

1. Intro. This program makes DLX data for MacMahon's problem of putting his 24 four-colored triangles into a hexagon, matching colors at the edges. The outer edge color is forced to be **a**. (It's a rewrite of the program that I wrote in September 2004.)

Actually I might as well make it more general, by allowing the hexagon to be replaced by any of the twelve double-size hexiamonds. The coordinates of the hexiamonds are specified on the command line.

I use the following coordinates for triangles: Those with apex at the top are (x, y) ; those with apex at the bottom are $(x, y)'$. If we think of a clock placed in the center of triangle (x, y) , it has edge neighbors $(x, y)'$ at 2 o'clock, $(x, y - 1)'$ at 6 o'clock, $(x - 1, y)'$ at 10 o'clock; it sees its nearest upright neighbors $(x, y + 1)$ at 1 o'clock, $(x + 1, y)$ at 3 o'clock, $(x + 1, y - 1)$ at 5 o'clock, $(x, y - 1)$ at 7 o'clock, $(x - 1, y)$ at 9 o'clock, $(x - 1, y + 1)$ at 11 o'clock. The transformation $(x, y) \mapsto (-y, x + y)'$, $(x, y)' \mapsto (-y, x + y + 1)$ rotates 60° about the lower left corner point of $(0, 0)$. (Putting (x, y) and $(x, y)'$ together in a parallelogram, then slanting the parallelogram into a square, gives normal Cartesian coordinates for the squares.)

The hexagon consists of Δ triangles (x, y) for $0 \leq x, y \leq 3$ and $2 \leq x + y \leq 5$, together with the ∇ triangles $(x, y)'$ for $0 \leq x, y \leq 3$ and $1 \leq x + y \leq 4$. To specify it on the command line, say this:

```
macmahon-triangles-dlx 00+ 10 10+ 01 01+ 11
```

[It's inconvenient to use the character `'` in a command line, so we use `+`.]

With change files I'll adapt the rules for edge matching. So I use a *mate* table that presently does nothing.

```
#include <stdio.h>
#include <stdlib.h>
char piece[24][4];
char occ[6][6], occp[6][6], edgeh[7][7], edgel[7][7], edger[7][7];
char mate[256];
main(int argc, char *argv[])
{
    register int i, j, k, l, x, y, z;
    <Set up the mate table 2>;
    <Generate the piece table 3>;
    <Process the command line 4>;
    <Output the item-name line 7>;
    for (j = 0; j < 6; j++)
        for (k = 0; k < 6; k++) {
            if (occ[j][k]) <Output the options for triangle (j, k) 8>;
            if (occp[j][k]) <Output the options for triangle (j, k)' 9>;
        }
    <Output the options for the boundary 10>;
}
```

2. <Set up the *mate* table 2> \equiv

```
mate['a'] = 'a';
mate['b'] = 'b';
mate['c'] = 'c';
mate['d'] = 'd';
```

This code is used in section 1.

```

3.  ⟨Generate the piece table 3⟩ ≡
    for (i = 0, j = 'a'; j ≤ 'd'; j++) {
        piece[i][0] = piece[i][1] = piece[i][2] = j, i++;
        for (k = 'a'; k ≤ 'd'; k++)
            if (j ≠ k) piece[i][0] = piece[i][1] = j, piece[i][2] = k, i++;
        for (k = j + 1; k ≤ 'd'; k++)
            for (l = k + 1; l ≤ 'd'; l++) {
                piece[i][0] = j, piece[i][1] = k, piece[i][2] = l, i++;
                piece[i][0] = j, piece[i][1] = l, piece[i][2] = k, i++;
            }
    }

```

This code is used in section 1.

```

4.  ⟨Process the command line 4⟩ ≡
    if (argc ≠ 7) {
        fprintf(stderr, "Usage: %s_t1_t2_t3_t3_t4_t5_t6\n", argv[0]);
        exit(-1);
    }
    for (j = 1; j ≤ 6; j++) {
        x = 2 * (argv[j][0] - '0'), y = 2 * (argv[j][1] - '0');
        if (argv[j][2] ≡ '\0') z = 0;
        else if (argv[j][2] ≡ '+') z = 1;
        else {
            fprintf(stderr, "Triangle '%s' should have the form xy or xy+!\n", argv[j]);
            exit(-2);
        }
        if (x < 0 ∨ x > 4 ∨ y < 0 ∨ y > 4) {
            fprintf(stderr, "Triangle '%s' should have coordinates between 0 and 3!\n", argv[j]);
            exit(-3);
        }
        ⟨Set the occupied table from x and y 5⟩;
    }
    ⟨Set the edge tables from occ and occp 6⟩;
    printf(" | %s %s %s %s %s %s\n", argv[0], argv[1], argv[2], argv[3], argv[4], argv[5], argv[6]);

```

This code is used in section 1.

```

5.  ⟨Set the occupied table from x and y 5⟩ ≡
    if (occ[x + z][y + z]) {
        fprintf(stderr, "Triangle '%s' has been specified twice!\n", argv[j]);
        exit(-4);
    }
    occ[x + z][y + z] = occp[x + z][y + z] = 1;
    if (z) occp[x][y + 1] = occp[x + 1][y] = 1;
    else occ[x][y + 1] = occ[x + 1][y] = 1;

```

This code is used in section 4.

