PROBABILITY AND STATISTICS (UCS401)

Lecture-10

(Binomial Distribution with illustrations)

Random Variables and their Special Distributions(Unit –III & IV)

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Discrete distribution

① Bernoli distribution –:

A random variable X is said to be Bernoli distribution if its probability mass function (p.m.f.) is given by –

$$P[X=x] = p_X(x) = \begin{cases} p^x (1-p)^{1-x} = p^x q^{1-x}, & x = 0,1 \\ 0 & o/w \end{cases}$$

$p \to$ success failure $\to 1-p = q$

$X \sim B(1, p)$.

Characteristic –: There are following characteristic for probability mass function:

(i) Mean
(ii) Varience
(iii) Moment generating function (MGF)
(iv) characteristic function
(v) Probability generating function.

Note that –:

(*) If $x_1, x_2, x_3, x_4, \ldots, x_n$ are n variables

$$\text{Mean} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$$

$$\text{Varience} = Var(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

In terms of frequency :

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \quad ; \quad Var(X) = \frac{\sum_{i=1}^{n} f_i (x_i-\bar{x})^2}{\sum_{i=1}^{n} f_i}.$$

$$\text{standard deviation} = +\sqrt{\text{varience}}$$

$$sd = +\sqrt{Var(X)}$$

Mathematical expectation – (Mean)

The expectation of random variable X is denoted by $E(X)$ and defined as :

$$\text{Mean} = E(X) = \begin{cases} \sum_{x} x P(x) & ; \text{ discreate distribution} \\ \int_{-\infty}^{\infty} x f(x)\,dx & ; \text{ Continuous distribution} \end{cases}$$

characteristic of Bernoli distribution –:

① Mean –:

$$\text{Mean} = E(X) = \sum_{x=0}^{1} x\, P(X=x)$$

$$= \sum_{x=0}^{1} x p^x (1-p)^{1-x} = \sum_{x=0}^{1} x p^x q^{1-x}$$

$$= 0\, p^0 q^{1-0} + 1\, p^1 q^{1-1}$$

$$\boxed{\text{Mean} = E(X) = p}$$

i.e., if $X \sim B(1, p)$

Mean $= E(X) = p$]

(ii) Varience :-

We know that

$$Var(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2$$

Now in case of Bernoli distribution:

$$E(X^2) = \sum_{x=0}^{1} x^2 P(X=x)$$

$$= \sum_{x=0}^{1} x^2 p^x q^{1-x}$$

$$= 0\, p^0 q^{1-0} + 1\, p^1 q^{1-1}$$

$$\boxed{E(X^2) = p}$$

Which $\Rightarrow$ $Var(X) = E(X^2) - (E(X))^2 = p - p^2$

$$= p(1-p)$$

$$= pq$$

$$\boxed{Var(X) = pq}$$

$\because p + q = 1$

i.e., if

③ Moment generating function (MGF) for Bernoli distribution —:

The moment generating function $M_X(t)$ is defined as —

$$M_X(t) = E(e^{tX}) = \begin{cases} \sum_x e^{tx} P[X=x] & ; \text{ discrete distribution} \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx & ; \text{ continuous distribution} \end{cases}$$

Here

$$M_X(t) = E(e^{tx}) = \sum_{x=0}^{1} e^{tx} p^x q^{1-x}$$

$$= e^{t0} p^0 q^{1-0} + e^t p^1 q^0$$

$$= q + pe^t$$

$$\boxed{M_X(t) = E(e^{tX}) = q + pe^t}$$

Question —: If X follow the Bernoli-distribution $X \sim B(1, p)$ and its moment generating function is

$$M_X(t) = \frac{1}{2} + \frac{1}{2}e^t,$$ then find its mean and variance.

Solution —: We know that for Bernoli-distribution the Moment generating function is given by

$$M_X(t) = q + pe^t$$

Mean $= E(X) = p$ $\qquad$ $Var(X) = pq$.

Given, $M_X(t) = \frac{1}{2} + \frac{1}{2}e^t$

$\Rightarrow p = \frac{1}{2}$, $q = \frac{1}{2}$

$\Rightarrow$ Mean $= E(X) = \frac{1}{2}$, $Var(X) = 1/4$ Ans

Question-: Show that

$$P[X=x] = p_X(x) = \begin{cases} p^x q^{1-x} & x = 0,1 \\ 0 & o/w \end{cases}$$

$p+q=1$

is a probability mass function (p.m.f.).

