

(Downloaded from <https://cs.stanford.edu/~knuth/programs.html> and typeset on May 28, 2023)

1. Introduction. This program inputs an undirected graph and the names of two vertices in that graph (the “source” and “target” vertices). It outputs a not-necessarily-reduced binary decision diagram for the family of all simple paths from the source to the target.

The format of the output is described in another program, SIMPATH-REDUCE. Let me just say here that it is intended only for computational convenience, not for human readability.

I’ve tried to make this program simple, whenever I had to choose between simplicity and efficiency. But I haven’t gone out of my way to be inefficient.

```
#define maxn 255      /* maximum number of vertices; at most 255 */
#define maxm 2000     /* maximum number of edges */
#define logmemsize 27
#define memsize (1 << logmemsize)
#define loghtsize 25
#define htsize (1 << loghtsize)
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "gb_graph.h"
#include "gb_save.h"
unsigned char mem[memsize]; /* the big workspace */
unsigned long tail, boundary, head; /* queue pointers */
unsigned int htable[htsize]; /* hash table */
unsigned int htid; /* “time stamp” for hash entries */
int htcount; /* number of entries in the hash table */
int wrap = 1; /* wraparound counter for hash table clearing */
Vertex *vert[maxn + 1];
int arcto[maxm]; /* destination number of each arc */
int firstarc[maxn + 2]; /* where arcs from a vertex start in arcto */
unsigned char mate[maxn + 3]; /* encoded state */
int serial, newserial; /* state numbers */
⟨Subroutines 13*⟩
main(int argc, char *argv[])
{
    register int i, j, jj, jm, k, km, l, ll, m, n, t, hash;
    register Graph *g;
    register Arc *a, *b;
    register Vertex *u, *v;
    Vertex *source =  $\Lambda$ , *target =  $\Lambda$ ;
    ⟨Input the graph 2⟩;
    ⟨Renumber the vertices 3⟩;
    ⟨Reformat the edges 4⟩;
    ⟨Do the algorithm 5⟩;
}
```

```

2. ⟨Input the graph 2⟩ ≡
  if (argc ≠ 4) {
    fprintf(stderr, "Usage: %s foo.gb source target\n", argv[0]);
    exit(-1);
  }
  g = restore_graph(argv[1]);
  if (¬g) {
    fprintf(stderr, "I can't input the graph %s (panic code %ld)!\n", argv[1], panic_code);
    exit(-2);
  }
  n = g→n;
  if (n > maxn) {
    fprintf(stderr, "Sorry, that graph has %d vertices;\n", n);
    fprintf(stderr, "I can't handle more than %d!\n", maxn);
    exit(-3);
  }
  if (g→m > 2 * maxm) {
    fprintf(stderr, "Sorry, that graph has %ld edges;\n", (g→m + 1)/2);
    fprintf(stderr, "I can't handle more than %d!\n", maxm);
    exit(-3);
  }
  for (v = g→vertices; v < g→vertices + n; v++) {
    if (strcmp(argv[2], v→name) ≡ 0) source = v;
    if (strcmp(argv[3], v→name) ≡ 0) target = v;
    for (a = v→arcs; a; a = a→next) {
      u = a→tip;
      if (u ≡ v) {
        fprintf(stderr, "Sorry, the graph contains a loop %s--%s!\n", v→name, v→name);
        exit(-4);
      }
      b = (v < u ? a + 1 : a - 1);
      if (b→tip ≠ v) {
        fprintf(stderr, "Sorry, the graph isn't undirected!\n");
        fprintf(stderr, "(%s->%s has mate pointing to %s)\n", v→name, u→name, b→tip→name);
        exit(-5);
      }
    }
  }
  if (¬source) {
    fprintf(stderr, "I can't find source vertex %s in the graph!\n", argv[2]);
    exit(-6);
  }
  if (¬target) {
    fprintf(stderr, "I can't find target vertex %s in the graph!\n", argv[3]);
    exit(-7);
  }
}

```

This code is used in section 1.

