PROBABILITY AND STATISTICS (UCS401)

Lecture-33

Student's t-test- one sample mean

Testing of Hypothesis (Unit –VII)

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Student's t-test -- one sample mean

When sample size is small ($n < 30$), the student's t-statistics is given by

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}.$$

Here,

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)} \quad \text{--- (1)}$$

is called as unbiased sample variance and $(n-1)$ is called degree of freedom (df) associated with S^2.

Whereas, for large sample ($n \geq 30$), the estimate provided by sample variance s^2 and normal test, i.e., Z-test applied

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \quad \text{--- (2)}$$

(1) & (2) $\Rightarrow$

$$S^2 = \left(\frac{n}{n-1}\right) s^2$$

$$\Rightarrow \boxed{S = s\sqrt{\frac{n}{n-1}}}$$

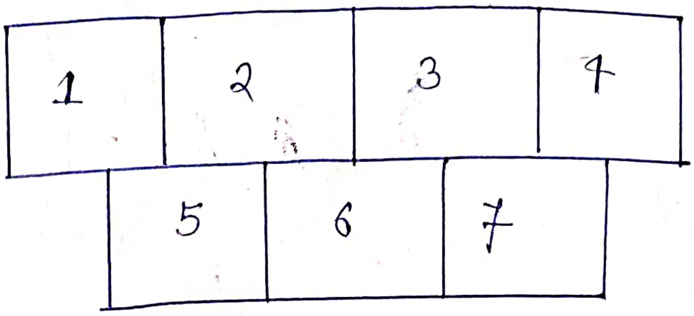
as $n \to \infty$ ($n \geq 30$)

$$s^2 \simeq S^2.$$

Degree of freedom -:

Consider a seven boxes and the task is to choose <u>one box</u> in a day.

Day-1	choices-7
Day-2	Choices-6
Day-3	choices-5
Day-4	choices-4
Day-5	choices-3
Day-6	choices 2
Day 7	No choice



If n sample $\Rightarrow$ $(n-1)$ dof.

Test for single mean - Hypothesis testing -:

(i) Define the Hypothesis -:

Null Hypothesis : $H_0 : \mu = D$, where D is some specified task that you wish to test.

Alternative Hypothesis -:

One tailed test	Two-tailed test
$H_1 : \mu > D$ or $H_1 : \mu < D$	$H_1 : \mu \neq D$

② Test - statistics :

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

where,

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \Rightarrow \text{unbiased estimator of S.D.}$$

