Name: ______________________ RollNo.: ______________________

THAPAR INSTITUTE OF ENGINEERING & TECHNOLOGY-PATIALA
Department of Computer Science and Engineering
U-grade / Axillary Exam (March 11, 2022)

Probability and Statistics (UCS410) **M.M. 45 Time: 2 Hours**
Instructors: AMT

Note: Attempt any FIVE questions. Assume missing data suitably, if any.
Some useful data $P(Z > 1.67) = 0.04746$, $P(Z < -3) = 0.00135$, $P(Z < 3) = 0.99865$, $P(Z < 0) = 0.5$, $P(Z < -1.5) = 0.06681$, $P(-2 < Z < 2) = 0.9544$, $P(-1 < Z < 1) = 0.6826$ Here, Z is the standard Normal variable.

Q1 (a)	Two computers A and B are to be marketed. A salesman who is assigned the job of finding customers for them has 70% and 40% chances respectively of succeeding in case of computer A and B. The two computers can be sold independently. Given that he was able to sell at least one computer, what is the probability that computer A has been sold?	**4 Marks**
Q1 (b)	(i) State Bayes Theorem. (ii) Suppose that a product is produced in three factories X, Y, and Z. It is known that factory X produces thrice as many items as factory Y, and that factories Y and Z produce the same number of items. Assume that it is known that 2 per cent of the items produced by each factories X and Z are defective while 4 percent of those manufactured by factory Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random. • What is the probability that this item is defective? • If an item selected at random is found to defective, what is the probability that it was produced by factory X, Y and Z respectively?	**5 Marks**
Q2(a)	Consider a random variable X with possible outcomes as 0, 1, 2, 3 … Suppose that $P(X = j) = (1-a)a^j, j = 0, 1, 2, 3 \ldots$ a) Find the values of 'a' so that this model represents a legitimate probability mass function. b) Show that the probability $P(X > s + t \mid X > s) = P(X \geq t)$.	**3+3 Marks**
Q2 (b)	Suppose X has exponential distribution with parameter $\lambda = 4$. Find the Cumulative Distribution Function (CDF) $F(t)$ of this distribution and hence calculate $P(X \geq 1)$ using this $F(t)$.	**3 Marks**
Q3 (a)	Find Moment Generating Function(MGF) for exponential distribution.	**3 Marks**
Q3 (b)	A component exhibits Normal Distribution for failure rate with mean of 3750 hrs. and standard deviation of 500 hrs. What percentage of parts will survive up to 4500 hrs.?	**4 Marks**

Q3(c) What is the probability that at least two out of n people have the same birthday? Assume 365 days in a year and that all days are equally likely. **2 Marks**

P.T.O

Q4 (a) Fit the curve $y = ax^b$ for the following data **6 Marks**

x:	1	2	3	4	5	6
y:	1200	900	600	200	110	50

Q4 (b) Suppose X has exponential distribution with parameter $\lambda = 4$. Find the Cumulative Distribution Function (CDF) $F(t)$ of this distribution and hence calculate $P(X \geq 1)$ using this $F(t)$. **3 Marks**

Q5 (a) Random variable X follows the continuous uniform distribution over the interval 0 to 1 i.e. [0,1]. Find the probability density function (pdf) of $Y = \frac{1}{X}$? **4 Marks**

Q5(b) The first four moments of a distribution about the value 4 of the variable are -1·5, 17, -30 and 108. Find the first four moments about the mean. **5 Marks**

Q6 Let the joint probability density function for (X , Y) be **9 Marks**

$$f(x , y) = \begin{cases} \frac{x+y}{2}, & 0 < x,\ 0 < y,\ and\ 3x + y < 3 \\ 0, & Otherwise \end{cases}$$

i. Find the probability P(X < Y) . Draw the clear target region under consideration
ii. Find the marginal probability density function of X.
iii. Find the marginal probability density function of Y.
iv. Are X and Y independent? If not, find Cov (X, Y).

Q7 (a) Suppose that the marks of students in a course are normally distributed with mean 55 and standard deviation 17. We take a sample of size 35, and note that the marks of students are: 82, 70, 45, 80, 49, 52, 59, 43, 36, 76 ,72 ,48 ,64, 60, 38, 54, 46, 68, 35, 55, 61, 58, 76, 56, 62, 38, 63, 37 ,78, 37, 60, 47, 38, 14, 32. Find the 95.44% confidence interval μ. **5 Marks**

Q7(b) Define a Sample and Sample Mean. Also, Show that Sample Mean is an unbiased estimate of population mean . **4 Marks**