3. If the source vertex is the first vertex in the graph, I'll process vertices according to the graph's own ordering.

Otherwise, I use a simple breadth-first strategy to number the vertices: The source is vertex 1. Then, for each $j \geq 1$, I run through the arcs from vertex j and assign the first unused number to any of its neighbors that haven't already got one.

#define *num* *z.I*

\langle Renumber the vertices **3** $\rangle \equiv$

```

if (source  $\equiv$  g-vertices) {
    for ( $k = 0; k < n; k++$ ) (g-vertices +  $k$ )-num =  $k + 1$ , vert[ $k + 1$ ] = g-vertices +  $k$ ;
} else {
    for ( $k = 0; k < n; k++$ ) (g-vertices +  $k$ )-num = 0;
    vert[1] = source, source-num = 1;
    for ( $j = 0, k = 1; j < k; j++$ ) {
        v = vert[ $j + 1$ ];
        for (a = v-arcs; a; a = a-next) {
            u = a-tip;
            if (u-num  $\equiv$  0) u-num = ++k, vert[k] = u;
        }
    }
}
if (target-num  $\equiv$  0) {
    fprintf(stderr, "Sorry, there's no path from %s to %s in the graph!\n", argv[2], argv[3]);
    exit(-8);
}
if ( $k < n$ ) {
    fprintf(stderr, "The graph isn't connected (%d<%d)!\n", k, n);
    fprintf(stderr, "But that's OK; I'll work with the component of %s.\n", argv[2]);
    n = k;
}
}

```

This code is used in section 1.

4. The edges will be considered as arcs $j \rightarrow k$ between vertex number j and vertex number k , when $j < k$ and those vertices are adjacent in the graph. We process them in order of increasing j ; but for fixed j , the values of k aren't necessarily increasing.

The k values appear in the *arcto* array. The edges for fixed j occur in positions *firstarc*[j] through *firstarc*[$j + 1$] - 1 of that array.

After this step, we forget the GraphBase data structures and just work with our homegrown integer-only representation.

⟨Reformat the edges 4⟩ ≡

```

for ( $m = 0, k = 1; k \leq n; k++$ ) {
    firstarc[ $k$ ] =  $m$ ;
     $v = \text{vert}[k]$ ;
    printf("%ld(%s)\n",  $v\text{-num}$ ,  $v\text{-name}$ );
    for ( $a = v\text{-arcs}; a; a = a\text{-next}$ ) {
         $u = a\text{-tip}$ ;
        if ( $u\text{-num} > k$ ) {
            arcto[ $m++$ ] =  $u\text{-num}$ ;
            if ( $a\text{-len} \equiv 1$ ) printf("_->_%ld(%s)_#%d\n",  $u\text{-num}$ ,  $u\text{-name}$ ,  $m$ );
            else printf("_->_%ld(%s,%ld)_#%d\n",  $u\text{-num}$ ,  $u\text{-name}$ ,  $a\text{-len}$ ,  $m$ );
        }
    }
}
firstarc[ $k$ ] =  $m$ ;

```

This code is used in section 1.

5. The algorithm. Now comes the fun part. We systematically construct a binary decision diagram for all simple paths by working top-down, considering the arcs in *arcto*, one by one.

When we're dealing with arc *i*, we've already constructed a table of all possible states that might arise when each of the previous arcs has been chosen-or-not, except for states that obviously cannot be part of a simple path.

Arc *i* runs from vertex *j* to vertex $k = \text{arcto}[i]$. Let *l* be the maximum vertex number in arcs less than *i*. (If the breadth-first ordering option was taken above, we'll always have $k \leq l + 1$, because of the way we did the numbering and reformatting; but that method is not always best.)

The state before we decide whether or not to include arc *i* is represented by a table of values *mate*[*t*], for $j \leq t \leq l$, with the following significance: If *mate*[*t*] = *t*, the previous arcs haven't touched vertex *t*. If *mate*[*t*] = *u* and $u \neq t$, the previous arcs have connected *t* with *u* by a simple path. If *mate*[*t*] = 0, the previous arcs have "saturated" vertex *t*; we can't touch it again.

