

Image Processing: Image Enhancement

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Introduction

- image processing
 - $f(x,y) \rightarrow f'(x,y)$
- image analysis
 - $f(x,y) \rightarrow$ image features
- image understanding
 - $f(x,y) \rightarrow$ high-level image descriptors

Relationships Between Pixels

- A single pixel considered in isolation conveys information on the intensity at a single location in an image
- Perform calculations over regions of an image where the new value of a pixel must be computed from its old value and the values of pixels in its vicinity → neighborhood

Neighbours of a Pixel

- A pixel p at coordinates (x,y) has four horizontal and vertical neighbors
 - The coordinates are given by:
 $(x+1,y)$, $(x-1,y)$, $(x,y+1)$, $(x,y-1)$
 - This set of pixels are called the *4-neighbours*
 - Some of the neighbours of p lie outside the image if (x,y) is on the border of the image.
- The four diagonal neighbours of p have coordinates:
 - $(x+1,y+1)$, $(x+1,y-1)$, $(x-1,y+1)$, $(x-1,y-1)$
 - Together with the 4-neighbours, they are called the *8-neighbours* of p .

Distance Measures

- For pixels p and q with coordinates (x_1, y_1) and (x_2, y_2) respectively:
 - The **Euclidean** distance between p and q is defined as:

$$D_e(p, q) = \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 \right]^{\frac{1}{2}}$$

- Pixels having a distance less than or equal to some value r from (x_1, y_1) are the points contained in a disk of radius r centred at (x_1, y_1) .

Distance Measures

- The \mathbf{D}_4 distance (**city-block**) between p and q is defined as:

$$D_4(p, q) = |x_1 - x_2| + |y_1 - y_2|$$

- The pixels having \mathbf{D}_4 distance from (x_1, y_1) less than or equal to some value r form a diamond centred at (x_1, y_1) .
- The \mathbf{D}_8 distance (**chessboard**) between p and q is defined as:

$$D_8(p, q) = \max(|x_1 - x_2|, |y_1 - y_2|)$$

Distance Measures

D₄

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

D₈

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

On a per pixel basis

- “adding two images together”
 - This means that the addition is carried out between *corresponding* pixels in the two images.

$$g = f_1 + f_2$$

$$g(x, y) = f_1(x, y) + f_2(x, y)$$

Image Enhancement

- The goal of **image enhancement** is to improve the visual appearance of an image.
 - Visual evaluation of image quality is a highly subjective process
- Approaches to image enhancement fall into two broad categories:
 - *Spatial domain*: direct modification of the pixels in an image.
 - *Frequency domain*: modification of the Fourier transform of an image.

Spatial Domain

- Characterized by *point-by-point* and *neighborhood* transformations

Image Noise

- The process of image acquisition frequently leads (inadvertently) to image degradation.
- **Noise** is an unexplained variation in intensity values:
 - Noise manifests itself as an unevenness in background and foreground regions
 - Imparts a bumpy or jagged appearance to otherwise smooth regions of intensity

Image Noise

- Spatial noise descriptors:
 - Gaussian noise
 - Rayleigh noise
 - Gamma noise
 - Exponential noise
 - Uniform noise
 - Impulse (salt-and-pepper, shot, spike) noise

Image Noise

Image Noise

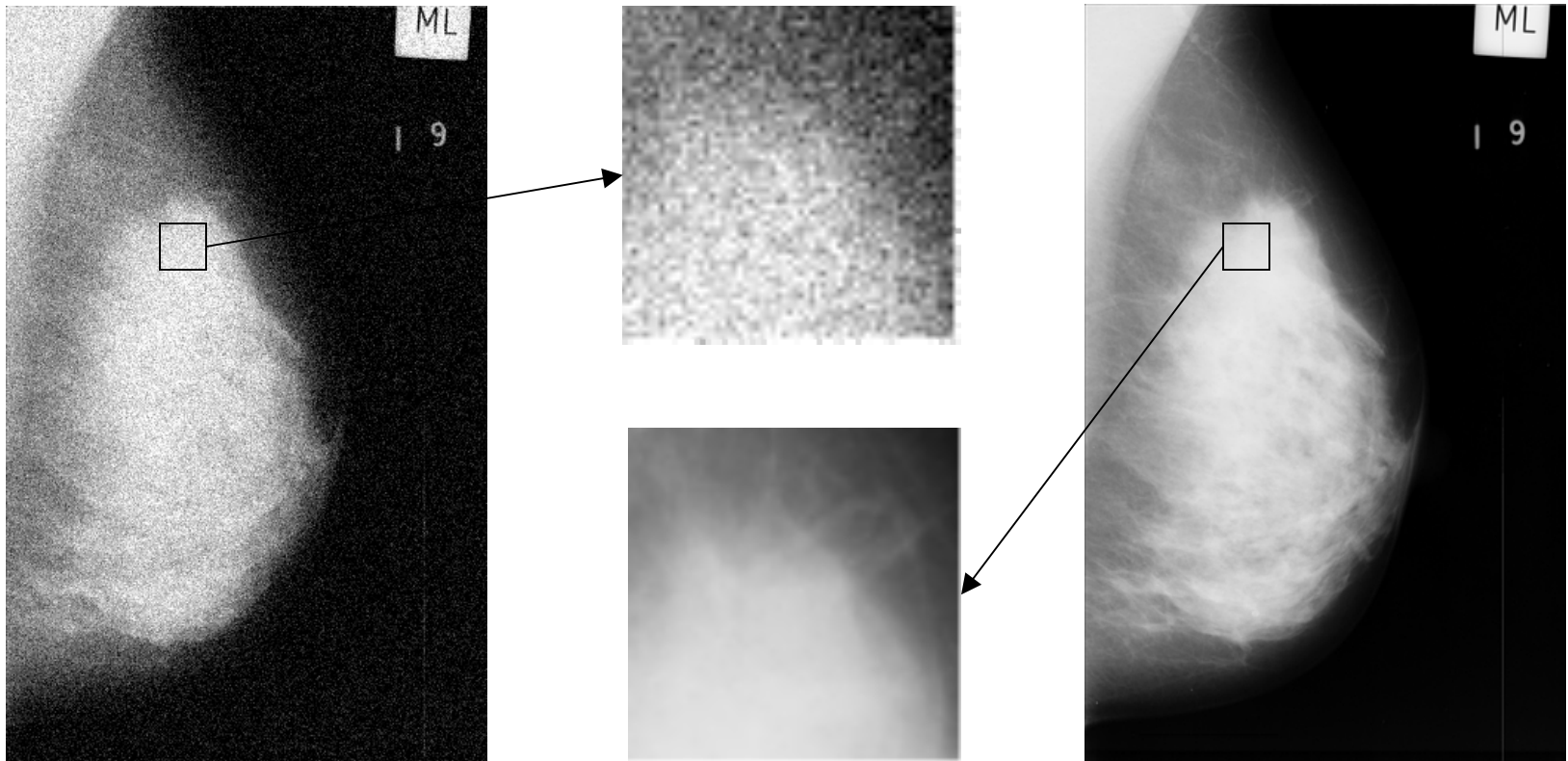


Image Contrast

- For an object to be visible on an image, it is not enough to have sufficient resolution but sufficient contrast also.
- Contrast quantifies the visibility of an object on the background:

$$C = \frac{I_o - I_b}{I_o + I_b}$$

where I_o is the intensity (brightness) of the object and I_b is the intensity (brightness) of the background.

Image Contrast

- Several variations of are frequently used also:

$$C = \left| \frac{I_o - I_b}{I_o + I_b} \right| \quad C = \frac{I_b - I_o}{I_o + I_b} \quad C = \frac{I_o - I_b}{I_b} \quad C = \frac{I_b - I_o}{I_o}$$

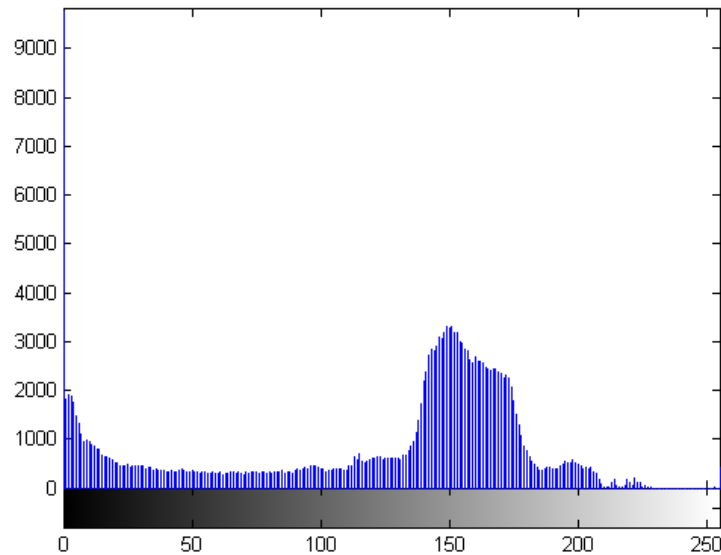
Image Contrast

- Contrast can be clearly illustrated using a histogram
 - **Low contrast** appears as a narrow set of intensity values, leaving other intensity levels minimally or completely unoccupied.
 - **High contrast** appears as a broad range of intensity values, with a near uniform distribution.

$$Contrast = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \leq 1.0$$

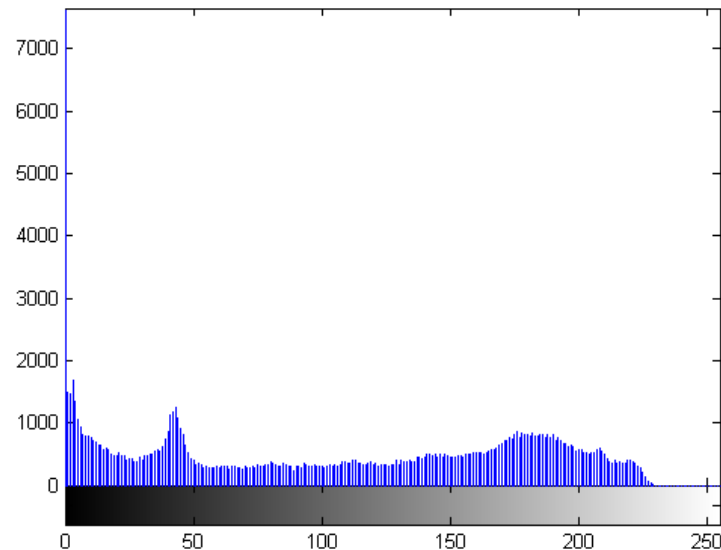
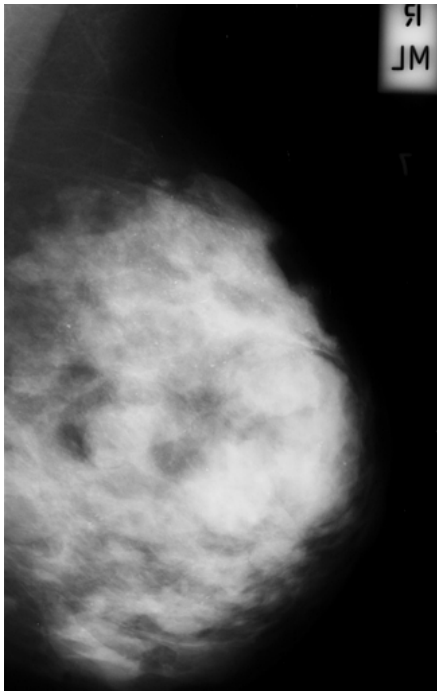
Low-contrast image

- A histogram that is narrowed and centred towards the middle.



