(See https://cs.stanford.edu/~knuth/programs.html for date.)

1. Introduction. This program inputs an undirected graph and the names of two vertices in that graph (the "source" and "target" vertices). It outputs a not-necessarily-reduced binary decision diagram for the family of all simple paths from the source to the target.

The format of the output is described in another program, SIMPATH-REDUCE. Let me just say here that it is intended only for computational convenience, not for human readability.

I've tried to make this program simple, whenever I had to choose between simplicity and efficiency. But I haven't gone out of my way to be inefficient.

```
#define maxn 255   /* maximum number of vertices; at most 255 */
#define maxm 2000   /* maximum number of edges */
#define logmemsize 27
#define memsize (1 << logmemsize)
#define loghtsize 25
#define htsize (1 << loghtsize)
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "gb_graph.h"
#include "gb_save.h"
  unsigned char mem[memsize];   /* the big workspace */
  unsigned long long tail, boundary, head;   /* queue pointers */
  unsigned int htable[htsize];   /* hash table */
  unsigned int htid;   /* "time stamp" for hash entries */
  int htcount;   /* number of entries in the hash table */
  int wrap = 1;   /* wraparound counter for hash table clearing */
  Vertex *vert[maxn + 1];
  int arcto[maxm];   /* destination number of each arc */
  int firstarc[maxn + 2];   /* where arcs from a vertex start in arcto */
  unsigned char mate[maxn + 3];   /* encoded state */
  int serial, newserial;   /* state numbers */
  ⟨Subroutines 13⟩
  main(int argc, char *argv[])
  {
    register int i, j, jj, jm, k, km, l, ll, m, n, t, hash;
    register Graph *g;
    register Arc *a, *b;
    register Vertex *u, *v;
    Vertex *source = Λ, *target = Λ;
    ⟨Input the graph 2⟩;
    ⟨Renumber the vertices 3⟩;
    ⟨Reformat the edges 4⟩;
    ⟨Do the algorithm 5⟩;
  }
```

2. ⟨Input the graph 2⟩ ≡

```
if (argc ≠ 4) {
  fprintf(stderr, "Usage:␣%s␣foo.gb␣source␣target\n", argv[0]);
  exit(−1);
}
g = restore_graph(argv[1]);
if (¬g) {
  fprintf(stderr, "I␣can't␣input␣the␣graph␣%s␣(panic␣code␣%ld)!\n", argv[1], panic_code);
  exit(−2);
}
n = g→n;
if (n > maxn) {
  fprintf(stderr, "Sorry,␣that␣graph␣has␣%d␣vertices;␣", n);
  fprintf(stderr, "I␣can't␣handle␣more␣than␣%d!\n", maxn);
  exit(−3);
}
if (g→m > 2 ∗ maxm) {
  fprintf(stderr, "Sorry,␣that␣graph␣has␣%ld␣edges;␣", (g→m + 1)/2);
  fprintf(stderr, "I␣can't␣handle␣more␣than␣%d!\n", maxm);
  exit(−3);
}
for (v = g→vertices; v < g→vertices + n; v++) {
  if (strcmp(argv[2], v→name) ≡ 0) source = v;
  if (strcmp(argv[3], v→name) ≡ 0) target = v;
  for (a = v→arcs; a; a = a→next) {
    u = a→tip;
    if (u ≡ v) {
      fprintf(stderr, "Sorry,␣the␣graph␣contains␣a␣loop␣%s--%s!\n", v→name, v→name);
      exit(−4);
    }
    b = (v < u ? a + 1 : a − 1);
    if (b→tip ≠ v) {
      fprintf(stderr, "Sorry,␣the␣graph␣isn't␣undirected!\n");
      fprintf(stderr, "(%s->%s␣has␣mate␣pointing␣to␣%s)\n", v→name, u→name, b→tip→name);
      exit(−5);
    }
  }
}
if (¬source) {
  fprintf(stderr, "I␣can't␣find␣source␣vertex␣%s␣in␣the␣graph!\n", argv[2]);
  exit(−6);
}
if (¬target) {
  fprintf(stderr, "I␣can't␣find␣target␣vertex␣%s␣in␣the␣graph!\n", argv[3]);
  exit(−7);
}
```

This code is used in section 1.

