PROBABILITY AND STATISTICS (UCS401)

Lecture-13

(Poisson distribution with illustrations)

Random Variables and their Special Distributions (Unit –III & IV)

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Poisson distribution

(i) When n is very large, i.e., $n \to \infty$

(ii) When probability of success p is very small, i.e., $p \to 0$

(iii) $np = \lambda$ (finite)

Poisson distribution —:

A discrete random variable X, which has the following probability mass function

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad ; \quad x = 0, 1, 2, 3, \dots, \infty$$

is called poisson variate and its distribution is called Poisson distribution.

or

A random variable X is said to follow poisson distribution $(X \sim P(\lambda))$ with parameter λ $(\lambda > 0)$ if its probability mass function (p.m.f.) is given by :

$$P[X=x] = p_X(x) = \begin{cases} \dfrac{e^{-\lambda} \lambda^x}{x!} & x = 0, 1, 2, 3, \dots \infty \\ 0 & o/w \end{cases}$$

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Question:- Show that $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots \infty$ is probability mass function for poisson distribution.

Solution:- For Poisson distribution

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, \ldots \infty$$

We know by definition $p(x)$ is p.m.f

if (i) $p(x) \geq 0$

(ii) $\sum_{x=0}^{\infty} p(x) = 1$

$$\therefore \sum_{x=0}^{\infty} p(x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda}\left(\sum_{x=0}^{\infty} \frac{\lambda^x}{x!}\right)$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \infty\right]$$

$$= e^{-\lambda} e^{\lambda} = e^{-\lambda+\lambda} = e^0 = 1$$

$$\Rightarrow \sum_{x=0}^{\infty} p(x) = 1$$

$\Rightarrow p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ is p.m.f. for Poisson distribution. #

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Question:- Prove that Poisson distribution is a limiting case of Bionomial distribution under following conditions :

(i) $n \to \infty$

(ii) $p \to 0$

(iii) $np = d$ (finite).

Solution:- The probability mass function (p.m.f.) for Bionomial distribution is given by

$$P(x) = {}^nC_x p^x q^{n-x}, \quad p+q=1$$

$$x = 0,1,2,---n$$

$$\Rightarrow \lim_{n\to\infty} P(x) = \lim_{n\to\infty} {}^nC_x p^x (1-p)^{n-x}$$

$$= \lim_{n\to\infty} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \lim_{n\to\infty} \frac{n(n-1)(n-2)(n-3)---(n-(x-1))\cancel{(n-x)!}}{x!\cancel{(n-x)!}} p^x(1-p)^{n-x}$$

$\because np = d \Rightarrow p = d/n$

$$= \lim_{n\to\infty} \frac{n(n-1)(n-2)(n-3)\dots(n-(x-1))}{x!}\left(\frac{d}{n}\right)^x\left(1-d/n\right)^{n-x}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^x}\left[(1-1/n)(1-2/n)(1-3/n)\cdots\left(1-\frac{x-1}{n}\right)\right]}{x!} \frac{d^x}{\cancel{n^x}}\left(1-d/n\right)^x$$

$$= \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right]\frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \lim_{n\to\infty}\left[\frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^n\right]$$

$$\times \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right]\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!}e^{-\lambda} \longrightarrow \text{p.m.f for poisson distribution}$$

Proved

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In Binomial distribution :

$$P(X=x) = \begin{cases} {}^nC_x\, p^x q^{n-x} & ; \quad x = 0, 1, 2, 3, \dots n \\ 0 & \quad \text{otherwise} \end{cases}$$

Assumptions are :

(i) No. of experiment conducted is finite "n".

(ii) Probability of success is p and constant for each experiment.

Poisson distribution –: Poisson distribution is a limiting case of the Binomial distribution under the following conditions :

(i) No. of experiment is infinite, "$n \to \infty$".

(ii) Probability of success is p and is infinitely small, i.e., $p \to 0$.

(iii) $np = \lambda$ (say) is finite.

Under these conditions, the Binomial probability function reduces to

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, 3, \dots \infty$$

is called Poisson distribution function.

* Only one parameter λ and is given as Mean.

