Artificial Neural Network

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Artificial Neural Networks

- Artificial neural networks are statistical learning models, inspired by biological neural networks (central nervous systems, such as the brain) and are used in machine learning.
- These networks are represented as systems of interconnected “neurons”, which send messages/data to each other.
- The connections within the network can be systematically adjusted based on inputs and outputs, making them ideal for supervised learning.

Processing of ANN depends upon the following three building blocks

- Network Topology
- Adjustments of Weights or Learning
- Activation Functions

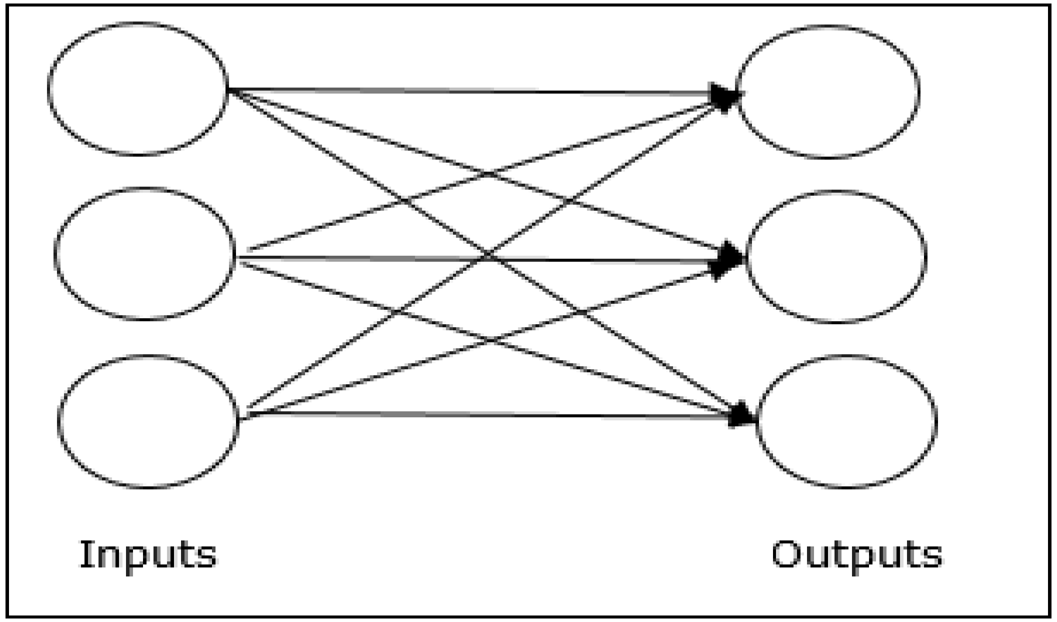
Network Topology

A network topology is the arrangement of a network along with its nodes and connecting lines. According to the topology, ANN can be classified as the following kinds –

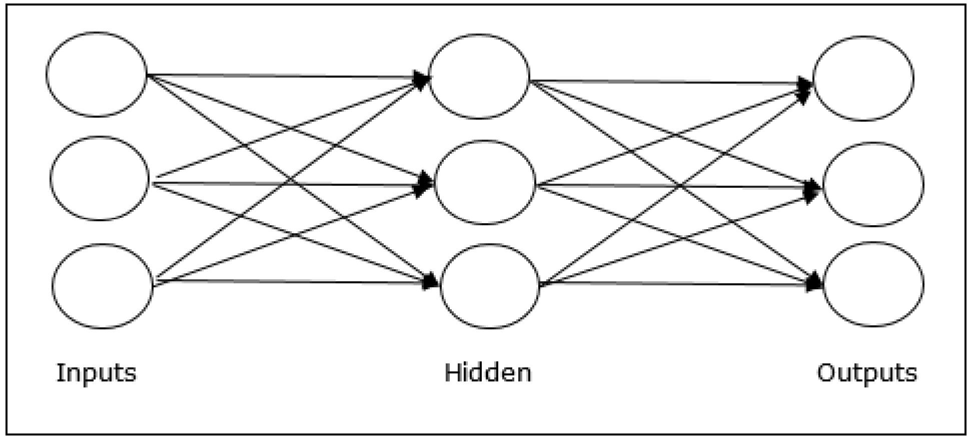
Feedforward Network

It is a non-recurrent network having processing units/nodes in layers and all the nodes in a layer are connected with the nodes of the previous layers. The connection has different weights upon them. There is no feedback loop means the signal can only flow in one direction, from input to output. It may be divided into the following two types –

Single layer feedforward network – The concept is of feedforward ANN having only one weighted layer. In other words, we can say the input layer is fully connected to the output layer.