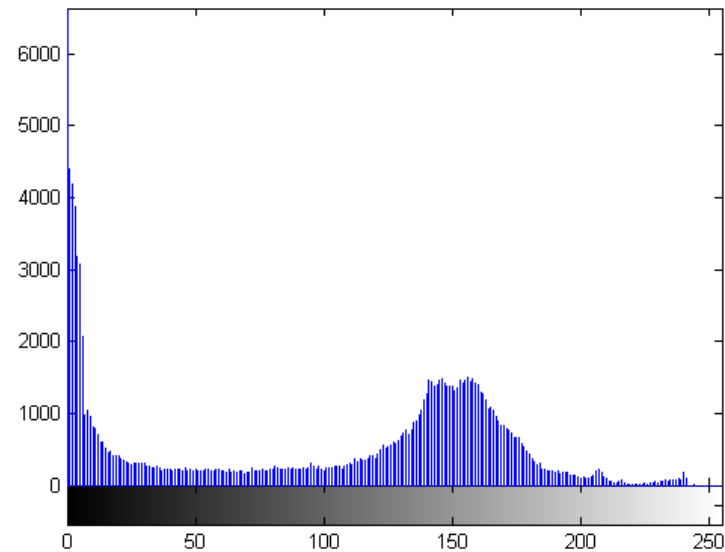
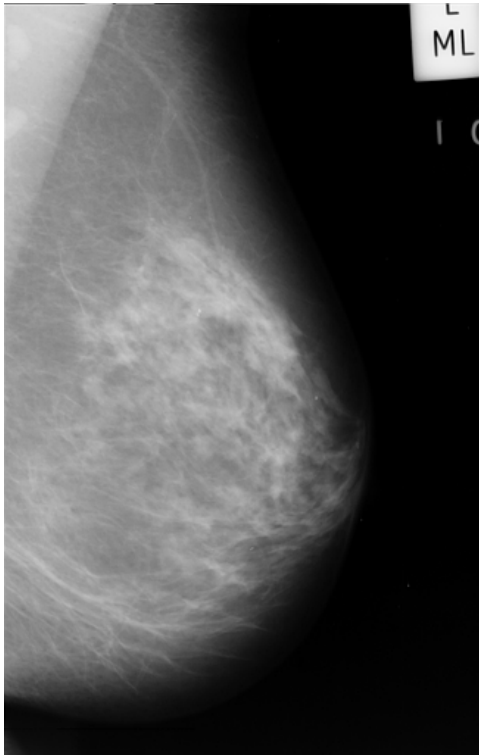
High-contrast image

- A histogram that covers a broad range of the intensity values, and a uniform distribution.



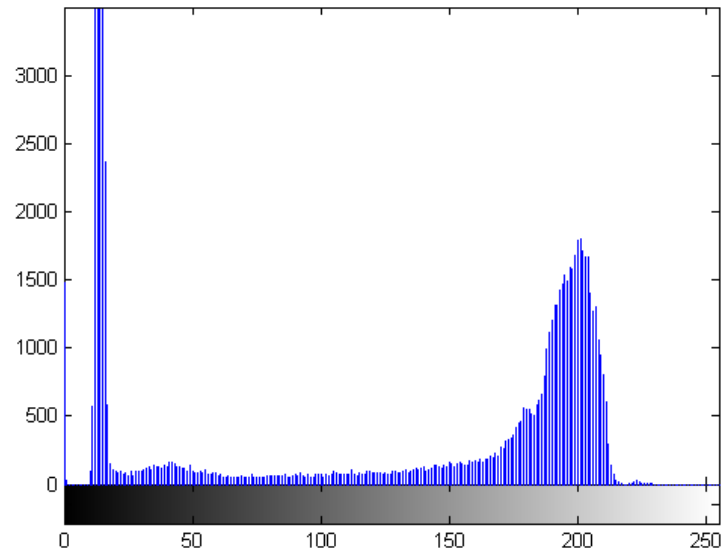
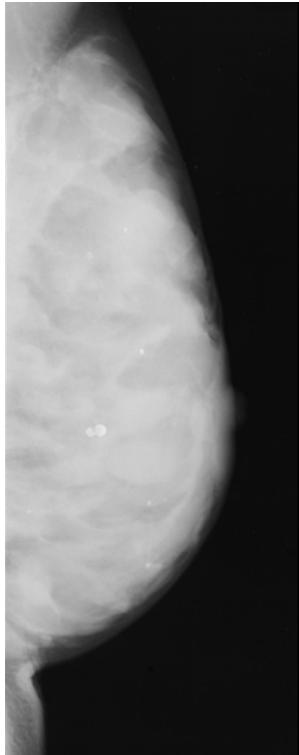
Low-intensity image

- Biased towards low intensities



High-intensity image

- Biased towards high intensities



Intensity Transformations

- Spatial transformations are of the form:

$$g(x, y) = T[f(x, y)]$$

where T is an intensity transformation.

- The simplest form of T is when the neighborhood is of size 1×1
 - This is known as **point**, or **point-by-point** processing

Point Processing

- There are various methods of pixel manipulation through intensity mapping functions
 - histogram
 - sliding, stretching, equalization
 - linear (negative)
 - logarithmic (log, inverse log, exponential)
 - power-law (n^{th} power and n^{th} root, square-root)

Image Negatives

- The negative of an image with grayscale values in the range $[0, L-1]$ is obtained by using the negative transformation:

$$g(x, y) = (L - 1) - f(x, y)$$

Logarithmic

- The general form of the log transformation is:

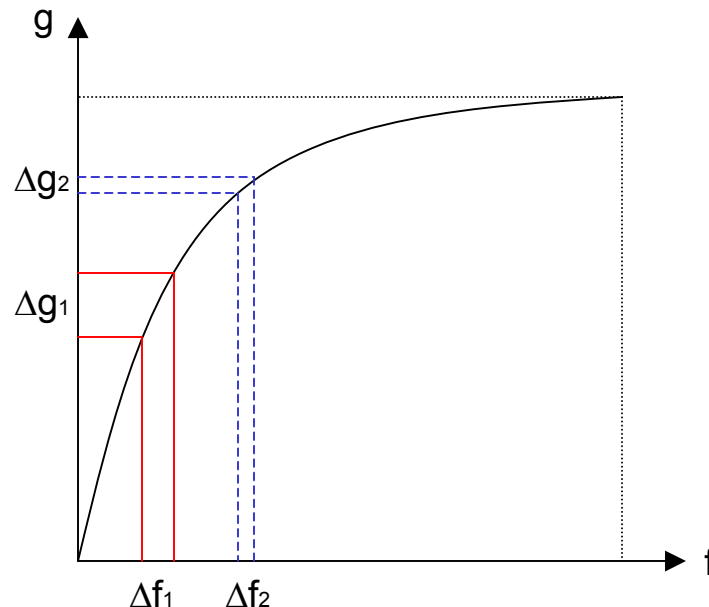
$$g(x,y) = c \log(1 + f(x,y))$$

where c is a constant, and it is assumed that $f(x,y) \geq 0$.

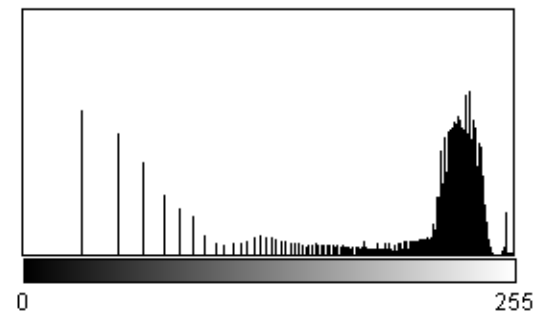
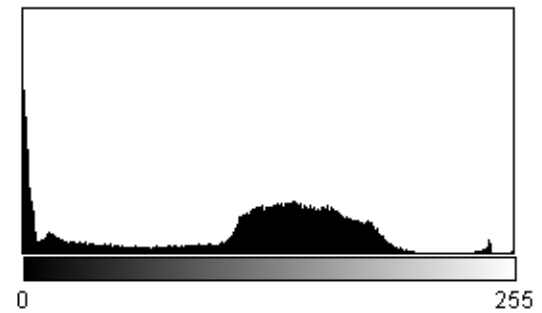
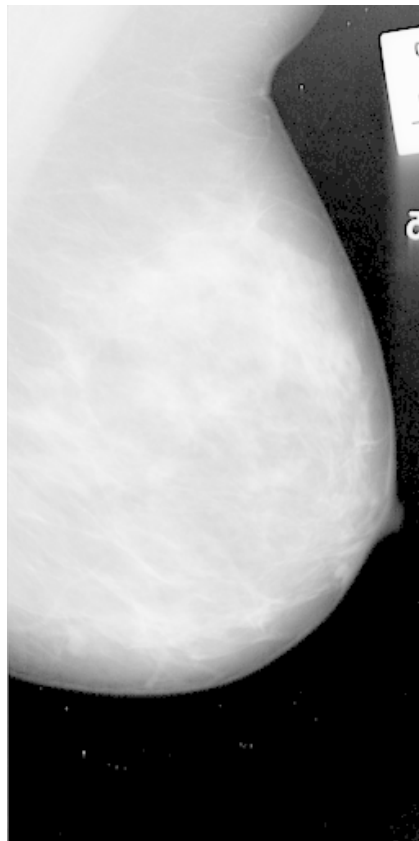
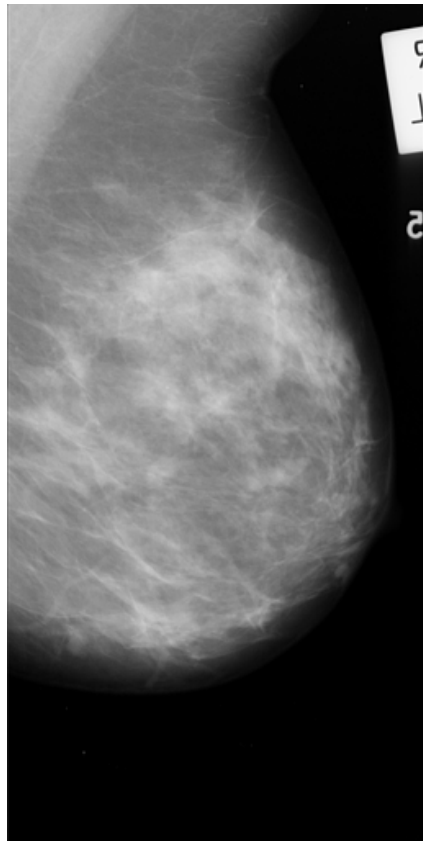
- This transformation maps a narrow range of low grayscale values in the input image into a wider range of output values.

