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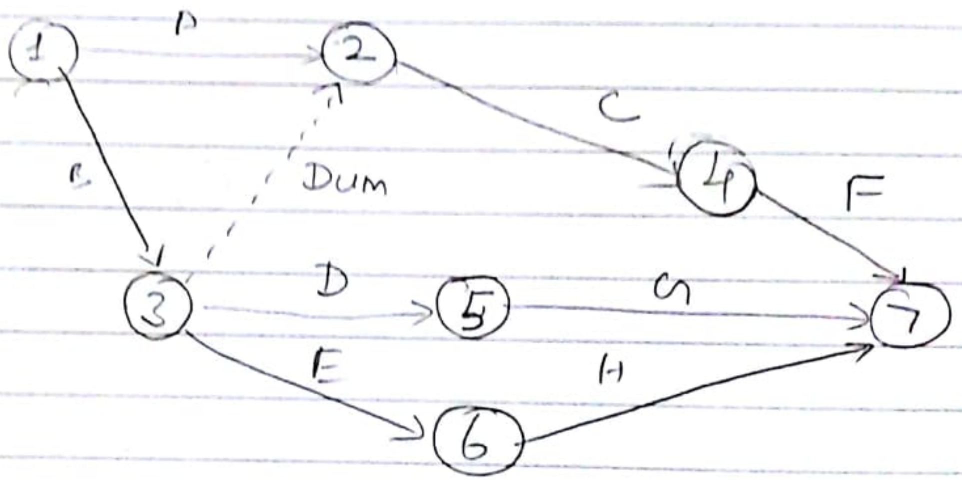
Network diagram Representation

Basic Definitions.

1) Activity : Any individual operation which utilizes resources and has a begining and an end is called an activity



For example A, B, C etc are activities in the above network diagram

They are classified as

A) predecessor activity

Activities that must be completed prior to the start of another activity.

Ex:- A is the predecessor activity to C.

B) Successor Activity

Activities That cannot be started until one or more other activities are completed. But immidiately succeed them are called successor activities.

C is The successor activity of A and B.

c) Concurrent Activity

Activities which can be accomplished concurrently.
For example: A & B as well as D & E
are concurrent Activities

D) Dummy Activity
An activity which does not consume any kind of resource but mearly depicts the technical dependence is called dummy activity.
Dumm Activity is inserted in the following situations.
1) To make activities with common starting and finishing points distinguishable.

2) To identify and maintain the proper precedence relationship between activities that are connected by events.
For example: In the above network Activity C is dependent both on A & B. But D & E are dependent only on B. In such a situation, dummy activities are inserted.

2) Event: An event represents a point in time which marks the begining and ending of activities.
It is represented by a circle ○.
For ex-: Activity A is marked by 2 events 1 and 2, 1 marks the begining of activity A & 2 marks the end.

They are classified into 3 types:

1) Merge event:

Eg: 7

When more than 1 activity comes and joins an event.

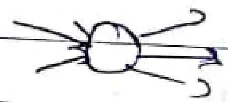
2.) Burst event:

When more than 1 activity leaves an event.

Eg: 1

3) Merge and Burst event:

An activity which is both the merge and burst.



eg: 3

4)

3) Sequencing: is to maintain the precedence relationships.

The foll. points must be considered during the development of a network:

1.) What jobs precede
2) Which jobs run concurrently.
3) Which jobs follow
4) What controls the start and finish of a job.

For ex:- Consider the following precedence table and construct the network.

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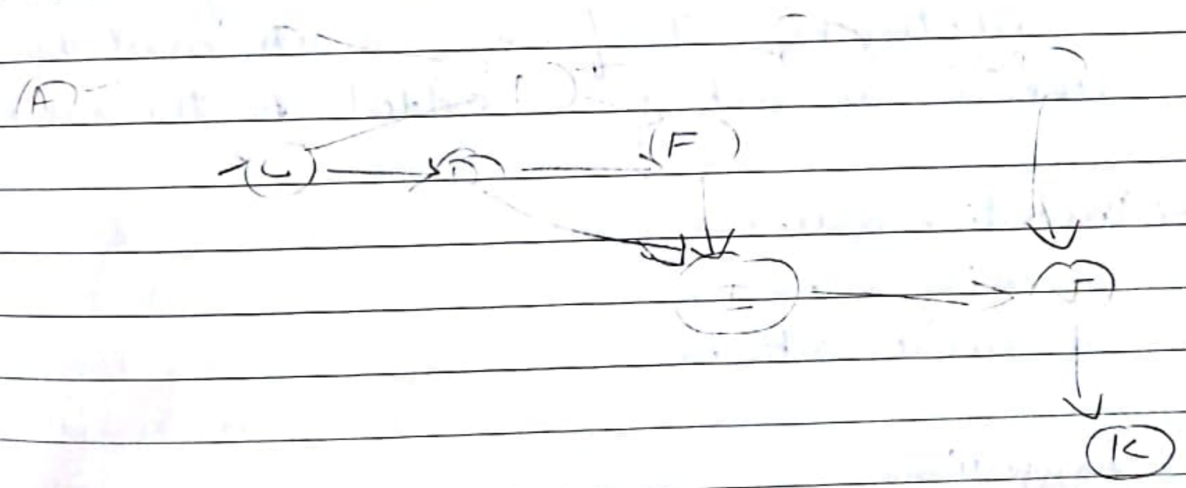
Activity	Preceding Activity.
A	—
B	A
C	A
D	C
E	B, C
F	D, E
G	E
H	G
I	D, F
J	I, H
K	J



Rules for drawing network diagram.