We also use a (slick?) trick: We imagine that an edge between the source and target has already been included. Then the final arc of a simple path will be an arc that completes a cycle, when no other incomplete paths are present. (Think about it.)

The *mate* information is all that we need to know about the behavior of previous arcs. And it's easily updated when we add the *i*th arc (or not). So each "state" is equivalent to a *mate* table, consisting of $l + 1 - j$ numbers.

The states are stored in a queue, indexed by 64-bit numbers *tail*, *boundary*, and *head*, where $\text{tail} \leq \text{boundary} \leq \text{head}$. Between *tail* and *boundary* are the pre-arc-*i* states that haven't yet been processed; between *boundary* and *head* are the post-arc-*i* states that will be considered later. The states before *boundary* are sequences of $s = l + 1 - j$ bytes each, and the states after *boundary* are sequences of $ss = ll + 1 - jj$ bytes each, where *ll* and *jj* are the values of *l* and *j* for arc *i* + 1.

Bytes of the queue are stored in *mem*, which wraps around modulo *memsize*. We ensure that $\text{head} - \text{tail}$ never exceeds *memsize*.

```

< Do the algorithm 5 > ≡
  < Initialize the mate table 6 >;
  < Initialize the queue 7 >;
  for (i = 0; i < m; i++) {
    printf("#%d:\n", i + 1);      /* announce that we're beginning a new arc */
    fprintf(stderr, "Beginning arc %d (serial=%d, head-tail=%lld)\n", i + 1, serial, head - tail);
    fflush(stderr);
    < Process arc i 8 >;
  }

```

This code is used in section 1.

```

6. < Initialize the mate table 6 > ≡
  for (t = 2; t ≤ n; t++) mate[t] = t;
  mate[target-num] = 1, mate[1] = target-num;

```

This code is used in section 5.

```

7. < Initialize the queue 7 > ≡
  jj = ll = 1;
  mem[0] = mate[1];
  tail = 0, head = 1;
  serial = 2;

```

This code is used in section 5.

8. Each state for a particular arc gets a distinguishing number. Two states are special: 0 means the losing state, when a simple path is impossible; 1 means the winning state, when a simple path has been completed. The other states are 2 or more.

The output format on *stdout* simply shows the identifying numbers of a state and its two successors, in hexadecimal.

```
#define trunc(addr) ((addr) & (memsize - 1))
⟨ Process arc i 8 ⟩ ≡
    boundary = head, htcount = 0, htid = (i + wrap) << logmemsize;
    if (htid ≡ 0) {
        for (hash = 0; hash < htsize; hash++) htable[hash] = 0;
        wrap++, htid = 1 << logmemsize;
    }
    newserial = serial + ((head - tail)/(ll + 1 - jj));
    j = jj, k = arcto[i], l = ll;
    while (jj ≤ n ∧ firstarc[jj + 1] ≡ i + 1) jj++;
    ll = (k > l ? k : l);
    while (tail < boundary) {
        printf("%x:", serial);
        serial++;
        ⟨ Unpack a state, and move tail up 9 ⟩;
        ⟨ Print the successor if arc i is not chosen 11 ⟩;
        printf(",");
        ⟨ Print the successor if arc i is chosen 10 ⟩;
        printf("\n");
    }
```

This code is used in section 5.

9. If the target vertex hasn't entered the action yet (that is, if it exceeds *l*), we must update its *mate* entry at this point.

```
⟨ Unpack a state, and move tail up 9 ⟩ ≡
    for (t = j; t ≤ l; t++, tail++) {
        mate[t] = mem[trunc(tail)];
        if (mate[t] > l) mate[mate[t]] = t;
    }
```

This code is used in section 8.

10. Here's where we update the mates. The order of doing this is carefully chosen so that it works fine when *mate*[*j*] = *j* and/or *mate*[*k*] = *k*.

```
⟨ Print the successor if arc i is chosen 10 ⟩ ≡
    jm = mate[j], km = mate[k];
    if (jm ≡ 0 ∨ km ≡ 0) printf("0"); /* we mustn't touch a saturated vertex */
    else if (jm ≡ k) ⟨ Print 1 or 0, depending on whether this arc wins or loses 12* ⟩
    else {
        mate[j] = 0, mate[k] = 0;
        mate[jm] = km, mate[km] = jm;
        printstate(j, jj, ll);
        mate[jm] = j, mate[km] = k, mate[j] = jm, mate[k] = km; /* restore original state */
    }
done:
```

This code is used in section 8.