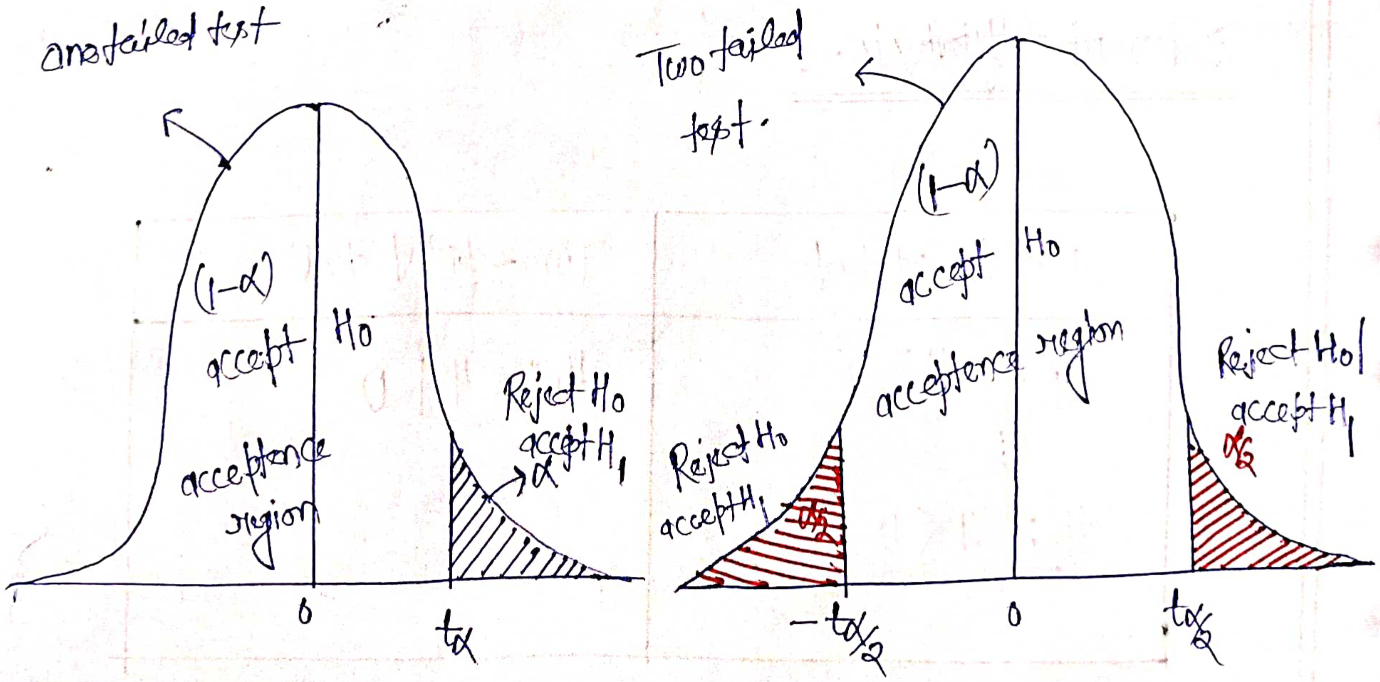
$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \Rightarrow \text{Biased estimator of S.D.}$$

$$\boxed{S = s\sqrt{\frac{n}{n-1}}}$$

③ Rejection region :

Let α is the level of significance,

One tailed test	Two tailed test
$t > t_\alpha$ or $t < -t_\alpha$	$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$



or

p-value

One tailed test	Two tailed test
p value $= P(T>t)$ or $P(T<-t)$	p value $= P(T>t)+P(T<-t)$

The critical value of t, t_α, $t_{\alpha/2}$ are based on $(n-1)$ degree of freedom.

(f) Conclusion -:

- Reject H_0 and conclude that H_1 is true.
- Accept (do not reject) H_0 as true.

Note that -: For a t-distribution, with 5-degree of freedom, the value of t that has area 0.05 to the right (i.e., one tailed) is found in row 5 in the column marked $t_{0.05}$.

sample
$n=6$
$Df=n-1$
$Df=5$
$t_5(0.05)$ or $t_{0.05}(5)$
$t_{df}(\alpha)$ or $t_\alpha(df)$

$t_5(0.05) = 2.015$ 1- tailed

$t_5(0.10) = 2.015$ 2- tailed

Question-: Ten Cartons are taken at random from an automatic filling machine. The mean net weight of 10 Cartons is 11.8 units and standard deviation is 0.15 unit. Does the sample mean differ significantly from the intended weigth of 12 unit?

Solution-: Given that

$n = 10, \quad \bar{x} = 11.8$

$s = 0.15 \rightarrow$ biased estimator S.D.

(i) Define the hypothesis-:

Null hypothesis : $H_0 \quad \mu = 12$

Alternative hypothesis : $H_1 \quad \mu \neq 12$ (a two tailed)

since $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ and $S^2 = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$

$$S^2 = s^2\left(\frac{n}{n-1}\right)$$

$$S^2 = (0.15)^2\left(\frac{10}{10-1}\right) = 0.025$$

$$\boxed{S = 0.15811}$$

Hence t-statistics $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}} = \dfrac{11.8 - 12}{0.15811/\sqrt{10}}$

$$\Rightarrow \boxed{t = -4}$$

③ Rejection region -:

