

CS 101: Computer Programming and Utilization

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(Abhiram Ranade's slides, borrowed and edited)
Lecture 18

Today's Lecture

- Recursion
- Examples

Euclid's theorem on GCD

THEOREM: If $m \% n == 0$, then $\text{GCD}(m, n) = n$, else $\text{GCD}(m, n) = \text{GCD}(n, m \% n)$.

The theorem looks like a program!

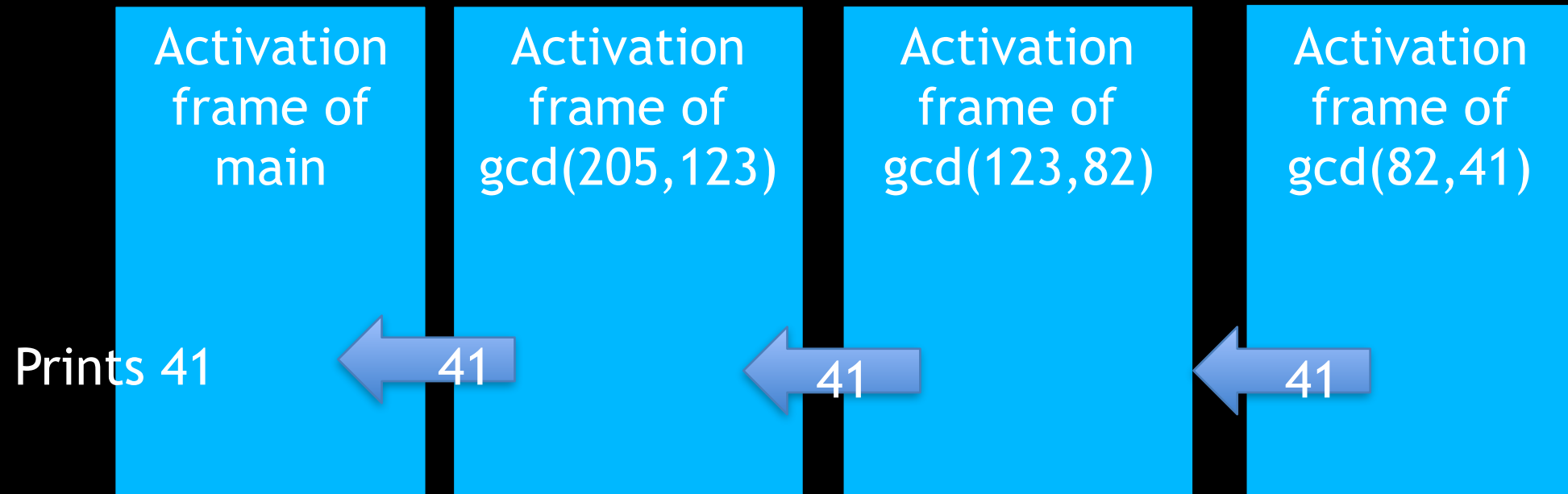
```
int gcd(int m, int n){  
    if (m % n == 0) return n;  
    else return gcd(n, m % n);  
}
```

Will this work?

Execution

```
int gcd(int m, int n){  
    if(m % n == 0) return n;  
    else return gcd(n, m%n);  
}
```

```
main_program{  
    cout <<  
    gcd(205,123)  
    << endl;  
}
```



Recursion

- Recursion = The phenomenon of a function calling itself
 - Seems like we are defining the function in terms of itself
 - But no circularity if the arguments to the new call are different from the arguments in the original call.
- Each call executes in its own activation frame.
- Some call must return without another recursive call
 - Otherwise infinite recursion (error!)
- In the body of gcd there was just one recursive call. We can have several calls if we wish. Examples soon.

Comparison of recursive and non-recursive gcd

```
int gcd(int m, int n){  
    if (m % n == 0) return n;  
    else return gcd(n, m % n);  
}
```

Recursive calls in gcd(205,123):
gcd(123,82)
gcd(82,41)

```
int gcd(int m, int n){  
    while(m % n != 0){  
        int r = m%n;  
        m = n;  
        n = r;  
    }  
    return n;  
}
```

- Values of m,n in consecutive iterations of gcd(205,123):
 - 205, 123
 - 123, 82,
 - 82, 41
- The two programs are "really" doing the same calculations!
- But on the surface they look very different.

Remarks

- Recursion often produces compact, elegant programs.
 - Recursive programs might be slightly slower because they need to create activation frames etc.
- Recursion is also a way to discover algorithms.

Euclid quite possibly thought to himself:

- “Instead of doing laborious computation to find the gcd of 205 and 123, can I find two smaller numbers whose gcd is the same as that of 205 and 123?”
- This is recursive thinking! It is common in mathematics.
- We will see more examples soon.

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