Multilayer feedforward network – The concept is of feedforward ANN having more than one weighted layer. As this network has one or more layers between the input and the output layer, it is called hidden layers.



A neural network is a collection of “neurons” with “synapses” connecting them.

The collection is organized into three main parts:

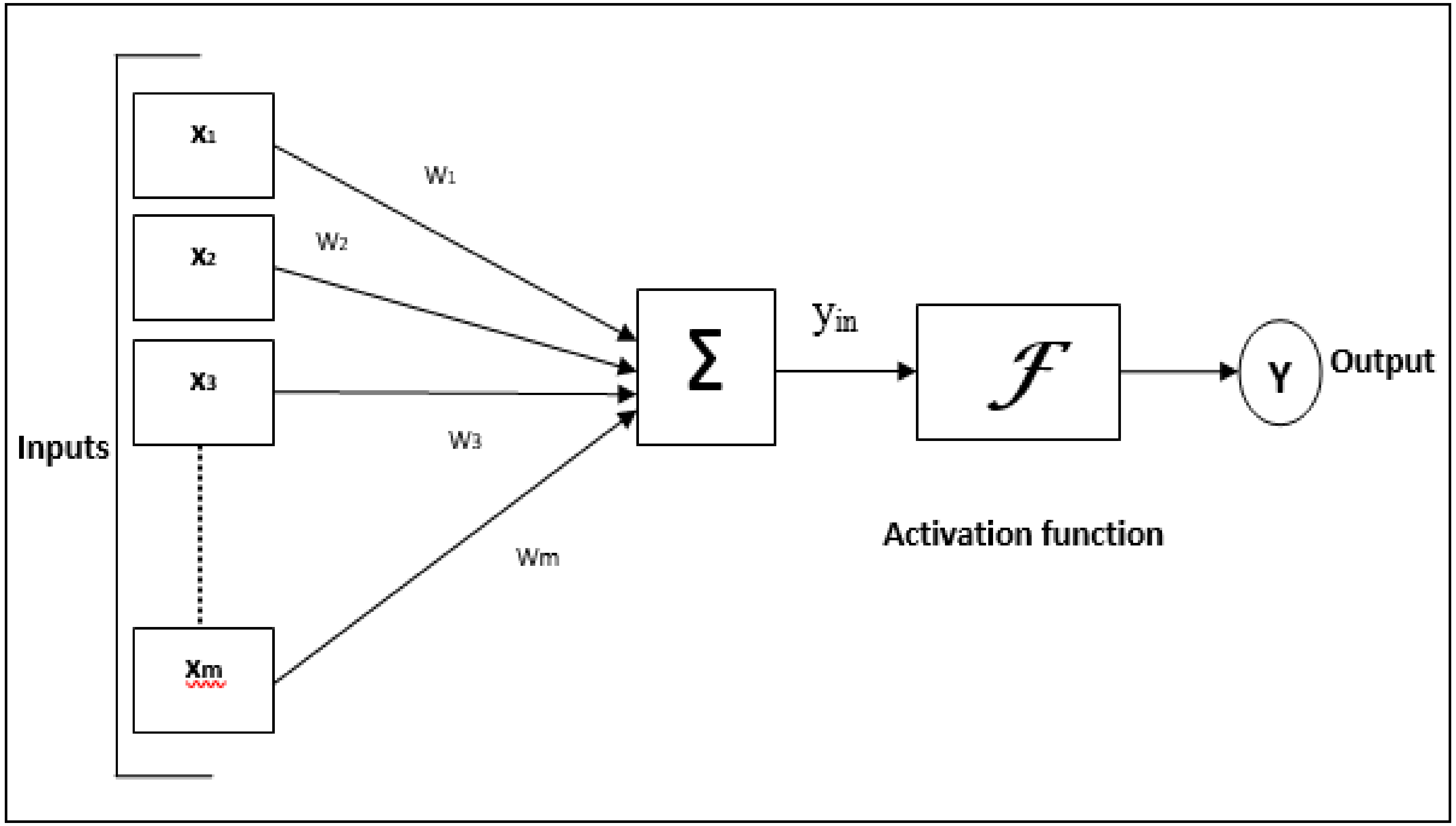
1. input layer,
2. hidden layer(s)
3. output layer.

