

(Downloaded from <https://cs.stanford.edu/~knuth/programs.html> and typeset on May 28, 2023)

**1. Intro.** I'm hurriedly experimenting with a new(?) way to explore the complexity of 4-variable Boolean functions. Namely, I calculate the “footprint” of each function, the set of all first steps by which I know how to evaluate the function in  $k$  steps. Then, if the footprints of  $f$  and  $g$  overlap, I can compute  $f \circ g$  in  $\text{cost}(f) + \text{cost}(g)$  steps.

I can restrict consideration to the  $2^{15}$  functions that take  $(0, 0, 0, 0) \mapsto 0$ .

This program extends FCHAINS4 by allowing several additional functions to be precomputed. Those functions appear on the command line, in hexadecimal form.

```
#define footsize 100
#include <stdio.h>
#include <stdlib.h>
typedef struct node_struct {
    unsigned int footprint[footsize];
    int parent;
    int cost;
    struct node_struct *prev, *next;
} node;
node func[1 << 15];
node head[9];
int x[100];
char buf[100]; /* lines of input */
char name[32 * footsize][16];
unsigned int ttt; /* truth table found in input line */
main(int argc, char *argv[])
{
    register int c, j, k, r, t, m, mm, s;
    register unsigned int u;
    register node *p, *q, *pp;
    <Read the initial functions 2>;
    <Initialize the tables 8>;
    for (r = 2; c; r++)
        for (k = (r - 1) >> 1; k ≥ 0; k--) <Combine all functions of costs k and r - 1 - k 3>;
    <Answer queries 12>;
}
```

**2.** <Read the initial functions 2>  $\equiv$

```
m = argc + 3;
for (k = 1; k ≤ m; k++) {
    if (k ≤ 4) x[k] = #ffff / ((1 << (1 << (4 - k))) + 1);
    else if (sscanf(argv[k - 4], "%x", &x[k]) ≠ 1) {
        fprintf(stderr, "Parameter %s should have been hexadecimal!\n", argv[k - 4]);
        exit(-1);
    }
    if (x[k] > #ffff) {
        fprintf(stderr, "Parameter %s is too big!\n", argv[k - 4]);
        exit(-1);
    }
    if (x[k] ≥ #8000) x[k] ⊕= #ffff;
}
```

This code is used in section 1.

```

3.  ⟨ Combine all functions of costs  $k$  and  $r - 1 - k$  3 ⟩ ≡
    for ( $p = \text{head}[k].\text{next}$ ;  $p\text{-parent} \geq 0$ ;  $p = p\text{-next}$ )
      for ( $q = \text{head}[r - 1 - k].\text{next}$ ;  $q\text{-parent} \geq 0$ ;  $q = q\text{-next}$ ) {
        for ( $j = 0$ ;  $j < mm$ ;  $j++$ )
          if ( $p\text{-footprint}[j] \& q\text{-footprint}[j]$ ) ⟨ Try for breakthru and goto pqdone 6 ⟩
        ⟨ Try for new function 4 ⟩;
      }
    pqdone: continue;
  }

```

This code is used in section 1.

```

4.  #define fun(p) ((p) - func)
    ⟨ Try for new function 4 ⟩ ≡
    {
      t = fun(p) & fun(q);
      if (func[t].cost ≥ r) ⟨ Update the table for cost r 5 ⟩;
      t = fun(p) & (~fun(q));
      if (func[t].cost ≥ r) ⟨ Update the table for cost r 5 ⟩;
      t = (~fun(p)) & fun(q);
      if (func[t].cost ≥ r) ⟨ Update the table for cost r 5 ⟩;
      t = fun(p) | fun(q);
      if (func[t].cost ≥ r) ⟨ Update the table for cost r 5 ⟩;
      t = fun(p) ⊕ fun(q);
      if (func[t].cost ≥ r) ⟨ Update the table for cost r 5 ⟩;
    }

```

This code is used in section 3.

```

5.  ⟨ Update the table for cost r 5 ⟩ ≡
    {
      pp = &func[t];
      if (pp-cost > r) {
        if (pp-cost ≡ 8) c--;
        pp-next-prev = pp-prev, pp-prev-next = pp-next;
        pp-cost = r, pp-parent = (fun(p) ≪ 16) + fun(q);
        for (j = 0; j < mm; j++) pp-footprint[j] = 0;
        pp-next = head[r].next, pp-prev = &head[r];
        pp-next-prev = pp, pp-prev-next = pp;
      }
      for (j = 0; j < mm; j++) pp-footprint[j] |= p-footprint[j] | q-footprint[j];
    }

```

This code is used in section 4.

6.  $\langle \text{Try for breakthru and goto } p\text{qdone } 6 \rangle \equiv$

```

{
  t = fun(p) & fun(q);
  if (func[t].cost ≥ r - 1)  $\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle$ ;
  t = fun(p) & (~fun(q));
  if (func[t].cost ≥ r - 1)  $\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle$ ;
  t = (~fun(p)) & fun(q);
  if (func[t].cost ≥ r - 1)  $\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle$ ;
  t = fun(p) | fun(q);
  if (func[t].cost ≥ r - 1)  $\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle$ ;
  t = fun(p) ⊕ fun(q);
  if (func[t].cost ≥ r - 1)  $\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle$ ;
  goto pqdone;
}

```

This code is used in section 3.

