Dynamic Portfolio Optimization

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Abstract

Portfolio management is an essential aspect of the financial sector. All portfolio managers need up-to-date tools to make the best investment decisions while ensuring minimum risk to the investor. This paper focuses on dynamic portfolio optimization by integrating advanced derivatives and stochastic calculus. The derivatives espoused in the paper are Forward swaps, interest rate swaps, and currency hedging. This paper provides an extensive analysis of financial risk based on multinational companies, specifically concentrating on magnificent firms: Tesla, Google, Meta, Apple, Microsoft, Nvidia, and Amazon. Using historical data, the main risks analyzed here are credit, currency, interest rate, and market risks. The correlation matrix and hierarchical clustering of market indices show significant correlations with major US indices sensitive to interest rate changes. Currency fluctuations are also correlated with market indices, influencing the performance of global companies. We need a thorough understanding of these linkages to adopt suitable mitigation techniques, which can be achieved by utilizing statistical analysis of historical data and using the insights from them to build a better risk-adjusted portfolio. This project underscores the importance of multifaceted risk management strategies for multinational companies, including hedging, diversification, and real-time adjustment of portfolios based on market conditions. Companies can enhance their resilience and secure their financial stability by analyzing historical data and utilizing sophisticated risk management tools. Investors must assess these risk factors when constructing portfolios, making well-considered decisions based on thorough risk analysis to ensure stability and growth opportunities in a fluctuating economic environment. We've tried to build dynamic portfolio optimization to achieve real-time adjustment to our portfolio using stochastic calculus.

Keywords: advanced derivative, Interest rate swap, FX Swap, bench market interest rates, dynamic portfolio optimization, stochastic calculus, SONIA, SOFR

1. Introduction

1.1. Problem Statement

Portfolio managers can achieve risk-adjusted returns using dynamic portfolio optimization by integrating advanced derivatives, including interest rate swaps, to hedge currency risk. In the finance business, both novel and old mathematical models can be used to achieve this integration.The models can respond to the ever-dynamic financial markets, illustrate the balance between return and risk trade-off, and efficiently adapt to market volatilities and interest rate changes.

Portfolio optimization in the quant finance domain missed the effective adaptation to be dynamic in the fast-growing and complex derivative markets. The literature review illustrates a shortage of one-stop market prediction models integrated with derivatives to consider an investor's position maximally in real-time. It also notes no in-depth incorporation of the primary benchmark interest rates, i.e., Sterling Overnight Index Average (SONIA) and Secured Overnight Financing Rate (SOFR), in accessing their impact on the risk of portfolio and derivative pricing.

There is a need to develop a sophisticated model that can provide the financial market stochastically, incorporate different derivatives efficiently, and react to market alteration and benchmark interest rates. The difficulty is to build a system that predicts and responds to market situations, coordinates with the associated risk of the investor's return expectations, and secures optimized portfolio performance in different market conditions.

The goal of this thesis is to construct a stochastic portfolio optimization model that balances the risk and return profile in line with benchmark interest rates and incorporates a variety of derivatives suited to real-time market fluctuation. This can facilitate better risk-return management, better decision-making and oversight of the portfolios, which can be adjusted to account for fluctuations in the market.

1.2. Goals and Objectives

This project's primary goal is to develop a dynamic portfolio that minimizes risk, maximizes return and reacts instantly to changes in the benchmark interest rate in real time, all while using stochastic calculus as a modeling technique. To do this, the following goals will be implemented:

I. Develop an asset allocation model using descriptive and predictive tools to allocate assets to a portfolio in real-time. The model should be able to integratively classify assets in the most suitable portfolios based on emerging market forecasts to achieve optimal gains.
II. Deliberately include financial derivatives such as options and futures in a portfolio to holistically use their risk-return robustness to protect it from extreme volatility.
III. Analyze and comprehend how the Sterling Overnight Index Average (SONIA), the Secured Overnight Financing Rate (SOFR), and other interest benchmarks affect the pricing of derivatives and the portfolio as a whole.
IV. The stochastic model is a method for simulating unpredictable financial elements, like interest rate swings, market volatility, and other potential economic indicators.
V. Maintaining investors' risk tolerance while adjusting the model's flexibility to suit shifting demands and preferences requires careful calibration.
VI. Carefully calibrate the model's versatility to adapt to changing investor needs and preferences while maintaining investors’ risk tolerance.

1.3. Clear Project Description and Plan

The financial market is filled with robust models for portfolio management and portfolio optimization (Fernholz 1). The market needs to improve in a dynamic and responsive model to fluctuating markets and ever-changing interest rates. It notes that the conventional models used for portfolio optimization do not allow the integration of advanced derivatives in the incoming real-time data to produce timely market information for portfolio managers.

This project paper aims to develop an innovative portfolio optimization model that uses stochastic calculus for dynamic and effective asset allocation within a portfolio. It has shown the plans to incorporate hedging as a risk control measure against market volatility in the financial derivatives included in the portfolio. An important component of our model is using benchmarked interest rates like SOFR and SONIA to understand the impact of financial derivatives on the portfolio optimization model. The methodology plan to follow is illustrated here.

The first step is to review the literature, perform a thorough and relative review of the available models close to what the model aims to achieve, and consequently identify the models' constraints. It has been difficult to understand how much emphasis real-time data market adaptation uses the derivatives selected for the portfolio.

The second step is to build a model that emulates the portfolio's behaviors during market fluctuations while incorporating stochastic calculus. It aims to rely heavily on hedging as a risk control measure by ensuring the model comprises options and futures.

The third step is to collect the data types required for testing, training, and validating our model. This data includes benchmark interest rates such as SOFR, and SONIA and the main macro- and micro-economic indicators such as inflation and unemployment. These data types will be the powerhouse of the model optimization process.

The fourth step is to determine and indicate the precise investors' risk acceptance of the market based on the fine-tuning of the model developed above. This step will assess the model's performance based on various market cases and improve it for optimal risk-adjusted returns. The model is incorporated into the current financial system, with real-time monitoring and regulations.

The following output is expected from the above methodology. A robust, adaptable, dynamic portfolio optimization model can transform the real-time market and interest rate variation. Portfolio managers' decision-making is improved due to the performance of risk adjustment returns. A structure for portfolio management with advanced derivative integration is needed, which supports enhanced protection against uncertainties using hedging.

1.4. Project/Research Idea.

The main purpose of this project is to build an optimization model that dynamically allocates assets while incorporating advanced derivatives to reduce investors' market risk exposure.

This model aims to highlight the real-time reflection of the market fluctuations and tactical utilization of advanced derivatives such as options and futures to improve portfolio performance and resilience, unlike the existing models. The inculcation of stochastic calculus in the model helps provide precise portfolio risk prediction caused by market volatility.

1.5. Pain Points and Solutions

This section illustrates the pain points and the solutions available from our work. The available static models cannot adjust to the fluctuation in the real-time market, providing discrepancy to the current market situation. Many models do not completely utilize advanced derivatives for risk management and decent hedging, and since they are not optimized, they cannot provide the best return against market volatility. Few models utilize the variation in benchmark interest rate, influencing advanced derivatives and risk management.

The solutions for curing the above pain points include a dynamic model of the market situation that can be achieved by incorporating stochastic calculus into real-time data and providing a more precise and adjusted portfolio. The main element of the model utilizes advanced derivatives to enhance return and risk management in the portfolio. Integrating the bench market interest rates such as SOFR and SONIA and utilizing the valuation of derivatives within the models improves portfolio risk-return adjustment in the real-time market.

1.6. Possible Obstacles and Impediments

The enhanced stochastic models and their incorporation into the real-time financial data may provide complexity and challenge for continuous maintenance. Another issue is that obtaining real-time and trustworthy data is very important, and the precision of the model may be impacted by unreliable data or constraints on accessing the data. The model might be required to comply with financial regulations, which is even more important when incorporating advanced derivatives. Implementing the model may require a significant update in the technology utilized by the current financial models when incorporating the current financial model with the enhanced model. The model might experience a challenge with unpredictable situations for an enhanced model, such as when adopting financial downturns or unique market situations. A high level of adaptability may provide complexity for model personalization to fit different investor portfolios and risk tolerance.

2. Literature Review

Incorporating derivatives into portfolio management is an advanced idea in financial engineering. This provides a better risk-return balance within market volatility. Major research has been conducted recently on this topic, with different research and elaborate strategies accentuating the crucial requirement for dynamic models that integrate derivatives into portfolio optimization.

Stochastic volatility and the pricing of financial derivatives: A good model that can raise stochastic calculus to suggest the dynamic financial markets was offered by Van der Ploeg (2006). It focuses on portfolio optimization with varying market conditions and interest rate benchmarks. A multifactor stochastic volatility model for financial derivatives pricing was proposed in the study. It brought together the ever-changing financial markets and offered a risk profile and realistic market behavior. The model is able to adjust to changing market conditions and interest rate volatility. He offered a flexible platform with improved portfolio performance that can adjust to changes in the market.

This research paper illustrates a new technique to incorporate derivatives into portfolios, concentrating on risk-return optimization of the portfolios. The model introduced a technique to utilize hedging and manage to balance the risk-return by utilizing the financial derivatives. One of the main characteristics of van der Ploeg's model is its incorporation of benchmark interest rates such as SONIA and SOFR. It also included these interest rates that affect the pricing of the derivatives and risk of the portfolio. The author illustrated better decision-making and portfolio monitoring. This has been done by providing some constraints to the available models, which provide a portfolio optimization method that can react to the benchmark interest rate and market fluctuation. Most of the available models need more resilience to real-time market volatility. The opportunity to incorporate advanced stochastic models provides a breakthrough in portfolio management. The problem is the uncertainty of the stochastic models. The regulation may affect the use of the derivatives. Van der Ploeg's (2006) multifactor stochastic volatility model suggested a significant gap in the way portfolios are currently optimized, particularly in the area of risk management and dynamic market volatility.

Optimization in a Regime-Switching Market with Derivatives: Black and Scholes, Markowitz, and Merton are a few of the models and methods for portfolio optimization that J. Wei et al. (2014) covered. They discussed the option pricing by Black and Scholes, mean-variance optimization by Markowitz, and finally the continuous-time portfolio selection model by Merton. The authors went deep into the regime-switching model's application and development of financial markets, provided by Hamilton, Gray, and Bollen et al. The paper emphasizes the challenges of derivative integration in portfolio optimization models. This was mainly based on the regime-switching concept. It illustrated the failure of the traditional model to provide the perplexity of the Regime switching of the market and derivatives.

The paper includes recent techniques such as Kraft's elasticity and the Çelikyurt and Özekici's regime-switching. The authors studied how these techniques addressed the complexity of derivative incorporation and regime switching into portfolio optimization. They concluded that asset allocation in a regime-switching market, which includes derivatives, can be optimized.

The model can integrate the derivatives and regime-switching dynamics to portfolio optimization and provides market behavior comprehension and risk management. One of the drawbacks was using the complex model, which caused the practicality of using the model. There is some opportunity to utilize the model for robo-advisory services and algorithmic trading, in which dynamic and adaptive portfolio management is important. The fast technological advancement and the integration of financial instruments could provide long-term relevance.

The paper can be fitted to the market as a progressive technique to optimize asset allocation for portfolio managers. It implies a crucial requirement for models that can dynamically adapt to regime switching and integrating derivatives and provide an advance in portfolio optimization and risk management techniques.

The role of financial derivatives in emerging markets: Anna IIyina (2004) examines the role derivatives play in the growth and development of emerging economies. Financial derivatives often manage different financial exposures in interest rates, foreign exchange, and credit risks (IIyina, 2004). They allow cross-border cash flows and investors to diversify their portfolios.

The various types of derivatives examined in detail include currency, fixed-income, equity, and credit derivatives (IIyina, 2004). The paper agrees that financial derivatives have been instrumental in enhancing the growth of emerging economies, especially in countries that have eliminated capital controls and strengthened the underlying security markets.

Financial derivatives have been misused by market participants who create excessive leverage, avoid regulations, and flaunt accounting rules to beat internal risk management systems (IIyina, 2004). This misuse has in the past been captured to have resulted in major economic distresses such as The Mexican Crisis of 1994, The Asian Crisis of 1997-1998, Russia's Default and Devaluation of 1998, and Argentina's Default and Devaluation of 2001. Financial derivatives have been noted to have propelled economic growth in emerging markets at an interestingly fast rate in the presence of the right fundamental policy controls.

Stochastic Portfolio Theory: E. R. Fernholz (2002) introduced a new approach for analyzing portfolio behavior and stock markets. Stochastic Portfolio Theory follows a descriptive rather than the conventionally identified normative approach of traditional mathematical and quantitative finance (Fernholz, 2002). SPT is a versatile approach applicable under various assumptions (Fernholz, 2002). SPT follows in the footsteps of other descriptive scientific theories. It can be seen from the literature that SPT can adapt to the information in the real markets and produce real-time feedback that investors can use promptly.

The stochastic nature of SPT optimization allows it to be consistent with both equilibrium and disequilibrium, as well as with either arbitrage or no arbitrage (Fernholz, 2002). An important distinction in the work of Fernholz (2002) is walking away from the normal arithmetic rate of return model to a robust growth rate or logarithmic rate of return model where even the obscured aspects of a portfolio's behavior can be singled out (Fernholz, 2002). In the book's last chapter, Fernholz (2002) outlines the applications of SPT, emphasizing how the first-order optimization model is used in estimating variances and growth rates. The first-order model makes stochastic portfolio theory more effective than the normal classical portfolio theory, which struggles with estimating important parameters like growth rates and variances (Fernholz, 2002).

The general flow of the results shows that change in portfolio diversity is an important factor in the performance of portfolio managers. The change in diversity explains more than half of the variations in portfolio manager performance from 1971 to 1998.

3. Risk Analysis

Let's take an example of a global tech company like Apple and use that to explain the major risks that Multinational firms (i.e. also a portfolio that is investing in different regions/countries of the world) is exposed to. Apple Inc., a global leader in technology and consumer electronics, operates in multiple regions including the US, Europe, and the UK. Companies like Apple that have operations in multiple countries are subject to several risks that could negatively affect their operating capital and general financial stability. These risks include market, credit, currency, and interest rate risk. We plotted each country's repo rate to examine this risk (see Figures 3 (USD Rates), 4 (EUR Rates), and 5 (GBP Rates). The data on repo rates offers valuable insights into the interest rate landscape of each nation.