Note that you can have ***n*** hidden layers, with the term “deep” learning implying multiple hidden layers.

Training a neural network basically means calibrating all of the “weights” by repeating two key steps, forward propagation and back propagation.

1. In forward propagation, we apply a set of weights to the input data and calculate an output. For the first forward propagation, the set of weights is selected randomly.

2. In back propagation, we measure the margin of error of the output and adjust the weights accordingly to decrease the error.



Inputs
x_1
x_2
x_3
x_m
W_1
W_2
W_3
W_m
Σ
y_{in}
$\mathcal{F}$
Activation function
Y
Output

For the general model of artificial neural network, the net input can be calculated as follows

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$$y_{in} = x_1.w_1 + x_2.w_2 + x_3.w_3 \dots x_m.w_m$$

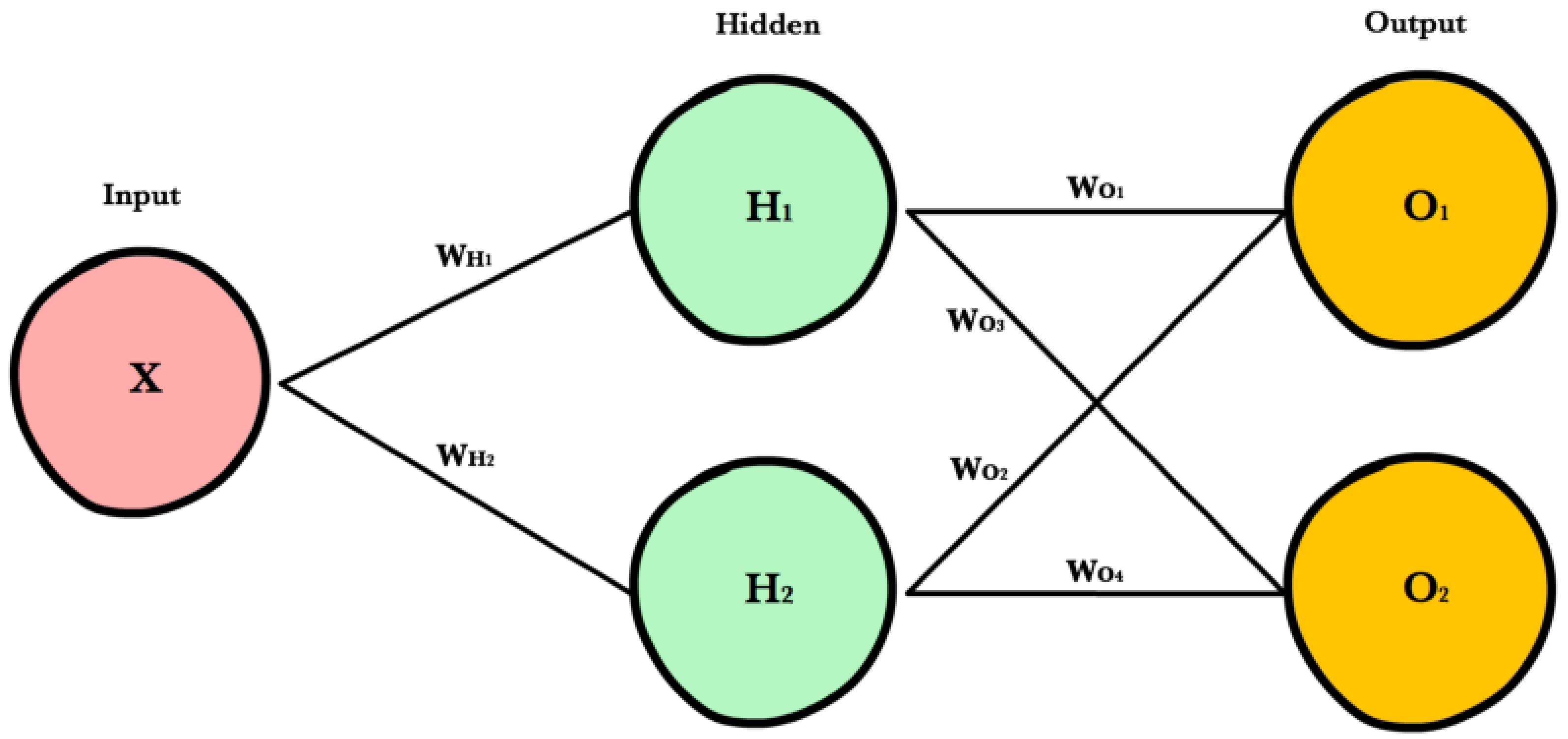
i.e., Net input $y_{in} = \sum_i^m x_i.w_i$