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Mean -:

$$\text{Mean} = E(X) = \sum_{x} x\, P(x)$$

$$= \sum_{x=0}^{\infty} x \frac{\lambda^{x}}{x!} e^{-\lambda}$$

$$= \sum_{x=0}^{\infty} \cancel{x} \frac{\lambda^{x}}{\cancel{x}(x-1)!} e^{-\lambda}$$

$$= \sum_{x=0}^{\infty} \lambda \frac{\lambda^{x-1}}{(x-1)!} e^{-\lambda}$$

$$= \lambda e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = \lambda e^{-\lambda}\left(1+\frac{\lambda}{1!}+\frac{\lambda^{2}}{2!}+\cdots\right)$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$\boxed{\text{Mean} = E(X) = \lambda}$$

i.e., $X \sim P(\lambda)$

$E(X) = \text{Mean} = \lambda$ (parameter).

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(ii) Varience —: The varience is defined as

$$V(X) = E(X^2) - (E(X))^2$$

$\because \quad X \sim P(\lambda) \quad E(X) = \lambda$

To find $E(X^2) = \sum_{x=0}^{\infty} x^2 P(x)$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!}$$

$\because \quad x^2 = x(x-1) + x$

$$= \sum_{x=0}^{\infty} (x(x-1)+x) \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)}\, e^{-\lambda}\lambda^x}{\cancel{x(x-1)}(x-2)!} + \sum_{x=0}^{\infty} \frac{\cancel{x}\, e^{-\lambda}\lambda^x}{\cancel{x}(x-1)!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^2\lambda^{x-2}}{(x-2)!} + \sum_{x=0}^{\infty} \frac{\lambda e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$

$$= \lambda^2 e^{-\lambda}\left(\sum_{x=0}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}\right) + \lambda e^{-\lambda}\left(\sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}\right)$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda e^{-\lambda} e^{\lambda}$$

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$$\Rightarrow \boxed{E(X^2) = \lambda^2 + \lambda}$$

Which $\Rightarrow$ $Var(X) = E(X^2) - (E(X))^2$

$$= (\lambda^2 + \lambda) - \lambda^2$$

$$\boxed{Var(X) = \lambda}$$

Note that :- It is only distribution in which

$$\boxed{\text{Mean} = \text{Varience} = \text{parameter}}$$

i.e., $X \sim P(\lambda)$

$$\boxed{E(X) = Var(X) = \lambda}$$

Moment generating function for Poisson distribution :-

The moment generating function (MGF) for poisson distribution is given by

$$M_X(t) = E(e^{tX}) = \sum_x e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{xt} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t}$$

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$$\Rightarrow \boxed{M_X(t) = E(e^{tX}) = e^{\lambda(e^t-1)}}$$

Characteristic function :-

The characteristic function is denoted by $\phi_X(t)$ and is defined as :

$$\phi_X(t) = E(e^{itX}) = \sum_{x=0}^{\infty} e^{itx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{itx} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{it})^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^{it}}$$

$$\boxed{\phi_X(t) = E(e^{itX}) = e^{\lambda(e^{it}-1)}}$$

Probability generating function :-

The probability generating function is denoted by $Z_X(t)$ and defined as

$$Z_X(t) = E(z^X) = \sum_{x} z^x p(x)$$

$$= \sum_{x=0}^{\infty} z^x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda z)^x}{x!}$$

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$$= e^{-\lambda} e^{\lambda z}$$

$$= e^{\lambda(z-1)}$$

$$\boxed{Z_X(t) = E(Z^X) = e^{\lambda(z-1)}}$$ #

~~mean and var~~

Question :- If X follows the poisson distribution, i.e., $X \sim P(\lambda)$ and its moment generating function is given by $M_X(t) = e^{4(e^t-1)}$. Then find mean, varience and standard deviation (σ).

