

ALGEBRA ASSIGNMENT 5- (Quadratic 3- Medium Level)

- 1) $P(x)$ is a third degree polynomial and the coefficients of $P(x)$ are rational. If the graph of $P(x)$ touches the x -axis, then how many rational roots does $P(x) = 0$ have
(a) 0 (b) 1 (c) 2 (d) 3 (e) none
- 2) If a, b, c, d and e are the five positive roots of $x^5 - 5x^4 + kx^3 + mx^2 + nx - 1 = 0$, what is the product of k, m and n ?
a) More than 77 b) Less than -457 c) 0 d) Multiple of 7 e) Cannot be determined
- 3) If $f(x) = x^5 - x^4 - x^2 - x$, and a, b, c are the roots of the cubic equation $x^3 - x^2 - 1 = 0$, then what is the sum of $f(a), f(b)$ and $f(c)$?
a) 0 b) 3 c) 1 d) -1 e) -3
- 4) If $2a + t, b, 2c + t$ are in geometric progression and the equation $ax^2 + bx + c = 0$ has repeated roots. How many values 't' can take?
(a) 0 (b) 1 (c) 2 (d) 3
- 5) If 'a' and 'b' are the roots of the equation $mx^2 + nx + p = 0$, and ' a^3 ' and ' b^3 ' are the roots of the equation $1000x^2 - 133x + 1 = 0$, then find the roots of the equation $px^2 + nx + m = 0$ options are :-
a) 2 and 3 b) 2 and 5 c) 3 and 5 d) 4 and 5 e) None of these
- 6) Find the real roots of the equation $(x+5)(x+6)(x+7)(x+8) = 960$
- 7) Which of the following statements is true about roots of the equation $(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$?
a) all are necessarily imaginary b) there is atleast one positive root
c) there is atleast one negative root. d) none of these
- 8) The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is
(a) 1 (b) 0 (c) 3 (d) 2
- 9) If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
a) 2 (b) 3 (c) 2 (d) 1
- 10) 50 persons are seated in a row, each assigned a number from 1 to 50 corresponding to his seating position. Every time, a count is made on the seats, and any person sitting on a seat corresponding to a prime number is removed and the seat numbers are rearranged beginning with one. This procedure is repeated until only 3 persons are left. What is the original seat number corresponding to the 3rd person

11) What relation should be there between p and q so that the equation $x^4 + px^2 + q = 0$ has four real solutions forming an arithmetic progression

12) $f(x) = x^2 + bx + c$ The equation $f(x) = 0$ has two distinct roots which are from the set $\{-3, -2, -1, 0, 1, 2, 3\}$. How many different expressions of $f(x)$ are possible such that $f(0)$ is non-negative?

- (a) 3 (b) 6 (c) 12 (d) Infinite

13) Let r be a root of $x^2 + 5x + 7 = 0$. Compute $(r-1)(r+2)(r+6)(r+3)$

- a) 0 b) 6 c) -6 d) 13 e) -13

14) $x^4 - 40x^2 + q = 0$ is a equation having four real solutions which forms arithmetic progression. How many different values 'q' can take

15) The total ordered pair of positive integers (p, q) such that the roots of $x^2 - px + p + q - 3 = 0$ and $x^2 - qx + p + q - 3 = 0$ are also positive integers are

16) If the minimum possible value of $(1+p)(1+q)(1+r)(1+s)$ is $[1 + k(p.q.r.s)^m]$, where, 'k' and 'm' are constants, and p, q, r and s are positive real numbers, what is the value of product of k and m ?

17) For positive real numbers a, b, c . find the minimum integer value possible of the following equation: $6a^3 + 9b^3 + 32c^3 + 1/(4abc)$

18) if $x, y, z \in \mathbb{R}$, $x+y+z=4$ and $x^2+y^2+z^2=6$, den find the sum of maximum and minimum possible value of z

19) If x and y are positive real numbers such that $6xy+10x+15y=39$. Find the min value of $2x+3y$?

20) Which is smaller 51^{101} or $101!$

ANSWER KEYS

1	d
2	b
3	d
4	c
5	b
6	2
7	d
8	a
9	d
10	1,48,50
11	$q=(9p^2)/100$
12	c
13	d
14	1
15	4
16	8
17	6
18	$8/3$
19	6
20	101!