Lab Report 2

Aim

Using Discrete Fourier Transform (DFT) to analyse images and operate various filters on them.

Theory

- The two-dimensional discrete Fourier transform (DFT) of an image f(x,y) of size M x N is represented by:

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

- The corresponding inverse of the above discrete Fourier transform is given by the following equation

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

- The magnitude and phase spectrum of an image f (x, y) is represented by

$$F(u,v) = |F(u,v)|\, e^{j\, arg\, \{F(u,v)\}}$$

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

- where R(u, v) and I(u, v) are the real and imaginary components of the spectrum F(u, v). Similarly, the power spectrum is represented by

$$\begin{aligned} P(u,v) &= |F(u,v)|^2 \\ &= R^2(u,v) + I^2(u,v) \end{aligned}$$

Translation Property:

$$f(x,y)(-1)^{x+y} \xleftrightarrow{\text{FT}} F(u-N/2, v-N/2)$$

Rotation Property:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad u = \omega\cos\varphi, \quad v = \omega\sin\varphi$$

$$f(r, \theta+\theta_0) \xleftrightarrow{\text{FT}} F(\omega, \varphi+\varphi_0)$$

- We use *fft2()* and *fftshift()* for DFT and it’s translation property respectively

CODE

```
% Spectrum of an image
% Create an image with a white rectangle and black background.
clear; close all; clc;

% Generate an image
im = zeros(30,30);
%%
im(5:24,13:17)=1;
%%
figure();
imshow(im); title('Original Image'); axis on
%%
% display('Spectrum of the image');
% display('Press any Key');
% pause

% Find the Spectrum using FFT
imF = fft2(im);
%%
imF_mag = abs(imF);
figure(); imshow(imF_mag,[]);title('Magnitude Spectrum'); axis on
%%
% display('Spectrum of the image with fftshift');
% display('Press any Key');
%
% pause

% The zero-frequency coefficient is displayed in the upper left hand
corner.
% To display it in the center, you can use the function fftshift.
imF_mag = fftshift(imF);

imF_mag = abs(imF_mag);
figure(); imshow(imF_mag,[]);title('Magnitude Spectrum with
fftshift'); axis on
%%
```

```
% display('Spectrum of the image with zero padding');
% display('Press any Key');
% pause

% To create a finer sampling of the Fourier transform,
% you can add zero padding to im when computing its DFT.

imF=fft2(im, 256,256);
imF_mag = abs(fftshift(imF));
figure(); imshow(imF_mag,[]); title('Magnitude Spectrum with Zero padding'); axis on
%%
% display('Spectrum of the image with log magnitude');
% display('Press any Key');
% pause

% To brighten the display, you can use a log function
imF_log_mag=log(1+imF_mag);
figure,imshow(imF_log_mag,[]);title('Log Magnitude  Spectrum'); axis on

disp('End of the program');
```

