PROBABILITY AND STATISTICS (UCS401)

Lecture-3

(Introduction to Probability (Sample Space, Events))

Introduction to Probability (Unit -II)

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Introduction to Probability (Sample space, Events)

(1) Random experiment:

If in each trail of an experiment conducted under identical conditions, the outcome is not unique, but may be any of the possible outcomes, then such experiment is called a random experiment.

Examples:

(i) Tossing a coin is a random experiment.

H or T → Not unique ⇒ random experiment.

(ii) Throwing a die ⇒ Random experiment.

$S = \{1, 2, 3, 4, 5, 6\}$ → Not unique

(iii) Selecting a card from a pack of 52 cards.

K, Q, J or Ace ⟹ Random experiment.

⇓

not unique output

(iv) A bag contains 3 red, 4 white, balls in which 2 balls are drawn at random.

$\Downarrow$

2R or 2W or 1R/1W $\Rightarrow$ Random experiment.

$\Downarrow$

output is not unique

Outcomes/Sample space :

The result of the random experiment is called outcomes.

(i) Tossing a coin : $\rightarrow$ H, $\rightarrow$ T $\quad S = \{H, T\} \Rightarrow$ outcomes

(ii) Throwing a die : outcomes $= \{1, 2, 3, 4, 5, 6\}$

(iii) A bag contains 2R, 2W balls in which 1 ball is drawn at random.

outcomes $= \{1R, 1W\}$

Trail / Event : Any particular performance of a random experiment is called trail and outcomes or combination of outcomes, are termed as events.

(i) Tossing a coin $\left[\begin{array}{l} H \\ \text{or} \\ T \end{array}\right.$ Trail $= \{H, T\}$

(ii) Throwing a die Trail $= \{1, 2, 3, 4, 5, 6\}$.

Note that : If a coin is tossed repeatedly, result is either head or tail is called trail.

Exhaustive events : The total number of possible outcomes of a random experiment is called Exhaustive events.

Examples : (i) In tossing a coin outcomes $= \{H, T\}$, i.e., there are two exhaustive cases.

(ii) In throwing a die, there are six exhaustive cases. $S = \{1, 2, 3, 4, 5, 6\}$.

(iii) A bag containing 3 Red and 2 white balls, in which 2 balls are drawn at random

$\Rightarrow$ $S = \{2R, 2W, 1R/1W\}$

Total balls $= 5$

You pick $= 2$

$\Downarrow$

${}^{5}C_{2} \Rightarrow$ Exhaustive cases.

Mutually exclusive/ disjoint events :

Two events are said to be mutually exclusive if happening of any one of them exclude the happening of all the others.

Examples : (i) In throwing a die, $S = \{1, 2, 3, 4, 5, 6\}$, all the six faces are mutually exclusive.

(ii) In tossing a coin, events, Head and tail are mutually exclusive.

Independent events :

Two events are said to be independent, if happening of one is not affected by the happening of others.

Examples : In tossing an unbiased coin, event of getting head in first toss is independent of getting a head in the second toss, third and subsequent throwan.

(i.) Coin 1^{st} toss $\longrightarrow$ H

2^{nd} toss $\longrightarrow$ H/T $\Rightarrow$ H & T is independent events.

(ii) 3W & 2R balls in which 2 balls are drawn at random.

Outcomes = { 2R or 2W or 1R & 1W }

(1) (2) (3)

Exclusive / disjoint events.

and $\Rightarrow$ independent events.

or $\Rightarrow$ Exclusive events.

Equally likely event: Two events are said to be equally likely events if chances of occurence of events are equal.

Mathematical / Classical probability: If a random experiment, results, 'N', exhaustive (total), exclusive (disjoint), and equally likely (same) outcomes then probability of event A is defined as:

$$\boxed{P(A) = \frac{\text{Number of favourable cases}}{\text{Total number of exhaustive cases}}}$$

Examples : (i) Tossing a Coin

$S = \{H, T\}$

Exhaustive cases $= 2$

$P(\text{tail}) = \frac{1}{2}$ & $P(\text{head}) = \frac{1}{2}$.

(ii) Throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$

Exhaustive cases $= 6$; $P(3) = \frac{1}{6}$

& $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$.

Question[1] Two unbiased dice are thrown. Find the probability that

(i) Both the die shows the same number.

(ii) The first die shows 6.

(iii) The total of the numbers on the dice is 8.

