(See https://cs.stanford.edu/~knuth/programs.html for date.)

1. Introduction. I'm reregenerating the illustrations for my paper in the Transactions on Graphics. This program has little generality, but it could be easily modified.

```
#define m 360   /* this many rows */
#define n 250   /* this many columns */
#define lisacode 1   /* say 1 for Mona Lisa */
#define spherecode 2   /* say 2 for the sphere */
#define fscode 1   /* say 1 for Floyd-Steinberg */
#define odithcode 2   /* say 2 for ordered dither */
#define ddiffcode 3   /* say 3 for dot diffusion */
#define sdiffcode 4   /* say 4 for smooth dot diffusion */
#define ariescode 5   /* say 5 for ARIES */

#include <gb_graph.h>
#include <gb_lisa.h>
#include <math.h>
#include <time.h>
  ⟨ Preprocessor definitions ⟩
  time_t clokk;
  double A[m + 2][256];   /* pixel data (darknesses), bordered by zero */
  int board[10][10];
  Graph *gg;
  int kk;
  ⟨ Global variables 6 ⟩
  ⟨ Subroutines 7 ⟩
  main(argc, argv)
      int argc;
      char *argv[];
  {
    register int i, j, k, l, ii, jj;
    register double err;
    register Graph *g;
    register Vertex *u, *v;
    register Arc *a;
    int imagecode, sharpcode, methodcode;
    ⟨ Scan the command line, give help if necessary 2 ⟩;
    ⟨ Input the image 3 ⟩;
    ⟨ Sharpen if requested 4 ⟩;
    ⟨ Generate and print the base matrix, if any 5 ⟩;
    ⟨ Compute the answer 33 ⟩;
    ⟨ Spew out the answers 29 ⟩;
    ⟨ Print relevant statistics 34 ⟩;
  }
```

2. ⟨ Scan the command line, give help if necessary 2 ⟩ ≡

```
if (argc ≠ 4 ∨ sscanf(argv[1], "%d", &imagecode) ≠ 1 ∨
        sscanf(argv[2], "%d", &sharpcode) ≠ 1 ∨
        sscanf(argv[3], "%d", &methodcode) ≠ 1) {
usage: fprintf(stderr, "Usage:␣%s␣imagecode␣sharpcode␣methodcode\n", argv[0]);
  fprintf(stderr, "␣Mona␣Lisa␣=␣%d,␣Sphere␣=␣%d\n", lisacode, spherecode);
  fprintf(stderr, "␣unretouched␣=␣0,␣edges␣enhanced␣=␣1\n");
  fprintf(stderr, "␣Floyd-Steinberg␣=␣%d,␣ordered␣dither␣=␣%d,\n", fscode, odithcode);
  fprintf(stderr, "␣dot␣diffusion␣=␣%d,␣smooth␣dot␣diffusion␣=␣%d,\n", ddiffcode, sdiffcode);
  fprintf(stderr, "␣ARIES␣=␣%d\n", ariescode);
  exit(0);
}
```

This code is used in section 1.

3. ⟨ Input the image 3 ⟩ ≡

```
if (imagecode ≡ lisacode) { Area workplace;
  register int *mtx = lisa(m, n, 255, 0, 0, 0, 0, 0, 0, workplace);
  for (i = 0; i < m; i++)
    for (j = 0; j < n; j++) A[i + 1][j + 1] = pow(1.0 − (*(mtx + i * n + j) + 0.5)/256.0, 2.0);
  fprintf(stderr, "(Mona␣Lisa␣image␣loaded)\n");
}
else if (imagecode ≡ spherecode) {
  for (i = 1; i ≤ m; i++)
    for (j = 1; j ≤ n; j++) {
      register double x = (i − 120.0)/111.5, y = (j − 120.0)/111.5;
      if (x * x + y * y ≥ 1.0) A[i][j] = (1500.0 * i + j * j)/1000000.0;
      else A[i][j] = (9.0 + x − 4.0 * y − 8.0 * sqrt(1.0 − x * x − y * y))/18.0;
    }
  fprintf(stderr, "(Sphere␣image␣loaded)\n");
}
else goto usage;
```

This code is used in section 1.