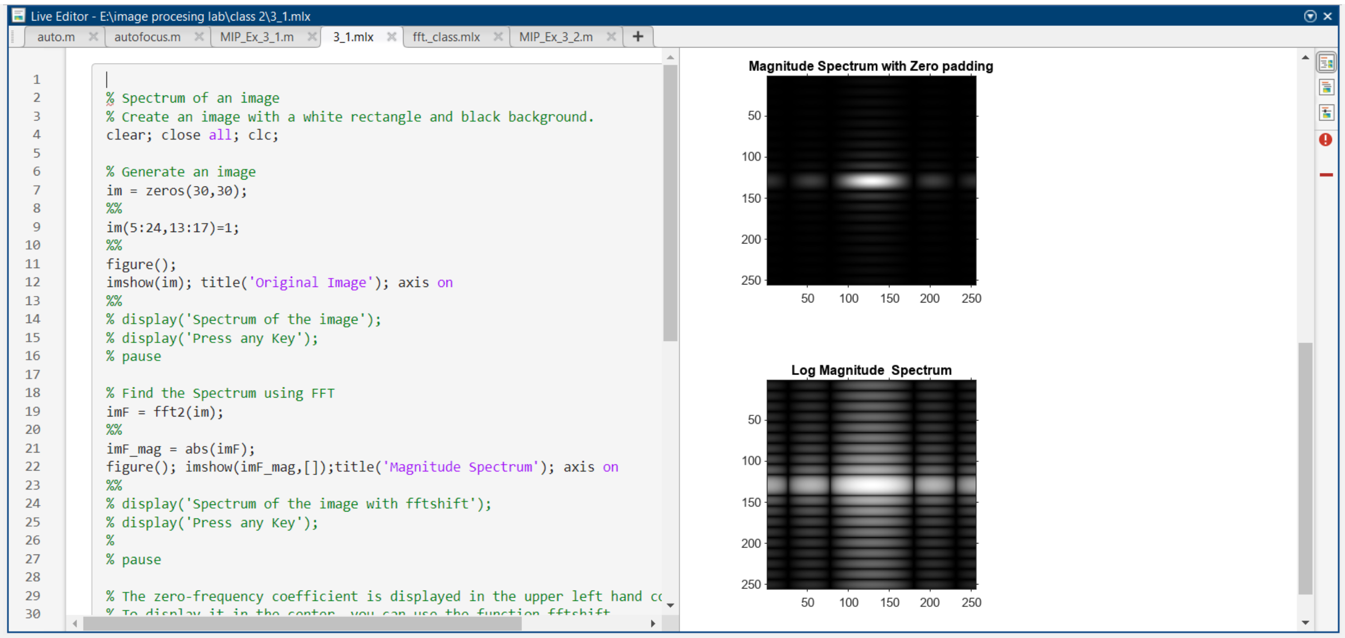


Figure 1: use of fft2( ) and fftshift( ) function in MATLAB

CODE

```
% Example 2: Spectrum and reconstruction of an image with magnitude and
% phase spectrums

clear; close all; clc;
a=zeros(256,256);
a(78:178,78:178)=1;

figure();
subplot(2,2,1); imshow(a);title('Original Image'); axis on;
%%
af=fftshift(fft2(a));
subplot(2,2,2);imshow(abs(af));title('Spectrum of Image');
%%
% Now rotated the image by 45 degrees
[x,y] = meshgrid(1:256,1:256);
b=(x+y<329)&(x+y>182)&(x-y>-67)&(x-y<73);
subplot(2,2,3);imshow(b);title('Rotated Image');axis on;
%%
bf = abs(fftshift(fft2(b)));
subplot(2,2,4);imshow(bf);title('Spectrum of Rotated Image');
```

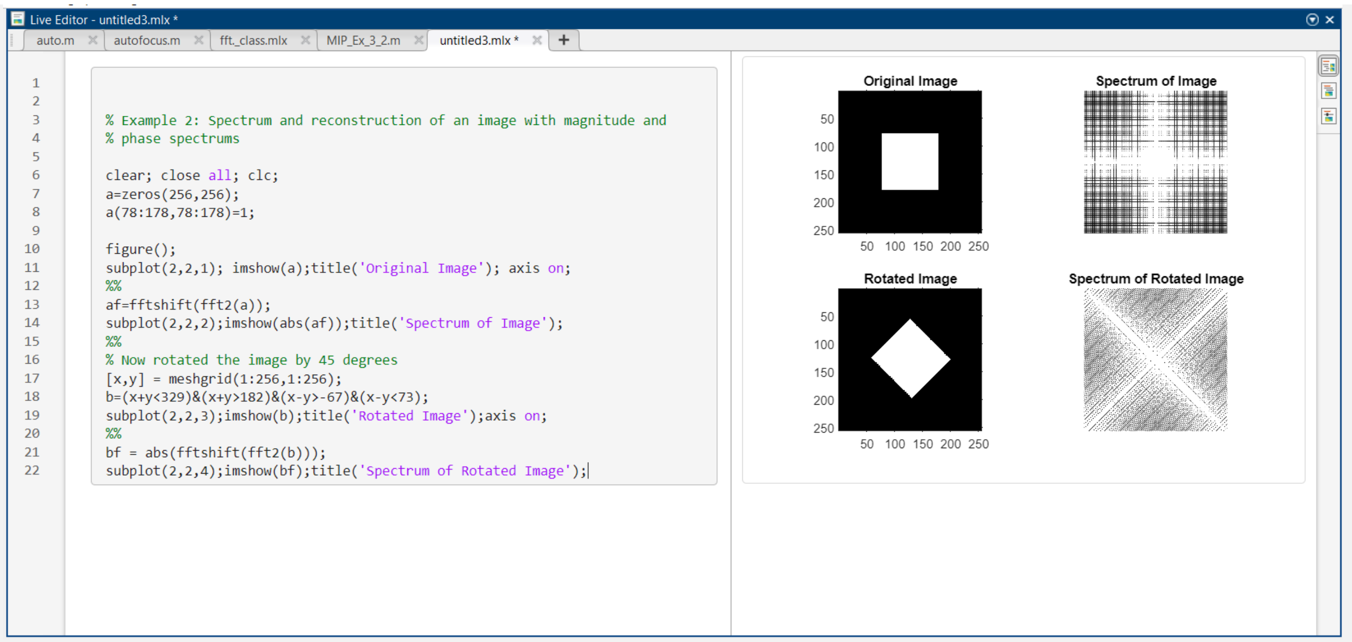


Figure 2: use of fft2( ) and fftshift( ) function in MATLAB

CODE

```
% Example 3     % Explore the FFT of an image

clear;close all;clc;

im = imread('hand-x-ray.jpg');
[m n] = size(im);
%%
% Spectrum calculations
imF = fft2(im);          % 2D FFT
imF_mag = abs(imF);         % Magnitude Spectrum
s = log(1+abs(fftshift(imF)));% Log Magnitude Spectrum
imF_ph=angle(imF);     % Phase Spectrum

figure();
subplot(1,3,1); imshow(im); title('Original Image');
subplot(1,3,2); imshow(s,[]); title('Log Magnitude Spectrum');
subplot(1,3,3); imshow(imF_ph); title('Phase Spectrum Image');
%%
% Reconstruction

% Reconstruction by combining both magnitude and phase spectrum
imr = ifft2(imF_mag.*exp(1i*imF_ph))/(m*n);
%%
% Reconstruction by only magnitude spectrum
imr_mag = abs(ifftshift(ifft2(imF_mag)));
% imr_mag = abs((ifft2(imF_mag)));

% Reconstruction by only phase spectrum
imr_ph = ifft2(exp(1i*imF_ph))/(m*n);



figure();
subplot(1,3,1); imshow(imr,[]);   title('Recon. Magn and Phase');
subplot(1,3,2); imshow(uint8(imr_mag),[]);  title('Recon.with Mag
Spectrum only');
subplot(1,3,3); imshow(imr_ph,[]);title('Reconstruction with Phase
Spectrum only');
```

