

(Downloaded from <https://cs.stanford.edu/~knuth/programs.html> and typeset on May 28, 2023)

1. Introduction. This is a quick-and-dirty program related to exercise 3.6–14. I’m finding how many terms appear in the representation of z^n with respect to bases of the form $z^0, \dots, z^{t-1}, z^{n-r+t}, \dots, z^{n-1}$, modulo $z^r + z^{r-s} + 1$ and mod 2, where $1 \leq t \leq r$.

```
#define r 100      /* the longer lag */
#define s 37       /* the shorter lag */
#define n 400      /* the number of elements generated simultaneously by ran_array */
#include <stdio.h>
  (Global variables 2) main()
  {
    register int i, j, k, m, t;
    (Initialize for the case  $t = r$  3);
    while (t) {
      (Gather statistics for case  $t$  5);
      t--;
      (Change the basis to eliminate  $z^t$  4);
    }
    (Print the statistics 8);
  }
```

2. The representation of $z^k = a_{k0}z^{b_0} + \dots + a_{k(r-1)}z^{b_{r-1}}$ appears in arrays a and b . The largest power of z less than z^n that is not in the basis is z^m .

```
(Global variables 2) ≡
  int a[n+1][r]; /* I could make this char, but int aids debugging */
  int b[r];      /* identifies the basis */
  int c[r], d[n+2]; /* for working storage */
  int p[n];      /* is this power of z in the basis? */
```

See also section 6.

This code is used in section 1.

```
3. (Initialize for the case  $t = r$  3) ≡
  for (k = 0; k < r; k++) {
    a[k][k] = 1;
    b[k] = k;
    p[k] = 1;
  }
  for (; k ≤ n; k++) {
    for (j = 1; j < r; j++) a[k][j] = a[k-1][j-1]; /*  $z^k = z \cdot z^{k-1}$  */
    if (a[k-1][r-1]) {
      a[k][0] = 1;
      a[k][r-s] ⊕= 1;
    }
  }
  m = n - 1;
  t = r;
```

This code is used in section 1.

4. $\langle \text{Change the basis to eliminate } z^t \text{ 4} \rangle \equiv$
for ($k = m$; $a[k][t] \equiv 0$; $k--$) ;
 $b[t] = k$;
for ($j = 0$; $j < r$; $j++$) $c[j] = a[k][j]$;
 $c[t] = 0$;
 $p[t] = 0$;
 $p[k] = 1$;
for (; $k \geq t$; $k--$)
 if ($a[k][t]$)
 for ($j = 0$; $j < r$; $j++$) $a[k][j] \oplus= c[j]$;
if ($a[n][t]$)
 for ($j = 0$; $j < r$; $j++$) $a[n][j] \oplus= c[j]$;
while ($p[m] \equiv 1$) $m--$;

This code is used in section 1.

5. We are interested in the number of nonzero coefficients in the representation of z^n . However, if this representation depends on any of the “forbidden” powers $z^t, \dots, z^{n-r+t-1}$, we want rather to exhibit the representation of z^m .

$\langle \text{Gather statistics for case } t \text{ 5} \rangle \equiv$
{
 register int *forbidden* = 0;
 for ($j = 0, i = 0$; $j < r$; $j++$)
 if ($a[n][j]$) {
 if ($b[j] < n - r + t \wedge b[j] \geq t$) *forbidden* = 1;
 else $i++$;
 }
 if (*forbidden*) $\langle \text{Print out an interesting linear dependency 7} \rangle$
 else $stat[i]++$;
}

This code is used in section 1.

6. $\langle \text{Global variables 2} \rangle + \equiv$
int $stat[r + 1]$; /* the number of cases with a given number of nonzero terms */

7. \langle Print out an interesting linear dependency 7 $\rangle \equiv$

```
{
  for (i = 0; i < n; i++) d[i] = 0;
  for (j = 0; j < r; j++)
    if (a[m][j]) d[b[j]] = 1;
  d[m] = 1;
  d[n] = 1;
  printf("%d:", t);
  for (i = 0; ; ) {
    while (d[i] == 0) i++;
    if (i == n) break;
    printf("␣%d", i);
    while (d[i] == 1) i++;
    if (i > n) i = n;
    printf("..%d", i - 1);
  }
  printf("\n");
}
```

This code is used in section 5.

8. \langle Print the statistics 8 $\rangle \equiv$

```
for (j = 0; j ≤ r; j++) printf("␣%3d:␣%d\n", j, stat[j]);
```

This code is used in section 1.

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