ASSIGNMENT PROBLEM

ASSIGNMENT PROBLEM

A special case of Transportation Problem.

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

s.t.:-

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2\cdots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\cdots n$$

$$x_{ij} \geq 0$$

— Transportation Problem

Assignment Problem

when;

$m = n$

$a_i = 1 \quad b_j = 1$

$x_{ij} = 0$ or 1

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

s.t.:-

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2\cdots m$$

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1,2,\cdots n$$

$$x_{ij} \in [0,1] \text{ \& } m = n$$

Assignment Problem.

Steps :-

① Check if Max/Min
   ↳ if max ⟶ convert to min

② Check if Unbalanced ($m \neq n$) or Balanced ($m = n$) Ass. Prob.
   ↳ convert unbalanced to balanced

③ HUNGARIAN METHOD

(1) find row minima - find minimum element from each row & subtract it from the corresponding element of that row.

(2) find column minia - find min element from each colum & subtract 'it' from corresponding element of that row.

(3) Draw minimum no. of horizontal or vertical lines 'N' to cover all the zeroes.

(4) $N = n = m$, then do the assignment otherwise ($N < n$)

(5) find min. element from uncovered element & subtract this element from the uncoved elements and add it to intersection of horizontal & vertical lines.

(6) Repeat steps 3-5 until $N = n$

(4) How to do assignment
   ↳ assign zero starting from min. zero in concerned row/column & cut all other zeros in the same row or column.

Q Which job should be assigned to which person so that total profit is maximum.

Persons \ Jobs	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

① This is a maximizing problem → convert to minimize

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	13	22	27
D	18	22	26	30

← We subtract each element from 42 (max)

← This is a BAP (m=n)

① finding row minima & subtract

	I	II	III	IV	
A	0	7	14	21	0
B	0	5	10	15	12
C	0	5	10	15	12
D	0	4	8	12	18

② Col^n min & Subtract

	I	II	III	IV
A	0	3	6	9
B	0	1	2	3
C	0	1	2	3
D	0	0	0	0
	0	4	8	12

③ Draw min no. of horz & vertical lines to cover all zeroes

	I	II	III	IV
A	0	3	6	9
B	0	1	2	3
C	0	1	2	3
D	0	0	0	0
	0	4	8	12

intersection

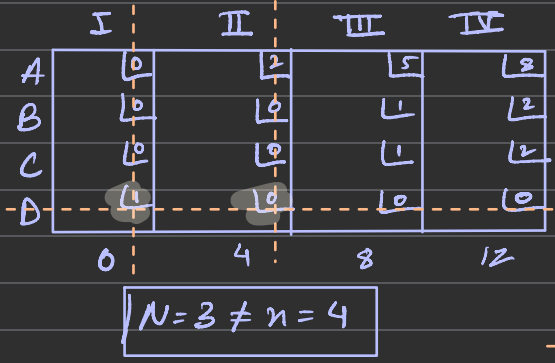
④ Lines drawn = N = 2

N ≠ n (2 ≠ 4)

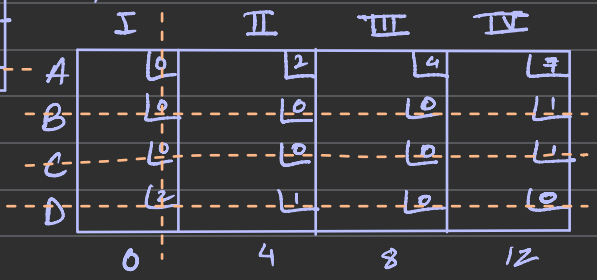
↑

Not optimal

(v) find min. element from uncovered element & subtract this element from the uncoved elements and add it to intersection of horizontal & vertical lines



repeat (v), (iii), (iv)

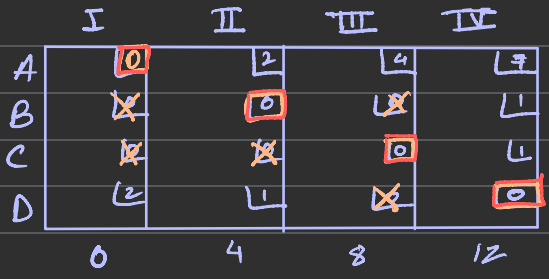


(N = 4) = (n = 4)

↑ OPTIMAL

(4) Performing assignment.

(1) Start from the row/col with min. no. of zeroes



Ans :-

A → I	← final assignment.
B → II	
C → III	
D → IV	

Max profit = 42 + 25 + 20 + 12 = 99 Ans

Remarks :-

(1) What will happen if we can not assign Job IV to 3rd person
↳ No change in the optimal as in optimal sol Job IV was NOT assigned to 3rd person.

(2) What will happen if IV job can not be assigned to 4th person.
↳ This will cause our optimal sol^{n} to change as we did assign IV Job to 4th person.

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30 → ∞

replace that cell with '∞' so that 4th person can never have Job IV.

↑ Then proceed as above.

Ans:
A → I
B → II
C → IV
D → III

(3) What will happen if 4th job should always be assigned to IV person? → ith & jth col/row delete

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30

Remove 4th job & 4th person & apply the method

⇒

	I	II	III
A	0	7	14
B	12	17	22
C	12	17	22

↑ Apply Hungarian NOW.

④ Suppose an addition person becomes available for performing all four jobs at respective 60, 40, 30 & 80 profit. Is it economical to replace one of the current 4 persons with the new one?

Add that person to the table & balance the table by adding a column. ← Add

	I	II	III	IV	Penny
A	50	50	—	20	0
B	70	40	20	30	0
C	90	30	50	—	0
D	20	20	60	70	0
Add → E	60	40	30	80	0

⇒ Ans:-

I → E

II → D

III → B

IV → A

V → C

Since there is no job V so no job is assigned to 'C' Hence, C can be replaced with the new person 'E'.