Solution :- Given that X follows the poisson distribution $X \sim P(\lambda)$ and its moment generating function is given by

$$M_X(t) = e^{\lambda(e^t-1)}$$

and its $\text{Mean} = E(X) = \lambda$

$$\text{Varience} = V(X) = \lambda$$

$$\text{Standard deviation } (\sigma) = \sqrt{V(X)}$$

Given that $M_X(t) = e^{4(e^t-1)}$

Here $\boxed{\lambda = 4}$

$\Rightarrow$ Mean = Varience = 4

$$sd = \sqrt{4} = 2$$

$\Rightarrow \boxed{sd = 2}$ Ans

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Mean and varience by Moment generating function (M.G.F) -:

We know that

$$\mu_r' = E(X^r) = \begin{cases} \left.\dfrac{d^r}{dt^r}\left(M_X(t)\right)\right|_{t=0} & \text{if } M_X(t) \text{ is given} \\[2ex] \left.(-i)^r \dfrac{d^r}{dt^r}\left(\phi_X(t)\right)\right|_{t=0} & \text{if } \phi_X(t) \text{ is given} \end{cases}$$

$$\text{Mean} = E(X)$$

$$\text{Varience} = E(X^2) - \left(E(X)\right)^2 \; ;$$

For Poission dist^n

$$M_X(t) = e^{\lambda(e^t-1)}$$

Now

$$E(X) = \left.\frac{d}{dt}\left(M_X(t)\right)\right|_{t=0}$$

$$= \left.\frac{d}{dt}\left(e^{\lambda(e^t-1)}\right)\right|_{t=0}$$

$$= \left.\left(e^{\lambda(e^t-1)} \lambda e^t\right)\right|_{t=0}$$

$$= \lambda e^0 e^{\lambda(e^0-1)}$$

$$= \lambda$$

$$\Rightarrow \boxed{E(X) = \text{Mean} = \lambda}$$

$$E(X^2) = \left.\frac{d^2}{dt^2}\left(M_X(t)\right)\right|_{t=0}$$

$$= \left.\frac{d}{dt}\left(e^{\lambda(e^t-1)} \lambda e^t\right)\right|_{t=0}$$

$$= \left[e^{\lambda(e^t-1)}(\lambda e^t)^2 + \lambda e^t e^{\lambda(e^t-1)}\right]_{t=0}$$

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$$= \lambda^2+\lambda$$

$$E(X^2) = \lambda^2+\lambda$$

Now $\quad Var(X) = E(X^2) - (E(X))^2$

$$= \cancel{\lambda^2}+\lambda - \cancel{\lambda^2}$$

$$\Rightarrow \boxed{Var(X) = \lambda}$$

Mean and Varience by characteristic function —:

We know that for Poisson distribution, the characteristic function is given by

$$\phi_X(t) = e^{\lambda(e^{it}-1)}$$

$$E(X^r) = (-i)^r \frac{d^r}{dt^r}\left(\phi_X(t)\right)\Big|_{t=0}$$

$$Mean = E(X)$$

$$Var(X) = E(X^2) - (E(X))^2.$$

$$E(X) = (-i)^1 \frac{d}{dt}\left(e^{\lambda(e^{it}-1)}\right)\Big|_{t=0}$$

$$= (-i)\left[e^{\lambda(e^{it}-1)}(\lambda i e^{it})\right]_{t=0}$$

$$= -i^2\left[e^{\lambda(e^{0i}-1)}\lambda e^{0i}\right]$$

$$\boxed{E(X) = \lambda}$$

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$$E(X^2) = (-i)^2 \frac{d^2}{dt^2}\left[\phi_X(t)\right]_{t=0}$$

$$= (-1)\left[\frac{d}{dt}\left(e^{\lambda(e^{it}-1)}(\lambda e^{it} i)\right)\right]_{t=0}$$

$$= -\left[(\lambda i^2 e^{it})e^{\lambda(e^{it}-1)} + e^{\lambda(e^{it}-1)}(\lambda i e^{it})^2\right]_{t=0}$$

$$= -\left[-\lambda - \lambda^2\right]$$

$$\boxed{E(X^2) = \lambda^2 + \lambda}$$

$$Var(X) = E(X^2) - (E(X))^2 = \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$\Rightarrow \boxed{Var(X) = \lambda}$$

$$\upsilon_r = E(X^r) = \begin{cases} \left.\frac{d^r}{dt^r}(M_X(t))\right|_{t=0} \\ \left.(-i)^r \frac{d^r}{dt^r}(\phi_X(t))\right|_{t=0} \end{cases}$$

$\Downarrow$

Moment about origin

Question:- Given that 2% of fuses are manufactured by a firm are defective. Find the probability that a box containing 200 fuses has:

(i) At least one defective fuses

(ii) 3 or more defective fuses

(iii) No defective fuses.

