Thapar Institute of Engineering and Technology, Patiala
School of Mathematics
Probability and Statistics (UCS410)
<u>Practice Sheet: 3</u>

1. The probability density function of a random variable X is $f(x) = k\,x(1-x)$, $0 < x < 1$. Then find (i) k and (ii) a number *'b'* such that $P(X<b)=P(X>b)$
   Ans: k=6, b=1/2.
2. A fair coin is tossed 3 times and let X be difference of the number of heads and the number of tails. Find (a) the probability mass function, (b) the cumulative distribution function of X.
   Ans: P(X=-3) =1/8, P(X=-1) =3/8, P(X=1) =3/8, P(X=3) =1/8, CDF: 0, 1/8, 4/8, 7/8, 1.
3. A random variable X has the probability distribution defined as

X	1	2	3	4	5	6
P(X)	0.04	0.15	0.37	0.26	0.11	0.07

   Find (i) P (X Odd | X<5) (ii)P (X<5|X Odd) (iii) P (X=4 | X is not equal to 3)

   Ans: (i): 41/82=1/2 (ii) 41/52 (iii) 26/63.

4. Consider the function $f(x) = \begin{cases} C(x^2 - 2x), 0 < x < 5/2 \\ 0 \quad\quad \text{elsewhere} \end{cases}$, where C is any constant. Could $f(x)$ be a probability density function? Justify your answer.

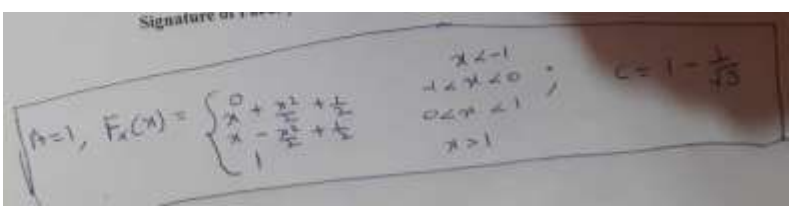
   Ans: Not possible. No value of C for which f(x) is always positive.

5. The probability density function of a random variable X is given by $f(x) = \begin{cases} A(1+x), -1 < x \le 0 \\ A(1-x), 0 < x < 1 \\ 0 \quad \text{elsewhere} \end{cases}$.

   Find (a) the value of A and plot $f(x)$, (b) the distribution function $F(x)$,

   (c) the point c such that P [X > c] =P [X< c]/2.

   Ans:



6. A continuous random variable X is defined as $f(x)=(ax+bx^2)$, $0<x<1$, 0 otherwise. If E[X]=0.6, then find (i) P[X<0.5], (ii) variance of X.

   Ans: a=3.6, b=-2.4, P=0.35, var=0.06

   The cumulative distribution function of a random variable X is given by $F(x) = (1 - e^{-2x^2})$, $x>0$. Find (a) P (0 <X < 3) (b) P (X >1) (c) P(X=5).

   Ans: (a) $1-e^{-18}$ (b) e^{-2} (c) Discuss in class.

7. A random variable X has the following probability function:

x	0	1	2	3	4
$p(x)$	k	$3k$	$5k$	$7k$	$9k$

find (a) k (b) c.d.f. (c) mean and variance of X and 4X+3 (d) P(X<3) and P(0<X<4).
Ans: k=1/25, mean (X): 70/25 Var (X): 850/25 Mean(4X+3)=280/25+3, Var(4X+3)=16*850/25 (d): 9/25, 15/25

8. Four unbiased coins are tossed and let X be the number of heads obtained. Write the probability mass function of X and find P(X>2).

[Ans.: 5/16]

9. A random variable X takes the values 1, 2, 3 and 4 such that P(X=1) = P(X=2) =2P(X=3) = 3P(X=4). Write the probability distribution of X and find

(i) P(X>2), (ii) P(1<X<4//X>2), (iii)P (X= 1or 2).

[Ans.: (i) 5/17 (ii) 3/5, (iii) 12/17]

10. A random variable X is equally probable to take even or odd integral values from 1 to 6 and has the following probability mass function:

X=x	1	2	3	4	5	6
P(X=x)	*k*	2k	*2k*	3k	*?*	*k/2*

Find (i) value of *k*, (ii) P(X=5) (iii) P (X< 4/X > 2) (iv) F (2), (v) F(x)

[Ans.: (i) 1/11, (ii) 5/22 (iii) 4/13, (iv) 3/11]

11. Differentiate between Binomial and Poisson distributions. Also derive the Expressions for mean, variance and MGF for these distributions.

