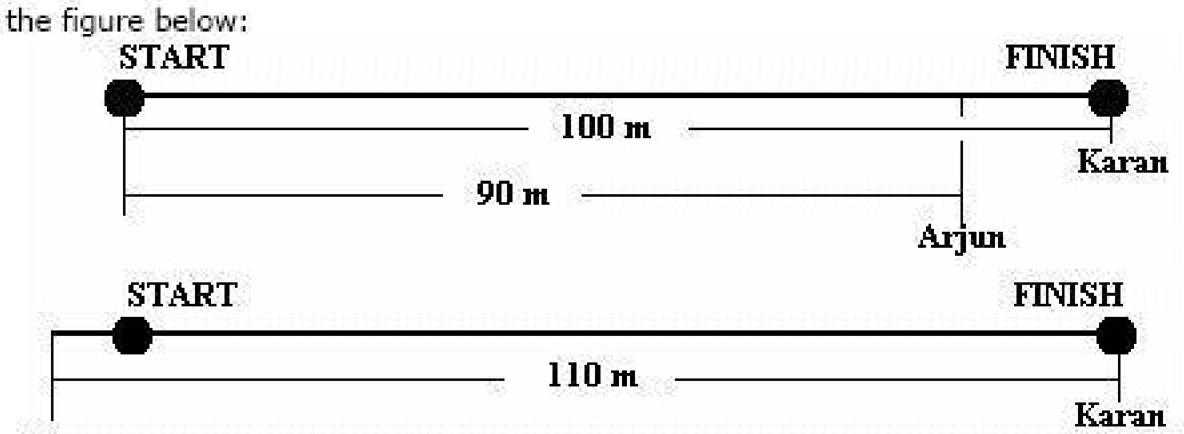
In the previous question, the two cars do not stop after meeting but continue on their way. After reaching the opposite end points, they again return along the path. At what distance from A will the cars meet for the second time?
Answer: The situation is summarized in the figure shown below:



Let the two cars meet at a distance D from A for the second time. The car from A has traveled a total distance of 1800 + 1800 – D. The car from B has traveled a total distance of 1800 + D.
Therefore, the total distance traveled by the two cars = 1800 + 1800 – D + 1800 + D = 3 × 1800 = 5400 km. Since the two cars are still traveling for the same time, the distances traveled will again be in the ratio of 4: 5. Therefore, the distance traveled by the car from B is equal to $\frac{5}{4+5} \times 5400 = 3000$. 3000 = 1800 + 1200. Therefore, the car from B travels 1800 from B to A and then a further distance of 1200 km. Therefore, the two cars meet at a distance of 1200 km from A.

Karan and Arjun run a 100-metre race, where Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in a second 100-metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race? **(CAT 2004)**
1. Karan and Arjun reach the finishing line simultaneously.
2. Arjun beats Karan by 1 metre.
3. Arjun beats Karan by 11 metres.
4. Karan beats Arjun by 1 metre.

Answer: The situation is shown in the figure below:



In the first case, Karan travels 100 m and Arjun travels 90 m in the same time. Therefore, the ratio of their speeds is 10: 9. In the second case, Karan is traveling 110 m to reach the finish line. In the same time, the distances traveled by both of them will again be in the same ratio as that of their speeds.

$$\frac{D_{Arjun}}{D_{Karan}} = \frac{V_{Arjun}}{V_{Karan}} \Rightarrow \frac{D_{Arjun}}{110} = \frac{9}{10} \Rightarrow D_{Arjun} = 99 \text{ m}$$

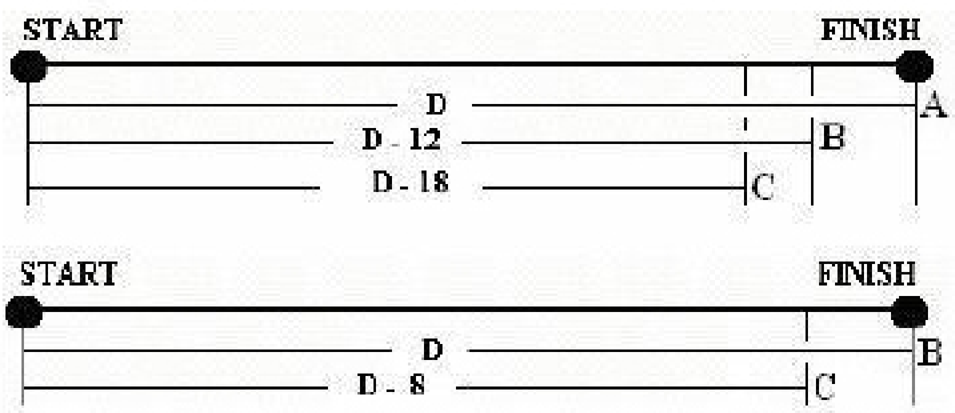
Therefore, Arjun has traveled only 99 m when Karan reaches the finish line. Hence, Karan beats Arun by 1 m.