4. ⟨ Sharpen if requested 4 ⟩ ≡

```
if (sharpcode ≡ 1) {
  for (i = 1; i ≤ m; i++)
    for (j = 1; j ≤ n; j++) A[i − 1][j − 1] = 9 * A[i][j] −
          (A[i − 1][j − 1] + A[i − 1][j] + A[i − 1][j + 1] + A[i][j − 1] +
          A[i][j + 1] + A[i + 1][j − 1] + A[i + 1][j] + A[i + 1][j + 1]);
  for (i = m; i > 0; i−−)
    for (j = n; j > 0; j−−)
      A[i][j] = (A[i − 1][j − 1] ≤ 0.0 ? 0.0 : A[i − 1][j − 1] ≥ 1.0 ? 1.0 : A[i − 1][j − 1]);
  for (i = 0; i < m; i++) A[i][0] = 0.0;
  for (j = 1; j < n; j++) A[0][j] = 0.0;
  fprintf(stderr, "(with␣enhanced␣edges)\n");
}
else if (sharpcode ≡ 0) fprintf(stderr, "(no␣sharpening)\n");
else goto usage;
```

This code is used in section 1.

5. ⟨ Generate and print the base matrix, if any 5 ⟩ ≡

```
switch (methodcode) {
case fscode: fprintf(stderr, "(using␣Floyd-Steinberg␣error␣diffusion)\n"); goto done;
case odithcode: fprintf(stderr, "(using␣ordered␣dithering)\n");
  for (i = 0; i < 4; i++)
    for (j = 0; j < 4; j++)
      for (k = 0; k < 4; k++) {
        ii = 4 * di[k] + 2 * di[j] + di[i] + 2;
        jj = 4 * dj[k] + 2 * dj[j] + dj[i] + 2;
        kk = 16 * i + 4 * j + k;
        board[8 − (jj & 7)][1 + (ii & 7)] = kk;
      }
  goto finishit;
case ddiffcode: fprintf(stderr, "(using␣dot␣diffusion)\n"); break;
case sdiffcode: fprintf(stderr, "(using␣smooth␣dot␣diffusion)\n"); break;
case ariescode: fprintf(stderr, "(using␣ARIES)\n"); break;
default: goto usage;
}
⟨ Set up the board for dot diffusion 9 ⟩;
finishit:
for (i = 1; i ≤ 8; i++) board[i][0] = board[i][8], board[i][9] = board[i][1];
for (j = 0; j ≤ 9; j++) board[0][j] = board[8][j], board[9][j] = board[1][j];
if (methodcode ≥ ddiffcode) ⟨ Install the vertices and arcs of the control graph 11 ⟩;
⟨ Print the board 10 ⟩;
done:
```

This code is used in section 1.

6. ⟨ Global variables 6 ⟩ ≡

```
int di[4] = {0, 1, 0, 1};
int dj[4] = {0, 1, 1, 0};
```

See also sections 8, 14, 16, 19, and 26.

This code is used in section 1.

7. ⟨ Subroutines 7 ⟩ ≡

```
void store(i, j)
    int i, j;
{
  Vertex *v;
  if (i < 1) i += 8; else if (i > 8) i -= 8;
  if (j < 1) j += 8; else if (j > 8) j -= 8;
  board[i][j] = kk;
  v = gg→vertices + kk;
  sprintf(name_buffer, "%d", kk);
  v→name = gb_save_string(name_buffer);
  v→row = i; v→col = j;
  kk++;
}
void store_eight(i, j)
    int i, j;
{
  store(i, j); store(i − 4, j + 4); store(1 − j, i − 4); store(5 − j, i);
  store(j, 5 − i); store(4 + j, 1 − i); store(5 − i, 5 − j); store(1 − i, 1 − j);
}
```

See also section 25.