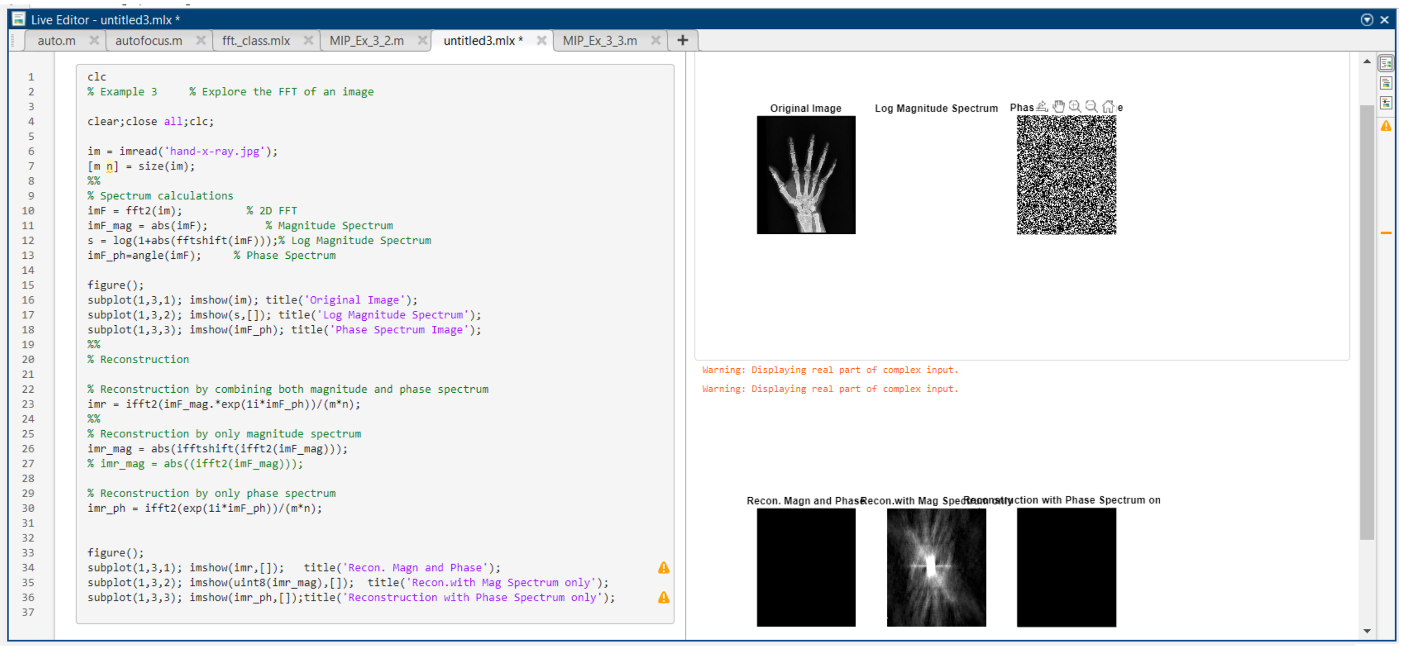
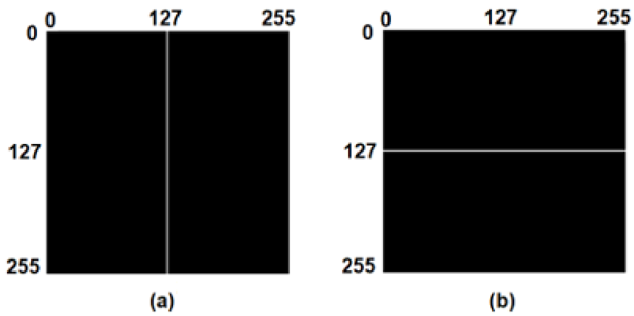


Figure 3: use of fft2( ) and fftshift( ) function in MATLAB

Exercise

Exercise1: (a) Write a Matlab code to generate the following images. Assume that the width of the white pixel for Fig(a) and height of the white pixel Fig(b) are unity.



(b) Find and display the magnitude and phase spectrums.

(c) Suppose the vertical line in Fig(a) and horizontal line in Fig(b) are rotated by (i) ±30°, (ii) ±45° and (iii) ±90°. Find and display the magnitude and phase spectrums. Comment on the results.

CODE

```
% x=zeros(255,255);
y=zeros(255,255);


    y(127:128,1:255)=1
    x(1:255,127:128)=1;



subplot(3,2,1);imshow(x);title('horizontal white line of height 1px');
```

```
subplot(3,2,2);imshow(y);title('vertical white line of width 1px');


fft_x=fft2(x);

fft_shift_x=fftshift(fft_x);

abs_fft_shift_x=abs(fft_shift_x);

abs_fft_x=abs(fft_x);

fft_y=fft2(y);

fft_shift_y=fftshift(fft_y);

abs_fft_shift_y=abs(fft_shift_y);

abs_fft_y=abs(fft_y);

subplot(3,2,3);imshow(abs_fft_x);title('horizontal mag spectrum without shift');

subplot(3,2,4);imshow(abs_fft_y);title('vertical mag spectrum without shift');

subplot(3,2,5);imshow(abs_fft_shift_x);title('horizontal  mag spectrum with shift');

subplot(3,2,6);imshow(abs_fft_shift_y);title('vertical  mag spectrum with shift');
```