Logarithmic

- It expands the values of low-intensity pixels, compressing higher-level intensities.
 - $\Delta f_1 < \Delta g_1$ contrast is increased
 - $\Delta f_2 > \Delta g_2$, contrast is reduced



Logarithmic

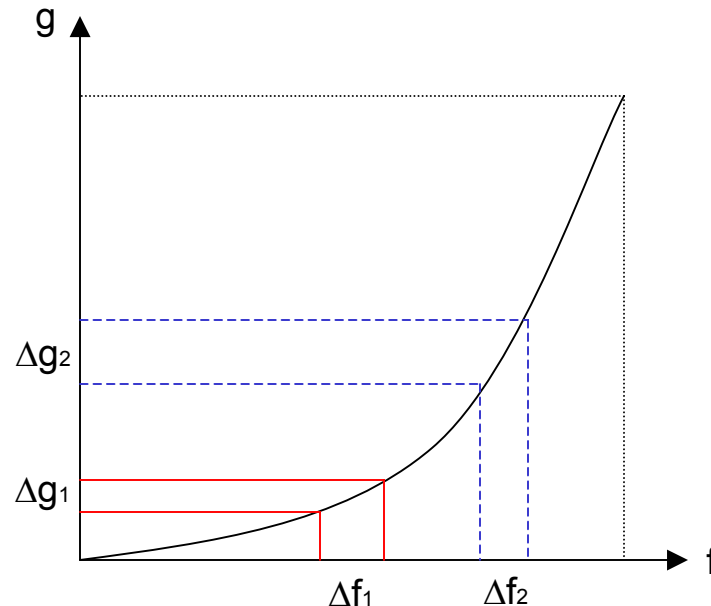


Inverse Logarithmic

- The inverse logarithmic transformation compresses the dynamic range of images with large variations in pixel values.

Exponential

- Uses an exponential function to perform the intensity mapping.
 - Enhances low-intensity regions
 - $\Delta f_1 > \Delta g_1$ contrast is reduced
 - $\Delta f_2 < \Delta g_2$, contrast is increased



Power-law

- Power-law transformations have the following form:

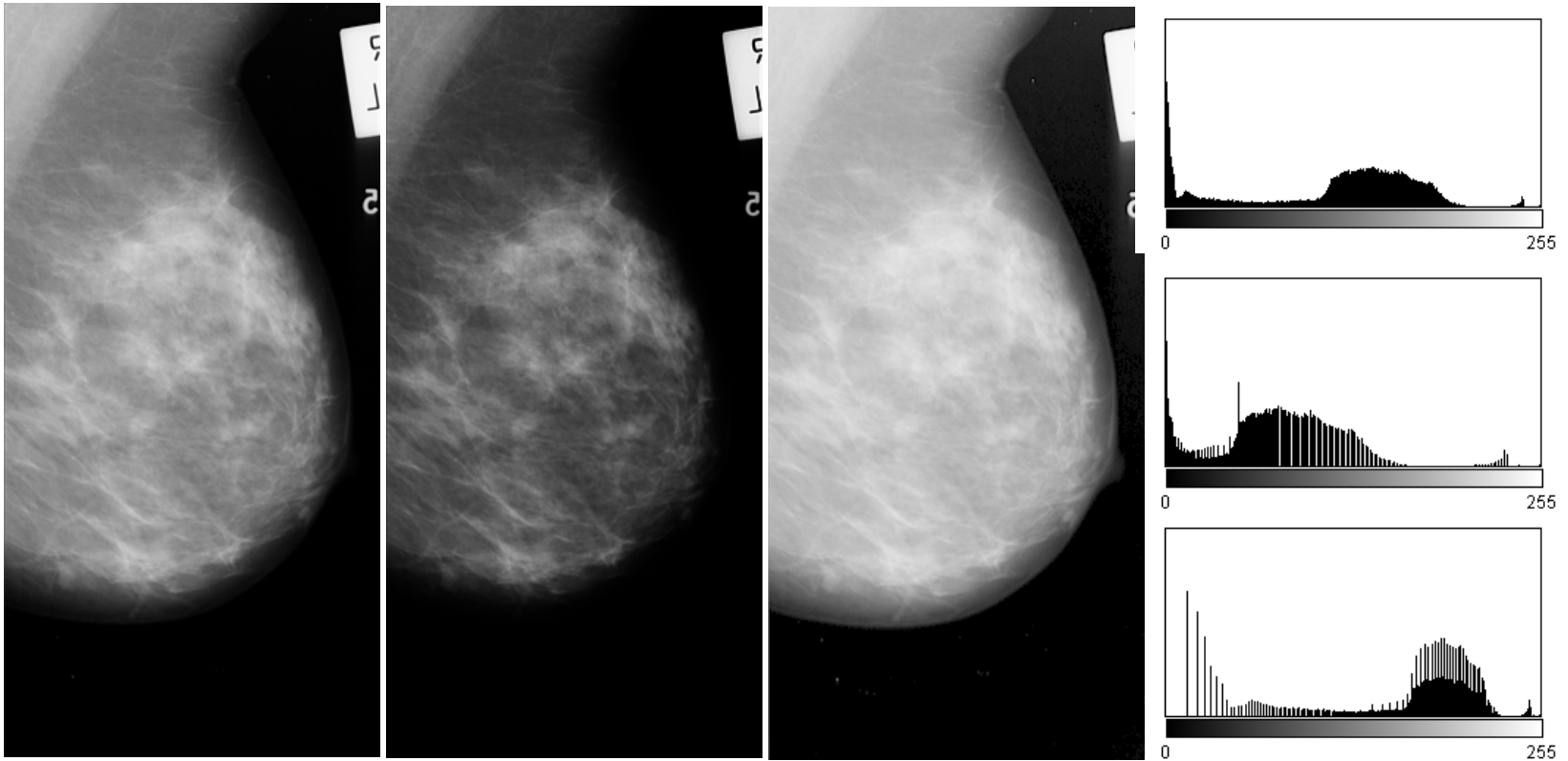
$$g(x, y) = cf(x, y)^\gamma$$

where c and γ are positive constants

e.g. *square-root* $\gamma = 0.5$

Power-law

Squared versus SQRT



Linear vs. Nonlinear Transformations

- A transformation T , is said to be linear if, for any two images f and g and two scalars a and b :

$$T(af + bg) = aT(f) + bT(g)$$

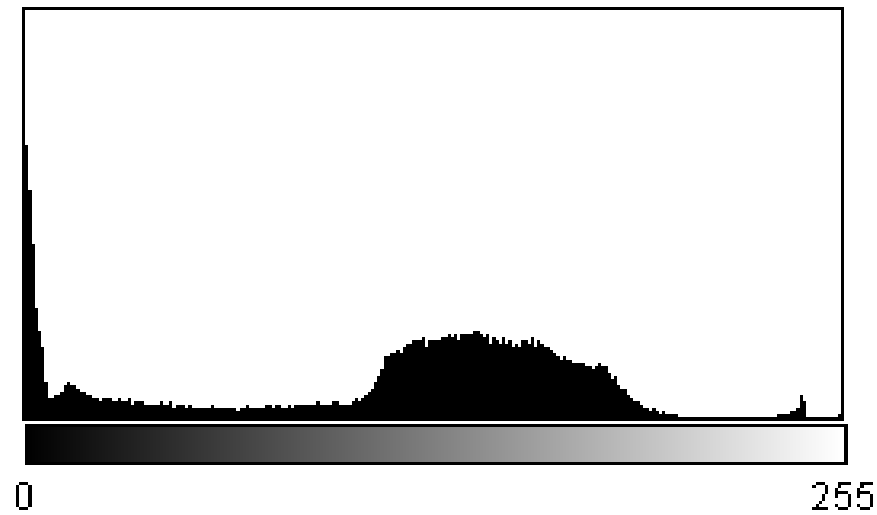
- e.g. An operation to compute the sum of K images is *linear*
- e.g. An operation to compute the absolute value of the difference of two images is not \rightarrow *nonlinear*

Local vs. Global Transformations

- A *global* transformation is performed using the entire image.
- A *local* transformation is performed by dividing the image into sub-images and processing each of these independently.
 - Also called *adaptive* or *variable*

Histogram

- A **histogram** represents the frequency distribution of intensity values in an image.
 - For example a histogram of an 8-bit image has 256 “bins” indexed from 0 to 255



Histogram

- The histogram of an image with grayscale values in the range $[0, L-1]$ is a function of the form:

$$h(r_k) = n_k$$

where r_k is the k^{th} intensity value and n_k is the number of pixels in the image having intensity value k .