12. Are the negative Binomial and Geometric distributions related? Describe with examples.

13. A communication system consists of n components, each of which will independently function with probability *p*. The total system will be able to operate effectively if at least one-half of its components function. For what values of *p* is a 5-component system more likely to operate effectively than a 3- component system?

Ans: $p > \left(\frac{1}{2}\right)$.

14. A space craft has 100,000 components. The probability of any one component being defective is $2x10^{-5}$. The mission will be in danger if five or more components become defective. Find theprobability of such an event.
Ans: $\lambda = 2; 1 - 7e^{-2}$

15. A shipment of 100 tape recorders contains 25 that are defective. If 10 of them are randomly chosenfor inspection, what is the probability that 2 of the 10 will be defective?
Ans: 0.28157

16. In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.
Ans: $e^{-3.75}$

17. A boy is throwing stones at a target, what is the probability that his 10^{th} throw is his 5^{th} hit, if the probability of hitting the target at any trial is 0.05.

Ans: $126 \times \frac{(19)^5}{(20)^{20}}$

18. If the probability that a certain test yields a positive reaction equals 0.4, what is the probability that fewer than 5 negative reactions occur before the first positive one?
    Ans: 0.92224

19. An experimental trial is performed until the first success is achieved. Assuming that the experiments are independent and the probability of success is p, find the value of p so that the probability that an odd number of experiments are required is equal to 0.6.
    Ans: 1/3

20. The number of blackflies on a board bean leaf follows a Poisson distribution with mean 2. A plant inspector, however, records the number of flies on a leaf only if at least 1 fly is present. What is the probability that he records 1 or 2 flies on a randomly chosen leaf? What is the expected number of flies recorded per leaf?
    Ans: 0.626; 2.3

21. An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives?
    Ans: $1-\left(\frac{5}{100}\right)^2 - {}^2C_1\left(\frac{5}{100}\right)^2\left(\frac{95}{100}\right)$

22. Suppose that during practice, a basketball player can make a free throw 80% of the time. Furthermore, assume that a sequence of free-throw shooting can be thought of as independentBernoulli trials. Let X = the minimum number of free throws that this player must attempt to make a total of ten shots.

    a. What is the p.m.f of X?
    b. What is the expected value and variance of X?
    c. What is the probability that the player must attempt 12 shots in order to make ten?

    Ans: mean $= 12.5$; variance $= 3.125$; 0.2362

23. Define the hypergeometric distribution and also find its mean.

24. A small voting district has 101 female and 95 male voters. A random sample of 10 voters is drawn. What is the probability that exactly 7 of the voters will be female?
    Ans- 0.13.

25. A group of 10 individuals are used for biological test with the following blood types, type O-3 people, type A-4 people and type B-3 people. What is the probability that a random sample of 5 people will contains 1-type O, 2-type A and 2-type B ?
    Ans: $\frac{3}{14}$.

26. Define the uniform/rectangular distribution. Also find the cumulative distribution function (C.D.F.), mean and variance.

27. If X is uniformly distributed in [-2, 2], then find the P(X<0) and $P|X-1| \geq \frac{1}{2}$ using P.D.F. and C.D.F. approach.
    Ans: (i) $\frac{1}{2}$, (ii) $\frac{3}{4}$.

28. Define the exponential distribution. Also find its mean, variance and moment generation function.

29. The time (in hours) required to repair a machine is exponentially distributed with parameter $\frac{1}{3}$. What is the probability that the repair time exceeds 3 hours?

    Ans: $\frac{1}{e}$.

30. The life length (in months) of an electric component follows an exponential distribution with parameter $\frac{1}{2}$. What is the probability that the component survives at least 10 months, given that already it had survived for more than 9 months?

    Ans: $e^{-\frac{1}{2}}$.

31. Define the normal distribution. Also find its mean, variance and moment generating function for the random variable X.

32. The saving bank account of a customer showed an average balance of $150 and standard deviation of $50. Assuming that the account balance(s) are normally distributed, find the percentage of the account(s) (i) over $ 200, (ii) between $120 and $170. Given that P(0<Z<1) =0.3413, P(Z<0.4) =0.6554 and P(Z<-0.6) =0.2743.

    Ans: (i) 0.1587, (ii) 0.3811.

33. In normal distribution, 31 % of the items are under 45 and 8 % are over 64. Find the mean and standard deviation. Given that P(Z<-1.4) =0.08 and P(Z>0.5) =0.31.

    Ans: $\mu = 50 \; and \; \sigma = 10$.