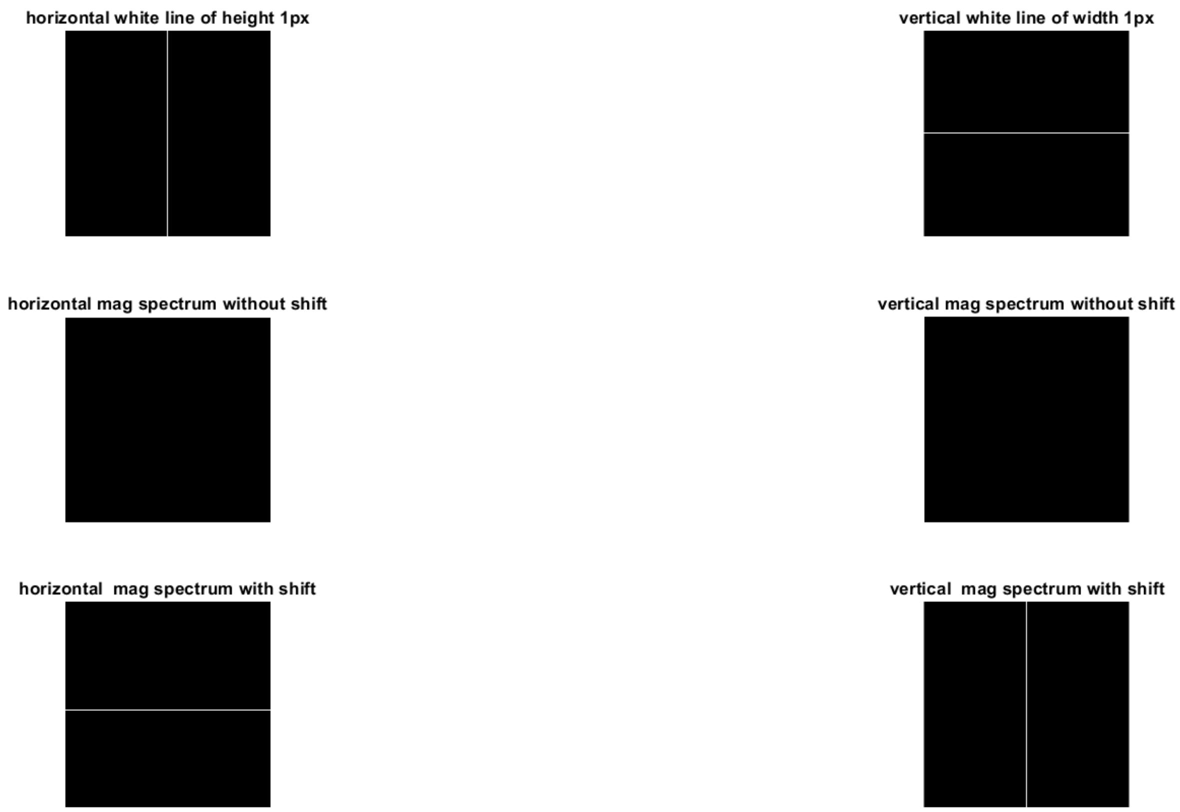


Figure 4: use of fft2( ) and fftshift( ) function in MATLAB

CODE

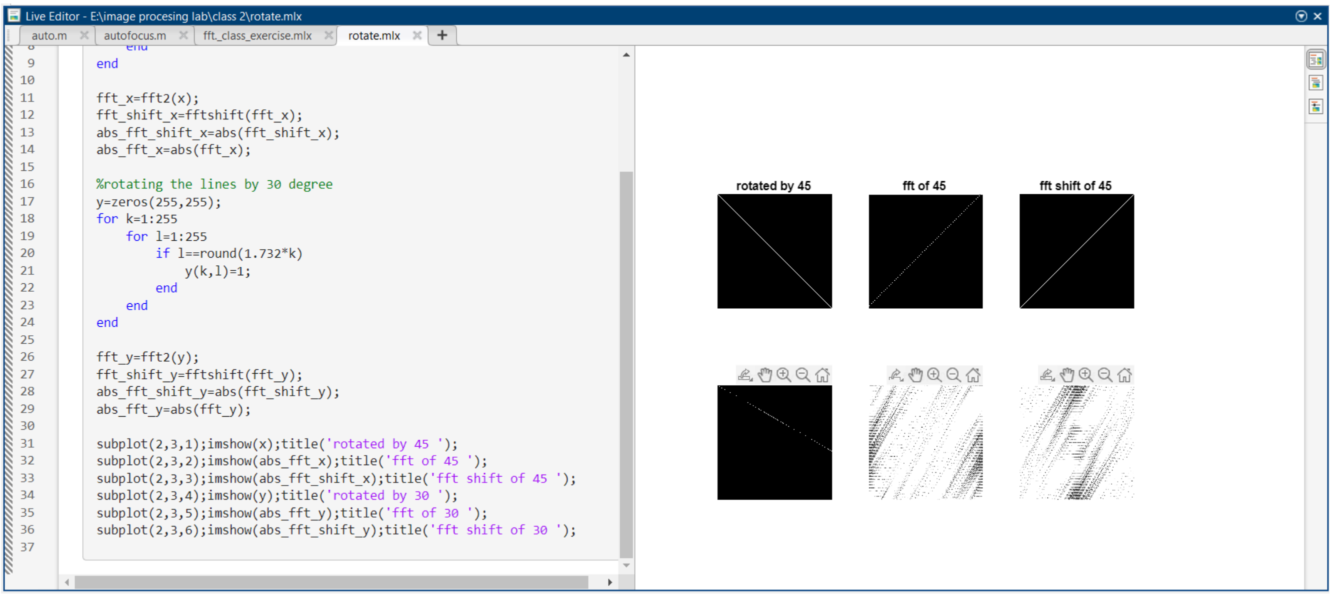
```
% %rotating the lines by 45 degree
x=zeros(255,255);
for i=1:255
    for j=1:255
        if j==i
            x(i,j)=1;
        end
    end
end

fft_x=fft2(x);
fft_shift_x=fftshift(fft_x);
abs_fft_shift_x=abs(fft_shift_x);
abs_fft_x=abs(fft_x);

%rotating the lines by 30 degree
y=zeros(255,255);
for k=1:255
    for l=1:255
        if l==round(1.732*k)
            y(k,l)=1;
        end
    end
end

fft_y=fft2(y);
fft_shift_y=fftshift(fft_y);
abs_fft_shift_y=abs(fft_shift_y);
abs_fft_y=abs(fft_y);