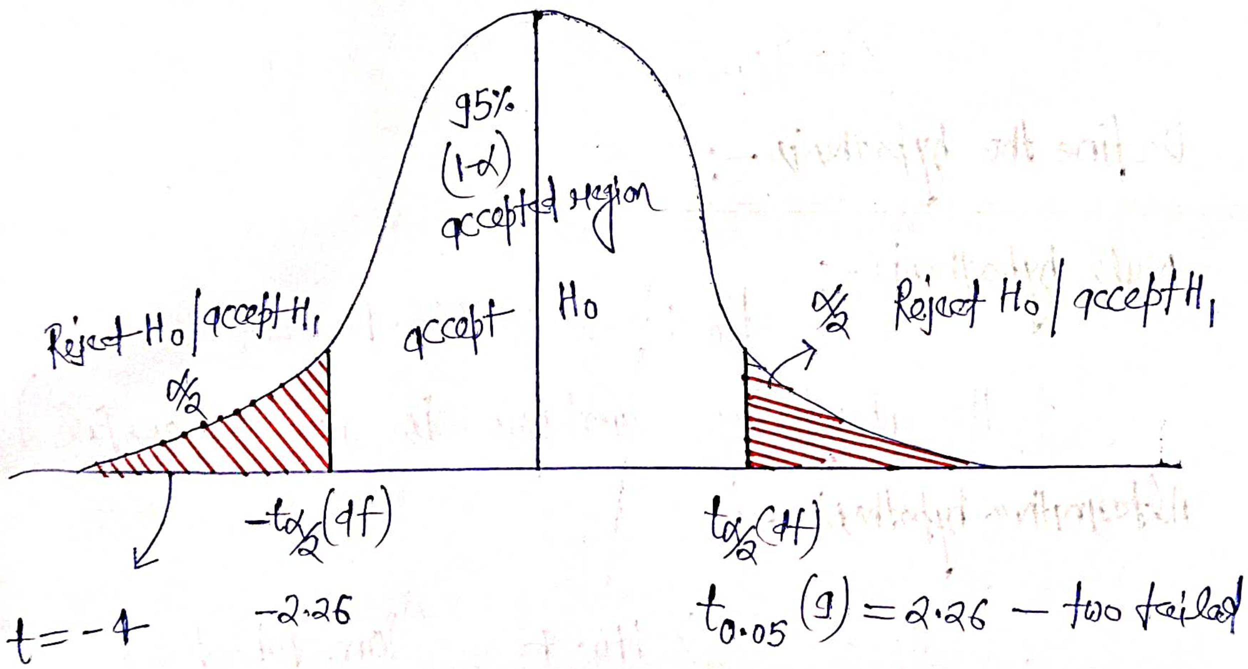
Degree of freedom $= 10-1 = 9$

The critical value at 9 degree of freedom with 5% level of significance is

$t_9(0.05) = 2.26$ for two-tailed test.

or

$t_{0.05}(9) = 2.26$ for two-tailed test



④ Conclusion -: Since t- state value (-4) lies in rejection region of H_0, so it is highly significant.

Hence, H_0 is rejected at 5% level of significance and conclude that sample mean differs significantly from population mean of 12. #

Question–: A mean weakly sales of the candy bar in a candy stores was 146.4 bars per store. After a advertising campaign the mean weakly sales in 22 stores for a typical week increases to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

Solution–: Given that

$$n = 22, \quad \bar{x} = 153.7,$$

$$s = 17.2.$$

(i) Define the hypothesis –:

Null hypothesis –: $H_0 : \mu = 146.4$

(the advertising campaigns is not successful).