The output can be calculated by applying the activation function over the net input.

$$Y = F(y_{in})$$

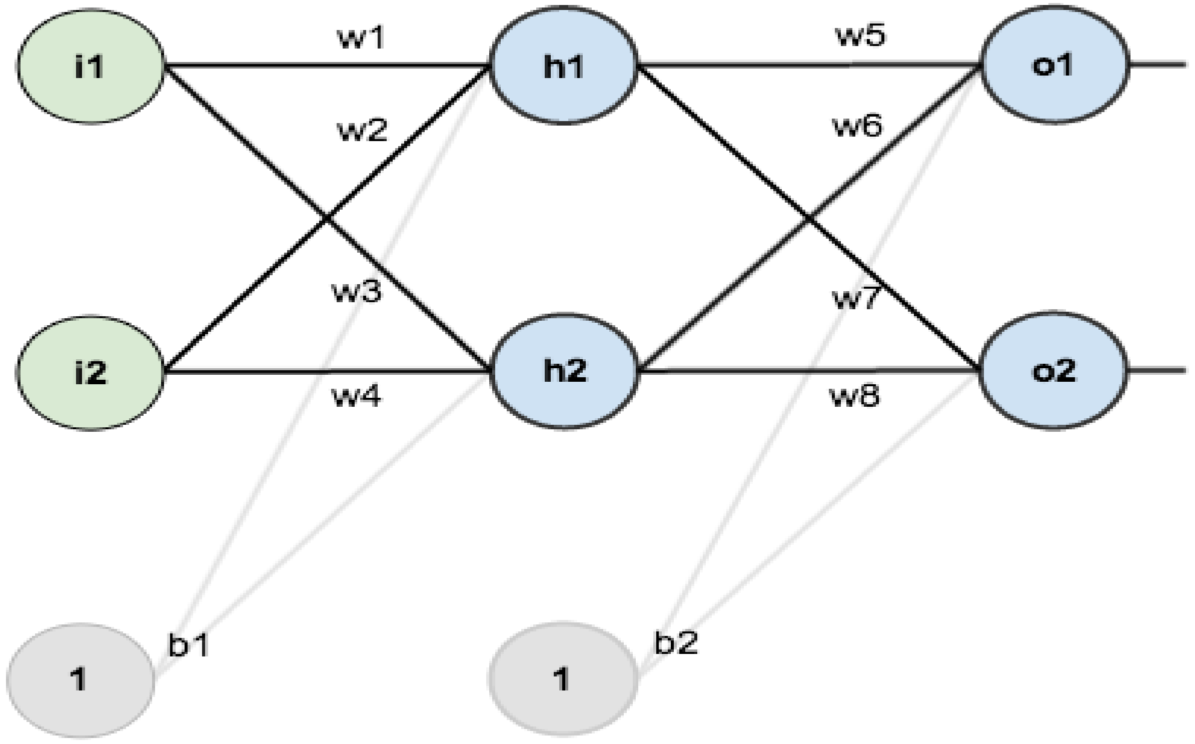
Output = function (net input calculated)