1. Each activity is represented by one and only arrow in the network i.e no activity can appear twice in ~~For ex~~ a network

2)(a) It must be represented in one of the following ways

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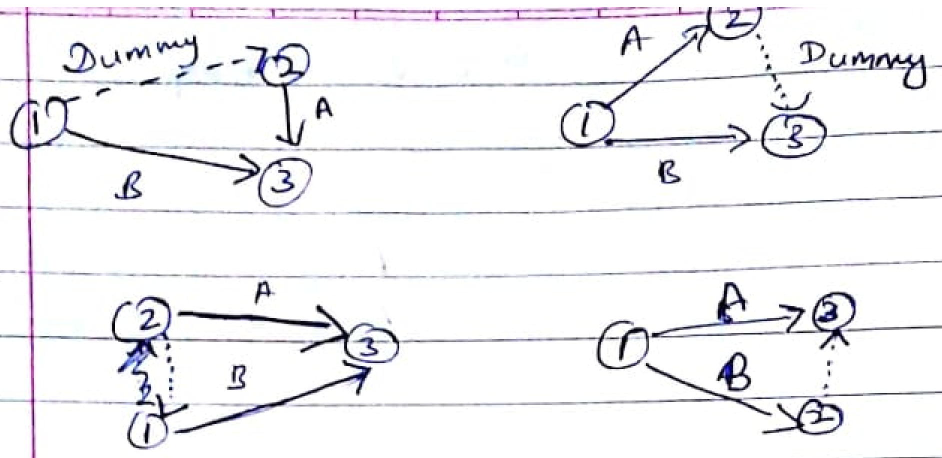


2) Rule 2 : No 2 activities can be identified by the same end events
Dummy activities has to be introduced as shown in the above diagrams.

3) Rule 3 :- In order to ensure correct precedence relationship, the following points must be checked whenever an activity is added to the network.

1) Preceeding activities
2) Succeeding activities
3) Concurrent activities

Suggestions

1) ~~Abort~~ Avoid arrows which cross each other.

2) Use straight arrows.

3) Use arrows from left to right or right to left

4) Avoid mixing 2 directions.

4) Do not represent duration of the activity by its arrow length

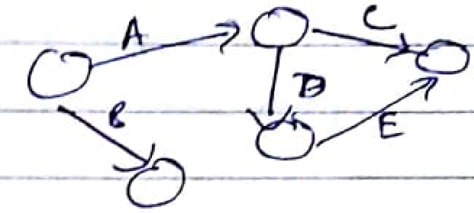
5) Use dummy activities as many as required in the draft network but the final network must not have any redundant dummies.

6) A network must have only one entry pt and 1 ~~exit~~ end point.

7) Common errors are :-

1) Incomplete network

for ex.



This is called a dangling networks as there are 2 endpoints

2) Looping Error

A network should not have any loops or cycles.

3) Redundancy

Unnecessarily inserting dummy activities may lead to redundancy.

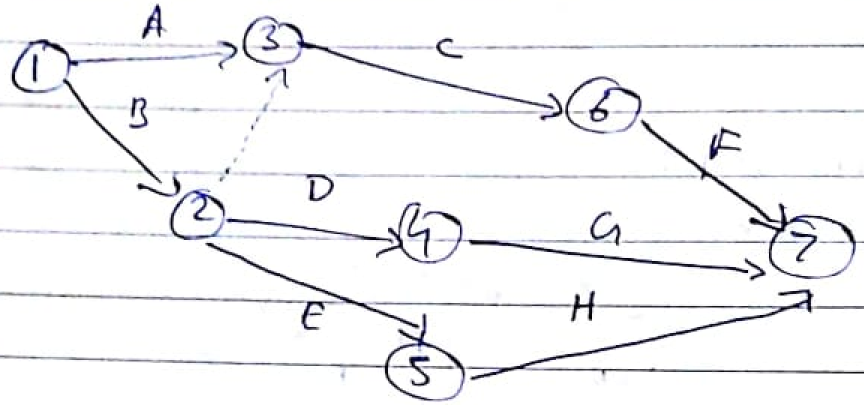
<u>Labelling a Network Using Fulkerson's I-J Rule</u>