Histogram Normalization

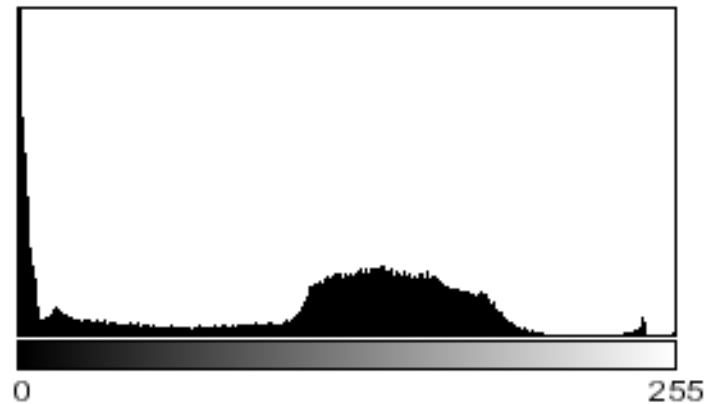
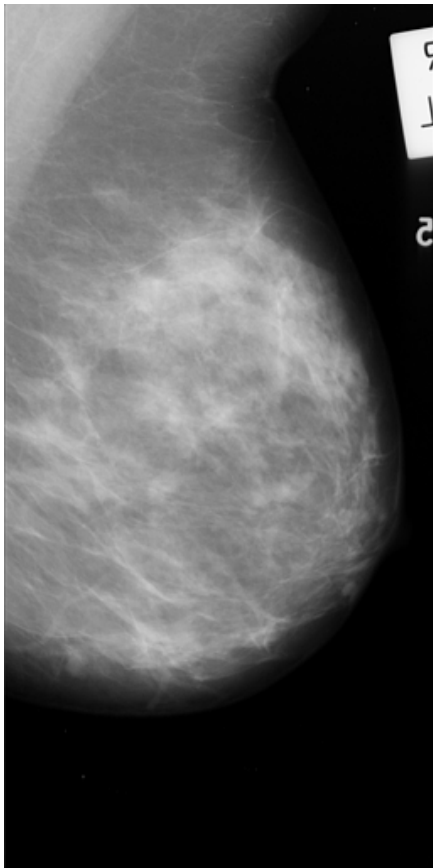
- A histogram can be normalized by dividing each of its values by the total number of pixels in the image, denoted by n .

$$p(k) = h_k / n$$

$p(k)$ gives an estimate of the probability of occurrence of intensity value k .

- The sum of all the bins in a normalized histogram is equal to 1.

Histogram



Count: 132096 Min: 0
Mean: 84.796 Max: 255
StdDev: 72.334 Mode: 0 (33335)

ALGORITHM: Calculation of an image histogram

Create an array *histogram* with 2^b elements

```
for i = 1:max_intensity
    histogram[i] = 0;
end
for x = 1:nrows
    for y = 1:ncolumns
        D = image[x,y];
        histogram[D] = histogram[D] + 1;
    end
end
```

Histogram Descriptors

- A number of global image statistical parameters can be derived from the histogram.
- **Maximum, Minimum**
 - The pixels with the highest and lowest intensity values.
- **Mean**
 - The average intensity value of the image.

$$m = \frac{\sum_{i=0}^{L-1} ih(i)}{n}$$

Histogram Descriptors

- Variance

- A measure of the width of the histogram.

$$v = \sum_{i=0}^{L-1} (h(i) - m)^2$$

- Skewness

- A measure of the non-symmetric distribution of the histogram.

$$sk = \sum_{i=0}^{L-1} (h(i) - m)^3$$

Histogram Descriptors

- Kurtosis

- A measure of the deviation of the histogram from a Gaussian profile.

$$k = \sum_{i=0}^{L-1} (h(i) - m)^4 - 3$$

- Entropy

- A measure of “choas” or noise in the image.

$$e = \sum_{i=0}^{L-1} \left\{ \frac{h(i)}{(n^2 \times m)} \cdot \log_2 \left(\frac{h(i)}{n^2 \times m} \right) \right\}$$

Histogram Descriptors

- Standard Deviation
 - A measure

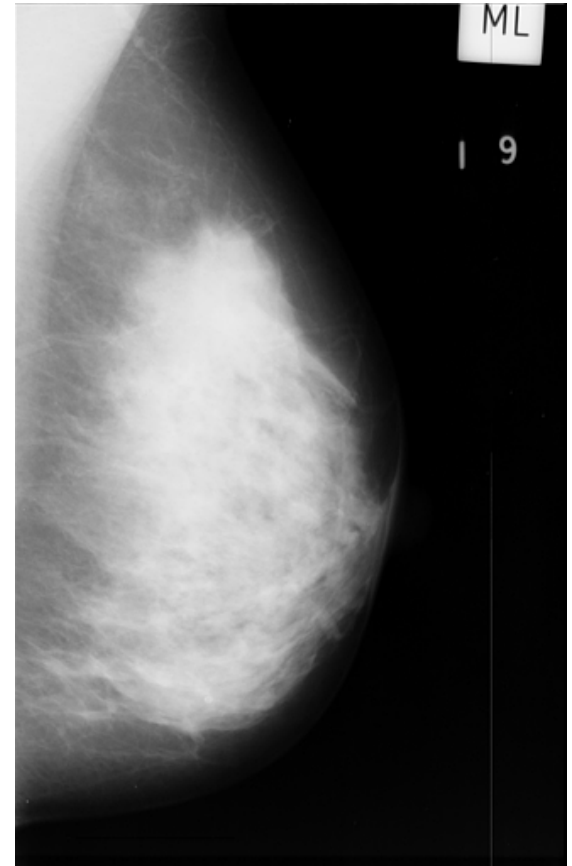
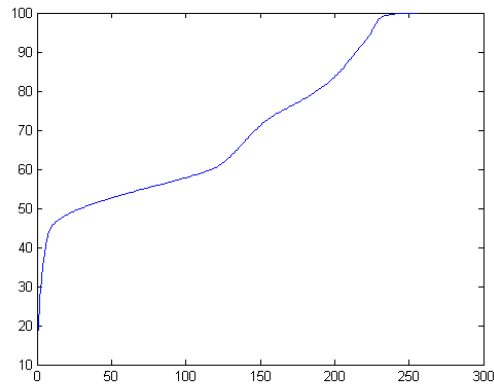
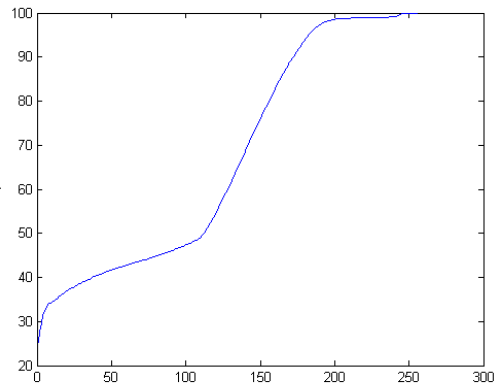
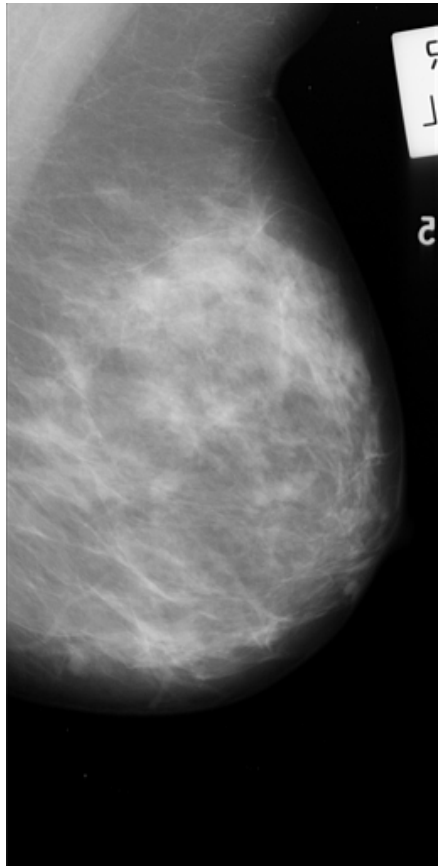
$$sd = \sqrt{v}$$

Cumulative Histogram

- A **cumulative histogram** represents the cumulative frequency distribution of intensity values in an image.
 - The cumulative frequency of an intensity value, i , is the number of times that an intensity value less than or equal to i occurs in an image.
 - Cumulative frequencies, c_i , are computed from histogram counts, $h(i)$:

$$c_j = \sum_{i=0}^j h(i) \quad j = 0, 1, \dots, L-1$$

Cumulative Histogram



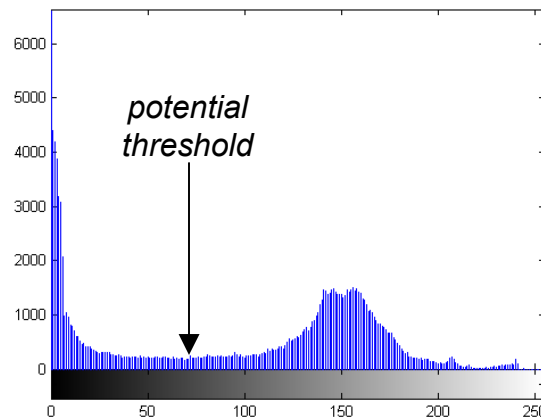
ALGORITHM: Calculation of a cumulative histogram

Create an array *histogram* with 2^b elements

```
sum = 0;  
for i = 1:max_intensity  
    sum = sum + histogram[i];  
    cumulative_histogram[i] = sum;  
end
```

Histograms

- One of the principal uses of the histogram is the selection of threshold parameters.
- It is useful to plot $h(i)$ as a function of i .
 - From this graph a suitable position for the threshold can be related directly to the position of the *foot of a hill* or *valley* in the histogram.