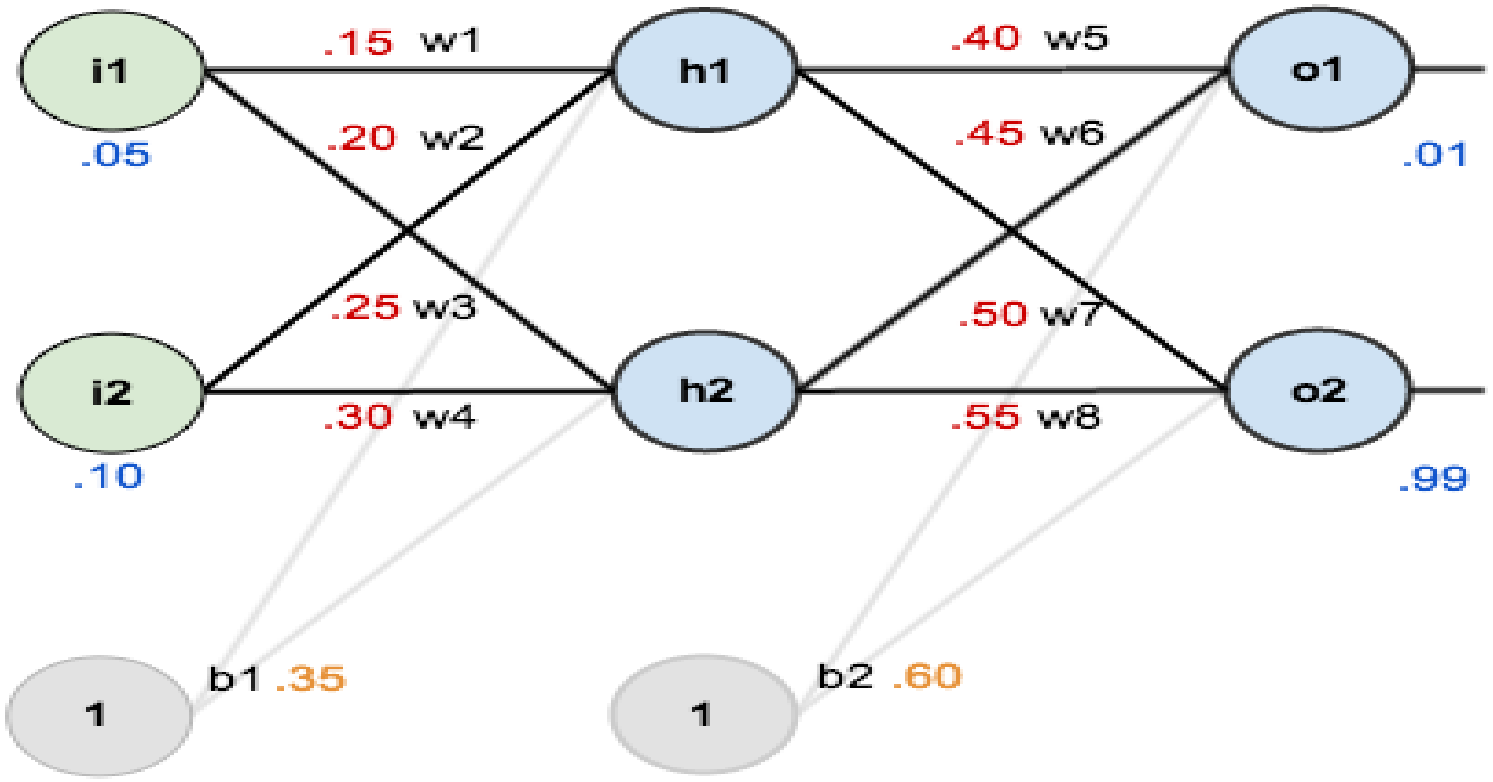
Neural Network with one input, two hidden neurons, two output neurons



Neural Network with two inputs, two hidden neurons, two output neurons

Additionally, the hidden and output neurons may include a bias.





we’re going to work with a single training set: given inputs 0.05 and 0.10, we want the neural network to output 0.01 and 0.99.

The Forward Pass

To begin, lets see what the neural network currently predicts given the weights and biases and inputs of 0.05 and 0.10. To do this we’ll feed those inputs forward though the network.