Ram and Shyam run a race between points **A** and **B**, 5 km apart. Ram starts at 9 a.m. from **A** at a speed of 5 km/hr, reaches **B**, and returns to **A** at the same speed. Shyam starts at 9:45 a.m. from **A** at a speed of 10 km/hr, reaches **B** and comes back to **A** at the same speed. **(CAT 2005)**
At what time do Ram and Shyam first meet each other?
1. 10 a.m.
2. 10:10 a.m.
3. 10:20 a.m.
4. 10:30 a.m.

Answer: Ram takes one hour to travel 5 km and hence reaches **B** at 10:00 am. By 10:00 am, Shyam has traveled 2.5 km and is therefore, halfway through. After 10:00 am, Ram will start traveling back towards **A** and Shyam will be traveling towards **B.** Now, when they meet, they would have taken the same time for travel (after 10:00 am that is). Therefore, distance traveled by Shyam would be twice the distance traveled by Ram (Shyam's speed is twice that of Ram's speed). The initial gap between them at 10:00 am is half of **AB.** Now Shyam will travel 2/3rd of that distance and Shyam will travel 1/3rd of that distance. As Shyam travels half the distance in 15 minutes, he will travel 2/3rd of that distance in 2/3) × 15 = 10 min. Therefore, they will meet at 10:10 am.

Three runners A, B and C run a race, with runner A finishing 12 m ahead of runner B and 18 m ahead of runner C, while runner B finishes 8 m ahead of runner C. Each runner travels the entire distance at a constant speed. What was the length of the race? **(CAT 2001)**
1. 36 m
2. 48 m
3. 60 m
4. 72 m

Answer: The two cases are shown in the figure below:



When A reaches the finish line, B is D – 12 m behind and C is D – 18 m behind. Since the time taken is same the ratio of speeds of B and C is $\frac{D-12}{D-18}$. When B reaches the finish line, C is D – 8 m behind. The ratio of speeds of B and C is $\frac{D}{D-8}$.

$$\frac{D-12}{D-18} = \frac{D}{D-8} \Rightarrow D = 48 \text{ m}$$

A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is 3/8 of the distance AB measured from the entrance A. When the train whistles the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat exactly at the exit. The speed of the train is greater than the speed of the cat by what order? **(CAT 2002)**
1. 3: 1
2. 4: 1
3. 5: 1
4. None of these

Answer: Let the train be at a distance y from A. Let the length of the tunnel AB be 8x. Therefore, the cat is at 3x from A. Now both the conditions given in the questions assume 'same time' scenario. Therefore, the ratio of the speeds of the cat and the train will be equal to the ratio of the distances traveled by them.

The ratio $= \frac{y}{3x} = \frac{y+8x}{5x} \Rightarrow y = 12x$. Therefore, the ratio of speeds $= \frac{y}{3x} = 4:1$

A train X departs from station A at 11.00 a.m. for station B, which is 180 km away. Another train Y departs from station B at 11.00 a.m. for station A. Tram X travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for 15 minutes at station C, which is 60 km away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other? **(CAT 2001)**

1. 112 2. 118 3. 120 4. None of these

Answer: This question can be easily solved through a smart move. Train Y stops for 15 minutes. In 15 minutes it would have traveled 12.5 km. Therefore, let's us start train Y 12.5 km behind station B. Therefore, the total distance between the trains is 192.5 km. Now we can divide this distance in the ratio of their speeds, 7: 5. Therefore, the train X travels $\frac{7}{7+5} \times 192.5 \approx 112$ km

I conclude this chapter over here itself with only one thought for you all to ponder- I did not use equations in a single question. In the next chapter Iâ€™ll cover relative speed and circular motion. Till then, chew the cud over these problems.

Total Gadha