6/9/18

Tutorial - 1

Teacher's Sign / Remarks

(2)

Raw material	Requirements per unit of given model			
	I	II	III	
A	2	3	5	→ 4000
B	4	2	7	→ 6000.
Profits	60/-	40/-	100/-	

Decision Variables → x_1, x_2, x_3.

Objective f'n → max. profit.

$$\text{Max } z = 60x_1 + 40x_2 + 100x_3.$$

Subject to $x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 \leq 2500$

Rawmaterials Constraints

$$2x_1 + 3x_2 + 5x_3 \leq 4000$$
$$4x_1 + 2x_2 + 7x_3 \leq 6000.$$

$x_1 \geq 500$, $x_2 \geq 500$, $x_3 \geq 375$

ratios of no. of units produced ⇒ 3:2:5

$$\Rightarrow 3x_1 = 2x_2$$
$$2x_2 = 5x_3$$
$$5x_3 = 3x_1$$

Non-negativity Constraints

$$x_1, x_2, x_3 \geq 0.$$

(3) Decision Variables $= x_1, x_2, x_3$ → x_1 → # of acres of tomatoes grown
x_2 → " " " lettuce "
x_3 → " " " raddishes "

Total Sales /acre $(1 \times 2000)x_1 + (0.75 \times 3000)x_2 + (2 \times 1000)x_3$

Fertilizer Cost /acre $= (0.5 \times 100)x_1 + (0.5 \times 100)x_2 + (0.5 \times 50)x_3$

Labor Cost /acre $= 20 \times (5x_1 + 6x_2 + 5x_3)$

Objective function:

Max P = (Total Sales /acre) − (Fertilizer Cost /acre + labor cost /acre)

$$= (2000x_1 + 2250x_2 + 2000x_3) - (50x_1 + 50x_2 + 25x_3 + 100x_1 + 120x_2 + 100x_3)$$

$$P = 1850x_1 + 2080x_2 + 1875x_3$$

Subject to $x_1 + x_2 + x_3 \leq 100.$

$5x_1 + 5x_2 + 6x_3 \leq 400$

Non-negativity Constraints: $x_1, x_2, x_3 \geq 0$.

(4) Decision Variables → x_1, x_2

objective fn. max $P = 100x_1 + 150x_2$

Date / /20

Subject to

$4x_1 + 10x_2 \le 8000$ (diodes)

$4x_1 + 2x_2 \le 3000$ (registers)

$12x_1 + 9.6x_2 \le 160 \times 60$. (time)

$12x_1 + 9.6x_2 \le 9600$

Non-negativity Constrains $x_1, x_2, \cancel{x_3} \ge 0$.

(5)

~~$x_1 \rightarrow$ # units of Cost of Protein~~

~~$x_2 \rightarrow$ " " fat.~~

~~$x_3 \rightarrow$ " " Carbohydrate~~

x_1 : # of units of food type 1

x_2 : " 2

x_3 : " 3

x_4 : " 4

Decision Variables → x_1, x_2, x_3, x_4

Objective function (minimize Cost)

Min $P = 45x_1 + 40x_2 + 85x_3 + 65x_4$

Subject to

$3x_1 + 4x_2 + 8x_3 + 6x_4 \ge 800$

$2x_1 + 2x_2 + 7x_3 + 5x_4 \ge 200$

$6x_1 + 4x_2 + 7x_3 + 4x_4 \ge 700$.

Non-negativity Constrants $x_1, x_2, x_3, x_4 \ge 0$

(6) Decision Variables : $x_1, x_2 / \cancel{x_3}$

x_1 → Cost of 1 jar of dry chemical

x_2 → Cost of 1 jar of liquid Chemical

Type	A	B	C	Cost/jar
liquid type	5	2	1	3
dry	1	2	4.	2
	10	12	12	

Subject to

$5x_1 + x_2 \ge 10$

$2x_1 + 2x_2 \ge 12$

$x_1 + 4x_2 \le 12$

Min $p : 3x_1 + 2x_2$

Non-negativity Constraints : $x_1, x_2 \ge 0$.

Decision Variables — x_1, x_2

(7)

Date / /20

x_1 : no. of old hens

x_2 : " " Young hens.

Profit through eggs ↑ — maintanence fee ↗

Objective fn Max $P = [2(3x_1 + 5x_2)] - [5(x_1 + x_2)]$

$= x_1 + 5x_2$

Subject to :

$50x_1 + 100x_2 \leq 2000$

$x_1 + x_2 \leq 40$

Non-negativity Constraints : $x_1, x_2 \geq 0$.

(1)

M/c \ Products	Rs 2 Profit/unit A (x_1 Units)	Rs 3 Profit/unit B (x_2 Units)	Available time (in min)
G	1	1	400
H	2	1	600
Profit	Rs 2	Rs 3	

$x_1, x_2 \rightarrow$ decision Variables

$x_1 \rightarrow$ no. of Units of Product A

$x_2 \rightarrow$ no. of Units of Product B

Max $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 400$

$2x_1 + x_2 \leq 600$

Non-negativity Constraints : $x_1, x_2 \geq 0$.

5/5

20/9/18

Tutorial - 2

Date / /20 (2)

Graphical linear programming

(1)

M/c \ Product	A (x_1 units)	B (x_2 units)	Available time (in min)
X	2	2	330.
Y	3	2	480.
Profit	₹4	₹5.	

$x_1 \rightarrow$ no. of units of A

$x_2 \Rightarrow$ no. of units of B

objective function :-

$$\text{Max } z = 4x_1 + 5x_2$$

subject to

(i) $2x_1 + 2x_2 \le 330$

(ii) $3x_1 + 2x_2 \le 480$

Non-negativity constraints

$x_1, x_2 \ge 0$

(i) $2x_1 + 2x_2 \le 330.$

$2x_1 + 2x_2 = 330$

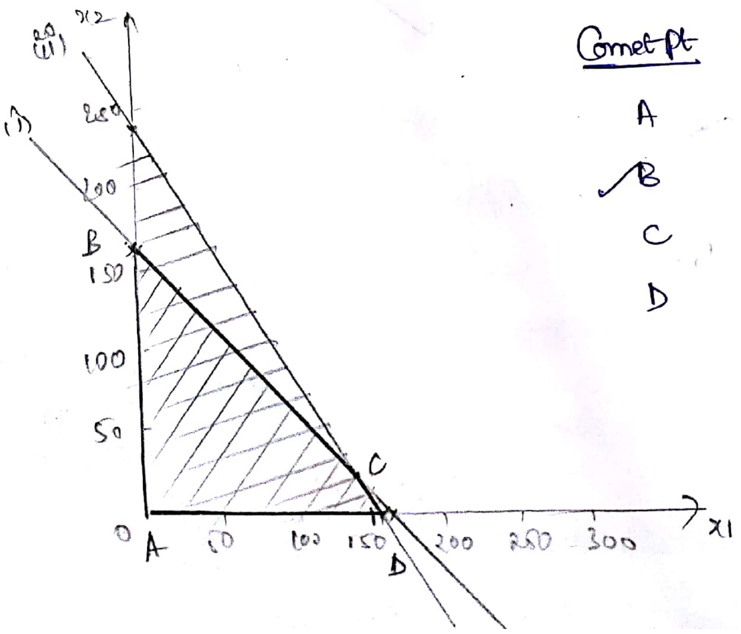
$x_1 = 0 \Rightarrow x_2 = 165$

$x_2 = 0 \Rightarrow x_1 = 165$

(ii) $3x_1 + 2x_2 = 480$

$x_1 = 0 \Rightarrow x_2 = 240$

$x_2 = 0 \Rightarrow x_1 = 160.$



Corner pt	(x_1, x_2)	$Z(4x_1+5x_2)$
A	(0,0)	0
B	(0,165)	825
C	(150,15)	675
D	(160,0)	640.

∴ Z is maximum for B with $x_1 = 0$ and $x_2 = 165$

optimal solution

Z = 825

(2) Decision Variables → x_1, x_2

x_1 → no. of old hens.

x_2 → no. of young hens

objective function

Profit through eggs → maintanence cost

$$\text{Max } p = [\,0.3(3x_1 + 5x_2) - [1\cdot(x_1 + x_2)]\,]$$

$$= [\,0.9x_1 + 1.5x_2 - x_1 - x_2\,]$$

$$= -0.1x_1 + 0.5x_2$$

$$\text{Max } p = 0.5x_2 - 0.1x_1 \geq 6.$$

Subject to

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

Non-negativity constraints : $x_1, x_2 \geq 0$.

(i) $2x_1 + 5x_2 = 80$

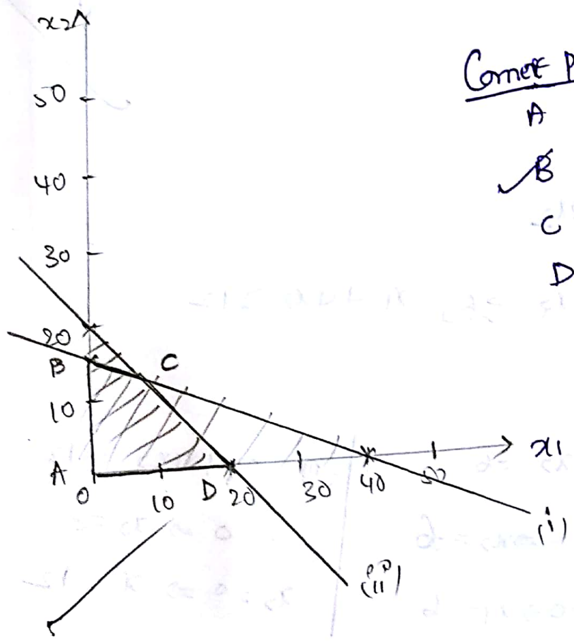
$x_1 = 0 \Rightarrow x_2 = 16$

$x_2 = 0 \Rightarrow x_1 = 40$.

