1. Introduction. This is a hastily written implementation of hull insertion.

```
format Graph int      /* gb_graph defines the Graph type and a few others */
format Vertex int
format Arc int
format Area int
#include "gb_graph.h"
#include "gb_miles.h"
  int n = 128;
  ⟨Global variables 2⟩
  ⟨Procedures 11⟩
  main()
  {
    ⟨Local variables 6⟩
    Graph *g = miles(128, 0, 0, 0, 0, 0, 0);
    mems = ccs = 0;
    ⟨Find convex hull of g 7⟩;
    printf("Total␣of␣%d␣mems␣and␣%d␣calls␣on␣ccw.\n", mems, ccs);
  }
```

2. I'm instrumenting this in a simple way.

```
#define o mems++
#define oo mems += 2
⟨Global variables 2⟩ ≡
  int mems;      /* memory accesses */
  int ccs;      /* calls on ccw */
  int serial_no = 1;      /* used to disambiguate entries with equal coordinates */
```

See also section 4.

This code is used in section 1.

3. Data structures. For now, each vertex is represented by two coordinates stored in the utility fields $x.I$ and $y.I$. I'm also putting a serial number into $z.I$, so that I can check whether different algorithms generate identical hulls.

A vertex v in the convex hull also has a successor $v\to succ$ and and predecessor $v\to pred$, stored in utility fields u and v.

This implementation is the simplest one I know; it simply walks around the current convex hull each time, therefore not really bad if the current hull never gets big.

```
#define succ u.V
#define pred v.V
```

4. ⟨ Global variables 2 ⟩ +≡

```
Vertex *rover;   /* one of the vertices in the convex hull */
```

5. We assume that the vertices have been given to us in a GraphBase-type graph. The algorithm begins with a trivial hull that contains only the first two vertices.

⟨ Initialize the data structures 5 ⟩ ≡

```
o, u = g→vertices;
v = u + 1;
u→z.I = 0;
v→z.I = 1;
oo, u→succ = u→pred = v;
oo, v→succ = v→pred = u;
rover = u;
if (n < 150) printf("Beginning␣with␣(%s;␣%s)\n", u→name, v→name);
```

This code is used in section 7.

6. We'll probably need a bunch of local variables to do elementary operations on data structures.

⟨ Local variables 6 ⟩ ≡

```
Vertex *u, *v, *vv, *w;
```

This code is used in section 1.

7. Hull updating. The main loop of the algorithm updates the data structure incrementally by adding one new vertex at a time. If the new vertex lies outside the current convex hull, we put it into the cycle and possibly delete some vertices that were previously part of the hull.

```
⟨Find convex hull of g 7⟩ ≡
  ⟨Initialize the data structures 5⟩;
  for (oo, vv = g→vertices + 2; vv < g→vertices + g→n; vv++) {
    vv→z.I = ++serial_no;
    ⟨Go around the current hull; continue if vv is inside it 9⟩;
    ⟨Update the convex hull, knowing that vv lies outside the consecutive hull vertices u and v 10⟩;
  }
  ⟨Print the convex hull 8⟩;
```

This code is used in section 1.

8. Let me do the easy part first, since it's bedtime and I can worry about the rest tomorrow.

```
⟨Print the convex hull 8⟩ ≡
  u = rover;
  printf("The␣convex␣hull␣is:\n");
  do {
    printf("␣␣%s\n", u→name);
    u = u→succ;
  } while (u ≠ rover);
```

This code is used in section 7.

9. ⟨Go around the current hull; **continue** if *vv* is inside it 9⟩ ≡

```
  u = rover;
  do {
    o, v = u→succ;
    if (ccw(u, vv, v)) goto found;
    u = v;
  } while (u ≠ rover);
  continue;
found: ;
```

This code is used in section 7.

10. ⟨ Update the convex hull, knowing that vv lies outside the consecutive hull vertices u and v 10 ⟩ ≡

```
if (u ≡ rover) {
  while (1) {
    o, w = u→pred;
    if (w ≡ v) break;
    if (ccw(vv, w, u)) break;
    u = w;
  }
  rover = w;
}
while (1) {
  if (v ≡ rover) break;
  o, w = v→succ;
  if (ccw(w, vv, v)) break;
  v = w;
}
oo, u→succ = v→pred = vv;
oo, vv→pred = u; vv→succ = v;
if (n < 150) printf("New␣hull␣sequence␣(%s;␣%s;␣%s)\n", u→name, vv→name, v→name);
```

This code is used in section 7.

11. Determinants. I need code for the primitive function *ccw*. Floating-point arithmetic suffices for my purposes.

We want to evaluate the determinant

$$ccw(u,v,w) = \begin{vmatrix} u(x) & u(y) & 1 \\ v(x) & v(y) & 1 \\ w(x) & w(y) & 1 \end{vmatrix} = \begin{vmatrix} u(x)-w(x) & u(y)-w(y) \\ v(x)-w(x) & v(y)-w(y) \end{vmatrix}.$$

```
⟨ Procedures 11 ⟩ ≡
  int ccw(u, v, w)
      Vertex *u, *v, *w;
  { register double wx = (double) w→x.I, wy = (double) w→y.I;
    register double det = ((double) u→x.I − wx) * ((double) v→y.I − wy) − ((double)
        u→y.I − wy) * ((double) v→x.I − wx);
    Vertex *uu = u, *vv = v, *ww = w, *t;
    if (det ≡ 0) {
      det = 1;
      if (u→x.I > v→x.I ∨ (u→x.I ≡ v→x.I ∧ (u→y.I > v→y.I ∨ (u→y.I ≡ v→y.I ∧ u→z.I > v→z.I)))) {
        t = u; u = v; v = t; det = −det;
      }
      if (v→x.I > w→x.I ∨ (v→x.I ≡ w→x.I ∧ (v→y.I > w→y.I ∨ (v→y.I ≡ w→y.I ∧ v→z.I > w→z.I)))) {
        t = v; v = w; w = t; det = −det;
      }
      if (u→x.I > v→x.I ∨ (u→x.I ≡ v→x.I ∧ (u→y.I > v→y.I ∨ (u→y.I ≡ v→y.I ∧ u→z.I < v→z.I)))) {
        det = −det;
      }
    }
    if (n < 150)
      printf("cc(%s;␣%s;␣%s)␣is␣%s\n", uu→name, vv→name, ww→name, det > 0 ? "true" : "false");
    ccs++;
    return (det > 0);
  }
```

This code is used in section 1.