This code is used in section 1.

8. ⟨ Global variables 6 ⟩ +≡

```
char name_buffer[] = "99";
```

9.

```
#define row u.I
#define col v.I
#define weight w.I
#define del_i a.I
#define del_j b.I
```

⟨ Set up the board for dot diffusion 9 ⟩ ≡

```
kk = 0;
gg = g = gb_new_graph(64);
store_eight(7, 2); store_eight(8, 3); store_eight(8, 2); store_eight(8, 1);
store_eight(1, 4); store_eight(1, 3); store_eight(1, 2); store_eight(2, 3);
```

This code is used in section 5.

10. ⟨ Print the board 10 ⟩ ≡

```
for (i = 1; i ≤ 8; i++) {
  for (j = 1; j ≤ 8; j++) fprintf(stderr, "␣%2d", board[i][j]);
  fprintf(stderr, "\n");
}
```

This code is used in section 5.

11. ⟨Install the vertices and arcs of the control graph 11⟩ ≡

```
if (methodcode ≡ ddiffcode) {   /* dot diffusion, two dots per 8 × 8 cell */
  for (v = g→vertices; v < g→vertices + 64; v++) {
    i = v→row;
    j = v→col;
    v→weight = 0;
    for (ii = i − 1; ii ≤ i + 1; ii++)
      for (jj = j − 1; jj ≤ j + 1; jj++) {
        u = g→vertices + board[ii][jj];
        if (u > v) {
          gb_new_arc(v, u, 0);
          v→arcs→del_i = ii − i;
          v→arcs→del_j = jj − j;
          v→weight += 3 − (ii − i) * (ii − i) − (jj − j) * (jj − j);
        }
      }
  }
}
else {   /* each vertex has a neighborhood covering 32 classes */
  for (v = g→vertices; v < g→vertices + 64; v++) {
    i = v→row;
    j = v→col;
    for (jj = j − 3; jj ≤ j + 3; jj++) { register int del = (jj < j ? j − jj : jj − j);
      for (ii = i − 3 + del; ii ≤ i + 4 − del; ii++) {
        u = g→vertices + board[ii & 7][jj & 7];
        if (u > v) {
          gb_new_arc(v, u, 0);
          v→arcs→del_i = ii − i;
          v→arcs→del_j = jj − j;
        }
      }
    }
  }
  for (i = 0; i < 10; i++)
    for (j = 0; j < 10; j++) board[i][j] ≫= 1;
}
```

This code is used in section 5.

12. Error diffusion. The Floyd-Steinberg algorithm uses a threshold of 0.5 at each pixel and distributes the error to the four unprocessed neighbors.

```
#define alpha 0.4375    /* 7/16, error diffusion to E neighbor */
#define beta 0.1875     /* 3/16, error diffusion to SW neighbor */
#define gamma 0.3125    /* 5/16, error diffusion to S neighbor */
#define delta 0.0625    /* 1/16, error diffusion to SE neighbor */
#define check(i, j)
        {
          if (A[i][j] < lo_A) lo_A = A[i][j];
          if (A[i][j] > hi_A) hi_A = A[i][j];
        }
```

```
⟨Do Floyd-Steinberg 12⟩ ≡
  for (i = 1; i ≤ m; i++)
    for (j = 1; j ≤ n; j++) {
      err = A[i][j];
      if (err ≥ .5) err −= 1.0;
      A[i][j] −= err;    /* now it's 0 or 1 */
      A[i][j + 1] += err * alpha; check(i, j + 1);
      A[i + 1][j − 1] += err * beta; check(i + 1, j − 1);
      A[i + 1][j] += err * gamma; check(i + 1, j);
      A[i + 1][j + 1] += err * delta; check(i + 1, j + 1);
    }
```

This code is used in section 33.