(ii) $x_1 + x_2 = 20$

$x_1 = 0 \Rightarrow x_2 = 20$

$x_2 = 0 \Rightarrow x_1 = 20$.



Corner pt	(x_1, x_2)	z $(-0.1x_1 + 0.5x_2)$
A	(0, 0)	0
B	(0, 16)	8
C	(6.66, 13.33)	5.999
D	(20, 0)	-2

∴ z is maximum for B with $x_1 = 0$ and $x_2 = 16$

(3) Max $z = 3x + 4y$

Subject to

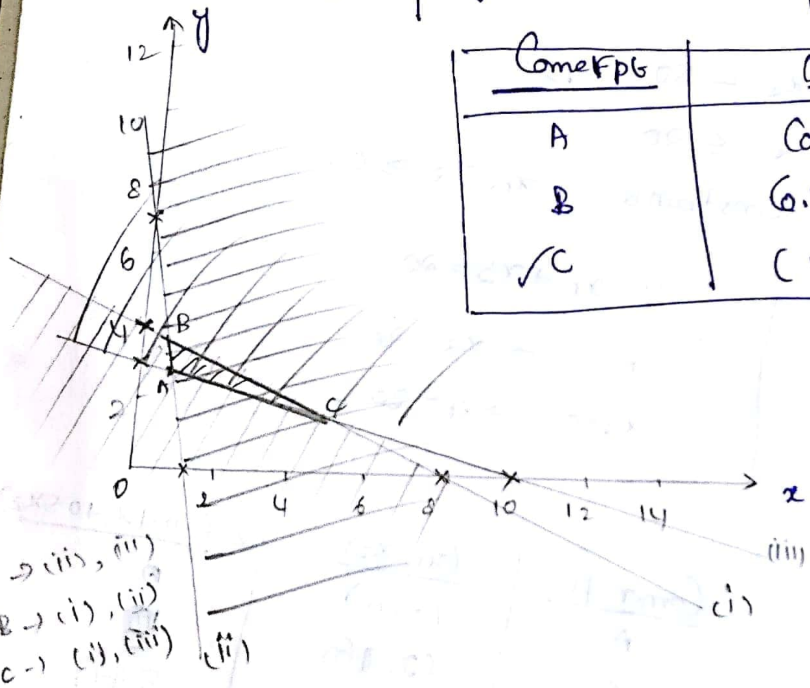
$4x + 8y \le 32$

$9x + 2y \ge 14$

$1.5x + 5y \ge 15$

Non negativity Constraints $x, y \ge 0$.

(i) $4x+8y=32$	(ii) $9x+2y=14$	(iii) $1.5x+5y=15$
$x=0 \Rightarrow y=4$	$x=0 \Rightarrow y=7$	$x=0 \Rightarrow y=3$
$y=0 \Rightarrow x=8$	$y=0 \Rightarrow x=1.55$	$y=0 \Rightarrow x=10$



A → (ii), (iii)

B → (i), (ii)

C → (i), (iii)

Corner pt	(x, y)	$(z) = 3x+4y$
A	(0.952, 2.714)	13.712
B	(0.75, 3.625)	16.75
✓C	(5, 1.5)	21

∴ z is maximum for c with $x_1 = 5, y = 1.5$

(4). Minimize $z = 3x_1 + 2x_2$

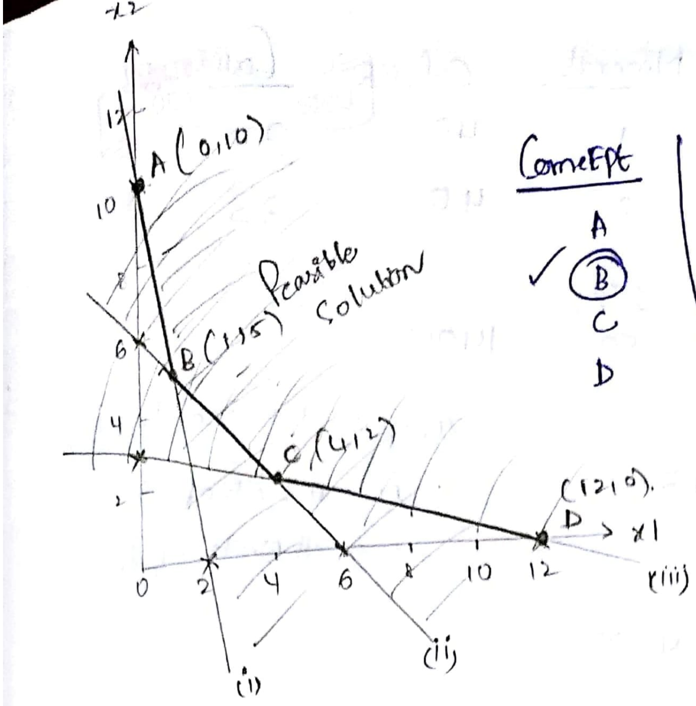
$5x_1 + x_2 \ge 10, \quad x_1 + x_2 \ge 6, \quad x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

(i) $5x_1 + x_2 = 10$	(ii) $x_1 + x_2 = 6$	(iii) $x_1 + 4x_2 = 12$
$x_1 = 0 \Rightarrow x_2 = 10$	$x_1 = 0 \Rightarrow x_2 = 6$	$x_1 = 0 \Rightarrow x_2 = 3$
$x_2 = 0 \Rightarrow x_1 = 2$	$x_2 = 0 \Rightarrow x_1 = 6$	$x_2 = 0 \Rightarrow x_1 = 12$

3+10 12
4

Mangal Date / /20



Corner pt	(x_1, x_2)	$z\ (3x_1+2x_2)$
A	(0,10)	20
✓ B	(1,5)	13
C	(4,2)	16
D	(12,0)	36

$\therefore$ z is minimum at Point B with $x_1 = 1$ & $x_2 = 5$

(5) Minimize $z = -x_1 + 2x_2$ S.T

$-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_1 \ge 0,\ x_2 \ge 0$

(i) $-x_1 + 3x_2 = 10$

$x_1 = 0 \Rightarrow x_2 = 3.33$

$x_2 = 0 \Rightarrow x_1 = -10$

(ii) $x_1 + x_2 = 6$

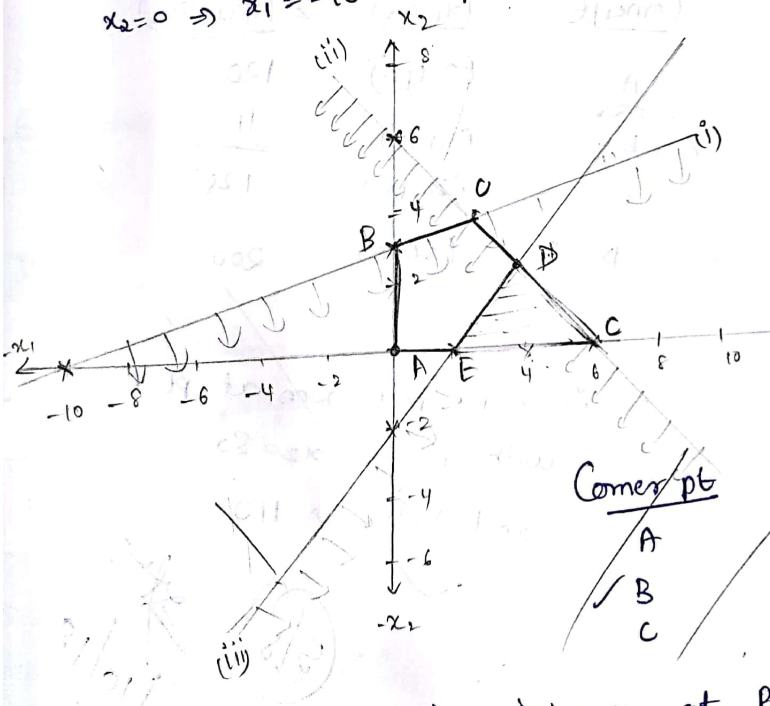
$x_1 = 0 \Rightarrow x_2 = 6$

$x_2 = 0 \Rightarrow x_1 = 6$

(iii) $x_1 - x_2 = 2$

$x_1 = 0 \Rightarrow x_2 = -2$

$x_2 = 0 \Rightarrow x_1 = 2$



pt	(x_1, x_2)	$z\ (-x_1+2x_2)$
A	(0,0)	0
B	(0,3.33)	6.66
C	(2,4)	6
D	(4,2)	0
✓ E	(2,0)	-2

~~Corner pt~~	~~(x_1, x_2)~~	~~$z = -x_1 + 2x_2$~~
~~A~~	~~(0,0)~~	~~0~~
~~B~~	~~(4,2)~~	~~0~~
~~C~~	~~(6,0)~~	~~-6.~~

$\therefore$ z is minimum at Point E with $x_1 = 2$, $x_2 = 0$ and Value is -2

(6)

Food type	Vitamins	Minerals	Calories	Cost
A (x_1)	200	1	40	24
B (x_2)	100	2	40.	23
Cost	~~2/4~~ 4000	~~2B~~. 50	1400.	

