PROBABILITY AND STATISTICS (UCS401)

Lecture-6

(Independent Events with illustrations)

Introduction to Probability (Unit -II)

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

Dr. Rajanish Rai
Assistant Professor
School of Mathematics
Thapar Institute of Engineering and Technology, Patiala

Rajanish Rai

[4] Independent events with illustration

Independent events :-

Events are said to be independent to each other if happening of one of them is not affected by and does not affect the happening of any one of others.

66 If A and B are independent events so that the occurrence or non-occurrence of A is not affected by occurrence or non-occurrence of B,

$$P(A/B) = P(A) \quad \text{or} \quad P(B/A) = P(B)$$

$A/B \rightarrow$ B is independent of A.

$B/A \rightarrow$ A is independent of B.

Multiplication theorem for independent events :-

Two events are independent if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

Proof :- Assume firstly, A and B are independent,

$$P(A/B) = P(A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow \quad P(A \cap B) = P(A)\, P(B) \qquad \#$$

Conversely, assume that

$$P(A \cap B) = P(A)\, P(B) \quad \text{--- (1)}$$

Claim! A & B are independent.

For this, we need to prove that

$$P(A/B) = P(A)$$

$$(1) \Rightarrow \quad \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow \quad P(A/B) = P(A)$$

$\Rightarrow$ A and B are independent. Hence proved

Generalization of n events :-

The events $A_1, A_2, A_3, \ldots, A_n$ are independent if and only if

$$P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) = P(A_1)\, P(A_2) \cdots P(A_n).$$

Difference between mutually exclusive (disjoint) & independent events :-

$$\text{Mutually exclusive (disjoint)} \not\Longrightarrow \text{independent}$$

$$\Downarrow$$

$$P(A/B) = P(A)$$

or

$$P(A \cap B) = P(A)\,P(B)$$

Let A and B be mutually exclusive (disjoint) events with positive probability $P(A) > 0$; $P(B) > 0$.

$$\therefore \quad A \cap B = \phi \Rightarrow P(A \cap B) = 0$$

By definition of conditional probability, we have

$$P(A \cap B) = P(A)\,P(B/A) \quad ; \quad P(A \cap B) = P(B)\,P(A/B)$$

$$P(A) \neq 0 \qquad ; \qquad P(B) \neq 0$$

$$\Rightarrow P(A)\,P(B/A) = 0 \quad ; \quad P(B)\,P(A/B) = 0$$

$$\Rightarrow P(B/A) = 0 \neq P(B) \quad ; \quad P(A/B) = 0 \neq P(A)$$

Thus, $P(B/A) \neq P(B)$ or $P(A/B) \neq P(A)$

$\Rightarrow$ A & B are <u>not independent</u> events, i.e.,

dependent event.

Conversely

$$\text{Independent} \not\Rightarrow \text{disjoint}$$
$$\Downarrow$$
$$A \cap B = \phi$$
$$P(A \cap B) = 0$$

i.e., if A and B are independent events with probability $P(A) > 0$, $P(B) > 0$, which implies that

$$P(A/B) = P(A) \quad \text{or} \quad P(B/A) = P(B)$$

Now,
$$P(A \cap B) = P(A)\, P(B/A)$$
$$= P(A)\, P(B) \neq 0$$

$\Rightarrow \quad P(A \cap B) \neq 0$

$\Rightarrow$ A & B are not mutually exclusive events.

Hence, two independent events can not be mutually exclusive.

Question :- Let A and B be two possible outcomes of an experiment and suppose $P(A) = 0.4$,
$$P(A \cup B) = 0.7 \quad \text{and} \quad P(B) = p$$

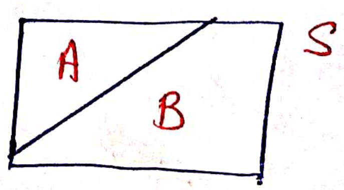
(i) For what choice of p, A and B are mutually exclusive and Exhaustive.

