Q.

Suppose 20 percent of the items produced by a factory are defective. Suppose 4 items are chosen at random. Find the Probability that

a) 2 are defective b) 3 are defective

c) none is defective

Hint

$$p = \frac{20}{100} = 0.2$$

$n = 4$

x: total no. of defective items

$X \sim$ Binomial $(4, 0.2)$

a) $P(x=2)$ b) $P(x=3)$ c) $P(x=0)$

Q.2

A team has probability $2/3$ of winning whenever it plays. Suppose A plays 4 games. Find the probability that A wins more than half of its games.

Hint

$n = 4$, $p = 2/3$

Binomial distribution

$P(x > 2)$

Q3 A family has 6 children. Find the Probability p that There are

a) 3 boys and 3 girls   b) fewer boys than girls

Hint

$p = \frac{1}{2}$ = { probability of any particular child being a boy is $\frac{1}{2}$

$n = 6$ Binomial (n, p)

a) $P(x=3)$ b) fewer boys than girls if there are 0, 1 or 2 boys

Hence, $P(x=0) + P(x=1) + P(x=2)$

Q4

Q: Suppose 300 misprints are distributed randomly throughout a book of 500 pages. Find the probability P that a given page contains

a) exactly 2 misprints b) 2 or more misprints

Hint

Poisson Distn.

$\lambda = \frac{300}{500} = 0.6$

a) $P(x=2)$ b) $P(x \geq 2)$

OR Binomial

$n = 300$, $p = \frac{1}{500}$

$np = 0.6$

Q: Suppose 2 percent of the items produced by a factory are defective. Find the probability p that there are 3 defective items in a sample of 100 items.

Hint

Poisson approximation

$\lambda = np = 100 \times 0.02 = 2$

$P(x=3)$

Q: A manufacturer, who produces medicine bottles, find that 0.1% of the bottles are defective. The bottles are packed in boxes, each containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Find how many boxes will contain

X defective

a) no defective b) atmost 3 defective c) atleast 2 defective bottles

Hint

$n = 500$, $p = 0.001$, $\lambda = np = 0.5$ Poisson approximation

a) $100 \times P(x=0)$ b) $100\, P(x \leq 3)$ c) $100\, P(x \geq 2)$

Q. Six coins are tossed 6400 times. find the prob. of getting six heads r times

Hint

$n = 6400, \; p = \left(\frac{1}{2}\right)^6$

prob. of getting 6 heads in one throw is $p = \left(\frac{1}{2}\right)^6$

$\lambda = np = 6400 \times \left(\frac{1}{2}\right)^6$

$= 100$

$P(X = r) =$

Poisson approximation

n large, p small

Such that $\lambda = np$

Q. Suppose that the number of telephone calls coming into a telephone exchange between 10 AM and 11 AM, say, X_1 is a random variable with Poisson distribution with parameter 2. Similarly the number of calls arriving between 11 AM and 12 noon, say, X_2 has a Poisson distribution with parameter 6. if X_1 & X_2 are independent, what is the Probability that more than 5 calls come in between 10 AM and 12 noon?

Hint

$X_1 \sim \text{Poisson}(2)$

$X_2 \sim \text{Poisson}(6)$

$\because$ given X_1 & X_2 are given Independent

$\therefore Y = X_1 + X_2 \sim \text{Poisson}(8)$

So $P(Y > 5) =$

Q. An insurance Company insures 5000 people against loss of limbs in a car accident. Based on previous data, The rates were computed on The assumption That on The average 40 persons in 2,00,000 will have car accident each year That result in This type of injury. What is The probability That more than 5 of The insured will Collect on Their policy in a given year?

Hint

$n = 5000$

$p = \dfrac{40}{200000} = 0.0002$

$\lambda = np = 5000 \times 0.0002 = 1$

$P(X > 5) =$

Poisson approximation

n - large

p - small

$np = \lambda$

Q. it is stated That 2% of mobile phones supplied by a manufacturer are defective. A random sample of 200 mobile phones is drawn from a lot. Find The Probability That

a) 3 or more are defective b) 4 or less are defective

Hint

$n = 200$

$p = \dfrac{2}{100} = 0.02$

$\lambda = np = 4$

a) $P(X \geq 3)$ b) $P(X \leq 4)$

Q. An automatic machine makes paper clips from coils of wire. On the average, 1 in 400 paper clips is defective if the paper clips are packed in boxes of 100, what is the probability that any given box of clips will contain

a) no defective b) one or more defective

c) less than two defectives?

Hint

$$p = \frac{1}{400}, \quad n = 100, \quad \lambda = np$$

Q. An item is produced in large numbers. The machine is Known to produce 5% defectives. A quality control inspector is examining the items by taking at random what is the probability that atleast 4 items are to be examined in order to get 2 defectives?

Hint

$p = 0.05$

$r = 2$

Negative Binomial

$$P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \; ; \; x = r, \ldots$$

$$P(X \geq 4) = \sum_{x=4}^{\infty} P(X=x)$$
$$= 1 - P(X=2) - P(X=3)$$

Q. if the probability that a target is destroyed on any one shot is 0.5, what is the Probability that it would be destroyed on 6^{th} attempt

Hint $p = \frac{1}{2} = 0.5$

$P(X=6) =$

Geometric distⁿ

$r=1$ → in Negative Binomial

Q. A die is cast until 6 appears. What is the Probability that it must be cast more than 5 times?

Hint $P(X>5) = 1 - P(X \leq 5)$

$X \sim$ Geometric(p)

$p = \frac{1}{6}$

Q. A Couple decides to have children until They have a male child. What is The probability distribution of The number of children They would have? if The probability of a male child in Their community is 1/3, how many children are They expected to have before The first male child is born?

