8.4

a)a-> (a,a)

f:: $\forall a. a -> (a, a)$

Let's assume, g :: (A,A) -> A' and x,y :: A

Then the free theorem is given as:-

f(g (x,y)) = (g(fx), g(fy))

b) (a,b) -> (b,a)

f:: $\forall a. \forall b. (a, b) -> (b, a)$

Let's assume, g :: (A,B) -> (A',B') and x :: A, y :: B

Then the free theorem is given as:-

f(g (x,y)) = g(f (x,y))

c) a -> a

f:: $\forall a. a -> a$

Let's assume, g :: A->B and x :: A

Then the free theorem is given as:-

f(g x) = g (f x)

Lets assume u,v are of type t.

u,v :: t and g :: Bool -> t

g False = u

g True = v

If x = False then f (g False) = g (f False)

i.e f u = g (f False)

This equation is only true if f is an identity function, i.e f x= x

Hence,

u = g False =u

True