13. ⟨Print boundary leakage and extreme values 13⟩ ≡

```
  if (methodcode ≠ sdiffcode) {
    for (i = 0; i ≤ m + 1; i++) edge_accum += fabs(A[i][0]) + fabs(A[i][n + 1]);
    for (j = 1; j ≤ n; j++) edge_accum += fabs(A[0][j]) + fabs(A[m + 1][j]);
  }
  fprintf(stderr, "Total␣leakage␣at␣boundaries:␣%.20g\n", edge_accum);
  fprintf(stderr, "Data␣remained␣between␣%.20g␣and␣%.20g\n", lo_A, hi_A);
```

This code is used in section 34.

14. ⟨Global variables 6⟩ +≡

```
  double edge_accum;
  double lo_A = 100000.0, hi_A = −100000.0;    /* record-breaking values */
```

15. Ordered dithering. The ordered dither algorithm uses a threshold based on the pixel's place in the grid.

```
⟨ Do ordered dither 15 ⟩ ≡
  for (i = 1; i ≤ m; i++)
    for (j = 1; j ≤ n; j++) {
      k = board[i & 7][j & 7];
      err = A[i][j];
      if (err ≥ (k + 0.5)/64.0) err −= 1.0;
      A[i][j] −= err;   /* now it's 0 or 1 */
      accum += fabs(err);   /* accumulate undiffused error */
      block_err[(i − 1) ≫ 3][(j − 1) ≫ 3] += err;   /* accumulate error in 8 × 8 block */
    }
```

This code is used in section 33.

16. ⟨ Global variables 6 ⟩ +≡

```
  double accum;
  double block_err[(m + 7) ≫ 3][(n + 7) ≫ 3];
  int bad_blocks;
```

17. ⟨ Print accumulated lossage 17 ⟩ ≡

```
  fprintf(stderr, "Total␣undiffused␣error:␣%.20g\n", accum);
  for (i = 0, accum = 0.0; i < m; i += 8)
    for (j = 0; j < n; j += 8) {
      if (fabs(block_err[i ≫ 3][j ≫ 3]) > 1.0) bad_blocks++;
      accum += fabs(block_err[i ≫ 3][j ≫ 3]);
    }
  fprintf(stderr, "Total␣block␣error:␣%.20g␣(%d␣bad)\n", accum, bad_blocks);
```

This code is used in section 34.

18. Dot diffusion. The dot diffusion algorithm uses a fixed threshold of 0.5 and distributes errors to higher-class neighbor pixels, except at baron positions.

⟨ Do dot diffusion 18 ⟩ ≡

```
for (v = g→vertices; v < g→vertices + 64; v++)
  for (i = v→row; i ≤ m; i += 8)
    for (j = v→col; j ≤ n; j += 8) {
      err = A[i][j];
      if (err ≥ .5) err −= 1.0;
      A[i][j] −= err;    /* now it's 0 or 1 */
      if (v→arcs) ⟨ Distribute the error to near neighbors 20 ⟩
      else {    /* baron */
        accum += fabs(err);
        barons++;
        if (fabs(err) > 0.5) bad_barons++;
        if (err < lo_err) lo_err = err;
        if (err > hi_err) hi_err = err;
      }
    }
```

This code is used in section 33.

19. ⟨ Global variables 6 ⟩ +≡

```
int barons;    /* how many barons are there? */
int bad_barons;    /* how many of them eat more than 0.5 error? */
double lo_err = 100000.0, hi_err = −100000.0;    /* record-breaking errors */
```

20. ⟨ Distribute the error to near neighbors 20 ⟩ ≡

```
for (a = v→arcs; a; a = a→next) {
  ii = i + a→del_i; jj = j + a→del_j;
  A[ii][jj] += err * (double)(3 − a→del_i * a→del_i − a→del_j * a→del_j)/(double) v→weight;
  check(ii, jj);
}
```

This code is used in section 18.