Solution-: We know that by definition $p_X(x)$ is p.m.f if (i) $p(x) \geq 0$

(ii) $\sum_{x=0}^{1} p(x) = 1$

$$\sum_{x=0}^{1} p(x) = \sum_{x=0}^{1} p^x q^{1-x} = p^0 q^{1-0} + p^1 q^{1-1}$$

$$= q + p = 1$$

$$\Rightarrow \sum_{x=0}^{1} p(x) = 1$$

$\Rightarrow$ $p(x)$ is p.m.f.

④ Characteristic function-:

The characteristic is denoted by $\phi_X(t)$ and defined as :

$$\phi_X(t) = E(e^{itX}) = \begin{cases} \sum_x e^{itx} P(X=x) & ; \text{discrete} \\ \int_x e^{itx} f(x)dx & ; \text{Continuous} \end{cases}$$

$P(X=x) \Rightarrow$ probability of random variable X with respect to x.

Here,

$$\phi_X(t) = E(e^{itX}) = \sum_{x=0}^{1} e^{itx} p^x q^{1-x}$$

$$= e^{it0} p^0 q^{1-0} + e^{it} p^1 q^{1-1}$$

$$\boxed{\phi_X(t) = q + pe^{it}}$$

Question :- If X follow the Bernoli-distribution $X \sim B(1, p)$ and its characteristic function is given by

$$\phi_X(t) = \frac{4}{5} + \frac{1}{5} e^{it}.$$

Then find mean, varience, and Moment generating function (MGF).

Solution :- If $X \sim B(1, p)$, then characteristic function is given by

$$\phi_X(t) = q + pe^{it}$$

$$\text{Mean} = E(X) = p$$

$$\text{Varience} = Var(X) = pq$$

$$MGF = q + pe^{t}.$$

Here, $\phi_X(t) = \frac{4}{5} + \frac{1}{5}e^{it} \Rightarrow p = \frac{1}{5}$ & $q = 4/5$

$\Rightarrow$ Mean $= E(X) = 1/5$

$\Rightarrow$ Variance $= Var(X) = \frac{1}{5} \cdot 4/5 = 4/25$

and Moment Generating function

$$\boxed{M_X(t) = 4/5 + \tfrac{1}{5}e^{t}}$$ <u>Ans</u>

<u>Probability Generating function :-</u>

The probability generating function is denoted by $Z_X(t)$ and defined as :

$$Z_X(t) = E(Z^X) = \sum_{x=0}^{1} z^x p^x q^{1-x}$$

$$= z^0 p^0 q^{1-0} + z^1 p^1 q^{1-1}$$

$$\boxed{Z_X(t) = q + zp}$$

Bionomial distribution

1. All the trails are independent.
2. Number of trails are finite
3. The probability p of sucess is same for each trail.

The probability mass function (p.m.f.) is given by

$$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x} \quad ; \quad p+q=1$$

$n =$ Number of trails

$p =$ probability of sucess

$q = 1-p =$ probability of failure.

or

A random variable X is said to follow Bionomial distribution $X \sim B(n,p)$ if its probability mass function (p.m.f.) is given by :

$$P[X=x] = p_X(x) = \begin{cases} \binom{n}{x} p^{x} q^{n-x} & x = 0,1,2,3,\ldots,n \\ & p+q=1 \\ 0 & o/\omega \end{cases}$$

Question:- A Coin is tossed trice ~~simultaneously~~, what is probability to Come head twice.

Solution:- Samplespace $S = \{HHH, HHT, HTH, THH, THT, TTH, HTT, TTT\}$

X (No. of Head)	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

⇒ probability distribution.

$$P(2\text{ Head}) = \frac{3}{8}$$

Here, $n = 3 \quad p = \frac{1}{2} \quad q = 1/2$

We know that for Bionomial distribution;

$$p(x) = {}^nC_x\, p^x q^{n-x}$$

$$p(x) = {}^3C_x \left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{3-x}$$

$$p(2\text{ Head}) = {}^3C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^1 = \frac{3\times 2}{2\times 1}\,\frac{1}{4}\,\frac{1}{2} = 3/8$$

$$\boxed{p(2\text{ Head}) = 3/8}$$ (Same value)

Question:- If a Coin is tossed 10 times ~~simultaneously~~, then what is probability of getting 5 times head.