(ii) For what choice of p, A and B are mutually exclusive.

(iii) For what choice of p, A and B are independent.

Solution :- (i) For mutually exclusive and exhaustive events A and B, it means that

$$A \cup B = S$$



$\Rightarrow \quad P(A) + P(B) = P(S)$

$\Rightarrow \quad 0.4 + P(B) = 1$

$\Rightarrow \quad 0.4 + p = 1$

$\Rightarrow \quad \boxed{p = 0.6}$

(ii) For mutualy exclusive (disjoint events)

$$A \cap B = \phi$$

$$P(A \cap B) = 0$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.4 + p - P(A \cap B)$$

$$0.7 = 0.4 + p - 0$$

$\Rightarrow \quad \boxed{p = 0.3}$

(iii) For independence of A and B

$$P(A/B) = P(A)$$

or

$$P(A \cap B) = P(A) . P(B)$$

$$P(A \cap B) = 0.4p.$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.7 = 0.4 + p - 0.4p$$

$\Rightarrow \quad 0.3 = 0.6p$

$\Rightarrow \quad \boxed{p = 0.5}$

Question-: Toss two coins and observe the outcomes.

Define these events

A; Head on the first coin.

B; Tail on the ~~first~~ second coin.

Are events A and B are independent?.

Solution-: Two coins are tossed. Thus, exhaustive

Cases $= 2^2 = 4$

Thus, the sample space is

$$S = \{HH, HT, TH, TT\}$$

Now, the events:

A; Head on the first coin $= \{HH, HT\}$

B; Tail on the second coin $= \{HT, TT\}$

$A \cap B = \{HT\}$

To show A and B are independent, we need to show

$$P(A \cap B) = P(A)\,P(B) \quad \text{or} \quad P(A/B) = P(A).$$

Thus, $P(A) = \frac{2}{4} = \frac{1}{2}$ & $P(B) = \frac{2}{4} = \frac{1}{2}$

$$P(A \cap B) = \frac{1}{4}$$

Since $P(A \cap B) = P(A)\,P(B)$

Thus, events A and B are independent.

or

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2} = P(A)$$

$\Rightarrow$ A and B are independent to each other.

Question:- In a telephone survey of 1000 adults, respondent were asked their opinion about the cost of a college education. The respondents were classified according to whether that had a child in college and whether they thought the loan burden for most college students is too high, the right amount, or two little. The proportions responding in each category are shown in below table;

	Too high (A)	Right amount (B)	Too little (C)
Child in College (D)	0·35	0·08	0·01
Not child in College (E)	0·25	0·20	0·11

Are event D and A independent? Explain.

Solution:-

	Too high (A)	Right amount (B)	Too little (C)	Sum
Child in college (D)	0·35	0·08	0·01	0·44
Not child in college (E)	0·25	0·20	0·11	0·55
Sum	0·60	0·28	0·12	1·00

must be one.

Claim!

$$P(A \cap D) = P(A)\,P(D)$$

or

$$P(A/D) = P(A)$$

or

$$P(D/A) = P(D)$$

$\Rightarrow$ A & D are independent events. Otherwise not

I^{st} method–:

From table we obtain that

$$P(A \cap D) = \frac{0.35}{1} \quad \& \quad P(A) = 0.35 + 0.25$$

$$P(A) = 0.60$$

$$P(D) = 0.35 + 0.08 + 0.01 = 0.44$$

Since $P(D) \times P(A) = 0.44 \times 0.60 = 0.264 \neq P(D \cap A)$

Thus, events A and D are not independent, i.e., dependent.

II^{nd} method–:

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0.35}{0.44} = 0.80 \neq P(A)$$

or

$$P(D/A) = \frac{P(D \cap A)}{P(A)} = \frac{0.35}{0.60} = 0.58 \neq P(D)$$

Thus, events A and D are dependent.