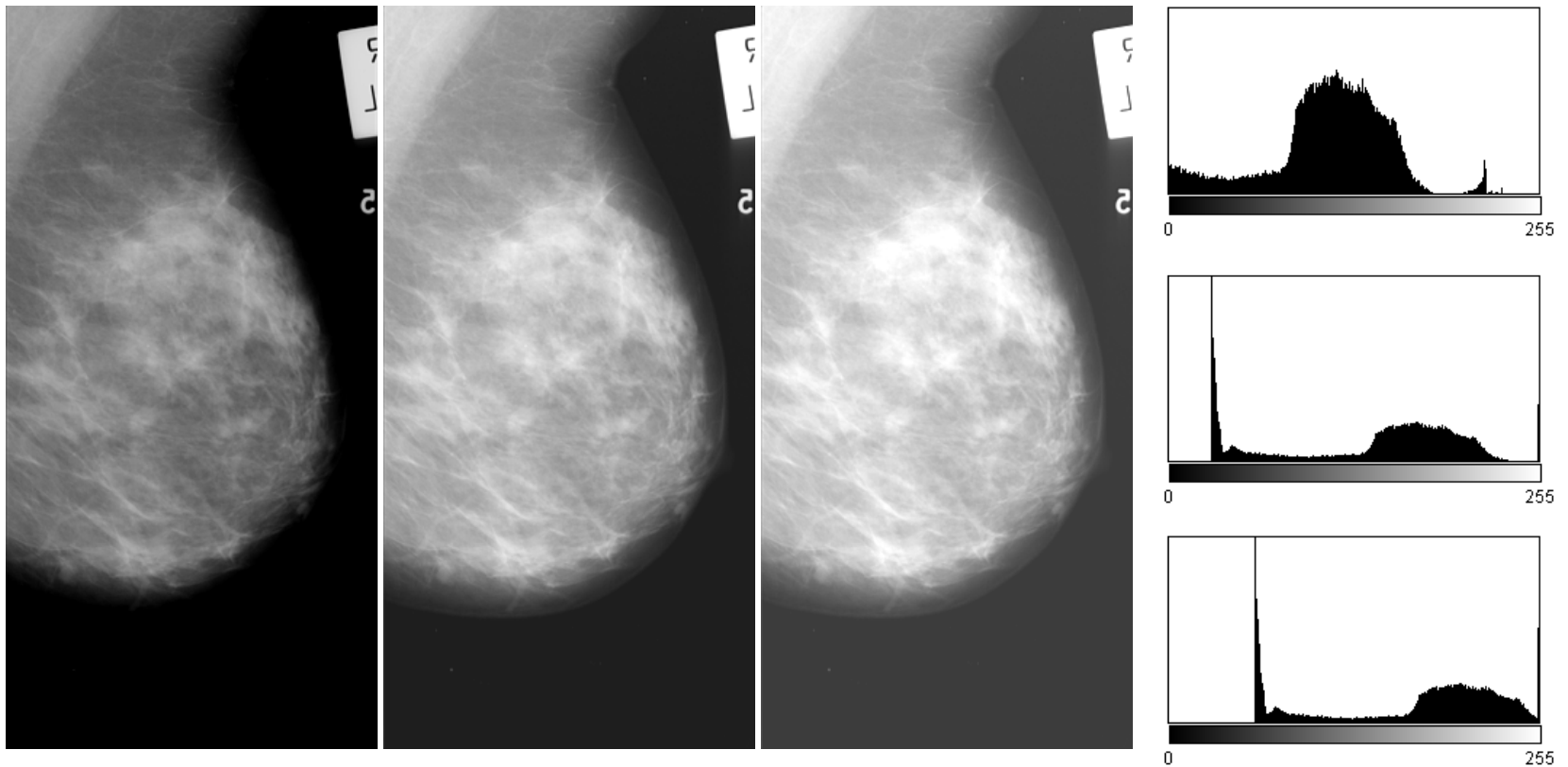
Histogram Sliding

- **Histogram sliding** involves the addition or subtraction of a constant value, b , to all the pixels in an image.
 - It is sometimes referred to as adding *bias* to the image intensity

$$g(x, y) = f(x, y) + b$$

- If $b > 0$, then brightness is increased; if $b < 0$, it is decreased.

Histogram Sliding



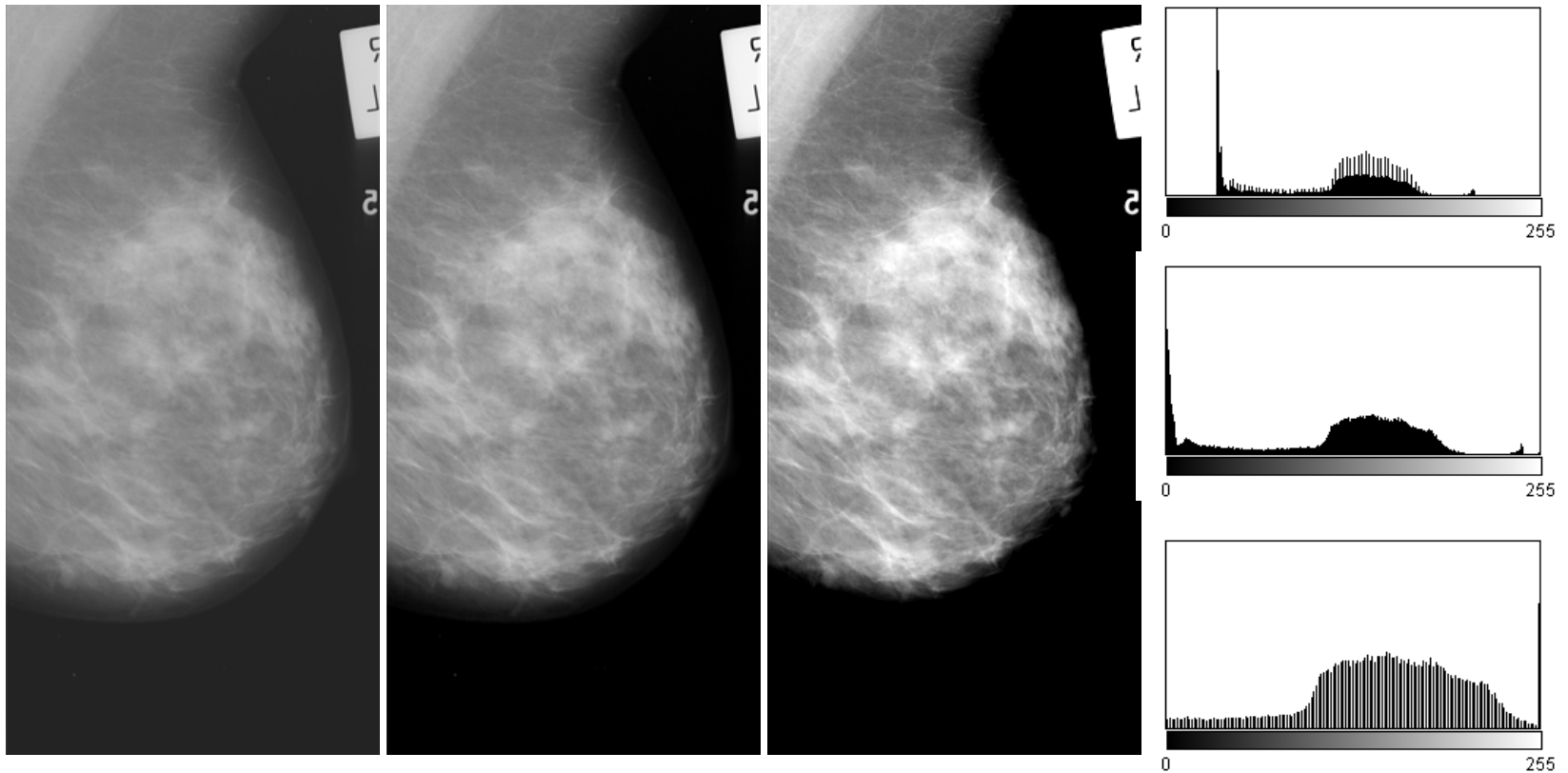
Histogram Stretching

- Histogram stretching involves the multiplication or division of all pixels in an image by a constant value, a .
 - It is sometimes referred to as adding *gain* to the image intensity

$$g(x, y) = af(x, y)$$

- If $a > 1$, then contrast is increased; if $a < 1$, it is reduced.

Histogram Stretching



Histogram Sliding & Stretching

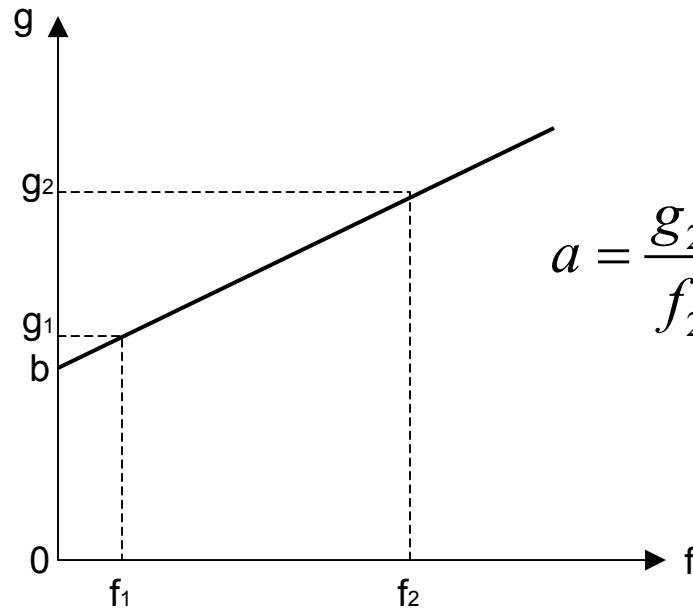
- Histogram stretching and sliding can be combined to give a general expression for brightness and contrast modification:

$$g(x, y) = af(x, y) + b$$

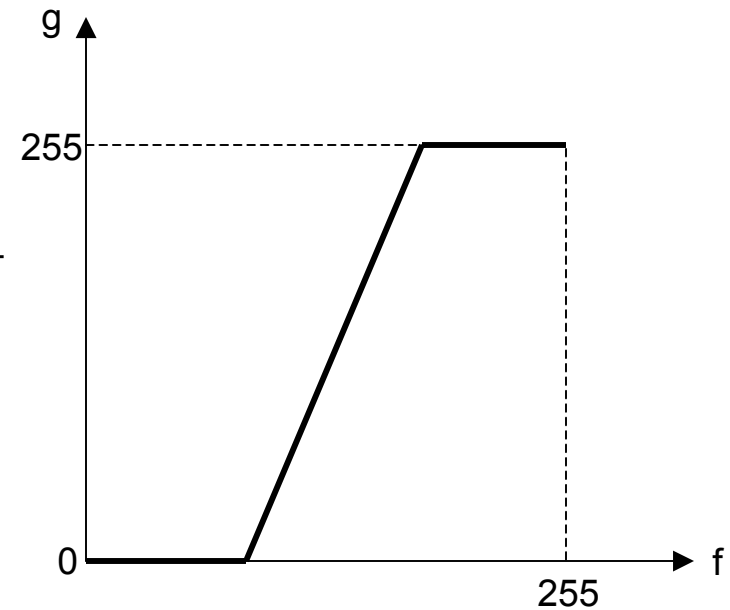
- To map a particular range of intensity values $[f_1, f_2]$ onto a new range $[g_1, g_2]$

$$g(x, y) = g_1 + \left(\frac{g_2 - g_1}{f_2 - f_1} \right) [f(x, y) - f_1]$$

Histogram Sliding & Stretching



$$a = \frac{g_2 - g_1}{f_2 - f_1}$$



Linear vs. Nonlinear Mappings

- In linear mapping the gain, a , is static
- In nonlinear mapping the gain, a , can vary.
 - The way in which contrast is modified depends on the input intensity value
e.g. logarithmic, exponential functions

Histogram Equalization

- Histogram equalization or linearization redistributes intensity values in an attempt to “flatten” the frequency distribution.
 - Involves normalizing the histogram
 - For each intensity level j in the original image histogram, the new intensity value K is calculated as:

$$k_j = \sum_{i=0}^j \frac{h(i)}{T}$$

$h(i)$ is the number of pixels with intensity value i .

