

(See <https://cs.stanford.edu/~knuth/programs.html> for date.)

1. Intro. A quickie to find a longest string that avoids the interesting set of “unavoidable” m -ary strings of length n constructed by Mykkeltveit in 1972.

His construction can be viewed as finding the minimum number of arcs to remove from the de Bruijn graph of $(n - 1)$ -tuples so that the resulting graph has no oriented cycles. (Because each n -letter string corresponds to an arc that must be avoided.)

This program constructs the graph and finds a longest path.

I hacked it from the previous program UNAVOIDABLE, which uses a different set of strings.

```
#define m 2      /* this many letters in the alphabet */
#define n 20     /* this many letters in each string, assumed greater than 2 */
#define space (1 << (n - 1)) /*  $m^{n-1}$  */
#include <stdio.h>
#include <math.h>
char avoid[m * space]; /* nonzero if the arc is removed */
int deg[space]; /* outdegree, also used as pointer to next level */
int link[space]; /* stack of vertices whose degree has dropped to zero */
int a[n + 1]; /* staging area */
double sine[n]; /* imaginary parts of the  $n$ th roots of unity */
int count; /* the number of vertices on the current level */
int code; /* an  $n$ -tuple represented in  $m$ -ary notation */

main()
{
    register int d, j, k, l, q;
    register int top; /* top of the linked stack */
    double u = 2 * 3.1415926535897932385 / (double) n;
    register double s;

    for (j = 0; j < n; j++) sine[j] = sin(j * u);
    ⟨Compute the avoid and deg tables 2⟩;
    for (d = 0; count; d++) {
        printf("Vertices at distance %d: %d\n", d, count);
        for (l = top, top = -1, count = 0; l ≥ 0; l = link[l])
            ⟨Decrease the degree of  $l$ 's predecessors, and stack them if their degree drops to zero 5⟩
    }
    ⟨Print out a longest path 6⟩;
}
```

2. Algorithm 7.2.1.1F gives us the relevant prime powers here.

```

⟨ Compute the avoid and deg tables 2 ⟩ ≡
  for (j = 0; j < space; j++) deg[j] = m;
  count = d = 0;
  top = -1;
  for (j = n; j; j--) a[j] = 0;
  a[0] = -1, j = 1;
  while (1) {
    if (n % j == 0) ⟨ Generate an n-tuple to avoid 3 ⟩;
    for (j = n; a[j] == m - 1; j--) ;
    if (j == 0) break;
    a[j]++;
    for (k = j + 1; k ≤ n; k++) a[k] = a[k - j];
  }
  printf("m=%d, n=%d: avoiding one arc in each of %d disjoint cycles\n", m, n, d);

```

This code is used in section 1.

3. At this point $\lambda = a_1 \dots a_j$ is a prime string and $\alpha = a_1 \dots a_n = \lambda^{n/j}$. The crux of Mykkeltveit's method is to compute an exponential sum $s(a) = \sum a_j \omega^{(j-1)}$, where $\omega = e^{2\pi i/n}$, and to avoid the "first" cyclic shift of the a array for which the imaginary part of $s(a)$ is positive. (If no such shift exists, an arbitrary shift is chosen.)

```

⟨ Generate an n-tuple to avoid 3 ⟩ ≡
{
  d++;
  if (j < n) q = n;
  else {
    for (q = 1; ; q++) {
      for (l = 1, s = 0.0; l ≤ n; l++) s += a[l] * sine[(l - 1 + n - q) % n];
      if (s < .0001) break;
    }
    for (q++; q < n + n; q++) {
      for (l = 1, s = 0.0; l ≤ n; l++) s += a[l] * sine[(l - 1 + n + n - q) % n];
      if (s ≥ .0001) break;
    }
    if (q > n) q -= n;
  }
  for (code = 0, k = q + 1; k ≤ n; k++) code = m * code + a[k];
  for (k = 1; k ≤ q; k++) code = m * code + a[k];
  ⟨ Avoid the n-tuple encoded by code 4 ⟩;
}

```

This code is used in section 2.

4. ⟨ Avoid the *n*-tuple encoded by code 4 ⟩ ≡
 avoid[code] = 1;
 q = code / m;
 deg[q]--;
 if (deg[q] == 0) deg[q] = -1, link[q] = top, top = q, count++;

This code is used in section 3.

5. \langle Decrease the degree of l 's predecessors, and stack them if their degree drops to zero 5 $\rangle \equiv$

```

for ( $j = m - 1$ ;  $j \geq 0$ ;  $j--$ ) {
     $k = l + j * space$ ;
    if ( $\neg avoid[k]$ ) {
         $q = k/m$ ;
         $deg[q]--$ ;
        if ( $deg[q] \equiv 0$ )  $deg[q] = l, link[q] = top, top = q, count++$ ;
    }
}

```

This code is used in section 1.

6. Here I apologize for using a dirty trick: The current value of k happens to be the most recent value of l , a vertex with no predecessors.

\langle Print out a longest path 6 $\rangle \equiv$

```

     $printf("Path:");$ 
    for ( $code = k, j = 1$ ;  $j < n$ ;  $j++$ ) {
         $code = code * m, q = code / space$ ;
         $printf("\%d", q)$ ;
         $code -= q * space$ ;
    }
    while ( $deg[k] \geq 0$ ) {
         $printf("\%d", deg[k] \% m)$ ;
         $k = deg[k]$ ;
    }

```

This code is used in section 1.

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UNAVOIDABLE2

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