Solution:- Given that

$$n = 200 \quad \& \quad p = 2\% = 0.02$$

$\because$ n is large and probability is small

$\Rightarrow$ Poisson distribution.

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$\therefore$ for Poission distribution

$$np = \lambda$$

$$\Rightarrow \lambda = 200 \times 0.02 = 4$$

$$\Rightarrow \boxed{\lambda = 4}$$

The probability mass function (p.m.f) for poisson distribution is given by

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x) = \frac{e^{-4}4^x}{x!}$$

(i) At least one defective fuses

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(0)$$

$$= 1 - \frac{e^{-4}4^0}{0!}$$

$$\boxed{P(X \geq 1) = 1 - e^{-4}}$$

(ii) 3 or more defective

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - \left(P(0) + P(1) + P(2)\right)$$

$$= 1 - \left(\frac{4^0e^{-4}}{0!} + \frac{4^1e^{-4}}{1!} + \frac{4^2e^{-4}}{2!}\right)$$

$$= 1 - e^{-4}(1 + 4 + 8)$$

$$P(X \geq 3) = 1 - 13e^{-4}$$ Ans

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(iii) No defective fuses

$$P(0) = \frac{e^{-4} 4^0}{0!}$$

$$\Rightarrow \boxed{P(0) = e^{-4}}$$ Ans

Question:- In certain factory turning out blades, there is small chance of 0.02 for any blade to be defective. The blade is supplied in packet of 10. Using Poisson distribution find approximate number of packets containing

① No defective blades
② One defective blade

In consignment of 10000 packets

Solution:- Given that

$$N = 10,0,00, \quad n = 10 \quad \& \quad p = 0.02$$

$\Rightarrow$ n is large and probability is small.

$\Rightarrow$ Poisson distribution.

$\because$ For poisson distribution

$$np = d$$

$$\Rightarrow d = 10 \times 0.002$$

$$\Rightarrow \boxed{d = 0.02}$$

We know that p.m.f. for poisson distribution is given by

$$P(x) = \frac{e^{-d} d^x}{x!}$$

$$P(x) = \frac{e^{-0.02} (0.02)^x}{x!}$$

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For no defective blades

$$P(0) = \frac{e^{-0.02}(0.02)^0}{0!}$$

$$P(0) = e^{-0.02} = 0.9802$$

$\therefore$ No. of packets containing zero defective blade

$= 10000 \times P(0)$

$= 9802$ packets

(ii) One defective blades

$$P(1) = \frac{e^{-0.02}(0.02)^1}{1!}$$

$$= (0.02) \times 0.9802$$

$\therefore$ No. of packets containing one defective blade.

$= 10000 \times 0.9802 \times 0.02$

$= 9802 \times 0.02$

$= 196.04$ Ans

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Question:- If the probability of a bad reaction from a certain injection is 0.01, find the chance that out of 200 individuals more than two will get bad reaction.

Solution:- Given $n = 200$ & $p = 0.01$

Here n is very large and probability is very small

$\Rightarrow$ Poisson distribution

For Poission distribution

$$\lambda = np$$

$$\lambda = 200 \times 0.01 = 2$$

$$\boxed{\lambda = 2}$$

The probability mass function (p.m.f.) for Poission distribution is given by—

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x) = \frac{e^{-2} 2^x}{x!}$$

For individuals more than two get bad reaction.