Solution: Dice 1 times $\rightarrow 6$

Dice 2 times $\rightarrow 6^2$

Dice 3 times $\rightarrow 6^3$

$\therefore$ If two dice are thrown then

Exhaustive cases $= 6^2 = 36$.

$$S = \left\{\begin{matrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & (24) & (25) & (26) \\ (31) & (32) & (33) & (34) & (35) & (36) \\ (41) & (42) & (43) & (44) & (45) & (46) \\ (51) & (52) & (53) & (54) & (55) & (56) \\ (61) & (62) & (63) & (64) & (65) & (66) \end{matrix}\right\}$$

(i) When both dice show same number

favourable cases $= \{(1,1)\ (22)\ (33)\ (44)\ (55)\ (66)\} = 6$

Thus, the required probability

$$P(S) = \frac{6}{36} = \frac{1}{6}$$

$$\boxed{P(S) = \frac{1}{6}}$$

Ans

(ii) When first die shows 6

favourable cases $= \{(61), (62), (63)\ (64)\ (65)\ (66)\}$

$= 6$

Thus, required probability

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

$$\boxed{P(E) = \frac{1}{6}}$$

Ans

(iii) When total of the numbers on the dice is 8.

favourable cases $= \{(2,6)\ (3,5),\ (4,4),\ (5,3),\ (6,2)\}$

$= 5$

Thus, required probability $= \frac{5}{36}$

$$\boxed{P(E) = \frac{5}{36}}$$ Ans

Question (2): Four cards are drawn at random from a pack of 52 cards. Find the probability that

(i) They are king, a queen, a jack and an ace.

(ii) Two are kings and two are queens.

(iii) Two are black and two are red.

Solution-:

you have $= 52$ cards $\quad {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$

you pick $= 4$ cards

So exhaustive cases $= {}^{52}C_{4}$

(i) they are a king, a queen, a jack and an ace.

favourable cases $= {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1}$

$= 4 \times 4 \times 4 \times 4 = 256$

Thus, the required probability

$$\boxed{P(E) = \frac{256}{{}^{52}C_{4}}}$$

(ii) Two are kings and two are queens.

$$\text{favourable cases} = {}^4C_2 \times {}^4C_2$$

$$= \frac{4\times3}{2\times1} \times \frac{4\times3}{2\times1}$$

$$= 36$$

Thus, required probability

$$\boxed{P(E) = \frac{36}{{}^{52}C_4}}$$

Ans

(iii) Two are black and two are red.

black $\longrightarrow$ 13+13

red $\longrightarrow$ 13+13

$$\text{favourable cases} = {}^{26}C_2 \times {}^{26}C_2$$

Thus, required probability

$$\boxed{P(E) = \frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4}}$$

Ans

Question (3): A bag containing 6 white balls, 4 red and 3 black balls. If 3 balls are drawn at random. What is the probability that

(i) Two of the balls drawn are white.

(ii) None of the balls are red.

Solution:

We have

White $= 6$

Red $= 4$

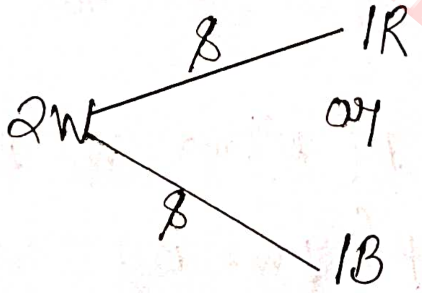
black $= 3$

Total balls $= 13$

you pick $= 3$ balls.

$\therefore$ Exhaustive cases $= {}^{13}C_3$

(i) two of the balls drawn are white



& → independent ×

or → Exclusive +

favourable cases $= (2W \,\&\, 1R) + (2W \,\&\, 1B)$

$$= {}^6C_2 \times {}^4C_1 + {}^6C_2 \times {}^3C_1$$

Thus, the required probability

$$\boxed{P(E) = \frac{{}^6C_2 \times {}^4C_1 + {}^6C_2 \times {}^3C_1}{{}^{13}C_3}}$$

Ans

(ii) None of the balls are red

favourable cases —
- → 3W
  or
- → 3B
  or
- → 2W & 1B
  or
- → 2B & 1W

favourable cases $= (3W) + (3B) + (2W \& 1B) + (2B \& 1W)$

$$= {}^{6}C_3 + {}^{3}C_3 + {}^{6}C_2 \times {}^{3}C_1 + {}^{3}C_2 \times {}^{6}C_1 .$$

Thus, required probability

$$\boxed{P(E) = \frac{{}^{6}C_3 + {}^{3}C_3 + {}^{6}C_2 \times {}^{3}C_1 + {}^{3}C_2 \times {}^{6}C_1}{{}^{13}C_3}}$$

Ans

or

$$P(\text{None is red}) = P(\text{all are } W \& B)$$

$$= P(9\,W \& B)$$

$$= \frac{{}^{9}C_3}{{}^{13}C_3}$$

$$\boxed{P(E) = \frac{{}^{9}C_3}{{}^{13}C_3}}$$

Ans

Question: If two dice are thrown, what is the probability that sum of dice is greater than 3.

Solution-: If two dice are thrown, then

Exhaustive cases $= 6^2 = 36$.

$$S = \{ (11)\ (12)\ (13)\ (14)\ (15)\ (16)$$
$$(21)\ (22)\ (23)\ (24)\ (25)\ (26)$$
$$\vdots$$
$$(61)\ (62)\ (63)\ (64)\ (65)\ (66) \}$$

Here, maximum sum $= 12$

minimum sum $= 2$

$$\therefore\ P(\text{sum} > 3) = P(\text{sum} = 4) + P(\text{sum} = 5) + \dots + P(\text{sum} = 12)$$

$$= 1 - P(\text{sum} \leq 3)$$

$$= 1 - [P(\text{sum} = 2) + P(\text{sum} = 3)]$$

favourable cases for

$\text{sum} = 2 \rightarrow \{(11)\}$

$\text{sum} = 3 \rightarrow \{(12)(21)\}$

$$P(\text{sum} > 3) = 1 - \left[\frac{1}{36} + \frac{2}{36}\right] = 1 - \frac{\cancel{3}}{\cancel{36}_{12}} = \frac{11}{12}$$

$$\boxed{P(\text{sum} > 3) = \frac{11}{12}}$$ Ans

Practice Questions

Question: A Committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of the sale department and 1 chartered accountant. Find the probability of forming the committee in the following manner:

(i) There must be one from each category.