Hint $p = \frac{1}{3}$

Negative Binomial

$\xrightarrow[r=1]{}$ geometric

Poisson Distribution as a limiting Case of Binomial Distⁿ under the following conditions:

(i) $n \to \infty$, the number of trials are large

(ii) $p \to 0$, the probability of success for each trial is indefinitely small

(iii) $np = \lambda$ (say) is finite

Thus $p = \frac{\lambda}{n}$

Now if $X \sim$ Binomial(n, p)

$$P(X = x \mid n, p) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x} \; ; \; x = 0, 1, \ldots n$$

$$= \frac{n!}{x!\,(n-x)!} \left(\frac{\lambda}{n}\right)^{x} \left(1 - \frac{\lambda}{n}\right)^{n} \frac{1}{\left(1 - \frac{\lambda}{n}\right)^{x}} \; ; \; x =$$

$$= \frac{n(n-1) \ldots (n-x+1)}{x!} \frac{(\lambda/n)^{x}}{(1-\lambda/n)^{x}} \left(1 - \frac{\lambda}{n}\right)^{n}$$

$$= \frac{n^{x}\, 1 \cdot \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) \ldots \left(1 - \frac{x+1}{n}\right)}{x!\left(1-\frac{\lambda}{n}\right)^{x}} \frac{\lambda^{x}}{n^{x}} \left(1 - \frac{\lambda}{n}\right)^{n}$$

$$= \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) \ldots \left(1 - \frac{x+1}{n}\right) \lambda^{x} \left(1 - \frac{\lambda}{n}\right)^{n}}{x!\,\left(1 - \frac{\lambda}{n}\right)^{x}}$$

$$= \frac{e^{-\lambda} \lambda^x}{x!}$$

as $n \to \infty$

$$\boxed{\begin{aligned} &\because \lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda} \\ &\lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^\alpha = 1, \ \alpha \text{ is not a function of } n \end{aligned}}$$

$$\therefore \lim_{n\to\infty} X \sim \text{Binomial}(n, p) \cong \text{Poisson}(\lambda)$$

when $n \to \infty$, $p \to 0$ and $np = \lambda$

Let $X \sim$ Hypergeometric

$$P(X = x \mid N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}} \quad ; \quad x = 0, 1, 2, \ldots K$$

$$M - (N-K) \le x \le M$$

As $N \to \infty$ and $\frac{M}{N} \to p$

$$P(X = x \mid N, M, K) = \frac{M!}{x!\,(M-x)!} \cdot \frac{(N-M)!}{(K-x)!\,(N-M-K+x)!} \cdot \frac{K!\,(N-K)!}{N!}$$

$$= \frac{M(M-1)\ldots(M-x+1)}{x!} \cdot \frac{(N-M)(N-M-1)\ldots(N-M-K+x+1)}{(K-x)!}$$

$$\cdot \frac{K!}{N(N-1)\ldots(N-K+1)}$$

$$= N^x \frac{M}{N}\left(\frac{M}{N}-\frac{1}{N}\right)\cdots\left(\frac{M}{N}-\frac{x-1}{N}\right)$$

$$\cdot N^{k-x}\left(1-\frac{M}{N}\right)\left(1-\frac{M}{N}-\frac{1}{N}\right)\cdots\left(1-\frac{M}{N}-\frac{k-x-1}{N}\right)$$

$$\cdot \frac{1}{N^k\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)\cdots\left(1-\frac{k-1}{N}\right)}$$

$$\cdot \frac{k!}{x!\,(k-x)!}$$

Proceeding to the limit as $N \to \infty$ and $\frac{M}{N} = p$, we get

$$\lim_{N\to\infty} \text{hypergeometric}(x \mid N, M, k) = \binom{k}{x} \cdot N^x \underbrace{p \cdot p \cdots p}_{x\text{-times}}$$

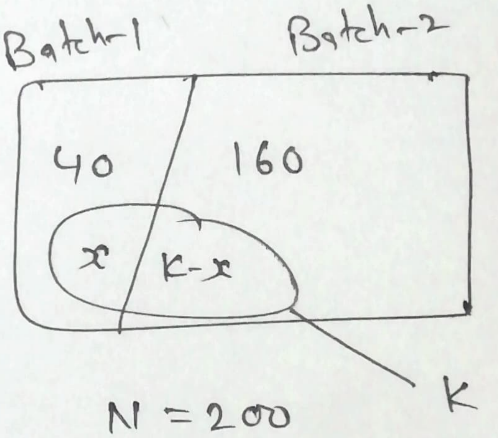
$$\cdot N^{k-x} \underbrace{(1-p)(1-p)\cdots(1-p)}_{k-x \text{ times}}$$

$$\cdot \frac{1}{N^k}.$$

$$= \binom{k}{x} p^x (1-p)^{k-x} \; ; \; x = 0, 1, \ldots k$$

$$\therefore X \sim \text{Binomial}(x \mid k, p)$$

Q. 200 students of the B Tech class in a certain University are divided at random into 2 Batches of 10 each for the annual practical examination in Statistics. Suppose the class consists of 40 resident students and 160 non-resident students; and let X denote the no. of resident students in the first batch. Find the probability that $X \geq 3$. {use binomial approximation}



Hint

$$P(X=x \mid N,M,K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$$

$N = 200$

$M = 40$, $K = 10$

$N-M = 160$

$$p = \frac{M}{N} = \frac{40}{200} = 0.2$$

$$P(X = x) = \binom{10}{x} (0.2)^x (0.8)^{10-x} \; ; \; x = 0,1 \ldots 10$$

$\cancel{x} = 0, 1, \ldots \cancel{K}$

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$