Solution:- We know that the probability mass function (p.m.f) for Bionomial distribution is given by

$$P(x) = {}^nC_x\, p^x q^{n-x}$$

Here, $n = 10$, $p = \frac{1}{2}$ $q = \frac{1}{2}$

$$P(X) = {}^{10}C_X \left(\frac{1}{2}\right)^X \left(\frac{1}{2}\right)^{10-X}$$

$$P(5) = {}^{10}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^5$$

$$= \frac{10!}{5!\,5!} \left(\frac{1}{4}\right)^5$$

$$P(5) = \frac{10 \times 9 \times 8 \times 7 \times 6}{5!} \left(\frac{1}{4}\right)^5$$

Ans

Question:– Show that $p(x) = {}^nC_x p^x q^{n-x}$ is a probability mass function (p.m.f) for Bionomial distribution.

Solution:– For Bionomial distribution

$$p(x) = {}^nC_x p^x q^{n-x}$$

We know by definition $p(x)$ is p.m.f if (i) $p(x) \geq 0$ (ii) $\sum_{x=0}^{n} p(x) = 1$

$$\sum_{x=0}^{n} p(x) = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x}$$

$$= q^n + {}^nC_1 p q^{n-1} + {}^nC_2 p^2 q^{n-2} + \dots + p^n$$

$$= (p+q)^n = 1 \qquad \because p+q = 1$$

$$\sum_{x=0}^{n} p(x) = 1$$

$\Rightarrow$ $p(x)$ is p.m.f.

① Mean for Bionomial distribution —:

$$\text{Mean} = E(X) = \sum_{x=0}^{n} x\, P[X=x]$$

$$= \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x}\, n!}{\cancel{x}(x-1)!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n\,(n-1)!\; p\; p^{x-1}}{(x-1)!\,\big((n-1)-(x-1)\big)!} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=0}^{n} \frac{(n-1)!}{(x-1)!\,\big((n-1)-(x-1)\big)!} p^{x-1} q^{((n-1)-(x-1))}$$

$$= np\,(p+q)^{n-1} \qquad \because p+q=1$$

$$\boxed{\text{Mean} = E(X) = np}$$

② Varience —:

The Variance of a random variable X is given by:

$$V(X) = E(X^2) - (E(X))^2$$

Now,

$$E(X^2) = \sum_{x=0}^{n} x^2 P[X=x]$$

$$= \sum_{x=0}^{n} x^2 \, {}^{n}C_{x} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x^2 \frac{n!}{x!\,(n-x)!} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x^2 \frac{n!}{x(x-1)(x-2)!\,(n-x)!} p^{x} q^{n-x}$$

$\because \quad x^2 = x(x-1) + x$

$$= \sum_{x=0}^{n} \left(x(x-1)+x\right) \frac{n!}{x(x-1)(x-2)!\,(n-x)!} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\, n!}{\cancel{x(x-1)}\,(x-2)!\,(n-x)!} p^{x} q^{n-x}$$

$$+ \sum_{x=0}^{n} \frac{\cancel{x}\, n!}{\cancel{x}(x-1)!\,(n-x)!} p^{x} q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=0}^{n} \frac{(n-2)!\; p^{x-2} q^{n-x}}{(x-2)!\,(n-x)!}$$

$$+ \; np \sum_{x=0}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} p^{x-1} q^{n-x}$$

$$= n(n-1)p^2(p+q)^{n-2} + np(p+q)^{n-1}$$

$$E(X^2) = n(n-1)p^2 + np$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np(1-p)$$

$$= npq$$

$$\Rightarrow \boxed{Var(X) = npq} \Rightarrow \text{Bionomial distribution.}$$

Question-: If X follows the Bionomial distribution $X \sim B(8, \frac{1}{3})$, then find mean and varience.