1. A start event is the one which has ~~arrows~~ only emerging arrows it must be numbered as 1.

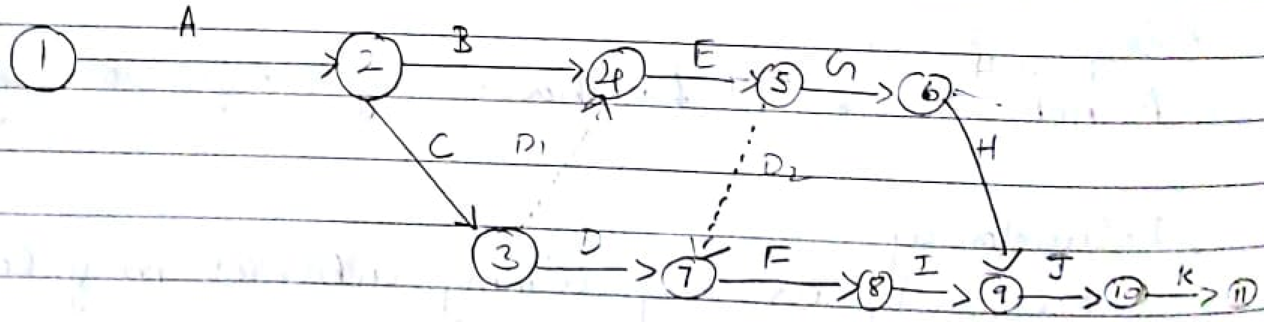
2. Delete all arrows emerging from all numbered events, This will create atleast one new start event.

3. Number all new start events as 2, 3 etc form top to bottom.

4. Repeat steps 2 & 3 till end is reached.



CQ 1. Construct the network diagram for the problem given previously with the precedence table



Type Estimates and Critical path in Network Analysis

After the network is constructed it is necessary to perform time analysis for planning various activities of the project.

This analysis helps in finding

1) Total completion time for the project

2) Earliest time for beginning an activity
3) Latest time for beginning an activity without delaying the total project.
4) Float for each activity which is the amount of time by which the completion of an activity can be delayed without delaying the total project completion
5) Identification of critical activities and critical path.

Basic computations and Notations

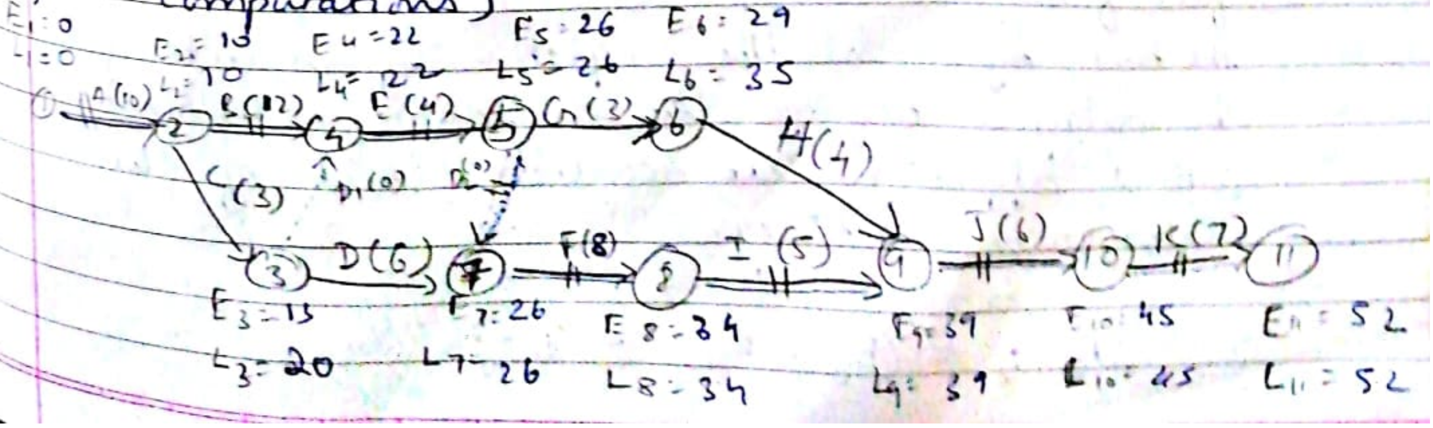
Notations (i, j)

1) (i, j) → Activity (i, j) from event i to event j

2. E_i or T_E - Earliest occurrence time of event i
3. L_j or T_L - Latest occurrence time of event j.
4. D_{ij} - Duration of activity i, j.
5. $(ES)_{ij}$ - Earliest starting time of activity (i, j).
6. $(EF)_{ij}$ - Earliest finishing time " "
7. $(LS)_{ij}$ - Latest starting " "
8. $(LF)_{ij}$ - " finishing time of activity (i, j)

Computation

1) Computation for earliest event time (E_i) (forward pass computations)



$E_1 = 0$

$E_j = \max [E_i + D_{ij}]$

2) 2nd Computation : Latest Event time (L_j) (backward pass computation).

$$L_n = E_n$$

$$L_i = \min (L_j - D_{ij})$$

3) Determination of floats and slacks time

There are 3 kinds of floats

1) Total float

Defn: The amt. of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time

Mathematically total float $(TF)_{ij} = (LS)_{ij} - (ES)_{ij}$

Latest Start - Earliest Start.

$$= (L_j - D_{ij}) - E_i$$

$$= L_j - E_i - D_{ij}$$ is the total float for each activity.