3.1. Interest Rate Risk

Interest rate risk covers the volatility in interest rates and its impact on the portfolio mainly fixed-income securities. This risk impacts the cost of borrowing and, hence, the value of the investments.

- Impact
    - Debt Financing: Debt financing gets directly affected by the shifts in interest rates. Either one is going for refinancing of the existing debt or getting into taking new debt it will lead to more cost because of higher interest rates.
    - Investment Returns: Having sizable cash reserves that can be deployed in a variety of financial instruments based on the changes in interest rates across various major economies may help achieve insulation from interest rate changes and get better results.
- Mitigation Strategies

- Hedging for Interest Rate Volatility: Insulation from interest rate fluctuations can be achieved using financial instruments like Interest Rate Swaps. We can stabilize interest expenses by exchanging the floating-rate interest payments for the fixed-rate payments using interest rate swaps.

- Diversified Investment Portfolio: Investing across economically stable regions in the world can help us have cushion from the risk of changing interest rates. We can also invest in various asset types and maturities across these regions.

3.2. Currency risk

It is also known as exchange rate risk or FX risk. It arises from volatility in the exchange rate of different currencies with respect to other currencies, but mostly w.r.t US Dollar. The variation in exchange rates directly affects the operations of any company that deals with multiple currencies further affecting the value of the company's assets and liabilities and their financial statements.

- Impact
  - Valuation Changes: Volatility among currencies can lead to profits or losses when the value of inter-country investments is converted back to the investor's base currency. For example, Apple generates a significant portion of its revenue outside the US, and changes in exchange rates can affect the dollar value of the revenues, affecting the company's financials.
  - Cost Variations: Apple incurs costs in different currencies, particularly for manufacturing and assembly in countries like China. Currency fluctuations can affect these costs when converted to USD.
- Mitigation Strategies
  - Natural Hedging: Matching revenue and costs in the same currency can naturally hedge against exchange rate fluctuations. For instance, using revenues generated in China to pay for manufacturing costs in China.
  - Currency Derivatives: Utilizing financial instruments like forward contracts, futures, and options to hedge against adverse currency movements.

3.3. Credit Risk:

Credit risk is the risk of arising from a counterparty's inability to meet its financial obligations. On a high level, we can associate it with the region's stability. For this, we have done a region-wise analysis of P/E ratio for their respective stock indices. We have done an analysis on the US and Canada, Latin America, East Asia, ASEAN and Oceania, South and West Asia, and Europe regions (Refer to Figure 12).

- Impact
  - Counterparty Risk: This risk deals with the probability of default from the suppliers, manufacturers, or the opposite party in a deal. The financial instability of the counterparty can be assessed from a high level by analyzing the instability of the region to which the counterparty belongs.
  - Customer Credit: Extending credit to distributors and customers, especially in emerging markets, exposes Apple to non-payment risk.
- Mitigation Strategies
  - Credit Analysis and Monitoring: Thorough credit assessments of counterparties is a must before going into any financial contract. Also, chose renowned credit rating agencies like Moody's, Fitch, and Standards and Poor for a reliable credit analysis.

    - Diversification: Diversification of assets into multiple regions leads to reduction in reliance on any single counterparty thus reducing the credit risk.
    - Credit Insurance: Investing in credit insurance instruments like credit default swaps might help reduce credit risk. This helps in covering losses in case of non-payment or bankruptcies.

3.4. Market Risk

Market risk is the possibility of an entity experiencing losses due to changes in factors that affects the performance of the financial markets. These include factors like GDP (real/nominal), global events, regulatory changes, infrastructure and technological developments, etc. We analyzed major market indices such as S&P 500, Nifty 50, FTSE, etc. each belonging to major countries for the duration between 2010 to 2020 to analyze market risks.

- Impact
    - Changes in Equity Price: Significant movement in prices of listed stocks leads to high volatility that directly affects the index prices and the functioning of the counterparties.
    - Commodity Price Changes: Variations in the prices of raw materials (e.g., rare earth elements used in electronics) can impact production costs.
    - Systemic Risk: Major economic and political events such as black swan events can lead to financial system disruptions, such as a financial crisis e.g. Bankruptcy events like Lehman Brothers caused by high selling of subprime mortgages. Events like these have devastating effects across all asset classes, leading to significant portfolio losses.
- Mitigation Strategies
    - Commodity Hedging: Getting into a forward commitment such as futures and forwards contracts to have fixed pricing for some time t reduces our exposure to fluctuating commodities prices.
    - Strong Brand and Innovation: A strong brand is mostly insulated from demand fluctuations even in economic downturns. Building a strong brand requires continuously investing in innovation gaining customers trust by providing unparalleled value to the customer, establishing and trust, and building a robust ecosystem.

4. Methodology

4.1. Portfolio Construction

Portfolio construction involves selecting a combination of assets and/or securities, such as stocks and bonds, and blending them to achieve specific financial goals while adhering to a set and investor-acceptable risk tolerance. The process of portfolio construction involves an elaborate procedure outlined here.

Constraints analysis introduces the process of portfolio construction. An investor has to understand the prevailing constraints when building a portfolio. Constraints are the different types of limitations an investor will likely encounter when building a portfolio. These constraints include that an investor should check whether the portfolio can still be sustainable by the investor even with current and constant income needs. The safety of the principal amount is important, especially when inflation and management fee rates are high. The existing taxation regime helps an investor determine the appropriate moment to build a portfolio. An investor's liquidity status helps avoid abandoning the portfolio before achieving the set goals.

An investor must understand the desired mark a portfolio must achieve. This concerns the dispensation of current income, the desired individual income growth, the intended level of principal amount appreciation, and the need to preserve personal capital from loss. The focus is on understanding risk and return analysis, the safety of the investor's principal amount, personal income growth, the maximum capital appreciation at any given moment, and the general individual objectives of getting specific assets and commodities in a portfolio. Portfolio diversification will depend on an investor's financial needs and risk tolerance. The investor reserves the right to continuously manage the portfolio and make necessary changes as economic regimes dictate.

4.2. Options

An option is a derivative, a contract that gives the buyer the right, but not the obligation, to buy or sell the underlying asset by a certain date i.e., expiration date at a specified price i.e., strike price.

Call options give the buyer the right, but not the obligation, to buy the underlying asset at the strike price specified in the option contract.

Puts give the buyer the right, but not the obligation, to sell the underlying asset at the strike price specified in the contract.

Option pricing is an important aspect of options; this requires that an investor carefully choose an optimal pricing model.

4.3. Forward Swap

A forward swap is a contract in which parties agree upon a date $t = 0$ to buy or sell the commodity or security at the date T for a set price F specified $t = 0$. Some key components of a forward swap are agreement terms, valuation, and risk (Haugh, 2016).

- Terms of the agreement: the agreement between parties is that one party pays for a commodity or an asset today, and the other delivers it at a future date.
- Management of risks: The main advantage of forward swaps is that participants can reduce uncertainty and protect against adverse movements by locking in rates or prices today.
- Valuation: Whenever counterparts have a forward swap, the future price, i.e, the forward price is fixed. The valuation of a forward swap affects predicting future cash flows and discounting them back to the present value.

4.3.1. Forward price formula

Forward price is an essential aspect of a Forward Swap contract; it considers the associated opportunity costs, interest or foregone interest, and any other costs related to the underlying asset and the current spot price. Below is the formula expressed as equation 1:

$$F = S_0 \, x \, e^{rT} \qquad \text{(Equation 1)}$$

Where: F = forward price, S_0 = current spot price for the underlying asset, r = risk-free rate, T = delivery time in years.

In exceptional circumstances where a forward contract has carrying costs, then the forward price is adjusted as follows:

$$F = S_0 x e^{(r+q)T} \quad \text{(Equation 2)}$$

Where q = carrying costs.

4.4. Interest Rate Swap

An interest rate swap is a contractual agreement between two counterparties where each party commits to make cyclic interest payments to the other party based on a known principal amount (Beckley, 2017). Whittaker (1987) states that a financial transaction exchanges a fixed interest rate for floating interest in the same currency. It is a financial derivative instrument that involves the exchange of one stream of future interest payments for another based on a specified principal amount. Interest rate swap is an essential financial derivative, and its key areas are highlighted below:

Contract Initiation: Two parties called counterparties agree to exchange interest rate cash flows, where typically one stream of payments is fixed rate, i.e., swap rate, while the other is floating rate. The counterpart who pays the swap rate is called a payer, while the counterparty who pays the variable rate and receives the swap rate is called a receiver.

Risk Hedging: Interest rate swaps are commonly used to hedge against or speculate on changes in interest rates. The basic tenet of an interest swap is “one participant exchanging an advantage in one credit market for an advantage available to another participant in a different credit market” (Whittaker, 1987). For instance, a company with a variable-rate loan might use a swap to secure a fixed rate.

Cash Flow Exchange: Throughout the swap's life, the payer and the receiver will periodically exchange cash flows on agreed-upon dates based on the difference between the contracted fixed rate and the prevailing floating rate.

Market Adaptation: The terms of the swap can be tailored to the needs of the parties involved, such as the frequency of payments, the notion of the principal amount (which typically does not exchange hands), and the duration of the swap.

Interest rate swaps have two legs: a floating leg (FLT) and a fixed leg (FIX).

The floating rate cash flows are expressed in the following equation:

$$S_i = \left(\frac{NAD_{FLT,i}}{NTD_{FLT,i}}\right) r_{FLT,i}$$

(Equation 3)

On the other hand, the fixed-rate cash flows are given by:

$$FS = \left(\frac{NAD_{FIX,i}}{NTD_{FIX,i}}\right) r_{FIX,i} \quad \text{(Equation 4)}$$

Where: r_{FLT} = floating rate for the time i, r_{FIX} = fixed swap rate, NAD_i= accrued days during the payment period, NTD_i = number of days in a year for cash flow i.

Once the two equations have been obtained, then the next step is computing the receive-fixed, pay-floating net cash flow as follows:

$$FS - S_i = AP(r_{FIX} - r_{FLT,\,i}) \quad \text{(Equation 5)}$$

In the same measure, the receive-floating, pay-fixed net cash flow is computed as follows:

$$S_i - FS = AP(r_{RFL,\,i} - r_{FIX})$$
(Equation 6)

Where; AP = Accrued Period.

4.5. Interest Rate Hedging

Hedging interest rates allude to an investment position that aims to reduce the risks related to volatility in interest rates. An interest rate hedge is secure against the risk.

4.6. Geometric Brownian Motion

This section describes the asset pricing model for financial markets in continuous time. Therefore, asset prices are modeled as continuous-time stochastic processes, utilizing the stochastic differential equation and diffusion processes as the main building blocks.

Geometric Brownian motion is one of our main building blocks for modeling asset prices, and it also occurs naturally in many other places (Hull, 2018) and (Sherve, 2004). The equation is one of two natural generalizations of the most straightforward linear ordinary differential equation. The following equation represents the stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad \text{(Equation 7)}$$

$$S_0 = s_0 \quad \text{(Equation 8)}$$

Where S_t represents the asset's price at a time t, μ represents the drift coefficient, which indicates the asset return, σ represents the volatility coefficient, the asset's return standard deviation, and W_t is the Weiner process.

It can be written as:

$$S_t' = \left(\mu + \sigma W_t'\right) S_t \quad \text{(Equation 9)}$$

The GBM equation 7 can be represented as a linear ordinary differential equation.

The main feature of the GBM is the log-normal distribution, which is the asset price logarithm, provided the asset price follows a log-normal distribution. The non-negative price that can be modeled is the Brownian motion exponential, which maintains favorable prices. Finally, the GBM is the Black-Scholes foundation of the option pricing model.

4.7. Martingales

It is a type of stochastic process that models a fair game i.e. the future expected value is equal to the present value given all the past information. By William (1991) and Karatzas (1998) ‘The process's future expected value is conditional on the present and past values and equal to the present value’

A process of $\left\{M_t\right\}_{t\geq 0}$ is Martingale to a filtration $\left\{F_t\right\}_{t\geq 0}$ if:

$$E\left[M_{t+s} | F_t\right] = M_t \quad \text{(Equation 10)}$$

Where for all t, $s \geq 0$, E represents the expectation, and F_t illustrates the information available for the time t.

The main characteristic of Martingale is its fair game property. The future value is equal to the current value on average, which provides a fair game with no systematic gain. Martingale is utilized in financial derivatives, specifically for risk-neutral measures, and the discounted asset prices are based on the Martingale.

4.8. GBM and Martingale Combination

Geometric Brownian motion can be simulated for risk-neutral measurement using the Martingale property in finance (Björk, 2009) and (Baxter, 1996). The discounted price process based on the risk-neutral measure can be calculated as follows:

$$S_t'' = e^{-rt} S_t \quad \text{(Equation 11)}$$

Where r represents the risk-free interest rate, this characteristic will provide the basis for financial derivative pricing theories.

5. Results Discussion

This section discusses the results of different parts of the code. The model read the data for major currencies and incorporated the forward swap to adjust the portfolio weights, which would be based on the forward swap rates. Figure 1 indicates the correlation between the key currencies, such as the US Dollar, Euro, Japanese yen, and Great Britain Pound. The matrix suggests a negative correlation between the US Dollar and the Euro between 2010 and 2020, 10 years. Also, it indicates no correlation with the Japanese Yen and a low positive correlation with the Great British pound.