3. If the source vertex is the first vertex in the graph, I'll process vertices according to the graph's own ordering.

Otherwise, I use a simple breadth-first strategy to number the vertices: The source is vertex 1. Then, for each $j \ge 1$, I run through the arcs from vertex j and assign the first unused number to any of its neighbors that haven't already got one.

```
#define num z.I

⟨Renumber the vertices 3⟩ ≡
  if (source ≡ g→vertices) {
    for (k = 0; k < n; k++) (g→vertices + k)→num = k + 1, vert[k + 1] = g→vertices + k;
  } else {
    for (k = 0; k < n; k++) (g→vertices + k)→num = 0;
    vert[1] = source, source→num = 1;
    for (j = 0, k = 1; j < k; j++) {
      v = vert[j + 1];
      for (a = v→arcs; a; a = a→next) {
        u = a→tip;
        if (u→num ≡ 0) u→num = ++k, vert[k] = u;
      }
    }
    if (target→num ≡ 0) {
      fprintf(stderr, "Sorry, there's no path from %s to %s in the graph!\n", argv[2], argv[3]);
      exit(−8);
    }
    if (k < n) {
      fprintf(stderr, "The graph isn't connected (%d<%d)!\n", k, n);
      fprintf(stderr, "But that's OK; I'll work with the component of %s.\n", argv[2]);
      n = k;
    }
  }
```

This code is used in section 1.

4. The edges will be considered as arcs $j \to k$ between vertex number j and vertex number k, when $j < k$ and those vertices are adjacent in the graph. We process them in order of increasing j; but for fixed j, the values of k aren't necessarily increasing.

The k values appear in the *arcto* array. The edges for fixed j occur in positions *firstarc*$[j]$ through *firstarc*$[j+1]-1$ of that array.

After this step, we forget the GraphBase data structures and just work with our homegrown integer-only representation.

⟨Reformat the edges 4⟩ ≡

```
for (m = 0, k = 1; k ≤ n; k++) {
  firstarc[k] = m;
  v = vert[k];
  printf("%ld(%s)\n", v→num, v→name);
  for (a = v→arcs; a; a = a→next) {
    u = a→tip;
    if (u→num > k) {
      arcto[m++] = u→num;
      if (a→len ≡ 1) printf("␣->␣%ld(%s)␣#%d\n", u→num, u→name, m);
      else printf("␣->␣%ld(%s,%ld)␣#%d\n", u→num, u→name, a→len, m);
    }
  }
}
firstarc[k] = m;
```

This code is used in section 1.

5. The algorithm. Now comes the fun part. We systematically construct a binary decision diagram for all simple paths by working top-down, considering the arcs in *arcto*, one by one.

When we're dealing with arc i, we've already constructed a table of all possible states that might arise when each of the previous arcs has been chosen-or-not, except for states that obviously cannot be part of a simple path.

Arc i runs from vertex j to vertex $k = arcto[i]$. Let l be the maximum vertex number in arcs less than i. (If the breadth-first ordering option was taken above, we'll always have $k \le l+1$, because of the way we did the numbering and reformatting; but that method is not always best.)

The state before we decide whether or not to include arc i is represented by a table of values $mate[t]$, for $j \le t \le l$, with the following significance: If $mate[t] = t$, the previous arcs haven't touched vertex t. If $mate[t] = u$ and $u \ne t$, the previous arcs have connected t with u by a simple path. If $mate[t] = 0$, the previous arcs have "saturated" vertex t; we can't touch it again.

We also use a (slick?) trick: We imagine that an edge between the source and target has already been included. Then the final arc of a simple path will be an arc that completes a cycle, when no other incomplete paths are present. (Think about it.)