subplot(2,3,1);imshow(x);title('rotated by 45 ');
subplot(2,3,2);imshow(abs_fft_x);title('fft of 45 ');
subplot(2,3,3);imshow(abs_fft_shift_x);title('fft shift of 45 ');
subplot(2,3,4);imshow(y);title('rotated by 30 ');
subplot(2,3,5);imshow(abs_fft_y);title('fft of 30 ');
subplot(2,3,6);imshow(abs_fft_shift_y);title('fft shift of 30 ');
```



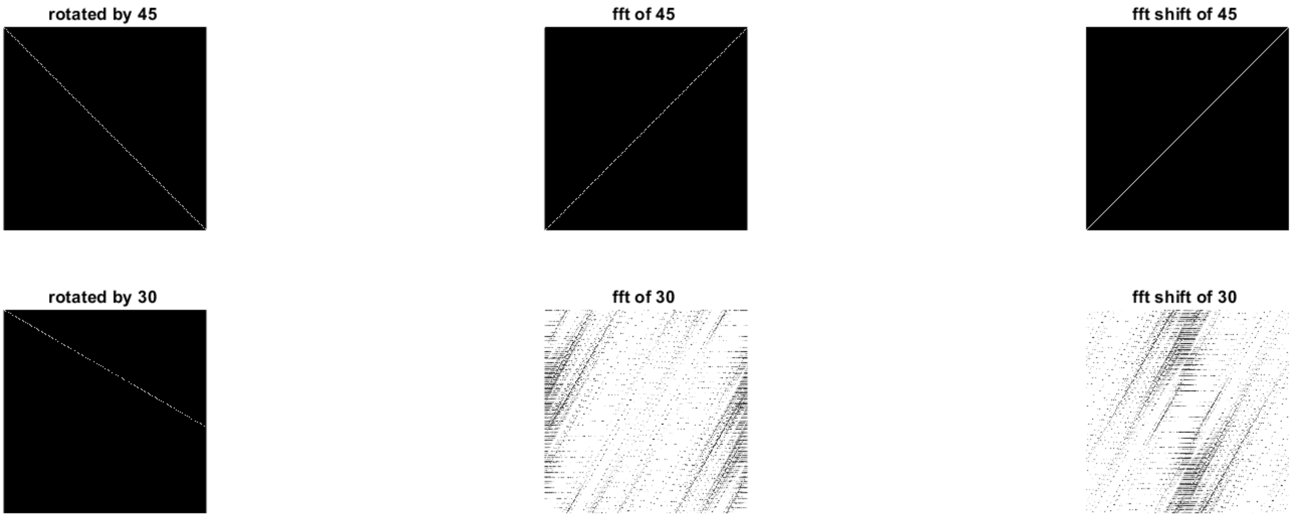
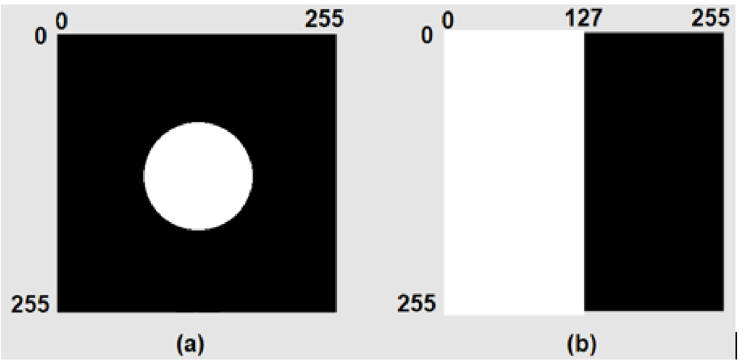


Figure 5: use of fft2( ) and fftshift( ) function in MATLAB

Exercise2: (a) Write a Matlab code to generate the following images. Assume that the radius of circle is 32 for Fig(a).

(b) Find and display the magnitude and phase spectrums.