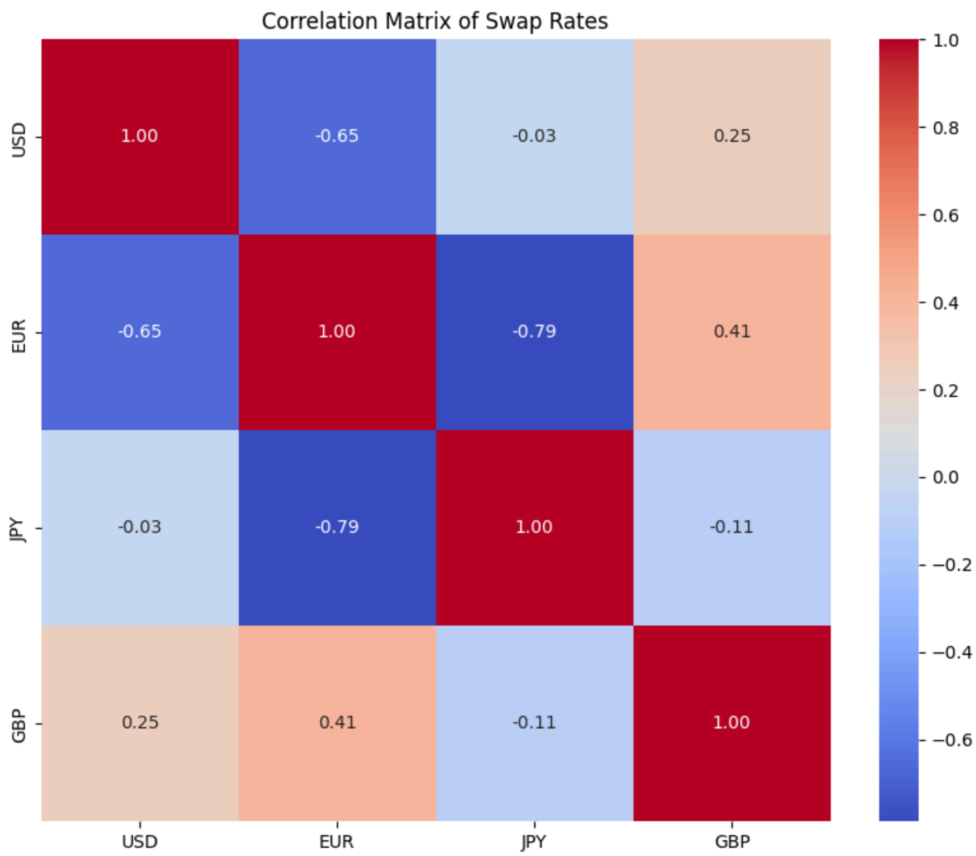

Figure 1: Currencies Correlations Matrix of Swap Rates

Figure 2 below shows the rolling correlation between the USD and the other three currencies. The plot indicates that the USD had a strong positive correlation from the end of 2014 to mid-2015, while it had a negative correlation with EUR almost during these ten years.

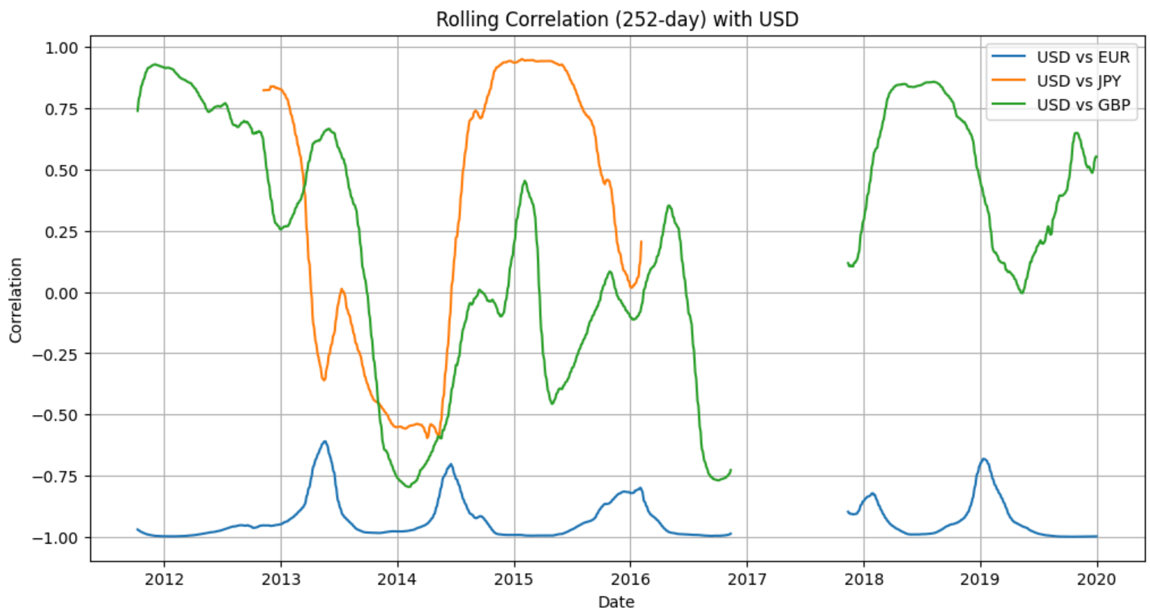

Figure 2: Rolling correlation between USD vs EUR, JPY, and GBP

Figure 3 shows the time series of USD rates based on the 10-year treasury over ten years. The lower rates indicate a low economy, which may require the central bank to intervene, while the higher rates indicate inflation. Some short-term volatilities can be seen in 2011, late 2013, and 2018. The overall fluctuation may indicate a cyclical economic pattern. The chart suggests a historically low around 2016, a period of low yield, and a higher pick around 2011, which may indicate a higher yield.

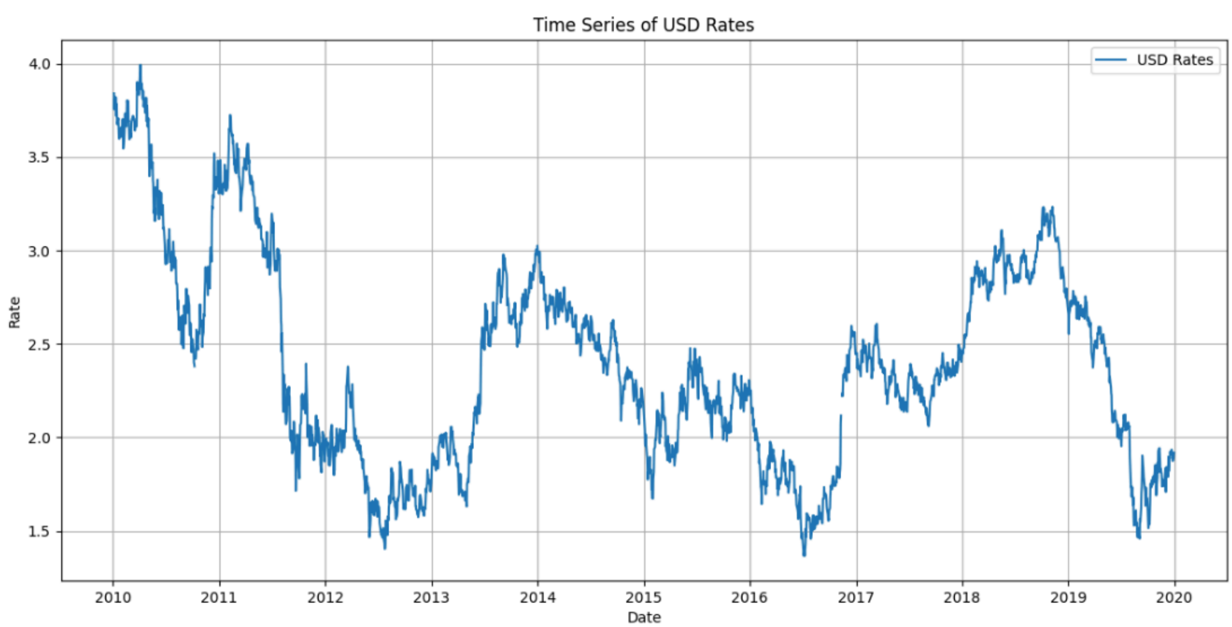

Figure 3. Time series of USD rates based on 10-year US treasury yield

Figure 4 shows the time series of the EUR rate based on the 7-10 years treasury ETF. The time series plot illustrates the long-term trends and volatility in EUR rates over a decade. The overall trend is upward, with periods of significant volatility. Such a time series analysis is critical for understanding the historical behavior of exchange rates, which can inform future financial and investment decisions. The EUR rates indicated an upward trend between 2010 and 2012, increasing from around 65 to 85. There was significant volatility during this period, with noticeable peaks and troughs. The rates continued to rise, reaching above 95 by mid-2014. There were periods of volatility, but the overall trend was upward. The EUR rates experienced more fluctuation and a minor downward trend, decreasing from around 95 to 85 from 2015 to 2017. The rates resumed their upward trend, with noticeable volatility, eventually reaching a peak of around 105 by late 2019. The rates remained high but indicated signs of stabilization, with volatility around the 105 mark.

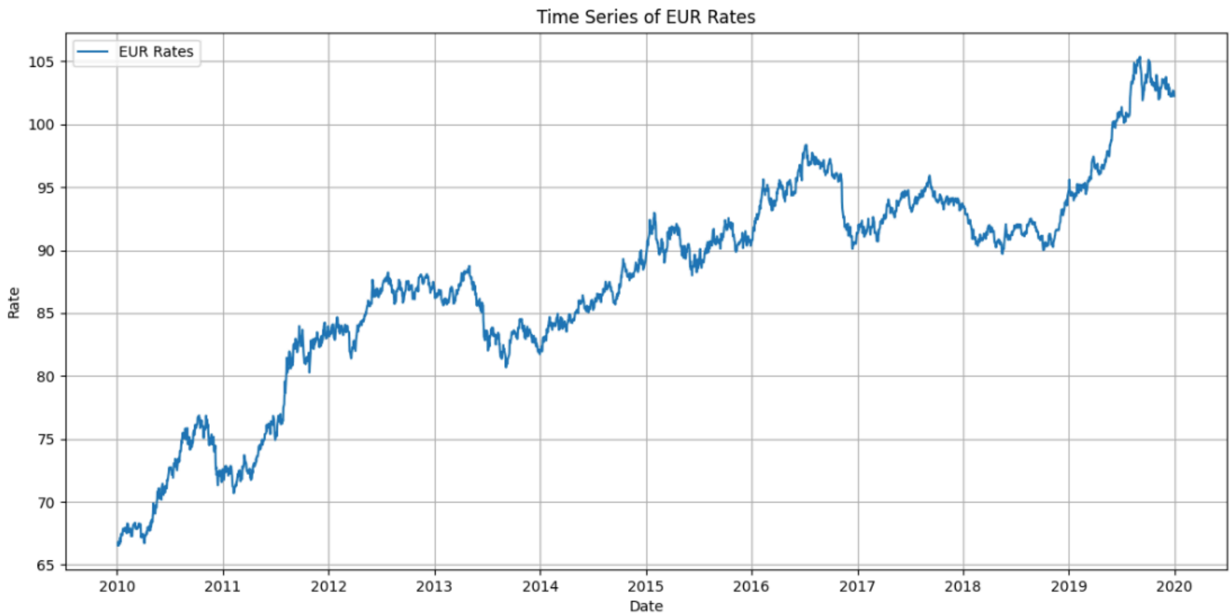

Figure 4. Times series of EUR rates based on the 7-10 year treasury

Figure 5 indicates the time series of the JPY rate based on the Japan government bond ETF. The decrease in the EFT rate over the ten years indicates an increase in the Japanese bond, which might be due to the deflation in Japan, which could be caused by economic stimuli by the Japanese bank. There were spikes in 2013 and 2016 due to the market response or global financial events. Japan is known to have near-zero yields, which is aligned with the deflationary issue the government is dealing with. It indicates the risk-averse behavior where capital assets go to government bonds.

Figure 5. Times series of JPY rates based on the Japan government bond ETF

Figure 6 shows the time series for the GBP rate based on the UK government bond ETF. It shows multiple peaks and valleys during the ten years, which change due to economic conditions and responses to political situations such as Brexit. There was a decrease in rates from 2010 to 2012 and during 2015, which would be translated to higher bonds or a reduction in yield. This could be caused by the financial crisis. A sharp rise and high volatility in the rate in early 2016 related to Brexit and the investors' concerns. The graph indicated a more stable rate during 2018 and 2019. There are some signs of volatility around the end of 2018.

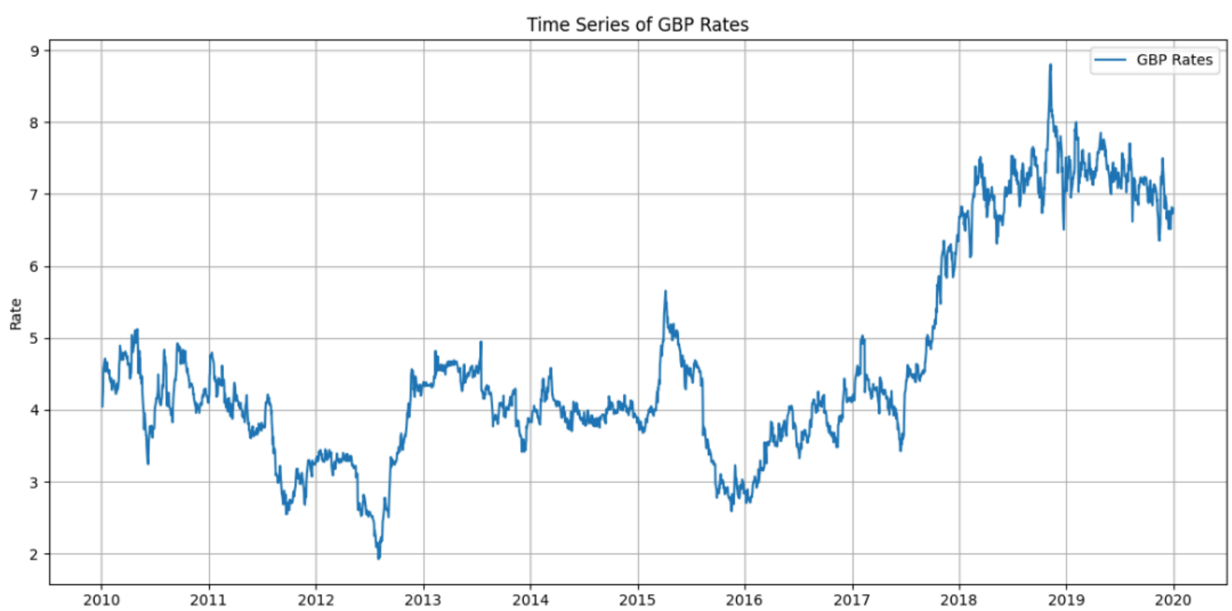

Figure 6. Times series of GBP rates based on the UK government bond ETF

Figure 7 shows the histogram of the Kernel density return in USD. The return distribution can be observed at around zero, indicating that for most of the day, there was no drastic movement in the price. The shape of the graph is close to a uniform distribution, which indicates the normal distribution of the daily return. It indicates a moderate level of volatility. There is no sign of skew, which needs to be confirmed with statistical analysis. There are very few outliers with no fat tails.

