PROBABILITY AND STATISTICS (UCS401)

Lecture-10[a]

Uniform Discrete Distribution

Random Variables and their Special Distributions(Unit –III & IV)

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Uniform distribution (Discrete):

A random variable X is said to follow uniform discrete distribution if its p.m.f. is given by

$$P(X=x) = \begin{cases} \frac{1}{N} ; & x \in \{1, 2, 3, \dots N\} \\ 0 ; & \text{elsewhere} \end{cases}$$

Clearly this is a valid p.m.f.

$$\because \sum_{x=1}^{N} \frac{1}{N} = \frac{1}{N} \times N = 1$$

$$E(X) = \sum_{x=1}^{N} x \frac{1}{N} = \frac{1}{N} \sum_{x=1}^{N} x = \frac{1}{\cancel{N}} \frac{\cancel{N}(N+1)}{2}$$

$$\boxed{E(X) = \frac{N+1}{2}}$$

$$E(X^2) = \sum_{x=1}^{N} x^2 \frac{1}{N} = \frac{1}{N} \sum_{x=1}^{N} x^2$$

$$= \frac{1}{N} \frac{1}{6} N(N+1)(2N+1)$$

$$\boxed{E(X^2) = \frac{(N+1)(2N+1)}{6}}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$Var(X) = \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2}\right)^2$$

$$= \frac{(N+1)\left[2(2N+1) - 3(N+1)\right]}{12}$$

$$Var(X) = \frac{(N+1)(N-1)}{12}$$

$$\boxed{Var(X) = \frac{N^2-1}{12}}$$

Moment generating function ($M_X(t)$) :

$$M_X(t) = E(e^{tX}) = \sum_{x=1}^{N} e^{tx} \frac{1}{N}$$

$$= \frac{1}{N}\left(e^t + e^{2t} + \cdots + e^{Nt}\right)$$

$$= \frac{1}{N} \frac{e^t(e^{Nt}-1)}{e^t-1}$$ provided $t \neq 0$

$$M_X(t) = \begin{cases} \dfrac{e^t(e^{Nt}-1)}{N(e^t-1)} & t \neq 0 \\ 1 & t = 0 \end{cases}$$