CODE

```
x x=zeros(255,255);
for i=1:255
```

```
    for j=1:127
        x(i,j)=1;
    end
end
fft_x=fft2(x);
fft_shift_x=fftshift(fft_x);
abs_fft_shift_x=abs(fft_shift_x);
abs_fft_x=abs(fft_x);

r=32;x_c=0;y_c=0;
[y,x]=ndgrid(-127:128,-127:128);
y= (x-x_c).^2+(y-y_c).^2 <= r^2;

fft_y=fft2(y);
fft_shift_y=fftshift(fft_y);
abs_fft_shift_y=abs(fft_shift_y);
abs_fft_y=abs(fft_y);
subplot(3,2,1);imshow(x);title('black and white rectangle');
subplot(3,2,2);imshow(y);title('circle of radius 32');
subplot(3,2,3);imshow(abs_fft_x);title('rectangle mag spectrum without shift');
subplot(3,2,4);imshow(abs_fft_y);title('circle mag spectrum without shift');
subplot(3,2,5);imshow(abs_fft_shift_x);title('rectangle  mag spectrum with shift');
subplot(3,2,6);imshow(abs_fft_shift_y);title('circle  mag spectrum with shift');
```

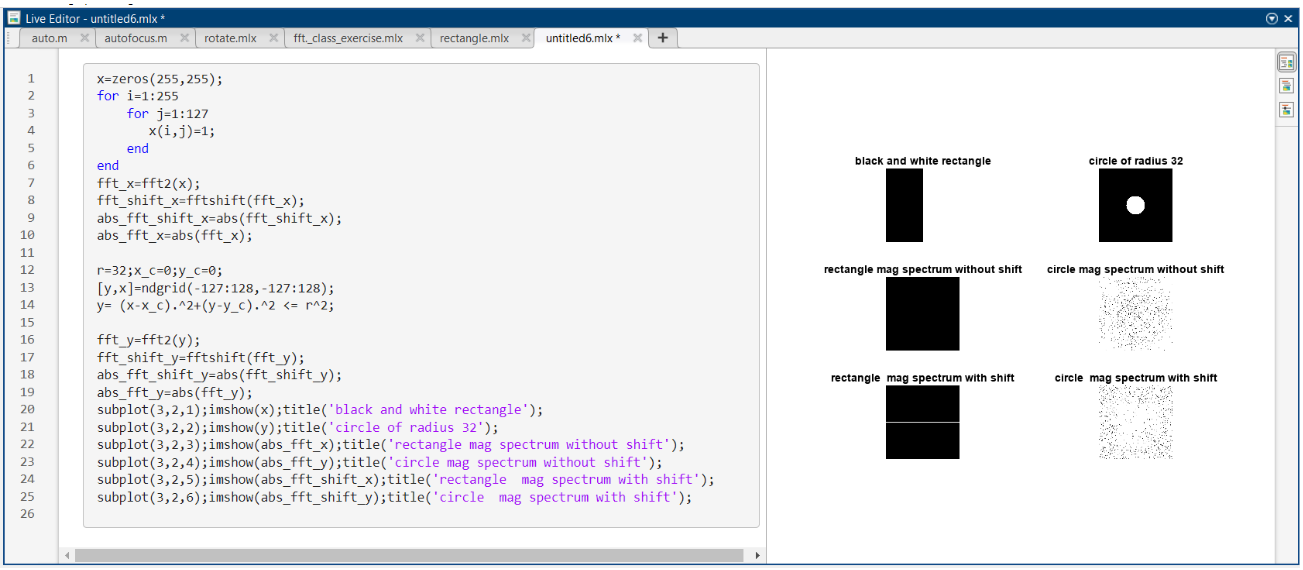


Figure 6: use of fft2( ) and fftshift( ) function in MATLAB

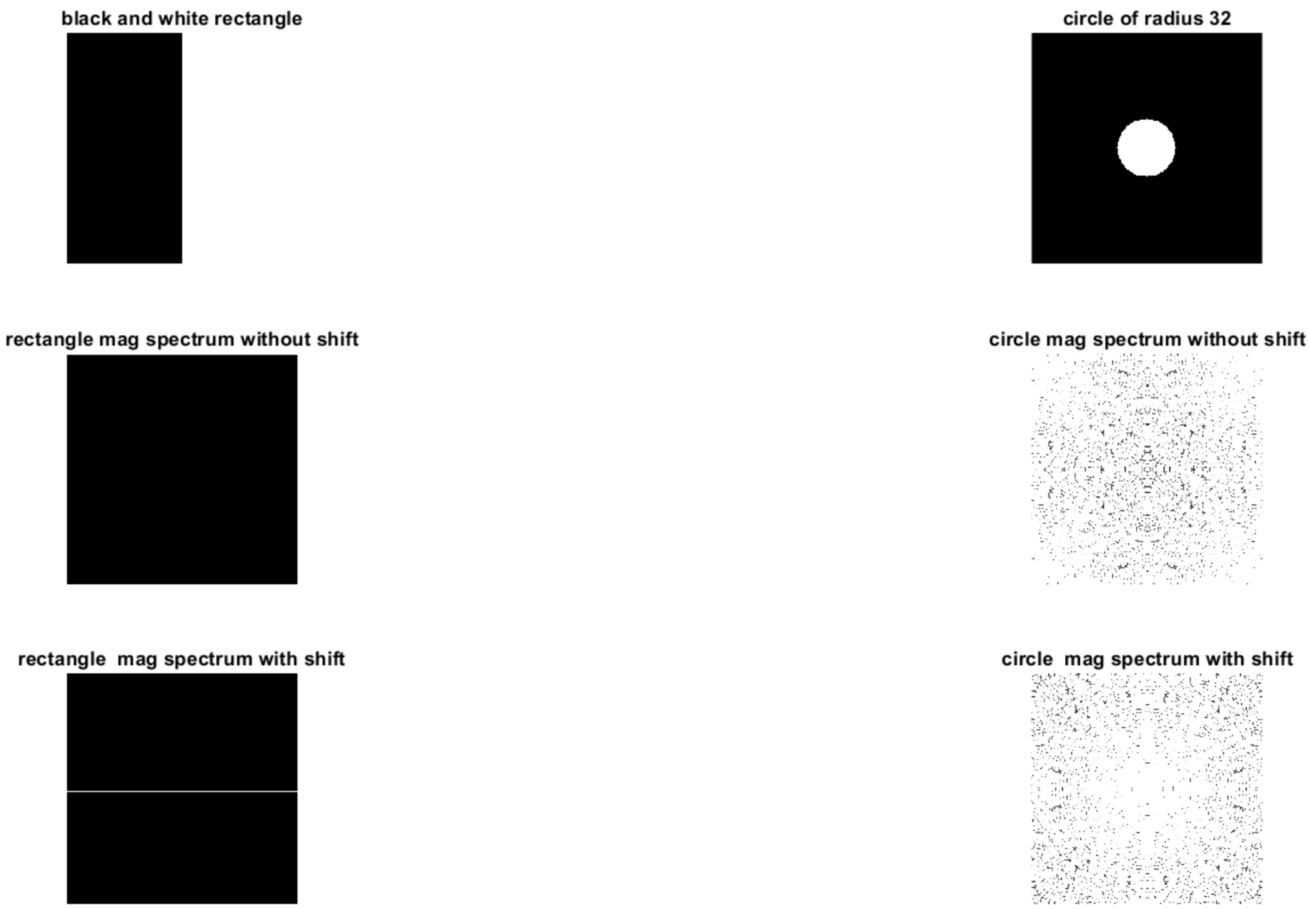
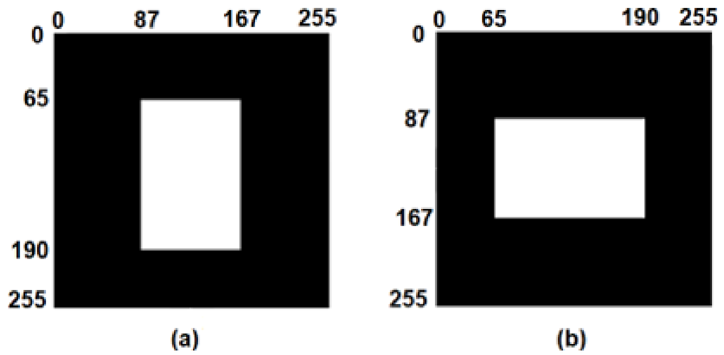


Figure 7: use of fft2( ) and fftshift( ) function in MATLAB