Objective function

$x_1, x_2 \to$ Decision Variables

$x_1 \to$ # units of A

$x_2 \to$ # units of B

Min $z = 4x_1 + 3x_2$

S.T

$$\begin{cases} 200x_1 + 100x_2 \geq 4000 \\ x_1 + 2x_2 \geq 50 \\ 40x_1 + 40x_2 \geq 1400 \end{cases}$$

Non negativity Constraints $x_1, x_2 \geq 0$.

(i) $200x_1 + 100x_2 = 4000$

$x_1 = 0 \Rightarrow x_2 = 40$

$x_2 = 0 \Rightarrow x_1 = 20$

(ii) $x_1 + 2x_2 = 50$.

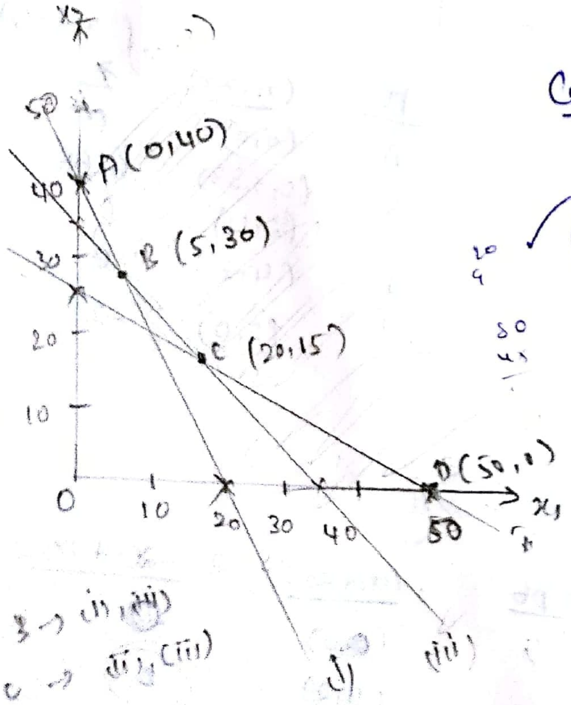
$x_1 = 0 \Rightarrow x_2 = 25$

$x_2 = 0 \Rightarrow x_1 = 50$

(iii) $40x_1 + 40x_2 = 1400$

$x_1 = 0 \Rightarrow x_2 = 35$

$x_2 = 0 \Rightarrow x_1 = 35$



B → (i), (iii)

C → (ii), (iii)

Corner pt	(x_1, x_2)	z ($4x_1+3x_2$)
A	(0,40)	120
B	(5,30)	110
C	(20,15)	125
D	(50,0)	200

∴ z is minimum at pt B with $x_1 = 5$, $x_2 = 30$ and Value is 110

5/5

4/10/18

04/10/... Tutorials

① Express the following LPP in equation form

(i) Max $3x_1 + 4x_2$

S.T

$2x_1 + 6x_2 \le 16$.

$5x_1 + 2x_2 \ge 20$.

$x_1, x_2 \ge 0$.

Max $z = 3x_1 + 4x_2$ S.T

$2x_1 + 6x_2 + S_1 = 16$

$5x_1 + 2x_2 - S_2 = 20$.

$x_1, x_2, S_1, S_2 \ge 0$

(ii) Min $z = 25x_1 + 30x_2$

$4x_1 + 7x_2 \ge 1$

$8x_1 + 5x_2 \ge 3$

$6x_1 + 9x_2 \ge -2 \Rightarrow -6x_1 - 9x_2 \le 2$

$x_1, x_2 \ge 0$

Min $z = 25x_1 + 30x_2$

$4x_1 + 7x_2 - S_1 = 1$

$8x_1 + 5x_2 - S_2 = 3$

$-6x_1 - 9x_2 + S_3 = 2$

$x_1, x_2, S_1, S_2, S_3 \ge 0$.

② Solve using Algebraic method.

(i) Max $z = 5x_1 + 3x_2$ S.T

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$.

$x_1, x_2 \ge 0$

In equation form

Max $z = 5x_1 + 3x_2$ S.T

$3x_1 + 5x_2 + S_1 = 15$

$5x_1 + 2x_2 + S_2 = 10$.

$x_1, x_2, S_1, S_2 \ge 0$

$n \to$ variables $\Rightarrow 4$

$m \to$ equations $\Rightarrow 2$.

No. of Corner points $= {}^{n}C_{m} = {}^{4}C_{2} = 6$ corner points.

Set $(n-m)$ variables to 0

Set ② variables to 0.

$3x_1+5x_2+S_1 = 15$, $5x_1+2x_2+S_2=10$.

$Z = 5x_1 + 3x_2$

Non-basic Variables (2=0)	Basic Variables	Basic Solution	Associated Corner point	feasible	sln
(x_1, x_2)	(S_1, S_2)	$(15, 10)$	A	Yes	$\rightarrow$ 0
(S_1, S_2)	(x_1, x_2)	$(20/19, \frac{45}{19})$	B	yes	$\rightarrow$ $235/19$
(x_1, S_1)	(x_2, S_2)	$(3, 4)$	C	yes	$\rightarrow$ 9
(x_2, S_2)	(x_1, S_1)	$(2, 9)$	D	yes	$\rightarrow$ 10
(x_1, S_2)	(x_2, S_1)	$(5, -10)$	E	No	
(x_2, S_1)	(x_1, S_2)	$(5, -15)$	F	No.	

(1.05, 2.368)

$\therefore$ $B(\frac{20}{19}, \frac{45}{19})$ is optimum solution with value $\frac{235}{19} = 12.3$

(ii) Max $z = 7x_1 + 5x_2$ S.T

$-x_1 - 2x_2 \geq -6. \Rightarrow x_1 + 2x_2 \leq 6$.

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$.

Equation form

Max $Z = 7x_1 + 5x_2$ S.T

$x_1 + 2x_2 + S_1 = 6$. $n \Rightarrow S_1, S_2, x_1, x_2 = 4$

$4x_1 + 3x_2 + S_2 = 12$ $m \rightarrow 2$ equations

$x_1, x_2, S_1, S_2 \geq 0$

$x_1 + 2x_2 = 6$
$4x_1 + 3x_2 = 12$
$4x_1 + 8x_2 = 24$
$-5x_2 = -12$
$x_2 = \frac{12}{5}$

No. of Corner Pts = $^nC_m = {}^4C_2 = 6$.

Non-basic Variables	Basic Variables	Basic Solution	Associated Corner point	feasible	Solution
(x_1, x_2)	(S_1, S_2)	$(6, 12)$	A	Yes	~~7(6)+5(12) = 102~~ 0
(S_1, S_2)	(x_1, x_2)	$(\frac{6}{5}, \frac{12}{5})$	B	Yes	$7(\frac{6}{5})+5(\frac{12}{5}) = 20.4$
(x_1, S_1)	(x_2, S_2)	$(3, 3)$	C	Yes	$7(0)+5(3) = 15$
(x_2, S_2)	(x_1, S_1)	$(3, 3)$	D	Yes	$7(3)+5(0) = \underline{\underline{21}}$
(x_1, S_2)	(x_2, S_1)	$(4, -2)$	E	No	—
(x_2, S_1)	(x_1, S_2)	$(6, -12)$	F	No	—

$\therefore$ D (3, 0) is optimum solution with value 21 //

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Date / /20

(3) Solve using simplex method

i) Max $z = 3x_1 + 2x_2$ S.T

$x_1 + x_2 \le 4$

$x_1 - x_2 \le 2$

$x_1, x_2 \ge 0$.

Equation form Max $z = 3x_1 + 2x_2 \Rightarrow z - 3x_1 - 2x_2 - 0s_1 - 0s_2 = 0$.

S.T $x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

$x_1, x_2, s_1, s_2 \ge 0$.

entering variable, pivot column

Basic Variable	z	x_1	x_2	s_1	s_2	Solution
z	1	-3	-2	0	0	0
s_1	0	1	1	1	0	$4/1 = 4$
s_2 (leaving variable)	0	1 (Pivot Element)	-1	0	1	$2/1 = 2$ min ← pivot row
z	1	0	-5	0	3	6.
s_1	0	0	2	1	-1	$2/2 = 1$
x_1	0	1	-1	0	1	2
z	1	0	0	5/2	1/2	11
x_2	0	0	1	1/2	-1/2	1
x_1	0	0	0	1/2	1/2	3

∴ optimum Solution with Values $z = 11$

$x_1 = 3$

$x_2 = 1$

$s_1, s_2 = 0$

(ii) Min $z = x_1 - 3x_2 + 2x_3$

S.T.

$3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$.

Equation form Min $z = x_1 - 3x_2 + 2x_3$

$-z = -x_1 + 3x_2 - 2x_3$

let $-z = z'$ $\Rightarrow$ $z' + x_1 - 3x_2 + 2x_3 = 0$ S.T

$3x_1 - x_2 + 3x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Basic Variable	z'	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z'	1	1	-3	2	0	0	0	0
s_1	0	3	-1	3	1	0	0	7
s_2	0	-2	4	0	0	1	0	12/4 = 3
s_3	0	-4	3	8	0	0	1	10/3 = 3.33
z'	1	-1/2	0	2	0	3/4	0	9
s_1	0	5/2	0	3	1	1/4	0	10 → 4
x_2	0	-1/2	1	0	0	1/4	0	3
s_3	0	-5/2	0	8	0	-3/4	1	1
z'	1	0	0	13/5	1/5	4/5	0	11
x_1	0	1	0	6/5	2/5	1/10	0	4
x_2	0	0	1	3/5	1/5	3/10	0	5
s_3	0	0	0	11	1	5/4	1	11

$\therefore$ optimum Solution $z' = 11 \Rightarrow -z = 11 \Rightarrow z = -11$

$x_1 = 4, \quad x_2 = 5, \quad x_3 = 0$

$s_1, s_2 = 0$

$s_3 = 11$

(iii) Max $z = x_1 + 2x_2 + 3x_3$ s.t.