(ii) It should have at least one from the purchase department.

(iii) The chartered accountant must be in the Committee.

Solution: 4 people to be appointed out of

→ 3 production dept.

→ 4 purches dept.

→ 2 sales dept.

→ 1 chartered a/c.

Total number of persons out of which four members are to be selected $= 10$

Exhaustive cases $= {}^{10}C_4 = \dfrac{10\times9\times8\times7}{4\times3\times2\times1} = 210.$

(i) favourable number of cases for committee to consist one member from each category

$= 1$ prod & 1 pur & 1 sales & 1 chart a/c

$= {}^{3}C_1 \times {}^{4}C_1 \times {}^{2}C_1 \times {}^{1}C_1$

$= 3\times4\times2\times1 = 24$

Thus, required probability

$$P(E) = \frac{24}{210} \quad \text{Ans.}$$

(ii) favourable cases for at least one from the purches department out of 4 appointment.

$=$ 1 from purches & 3 others

$+$ 2 from purches & 2 others

$+$ 3 from purches & 1 other

$+$ 4 from purches dept.

$$= {}^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^4C_4$$

$$= 4 \times \frac{6\times5\times4}{3\times2\times1} + \frac{4\times3}{2\times1}\cdot\frac{6\times5}{2\times1} + \frac{4\times3\times2}{3\times2\times1}\,6 + 1$$

$$= 80 + 90 + 24 + 1 = 195.$$

Thus, required probability

$$P(E) = \frac{195}{210}$$ Ans

or

Let X = Number of person from purches dept.

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(\text{No from purches dept.})$$

$$= 1 - \frac{{}^6C_4}{{}^{10}C_4}$$

$$= 1 - \frac{6\times5}{2\times1\times210} = 1 - \frac{15}{210}$$

$$\boxed{P(X \geq 1) = \frac{195}{210}}$$

Ans

(iii) favourable number of cases to must include 1 chartered accountant out of 4 appointment.

$$= 1 \text{ chart. } \& \ 3 \text{ others}$$

$$= 1 \times {}^{9}C_{3} = \frac{\cancel{9}^{3}\times\cancel{8}^{4}\times 7}{\cancel{3}\times\cancel{2}\times 1} = 84$$

Thus, required probability

$$P(E) = \frac{\cancel{84}\ \cancel{4}\ 2}{\cancel{210}\ \cancel{10}\ 5}$$

$$\boxed{P(E) = \frac{2}{5}}$$

Ans

Question—: A box contains 6 white, 4 red, and 9 black balls. If 3 balls are drawn at random, find the probability that

(i) two of balls drawn are white.
(ii) one is of each color.
(iii) none is red.
(iv) At least one is white.

Solution—: We have

A box contains →
- 6 white
- 4 red
- 9 black

Total balls = 19

you pick = 03

Exhaustive cases = $^{19}C_3$

(i) two balls drawn are white.

6W, 4R, 9B

$2W \xrightarrow{\&} 1R$ or $2W \xrightarrow{\&} 1B$

favourable cases = (2W & 1R) + (2W & 1B)

$= {}^6C_2 \times {}^4C_1 + {}^6C_2 \times {}^9C_1$

Thus, required probability

$$P(E) = \frac{{}^6C_2 \times {}^4C_1 + {}^6C_2 \times {}^9C_1}{{}^{19}C_3}$$ An

(ii) when one is of each colour

favourable cases = 1W & 1R & 1B

$= {}^6C_1 \times {}^4C_1 \times {}^9C_1$

Thus, required probability

$$\boxed{P(E) = \frac{{}^6C_1 \times {}^4C_1 \times {}^9C_1}{{}^{19}C_3}}$$ Ans

(iii) $P(\text{None is red}) = P(\text{all are White \& Black})$

$$= P((6+9)\, W \,\&\, B)$$

$$= \frac{^{15}C_3}{^{19}C_3}$$

$$\therefore P(\text{None is red}) = \frac{^{15}C_3}{^{19}C_3}$$ Ans

(iv) at least one white

$$P(W \geq 1) = 1 - P(W < 1)$$

$$= 1 - P(\text{none is white})$$

$$= 1 - P(\text{all } R \,\&\, B)$$

$$= 1 - P((4+9)\, R \,\&\, B)$$

$$= 1 - \frac{^{13}C_3}{^{19}C_3}$$

$$\therefore \boxed{P(E) = 1 - \frac{^{13}C_3}{^{19}C_3}}$$ Ans