6. \langle Set the edge tables from *occ* and *occp* 6 $\rangle \equiv$

```

for ( $x = 0$ ;  $x < 6$ ;  $x++$ )
  for ( $y = 0$ ;  $y < 6$ ;  $y++$ ) {
     $edgeh[x][y] += occ[x][y]$ ,  $edgel[x][y] += occ[x][y]$ ,  $edger[x][y] += occ[x][y]$ ;
     $edgeh[x][y+1] += occp[x][y]$ ,  $edgel[x][y] += occp[x][y]$ ,  $edger[x+1][y] += occp[x][y]$ ;
  }

```

This code is used in section 4.

7. There's a primary item $*$ for forcing the boundary condition. There's a primary item xy or xy' for each triangle. There's a primary item abc for each piece. There's a secondary item for each edge, denoting the color on that edge; the edges are $-xy$, $/xy$, $\backslash xy$ for the horizontal, forward-leaning, or backward-leaning edges that surround triangle (x, y) .

\langle Output the item-name line 7 $\rangle \equiv$

```

   $printf("*\square");$ 
  for ( $j = 0$ ;  $j < 6$ ;  $j++$ )
    for ( $k = 0$ ;  $k < 6$ ;  $k++$ ) {
      if ( $occ[j][k]$ )  $printf("%d\%d\square", j, k)$ ;
      if ( $occp[j][k]$ )  $printf("%d\%d'\square", j, k)$ ;
    }
  for ( $i = 0$ ;  $i < 24$ ;  $i++$ )  $printf("%s\square", piece[i])$ ;
   $printf(" | ");$ 
  for ( $j = 0$ ;  $j < 7$ ;  $j++$ )
    for ( $k = 0$ ;  $k < 7$ ;  $k++$ ) {
      if ( $edgeh[j][k]$ )  $printf("\square-\%d\%d", j, k)$ ;
      if ( $edger[j][k]$ )  $printf("\square/\%d\%d", j, k)$ ;
      if ( $edgel[j][k]$ )  $printf("\square\backslash\%d\%d", j, k)$ ;
    }
   $printf("\n");$ 

```

This code is used in section 1.

8. \langle Output the options for triangle (j, k) 8 $\rangle \equiv$

```

  for ( $i = 0$ ;  $i < 24$ ;  $i++$ ) {
     $printf("%d\%d\square\square-\%d\%d:\%c\square/\%d\%d:\%c\square\backslash\%d\%d:\%c\n", j, k, piece[i], j, k, piece[i][0], j, k, piece[i][1], j, k,$ 
       $piece[i][2])$ ;
    if ( $piece[i][1] \neq piece[i][2]$ ) {
       $printf("%d\%d\square\square-\%d\%d:\%c\square/\%d\%d:\%c\square\backslash\%d\%d:\%c\n", j, k, piece[i], j, k, piece[i][1], j, k, piece[i][2], j,$ 
         $k, piece[i][0])$ ;
       $printf("%d\%d\square\square-\%d\%d:\%c\square/\%d\%d:\%c\square\backslash\%d\%d:\%c\n", j, k, piece[i], j, k, piece[i][2], j, k, piece[i][0], j,$ 
         $k, piece[i][1])$ ;
    }
  }

```

This code is used in section 1.

9. \langle Output the options for triangle $(j, k)'$ 9 $\rangle \equiv$

```

for ( $i = 0$ ;  $i < 24$ ;  $i++$ ) {
    printf ("%d%d' %s-%d%d:%c_/%d%d:%c_\\%d%d:%c\n",  $j, k, piece[i], j, k + 1, mate[piece[i][0]], j + 1, k,$ 
            $mate[piece[i][1]], j, k, mate[piece[i][2]]$ );
    if ( $piece[i][1] \neq piece[i][2]$ ) {
        printf ("%d%d' %s-%d%d:%c_/%d%d:%c_\\%d%d:%c\n",  $j, k, piece[i], j, k + 1, mate[piece[i][1]], j + 1,$ 
               $k, mate[piece[i][2]], j, k, mate[piece[i][0]]$ );
        printf ("%d%d' %s-%d%d:%c_/%d%d:%c_\\%d%d:%c\n",  $j, k, piece[i], j, k + 1, mate[piece[i][2]], j + 1,$ 
               $k, mate[piece[i][0]], j, k, mate[piece[i][1]]$ );
    }
}

```

This code is used in section 1.

10. The boundary edges all are colored a. (A text editor could change this.)

\langle Output the options for the boundary 10 $\rangle \equiv$

```

printf ("*");
for ( $j = 0$ ;  $j < 7$ ;  $j++$ )
    for ( $k = 0$ ;  $k < 7$ ;  $k++$ ) {
        if ( $edgeh[j][k] \equiv 1$ ) printf ("%d-%d:%c",  $j, k, \neg occ[j][k] ? mate['a'] : 'a'$ );
        if ( $edgel[j][k] \equiv 1$ ) printf ("%d\\%d%d:%c",  $j, k, \neg occ[j][k] ? mate['a'] : 'a'$ );
        if ( $edger[j][k] \equiv 1$ ) printf ("%d/%d%d:%c",  $j, k, \neg occ[j][k] ? mate['a'] : 'a'$ );
    }
printf ("\n");

```

This code is used in section 1.

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