2) free float

The time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of the subsequent activity

$$(F_f)_{ij} = (E_j - E_i) - D_{ij}$$

3) Independant float :-

The amt of time by which the start of an activity can be delayed without affecting the earliest start time of any subsequent activities assuming that the preceding activity has finished at its latest finish time.

$$(If)_{ij} = (E_j - L_i) - D_{ij}$$

4) Interfering float

is that part of total float which causes a reduction in the float of successor activities.

5) Event slacks

is defined as the difference b/w the latest event and earliest event times

$L_i - E_i$

Determination of critical path

1. Critical Event :-

Those events for which latest and earliest times are the same. i.e $E_i = L_i$.

2. Critical Activity :-

The activities with zero total float are known as critical activities.

3) Critical path :

The sequence of critical activities in a network is called a critical path.

It is the longest path in the network from the starting event to the ending event and defines the minimum time required to complete the project.

For example:

In the previous network,

1 - 2 - ~~2~~4 - 5 - 7 - 8 - 9 - 10 - 11

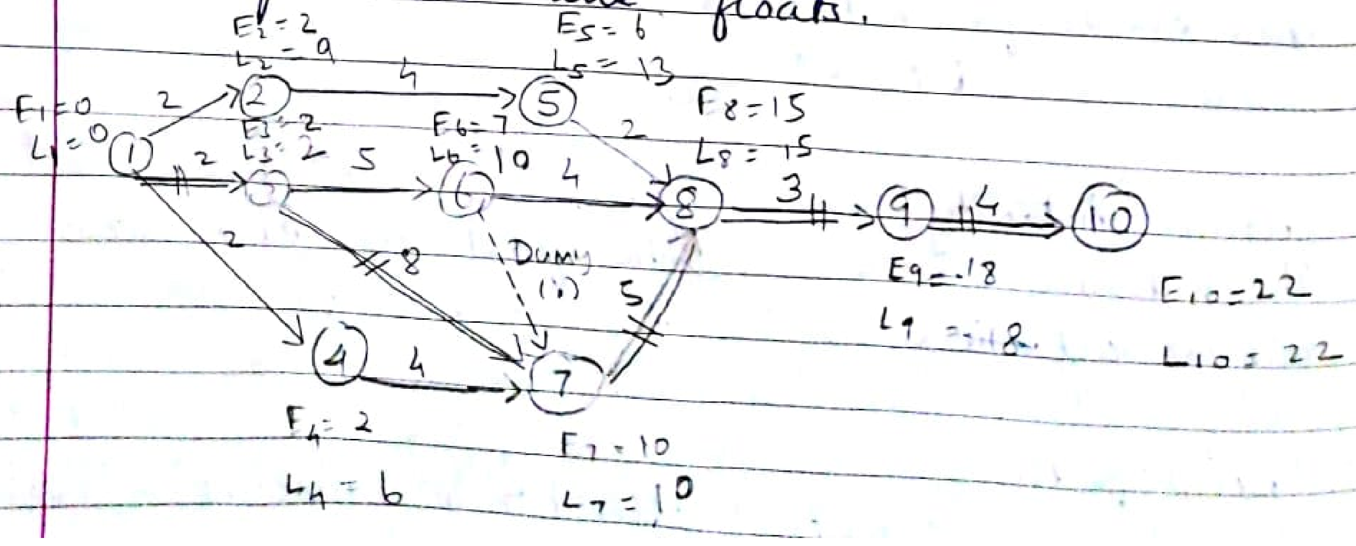
and critical path length is 52

Features of critical path.

1. If the project have to be shortened then some of the activities in that path must be shortened.

2. The variation in actual performance from the expected activity duration time will be completely reflected in ~~1 2 1~~ one to one manner in the anticipated completion of the whole project