$x_1 + 2x_2 + 3x_3 \le 10$

$x_1 + x_2 \le 5$

$x_1 \le 1$

$x_1, x_2, x_3 \ge 0.$

Mangal Date / /20

Equation form Max $z = x_1 + 2x_2 + 3x_3 \Rightarrow z - x_1 - 2x_2 - 3x_3 = 0$

$x_1 + 2x_2 + 3x_3 + s_1 = 10$

$x_1 + x_2 + s_2 = 5$

$x_1 + s_3 = 1$

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0.$

entering Variable — x_3 (Pivot Column); leaving variable — s_1

Basic Variables	z	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	1	-1	-2	-3	0	0	0	0
s_1	0	1	2	3	1	0	0	$10/3 = 3.33$
s_2	0	1	1	0	0	1	0	5
s_3	0	1	0	0	0	0	1	1
z	1	0 ($-1-(-3)1/3$)	0 ($-2-(-3)(2/3)$)	0	1	0	0	10
x_3	0	$1/3$	$2/3$	1	$1/3$	0	0	$10/3 = 10$
s_2	0	1	1	0	0	1	0	$5 = 5$
s_3	0	1	0	0	0	0	1	$1 = 1$

$\therefore$ optimum Solution $x_3 = 10/3$, $s_2 = 5$, $s_3 = 1$, $z = 10$ exists.

Alternative optimum Solution

Alternative optimum solution	z	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	1	0	0	0	1	0	0	10
x_3	0	0	$2/3$	1	$1/3$	0	$-1/3$	$\frac{10}{3} - \frac{1}{3} = 3$
s_2	0	0	1	0	0	1	-1	4.
x_1	0	1	0	0	0	0	1	1

Alternative optima Solution	z	x_1	x_2	x_3	S_1	S_2	S_3	Solution
z	1	0	0	0	0	1	0	10
x_3	0	0	0	1	1/3	-2/3	1	1/3
x_2	0	0	1	0	0	1	-1	4
x_1	0	1	0	0	0	0	1	1

$\therefore$ Alternative optimal solution is

$z=10,\ x_1=1,\ x_2=4,\ x_3=1/3$

$S_1, S_2, S_3 = 0$.

(iv) Max $z = 5x_1+7x_2$

ST $x_1+x_2 \le 4$

$3x_1+8x_2 \le 24$

$10x_1+7x_2 \le 35$

$x_1, x_2 \ge 0$

Equation form

$z-5x_1-7x_2=0$

$x_1+x_2+S_1=4$

$3x_1+8x_2+S_2=24$

$10x_1+7x_2+S_3=35$.

Basic Variables	z	x_1	x_2	x_3	S_1	S_2	S_3	Solution
z	1	-5	-7	0	0	0	0	0
S_1	0	1	1	0	1	0	0	4/1=4
S_2	0	3	8	0	0	1	0	24/8=3
S_3	0	10	7	0	0	0	1	35/7=5
z	1	-19/8	0		0	7/8	0	21
S_1	0	5/8	0		1	-1/8	0	1 → 8/5
x_2	0	3/8	1		0	1/8	0	3 → 8
S_3	0	59/8	0		0	-7/8	1	14 → 1.8

Basic Variables	z	x_1	x_2	s_1	s_2	s_3	Soln
z	1	0	0	19/5	2/5	0	124/5
x_1	0	1	0	8/5	-1/5	0	8/5
x_2	0	0	1	-3/5	1/5	0	12/5
s_3	0	0	0	-59/5	3/5	1	11/5

Date / /20

∴ optimum Solution is $z = \frac{124}{5} = 24.5$ at $\left(\frac{8}{5}, \frac{12}{5}\right)$

$s_1 = s_2 = 0 \quad s_3 = \frac{11}{5}.$

5/5

11/10/18

11/10/14

Tutorial-4

Special Cases in Simplex method

(i) Solve the following LPP

Max $z = 3x_1 + 9x_2$ S.T

$x_1 + 4x_2 \le 8$

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0$

Equation form

Max $z = 3x_1 + 9x_2 \Rightarrow z - 3x_1 - 9x_2 = 0$ S.T

$x_1 + 4x_2 + S_1 = 8$

$x_1 + 2x_2 + S_2 = 4$

$x_1, x_2, S_1, S_2 \ge 0.$

This is a case of degeneracy

	Basic Variables	z	x_1	x_2	S_1	S_2	Solution	
	z	1	-3	-9	0	0	0	
0	S_1	0	1	4	1	0	8	8/4 = 2
	S_2	0	1	2	0	1	4	4/2 = 2
	z	1	-3/4	0	9/4	0	18	
1	x_2	0	1/4	1	1/4	0	2	×4 = 8
	S_2	0	1/2	0	-1/2	1	0	×2 = 0
	z	1	0	0	3/2	3/2	18	
2	x_2	0	0	1	1/2	-1/2	2	
	x_1	0	1	0	-1	2	0	

Optimum Solution

$z = 18$

$x_1 = 0$, $x_2 = 2$

$\cancel{S_1}$ $\cancel{S_2}$

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Q) Solve the following LPP. Verify the Problem using graphical solution

Max $z = 3x_1 - x_2$ S.T

$x_1 - x_2 \le 10$

$x_1 \le 20$

$x_1, x_2 \ge 0.$

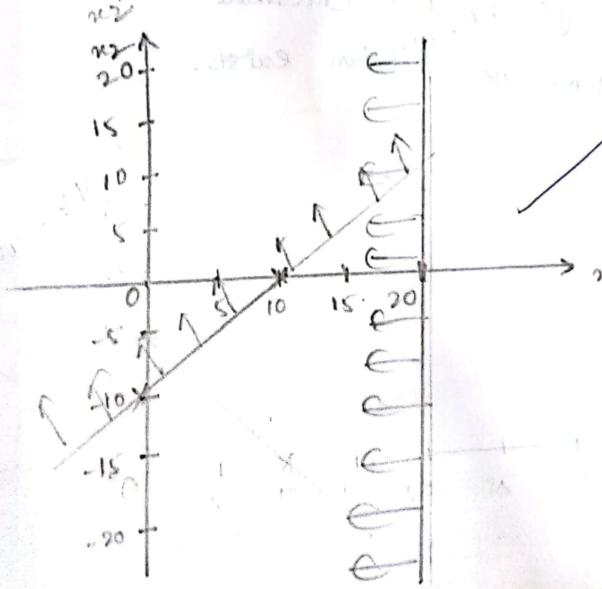
Equation form $\quad z = 3x_1 - x_2 \Rightarrow z - 3x_1 + x_2 = 0.$

$x_1 - x_2 + s_1 = 10$

$x_1 + s_2 = 20.$

$x_1, x_2, s_1, s_2 \ge 0.$

Iteration	Basic Variables	z	x_1	x_2	s_1	s_2	Soln	Min ratio
x_1 enters	z	1	-3	1	0	0	0	
0	s_1	0	1	-1	1	0	$10/1$	$= 10$
s_1 leaves.	s_2	0	1	0	0	1	$20/1$	$= 20$
x_2 enters.	z	1	0	-2	3	0	30.	
1	x_1	0	1	-1	1	0	10	
s_2 leaves	s_2	0	0	1	-1	1	$10/1$	$= 10$
x_2 enters	z	1	0	0	1	2	50	
2	x_1	0	1	0	0	1	20.	
s_2 leaves	x_2	0	0	1	-1	1	10	



∴ from graphical representations it says it has Unbounded Solution.

(iii) Solve the following LPP

Max $z = 2x_1 + x_2$ S.T.

$x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0.$

Equation form Max $z = 2x_1 + x_2 \Rightarrow z - 2x_1 - x_2 = 0$ S.T.

$x_1 - x_2 + S_1 = 10$

$2x_1 - x_2 + S_2 = 40$

$x_1, x_2, S_1, S_2 \geq 0.$

Iteration	Basic Variables	z	x_1	x_2	S_1	S_2	Soln	Min ratio
x_1 enters	z	1	-2	-1	0	0	0	
0	S_1	0	1	-1	1	0	10	$10/1 = 10$
S_1 leaves	S_2	0	2	-1	0	1	40	$40/2 = 20$
x_2 enters	z	1	0	-3	2	0	20	
1	x_1	0	1	-1	1	0	10	
S_2 leaves	S_2	0	0	1	-2	1	20	$20/1 = 20$
	z	1	0	0	-4	3	80	
2	x_1	0	1	0	-1	1	30	
	x_2	0	0	1	-2	1	20	

All the Coefficients of Constraints are negative and hence it says it has unbounded Solution and hence no Solution exists.

(iv) Solve the LPP

Max $z = 2x_1 + 3x_2$ S.T

$-x_1 + 2x_2 \le 4$

$2x_1 - x_2 \le 40$

$x_1, x_2 \ge 0$ and x_1, x_2 are Unrestricted.