T is the total number of pixels in the image and $j = 0, 1, 2, \dots, L-1$

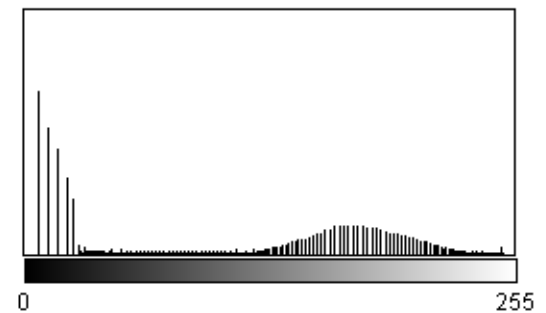
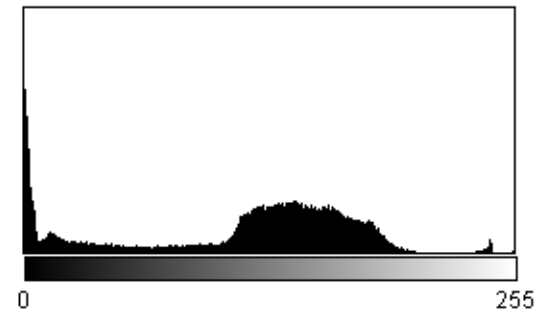
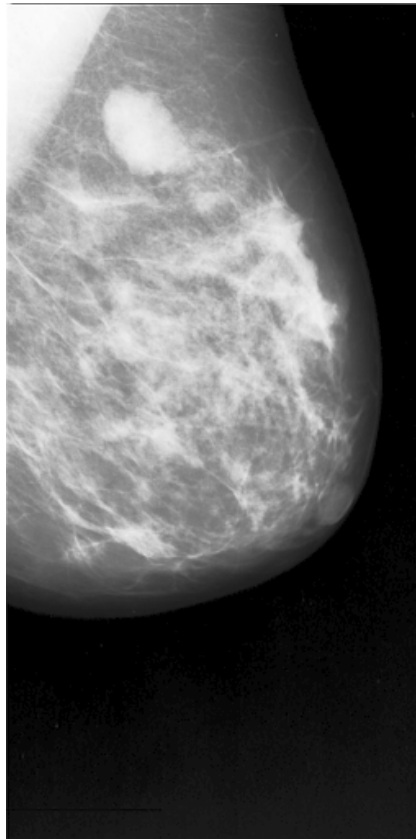
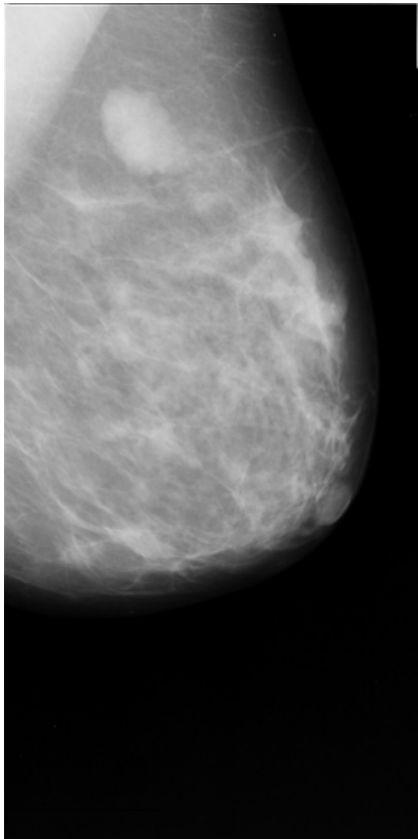
ALGORITHM: Histogram equalization

Compute a scaling factor, $b=255/\text{number of pixels}$

Calculate histogram

```
c[1] = b * histogram[1];  
for i = 2:max_intensity  
    c[i] = c[i-1] + b*histogram[i];  
end  
for x = 1:nrows  
    for y = 1:ncolumns  
        D = image[x,y];  
        g[x,y] = c[D+1];  
    end  
end
```


Histogram Equalization



Histogram Equalization

Histogram Specification

- A method which generates an image that has a *specified* histogram.

Convolution

- One of the fundamental operations in image processing
 - Enhancement techniques based on this type of approach are often referred to as *spatial filtering*.

Convolution

- In convolution, the calculation performed at a pixel is a weighted sum of intensity values from a **neighborhood** surrounding a pixel.
 - If a neighborhood is centred on a pixel then it must have odd dimensions
 - Intensity values from a neighborhood are weighted by coefficients that come from a **convolution kernel**, or **mask**

Convolution

- The kernel is usually small relative to the image

e.g. 3x3 is most common

$i-1, j-1$	$i-1, j$	$i-1, j+1$
$i, j-1$	i, j	$i, j+1$
$i+1, j-1$	$i+1, j$	$i+1, j+1$

Convolution

- During convolution, each kernel coefficient is taken in turn and multiplied by a value from the neighborhood of the image lying under the kernel.

$$g(x, y) = \sum_{j=-1}^1 \sum_{i=-1}^1 w(i, j) f(x - i, y - j)$$

- For example:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Convolution

- For a kernel of width m and height n , both odd:

$$g(x, y) = \sum_{j=-n_2}^{n_2} \sum_{i=-m_2}^{m_2} w(i, j) f(x - i, y - j)$$

the kernel half-width m_2 , and half-height n_2 are given as:

$$m_2 = \lfloor m/2 \rfloor, \quad n_2 = \lfloor n/2 \rfloor$$

Kernels

- An **omnidirectional** kernel is one whose response is the same, whatever the direction in which intensity values vary.
- A **uniform** kernel has coefficients which all have the same weight.
- A **nonuniform** kernel has coefficients which have differing weights.

Kernel Shapes

- Rectangular versus circular (pill-box) uniform (smoothing) kernels

$$w(i, j) = \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$w(i, j) = \frac{1}{21} \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Kernel Shapes

- **Pyramidal** versus **cone** nonuniform (smoothing) kernels

$$w(i, j) = \frac{1}{81} \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$

$$w(i, j) = \frac{1}{25} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Separable Kernels

- A **separable** $n \times n$ kernel is represented as a vector product of two orthogonal 1D kernels, each of width n .
 - One is applied down the columns of an image, generating an intermediate result.
 - The other kernel is then applied along the rows of the intermediate image, producing the final result.

The Convolution Process

1. Place the centre of the kernel over a pixel, P , in the input image.
2. Multiply each pixel in the $k \times k$ neighborhood by the appropriate filter kernel coefficient superimposed on it.
3. Sum all the products.
4. Place the suitably normalized sum into the pixel position, P , in the output image

ALGORITHM: Image convolution

Create a kernel, w , with dimensions $m \times n$

Compute kernel half-width, m_2

Compute kernel half-height, n_2

Create an $M \times N$ output image, g

```
for x = 1:nrows
    for y = 1:ncolumns
        g[x,y] = 0;
    end
end
```

ALGORITHM: Image convolution

```
for x = m2:M-m2
    for y = n2:N-n2
        sum = 0;
        for i = -m2:m2
            for j = -n2:n2
                sum = sum+w[i+m2,j+n2]*image[x-i,y-j];
            end
        end
        g[x,y] = sum;
    end
end
```

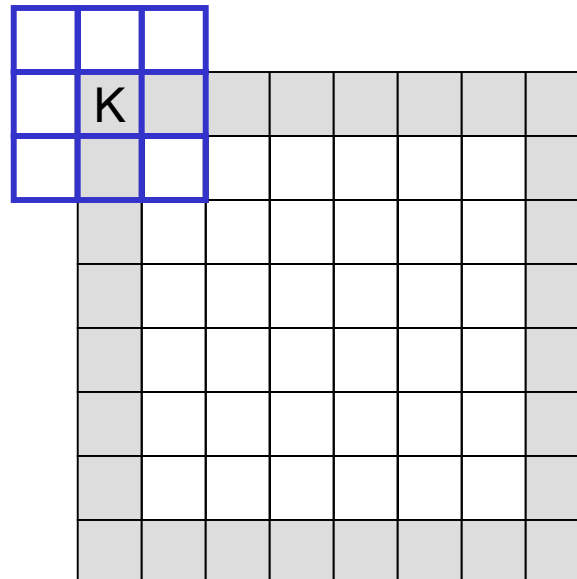
Image Borders

- Along the borders of an image it is not possible to compute a convolution, because part of the kernel lies beyond the image.
 - True of any neighborhood operation
- The size of the region in which normal convolution is possible is dictated by the dimensions of the convolution kernel.
 - e.g. for a 3x3 kernel: 1-edge border

Image Borders

- For certain values of x and y , one or both of the expressions $(x-i)$ and $(y-j)$ will give a value outside the allowed range $[0, M-1][0, N-1]$

3×3 kernel



image

Image Borders

- For an image $f(x,y)_{M \times N}$ the region to which the kernel applies is $(M-2) \times (N-2)$ with an origin at $(1,1)$
- In general an origin at (m_2, n_2) and dimensions of $(M-2m_2) \times (N-2n_2)$
- A number of different strategies exist to deal with this problem

Image Borders

1. No processing at the border

- Ignore those pixels for which convolution is not possible

2. Copy of input pixels

- Copy the corresponding pixel value from the input image wherever it is not possible to carry out convolution.
- The image will have a border of unprocessed pixels

Image Borders

3. Truncation of the image

- Remove those pixels for which convolution is not possible.
- The resulting image is smaller than, and offset to, the input image.