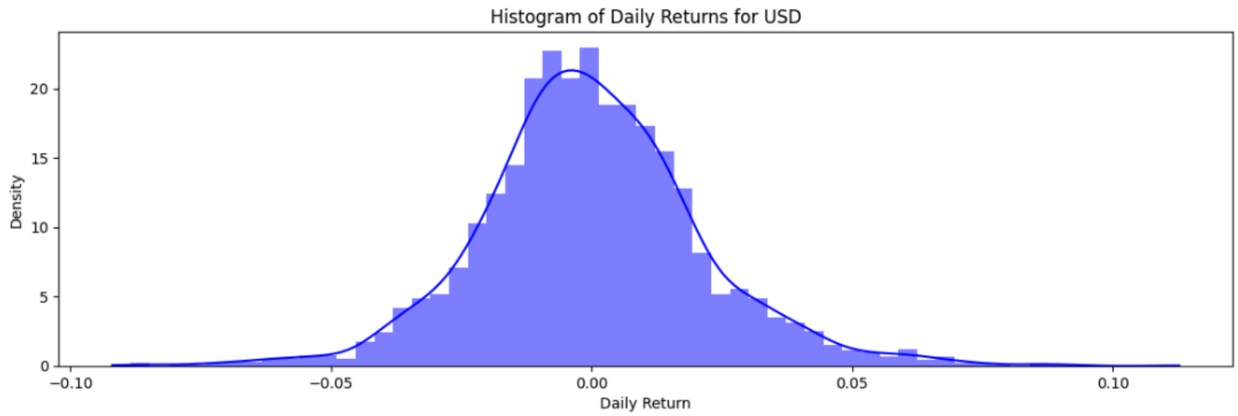

Figure 7. Kernel density return for USD

Figure 8 indicates the histogram of the Kernel density return in EUR, and the performance is very similar to that of the previous graph.

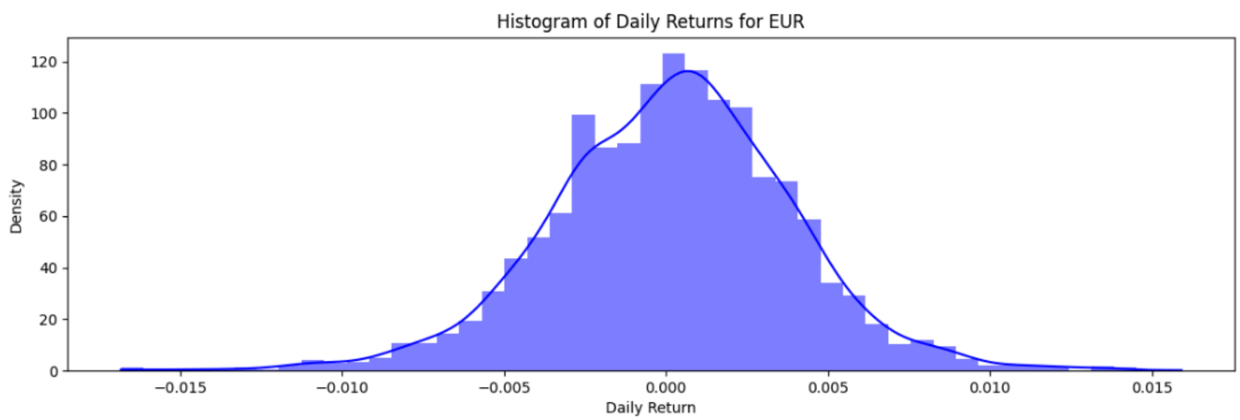

Figure 8. Kernel density return for EUR

Figure 9 shows the histogram of the Kernel density return in JPY. The highest return is concentrated around zero, which indicates a shallow price change. The graph shows a very low volatility in the return. There is no sign of outliers. It can be seen that the large spike at zero may indicate there is not much on the daily price change. This might be caused by missing data that is filled out by zero. There will be more investigation into this plot.

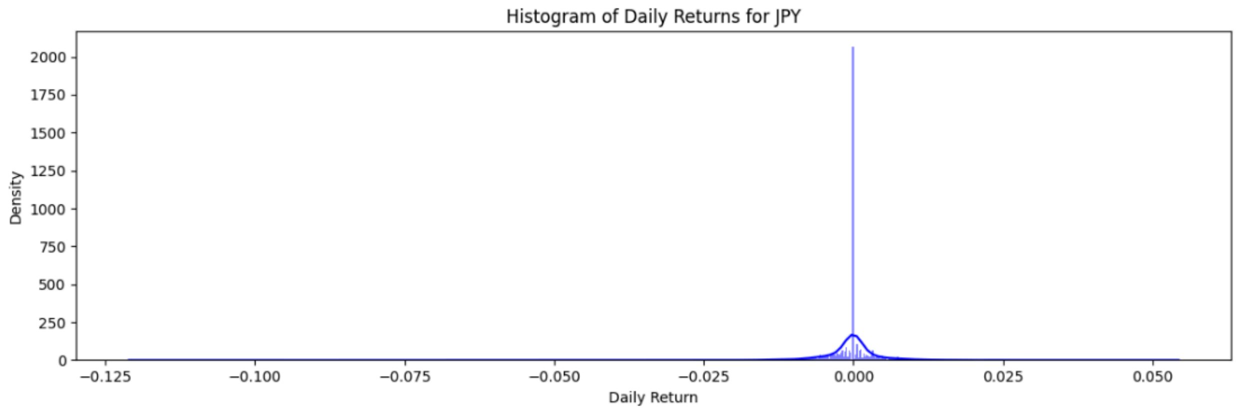

Figure 9. Kernel density return for JPY

Figure 10 illustrates the histogram of the Kernel density return in GBP, and the performance is also very similar to the USD and EUR graphs.

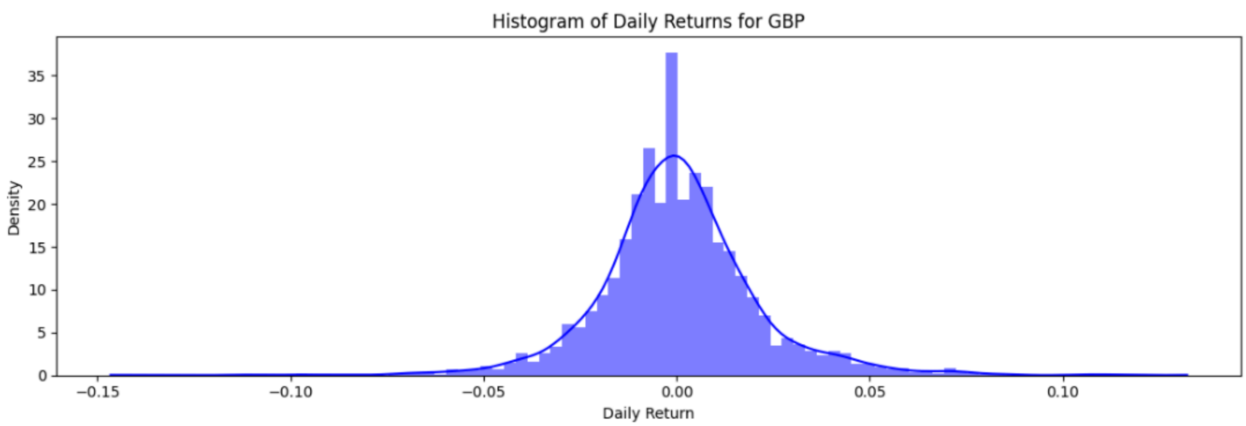

Figure 10. Kernel density return for GBP

Figure 11 shows the returns of major stock indices over different periods (6 months, 1 year, 3 years, 5 years, and 10 years) based on data from September 30th between 2010 and 2020. The plot shows significant variability in returns across major stock indices. Short-term returns (6MR and 1YR) show more variation, including some negative returns, whereas long-term returns (5YR and 10YR) are predominantly positive. This means that companies doing short-term deals across different markets will be exposed to higher volatility in the form of Market risk than those with long-term deals (i.e. >=3 years).

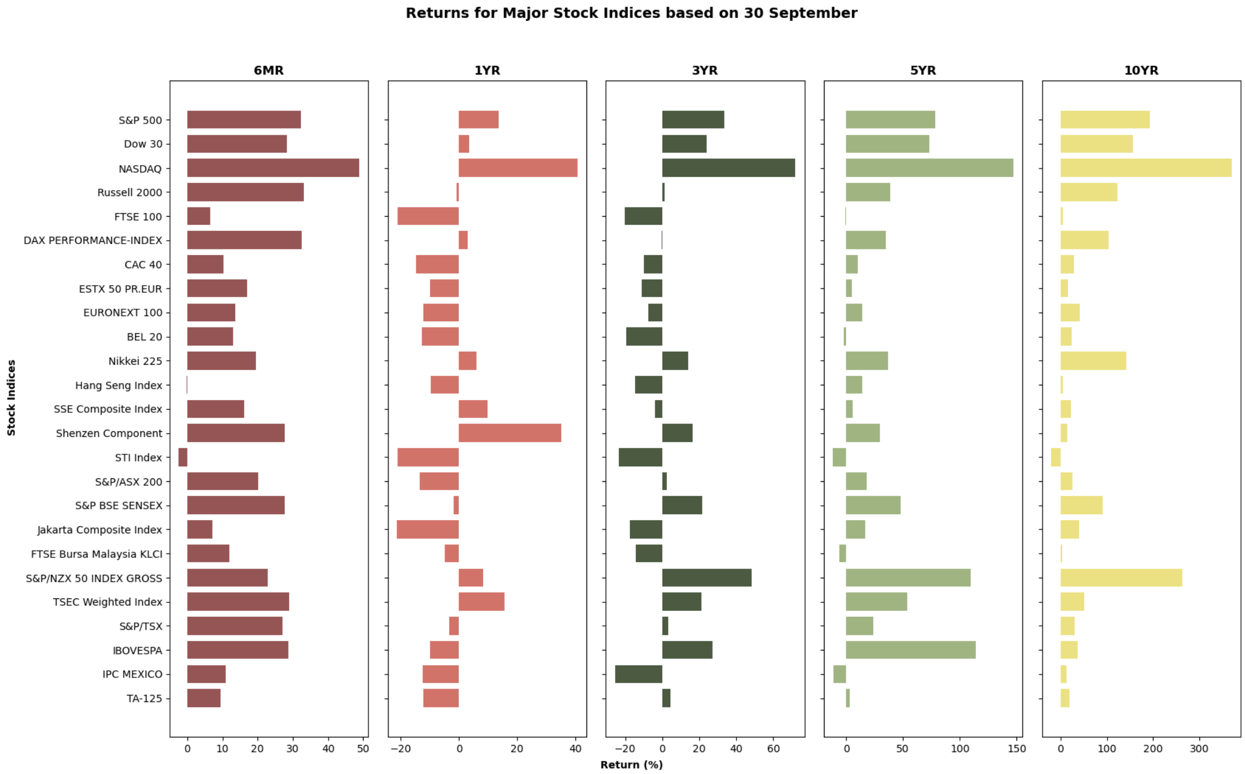

Figure 11: Returns of Major stock indices on September 30th between the period 2010 and 2020

Figure 12 illustrates the Price-to-earnings (P/E) ratios for significant stock indices across regions: US & Canada, Latin America, East Asia, ASEAN & Oceania, South & West Asia, and Europe. The P/E ratio is a critical financial metric that indicates the market's valuation of a company or index relative to its earnings. It is a crucial indicator to know the stability of markets (in our case regions). It is calculated by dividing the current market price by the earnings per share (EPS).

The variation in the P/E ratio shows trends in the major stock indices across different regions over the last five years. Key takeaways include:

- US & Canada: High market valuations with notable volatility.
- Latin America: Significant fluctuations reflecting economic and political instability.
- East Asia: Steady P/E ratios with occasional spikes in Japan.
- ASEAN & Oceania: Rising trends in Australia, stable elsewhere.
- South & West Asia: Consistent rise in India, volatility in Israel.
- Europe: High valuations with a dip in 2020 due to the pandemic.

