Problem Statement

The task is to **evaluate the performance of hospitals** by comparing their input resources (such as the number of beds and physicians) with their output resources (like surgery success rates, profit, and patient satisfaction). **The goal is to identify which hospitals are most efficient in converting their inputs into desirable outputs.** By applying Data Envelopment Analysis (DEA), an optimization model will be used to assess the efficiency of each hospital. This model will assign optimal weights to inputs and outputs, normalizing them in such a way that ensures no hospital's weighted outputs exceed its weighted inputs. The result will help distinguish hospitals that are performing efficiently from those that are not, providing insight into areas where hospitals can improve resource usage and overall performance. This analysis will guide hospital administrators in making informed decisions to enhance operational effectiveness.

Objective in Words:

Decide the efficiency or the performance of hospitals **so that th**e ratio of their weighted outputs or weighted outcomes (e.g., treatment success, financial returns, patient feedback) to their weighted inputs or weighted resources (e.g., number of beds/ physicians) is maximized,

subject to the following constraints:
• The total of the weighted outcomes for each hospital must not surpass the total of the weighted resources, ensuring a fair and consistent assessment of efficiency.
• The inputs and outputs of the hospital being assessed are standardized to normalize its efficiency score.
• The weights applied to the inputs and outputs are non-negative, allowing the model to determine the best possible combination for maximizing efficiency.

Sample Data:

Hospital	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	Output 4	Output 5	Output 6	Output 7
1	83.7033	2.994	8.048	51.1574	32.7153	66.2448	35.7387	49.023	49.2987	39.5518
2	91.5586	1.1948	77.4837	9.8605	70.3838	127.5586	59.4214	88.7819	46.5052	2.3613
3	43.7093	0.8526	185.2281	8.741	31.4517	2.281	31.1873	30.2933	18.9924	11.3759
4	79.9493	1.1131	63.5319	19.5161	0.5712	82.8274	105.6284	17.5802	33.5798	10.0547
5	20.2349	0.0052	10.0501	1.2792	8.8982	1.3348	7.9021	5.7841	0.8698	1.1927
6	54.1875	0.4116	159.9644	17.6729	292.7728	43.5424	107.0383	62.4587	41.8481	7.3327
7	488.3607	4.9761	144.1819	30.5292	472.6967	38.9624	13.8984	99.384	141.5574	13.424
8	28.2891	0.3203	56.8971	15.7713	79.7854	54.1642	80.0574	105.8141	72.7695	7.5359
9	347.0949	2.8801	2.9829	234.2355	7.4186	77.2056	138.601	10.4188	93.4526	55.0378
10	107.6619	1.1307	69.6467	26.0705	3.0298	79.1683	30.2544	54.5973	141.9813	8.0685

Decision Variables:

Let:
$x_i = weight\ of\ input\ i, for\ i \in I$
$y_j = weight\ of output\ j, for\ j \in J$

Data Definition

Let
$I = denotes\ the\ set\ of\ input\ variables$

$J = denotes\ the\ set\ of\ output\ variables$
$Z = represents\ the\ collection\ of\ hospitals\ involved\ in\ the\ analysis$
$k = denotes\ specific\ hospital\ being\ assessed\ for\ its\ operational\ efficiency$
$input\_data_{zi} = refers\ to\ the\ quantity\ of\ input\ \boldsymbol{i}\ utilized\ by\ hospital\ \boldsymbol{z}, where\ \boldsymbol{z} \in \boldsymbol{Z}\ and\ \boldsymbol{i} \in \boldsymbol{I}.$
$Output\ data_{zj} = refers\ to\ the\ quantity\ of output\ j\ utilized\ by\ hospital\ z, where\ \boldsymbol{z} \in \boldsymbol{Z}\ \boldsymbol{and}\ \boldsymbol{j} \in \boldsymbol{J}$

ALGEBRAIC FORMULATIONS:

$$\boldsymbol{Max} \sum_{\boldsymbol{j \in J}} \boldsymbol{output\_data_{kj}} * \boldsymbol{y_j}$$

CONSTRAINTS:

Efficiency Constraint

$$\sum_{j \in J} \boldsymbol{output\_data_{zj}} * \boldsymbol{y_j} \leq \sum_{\boldsymbol{i \in I}} \boldsymbol{input\_data_{zi}} * \boldsymbol{x_i}$$

Normalization Constraint:

$$\sum_{\boldsymbol{i \in I}} \boldsymbol{input\_data_{ki}} * \boldsymbol{x_i} = \boldsymbol{1}$$

Non-negativity Constraints:

$$\boldsymbol{x_i \geq 0\ for\ i \in I}$$
$$\boldsymbol{y_j \geq 0\ for\ j \in J}$$

Implementation:

A model implementation and solution have been provided below using Python and AMPL :

https://drive.google.com/drive/folders/1gcK7eTUTuhnyhWHDfsUpeNK1BIzlX2lb

Results

The analysis using the Data Envelopment Analysis (DEA) model identified **50 hospitals** as efficient, achieving **an efficiency score of 1.000.** These hospitals represent the benchmark for operational efficiency in the dataset, effectively converting their input resources into desirable outputs. Below are the key findings:

Detailed List of Efficient Hospitals

The hospitals achieving the maximum efficiency score are identified by their respective IDs: 9, 11, 12, 14, 42, 45, 47, 50, 51, 60, 80, 82, 83, 85, 95, 99, 106, 108, 114, 151, 167, 168, 178, 189, 193, 203, 209, 213, 223, 225, 231, 252, 256, 259, 279, 282, 284, 288, 291, 296, 297, 303, 306, 308, 319, 329, 334, 339, 344, 345.