The *mate* information is all that we need to know about the behavior of previous arcs. And it's easily updated when we add the ith arc (or not). So each "state" is equivalent to a *mate* table, consisting of $l+1-j$ numbers.

The states are stored in a queue, indexed by 64-bit numbers *tail*, *boundary*, and *head*, where $tail \le boundary \le head$. Between *tail* and *boundary* are the pre-arc-i states that haven't yet been processed; between *boundary* and *head* are the post-arc-i states that will be considered later. The states before *boundary* are sequences of $s = l+1-j$ bytes each, and the states after *boundary* are sequences of $ss = ll+1-jj$ bytes each, where *ll* and *jj* are the values of l and j for arc $i+1$.

Bytes of the queue are stored in *mem*, which wraps around modulo *memsize*. We ensure that $head - tail$ never exceeds *memsize*.

```
⟨ Do the algorithm 5 ⟩ ≡
  ⟨ Initialize the mate table 6 ⟩;
  ⟨ Initialize the queue 7 ⟩;
  for (i = 0; i < m; i++) {
    printf("#%d:\n", i + 1);      /* announce that we're beginning a new arc */
    fprintf(stderr, "Beginning␣arc␣%d␣(serial=%d,head-tail=%lld)\n", i + 1, serial, head − tail);
    fflush(stderr);
    ⟨ Process arc i 8 ⟩;
  }
```

This code is used in section 1.

6. ⟨ Initialize the *mate* table 6 ⟩ ≡

```
  for (t = 2; t ≤ n; t++) mate[t] = t;
  mate[target→num] = 1, mate[1] = target→num;
```

This code is used in section 5.

7. ⟨ Initialize the queue 7 ⟩ ≡

```
  jj = ll = 1;
  mem[0] = mate[1];
  tail = 0, head = 1;
  serial = 2;
```

This code is used in section 5.

8. Each state for a particular arc gets a distinguishing number. Two states are special: 0 means the losing state, when a simple path is impossible; 1 means the winning state, when a simple path has been completed. The other states are 2 or more.

The output format on *stdout* simply shows the identifying numbers of a state and its two successors, in hexadecimal.

```
#define trunc(addr) ((addr) & (memsize − 1))

⟨ Process arc i 8 ⟩ ≡
  boundary = head, htcount = 0, htid = (i + wrap) ≪ logmemsize;
  if (htid ≡ 0) {
    for (hash = 0; hash < htsize; hash++) htable[hash] = 0;
    wrap++, htid = 1 ≪ logmemsize;
  }
  newserial = serial + ((head − tail)/(ll + 1 − jj));
  j = jj, k = arcto[i], l = ll;
  while (jj ≤ n ∧ firstarc[jj + 1] ≡ i + 1) jj++;
  ll = (k > l ? k : l);
  while (tail < boundary) {
    printf("%x:", serial);
    serial++;
    ⟨ Unpack a state, and move tail up 9 ⟩;
    ⟨ Print the successor if arc i is not chosen 11 ⟩;
    printf(",");
    ⟨ Print the successor if arc i is chosen 10 ⟩;
    printf("\n");
  }
```

This code is used in section 5.

9. If the target vertex hasn't entered the action yet (that is, if it exceeds l), we must update its *mate* entry at this point.

```
⟨ Unpack a state, and move tail up 9 ⟩ ≡
  for (t = j; t ≤ l; t++, tail++) {
    mate[t] = mem[trunc(tail)];
    if (mate[t] > l) mate[mate[t]] = t;
  }
```

This code is used in section 8.

10. Here's where we update the mates. The order of doing this is carefully chosen so that it works fine when $mate[j] = j$ and/or $mate[k] = k$.

```
⟨ Print the successor if arc i is chosen 10 ⟩ ≡
  jm = mate[j], km = mate[k];
  if (jm ≡ 0 ∨ km ≡ 0) printf("0");    /* we mustn't touch a saturated vertex */
  else if (jm ≡ k) ⟨ Print 1 or 0, depending on whether this arc wins or loses 12 ⟩
  else {
    mate[j] = 0, mate[k] = 0;
    mate[jm] = km, mate[km] = jm;
    printstate(j, jj, ll);
    mate[jm] = j, mate[km] = k, mate[j] = jm, mate[k] = km;    /* restore original state */
  }
  done:
```

This code is used in section 8.