We figure out the *total net input* to each hidden layer neuron, *squash* the total net input using an *activation function*, then repeat the process with the output layer neurons.

Here’s how we calculate the total net input for hidden node h1

$$net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

$$net_{h1} = 0.15 * 0.05 + 0.2 * 0.1 + 0.35 * 1 = 0.3775$$

We then squash it using the sigmoid function to get the output of h1

$$out_{h1} = \frac{1}{1+e^{-net_{h1}}} = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

Carrying out the same process for hidden node h2, we get:

$$out_{h2} = 0.596884378$$

We repeat this process for the output layer neurons, using the output from the hidden layer neurons as inputs.

Here's the output for o1:

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$net_{o1} = 0.4 * 0.593269992 + 0.45 * 0.596884378 + 0.6 * 1 = 1.105905967$$

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}} = \frac{1}{1+e^{-1.105905967}} = 0.75136507$$

And carrying out the same process for o2 we get:

$$out_{o2} = 0.772928465$$

Calculating the Total Error

We can now calculate the error for each output neuron using the squared error function and sum them to get the total error:

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

The1/2 is included so that exponent is cancelled when we this function will be differentiated.

$$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2 = \frac{1}{2}(0.01 - 0.75136507)^2 = 0.274811083$$

$$E_{o2} = 0.023560026$$

$$E_{total} = E_{o1} + E_{o2} = 0.274811083 + 0.023560026 = 0.298371109$$

Back Propagation

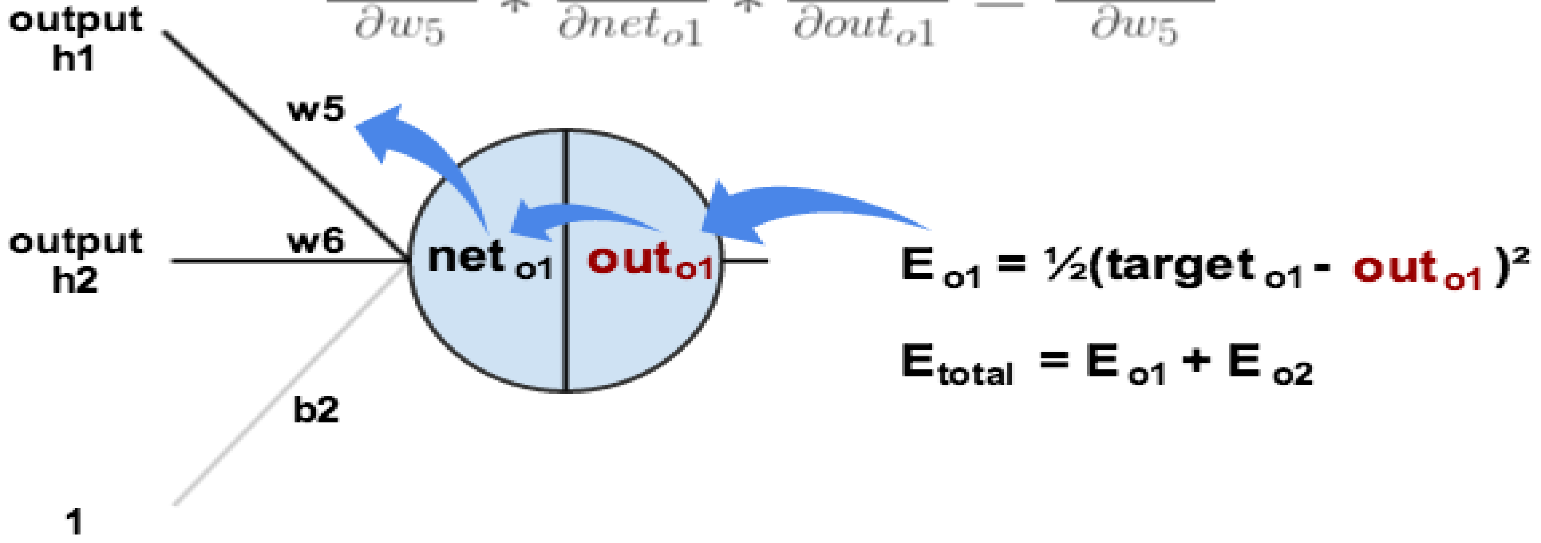
The objective of the backpropagation is to update each of the weights in the network so that they cause the predicted output to be closer to the target output, thereby minimizing the error for each output neuron and the network as a whole.