Equation form Max $z = 2x_1 + 3x_2$ $\Rightarrow$ $z - 2x_1 - 3x_2 = 0$. S.T.

$-x_1 + 2x_2 + S_1 = 4$

$2x_1 - x_2 + S_2 = 40$

$x_1, x_2 \ge 0$.

Iteration	Basic Variables	z	x_1	x_2	S_1	S_2	Soln	min ratio
x_2 enters	z	1	-2	-3	0	0	0	
0	S_1	0	-1	2	1	0	4/2	4/2 = 2
S_1 leaves	S_2	0	2	-1	0	1	40	
x_1 enters	z	1	-7/2	0	3/2	0	6	
1	x_2	0	-1/2	1	1/2	0	2	
S_2 leaves	S_2	0	3/2	0	1/2	1	42	$42 \times \frac{2}{3} = 28$
	z	1	0	0	8/3	7/3	104	
2	x_2	0	0	1	2/3	1/3	16	
	x_1	0	1	0	1/3	2/3	28	

$\therefore$ $z = 104$ with $x_1 = 28$, $x_2 = 16$.

2 - (-1)(-1/2)

$2 - \frac{1}{2}$

$-2 - (-3)(\frac{-1}{2})$

$0 - (-3)(\frac{1}{2})$

$0 - (-3)/2$

$40 - (-1)(2)$

(V) Solve the LPP

Max $z = 3x_1 + 5x_2 + 4x_3$ ST

$2x_1 + 3x_2 \leq 8$

$2x_1 + 5x_2 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0$

Date / /20

Equation form Max $z - 3x_1 - 5x_2 - 4x_3 = 0$ ST

$2x_1 + 3x_2 + s_1 = 8$

$2x_1 + 5x_2 + s_2 = 10$

$3x_1 + 2x_2 + 4x_3 + s_3 = 15$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Iteration	Basic Variable	z	x_1	x_2	x_3	s_1	s_2	s_3	Soln	min ratio
0	z	1	-3	-5	-4	0	0	0	0	
	s_1	0	2	3	0	1	0	0	8	8/3=
x_2 enters s_2 leaves	s_2	0	2	5	0	0	1	0	10	10/5=
	s_3	0	3	2	4	0	0	1	15	15/2=
x_3 enters	z	1	-1	0	-4	0	1	0	10	
1	s_1	0	4/5	0	0	1	-3/5	0	2	
s_3 leaves	x_2	0	2/5	1	0	0	1/5	0	2	
	s_3	0	11/5	0	4	0	-2/5	1	11	11/4
	z	1	6/5	0	0	0	3/5	1	21	
2	s_1	0	4/5	0	0	1	-3/5	0	2	
	x_2	0	2/5	1	0	0	1/5	0	2	
	x_3	0	11/20	0	1	0	-1/10	1/4	11/4	

∴ optimum solution $z = 21$

$s_1 = 2$ $s_2 = 0$

$(x_1, x_2, x_3) = (0, 2, 11/4)$

8/8

(vi) Max $z = \dots$

$$4x_1 - x_2 \le 8$$
$$4x_1 + 3x_2 \le 12$$
$$4x_1 + x_2 \le 8$$
$$x_1, x_2 \ge 0$$

Equation form max $z - 3x_1 - 2x_2 = 0$ S.T

$$4x_1 - x_2 + s_1 = 8$$
$$4x_1 + 3x_2 + s_2 = 12$$
$$4x_1 + x_2 + s_3 = 8$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0.$$

Iteration	Basic Variable	z	x_1	x_2	s_1	s_2	s_3	Soln	min ratio
x_1 enters	z	1	-3	-2	0	0	0	0	
0	s_1	0	4	-1	1	0	0	8/4	= 2.
s_1 leaves	s_2	0	4	3	0	1	0	12/4	= 3
	s_3	0	4	1	0	0	1	8/4	2.
x_2 enters	z	1	0	-11/4	3/4	0	0	6	
1	x_1	0	1	-1/4	1/4	0	0	2	
s_3 leaves	s_2	0	0	4	-1	1	0	4	4/4 = 1
	s_3	0	0	2	-1	0	1	0	0/2 = 0
s_1 enters	z	1	0	0	-5/8	0	11/8	6	
2	x_1	0	1	0	1/8	0	1/8	2	$2 \times \frac{8}{1} = 16$
s_2 leaves	s_2	0	0	0	1	1	-2	4	4/1 = 4
	x_2	0	0	1	-1/2	0	1/2	0	
3	z	1	0	0	0	5/8	1/8	17/2	
	x_1	0	1	0	0	-1/8	3/8	3/2	
	s_1	0	0	0	1	1	-2	4	
	x_2	0	0	1	0	1/2	-1/2	2	

Optimum soln $z = 17/2$, $(x_1\ x_2\ x_3) = \left(\frac{3}{2}, 2, 0\right)$

$s_1 = 4.$

25/10

* Tutorial-5

(1) Solve the following LP problems using Big M method.

(a) Min $z = 2x_1 + 9x_2 + x_3$, Subject to

$$x_1 + 4x_2 + 2x_3 >= 5$$
$$3x_1 + x_2 + 2x_3 >= 4$$
$$x_1, x_2, x_3 >= 0.$$

Standard form

Min $z = 2x_1 + 9x_2 + x_3 + Ma_1 + Ma_2$

Max $z' = -2x_1 - 9x_2 - x_3 - Ma_1 - Ma_2$ — (i)

S.T

$x_1 + 4x_2 + 2x_3 + a_1 - s_1 = 5$ — (ii)

$3x_1 + x_2 + 2x_3 + a_2 - s_2 = 4$ — (iii)

Before going to simplex approach, artificial variables are to be eliminated ie a_1 & a_2

(i) $z' = -2x_1 - 9x_2 - x_3 - \cancel{Ma_1} - Ma_2$

(ii) × M ⇒ $5M = Mx_1 + 4Mx_2 + 2Mx_3 + \cancel{Ma_1} - Ms_1$

+

$z' + 5M = (-2+M)x_1 + (4M-9)x_2 + (2M-1)x_3 - \cancel{Ma_2} - Ms_1$

(iii) × M ⇒ $4M = 3Mx_1 + Mx_2 + 2Mx_3 + \cancel{Ma_2} - Ms_2$

$z' + 9M = (4M-2)x_1 + (5M-9)x_2 + (4M-1)x_3 - M(s_1+s_2)$

objective function

$z' - (4M-2)x_1 - (5M-9)x_2 - (4M-1)x_3 + M(s_1+s_2) = -9M$

Nonbasic Variables ⇒ x_1, x_2, x_3, s_1, s_2

Basic Variables ⇒ a_1, a_2

Iteration	BV	z'	x_1	x_2	x_3	S_1	S_2	a_1	a_2	Soln	
0	z'	1	$2-4M$	$9-5M$	$1-4M$	M	M	0	0	$-9M$	
	a_1	0	1	4	2	-1	0	1	0	5	$5/4$
	a_2	0	3	1	2	0	-1	0	1	4	4
1	z'	1	$11M-25$	0	$6M-17$	M	$9-4M$	0	$5M-9$	$11M-36$	
	a_1	0	-11	0	-6	-1	4	1	-4	-11	
	x_2	0	3	1	2	0	-1	0	1	4	
1	z'	1	$\frac{-11M-1}{4}$	0	$\frac{-3M-7}{2}$	$\frac{9-M}{4}$	M	$\frac{5M-9}{4}$	0	$\frac{-11M-45}{4}$	
	x_2	0	$1/4$	1	$1/2$	$-1/4$	0	$1/4$	0	$5/4$	
	a_2	0	$\frac{11}{4}$	0	$3/2$	$1/4$	-1	$-1/4$	1	$\frac{11}{4}$	
2	z'	1	0	0	$-37/11$	~~100/44~~ $25/11$	$-1/11$	$\frac{-29M}{11}$	$\frac{11-8M}{4}$	1	
	x_2	0	0	1	$4/11$	$-3/11$	$1/11$	$\frac{3}{11}$	$\frac{-1}{11}$	1	
	x_1	0	1	0	$6/11$	$1/11$	$-4/11$	$-1/11$	$\frac{4}{11}$	1	
3	z'	1	$87/6$	0	0	$17/6$	$-7/3$	$\frac{M-16}{66}$	$\frac{M+2}{3}$	$-27/6$	
	x_2	0	$-2/3$	1	0	$-1/3$	$1/3$	$1/3$	$-1/3$	$1/3$	
	x_3	0	$11/6$	0	1	$1/6$	$-4/6$	$-1/6$	$4/6$	$11/6$	
4	z'	1	$3/2$	7	0	$1/3$	0	$3/6$	M	$-15/6 = -\frac{5}{2}$	
	S_2	0	-2	3	0	-1	1	$+1$	-1	1	
	x_3	0	$1/2$	2	1	$-1/2$	0	$1/2$	0	$15/6 = \frac{5}{2}$	