11. ⟨ Print the successor if arc i is not chosen 11 ⟩ ≡

```
printstate(j, jj, ll);
```

This code is used in section 8.

12. See the note below regarding a change that will restrict consideration to Hamiltonian paths. A similar change is needed here.

⟨ Print 1 or 0, depending on whether this arc wins or loses 12 ⟩ ≡

```
{
  for (t = j + 1; t ≤ ll; t++)
    if (t ≠ k) {
      if (mate[t] ∧ mate[t] ≠ t) break;
    }
  if (t > ll) printf("1");   /* we win: this cycle is all by itself */
  else printf("0");   /* we lose: there's junk outside this cycle */
}
```

This code is used in section 10.

13. The *printstate* subroutine does the rest of the work. It makes sure that no incomplete paths linger in positions j through $jj - 1$, which are about to disappear; and it puts the contents of $mate[jj]$ through $mate[ll]$ into the queue, checking to see if it was already there.

If '$mate[t] \neq t$' is removed from the condition below, we get Hamiltonian paths only (I mean, simple paths that include every vertex).

⟨ Subroutines 13 ⟩ ≡

```
void printstate(int j, int jj, int ll)
{
  register int h, hh, ss, t, tt, hash;
  for (t = j; t < jj; t++)
    if (mate[t] ∧ mate[t] ≠ t) break;
  if (t < jj) printf("0");   /* incomplete junk mustn't be left hanging */
  else if (ll < jj) printf("0");   /* nothing is viable */
  else {
    ss = ll + 1 − jj;
    if (head + ss − tail > memsize) {
      fprintf(stderr, "Oops, I'm out of memory (memsize=%d, serial=%d)!\n", memsize, serial);
      fflush(stdout);
      exit(−69);
    }
    ⟨ Move the current state into position after head, and compute hash 14 ⟩;
    ⟨ Find the first match, hh, for the current state after boundary 15 ⟩;
    h = trunc(hh − boundary)/ss;
    printf("%x", newserial + h);
  }
}
```

This code is used in section 1.

14. ⟨ Move the current state into position after *head*, and compute *hash* 14 ⟩ ≡

```
for (t = jj, h = trunc(head), hash = 0; t ≤ ll; t++, h = trunc(h + 1)) {
  mem[h] = mate[t];
  hash = hash * 31415926525 + mate[t];
}
```

This code is used in section 13.

15. The hash table is automatically cleared whenever *htid* is increased, because we store *htid* with each relevant table entry.

⟨ Find the first match, *hh*, for the current state after *boundary* 15 ⟩ ≡

```
for (hash = hash & (htsize − 1); ; hash = (hash + 1) & (htsize − 1)) {
  hh = htable[hash];
  if ((hh ⊕ htid) ≥ memsize) ⟨ Insert new entry and goto found 16 ⟩;
  hh = trunc(hh);
  for (t = hh, h = trunc(head), tt = trunc(t + ss − 1); ; t = trunc(t + 1), h = trunc(h + 1)) {
    if (mem[t] ≠ mem[h]) break;
    if (t ≡ tt) goto found;
  }
}
found:
```

This code is used in section 13.

16. ⟨ Insert new entry and **goto** *found* 16 ⟩ ≡

```
{
  if (++htcount > (htsize ≫ 1)) {
    fprintf(stderr, "Sorry,␣the␣hash␣table␣is␣full␣(htsize=%d,␣serial=%d)!\n", htsize, serial);
    exit(−96);
  }
  hh = trunc(head);
  htable[hash] = htid + hh;
  head += ss;
  goto found;
}
```

This code is used in section 15.

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SIMPATH