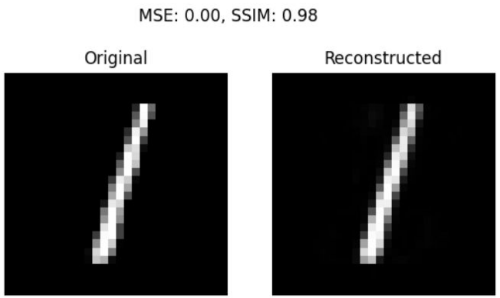
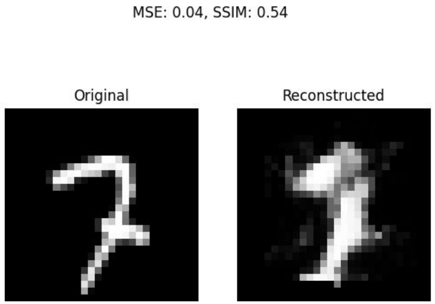
Structural Similarity Index Measure (SSIM)

What is Structural Similarity Index Measure (SSIM)?

- **quantifies image quality** degradation* caused by processing such as **data compression** or by losses in data transmission
- requires two images from the same image resolution: Original and processed Image
- cannot judge which of the two is better:
- assumption : both the **image resolution and the viewing distance** is uniform between the images





Mathematical representation

-X,Y with same image resolution

$$\mathrm{SSIM}(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

- μ_x the average of x;
- μ_y the average of y;
- σ_x^2 the variance of x;
- σ_y^2 the variance of y;
- σ_{xy} the covariance of x and y;
- $c_1 = (k_1 L)^2$, $c_2 = (k_2 L)^2$ two variables to stabilize the division with weak denominator;
- L the dynamic range of the pixel-values (typically this is $2^{\#bits\ per\ pixel} - 1$);
- $k_1 = 0.01$ and $k_2 = 0.03$ by default.

How does SSIM works?

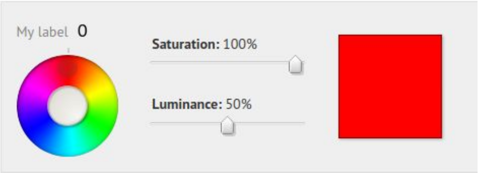
The SSIM incorporates three comparisons between the original and compressed:

1. Luminosity
2. Contrast
3. Structural similarity.

Luminosity:

luminance : average value of brightness

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i\,.$$



Luminosity similarity of two image is calculated by:

$$l(\mathbf{x},\mathbf{y}) = \frac{2\,\mu_x\,\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}\,.$$

C1: constant to avoid instability when the denominator is close to zero.

Contrast

Contrast is a measure of how much the **intensities are spread** in an image.

Very **high contrast** means there are very **bright** as well as very **dark** regions in an image

Low contrast indicates the pixels are in a **neighbourhood of similarity**.

$$\sigma_x = \left(\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu_x)^2 \right)^{1/2}$$

The spread of pixel is calculated with the standard deviation

Differences in contrast between two images is calculated by:

C2: constant to avoid instability when the denominator is close to zero.

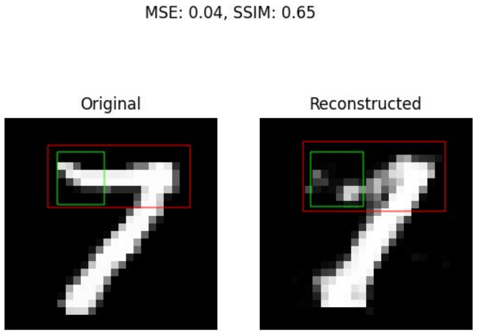
$$c(\mathbf{x}, \mathbf{y}) = \frac{2\,\sigma_x\,\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2},$$

Structural Similarity

Structural Similarity is determined by calculating **correlation between image intensities.**

Correlation measure how much two sets of numbers(in case of image: **image intensities**) move in the same direction.

$$s(\mathbf{x}, \mathbf{y}) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3}.$$