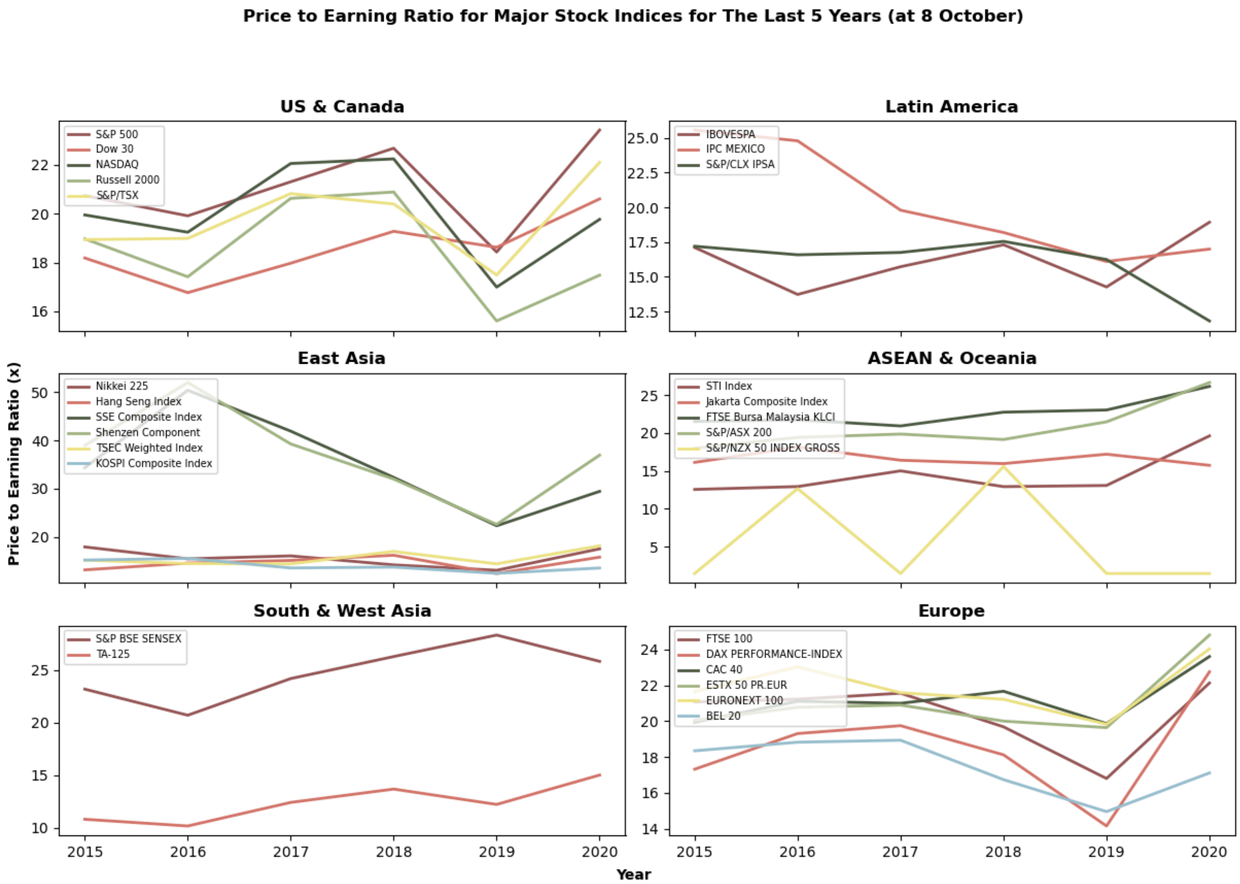

Figure 12: Price to Earnings Ratio for Major Stock Indices (2015-2020)

Figure 13 shows the historical rates of SOFR and SONIA from 2018 to the present. SOFR (blue line): This rate depicts higher volatility than SONIA, with significant fluctuations. There is a spike above 5 around 2020, a decline close to 0 around early 2020, and a steady rise starting around 2021, continuing upward through 2023 and leveling off slightly above 5 in 2024. On the other hand, SONIA (green line) indicates more gradual changes. SONIA remained relatively stable until 2020, with a major rise starting at the beginning of 2022 and continuing steadily upward, leveling off just below 5 in 2024. The main observations are that both rates indicate a period of stability from 2018 until early 2020, with a significant drop in SOFR at the beginning of 2020 coinciding with the onset of the COVID-19 pandemic. Post-2020, both rates increase, with SOFR showing more fluctuations and spikes. By 2024, both rates have stabilized, with SOFR slightly higher than SONIA.

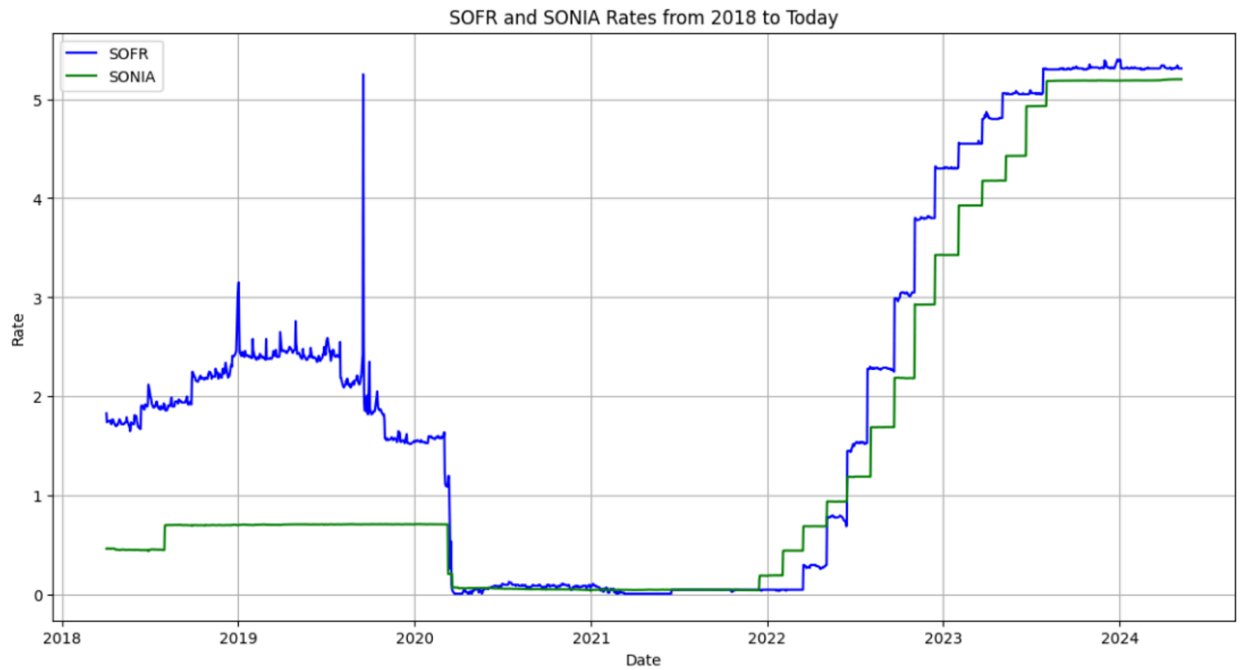

Figure 13. SOFR and SONIA rates

Figure 14 shows the normalized exchange rates of four main currencies (CNY, EUR, GBP, and JPY) against the US dollar from 2018 to the present. The graph depicts the relative performance of these major currencies against the US dollar over the last six years, highlighting periods of stability, decline, and recovery influenced by various global economic events.

The exchange rate of the Chinese Yuan against the US dollar, CNY=X (blue line), is generally stable with minor fluctuations from 2018 to 2020, a slight downward trend starting in 2020, stabilizing around mid-2021, and a gradual increase from 2022 onwards. The Euro's exchange rate against the US dollar EURUSD=X (orange line), indicates a decline from 2018 to mid-2020, a sharp dip around the beginning of 2020, possibly due to the COVID-19 pandemic, and more decline until early 2023, followed by a slight recovery. The exchange rate of the British Pound against the US dollar, GBPUSD=X (green line), shows a decline from 2018 to mid-2020, some recovery followed by fluctuations through 2021, and another decline in 2022, with a slight recovery in 2023. The exchange rate of the Japanese Yen against the US dollar, JPY=X (red line), indicates relative stability from 2018 to early 2020, a significant increase beginning in 2021, continuing through 2023, and reaching its peak in early 2024, indicating a strong rise in the value of the Yen against the US dollar. The main observations are that CNY indicates stability with a slightly increasing trend in recent years, EUR and GBP both depict a downward trend with some recovery attempts, and JPY indicates a significant upward trend from 2021 onwards, showing an increase in its value against the US dollar.

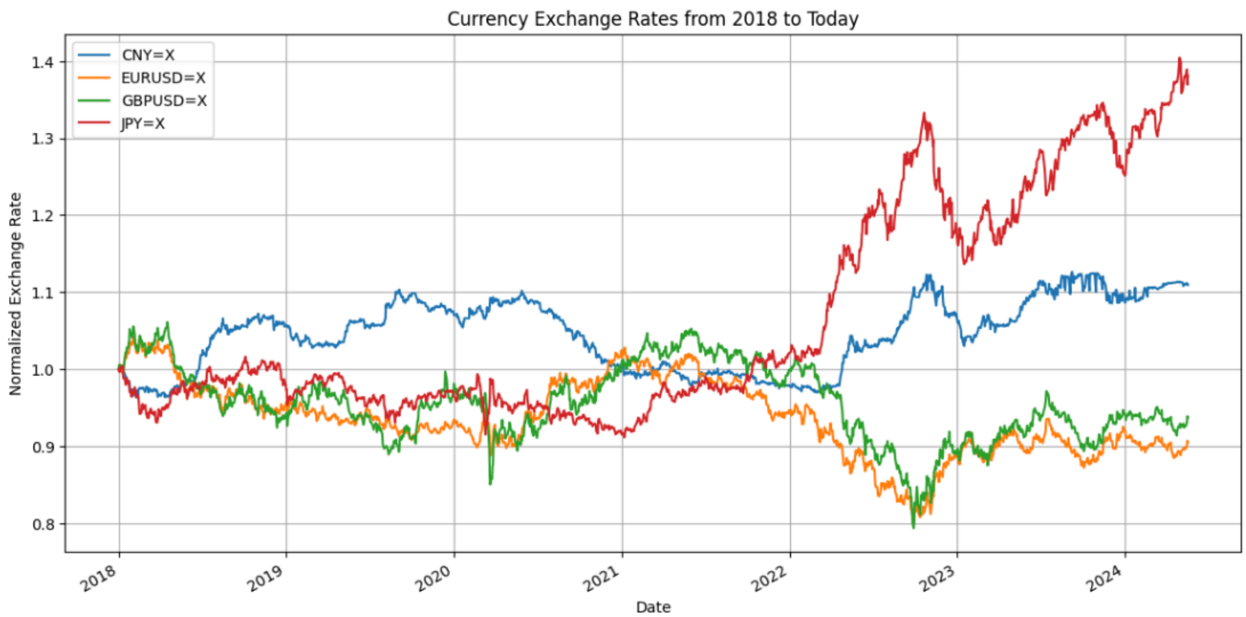

Figure 14. Currency exchange

Table 1 lists various companies' credit ratings and Credit Default Swap (CDS) spreads. Microsoft has the highest credit rating (AAA) and the lowest CDS spread (20 bps), which indicates that it is considered the most creditworthy with the lowest cost of default protection. Tesla has the lowest credit rating (BBB+) and the highest CDS spread (60 bps), indicating it is considered the least creditworthy among the listed companies with the highest cost of default protection. Apple and Google have a credit rating of AA+ with CDS spreads of 30 bps and 35 bps, respectively. Meta has a credit rating of A with a CDS spread of 50 bps, indicating a moderate level of credit risk compared to the other companies listed.

Table 1

Company	Credit Rating	CDS Spread
Apple	AA+	30
Microsoft	AAA	20
Amazon	AA	40
Google	AA+	35
Meta	A	50
Nvidia	AA-	45
Tesla	BBB+	60

Figure 15 shows the normalized levels of several leading market indices from 2018 to the present. It highlights the resilience of major market indices, indicating recovery and growth after the initial impact of the COVID-19 pandemic and subsequent market fluctuations. The Dow Jones Industrial Average, ^DJI (blue line), remains relatively stable with minor fluctuations from 2018 to early 2020, a sharp decline around early 2020, likely due to the COVID-19 pandemic, and a steady rise from mid-2020 onwards, with some fluctuations, reaching a higher level by 2024. The FTSE 100 Index, ^FTSE (orange line), shows less volatility than other indices and a dip around early 2020, followed by a gradual recovery and steady growth. The S&P 500 Index, ^GSPC (green line), indicates a pattern similar to the Dow Jones with a sharp decline in early 2020 and a strong recovery post-2020, continuing upwards with some fluctuations. The NASDAQ Composite Index, ^IXIC (red line), is the most volatile index among those shown, with a sharp decline in early 2020 followed by a significant rise and some fluctuations from 2021 to 2022, but continues to rise steeply. The Nifty 50 Index, ^NSEI (purple line), shows stability with minor fluctuations and a drop in early 2020, followed by a consistent upward trend with some fluctuations. The main observations are that all indices experienced a sharp decline around early 2020 due to the COVID-19 pandemic; post-2020, there was a strong recovery across all indices. The NASDAQ Composite (^IXIC) indicates the most major rise, showing strong performance in the technology sector. The FTSE 100 (^FTSE) indicates more stability with less volatility with respect to other indices. By 2024, all indices have indicated growth compared to their 2018 levels, with the NASDAQ Composite reaching the highest normalized level.

