School of Mathematics(SOM)

Thapar Institute of Engineering and Technology (TIET)

Probability and Statistics(UCS410) Practice Sheet 05

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(1) Suppose that a grocery store purchases 5 cartons of skim milk at the wholesale price of \$1.20 per carton and retails the milk at \$1.65 per carton. After the expiration date, the unsold milk is removed from the shelf and the grocer receives a credit from the distributor equal to three-fourths of the wholesale price. If the probability distribution of the random variable X, the number of cartons that are sold from this lot, is

x	0	1	2	3	4	5
$f(x)$	1/15	2/15	2/15	3/15	4/15	3/15

Find the expected profit.

(2) Let X be a random variable with the following probability distribution:

x	-3	6	9
$f(x)$	1/6	1/2	1/3

Find $E(X)$ and $E(X^2)$ and then, using these values, evaluate $E[(2X+1)^2]$.

(3) The total time, measured in units of 100 hours, that a teenager runs her hair dryer over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & elsewhere \end{cases}$$

evaluate the mean of the random variable $Y = 60X^2 + 39X$, where Y is equal to the number of kilowatt hours expended annually.

(4) If a random variable X is defined such that $E[(X-1)^2] = 10E$ and $E[(X-2)^2] = 6$, find μ and σ^2.

(5) Let X represent the number that occurs when a red die is tossed and Y the number that occurs when a green die is tossed. Find

(a) $E(X+Y)$;

(b) $E(X-Y)$;

(c) $E(XY)$.

(6) Let X represent the number that occurs when a green die is tossed and Y the number that occurs when a red die is tossed. Find the variance of the random variable

(a) $2X - Y$; (b) $X + 3Y - 5$.

(7) By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probability 0.7. What is this persons expected gain?

(8) The density function of coded measurements of the pitch diameter of threads of a fitting is

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)}, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find the expected value of X.

(9) For a laboratory assignment, if the equipment is working, the density function of the observed outcome X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find the variance and standard deviation of X.

(10) Suppose that the probabilities are $0.4, 0.3, 0.2$, and 0.1, respectively, that $0, 1, 2, or 3$ power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of power failures striking this subdivision.

(11) Given a random variable X, with standard deviation σ_x, and a random variable $Y = a + bX$, show that if $b < 0$, the correlation coefficient $\rho_{XY} = -1$, and if $b > 0, \rho_{XY} = 1$.

(12) Random variables X and Y follow a joint distribution

$$f(x,y) = \begin{cases} 2, & 0 < x \leq y < 1, \\ 0, & otherwise. \end{cases}$$

Determine the correlation coefficient between X and Y.

(13) A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshevs theorem, find

(a)$P(|X - 10| \geq 3)$;

(b) $P(|X - 10| < 3)$;

(c) $P(5 < X < 15)$;

(d) the value of the constant c such that $P(|X - 10| \geq c) \leq 0.04$.