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Probability and Statistics (UCS410)

Experiment 4: Mathematical Expectation, Moments and Functions of Random Variables

1. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

x	0	1	2	3	4
$p(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

(Try functions **sum( )**, **weighted.mean( )**, c(a **%*%** b) to find expected value/mean.

CODE (using weighted.mean( )):

```
#Q1:

#Using weighted.mean()
x <- c(0, 1, 2, 3, 4)
p_x <- c(0.41, 0.37, 0.16, 0.05, 0.01)
mean_imperf <- weighted.mean(x, p_x)
cat("The average number of imperfections per 10 meters of fabric is:", mean_imperf)
```

OUTPUT:

```
> x <- c(0, 1, 2, 3, 4)
> p_x <- c(0.41, 0.37, 0.16, 0.05, 0.01)
> mean_imperf <- weighted.mean(x, p_x)
> cat("The average number of imperfections per 10 meters of fabric is:", mean_imperf)
The average number of imperfections per 10 meters of fabric is: 0.88
```

CODE(sum()):

```
#Using sum()
x <- c(0, 1, 2, 3, 4)
p_x <- c(0.41, 0.37, 0.16, 0.05, 0.01)
mean_imperf <- sum(x * p_x)
cat("The average number of imperfections per 10 meters of fabric is:", mean_imperf)
```

OUTPUT:

```
> x <- c(0, 1, 2, 3, 4)
> p_x <- c(0.41, 0.37, 0.16, 0.05, 0.01)
> mean_imperf <- sum(x * p_x)
> cat("The average number of imperfections per 10 meters of fabric is:",
mean_imperf)
The average number of imperfections per 10 meters of fabric is: 0.88
```

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2. The time T, in days, required for the completion of a contracted project is a random variable with probability density function $f(t) = 0.1\ e^{(-0.1t)}$ for $t > 0$ and 0 otherwise. Find the expected value of T.
Use function **integrate( )** to find the expected value of continuous random variable T.

CODE:

```
#Q2.
pdf <- function(t){
  t*0.1*exp(-0.1*t)
}

expected_value <- integrate(pdf, lower = 0, upper = Inf)$value
cat("The expected value of T is:", expected_value)
```

OUTPUT:

```
>
> expected_value <- integrate(pdf, lower = 0, upper = Inf)$value
> cat("The expected value of T is:", expected_value)
The expected value of T is: 10
>
```

3. A bookstore purchases three copies of a book at \$6.00 each and sells them for \$12.00 each. Unsold copies are returned for \$2.00 each. Let X = {number of copies sold} and Y = {net revenue}. If the probability mass function of X is

x	0	1	2	3
$p(x)$	0.1	0.2	0.2	0.5

Find the expected value of Y.

CODE:

```
#Q3.
#Y = (12X+(3-X)2 - (6*3)) = (10X-12)
x<-c(0,1,2,3)
probab<-c(0.1,0.2,0.2,0.5)
print(weighted.mean(x,probab))
expval<-(10*weighted.mean(x,probab))-12
print(expval)

cat("The expected value of Y (net revenue) is:", expval)

#or

x<-c(0,1,2,3)
probabx<-c(0.1,0.2,0.2,0.5)
y<-10*x-12
probaby<-probabx
expval<-sum(y*probaby)
cat("The expected value of Y (net revenue) is:", expval)
```

OUTPUT:

```
> probaby<-probabx
> expval<-sum(y*probaby)
> cat("The expected value of Y (net revenue) is:", expval)
The expected value of Y (net revenue) is: 9
> 
```

4. Find the first and second moments about the origin of the random variable X with probability density function $f(x) = 0.5e^{-|x|}$, $1 < x < 10$ and 0 otherwise. Further use the results to find Mean and Variance.
(kth moment = $E(X^k)$, Mean = first moment and Variance = second moment – Mean^2.

CODE:

```
#Q4.

f1<-function(x){
  x*0.5*exp(-abs(x))
}

f2<-function(x){
  x^2*0.5*exp(-abs(x))
}

moment1<-integrate(f1,1,10)
moment2<-integrate(f2,1,10)

print(moment1$value)
print(moment2$value)

meanval<-moment1$value
print(meanval)

f3<-function(m1,m2){
  return (m2-(m1^2))
}
print(meanval)

varval<-f3(moment1$value,moment2$value)
print(varval)
```

OUTPUT:

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```
> print(moment1$value)
[1] 0.3676297
> print(moment2$value)
[1] 0.9169292
>
> meanval<-moment1$value
> print(meanval)
[1] 0.3676297
>
> f3<-function(m1,m2){
+   return (m2-(m1^2))
+ }
> print(meanval)
[1] 0.3676297
>
> varval<-f3(moment1$value,moment2$value)
> print(varval)
[1] 0.7817776
```

5. Let X be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1}, x = 1,2,3,\ldots$$

Write a function to find the probability distribution of the random variable $Y = X^2$ and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1,2,3,4,5.

CODE:

```
#Q5.
calculate_Y_distribution <- function(x, p_x) {
  y <- x^2
  p_y <- p_x
  return(data.frame(Y = y, Probability = p_y))
}

x <- 1:20
p_x <- (3/4) * (1/4)^(x - 1)
y_distribution <- calculate_Y_distribution(x, p_x)
probability_of_Y_for_X_3 <- y_distribution[y_distribution$Y == 9, "Probability"]

expected_values <- sapply(1:5, function(x) {
  sum(y_distribution[y_distribution$Y <= x^2, "Probability"] * y_distribution[y_distribution$Y <= x^2, "Y"])
})

variances <- sapply(1:5, function(x) {
  sum((y_distribution[y_distribution$Y <= x^2, "Probability"] * y_distribution[y_distribution$Y <= x^2, "Y"])^2) - (expected_values[x])^2
})

cat("Probability distribution of Y = X^2:\n")
print(y_distribution)

cat("\nProbability of Y for X = 3:", probability_of_Y_for_X_3, "\n")

cat("\nExpected Values of Y for X = 1, 2, 3, 4, 5:\n")
print(expected_values)

cat("\nVariances of Y for X = 1, 2, 3, 4, 5:\n")
print(variances)
```

OUTPUT:

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```
> cat("Probability distribution of Y = X^2:\n")
Probability distribution of Y = X^2:
> print(y_distribution)
     Y  Probability
1    1 7.500000e-01
2    4 1.875000e-01
3    9 4.687500e-02
4   16 1.171875e-02
5   25 2.929688e-03
6   36 7.324219e-04
7   49 1.831055e-04
8   64 4.577637e-05
9   81 1.144409e-05
10 100 2.861023e-06
11 121 7.152557e-07
12 144 1.788139e-07
13 169 4.470348e-08
14 196 1.117587e-08
15 225 2.793968e-09
16 256 6.984919e-10
17 289 1.746230e-10
18 324 4.365575e-11
19 361 1.091394e-11
20 400 2.728484e-12
>
```

```
> cat("\nProbability of Y for X = 3:", probability_of_Y_for_X_3, "\n")

Probability of Y for X = 3: 0.046875
>
> cat("\nExpected Values of Y for X = 1, 2, 3, 4, 5:\n")

Expected Values of Y for X = 1, 2, 3, 4, 5:
> print(expected_values)
[1] 0.750000 1.500000 1.921875 2.109375 2.182617
>
> cat("\nVariances of Y for X = 1, 2, 3, 4, 5:\n")

Variances of Y for X = 1, 2, 3, 4, 5:
> print(variances)
[1]  0.000000 -1.125000 -2.390625 -3.111328 -3.420319
>
```