Thapar Institute of Engineering and Technology, Patiala
SCHOOL OF MATHEMATICS
MST

Course No. UCS410 | *Course Name: PROBABILITY AND STATISTICS*
Time: 2 hours | *MM: 25*

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There are a total of 4 Questions each having two parts. Attempt all the questions. Non programmable calculators are allowed.

1 (a) A box contains 4 bad and 6 good tubes. Two are drawn out together. One of them is tested and found to be good. What is the probability that the other one is also good?

[2 marks]

(b) Three companies B_1, B_2 and B_3 produce 30%, 45%, and 25% of the cars, respectively. It is known that 2%, 3%, and 2% of these cars produced are defective. (a) What is the probability that a car produced is defective? (b) If a car purchased is found to be defective, what is the probability that this car is produced by company B_1? [3 marks]

2 (a) Given

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5}, & 0 < x < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Check if it can be a probability density function of any random variable X. If so, find the corresponding cumulative density function. [3 marks]

(b) If σ_X^2 and σ_Y^2 denotes the variance of random variables X and Y respectively and σ_{XY} denotes the covariance of X and Y, then show that

$$\sigma^2_{aX+bY+c} = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\sigma_{XY},$$

where a, b, c are the constants. [3 marks]

3 (a) The joint probability density of two random variables X and Y is

$$f(x,y) = \begin{cases} 4xy; & 0 \le x \le 1, 0 \le y \le 1 \\ 0; & \text{elsewhere.} \end{cases}$$

Check whether that X and Y are statistically independent or not? Justify the answer.

[3 marks]

P.T.O.

(b) Let X be discrete random variable with probability distribution

x	0	1	2	3	4
$f(x)$	0	$6k^2$	2k	k	2k

Find the value of k and hence, evaluate $P\left(\frac{1}{2} < X < \frac{5}{2} \middle| X > 1\right)$. [4 marks]

4 (a) It is known that 60% of computers in a lab get affected by a particular virus. If 5 computers are inspected, find the probability that (i) none of the computers are affected; (ii) more than 3 are affected. Use $\sum_{r=0}^{3} b(r; 5, 0.6) = 0.6630$, where $b(x; n, p)$ represents the binomial distribution. [3 marks]

(b) The probability that a person will die when he/she contracts an infection is 0.001 of the next 4000 people infected, what is the mean/average number of people who die? Also what is the probability that the number of people dying is at most equal to the mean value? Use $\sum_{r=0}^{4} p(r; 4) = 0.6288$ where $p(r, \mu)$ represents the Poisson distribution.

[4 marks]

THE END