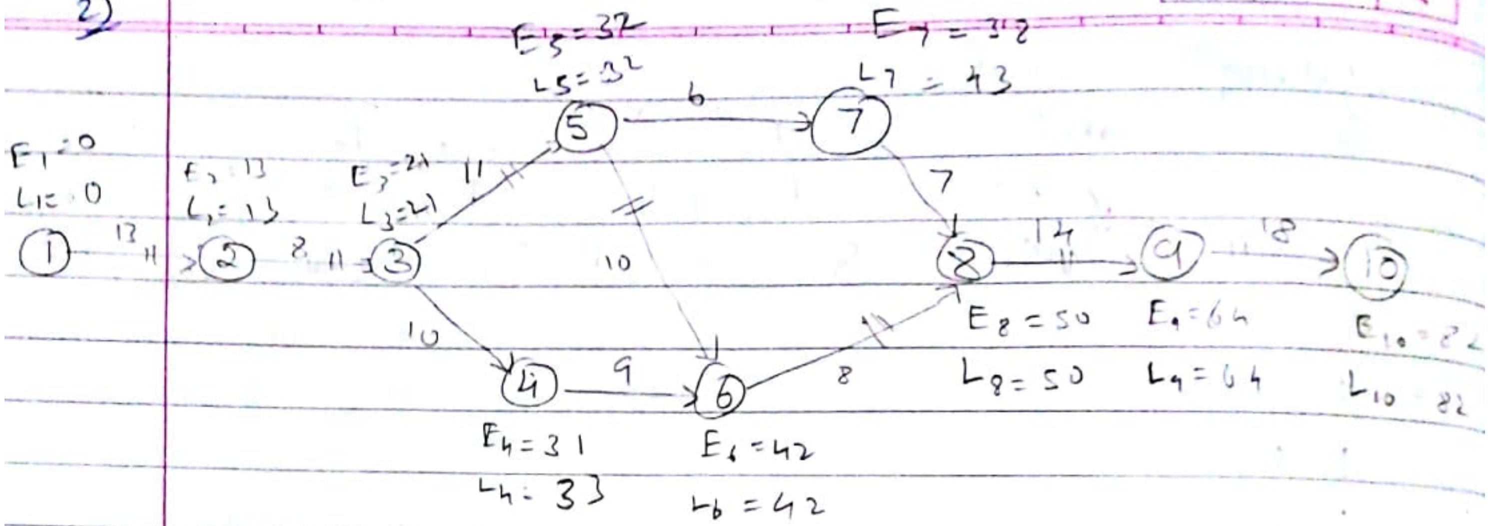
Consider the following network and compute the critical path and the floats.



Activity	Float total $L_j - E_i - D_{ij}$	Free $E_j - E_i - D_{ij}$	Indefinite $E_j - L_i - D_{ij}$
1-2	7	0	2
1-3	0	0	0
1-4	4	0	0
2-5	7	0	0
3-6	3	0	0
3-7	0	0	0
4-7	4	4	0
5-8	7	7	0
6-8	4	4	1
7-8	0	0	0
8-9	0	0	0
9-10	0	0	0

CP is 1 - 3 - 7 - 8 - 9 - 10

2)



$E_j = \max[E_i + D_{ij}]$

$L_i = \min(L_j - D_{ij})$

Critical path 1 - 2 - 3 - 5 - 6 - 8 - 9 - 10

Activity Float

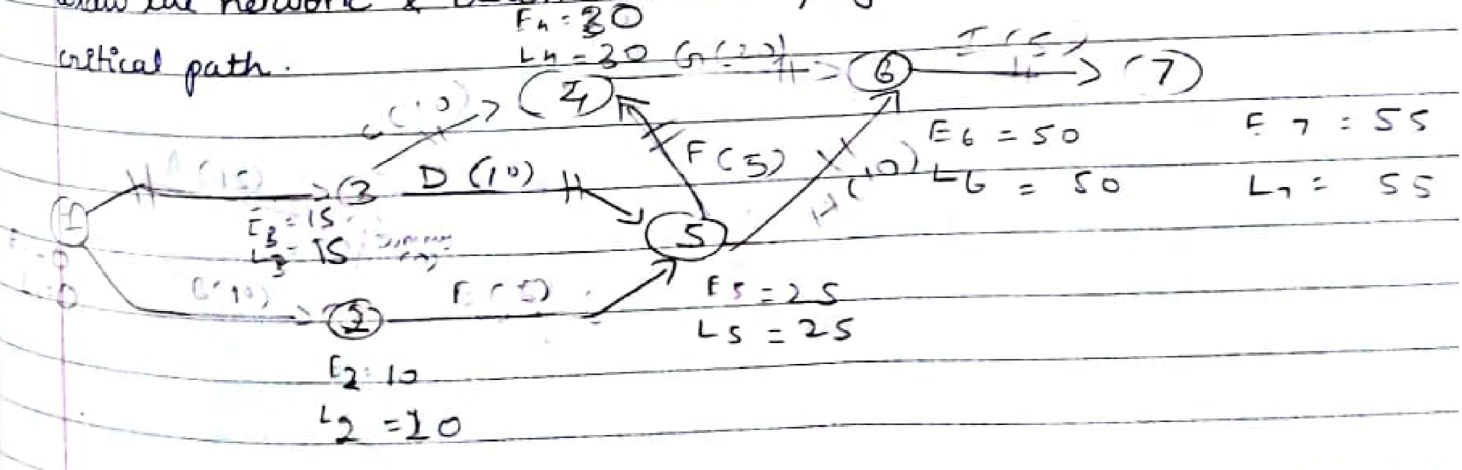
	Total $L_j - E_i - D_{ij}$	Free $E_j - E_i - D_{ij}$	Independent $E_j - L_i - D_{ij}$
1-2	0	0	0
2-3	0	0	0
3-5	0	0	0
3-4	2	0	0
5-6	-		0
5-7	5	0	-
6-8			0
7-8			
8-9			
9-10			

i) $B < E, F$; $C < G, L$; $E, G < H$; $F, G < H$; $L, H < I$, $L < M$; $H, M < N$; $H < J$; $I, J < P$; $P < Q$; $X < Y$ means X must be finished before Y can begin. Construct the network diagram.