Output Layer

Consider w5. We want to know how much a change in w5 affects the total error, aka $\frac{\partial E_{total}}{\partial w_5}$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



$$E_{total} = \frac{1}{2}(target_{o1} - out_{o1})^2 + \frac{1}{2}(target_{o2} - out_{o2})^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1}) = -(0.01 - 0.75136507) = 0.74136507$$ … (1)

The partial derivative of the logistic function is the output multiplied by 1 minus the output:

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}}$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1}) = 0.75136507(1 - 0.75136507) = 0.186815602$$ … (2)

Finally, how much does the total net input of o1 change with respect to w5?

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial w_5} = 1 * out_{h1} * w_5^{(1-1)} + 0 + 0 = out_{h1} = 0.593269992$$

Taken from forward propagation

Putting all these together:

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.74136507 * 0.186815602 * 0.593269992 = 0.082167041$$

You’ll often see this calculation combined in the form of the delta rule:

$$\frac{\partial E_{total}}{\partial w_5} = -(target_{o1} - out_{o1}) * out_{o1}(1 - out_{o1}) * out_{h1}$$

To decrease the error, we then subtract this value from the current weight (optionally multiplied by some learning rate, set to 0.5 in this example).

$$w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 * 0.082167041 = 0.35891648$$

Repeating for w6, w7 and w8

$$w_6^+ = 0.408666186$$

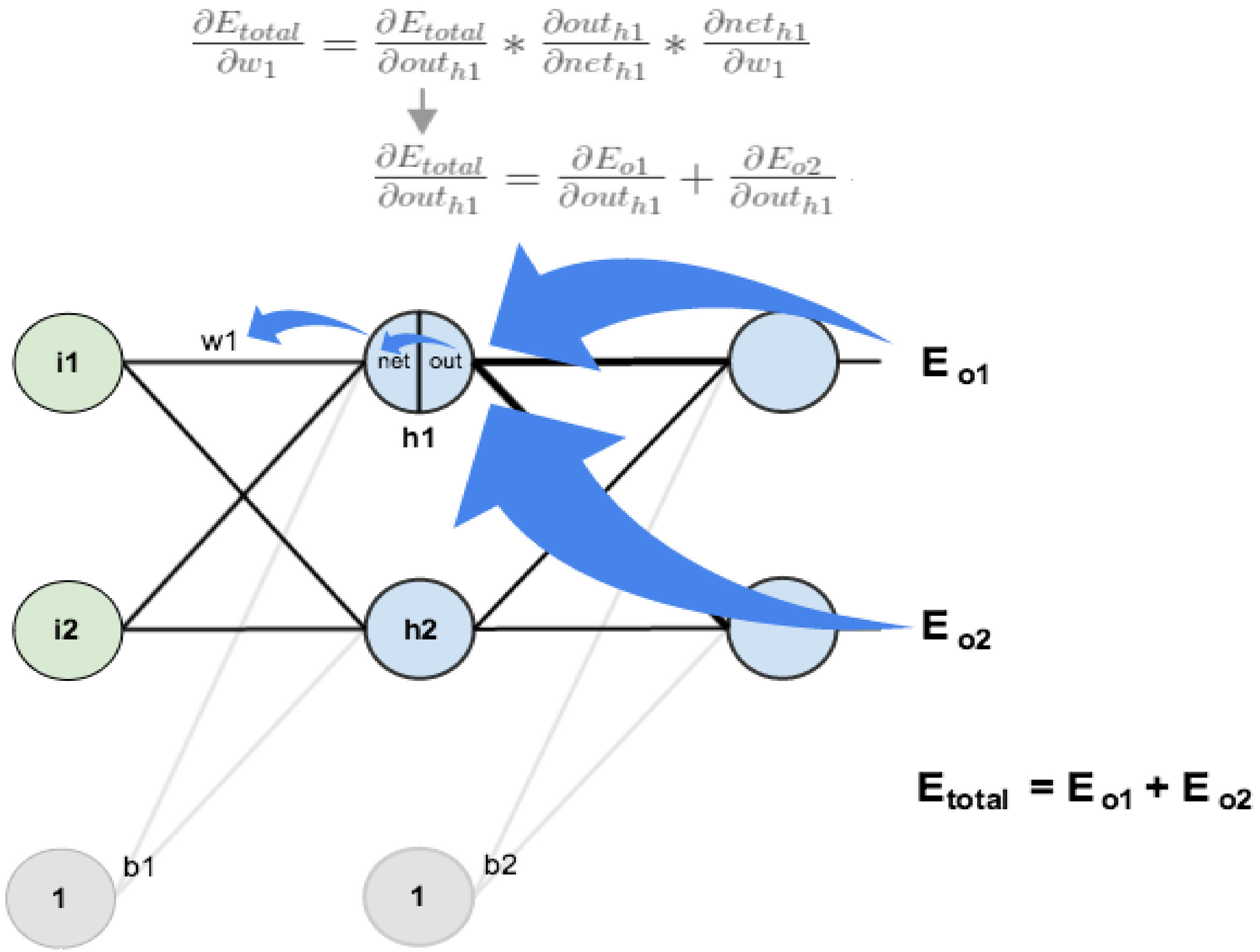
$$w_7^+ = 0.511301270$$

$$w_8^+ = 0.561370121$$

Hidden Layer

Next, we'll continue the backwards pass by calculating new values for w1, w2, w3 and w4

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$



$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$
$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$
i1
w1
net
out
h1
E o1
i2
h2
E o2
$E_{total} = E_{o1} + E_{o2}$
1
b1
1
b2

We're going to use a similar process as we did for the output layer, but slightly different to account for the fact that the output of each hidden layer neuron contributes to the output (and therefore error) of multiple output neurons. We know that out_{h1} affects both out_{o1} and out_{o2}