Image Borders

4. Truncation of the kernel

- Deal with the borders of the image as a special case and use a *modified kernel* to perform convolution.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

- Adds considerably to the complexity of convolution.

Image Borders

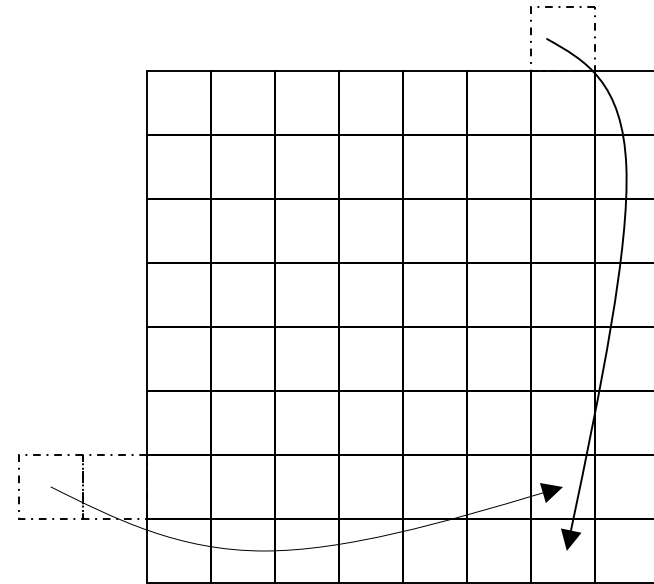
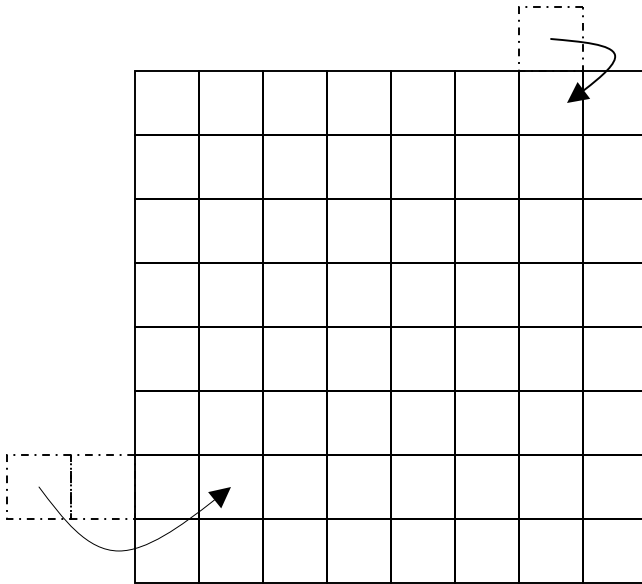
5. Reflected indexing

- Test $(x-i)$ to see whether it corresponds to a valid pixel value, and if it doesn't, the coordinate can be reflected-back into the image.
- The same can be done at $(y-j)$
- Simulates **mirroring** of the image at its borders

6. Circular indexing

- Imaging that the image repeats itself endlessly in all directions.

Image Borders



Reflected and Circular indexing

Spatial Frequency

- **Spatial frequency** is a measure of how rapidly intensity varies as an image is traversed.
 - Images in which intensity varies slowly and smoothly → **low spatial frequency**
 - Images with sudden intensity transitions, fine detail and strong texture → **high spatial frequency**

Linear Filtering

- Convolution can be used to carry out linear filtering of an image.
 - The response is given by a sum of products of the kernel coefficients and the corresponding image pixels in the area spanned by the filter mask.
 - There are two complementary types of linear filters: **spatial** or **frequency** filters.

Image Smoothing

- Image smoothing or low-pass filtering, allows low spatial frequencies to remain unchanged, but suppresses high frequencies.
 - A low-pass filter has the effect of smoothing or blurring the image, reducing noise but obscuring fine detail.

Image Smoothing

- The filter replaces the value of every pixel in an image by the average of the intensity values in the neighborhood defined by the kernel.
 - The resulting image has reduced “sharp” transitions in the intensity values.
 - Noise consists of sharp transitions → noise reduction
 - Edges are also characterized by sharp transitions in intensities, so smoothing filters blur edges.

Image Smoothing

- Any convolution kernel whose coefficients are all positive will act as a low-pass filter.
- In the simplest case, all coefficients are equal.

$$w = \begin{bmatrix} 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.04 \end{bmatrix}$$

- Note that the kernel has already been normalized. It's coefficients sum to 1.

Image Smoothing

- Factor out the normalization:

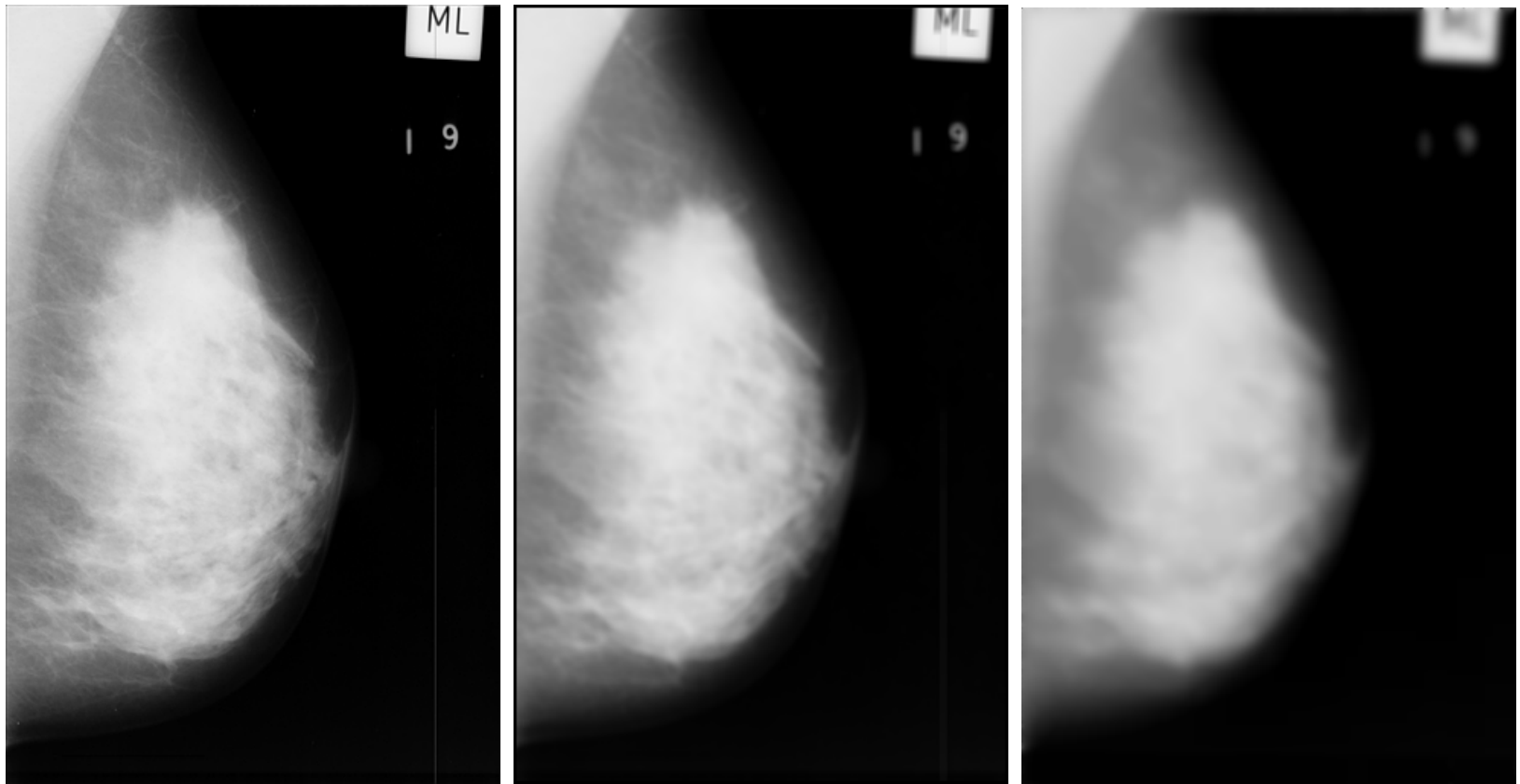
$$w(i, j) = \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Pixel values in the neighborhood are summed without being weighted, and the sum is divided by the number of pixels in the neighborhood.

Image Smoothing

- Computing the mean intensity value over the neighborhood defined by the kernel:
 - Often described as *mean filters*.
 - Large kernels, or the repeated application of a small kernel produces more pronounced smoothing.

Example of Image Smoothing



Gaussian Filters

- The **Gaussian** filter is a smoothing filter with a nonuniform kernel, whose coefficients are derived from a 2D Gaussian function.
- The kernel coefficients diminish in size with increasing distance from the kernels centre

Gaussian Filters

$$w(i, j) = \exp\left[\frac{-(i^2 + j^2)}{2\sigma^2}\right]$$

$$\text{for } i, j = -[3\sigma], \dots, [3\sigma]$$

- $[3\sigma]$ denotes the “integer part” of 3σ
- Limits of ± 3 are chosen because Gaussian weights are negligible beyond them

Gaussian Filters

- If $\sigma^2 = 1$ then the normalised Gaussian kernel is:

$$w(i, j) = \frac{1}{1000} \begin{bmatrix} 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 13 & 22 & 13 & 3 & 0 \\ 1 & 13 & 59 & 97 & 59 & 13 & 1 \\ 2 & 22 & 97 & 159 & 97 & 22 & 2 \\ 1 & 13 & 59 & 97 & 59 & 13 & 1 \\ 0 & 3 & 13 & 22 & 13 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