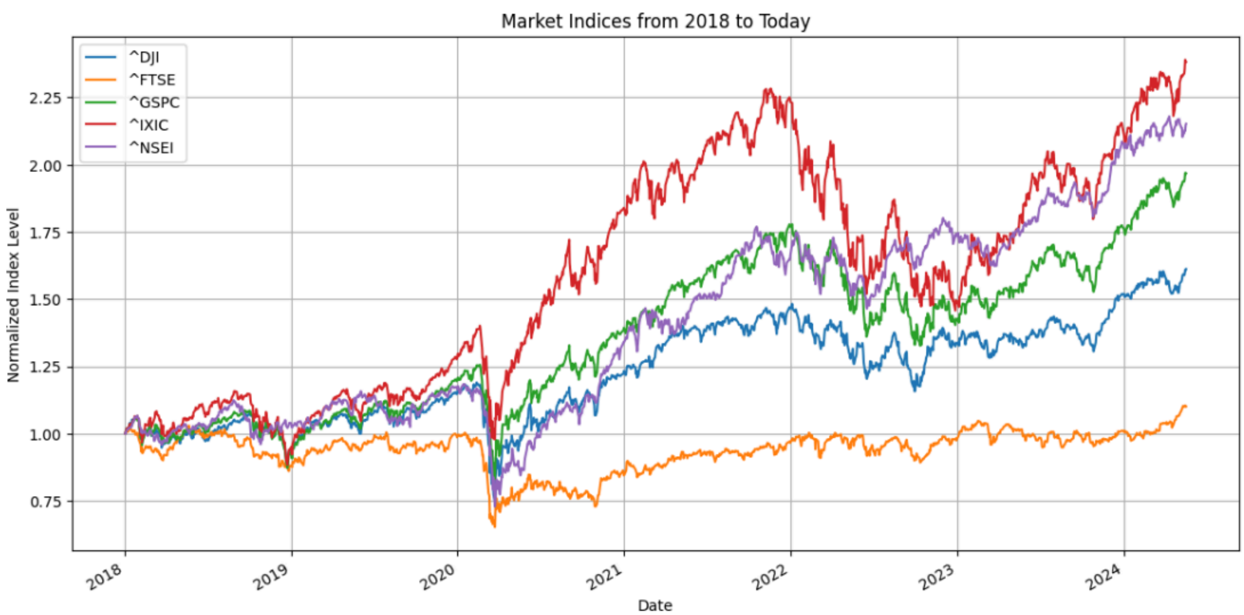

Figure 15. Market indices

Figure 16 shows a heatmap illustrating the correlation matrix of different market indices, which depicts the strength of the relationships between different market indices. It indicates that ^DJI, ^GSPC, and ^IXIC have strong positive correlations with each other, while ^FTSE has the weakest correlations with the other indices. The color scale on the right ranges from blue (low correlation) to red (high correlation), with numerical values from 0 to 1 indicating the degree of correlation. The values inside the squares show the correlation coefficient between the indices. A correlation coefficient of 1 (dark red) indicates a perfect positive correlation. In contrast, a correlation coefficient close to 0 (blue) indicates no correlation, and values between 0 and 1 indicate varying degrees of positive correlation. The main observations indicate a high correlation of ^DJI with ^GSPC (0.99), ^IXIC (0.94), and ^NSEI (0.95), and moderately correlated with ^FTSE (0.44). The ^FTSE indicates lower correlations with other indices, with the highest being ^NSEI (0.51), and shows a weak correlation with ^IXIC (0.17). The ^GSPC indicates a high correlation with ^DJI (0.99), ^IXIC (0.97), and ^NSEI (0.94) and a lower correlation with ^FTSE (0.36). The ^IXIC depicted a high correlation with ^DJI (0.94), ^GSPC (0.97), and ^NSEI (0.85) and a very low correlation with ^FTSE (0.17). The ^NSEI shows a high correlation with ^DJI (0.95), ^GSPC (0.94), moderately with ^IXIC (0.85), and a lower correlation with ^FTSE (0.51).

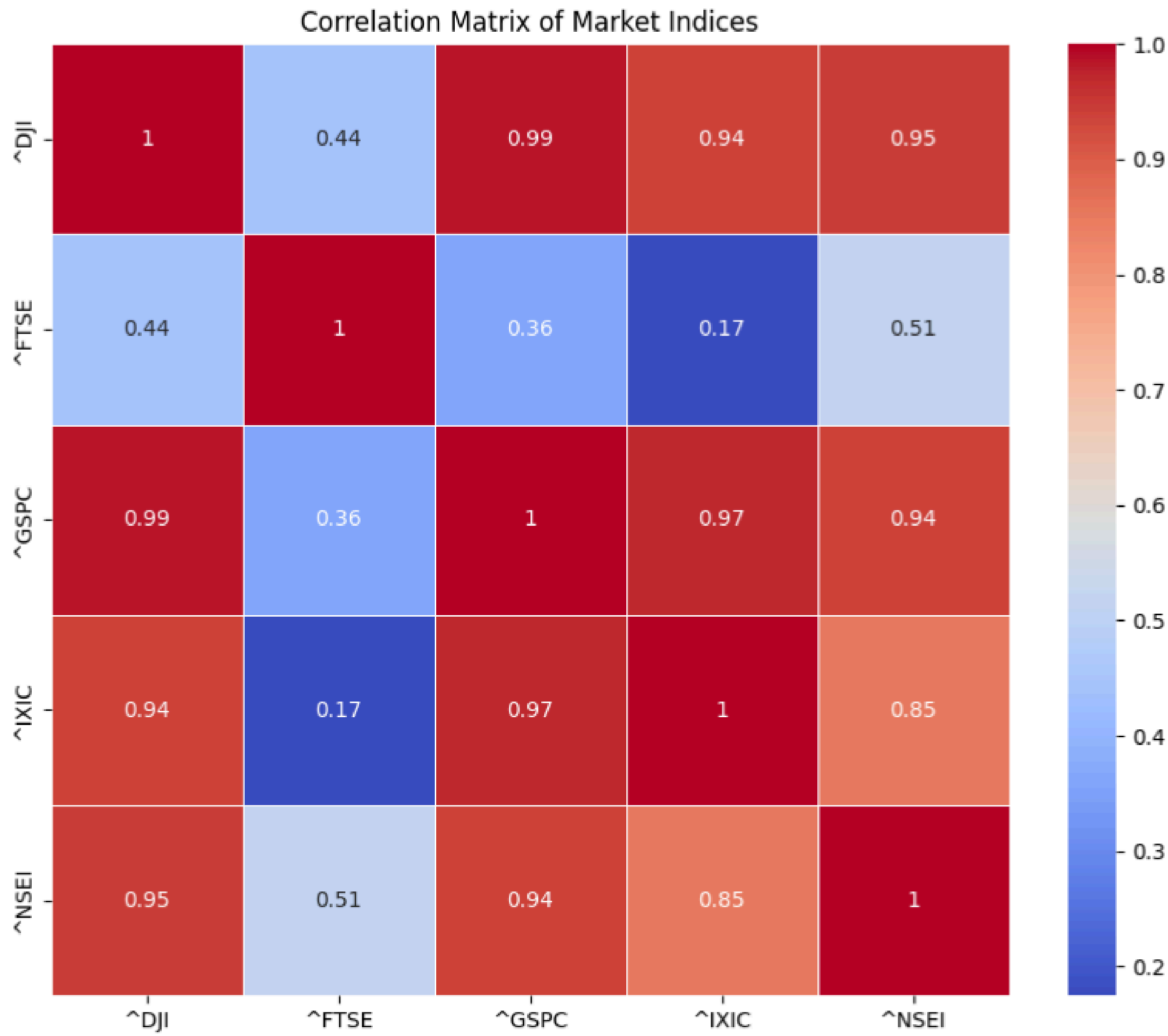

Figure 16. Correlation matrix of market indices

Table 2 indicates the cointegration tests between different market indices. Cointegration tests are utilized to determine whether two or more time series are statistically associated in the long run. No significant cointegration was found between the two indices except the S&P 500 and FTSE 100.

Table 2

Cointegration Test	p-value
^GSPC and ^DJI	0.8153
^GSPC and ^IXIC	0.0137
^GSPC and ^NSEI	0.2791
^GSPC and ^FTSE	0.0602
^DJI and ^IXIC	0.2876
^DJI and ^NSEI	0.3943
^DJI and ^FTSE	0.1532
^IXIC and ^NSEI	0.8313
^IXIC and ^FTSE	0.4104
^NSEI and ^FTSE	0.6815

Figure 17 shows a heatmap with hierarchical clustering, illustrating the correlation between different market indices. It indicates the strength and grouping of correlations between different market indices. It shows which indices move together and which ones have more independent movements. The color scale on the top left ranges from blue (low correlation) to red (high correlation), with numerical values from 0 to 1 indicating the degree of correlation. The dendrograms along the top and left sides depict the hierarchical clustering of the indices based on their correlation. Closer branches show a higher correlation between the indices. The colors within the heatmap squares define the correlation coefficient between the indices, with red exhibiting high correlation and blue exhibiting low correlation. The main observations are that the clusters ^DJI and ^GSPC form a close cluster, showing a very high correlation, ^IXIC and ^NSEI form another cluster with a high correlation, and ^FTSE stands apart, indicating a lower correlation with other indices. ^DJI and ^GSPC indicate the highest correlation (deep red) and ^IXIC and ^NSEI also show a high correlation (red). ^FTSE indicates relatively lower correlations with ^IXIC, ^NSEI, ^DJI, and ^GSPC (blue to light blue).

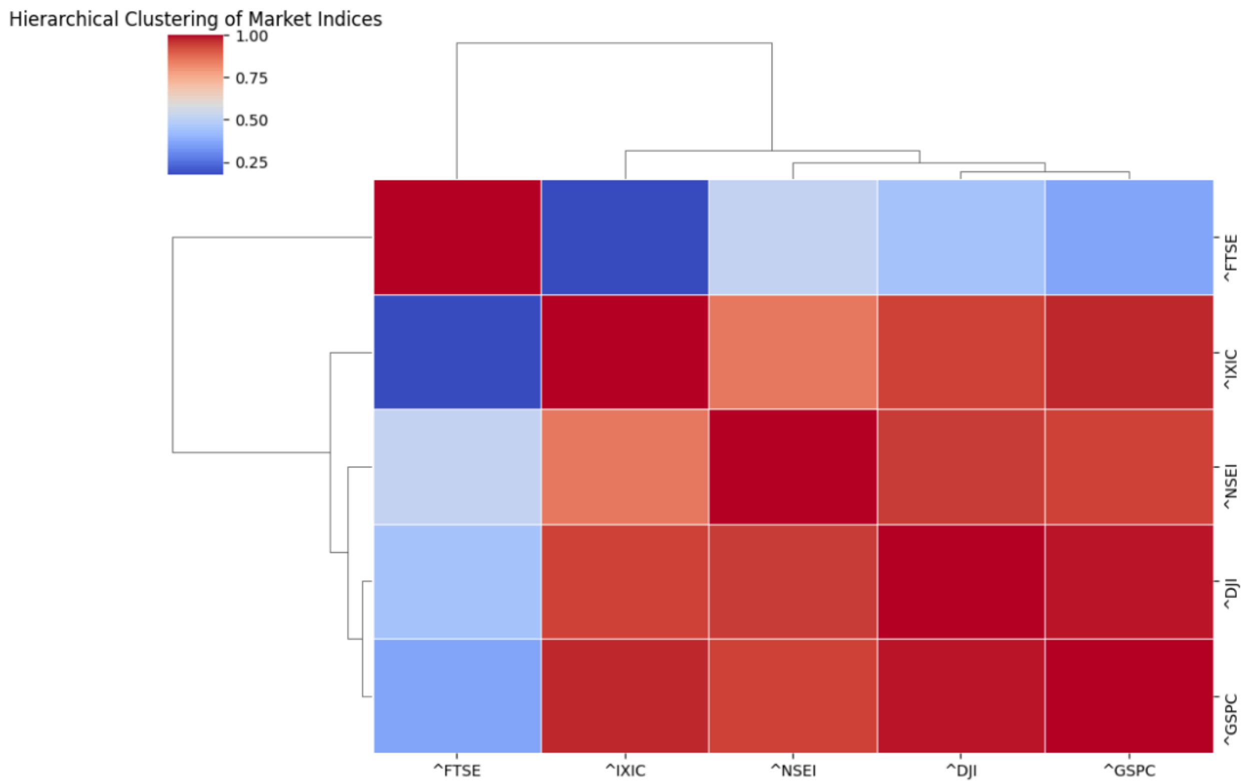

Figure 17. Hierarchical clustering of market indices

Table 3 indicates two financial instruments and their associated hedging strategies. These hedging strategies are designed to manage and mitigate risk associated with fluctuations in interest rates and foreign exchange rates, thereby providing more stability to the financial portfolio. Based on the interest rate swap instrument, this strategy utilizes a 50% hedge of SOFR and SONIA exposure. This strategy also involves using interest rate swaps to hedge 50% of the exposure to SOFR and SONIA (Sterling Overnight Index Average). An interest rate swap is a financial derivative contract where two parties exchange interest rate cash flows based on a specified notional amount over a specified period. FX swap utilizes 50% hedge of exposure in CNY, JPY, EUR, and GBP. This strategy involves utilizing FX swaps to hedge 50% of the exposure in various currencies: CNY, JPY, EUR, and GBP.

Table 3

Instrument	Strategy
Interest Rate Swap	Hedge 50% of SOFR and SONIA exposure
FX Swa[p	Hedge 50% of exposure in CNY, JPY, EUR. GBP

Table 4 shows the output of the HRP weights assigned to various stocks in a portfolio. The weights indicate the proportion of the total portfolio value allocated to each stock. The weights are utilized in the HRP strategy to optimize the portfolio by considering the returns and the risks associated with each stock, aiming to achieve a more balanced and diversified portfolio. Microsoft Corporation has the highest weight, showing that the HRP strategy allocates the largest portfolio to this stock. Apple Inc. has the second highest weight. NVIDIA Corporation has the lowest weight, showing the smallest allocation in the portfolio. The weights are spread across the stocks, but some have significantly higher allocations than others.

Table 4. HRP Weights

Ticker	Weight
AAPL	0.18022519318998162
AMZN	0.1445892720300246
GOOGL	0.16567684817219214
META	0.13550728268714532
MSFT	0.1840211934598403
NVDA	0.09476747178752881
TSLA	0.0952127386732872

Figure 18 illustrates the historical stock prices and ARIMA forecasts for seven major technology stocks from 2018 to the present. It depicted these main technology stocks' past performance and future forecasts, indicating overall growth with varying degrees of volatility. The ARIMA model's forecasts indicate continued growth for most of these stocks, with Apple (AAPL) indicating the most significant projected increase. The main observations are as follows.

- AAPL: Exhibits significant growth starting from 2023, with a sharp upward trend reaching close to 1000 by 2024. The forecast indicates a continued upward trend.
- MSFT: Exhibits steady growth with fluctuations, reaching over 300 by 2024. The forecast indicates a minor upward trend.
- AMZN: Exhibits growth with more volatility, reaching over 400 by 2024. The forecast illustrates a minor increase.
- GOOGL: Exhibits moderate growth, reaching around 150 by 2024. The forecast illustrates a slight increase.
- META: Exhibits moderate growth, reaching around 300 by 2024. The forecast illustrates a minor increase.
- NVDA: Exhibits significant growth and volatility, reaching around 500 by 2024. The forecast illustrates continued growth.
- TSLA: Exhibits growth with substantial volatility, reaching over 200 by 2024. The forecast illustrates a minor upward trend.