Solution-: We know that if

$$X \sim B(n, p)$$

$$\Rightarrow \text{Mean} = np, \quad \& \quad \text{Varience} = npq \; ; \; q = 1-p$$

Here, $X \sim B(8, \frac{1}{3}) \Rightarrow n = 8, \; p = \frac{1}{3}$

which $\Rightarrow$ Mean $= \frac{8}{3}$ Varience $= 8\left(\frac{1}{3}\right)\left(1 - \frac{1}{3}\right) = \frac{16}{9}$ Ans

Note that-: It is observed that for Bionomial distribution

$$\boxed{\text{Mean} > \text{varience}}$$

and for poisson distribution

$$\boxed{\text{Mean} = \text{Varience}}$$

* Suppose for Bionomial distribution

$$X \sim B(n, p)$$

having mean $= 8$ and varience $= 10$ (Is it possible?)

We know that for Bionomial distribution

$$X \sim B(n, p)$$

$$\text{Mean} = np \quad \& \quad \text{Varience} = npq$$

$$np = 8 \quad \& \quad npq = 10$$

$$\Rightarrow \quad 8q = 10$$

$$\Rightarrow \quad q = 10/8$$

which is not true as $0 < p\,q < 1$ (probability).

Thus it is not possible as Mean < varience

$\because$ for Bionomial distribution

$$\boxed{\text{Mean} > \text{varience}}$$

③ Moment generating function (Bionomial distribution) :-

The moment generating function is defined as

$$M_X(t) = E(e^{Xt}) = \sum_{x=0}^{n} e^{xt} P(x)$$

$$= \sum_{x=0}^{n} e^{xt}\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x\, (pe^t)^x q^{n-x}$$

$$= (q+pe^t)^n$$

$$\boxed{M_X(t) = M.G.F. = (q+pe^t)^n}$$

Charecteristic function (Bionomial distribution) -:

The charecteristic function is denoted by $\phi_X(t)$ and defined as :

$$\phi_X(t) = E(e^{iXt}) = \sum_{x=0}^{n} e^{ixt}\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (pe^{it})^x q^{n-x}$$

$$= (q+pe^{it})^n$$

$$\boxed{\phi_X(t) = (q+pe^{it})^n}$$

Probability Generating function -:

The probability generating function is denoted by $Z_X(t)$ and defined as :

$$Z_X(t) = \sum_{x=0}^{n} z^x\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (pz)^x q^{n-x}$$

$$\boxed{Z_X(t) = (q+pz)^n}$$ #

Mean and varience by Moment generating function :

we know that

$$v_r = E(X^r) = \begin{cases} \dfrac{d^r}{dt^r}\left(M_X(t)\right)\Big|_{t=0} & \text{if } M_X(t) \text{ is given} \\ (-i)^r \dfrac{d^r}{dt^r}\left(\phi_X(t)\right)\Big|_{t=0} & \text{if } \phi_X(t) \text{ is given.} \end{cases}$$

$$\text{Mean} = E(X)$$

$$Var(X) = E(X^2) - (E(X))^2$$

For Bionomial distribution

$$M_X(t) = (q+pe^t)^n$$

Now,

$$E(X) = \frac{d}{dt}\left(M_X(t)\right)\Big|_{t=0}$$

$$= \frac{d}{dt}\left((q+pe^t)^n\right)\Big|_{t=0}$$

$$= \left(n(q+pe^t)^{n-1} pe^t\right)\Big|_{t=0}$$

$$= n(q+pe^0)^{n-1} pe^0$$

$$= n(p+q)^{n-1} p$$

$$E(X) = np.$$

$$\Rightarrow \boxed{\text{Mean} = E(X) = np}$$

$$E(X^2) = \frac{d^2}{dt^2}\left(M_X(t)\right)\Big|_{t=0}$$

$$= \frac{d}{dt}\left(n(pe^t+q)^{n-1} pe^t \right)\Big|_{t=0}$$

$$= \left[pe^t\, n\left(q+pe^t\right)^{n-1} + n(n-1)\left(q+pe^t\right)^{n-2}(pe^t)^2 \right]_{t=0}$$

$$= \left[np + n(n-1)p^2 \right]$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= np + n(n-1)p^2 - (np)^2$$

$$= np + \cancel{n^2p^2} - np^2 - \cancel{n^2p^2}$$

$$= np(1-p) = npq$$

$$\Rightarrow \boxed{Var(X) = npq}$$

Question:- The probability that man aged 60 will live upto 70 is 0.65 out of ten man, now aged 60, find the probability:

(i) At least 7 will live upto 70

(ii) Exactly 9 will live upto 70

(iii) At most 9 will live upto 70.