Alternative hypothesis –:

$H_1 : \mu > 146.4$ (one failed).

t-statistics :

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}},$$

Since $s^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})}{n}$ and $S^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$

Thus $S^2 = s^2\left(\dfrac{n}{n-1}\right) = (17.2)^2\left(\dfrac{22}{22-1}\right) = 309.9276.$

Hence, $t = \dfrac{153.7 - 146.3}{\sqrt{309.9276}/\sqrt{22}} = 1.9716$

$$\boxed{t = 1.9716}$$

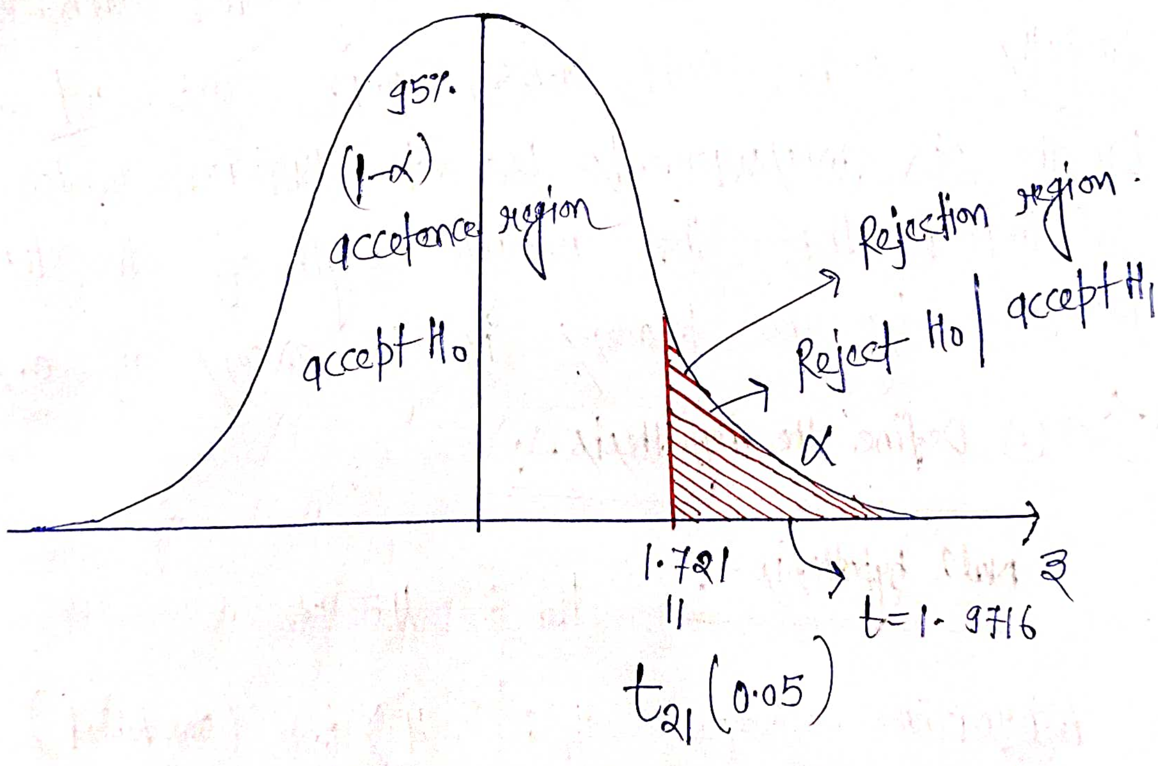
(3) Rejection region :-

Degree of freedom $= 22-1 = 21$

Critical value at 21 degree of freedom with 5% level of significance is

$t_{21}(0.05) = 1.721$ for one tailed.

or $t_{21}(0.10) = 1.721$ for two tailed.



④ Conclusion-:

Since the calculated t-stat value (1.9716) lies in rejection region of H_0, so it is highly significant.

Hence, H_0 is rejected at 5% level of significance and conclude that advertising campaigns was successful in promoting sales.

Question-③ A new process for producing synthetic diamonds can be operated at a profitable level only if the average weigth of the diamonds is greater than 0.5 Karat. To evaluate the probability of the process, six diamonds are generated with recorded weigths, 0.46, 0.61, 0.52, 0.48, 0.57 and 0.54 Karat. Do the six measurements present sufficient evidence to indicate that the average weigth of the diamonds produced by the process is in excess of 0.5 Karat.

solution-: ① Define the Hypothesis-:

Null hypothesis-: $H_0 : \mu = 0.5$

Alternative Hypothesis $H_1 : \mu > 0.5$ (one tailed).