7. This code is not executed when  $k = 0$ , because  $q$ 's footprint is zero in that case.

$\langle \text{Update the table for cost } r - 1 \text{ } 7 \rangle \equiv$

```

{
  pp = &func[t];
  if (pp-cost > r - 1) {
    if (pp-cost ≡ 8) c--;
    pp-next-prev = pp-prev, pp-prev-next = pp-next;
    pp-cost = r - 1, pp-parent = (fun(p) ≪ 16) + fun(q);
    for (j = 0; j < mm; j++) pp-footprint[j] = 0;
    pp-next = head[r - 1].next, pp-prev = &head[r - 1];
    pp-next-prev = pp, pp-prev-next = pp;
  }
  for (j = 0; j < mm; j++) pp-footprint[j] |= p-footprint[j] & q-footprint[j];
}

```

This code is used in section 6.

8.  $\langle \text{Initialize the tables } 8 \rangle \equiv$

```

for (p = &func[2]; p < &func[#8000]; p++) (p - 1)-next = p, p-prev = p - 1, p-cost = 8;
func[1].cost = 8;
for (k = 0; k ≤ 8; k++) head[k].parent = -1, head[k].next = head[k].prev = &head[k];
head[0].next = head[0].prev = &func[0];
func[0].next = func[0].prev = &head[0];
head[8].next = &func[1], func[1].prev = &head[8];
head[8].prev = &func[#7fff], func[#7fff].next = &head[8];
 $\langle \text{Initialize the functions of cost } 0 \text{ } 9 \rangle$ ;
 $\langle \text{Initialize the functions of cost } 1 \text{ } 10 \rangle$ ;

```

This code is used in section 1.

9.  $\langle \text{Initialize the functions of cost 0 } \textcolor{blue}{9} \rangle \equiv$

```

for ( $k = 1$ ;  $k \leq m$ ;  $k++$ ) {
     $p = \&func[x[k]]$ ;
    if ( $p \rightarrow cost \equiv 0$ ) continue;
     $p \rightarrow next \rightarrow prev = p \rightarrow prev, p \rightarrow prev \rightarrow next = p \rightarrow next$ ;
     $p \rightarrow cost = 0$ ;
     $p \rightarrow next = head[0].next, p \rightarrow prev = \&head[0]$ ;
     $p \rightarrow next \rightarrow prev = p, p \rightarrow prev \rightarrow next = p$ ;
}
 $c = (1 \ll 15) - 1 - m$ ;

```

This code is used in section 8.

10.  $\langle \text{Initialize the functions of cost 1 } \textcolor{blue}{10} \rangle \equiv$

```

 $s = 0$ ;
for ( $r = 2$ ;  $r \leq m$ ;  $r++$ )
    for ( $k = 1$ ;  $k < r$ ;  $k++$ ) {
         $t = x[k] \& x[r], \text{sprintf}(name[s], "\%d\&\%d(\%04x)", k, r, t)$ ;
         $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle$ ;
         $t = x[k] \& (\sim x[r]), \text{sprintf}(name[s], "\%d\>\%d(\%04x)", k, r, t)$ ;
         $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle$ ;
         $t = (\sim x[k]) \& x[r], \text{sprintf}(name[s], "\%d\<\%d(\%04x)", k, r, t)$ ;
         $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle$ ;
         $t = x[k] | x[r], \text{sprintf}(name[s], "\%d|\%d(\%04x)", k, r, t)$ ;
         $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle$ ;
         $t = x[k] \oplus x[r], \text{sprintf}(name[s], "\%d\^{\sim}\%d(\%04x)", k, r, t)$ ;
         $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle$ ;
    }
 $mm = (s + 31) / 32$ ;

```

This code is used in section 8.

11.  $\langle \text{Update for cost 1 } \textcolor{blue}{11} \rangle \equiv$

```

 $p = \&func[t]$ ;
if ( $p \rightarrow cost > 1$ ) {
    if ( $s \geq 32 * \text{footsize}$ ) {
         $\text{fprintf}(\text{stderr}, "Too\_many\_special\_functions\_(\text{footsize}=\%d)!\backslash n", \text{footsize})$ ;
         $\text{exit}(-3)$ ;
    }
     $p \rightarrow next \rightarrow prev = p \rightarrow prev, p \rightarrow prev \rightarrow next = p \rightarrow next$ ;
     $p \rightarrow cost = 1, p \rightarrow parent = (x[k] \ll 16) + x[r]$ ;
     $p \rightarrow footprint[s \gg 5] = 1 \ll (s \& \#1f)$ ;
     $p \rightarrow next = head[1].next, p \rightarrow prev = \&head[1]$ ;
     $p \rightarrow next \rightarrow prev = p, p \rightarrow prev \rightarrow next = p$ ;
     $s++$ ;
     $c--$ ;
}

```

This code is used in section 10.

12.  $\langle \text{Answer queries } 12 \rangle \equiv$

```

while (1) {
    printf("Truth_table_(hex):_");
    fflush(stdout);
    if (!fgets(buf, 100, stdin)) break;
    if (sscanf(buf, "%x", &ttt) != 1) break;
    printf("%04x_has_cost_", ttt);
    if (ttt & #8000) ttt ^= #ffff;
    printf("%d,_parents_(%04x,%04x),_and_footprint", func[ttt].cost, func[ttt].parent >> 16,
           func[ttt].parent & #ffff);
    for (j = 0; j < mm; j++)
        if (func[ttt].footprint[j]) {
            s = 32 * j;
            for (u = func[ttt].footprint[j]; u; u >>= 1, s++)
                if (u & 1) printf("_%s", name[s]);
        }
    printf("\n");
}

```

This code is used in section 1.

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