$\bigcirc \xrightarrow{B} \bigcirc$

Job	Predecessor	Time in days
A	–	15
B	–	10
C	A, B	10
D	A, B	10
E	~~D, E~~ B	5
F	~~C, E, F~~ D, E	5
G	~~D, E~~ C, F	20
H	D, E	10
I	G, H	5

Draw the network & determine the project duration. Also identify critical path.



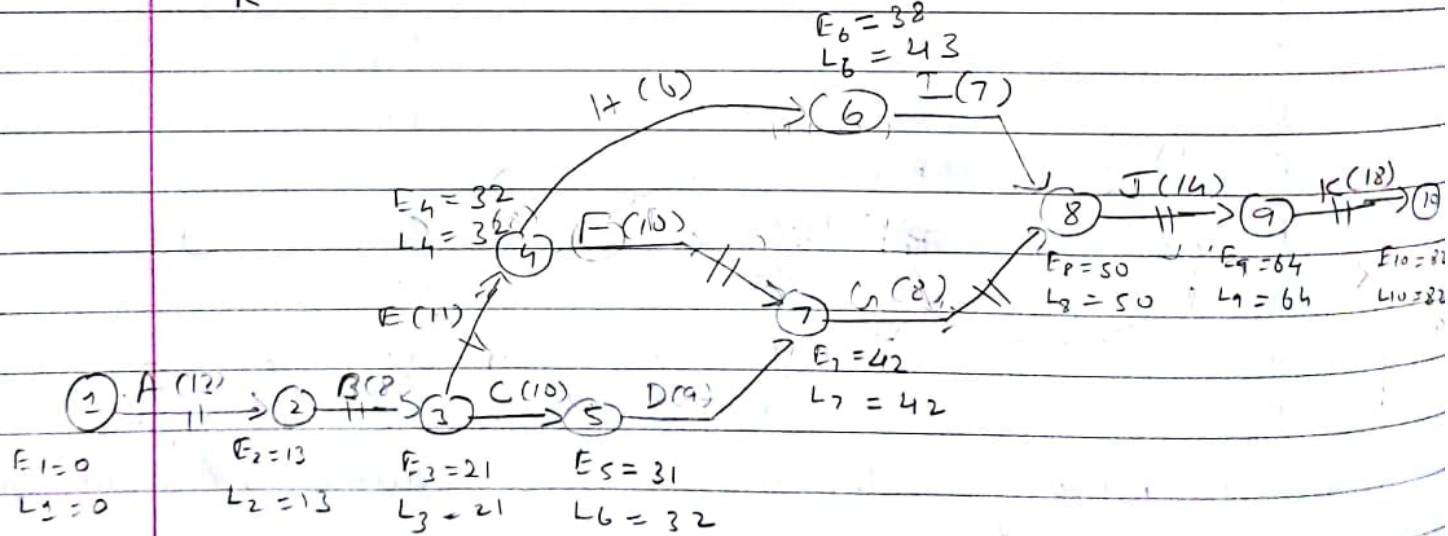
1 – 3 – 5 – 6 – 7 = 55

1 – 3 – 4 – 6 – 7 = 55

1 – 3 – 5 – 4 – 6 – 7 = 55

Q2

Job	Job time	Immediate Predecessor.
A	13	—
B	8	A
C	10	B
D	9	C
E	11	B
F	10	E
G	8	D, F
H	6	E
I	7	H
J	14	G, I
K	18	J



1 – 2 – 3 – 4 – 7 – 8 – 9 – 10 = 82

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Project Evaluation and Preview Technique (PERT)

In the network analysis discussed so far, the time duration values were deterministic. However in most of the projects, a reliable, time estimate is difficult to obtain. In such cases, 3 time values are associated with each activity.

