PROBABILITY AND STATISTICS (UCS401)

Lecture-25

(Log-normal Distribution with illustrations)

Random Variables and their Special Distributions(Unit –III & IV)

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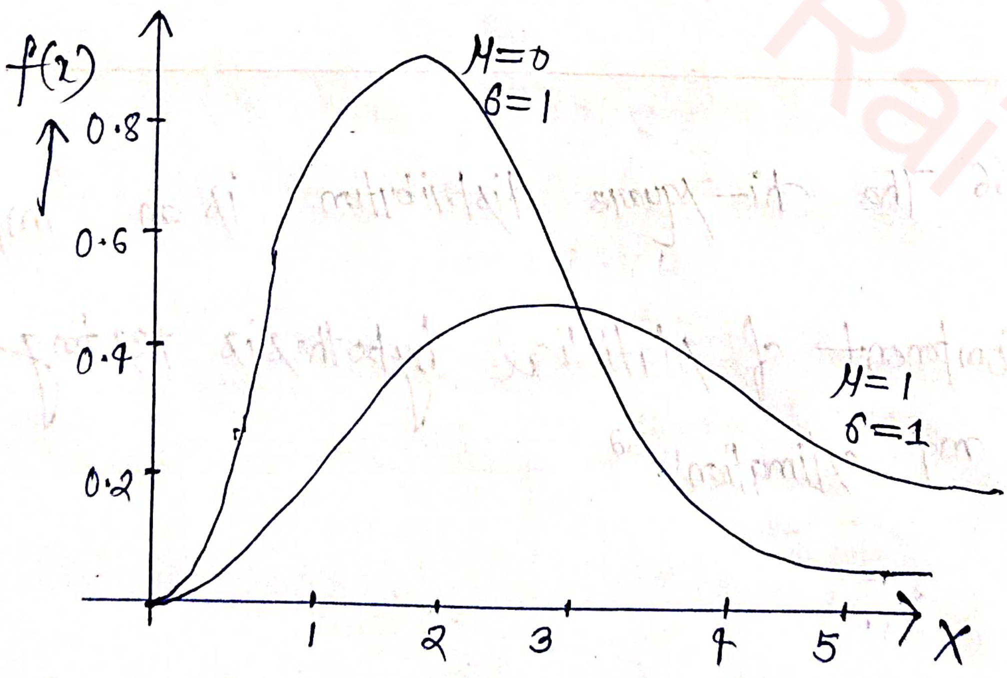
Lognormal distribution with illustrations

Lognormal distribution :

The continuous random variable X has a lognormal distribution if the random variable $Y = \log(X)$ has a normal distribution with mean μ and standard deviation σ.

The resulting density function of X is

$$f(x;\mu,\sigma) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\,\sigma x}\, e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$



Theorem : The mean and variance of the lognormal distribution are

$$\mu = E(X) = e^{\mu + \sigma^2/2}$$

and $Var(X) = \sigma^2 = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$.

Question : Concentrations of pollutants produced by chemical plants historically are known to exhibits behavior that resembles a lognormal distribution. This is important when one considers issues regarding compliance with government regulations. Suppose, it is assumed that the concentration of a certain pollutant, in parts per million, has a lognormal distribution with parameters $\mu = 3.2$ and $\sigma = 1$. What is the probability that the concentration exceeds 8 parts per million?

Solution :- Let the random variable X be pollutant concentration. Then

$$P(X > 8) = 1 - P(X \le 8).$$

Since $\log(X)$ has a normal distribution with mean $\mu = 3.2$ and standard deviation $\sigma = 1$.

$$P(X \leq 8) = P\left(\frac{\log x - 3.2}{1} \leq \frac{\log(8) - 3.2}{1}\right)$$

$$= P\left(Z \leq \frac{\log(8) - 3.2}{1}\right)$$

$$= \Phi\left[\frac{\log(8) - 3.2}{1}\right]$$

$$= \Phi(-1.12) = P(Z < -1.12)$$

$$P(X \leq 8) = 0.1314.$$

Thus, required probability

$$P(X > 8) = 1 - P(X \leq 8)$$

$$= 1 - 0.1314$$

$$\boxed{P(X > 8) = 0.8686}$$

Ans

Question: The life, in thousands of miles, of a certain type of electronic control for locomotives has an approximately lognormal distribution with $\mu = 5.149$ and $\sigma = 0.737$. Find the 5th percentile of the life of such an electronic control.

Solution: From table, we know that

$$P(Z < -1.645) = 0.05.$$

Denote by X the life of such an electronic control.

Since $\log(X)$ has a normal distribution with mean $\mu = 5.149$ and $\sigma = 0.737$,

$$Z = \frac{\log X - \mu}{\sigma} = \frac{\log(x) - 5.149}{0.737}$$

The 5th percentile of X can be calculated as

$$\frac{\log(x) - 5.149}{0.737} = -1.645$$

$$\log(x) = 5.149 + (0.737)(-1.645)$$

$$\log(x) = 3.937$$

Hence, $x = 51.265$

This means that only 5% of the controls will have lifetimes less than 51,265 miles.