11. $\langle \text{Print the successor if arc } i \text{ is not chosen } 11 \rangle \equiv$
`printstate(j, jj, ll);`

This code is used in section 8.

12* See the note below regarding a change that will restrict consideration to Hamiltonian paths. A similar change is needed here.

$\langle \text{Print 1 or 0, depending on whether this arc wins or loses } 12^* \rangle \equiv$

```

{
    for (t = j + 1; t ≤ ll; t++)
        if (t ≠ k) {
            if (mate[t]) break;
        }
    if (t > n) printf("1"); /* we win: this cycle is all by itself */
    else printf("0"); /* we lose: there's junk outside this cycle */
}

```

This code is used in section 10.

13* The `printstate` subroutine does the rest of the work. It makes sure that no incomplete paths linger in positions *j* through *jj* − 1, which are about to disappear; and it puts the contents of *mate*[*jj*] through *mate*[*ll*] into the queue, checking to see if it was already there.

If '*mate*[*t*] ≠ *t*' is removed from the condition below, we get Hamiltonian paths only (I mean, simple paths that include every vertex).

$\langle \text{Subroutines } 13^* \rangle \equiv$

```

void printstate(int j, int jj, int ll)
{
    register int h, hh, ss, t, tt, hash;
    for (t = j; t < jj; t++)
        if (mate[t]) break;
    if (t < jj) printf("0"); /* incomplete junk mustn't be left hanging */
    else if (ll < jj) printf("0"); /* nothing is viable */
    else {
        ss = ll + 1 − jj;
        if (head + ss − tail > memsize) {
            fprintf(stderr, "Oops, I'm out of memory (memsize=%d, serial=%d)!\n", memsize, serial);
            fflush(stdout);
            exit(−69);
        }
         $\langle \text{Move the current state into position after } head, \text{ and compute } hash \text{ } 14 \rangle$ ;
         $\langle \text{Find the first match, } hh, \text{ for the current state after } boundary \text{ } 15 \rangle$ ;
        h = trunc(hh − boundary)/ss;
        printf("%x", newserial + h);
    }
}

```

This code is used in section 1.

14. $\langle \text{Move the current state into position after } head, \text{ and compute } hash \text{ } 14 \rangle \equiv$

```

for (t = jj, h = trunc(head), hash = 0; t ≤ ll; t++, h = trunc(h + 1)) {
    mem[h] = mate[t];
    hash = hash * 31415926525 + mate[t];
}

```

This code is used in section 13*.

15. The hash table is automatically cleared whenever *htid* is increased, because we store *htid* with each relevant table entry.

```

⟨Find the first match, hh, for the current state after boundary 15⟩ ≡
  for (hash = hash & (htsize - 1); ; hash = (hash + 1) & (htsize - 1)) {
    hh = htable[hash];
    if ((hh ⊕ htid) ≥ memsize) ⟨Insert new entry and goto found 16⟩;
    hh = trunc(hh);
    for (t = hh, h = trunc(head), tt = trunc(t + ss - 1); ; t = trunc(t + 1), h = trunc(h + 1)) {
      if (mem[t] ≠ mem[h]) break;
      if (t ≡ tt) goto found;
    }
  }
found:

```

This code is used in section 13*.

```

16. ⟨Insert new entry and goto found 16⟩ ≡
{
  if (++htcount > (htsize ≫ 1)) {
    fprintf(stderr, "Sorry, the hash table is full (htsize=%d, serial=%d)!\n", htsize, serial);
    exit(-96);
  }
  hh = trunc(head);
  htable[hash] = htid + hh;
  head += ss;
  goto found;
}

```

This code is used in section 15.

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SIMPATH-HAM

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