Probability and Statistics (UCS410)   M.M. 50   Time: 2 Hours

Instructors: SWT

Q1. (8 marks)

For two groups of observations the following results were obtained: Find the mean, S.D.of the 45 observations obtained by combining the two groups.

Group I: $\sum(X-5)=8,\ \sum(X-5)^2=40,\ n_1=20$

Group II: $\sum(X-8)=10,\ \sum(X-8)^2=70,\ n_2=25$

Q2. (4+4 marks)

    a.) In a multiple choice examination, there are 20 questions. Each question has four alternative answers following it and the student must select the one correct answer. Four marks are given for the correct answer and one marks in deducted for every wrong answer. A student must secure at least 50% of maximum possible marks to pass the examination. Suppose that a student has not studied at all so that he decides to select the answers to the questions on a random basis. What is the probability that he will pass in the examination?

    b.) A certain type of electric motor fails either by seizure of the bearings, or by burning out of the electric windings, or by wearing out of the brushes. Suppose that seizure is twice as likely as burning out, which is four times as likely as brush wearout. What is the probability that failure will be by each of these three mechanisms?

Q3. (4+4 marks)

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x,y) = \begin{cases} \frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6} & 0 \le x \le 1, 0 \le y \le 2 \\ \\ 0 & \text{otherwise} \end{cases}$$

For 0≤y≤2, find

a. the conditional PDF of X given Y=y;

b .P(X<1/2|Y=y)

Q4. (10 marks)

Define rectangular distribution (Continuous Uniform distribution) and derive its expectation and variance.

Q5. (4+4+4+4 marks)

Let the joint probability density function for (X,Y) be

$$f(x,y)=\begin{cases} \frac{x+y}{2}, & 0 < x,\ 0 < y,\ and\ 3x+y <.3 \\ 0, & Otherwise \end{cases}$$

    i.   Find the probability P(X < Y) . Draw the clear target region under consideration

    ii.   Find the marginal probability density function of X.

    iii.   Find the marginal probability density function of Y.

    iv.   Are X and Y independent? If not, find Cov (X, Y).