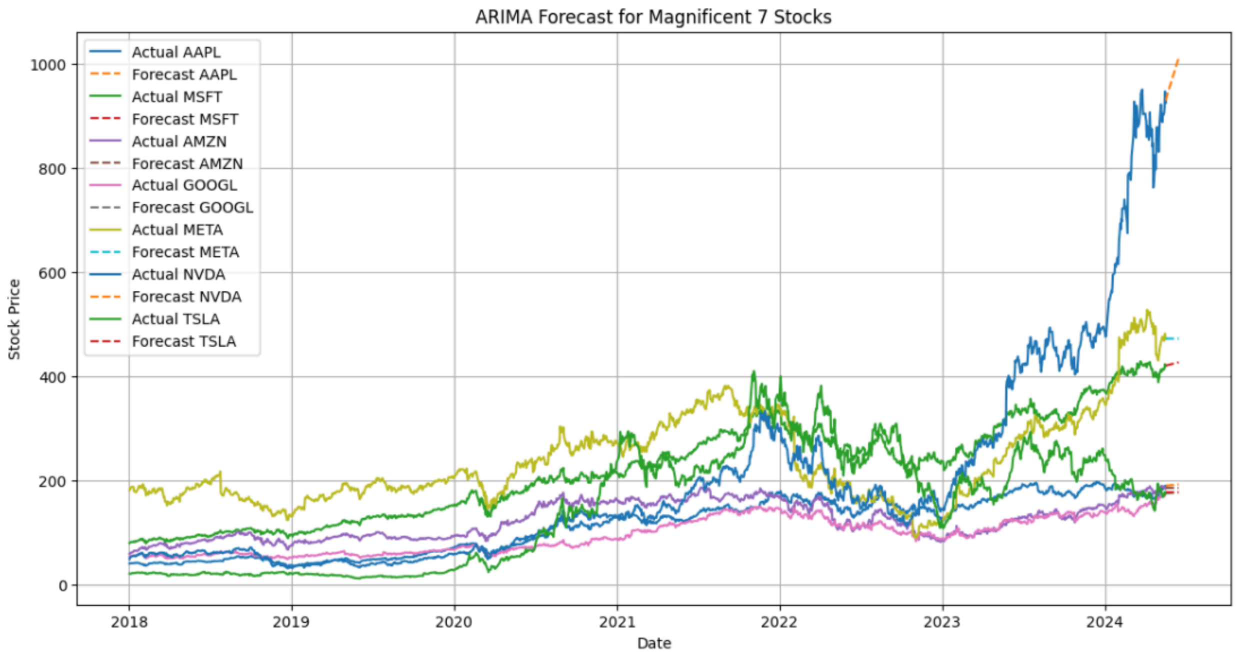

Figure 18. ARIMA forecast for magnificent 7 stocks

Figure 19 illustrates the historical index levels and ARIMA forecasts for several major market indices from 2018 to the present. It indicates overall growth trends despite fluctuations caused by significant economic events like the COVID-19 pandemic. The ARIMA model forecasts suggest continued growth for all indices, with variations in the extent of the upward trends. The main observations are as follows:

- ^GSPC: Indicates steady growth from 2018, with a sharp dip around early 2020 due to the COVID-19 pandemic, strong recovery, and continued growth through 2023, and the forecast shows a minor upward trend.
- ^DJI: Similar pattern to ^GSPC, with growth, a valley in early 2020, recovery, and continued growth, and the forecast shows a minor upward trend.
- ^IXIC: Indicated steady growth from 2018, with a significant valley in early 2020, strong recovery and continued growth, the highest overall levels among the indices, and forecast indicates continued upward movement.
- ^NSEI: Shows steady growth with some fluctuations, a dip in early 2020, and a gradual recovery, and the forecast shows a minor upward trend.
- ^FTSE: Shoes are more stable with less growth than the other indices, with a dip in early 2020, followed by gradual recovery and the forecast shows a minor upward movement.

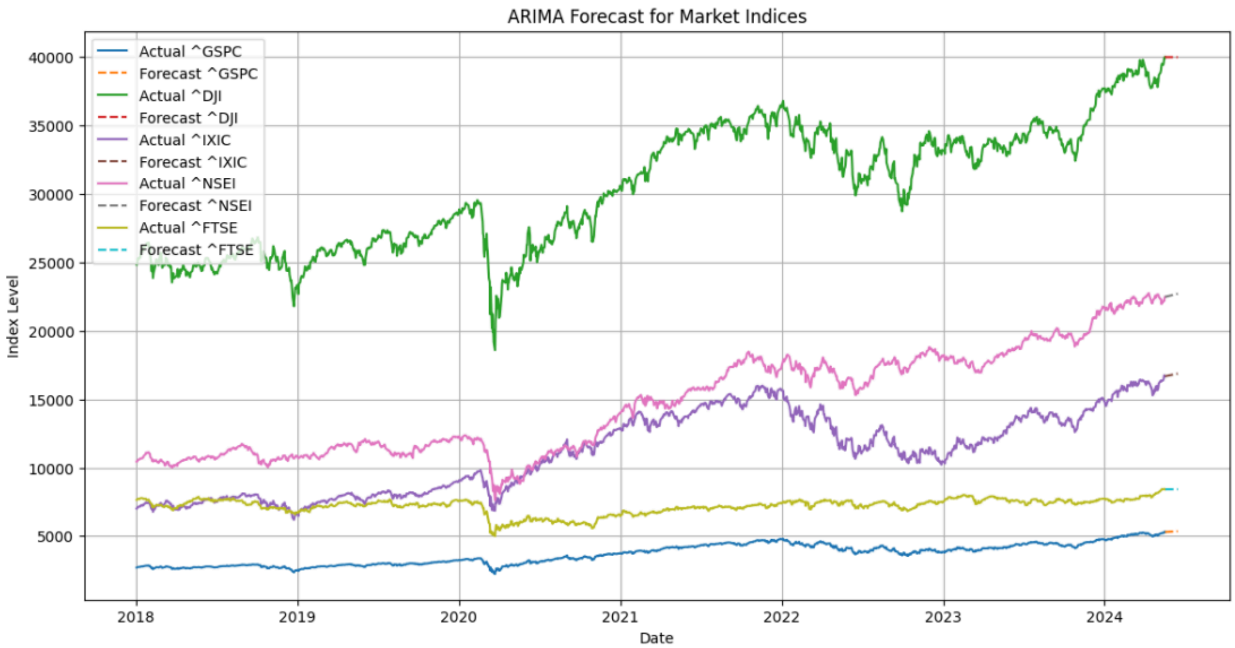

Figure 19. ARIMA forecast for market indices

Figure 20 indicates the daily returns of seven major technology stocks from 2018 to the present. The graph emphasizes the daily volatility of these significant technology stocks, with fluctuations indicating market responses to different economic events and company-specific news. It indicates that while most daily returns are near zero, there are notable periods of higher volatility, reflecting market uncertainty and major trading activity. The main observations indicate the percentage change in the stock prices from one day to the next, most daily returns fluctuate near zero, showing that most daily changes are relatively small. There are occasional spikes and dips, illustrating days with major positive or negative returns. The periods around 2020 and early 2021 indicate higher volatility, likely due to the impact of the COVID-19 pandemic and other market events. Tesla (TSLA) illustrates some of the most significant positive and negative fluctuations compared to the other stocks.

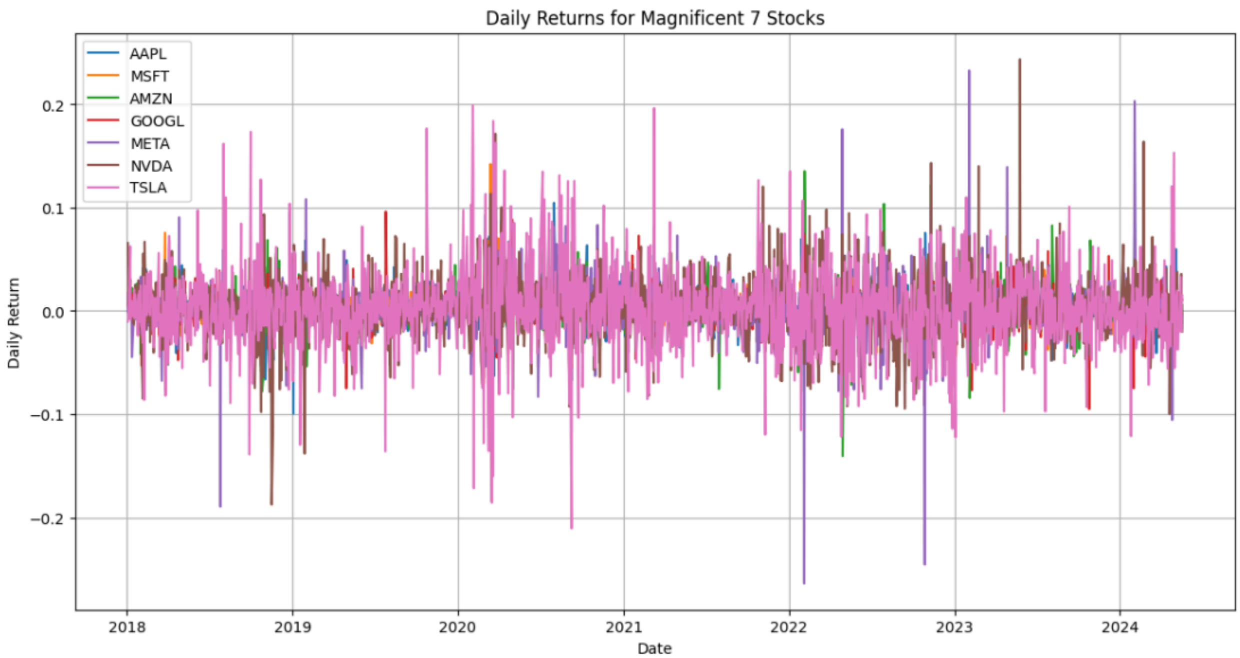

Figure 20. Daily returns for Magnificent 7 stocks

Figure 21 shows the daily returns with the volatility of several main market indices from 2018 to the present. It shows how they respond to different economic events. While most daily returns are near zero, periods of increased volatility reflect market uncertainty and major trading activity. The main observations depict the daily returns, showing the percentage change in the index levels from one day to the next. Most daily returns fluctuate near zero, showing that most changes are relatively small. There are occasional spikes and dips, depicting days with significant positive or negative returns. The period at the beginning of 2020 indicates higher volatility, likely due to the impact of the COVID-19 pandemic. The indices generally follow a similar pattern, with overlapping lines showing synchronized movements, particularly noticeable during significant market events.

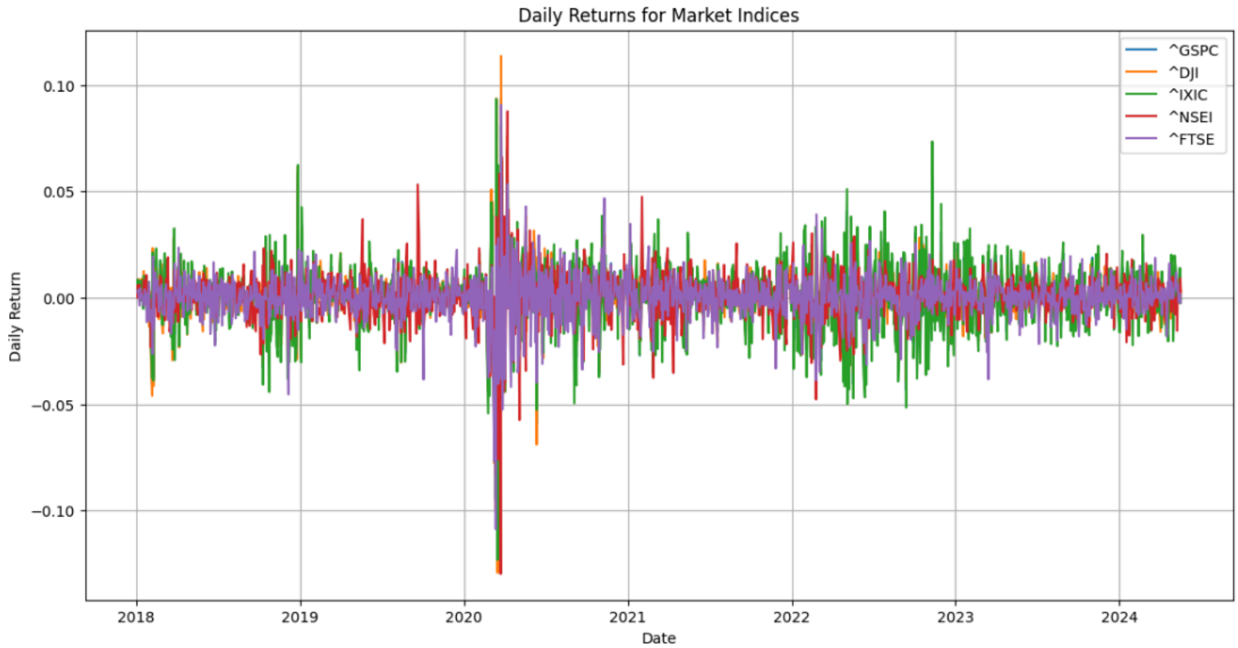

Figure 21. Daily returns for market indices

Figure 22 shows the results of multiple simulations of portfolio value over two years using Geometric Brownian Motion (GBM) with a control strategy for portfolio optimization and the uncertainty and potential variability in portfolio value over time. It indicates the range of possible outcomes and the probabilistic nature of portfolio growth under this model. Each blue line depicts a single simulation of the portfolio value over time. Various lines indicate a possible path that the portfolio value might take under the given GBM model and control strategy. The main observations indicate all simulations start with a portfolio value of 1 million at time zero. The portfolio values spread out, indicating the variability and possible outcomes of the portfolio under the model over time. Most portfolio values remain between 0.95 million and 1.10 million, with some simulations reaching values above 1.15 million or below 0.95 million. The density of the lines suggests that most portfolio values cluster around 1 million, with less frequent extreme values.