21. Smooth dot diffusion is similar, but it uses a class-based threshold and considers a larger neighborhood of size 32.

⟨ Do smooth dot diffusion 21 ⟩ ≡

```
for (v = g→vertices; v < g→vertices + 64; v++)
  for (i = v→row; i ≤ m; i += 8)
    for (j = v→col; j ≤ n; j += 8) {
      k = (v − g→vertices) ≫ 1;     /* class number */
      err = A[i][j];
      if (err ≥ .5/(double)(32 − k)) err −= 1.0;
      A[i][j] −= err;     /* now it's 0 or 1 */
      if (v→arcs) ⟨ Distribute the error to dot neighbors 22 ⟩
      else {     /* baron */
        accum += fabs(err);
        barons++;
        if (fabs(err) > 0.5) bad_barons++;
        if (err < lo_err) lo_err = err;
        if (err > hi_err) hi_err = err;
      }
    }
```

This code is used in section 33.

22. This pixel has $31 - k$ neighbors of higher classes; each shares equally in the distribution.

⟨ Distribute the error to dot neighbors 22 ⟩ ≡

```
for (a = v→arcs; a; a = a→next) {
  ii = i + a→del_i; jj = j + a→del_j;
  if (ii > 0 ∧ ii ≤ m ∧ jj > 0 ∧ jj ≤ n) {
    A[ii][jj] += err/(double)(31 − k); check(ii, jj);
  }
  else edge_accum += fabs(err);     /* error leaks out the boundary */
}
```

This code is used in section 21.

23. ⟨ Print baronial lossage 23 ⟩ ≡

```
fprintf(stderr, "Total␣undiffused␣error␣%.20g␣at␣%d␣barons\n", accum, barons);
fprintf(stderr, "␣␣(%d␣bad,␣min␣%.20g,␣max␣%.20g)\n", bad_barons, lo_err, hi_err);
```

This code is used in section 34.

24. Alias-Reducing Image-Enhancing Screening. The ARIES method works with 32-pixel dots and dithers them but adjusts the threshold by considering the average intensity in the dot.

⟨ Do ARIES 24 ⟩ ≡
  for $(i = -1;\ i \le m + 3;\ i \mathrel{+}= 4)$
    for $(j = (i \,\&\, 4)\ ?\ 2 : -2;\ j \le n + 3;\ j \mathrel{+}= 8)$ { **double** $s = 0.5$;
      $ll = 0$;   /* number of cells in current dot */
      for $(jj = j - 3;\ jj \le j + 3;\ jj{+}{+})$ { **register int** $del = (jj < j\ ?\ j - jj : jj - j)$;
        for $(ii = i - 3 + del;\ ii \le i + 4 - del;\ ii{+}{+})$
          if $(ii > 0 \wedge ii \le m \wedge jj > 0 \wedge jj \le n)$ $s \mathrel{+}= A[ii][jj], rank(ii, jj)$;
      }
      ⟨ Blacken the top $\lfloor s \rfloor$ pixels of the dot 27 ⟩;
    }

This code is used in section 33.

25. The ranking procedure sorts the entries by the key $a_{ij} - k/32$, where k is the class number of cell (i, j).

⟨ Subroutines 7 ⟩ +≡
  rank(i, j)
      int $i,\ j$;
  {
    register double $key = A[i][j] - board[i \,\&\, 7][j \,\&\, 7]/32.0$;
    register int l;
    for $(l = ll;\ l > 0;\ l{-}{-})$
      if $(key \ge val[l - 1])$ **break**;
      else $inxi[l] = inxi[l - 1], inxj[l] = inxj[l - 1], val[l] = val[l - 1]$;
    $inxi[l] = i;\ inxj[l] = j;\ val[l] = key;\ ll{+}{+}$;
  }

26. ⟨ Global variables 6 ⟩ +≡
  int ll;   /* the number of items in the ranking table */
  int $inxi[32],\ inxj[32]$;   /* indices of the ranked pixels */
  double $val[32]$;   /* keys of the ranked pixels */