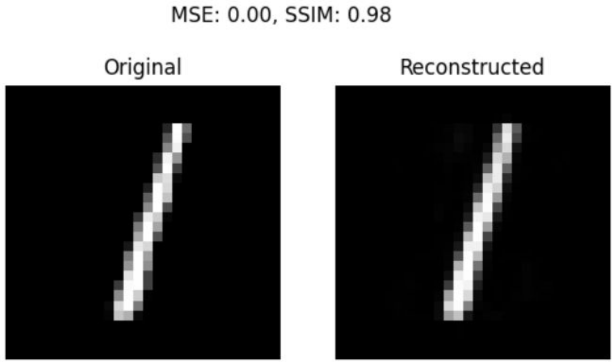


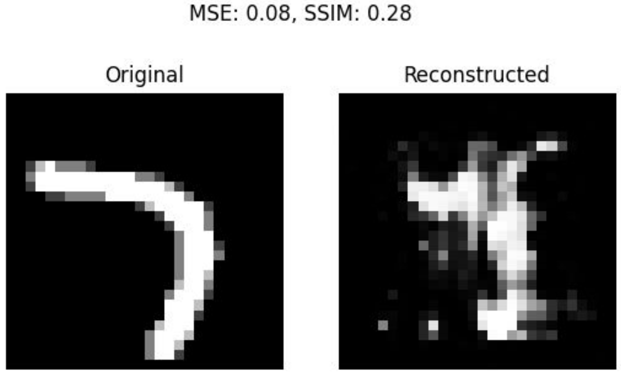
SSIM

SSIM = Luminosity X Contrast X Structural Similarity

$$\text{SSIM}(\mathbf{x}, \mathbf{y}) = [l(\mathbf{x}, \mathbf{y})]^{\alpha} \cdot [c(\mathbf{x}, \mathbf{y})]^{\beta} \cdot [s(\mathbf{x}, \mathbf{y})]^{\gamma}$$

$$\text{SSIM}(x, y) = \frac{(2\mu_x \mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$





MSE and SSIM

Fig:1

Fig:2

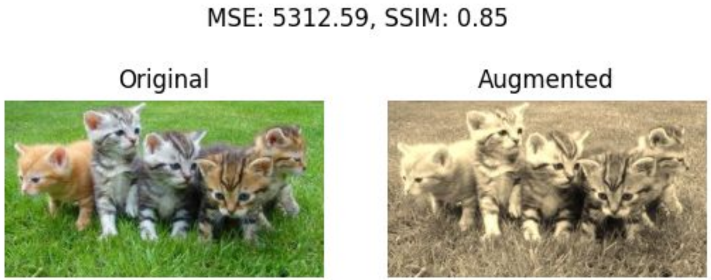


Fig:3

Summary

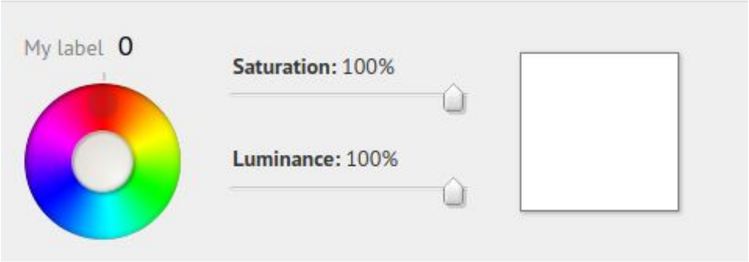
SSIM can be used to **measure and optimise neural networks** in several applications of d**enoising, deblurring** etc.

It has played a major role in **image quality assessment**

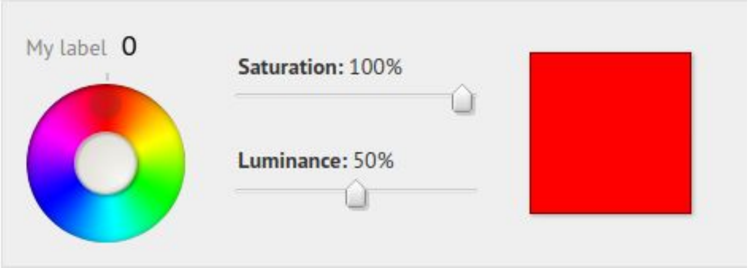
MSE is **not the most accurate measurement of distortions** to an image as they appear to our human eyes.

SSIM along with the MSE allows us leverage in key areas where the **MSE fails to capture** what we are looking for.

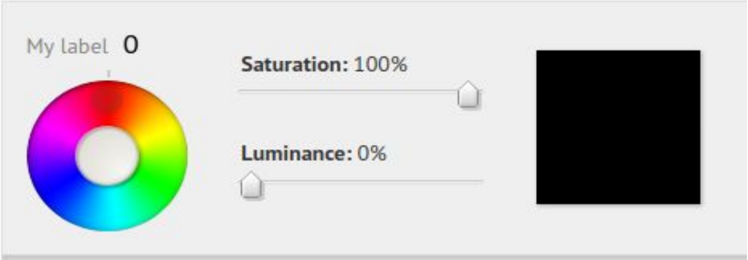
Luminosity: Img



"luminosity" within an image = perceived brightness distribution



[Example: HSL Color Picker - YUI Library]



Contrast: Img

Low Contrast

High Contrast

Structural Similarity: Img