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}}$$

Using (1) and (2)

$$\frac{\partial E_{o1}}{\partial net_{o1}} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} = 0.74136507 * 0.186815602 = 0.138498562$$ … (3)

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$ … (4)

Using (3) and (4)

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}} = 0.138498562 * 0.40 = 0.055399425$$

$$\frac{\partial E_{o2}}{\partial out_{h1}} = -0.019049119$$

Therefore

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}} = 0.055399425 + -0.019049119 = 0.036350306$$

Now that we have $\frac{\partial E_{total}}{\partial out_{h1}}$ we need to figure out $\frac{\partial out_{h1}}{\partial net_{h1}}$ and then $\frac{\partial net_{h1}}{\partial w}$ for each weight

$$out_{h1} = \frac{1}{1+e^{-net_{h1}}}$$

$$\frac{\partial out_{h1}}{\partial net_{h1}} = out_{h1}(1 - out_{h1}) = 0.59326999(1 - 0.59326999) = 0.241300709$$

We calculate the partial derivative of the total net input to h1 with respect to w1 the same as we did for the output neuron:

$$net_{h1} = w_1 * i_1 + w_3 * i_2 + b_1 * 1$$

$$\frac{\partial net_{h1}}{\partial w_1} = i_1 = 0.05$$

Putting it all together:

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.036350306 * 0.241300709 * 0.05 = 0.000438568$$

We can now update w1:

$$w_1^{+} = w_1 - \eta * \frac{\partial E_{total}}{\partial w_1} = 0.15 - 0.5 * 0.000438568 = 0.149780716$$

Repeating this for w2, w3, and w4

$$w_2^+ = 0.19956143$$

$$w_3^+ = 0.24975114$$

$$w_4^+ = 0.29950229$$

Finally, we've updated all of our weights! When we feed forward the 0.05 and 0.1 inputs originally, the error on the network was 0.298371109.

After first round of backpropagation, the total error is now down to 0.291027924. It might not seem like much, but after repeating this process 10,000 times, for example, the error plummets to 0.0000351085. At this point, when we feed forward 0.05 and 0.1, the two outputs neurons generate 0.015912196 (vs 0.01 target) and 0.984065734 (vs 0.99 target).

Thanks