Solution:- Given that $n=10$ $p=0.65$

$q=0.35$

The probability distribution for Bionomial distribution:

$$p(X) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$p(X) = {}^{10}C_{x}\, (0.65)^{x} (0.35)^{10-x}$$

(i) At least 7 will live upto 70

$$P(X \geq 7) = P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}C_{7}(0.65)^{7}(0.35)^{3} + {}^{10}C_{8}(0.65)^{8}(0.35)^{2} + {}^{10}C_{9}(0.65)^{9}(0.35) + {}^{10}C_{10}(0.65)^{10}$$

$$P(X \geq 7) = 0.5139$$ Ans

(ii) Exactly 9 will live upto 70

$$P(9) = {}^{10}C_{9}(0.65)^{9}(0.35)$$

$$\boxed{P(9) = 0.0725}$$ Ans

(iii) At most 9 will live upto 70.

$$P(X \leq 9) = 1 - P(X > 9)$$

$$= 1 - P(10)$$

$$= 1 - {}^{10}C_{10}(0.65)^{10}(0.35)^{0}$$

$$\boxed{P(X \leq 9) = 0.9865}$$ Ans

Question :- Out of 800 families with 5 children each. How many families could be expected to have

(i) 3 boys (iii) either 2 or 3 boys

(ii) 5 Girls (iv) at least two girls.

Sol.ⁿ Given $N = 800$, $n = 5$

Boys $p = \frac{1}{2}$ $q = \frac{1}{2}$

The probability distribution for Bionomial distribution

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

$$P(x) = {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$$

$$= {}^5C_x \left(\frac{1}{2}\right)^5$$

$$P(x) = {}^5C_x \left(\frac{1}{32}\right)$$

(i) 3 boys

$$P(3) = {}^5C_3 \left(\frac{1}{32}\right) = \frac{5 \times 4}{2 \times 1} \cdot \frac{1}{32} = \frac{5}{16}$$

$$P(3) = 5/16$$

$\therefore$ No. of families $= \frac{5}{16} \times 800 = 250$ #

(ii) 5 girls $\Rightarrow$ 0 boy

$$P(0) = {}^5C_0 \left(\frac{1}{32}\right) = \frac{1}{32}$$

$\therefore$ No. of families $= 800 \times \frac{1}{32} = 25$ #

(iii) Either 2 or 3 boys

$$= P(2) + P(3)$$

$$= {}^5C_2 \frac{1}{32} + {}^5C_3 \left(\frac{1}{32}\right)$$

$$= \frac{10}{32} + \frac{10}{32} = \cancel{\frac{20}{32}}\ \cancel{\frac{10}{16}}\ \frac{5}{8}$$

$$\boxed{\text{Either 2 or 3 boys} = 5/8}$$

(iv) At least 2 girls $\Rightarrow$ 2, 3, 4, 5

3, 2, 1, 0 (boys)

$\Rightarrow$ At most 3 boys

$\Rightarrow P(X \leq 3)$

$$= P(0) + P(1) + P(2) + P(3)$$

$$= {}^5C_0\left(\frac{1}{32}\right) + {}^5C_1\left(\frac{1}{32}\right) + {}^5C_2\left(\frac{1}{32}\right) + {}^5C_3 \frac{1}{32}$$

$$= 0.5$$

$\therefore$ Number of families $= 800 \times 0.5$

$= 400$ families Ans

Question:- 4 Coin are tossed 100 times & following data were obtained. Fit a bionomial distribution for data and Calculate theoretical frequency.

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x No. of Head	f frequency	xf	$P(X) = {}^nC_x p^x q^{n-x}$ $= {}^4C_x (0.49)^x (0.51)^{4-x}$	$100 \times P(x)$
0	5	0	$P(0) = {}^4C_0 (0.49)^0 (0.51)^4$ $= 0.0676$	$6.76 \approx 7$
1	29	29	$P(1) = {}^4C_1 (0.49)^1 (0.51)^3$ $= 0.2599$	$25.99 \approx 26$
2	36	72	$P(2) = {}^4C_2 (0.49)^2 (0.51)^2$ $= 0.3747$	$37.47 \approx 37$
3	25	75	$P(3) = {}^4C_3 (0.49)^3 (0.51)^1$ $= 0.2400$	24
4	05	20	$P(4) = {}^4C_4 (0.49)^4 (0.51)^0$ $= 0.05765$	$5.765 \approx 6$
$n = 4$	$\sum f = 100$	$\sum xf = 196$		