$x_1 = 0, \quad x_2 = 0, \quad x_3 = 5/2$

$\therefore$ Optimum soln. $\quad z' = -5/2$

$\Rightarrow z = 5/2$

(b) Max $R = 5x - 2y - z$ S.T

$$2x + 2y - z >= 2$$
$$3x - 4y >= 3$$
$$y + 3z >= 5$$
$$x, y, z >= 0$$

Standard form

$$\text{Max } R = 5x - 2y - z - Ma_1 - Ma_2 - Ma_3 \quad \text{S.T} \quad \text{—(i)}$$

$$2x + 2y - z + a_1 - s_1 = 2 \quad \text{—(ii)}$$
$$3x - 4y + a_2 - s_2 = 3 \quad \text{—(iii)}$$
$$y + 3z + a_3 - s_3 = 5 \quad \text{—(iv)}$$

(i) $\Rightarrow$ $5x - 2y - z - \cancel{Ma_1} - Ma_2 - Ma_3 = R.$ ~~$5Mx - 2My - Mz$~~

M × (ii) $\quad 2Mx + 2My - Mz + \cancel{Ma_1} - Ms_1 = 2M$

$$(5+2M)x + (2M-2)y - (M+1)z - \cancel{Ma_2} - Ma_3 - Ms_1 = R + 2M$$

(iii) × M $\Rightarrow$ $3Mx - 4My + \cancel{Ma_2} - Ms_2 = 3M$

$$(5+5M)x + (-2M-2)y - (M+1)z - \cancel{Ma_3} - Ms_1 - Ms_2 = R + 5M$$

(iv) × M $\Rightarrow$ $My + 3Mz + \cancel{Ma_3} - Ms_3 = 5M$

$$(5+5M)x + (-M-2)y + (2M-1)z - Ms_1 - Ms_2 - Ms_3 = R + 10M$$

$$\Rightarrow R + (-5M - 5)x + (M+2)y + (1-2M)z + Ms_1 + Ms_2 + Ms_3 = -10M$$

N.B.V $\rightarrow$ x, y, z, s_1, s_2, s_3

B.V $\rightarrow$ a_1, a_2, a_3.

(c) Food X Contains 6 Units of Vitamin A per gram and 7 Units of Vitamin B per gram & Cost 12 paise per gram. Food Y Contains 8 Units of Vitamin A per gram and 12 Units of Vitamin B and Costs 20 paise per gram. The daily minimum requirements of Vitamin A and B are 100 & 120 Units resp. Find the minimum product mix.

Date / /20

Soln

		x_1 A	x_2 B	
12	X	6	7	→ 100
20	Y	8	12	→ 120

objective function S.T

$$\text{Min } 12x_1 + 20x_2 = z.$$

$$6x_1 + 8x_2 \geq 100$$

$$7x_1 + 12x_2 \geq 120.$$

$$\text{Min } z = 12x_1 + 20x_2 + Ma_1 + Ma_2 \Rightarrow \text{Max } z' = -12x_1 - 20x_2 - Ma_1 - Ma_2 \quad \text{—(i)}$$

where $z' = -z$

$$6x_1 + 8x_2 + a_1 - s_1 = 100 \quad \text{—(ii)}$$

$$7x_1 + 12x_2 + a_2 - s_2 = 120 \quad \text{—(iii)}$$

(i) ⇒ $z' = -12x_1 - 20x_2 - \cancel{Ma_1} - Ma_2$

(ii)×M ⇒ $100M = 6Mx_1 + 8Mx_2 + \cancel{Ma_1} - Ms_1$

$$z' + 100M = (6M-12)x_1 + (8M-20)x_2 - \cancel{Ma_2} - Ms_1$$

(iii)×M ⇒ $120M = 7Mx_1 + 12Mx_2 + \cancel{Ma_2} - Ms_2$

$$z' + 220M = (13M-12)x_1 + (20M-20)x_2 - Ms_1 - Ms_2$$

$$z' + (12-13M)x_1 + (20-20M)x_2 + Ms_1 + Ms_2 = -220M$$

$a_1, a_2 \rightarrow$ B.V

200 +190M
30M-200

Mangal Date / /20

Minratio

$\frac{100}{7} = 14.28$

$\frac{20}{12} = 10$

BV	z'	x_1	x_2	S_1	S_2	a_1	a_2	Soln	minratio
z'	$-13M+12$	$20-20M$	M	M	0	0			
a_1									
a_2									

$\times\frac{12}{7} = \frac{360}{7}$

BV	z'	x_1	x_2	S_1	S_2	a_1	a_2	Soln	MR
z'	1	$12-18M$	$20-20M$	M	M	0	0	$-220M$	
a_1	0	6	8	-1	0	1	0	100	12.5
a_2	0	7	12	0	-1	0	1	120	10
z'	1	$\frac{4-16M}{12}$	0	M	$\frac{-8M+20}{12}$	0	$\frac{20M-20}{12}$		
a_1	0	$16/12$	0	-1	$8/12$	1	$-8/12$	20	
x_2	0	$7/12$	1	0	$-1/12$	0	$1/12$	10	
z'	1	0	0	$1/4$	8	$(M-1)/4$	$\frac{12M-18}{12}$	-205	
x_1	0	1	0	$-3/4$	$1/2$	$3/4$	$-1/2$	15	
x_2	0	0	1	$21/48$	$-9/24$	$-21/48$	$9/24$	$5/4$	

Optimum soln

$z' = -205 \Rightarrow z = 205$

when

$x_1 = 15$

$x_2 = 5/4$.

(1)

(b)

BV	R	x	y	z	s1	s2	s3	a1	a2	a3	soln
R	1	-5-5M	M+2	1-2M	M	M	M	0	0	0	-10M
a1	0	2	2	-1	-1	0	0	1	0	0	2
a2	0	3	-4	0	0	-1	0	0	1	0	3
a3	0	0	1	3	0	0	-1	0	0	1	5
R	1	0	$\frac{-14-17M}{3}$	1-2M	M	$\frac{-2M-5}{3}$	M	0	$\frac{5+5M}{3}$	0	5-5M
a1	0	0	14/3	-1	-1	2	0	1	-2/3	0	0
x	0	1	-4/3	0	0	-1/3	0	0	1/3	0	1
a3	0	0	1	3	0	0	-1	0	0	1	1
R	1	0	0	$\frac{-43M}{14}$	$\frac{-13M-4}{4}$	$\frac{7+38M}{21}$	M	$\frac{14+17M}{14}$	$\frac{28+18M}{21}$	0	-5M+5
y	0	0	1	-3/14	-3/14	3/7	0	3/4	-1/7	0	0
x	0	1	0	-2/7	-2/7	5/12	0	4/3	-5/9	0	1
a3	0	0	0	46/14	3/14	-3/7	-1	-1	2/3	1	5
R	1	0	0	0	$\frac{-31M-22}{28}$	$\frac{7+28M}{21}$	0	$\frac{14+3M}{14}$	M	$\frac{14M+3}{14}$	5
y	0	0	1	0	-1/5	2/5	-1/15	2/45	1/5	31/240	1/3
x	0	1	0	0	-28/105	163/420	-4/45	-67/135	4/215	56/45	13/9
z	0	0	0	1	1/15	-2/15	-14/45	28/138	-14/48	-14/48	14/9

Optimum soln

$R = 5, \quad x = 13/9, \quad y = 1/3, \quad z = 14/9$

Tutorial-6

Two-phase

Solve using Big-M method

(a) Min $z = 2x_1 + x_2$ S.T

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

$$3x_1 + x_2 + a_1 = 3 \quad \text{--- (1)}$$

$$4x_1 + 3x_2 - s_1 + a_2 = 6 \quad \text{--- (2)}$$

$$x_1 + 2x_2 + s_2 = 3 \quad \text{--- (3)}$$

Min $z = a_1 + a_2$

Max $z' = -a_1 - a_2$

$$z' + a_1 + a_2 = 0 \quad \text{--- (0)}$$

Eq (0) = Eq (0) − Eq (1) − Eq (2)

$$z' + a_1 + a_2 = 0$$

$$-3x_1 - x_2 - a_1 = -3$$

$$-4x_1 - 3x_2 + s_1 - a_2 = -6$$

$$z' - 7x_1 - 4x_2 + s_1 = -9$$

(+) (3) ⇒ ~~x1~~

~~$z' - 7x_1 - 4x_2 + s_1 + s_2 = -90$~~

$z' + a_1 + a_2 = 0$

$3x_1 + x_2 + a_1 = 3$

$z' - 3x_1 + a_2 - x_2 = -3$

~~$x_1 + 2x_2$~~

$4x_1 + 3x_2 - s_1 + a_2 = 6$

$z' - 7x_1 - 4x_2 + s_1 = -9$

BV	z'	x_1	x_2	s_1	s_2	a_1	a_2	Soln
z'	1	-7	-4	1	0	0	0	-9
a_1	0	3	1	0	0	1	0	3
a_2	0	4	3	-1	0	0	1	6
s_2	0	1	2	0	1	0	0	3
z'	1	0	-5/3	1	0	7/3	0	-2
a_1	0	1	1/3	0	0	1/3	0	1
a_2	0	0	5/3	-1	+0	-4/3	1	2
s_2	0	0	5/3	0	1	-1/3	0	2
z'	1	0	0	0	0	1	1	0
x_1	0	1	0	1/5	0	3/5	-1/5	3/5
x_2	0	0	1	-3/5	0	-4/5	3/5	6/5
s_2	0	0	0	5/3	1	17/9	-1	0.