27. I have to admit that I rather like this implementation of ARIES!

⟨ Blacken the top $\lfloor s \rfloor$ pixels of the dot 27 ⟩ ≡
  if (ll) { $barons{+}{+}$; $accum \mathrel{+}= fabs(s - 0.5 - (\textbf{int})\ s)$; }
  while $(ll > 0)$ {
    $ll{-}{-};\ s \mathrel{-}= 1.0$;
    $ii = inxi[ll];\ jj = inxj[ll]$;
    $err = A[ii][jj]$;
    if $(s \ge 0.0)$ $err \mathrel{-}= 1.0$;
    $A[ii][jj] \mathrel{-}= err$;   /* now it's 0 or 1 */
  }

This code is used in section 24.

28. ⟨ Print ARIES lossage 28 ⟩ ≡
  fprintf(*stderr*, "Total␣lossage␣%.20g␣in␣%d␣dots\n", *accum*, *barons*);

This code is used in section 34.

29. Encapsulated PostScript. When all has been done (but all has not necessarily been said), we output the matrix as a PostScript file with resolution 72 pixels per inch.

⟨ Spew out the answers 29 ⟩ ≡
  ⟨ Output the header of the EPS file 30 ⟩;
  ⟨ Output the image 31 ⟩;
  ⟨ Output the trailer of the EPS file 32 ⟩;

This code is used in section 1.

30. ⟨ Output the header of the EPS file 30 ⟩ ≡

```
printf("%%!PS\n");
printf("%%%%BoundingBox:␣0␣0␣%d␣%d\n", n, m);
printf("%%%%Creator:␣togpap\n");
clokk = time(0);
printf("%%%%CreationDate:␣%s", ctime(&clokk));
printf("%%%%Pages:␣1\n");
printf("%%%%EndProlog\n");
printf("%%%%Page:␣1␣1\n");
printf("/picstr␣%d␣string␣def\n", (n + 7) >> 3);
printf("%d␣%d␣scale\n", n, m);
printf("%d␣%d␣true␣[%d␣0␣0␣-%d␣0␣%d]\n", n, m, n, m, m);
printf("␣{currentfile␣picstr␣readhexstring␣pop}␣imagemask\n");
```

This code is used in section 29.

31. ⟨ Output the image 31 ⟩ ≡

```
for (i = 1; i ≤ m; i++) {
  for (j = 1; j ≤ n; j += 8) {
    for (k = 0, l = 0; k < 8; k++) l = l + l + (A[i][j + k] ? 1 : 0);
    printf("%02x", l);
  }
  printf("\n");
}
```

This code is used in section 29.

32. ⟨ Output the trailer of the EPS file 32 ⟩ ≡

```
printf("%%%%EOF\n");
```

This code is used in section 29.

33. Synthesis. And now to put the pieces together:

⟨ Compute the answer 33 ⟩ ≡
  switch (*methodcode*) {
  case *fscode*: ⟨ Do Floyd-Steinberg 12 ⟩; **break**;
  case *odithcode*: ⟨ Do ordered dither 15 ⟩; **break**;
  case *ddiffcode*: ⟨ Do dot diffusion 18 ⟩; **break**;
  case *sdiffcode*: ⟨ Do smooth dot diffusion 21 ⟩; **break**;
  case *ariescode*: ⟨ Do ARIES 24 ⟩; **break**;
  }

This code is used in section 1.

34. ⟨ Print relevant statistics 34 ⟩ ≡
  switch (*methodcode*) {
  case *odithcode*: ⟨ Print accumulated lossage 17 ⟩; **break**;
  case *ariescode*: ⟨ Print ARIES lossage 28 ⟩; **break**;
  case *ddiffcode*: **case** *sdiffcode*: ⟨ Print baronial lossage 23 ⟩;
  case *fscode*: ⟨ Print boundary leakage and extreme values 13 ⟩; **break**;
  }

This code is used in section 1.

35. Index.

TOGPAP