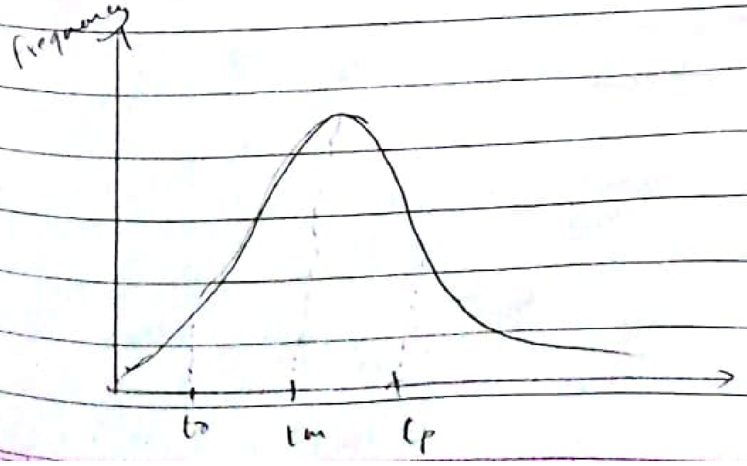
1. Optimistic value
2. Pessimistic value.
3. Most likely value.

These uncertainities associated with the activities are analyzed through PERT approach.

1. Optimistic time
It is the shortest possible time for completing the activity. denoted by t_o.

2. Most likely time
is the estimate of the normal time, the activity would take. This assumes normal delays. denoted by t_m

3. Pessimistic time
represents the longest time taken by the activity denoted by t_p.



4. Expected time
is the average time an activity will take, if it were to be repeated many times and is given by

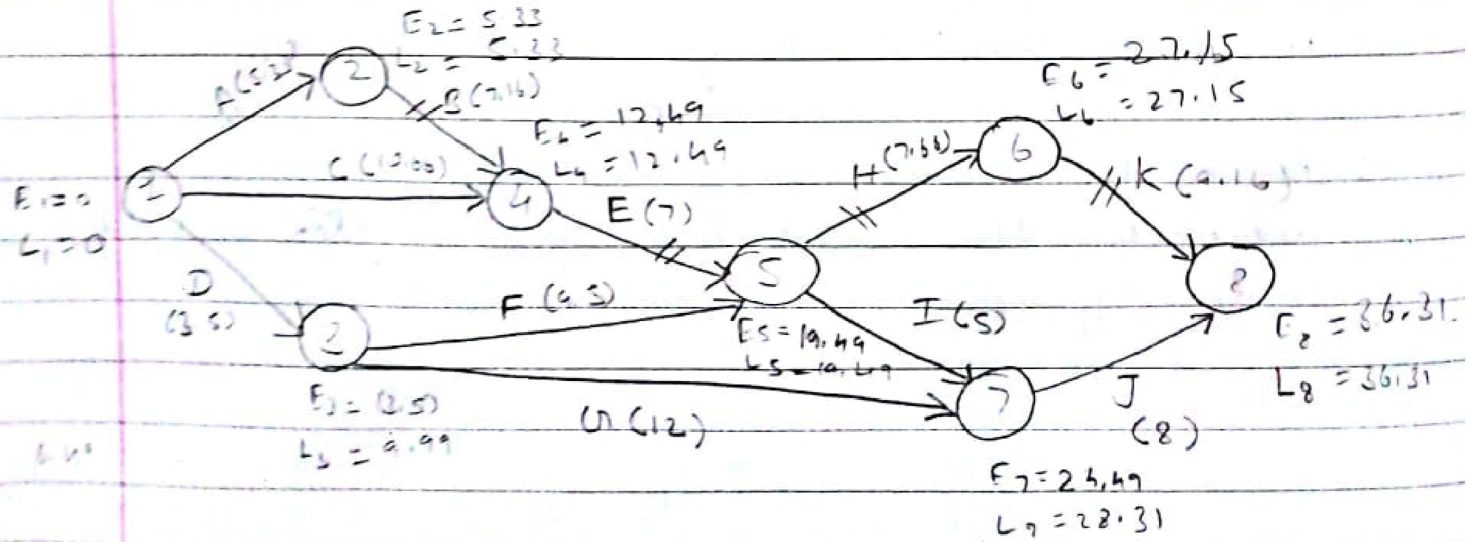
$$t_e = (t_o + 4t_m + t_p)/6$$

5 Variance
of an activity is given by

$$\sigma^2 = \left[(t_p - t_o)/6\right]^2$$

Q1. For the project represented by the following network diagram. Find the earliest and latest time to reach each node. Given the following data. Slack

	Task	A	B	C	D	E	F	G	H	I	J	K
t_o	Least time	4	5	8	2	4	6	8	5	3	5	6
t_p	Greatest time	8	10	12	7	10	15	16	9	7	11	13
t_m	Most likely time	5	7	11	3	7	9	12	8	5	8	9
t_e	Expected	5.33	7.166	10.66	3.5	7	9.5	12	7.66	5	8	9.16



1 - 2 - 4 - 5 - 6 - 8 , project time = 36.31

Slack

1 → 0 - 0 = 0

2 → 5.33 - 5.33 = 0

3 → 6.49

4 - 0

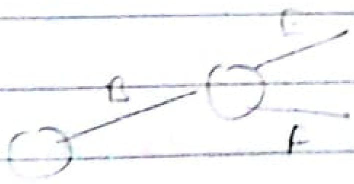
5 - 0

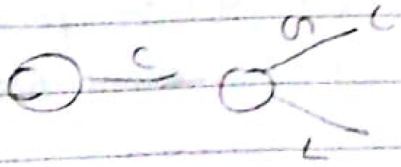
6 = 0

7 - 3.82

8 - 0

Q2) Find the critical path for the above network.