Max $z = 2x_1 + x_2$

$z - 2x_1 - x_2 = 0$ — (0)

Date / /20

$x_1 + s_1 = 3/5$

$x_2 + \frac{3s_1}{5} = 6/5$

For entering non basic Variables we bought in the Coefficients of non basic Variables

$z - 2x_1 - x_2 = 0$ — (0)

$x_1 + s_1 = 3/5$ — (1)

$x_2 + \frac{3s_1}{5} = 6/5$ — (2)

$\frac{5s}{3} + s_2 = 0$ — (3)

Eq (0) = (0) − 2(1) − (2)

$z - 2x_1 + x_2 = 0$

$2x_1 + 2s_1 = 6/5$

$-x_2 - \frac{3s_1}{5} = \frac{-6}{5}$

$z - \frac{s_1}{5} = \frac{12}{5}$

B.V	z	x_1	x_2	s_1	s_2	Soln
z	1	0	0	−1/5	0	12/5
x_1	0	1	0	1/5	0	3/5
x_2	0	0	1	3/5	0	6/5
s_2	0	0	0	5/3	1	0

Soln

$z = \frac{12}{5}$, $x_1 = \frac{3}{5}$, $x_2 = \frac{6}{5}$, $s_2 = 0$.

(b)

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1 + 2x_2 \leq 40$

$x_1, x_2 \geq 0.$

$3x_1 + x_2 - s_1 + a_1 = 30$ — (1)

$4x_1 + 3x_2 - s_2 + a_2 = 60$ — (2)

$x_1 + 2x_2 + s_3 = 40$ — (3)

Min $z = a_1 + a_2$

Max $z' = -a_1 - a_2$

$z' + a_1 + a_2 = 0$ — (0)

(0) − (1) − (2) $\Rightarrow$ $z' - 7x_1 - 4x_2 + s_1 + s_2 = -90.$

BV	z'	x_1	x_2	s_1	s_2	s_3	a_1	a_2	soln
z'	1	−7	−4	1	1	0	0	0	−90
a_1	0	3	1	−1	0	0	1	0	30
a_2	0	4	3	3	−1	0	0	1	60
s_3	0	1	2	2	0	1	0	0	40
z'	1	0	−5/3	−1/3	1	0	7/3	0	−20
x_1	0	1	−1/3	−1/3	0	0	1/3	0	20
a_2	0	0	5/3	4/3	−1	0	−4/3	1	25
s_3	0	0	5/3	1/3	0	1	−1/3	0	20
z'	1	0	0	1	0	0	1	1	0
x_1	0	1	0	−3/5	+1/5	0	3/5	−1/5	6
x_2	0	0	1	4/5	−3/5	0	−4/5	3/5	12
s_3	0	0	0	−1	1	1	1	−1	10

$x_1 = 6,\ x_2 = 12,\ s_3 = 10$

$z - 20x_1 - 10x_2 = 0$ — (0)

$x_1 - \frac{3x_1}{5} + \frac{s_2}{5} = 6$ — (1)

$x_2 + \frac{4s_1}{5} - \frac{3s_2}{5} = 12$ — (2)

$-s_1 + s_2 + s_3 = 10$ — (3)

(0) + 20 (1) + 10 (2) $\Rightarrow$ $z - \frac{4s_1}{5} - \frac{2s_2}{5} = 240$

B.V	z	x_1	x_2	s_1	s_2	s_3	Soln
z	1	0	0	$-4/5$	$-2/5$	0	240
x_1	0	1	0	$-3/5$	$1/5$	0	6
x_2	0	0	1	$4/5$	$-3/5$	0	12
s_3	0	0	0	-1	1	1	10

$\therefore x_1 = 6, \quad x_2 = 12, \quad s_3 = 10, \quad z = 240$

(2)(c) Max $P = 2x_1 + 3y + 5z$ ST

$3x + 10y + 5z \le 15$

$x + 2y + z \ge 4$

$33x - 10y + 9z \le 33$

$x, y, z \ge 0$

$3x + 10y + 5z + s_1 = 15$ — (1)

$x + 2y + z + a_1 - s_2 = 4$ — (2)

$33x - 10y + 9z + s_3 = 33$

$x, y, z, s_1, s_2, s_3, a_1 \ge 0$

Phase 1 :

Min $z = a_1$

Max $z' = -a_1 \Rightarrow z' + a_1 = 0$

B.V → s_1, a_1, s_3

$z' + \cancel{a_1} = 0$

eq (−2) ⇒ $-x - 2y - z - \cancel{a_1} + s_2 = -4$

$z' - x - 2y - z + s_2 = -4$

B.V

z'

s_1

a_1

s_3

z'

y

a_1

s_3

z'

y

a_1

x

BV	z'	x	y	z	s1	s2	s3	a1	Soln
z'	1	-1	-2	-1	0	1	0	0	-4
s1	0	3	10	5	1	0	0	0	15
a1	0	1	2	1	0	-1	0	1	4
s3	0	33	-10	9	0	0	1	0	23
z'	1	4/10	0	0	2/10	1	0	0	-1
y	0	3/10	1	1/2	1/10	0	0	0	3/2
a1	0	4/10	0	0	-2/10	-1	0	1	1
s3	0	36	0	14	1	0	1	0	48
z'	1	0	0	7/45	19/40	1	0	1/90	6
y	0	0	1	23/60	3/40	0	0	-1/20	0
a1	0	0	0	-7/45	-19/90	-1	0	-1/90	1
x	0	1	0	2/18	1/36	0	1	1/36	2

No feasible solution because artificial variable dont get eliminated at the end of simplex method

15/11/18

Tutorial - 7

I) Obtain the dual of the following :-

1) Max $3x_1 + 4x_2$

S.T

$2x_1 + 6x_2 \leq 16$

$5x_1 + 2x_2 \geq 20$

$x_1, x_2 \geq 0$.

In standard primal form,

$z = 3x_1 + 4x_2$

S.T

$2x_1 + 6x_2 \leq 16 \quad — y_1$

$\cancel{5x_1 + 2x_2 \leq 20 \quad — y_2}$

$-5x_1 - 2x_2 \leq -20 \quad — y_2$

$x_1, x_2 \geq 0$.

In ~~dual~~ equation form,

$z = 3x_1 + 4x_2$

S.T

$2x_1 + 6x_2 + s_1 = 16 \quad — y_1$

$5x_1 + 2x_2 + s_2 = 20 \quad — y_2$

$-5x_1 - 2x_2 + s_3 = -20 \quad — y_3$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

In dual form,

min $w = 16y_1 + 20y_2 \cancel{- 20y_3}$

S.T.

$2y_1 \cancel{+6y_2} - 5y_2 \geq 3$

$6y_1 \cancel{+2y_2} - 2y_2 \geq 4$

$y_1, y_2, \cancel{y_3} \geq 0.$

2) min $z = x_1 - x_2 + 3x_3$

S.T

$x_1 + x_2 + x_3 = 10$

$2x_1 - x_3 \leq 2$

$2x_1 - 2x_2 + 3x_3 \leq 6$

$x_1, x_2, x_3 \geq 0.$

Max $z' = -x_1 + x_2 - 3x_3$

In standard primal form,

$x_1 + x_2 + x_3 \leq 10 \quad — y_1$

$-x_1 - x_2 - x_3 \leq -10 \quad — y_2$

$2x_1 - x_3 \leq 2 \quad — y_3$

$2x_1 - 2x_2 + 3x_3 \leq 6 \quad — y_4$

$x_1, x_2, x_3 \geq 0.$

In dual form,

min $w = 10y_1 - 10y_2 + 2y_3 + 6y_4$

S.T

$y_1 - y_2 + 2y_3 + 2y_4 \geq -1$

$y_1 - y_2 - 2y_4 \geq 1$

$y_1 - y_2 - y_3 + 3y_4 \geq -3$

where. $y_1, y_2, y_3, y_4 \geq 0.$

3 Min $z = 3x_1 + x_2 + 4x_3 + x_4 + 9x_5$

S.T

$4x_1 - 5x_2 - 9x_3 + x_4 - 2x_5 \le 6$

$2x_1 + 3x_2 - 5x_4 + x_5 \le 9$

$x_1 + 5x_3 - 7x_4 + 11x_5 = 10$

x_1, x_2, x_3, x_4 is unrestricted, $x_5 \ge 0$.

Min $z = 3(x_1 - x_1') + (x_2 - x_2') + 4(x_3 - x_3') + (x_4 - x_4') + 9x_5$

In standard primal form,

Max $z = -3(x_1 - x_1') - (x_2 - x_2') - 4(x_3 - x_3') - (x_4 - x_4') - 9x_5$

S.T

y_1 — $4(x_1 - x_1') - 5(x_2 - x_2') - 9(x_3 - x_3') + (x_4 - x_4') - 2x_5 \le 6$

y_2 — $2(x_1 - x_1') + 3(x_2 - x_2') - 5(x_4 - x_4') + x_5 \le 9$

y_3 — $(x_1 - x_1') + 5(x_3 - x_3') - 7(x_4 - x_4') + 11x_5 \le 10$

y_4 $-(x_1 - x_1') - 5(x_3 - x_3') + 7(x_4 - x_4') - 11x_5 \le -10$

Min $w = 6y_1 + 9y_2 + 10y_3 - 10y_4$

S.T

$4y_1 + 2y_2 + y_3 - y_4 \ge -3$

$-4y_1 - 2y_2 - y_3 + y_4 \ge 3$

$-5y_1 + 3y_2 \cancel{+ 5y_3 - 6} \ge -1$

$5y_1 - 3y_2 \ge 1$

$-9y_1 \cancel{-5y_2} + 5y_3 - 5y_4 \ge -4$

$9y_1 \cancel{+5y_2} - 5y_3 + 5y_4 \ge 4$

$-5y_2 + y_1 - 7y_3 + 7y_4 \ge -1$

$+5y_2 - y_1 + 7y_3 - 7y_4 \ge 1$

$-2y_1 + y_2 + 11y_3 - 11y_4 \ge 9$

$y_1, y_2, y_3, y_4, \cancel{y_5} \ge 0$

II Use dual simplex method to solve:-

1. Max $z = -3x_1 - 2x_2$

S.T

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$.