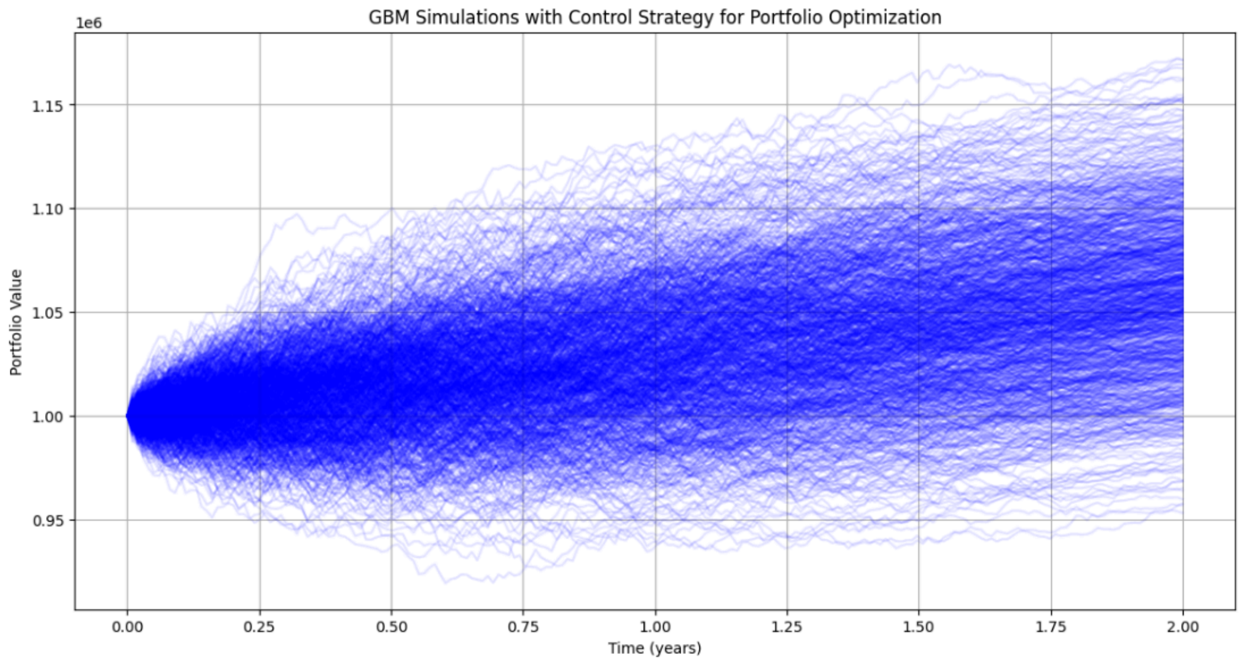

Figure 22. GBM simulations with control strategy for portfolio optimization

Figure 23 shows the results of multiple simulations of portfolio value over two years using Geometric Brownian Motion with a dynamic portfolio optimization strategy involving Hierarchical Risk Parity. It shows the uncertainty and possible variability in portfolio value over time when using a GBM model with a dynamic portfolio optimization strategy involving HRP. It indicates the range of possible outcomes and the probabilistic nature of portfolio growth under this model, indicating how the strategy can influence the distribution of portfolio values over time. Each blue line represents a single simulation of the portfolio value over time. Numerous lines show a potential path that the portfolio value might take under the given GBM model and HRP strategy. The main observations indicate all simulations begin with a portfolio value of 1 million at time zero. The portfolio values spread out, indicating the variability and possible outcomes of the portfolio under the model over time. Most portfolio values remain between 0.9 million and 1.3 million, with some simulations reaching values above 1.5 million or below 0.8 million. The density of the lines recommends that most portfolio values cluster around 1.1 million, with less frequent extreme values.

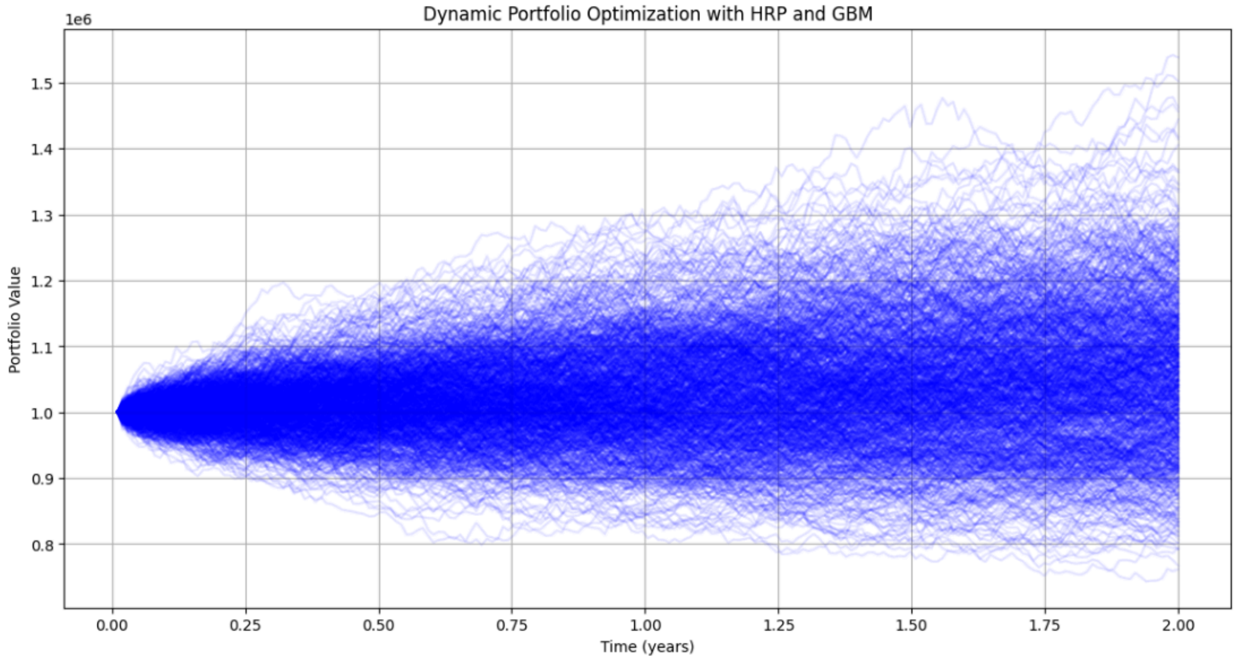

Figure 23. Dynamic portfolio optimization with HRP and GBM

The two graphs Figures 22 and 23 indicate some differences and recommend a possible improvement in portfolio outcomes when utilizing the dynamic portfolio optimization with HRP and GBM compared to GBM simulation with a control strategy. The GBM simulation with strategy indicates the spread of portfolio values around 0.95 million to 1.15 million, and it is relatively narrow, which shows limited variability and possible lower returns. It indicates the concentration is around 1 million, indicating less upward movement, and there are fewer occurrences that the portfolio indicates significantly exceeds the initial value. With the dynamic optimization with HRP and GBM, the portfolio spread around 0.8 to 1.5 million, which is a wider spread and indicates higher variability and possibly higher returns. The portfolio values indicate a higher concentration near 1.1 million, recommending an overall improvement in the portfolio value. There are also more occurrence portfolio values significantly exceeding the initial value, with some simulations reaching as high as 1.5 million.

The HRP and GBM strategies indicate higher potential returns, with a broader spread and upper range of portfolio values. While the HRP and GBM strategy increases the range of potential outcomes, it also introduces more variability, suggesting a trade-off between higher potential returns and higher risk. The HRP and GBM strategy depicts positive skewness, with more portfolio paths increasing over time, compared to the more symmetric distribution in the GBM with the control strategy.

6. Conclusion

This research examines the financial risks experienced by international firms such as Magnificent 7. The major risks include interest rates, currency, credit, and market risks. A conclusion is provided for future risk mitigation techniques by studying historical data and trends from 2018 to 2024,

Companies borrowing and financial stability will be impacted by the interest rate. The SOFR and SONIA indicated high fluctuation since 2018. A significant drop in rates can be observed in the early stages of the pandemic around the beginning of 2020. It rebound can be observed at the start of 2021. The increase in SOFR and SONIA rates necessitates companies subjected to these rates implement resilience hedging strategies to address possible hikes in borrowing costs. It has been shown that interest rate swaps can effectively hedge against those risks, ensuring these companies maintain stable financing costs even in a financially volatile environment.

Currency risk is caused by volatility in exchange rates, impacting the valuation of international operations and financial results. The calibrated exchange rates for the CNY, EUR, GBP, and JPY against the USD show variations in volatility degree from 2018 to 2024. A significant variation can be seen during the early stages of the COVID-19 pandemic responding to global economic disruptions. International firms should consider utilizing FX swaps, forward contracts, or various hedging techniques to handle currency risk. These techniques can support ensuring cash flow consistency and protection from negative movements in exchange rates. A preemptive method to mitigate currency risk is required for sustaining financial stability and securing profit margins based on the persistent fluctuation in leading currencies.

The credit ratings and CDS spreads evaluate the Magnificent 7 creditworthiness. Microsoft indicated the highest credit rating of AAA and lowest CDS spread is considered as the most secure. In contrast, Tesla indicated the lowest with a credit rating of BBB+ and the highest CDS spread, which is shown as the least safe. The investors must track the CDS spread and the credit ratings to manage their credit risk.

Market risk entails the possibility of losses because of fluctuations in market prices, such as equity indices. The analysis of leading market indices from 2018 to 2024 indicates significant growth and volatility. The pandemic led to a significant downturn in early 2020, but most indices witnessed a substantial bounce afterward. The NASDAQ, specifically, illustrated major growth, demonstrating the strong performance of the technology sector.

Multinational companies should consider diversifying their investments across different indices and geographies to manage market risk. This approach can mitigate the impact of regional market downturns. Hedging techniques like options or futures can protect against adverse market movements. Tracking market dynamics and tailoring investment techniques are essential for managing market fluctuation. The correlation matrix and hierarchical clustering of market indices indicate strong correlations among major US indices. These indices are highly sensitive to interest rate changes, particularly in a rising rate environment. Similarly, currency movements are correlated with market indices, suggesting that currency fluctuations can significantly impact the performance of multinational companies.

Comprehensive risk analysis demands a broad understanding of the correlations. Firms can use statistical analysis and historical data to determine possible risks and apply suitable mitigation strategies. This technique assures that firms are better positioned against future uncertainties and can protect their financial stability.

Addressing these risks needs a comprehensive technique, including hedging techniques, diversification, and continuous monitoring of market conditions. Firms can increase their resilience and protect their financial health by analyzing historical trends and utilizing advanced risk management techniques.

Investors should consider the above risk factors while constructing their portfolios making decisions based on a thorough risk analysis. A diversified portfolio with robust risk management practices can establish stability and growth opportunities in a fluctuating economic environment. The insights derived from this comprehensive analysis offer valuable guidance for companies and investors in guiding the complexities of the global financial landscape.

Appendix A. Code

The project source code can be found on GitHub using the link below.

Parent Repository (Shreejit) -> https://github.com/shreejitverma/MScFE690-Capstone/

Forked Repository (Farbod) -> https://github.com/farbod145/MScFE690-Capstone

Forked Repository (Hillary) -> https://github.com/lulumusilu/MScFE690-Capstone

Folder Structure:

MScFE Capstone

```
├── LICENSE
├── README.md
├── advanced_derivatives
│   ├── advanced_derivatives.py
│   ├── fwd_swap_tracker.py
│   ├── interest_rate_hedging.py
│   └── main_IRS_tracker_demo.py
├── calendars
│   ├── __init__.py
│   ├── custom_date_types.py
│   ├── daycounts.py
│   └── holidays
│       ├── __init__.py
│       ├── factory.py
│       ├── libor
│       │   ├── __init__.py
│       │   ├── base.py
│       │   ├── eur_on.py
│       │   └── usd_on.py
│       ├── us
│       │   ├── __init__.py
│       │   ├── __pycache__
│       │   └── core.py
│       └── utils
│           ├── __init__.py
│           ├── __pycache__
│           ├── abstract_base.py
│           ├── anglorules.py
│           ├── constants.py
│           ├── international.py
│           └── observances.py
├── data
│   ├── Industrial_Productionvs_CPI.csv
│   ├── Summary.xlsx
│   ├── __init__.py
│   ├── getIndicesAnalysis.py
│   ├── getRepoRate.py
│   ├── getfreddata.py
│   ├── main_fred_demo.py
│   ├── majorStockIndices.csv
│   ├── repo_rate
│   │   └── US_repo_rate_data.csv
│   └── stock_data_fetcher.py
│
```

```
├── plots
│   ├── US Repo Rate Time Series.png
│   ├── US_repo_rate_data.csv
│   ├── __init__.py
│   ├── price_change_return.png
│   └── stock_indices_returns.png
├── portfolio
│   ├── __init__.py
│   ├── backtesting.py
│   ├── construction.py
│   ├── performance.py
│   └── strategy_example.py
└── stochastic
    ├── __init__.py
    ├── __main__.py
    ├── stochastic.py
    └── stochastic_analysis.py
```

Appendix B. Acronym

SPT	Stochastic Portfolio Theory
AAPL	Apple Stocks ticker symbol
AMZN	Amazon Stocks ticker symbol
ARIMA	Autoregressive Integrated Moving Average
CDS	Credit Default Swap
CNY	Chinese Yuan Renminbi
DJI	Dow Jones Industrial Average
ETF	Exchange Traded Fund
EUR	Euro currency
FTSE	Financial Times Stock Exchange
FX	Foreign Exchange
GBP	Great British Pound
GBM	Geometric Brownian Motion
GOOGL	Google Stocks ticker symbol
GSPC	Standard and Poor's 500 Index
HRP	Hierarchical Risk Parity
IXIC	Nasdaq Composite
JPY	Japanese Yen currency
META	Meta, former Facebook Stocks ticker symbol
MSFT	Microsoft Stocks ticker symbol
NSEI	National Stock Exchange of India
NVDA	Nvidia Stocks ticker symbol
SOFR	Secured Overnight Financing Rate
SONIA	Sterling Overnight Index Average
TSLA	Tesla Stocks ticker symbol
USD	United States Dollar

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