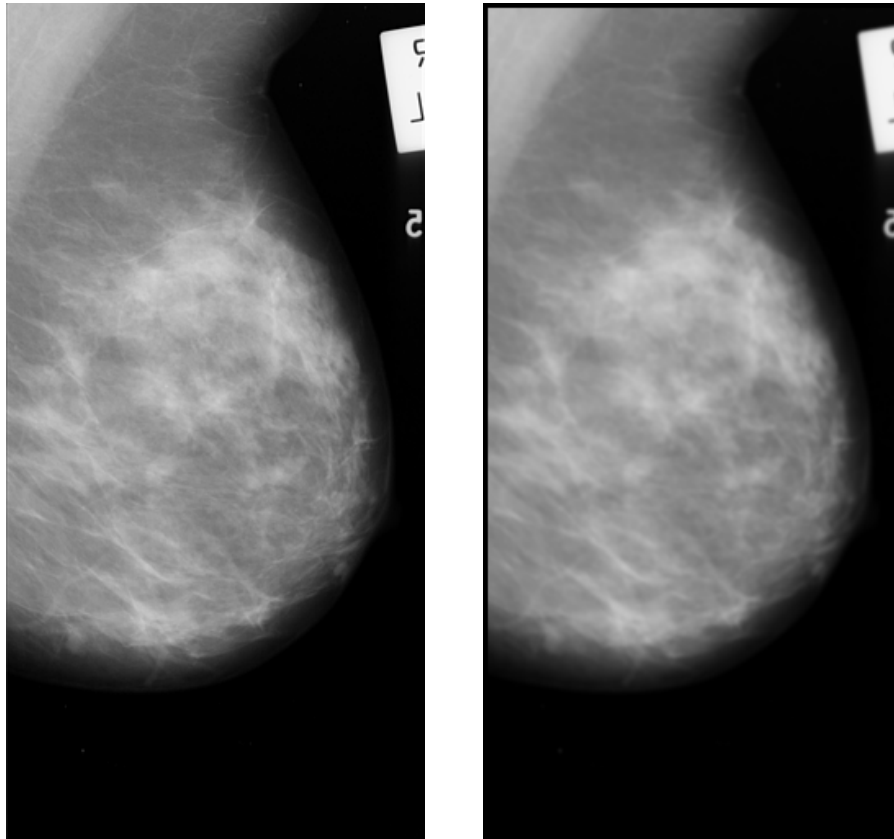
$$\sigma^2 = \frac{2}{3}, 2, 6\frac{2}{3}$$

Other common values for

Gaussian Filters

- Gaussian characteristics:
 - More weight is given to central pixels than to those in the periphery of the neighborhood
 - Large values of σ produce a wider peak → increased blurring
 - As σ increases the dimensions of the kernel also increase
 - The kernel is rotationally symmetric, so there is no directional bias in the amount of smoothing
 - The Gaussian kernel is separable.

Gaussian Filters



Median Filter

- Noise tends to spread outwards when convolution is applied.
- In a **median filter** pixels are replaced by the median value of the neighboring intensities in a $k \times k$ neighborhood.
- Efficient in eliminating isolated and impulse (i.e. salt-and-pepper) noise.

Median Filter

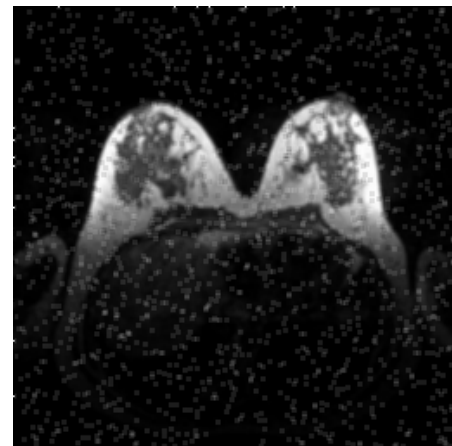
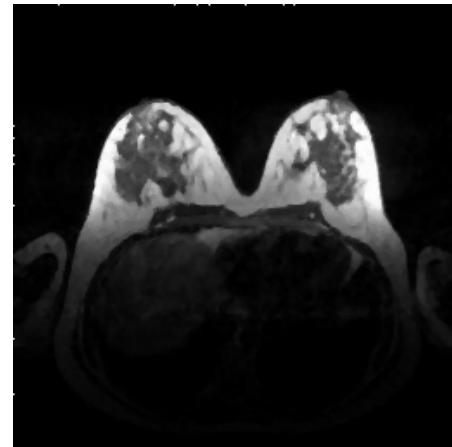
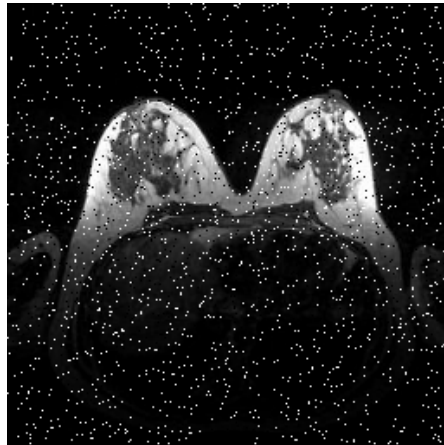
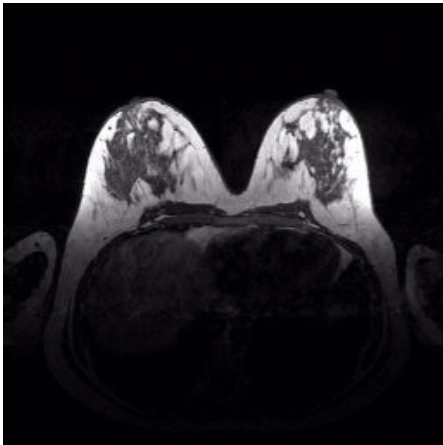


Image Sharpening

- Image sharpening, or high-pass filtering allows high spatial frequencies to remain unchanged, but suppresses low frequencies.
 - A high-pass filter has the effect of preserving sudden variations in intensity, such as those that occur at the boundaries of objects, but suppresses more gradual variations.
 - Makes noise more prominent (noise has a strong high-frequency component).

Image Sharpening

- A HPF convolution kernel contains a mixture of positive and negative values.
- An omnidirectional high pass filter should have positive coefficients near its centre and negative coefficients in the periphery of the kernel.

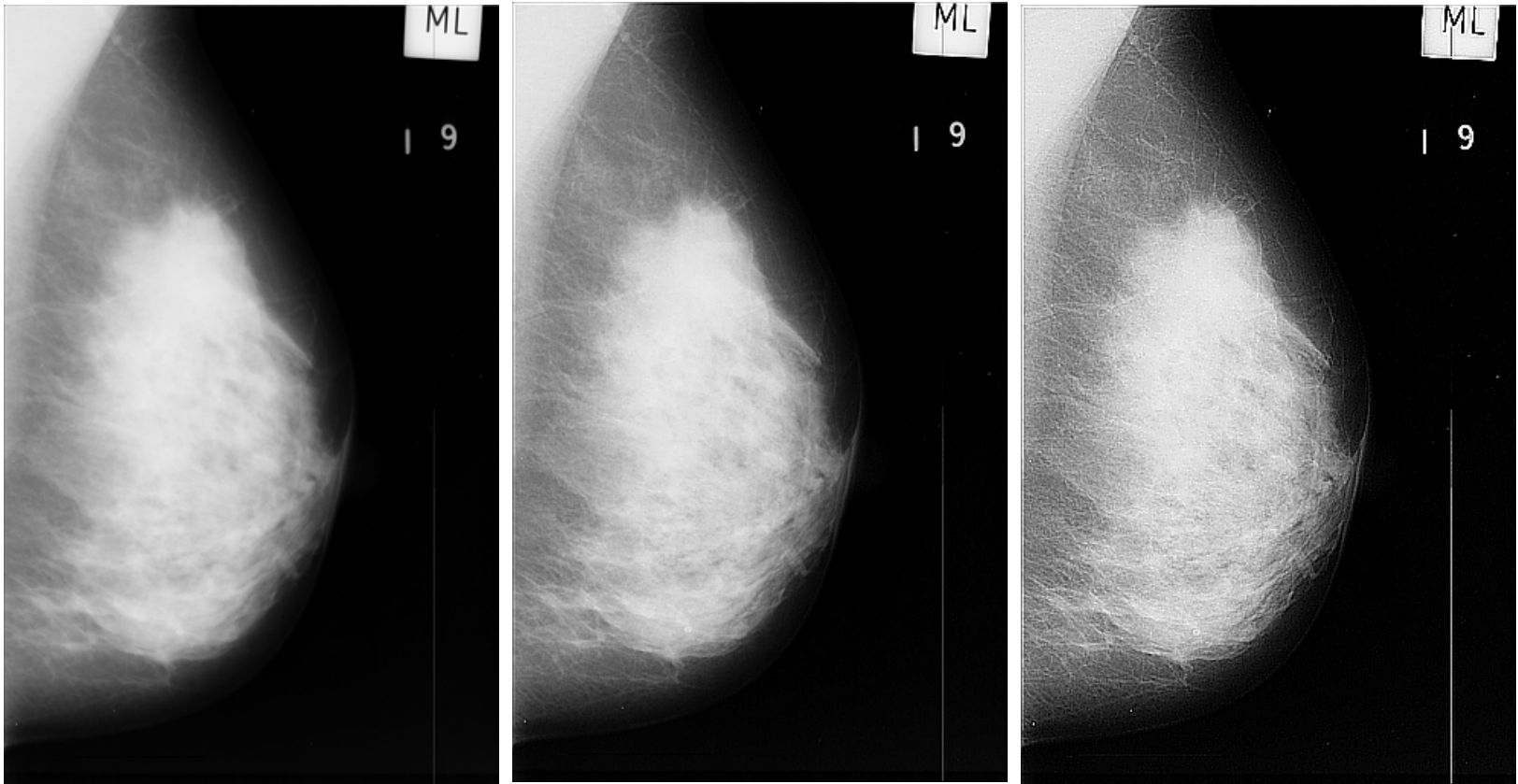
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

- The sum of the coefficients in this kernel is zero.

Image Sharpening

- When the kernel is over an area of constant or slow-changing intensity values the result of the convolution is zero (or small).
- When the intensity values vary rapidly within a neighborhood, the result of the convolution is a large number.
- The result can be positive or negative:
 - Map the pixel values onto a 0-255 range. A filter response of 0 maps onto the middle of the range.

Example of Image Sharpening



Unsharp Masking

- Subtracting from an image a “blurred” version of that image (thereby removing the low spatial frequencies) is known as *unsharp masking*. $\hat{f}(x, y) = f(x, y) - S(f(x, y))$

High-Boost Filter

- Compute a weighted sum of the original image and the output from a high-pass filter.
 - The result is an image in which high spatial frequencies are emphasized relative to lower frequencies.
 - The **high-boost filter** is used to sharpen an image.
 - Can be performed in a single convolution operation

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & c & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad (c > 8)$$