In equation form,

$z + 3x_1 + 2x_2 = 0$

S.T

$-x_1 - x_2 + s_1 = -1$

$x_1 + x_2 + s_2 = 7$

$x_1 + 2x_2 + s_3 = 10$

$x_2 + s_4 = \cancel{10}\ 3$

$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$.

	z	x_1	x_2	s_1	s_2	s_3	s_4	solution	iteration
z	1	3	2	0	0	0	0	0	0
s_1	0	-1	-1	+1	0	0	0	-1	min
s_2	0	1	1	0	1	0	0	7	(-1, 7, 10, 3)
s_3	0	1	2	0	0	1	0	10	max $\left(\frac{3}{1}\right.$
s_4	0	0	1	0	0	0	1	3	
z	1	1	0	2	0	0	0	-2	
x_2	0	1	1	-1	0	0	0	1	
s_2	0	0	0	1	1	0	0	6	
s_3	0	-1	0	+2	0	1	0	8	
s_4	0	-1	0	1	0	0	1	2	

Solution

$x_2 = 1$, $x_1 = 0$ $\quad z = -2$.

2) Max $z = -2x_1 - 2x_2 - 4x_3$

S.T

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0.$$

In standard LPP form,

$$z = -2x_1 - 2x_2 - 4x_3$$

S-T

$$-2x_1 - 3x_2 - 5x_3 \leq -2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0.$$

In equation form,

$$z + 2x_1 + 2x_2 + 4x_3 = 0$$

S-T

$$-2x_1 - 3x_2 - 5x_3 + s_1 = -2$$
$$3x_1 + x_2 + 7x_3 + s_2 = 3$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0.$$

	z	x_1	x_2	x_3	s_1	s_2	solution	iteration
z	1	2	2	4	0	0	0	0
s_1	0	-2	-3	-5	1	0	-2	min(-2, 3)
s_2	0	3	1	7	0	1	3	max(
z	1	2/3	0	2/3	2/3	0	-4/3	All are +ve
x_2	0	2/3	1	6/3	-1/3	0	2/3	
s_2	0	7/3	0	16/3	+1/3	~~1+1/3~~	7/3	

$z = -4/3$, $x_2 = \frac{2}{3}$, $x_1 = 0$, $x_3 = 0$.

3. $\min \ z = 6x_1 + x_2$

S.T

$2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$.

$\text{Max} \quad z' = -6x_1 - x_2$

In standard primal form,

$\text{Max} \quad z' = -6x_1 - x_2$

S.T

$-2x_1 - x_2 \leq -3$

$-x_1 + x_2 \leq 0$

$x_1, x_2 \geq 0$.

In equation form,

$\text{Max} \quad z' + 6x_1 + x_2 = 0$

S.T

$-2x_1 - x_2 + s_1 = -3$

$-x_1 + x_2 + s_2 = 0$

$x_1, x_2, s_1, s_2 \geq 0$.

z'	z'	x_1	x_2	s_1	s_2	solution	iteration
z'	1	6	1	0	0	0	0
s_1	0	-2	-1	1	0	-3	min(-3, 0)
s_2	0	-1	1	0	1	0	max $\left(\frac{6}{-2}, \frac{1}{-1}\right)$
z'	1	4	0	1	0	-3	1
x_2	0	2	1	-1	0	3	min(3, -3)
s_2	0	-3	0	1	1	-3	max $\left(\frac{4}{-3}\right)$
z'	1	0	0	7/3	4/3	-7	
x_2	0	0	1	-1/3	+2/3	1	
x_1	0	1	0	-1/3	-1/3	1	✓

$x_1 = 1, x_2 = 1$ $\qquad z' = -7$ $\qquad z = 7$

4. min $z = x_1 + x_2$

s.T

$2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$.

max $z' = -x_1 - x_2$

In standard primal form,

Max $z' = -x_1 - x_2$

$-2x_1 - x_2 \leq -2$

$+x_1 + x_2 \leq -1$

$x_1, x_2 \geq 0$.

In equation form,

max $z' + x_1 + x_2 = 0$

S.T

$-2x_1 - x_2 + s_1 = -2$

$x_1 + x_2 + s_2 = -1$

$x_1, x_2, s_1, s_2 \geq 0$.

	z'	x_1	x_2	s_1	s_2	solution	iteration
z'	1	1	1	0	0	0	0
s_1	0	-2	-1	1	0	-2	min(-2, -1)
s_2	0	1	1	0	1	-1	
z'	1	0	1/2	1/2	0	-1	
x_1	0	1	1/2	-1/2	0	1	
s_2	0	0	1/2	1/2	1	-2	

All are +ve in s_2, even though solution is −ve and hence it is called pseudo optima solution

$z' = -1$, $z = 1$, $x_1 = 1$.

5. $\min \; x_1 + 2x_2 + 3x_3$

S.T

$2x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \geq 8$

$x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$.

$\max \; z' = -x_1 - 2x_2 - 3x_3$

In standard primal form

$\text{Max } z' = -x_1 - 2x_2 - 3x_3$

S.T

$-2x_1 + x_2 - x_3 \leq -4$

$-x_1 - x_2 - 2x_3 \leq -8$

$-x_2 + x_3 \leq -2$

$x_1, x_2, x_3 \geq 0$.

In equation form

$\text{Max } z' + x_1 + 2x_2 + 3x_3 = 0$

S.T

$-2x_1 + x_2 - x_3 + s_1 = -4$

$-x_1 - x_2 - 2x_3 + s_2 = -8$

$-x_2 + x_3 + s_3 = -2$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$.

	z'	x_1	x_2	x_3	s_1	s_2	s_3	Solution	Iteration
z'	1	1	2	3	0	0	0	0	0
s_1	0	-2	1	-1	1	0	0	-4	min(-4, -8, -2)
s_2	0	-1	-1	-2	0	1	0	-8	
s_3	0	0	-1	1	0	0	1	-2	
z'	1	0	1	1	0	1	0	-8	
s_1	0	0	3	3	1	-2	0	12	
x_1	0	1	1	2	0	-1	0	8	min(8, -2).
s_3	0	0	-1	1	0	0	1	-2	
z'	1	0	0	2	0	1	1	-10	
s_1	0	0	0	6	1	-2	3	6	
x_1	0	1	0	3	0	-1	1	6	
x_2	0	0	1	-1	0	0	-1	2	

$z' = (1, 1, 2, 3, 0, 0, 0, 0) - (1)(0, 1, 1, 2, 0, -1, 0, 8)$

$s_1 = (0, -2, 1, -1, 1, 0, 0, -4) - (-2)(0, 1, 1, 2, 0, -1, 0, 8)$

$s_3 = (0, 0, -1, 1, 0, 0, 1, -2) -$

$z' = (1, 0, 1, 1, 0, 1, 0, -8) - 1(0, 0, -1, 1, 0, 0, 1, -2)$

$s_1 = (0, 0, 3, 3, 1, -2, 0, 12) - 3(0, 0, -1, 1, 0, 0, 1, -2)$

$x_2 = (0, 1, 1, 2, 0, -1, 0, 8) - (1)(0, 0, -1, 1, 0, 0, 1, -2)$

$z' = -10 \qquad z = 10.$

$x_1 = 6 \qquad x_2 = 2$

6. min $z = 10x_1 + 6x_2 + 2x_3$

s.t.

$-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0.$

max $z' = -10x_1 - 6x_2 - 2x_3$

In standard primal form,

$z' = -10x_1 - 6x_2 - 2x_3$

s.t.

$+x_1 - x_2 - x_3 \leq -1$

$-3x_1 - x_2 + x_3 \leq 2$

$x_1, x_2, x_3 \geq 0.$

In equation form,

$z' + 10x_1 + 6x_2 + 2x_3 = 0$

$x_1 - x_2 - x_3 + s_1 = -1$

$-3x_1 - x_2 + x_3 + s_2 = 2$

$x_1, x_2, x_3, s_1, s_2 \geq 0.$

	z'	x_1	x_2	x_3	s_1	s_2	solution	[illegible]
z'	1	10	6	2	0	0	0	
s_1	0	1	-1	-1	1	0	-1	0
s_2	0	-3	-1	1	0	1	-2	min(-[illegible]
z'_2	1	0	8/3	16/3	0	10/3	-20/3	max(
s_1	0	0	-4/3	-2/3	1	1/3	-5/3	1
x_1	0	1	1/3	-1/3	0	-1/3	2/3	min($\frac{5}{[illegible]}$
z'	1	0	0	4	2	4	-10	
x_2	0	0	1	1/2	-3/4	-1/4	5/4	2
x_1	0	1	0	-1/2	1/4	-1/4	1/4	

URBAN EDGE ___/___/___

$z' = -10$, $z = 10$

$x_1 = \frac{1}{4}$ $x_2 = \frac{5}{4}$

$\sqrt{17 - 5(5)}$

lim

$(-1, -2)$

$(\frac{5}{3}, \frac{2}{3})$