Exercise5: (a) Write a Matlab code to generate the following images.



(b) Find and display the magnitude and phase spectrums.

(c) Suppose the white rectangular images are rotated by (i) $\pm 45^{o}$ and (ii) $\pm 120^{o}$. Find and display the magnitude and phase spectrums. Comment on the results.

CODE

```
x=zeros(255,255);
y=zeros(255,255);


    y(65:190,87:167)=1
    x(87:167,65:190)=1


subplot(3,2,1);imshow(x);title('horizontal white line of height 1px');
subplot(3,2,2);imshow(y);title('vertical white line of width 1px');


fft_x=fft2(x);
fft_shift_x=fftshift(fft_x);
abs_fft_shift_x=abs(fft_shift_x);
abs_fft_x=abs(fft_x);
fft_y=fft2(y);
fft_shift_y=fftshift(fft_y);
abs_fft_shift_y=abs(fft_shift_y);
abs_fft_y=abs(fft_y);


%ploting all the images and there fft
subplot(3,2,3);imshow(abs_fft_x);title('horizontal mag spectrum without shift');
subplot(3,2,4);imshow(abs_fft_y);title('vertical mag spectrum without shift');
subplot(3,2,5);imshow(abs_fft_shift_x);title('horizontal  mag spectrum with shift');
subplot(3,2,6);imshow(abs_fft_shift_y);title('vertical  mag spectrum with shift');end
```