$\sum 100 \times P(x) = 100$ Ans

For Bionomial distribution

$$np = \bar{x} = \frac{\sum fx}{\sum f} = \frac{196}{100} = 1.96$$

$$n = 4 \Rightarrow p = \frac{1.96}{4} = 0.49$$

$$\Rightarrow q = 1 - p = 0.51$$

This is called fitting of Bionomial distribution.

#

Question (4) Assume that half of the population is vegetarian so chance of an individual being a vegetarian is $\frac{1}{2}$. Assuming that 100 investigators each take sample of 10 individuals to see whether they are vegetarians, How many investigators would you expect to report that three people or less were vegetarian?

Solution :- $X =$ No. of vegetarian

Required probability

$$P(X \leq 3)$$

p $\quad n = 10$

$$p(\text{vegetarian}) = \frac{1}{2}, \quad n = 10 \;\&\; N = 100$$

Thus, total no. of investigator to report that three or less were vegetarian $= 100 \times P(X \leq 3)$.

For, Binomial distribution, the p.m.f. is given by

$$P(x) = P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= {}^{10}C_{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{10-x}$$

$$P(X = x) = {}^{10}C_{x} \left(\frac{1}{2}\right)^{10} \quad \text{---(1)}$$

$$\because P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3\right]$$

Total No. of investigator to report that three or less were vegetarian

$$= 100 \times \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3\right]$$

Ans

Question:- A Binomial distribution with parameter $n=5$ satisfy the property

$$8\,P(X=4) = P(X=2).$$

Find

(i) value of p

(ii) $P(X \geq 1)$

Solution:- Given $n=5$

For Binomial distribution, the p.m.f is given by

$$P(x) = P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$P(X=x) = {}^{5}C_{x}\, p^{x}\, q^{5-x} \quad \text{---(1)}$$

$$\because \quad 8\,P(X=4) = P(X=2)$$

$$8\; {}^{5}C_{4}\, p^{4} q^{1} = {}^{5}C_{2}\, p^{2} q^{3}$$

$$\Rightarrow \quad 40\, p^{4} q = 10\, p^{2} q^{3}$$

$$\Rightarrow \quad 40\, p^{4} q - 10\, p^{2} q^{3} = 0.$$

$$\Rightarrow \quad \underbrace{10\, p^{2} q}_{\neq 0}\,(4p^{2} - q^{2}) = 0$$

$\because \quad 0 < p, q < 1$

$\Rightarrow \quad 4p^2 - q^2 = 0$

$\Rightarrow \quad \boxed{q = +2p}$

$\because \quad p + q = 1$

$p + 2p = 1 \quad \Rightarrow \quad \boxed{p = 1/3} \;\&\; \boxed{q = 2/3}$

(ii) The required probability

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^5C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5$$

$$\boxed{P(X \geq 1) = 1 - \left(\frac{2}{3}\right)^5}$$ Ans

Question:- The probability that a bomb hits a target is given by 0.6. If 2 bombs are enough to destroy the bridge, what is the probability that out of 10 bombings, the bridge will destroy?

Solution:- X: No. of bombs hitting on target $\rightarrow$ max $= 10$

$\because$ 2 bombs are enough to destroy the target.

Thus, required probability $= P(X \geq 2)$

Here, $P(\text{hitting the target}) = 0.6$

$p = 0.6 \quad \& \quad n = 10 \rightarrow$ finite (Binomial)

$q = 0.4$

$\because$ For Binomial distribution, the p.m.f is given by :-

$$p(x) = P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$P(X=x) = {}^{10}C_{x}\, (0.6)^{x}\, (0.4)^{10-x} \quad \text{--- (1)}$$

$$\because \quad P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - [{}^{10}C_{0}(0.6)^{0}(0.4)^{10} + {}^{10}C_{1}(0.6)^{1}(0.4)^{9}]$$

$$P(X \geq 2) = 1 - [(0.4)^{10} + 10\,(0.6)(0.4)^{9}]$$