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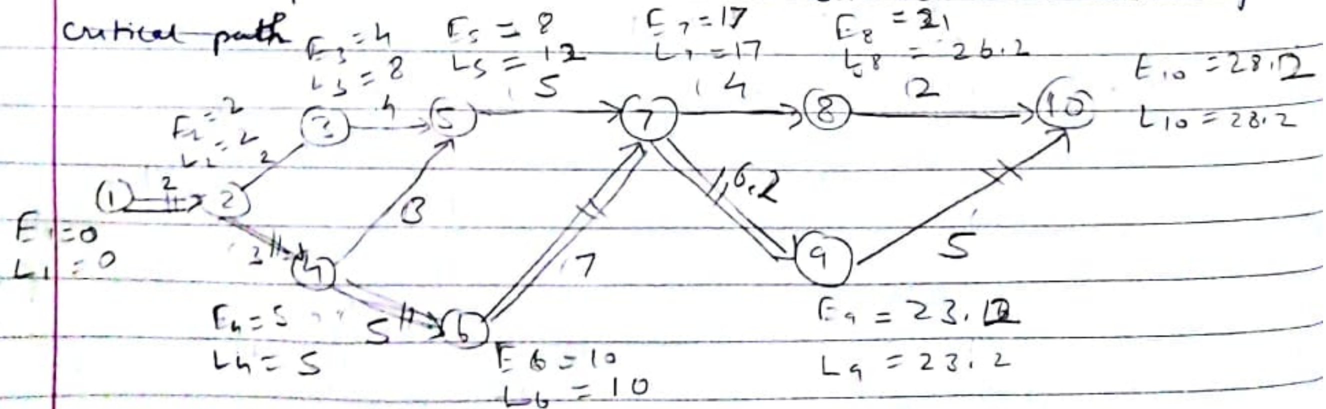
Activity	Most optimise time t_o	Most Pessimistic time t_p	Most likely time t_m	te
1 − 2	1	5	1.5	12 = 2
2 − 3	1	3	2	12 = 2
2 − 4	1	5	3	18 = 3
3 − 5	3	5	4	24 = 4
4 − 5	2	4	3	18 = 3
4 − 6	3	7	5	30 = 5
5 − 7	4	6	5	30 = 5
6 − 7	6	8	7	42 = 7
7 − 8	2	6	4	24 = 4
7 − 9	5	8	6	~~6+16~~ 37 6.2
8 − 10	1	3	2	12 2
9 − 10	3	7	5	30 5

A project has the following characteristics. Construct a pert network, Find critical path and variance for each activity. Find the project duration at 95% probability. Find the ~~expected time~~ and variance at each event also along the critical path



critical path $1-2-4-6-7-9-10 = 28.2$

σ^2

$\left(\frac{t_p - t_o}{6}\right)^2$

Variance at each event

σ^2	Nodes	Variance
	1	0
4/9	2	4/9
1/4	3	5/9
4/9	4	8/9
1/9	5	1
1/9	6	12/9
4/9	7	13/9
1/9	8	17/9
1/9	9	61/36
4/9	10	77/36
1/4		
1/9		
4/9		

$\frac{13}{9} + \frac{1}{4}$

$\frac{52+9}{36}$

61

$\frac{61}{36} + \frac{4}{9}$

$\frac{61+16}{36}$

The expected duration of the project is 28.2 days and the variance of this path is $\frac{77}{36}$ and

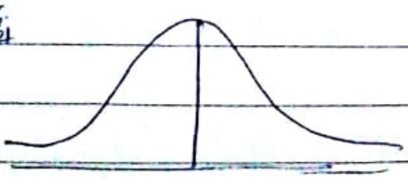
61
16
77

Standard deviation $\sigma = \sqrt{\frac{77}{6}} = 1.46$

$\sigma = 1.46$

$\mu = 28.2$ days.

Ex



From the cummulative distribution table

$F(z) = 0.95 \quad = (1 - 0.95) + 1.6$

$z = 1.65$

$z = \frac{x - \mu}{\sigma}$

$x = z\sigma + \mu = (1.65)(1.46) + 28.2$

Q2 Find the probability of completing the project in 32 d.

$x = 32$ day

$x = 32$

$$z = \frac{32 - 28.2}{1.46} = 2.602$$

$$F(z) = F(2.602) = 0.99598$$

$$= 99.5\%$$

$x = 25$ day

$$z = \frac{25 - 28.2}{1.46} = \cancel{x} \quad \cancel{\text{not posst}} \; -2.19$$

$$F(-2.19) = (0.985) \; (1 - 0.995$$

$$= \cancel{0.485} \quad 0.015$$

F(

$$= 1.5\%$$