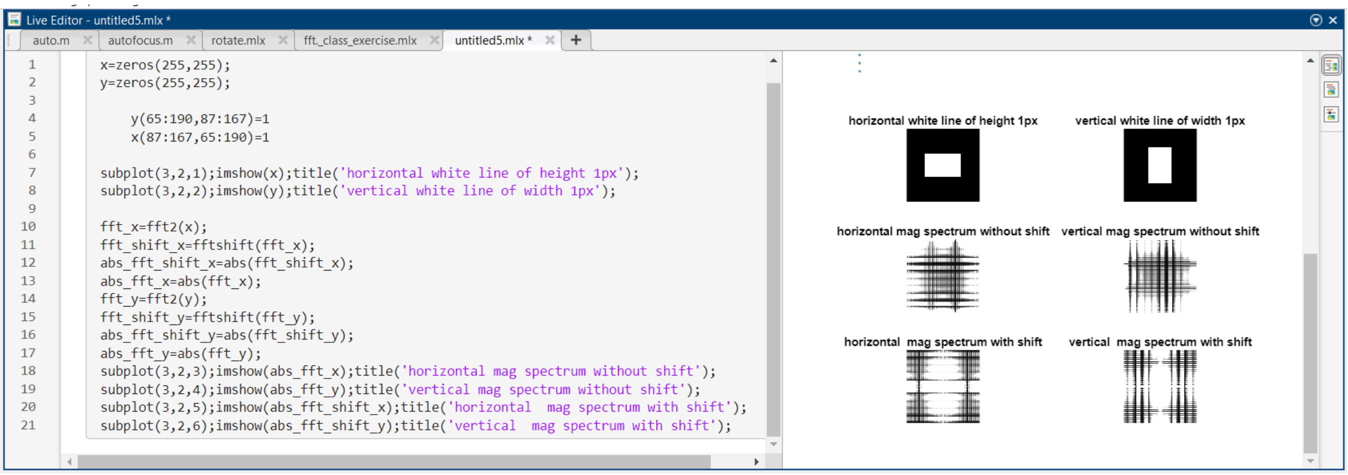


Figure 8: use of fft2( ) and fftshift( ) function in MATLAB

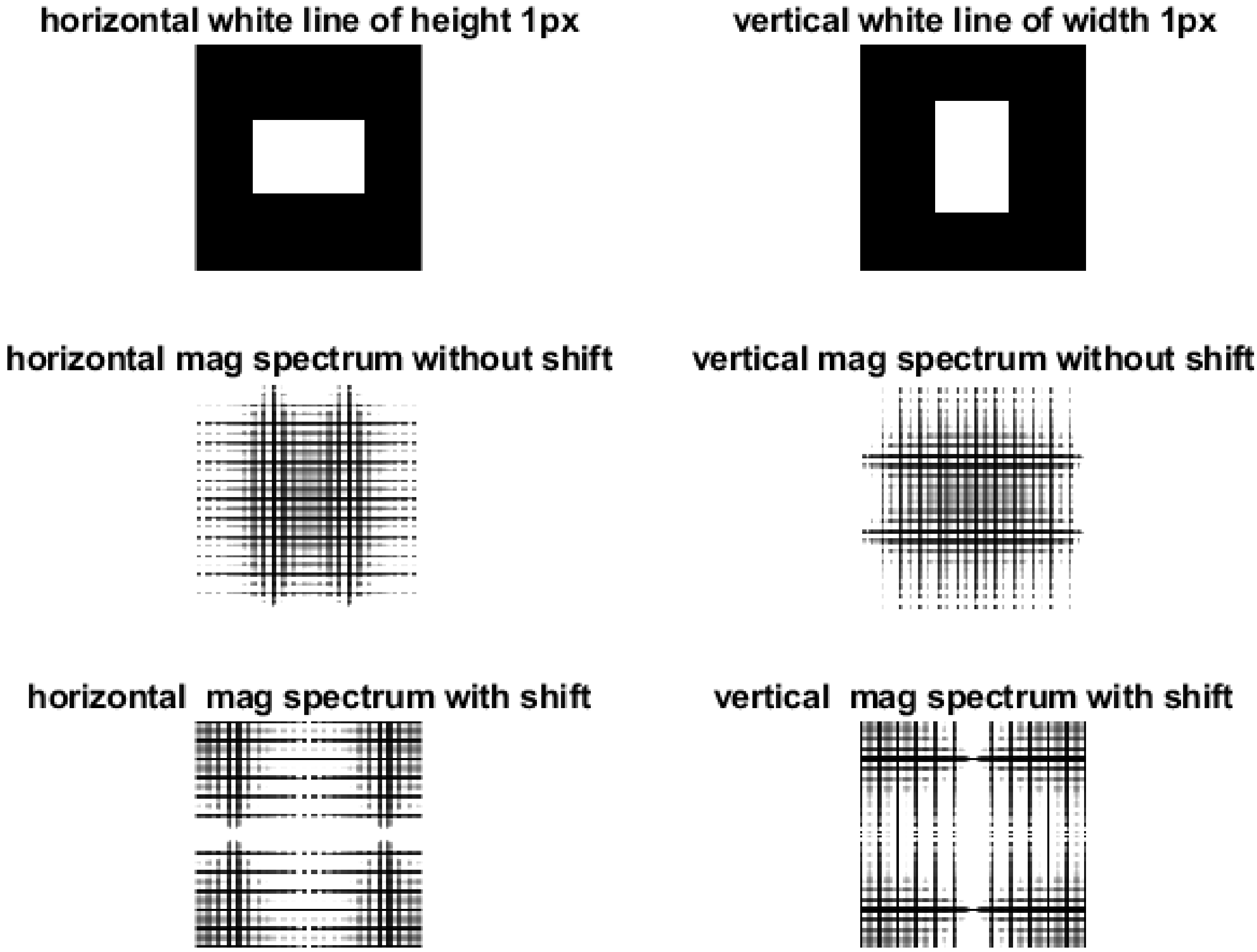


Figure 9: use of fft2( ) and fftshift( ) function in MATLAB

Shreenandan Sahu |120BM0806