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - \left(P(0) + P(1) + P(2)\right)$$

$$= 1 - \left(\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}\right)$$

$$= 1 - e^{-2}(1+2+2)$$

$$\boxed{P(X>2) = 1 - 5e^{-2}}$$

Ans

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Fitting of Poission distribution

Question -: A skilled typist, on rutine work, kept a record of mistake made per day during 300 working days

Mistake/day	0	1	2	3	4	5	6
No. of days	143	90	42	12	9	3	1

x mistake/day	f No. of days	xf	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.89}(0.89)^x}{x!}$	$P(x) \times 300$
0	143	0	$P(0) = e^{-0.89}\frac{(0.89)^0}{0!} = 0.411$	$123.3 \approx 123$
1	90	90	$P(1) = e^{-0.89}\frac{(0.89)^1}{1!} = 0.365$	$109.5 \approx 110$
2	42	84	$P(2) = e^{-0.89}\frac{(0.89)^2}{2!} = 0.163$	$48.9 \approx 49$
3	12	36	$P(3) = e^{-0.89}\frac{(0.89)^3}{3!} = 0.048$	$14.4 \approx 14$
4	9	36	$P(4) = e^{-0.89}\frac{(0.89)^4}{4!} = 0.011$	$3.3 \approx 3$
5	3	15	$P(5) = e^{-0.89}\frac{(0.89)^5}{5!} = 0.002$	$0.6 \approx 1$
6	1	6	$P(6) = e^{-0.89}\frac{(0.89)^6}{6!} = 0.0003$	$0.09 \approx 0$
	$\sum f = 300$	$\sum fx = 267$		$\sum P(x) \times 300 = 300$ Ans

$\Downarrow$ fitting of poission distribution.

$\sum f = 300$, $\sum fx = 267$

For Poission distribution

$$\lambda = \bar{x} = \frac{\sum fx}{\sum f}$$

$$\lambda = \frac{267}{300} = 0.89$$

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Question: Assuming that one in 80 births in a case of twins, calculate the probability of 2 or more sets of twins on a day when 30 births occur.

Solution: X : No. of twins

The required probability

$P(X \geq 2)$

$p(\text{twins}) = \frac{1}{80}$ No. of births $n = 30$

Here $n = 30$ (large), $p = \frac{1}{80}$ (small)] Poisson distribution

$\lambda = np = 30 \times \frac{1}{80} = \frac{3}{8}$

$$\boxed{\lambda = 3/8}$$

$\therefore$ For Poisson distribution, the p.m.f. is given by

$$P(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3/8}(3/8)^x}{x!}$$

$$P(X \geq 2) = 1 - P(X<2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{e^{-3/8}(3/8)^0}{0!} + \frac{e^{-3/8}(3/8)^1}{1!}\right]$$

$$= 1 - e^{-3/8}\left[1 + 3/8\right]$$

$$P(X \geq 2) = 1 - \frac{11}{8}e^{-3/8} = 0.055$$

Ans

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Question :- A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.5. Calculate the proportion of days on which

(i) Neither car is used.
(ii) Some demand is refused.

Solution :- X : No. of cars used

For Poisson distribution

$$\boxed{\text{Mean} = \lambda = 1.5}$$

(i) Neither car is used, i.e., $P(X=0)$

For Poisson distribution, the p.m.f is given by

$$P(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=x) = \frac{e^{-1.5}(1.5)^x}{x!}$$

$$\therefore \quad P(X=0) = \frac{e^{-1.5}(1.5)^0}{0!}$$

$$\boxed{P(X=0) = e^{-1.5}} \quad \text{Ans}$$

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(ii) Some demand is refused, i.e., when demanded more than two cars

i.e., $P(X>2)$

$$P(X>2) = 1 - P(X \le 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$P(X>2) = 1 - e^{-1.5}\left[1 + \frac{1.5}{1!} + \frac{(1.5)^2}{2!}\right]$$

Ans

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Question-: *** The m.g.f. of a random variable X is given by $M_X(t) = e^{3(e^t-1)}$

Then find $P(X=1)$

Solution-: We know that the m.g.f. for Poission distribution is

$$M_X(t) = e^{\lambda(e^t-1)}$$

Here $$M_X(t) = e^{3(e^t-1)}$$

$$\Rightarrow \boxed{\lambda = 3}$$

The probability mass function (p.m.f.) for poission distribution is given by

$$P(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=x) = \frac{e^{-3}3^x}{x!}$$

$$\therefore \quad P(X=1) = \frac{e^{-3}3^1}{1!}$$

$$\boxed{P(X=1) = \frac{3}{e^3}}$$

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