② T- statistics -:

Compute

$$\bar{x} = \frac{\sum_{i=1}^{6} x_i}{n} = \frac{0.46 + 0.61 + 0.52 + 0.48 + 0.57 + 0.51}{6}$$

$$\boxed{\bar{x} = 0.53}$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = 0.0559.$$

Thus,

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{0.53 - 0.50}{0.0559/\sqrt{6}} = 1.32.$$

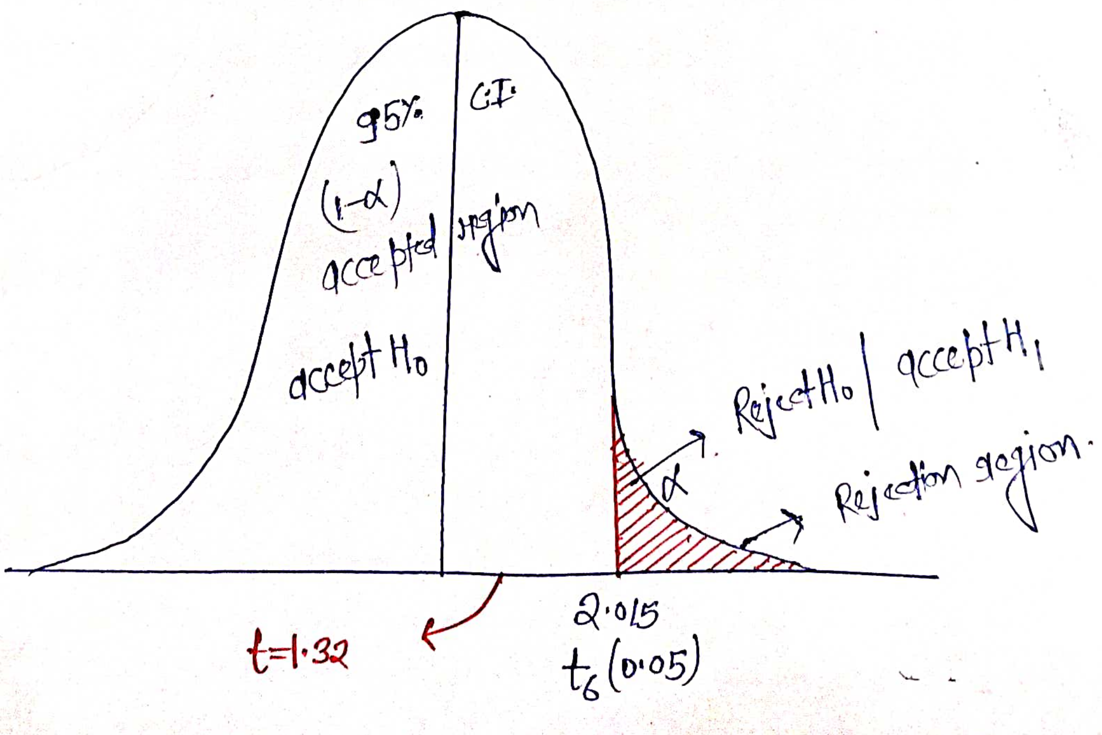
$$\boxed{t = 1.32}$$

③ Rejection region -:

Degree of freedom $= 6 - 1 = 5$

Critical value at 5 degree of freedom with 5% level of significance is

$t_5(0.05) = 2.015$ for one tailed.



④ Conclusion -:

Since the calculated t-statistics value (1.32) lies in acceptance region of H_0.

Hence H_0 can not be rejected at 5% level of significance and conclude that the data DO NOT present sufficient evidence to indicate that the mean diamond weigth exceeds.