Roll Number:

School of Mathematics, TIET, PATIALA
End Semester Examination, December 11, 2023

B.E(Sem III)
Course Name: Optimization Techniques **Course Code:** (UMA035/UMA031)
TIME: 3 Hours. MAXIMUM MARKS : 35
Faculty: MKS
NOTE: Attempt any five questions.

Q1(a). By using algebraic method, find all the basic solutions and identify the basic feasible solutions of the following linear programming problem (LPP)
$Max\ z = 4x_1 + 3x_2$, subject to $x_1 + x_2 \leq 8,\ \ 2x_1 + x_2 \leq 10, x_1, x_2 \geq 0.$ **3.5**

(b) For a linear programming problem (LPP), explain the following with examples by considering, number of variables n=6, number of constraints m=3 and rank of constraint matrix is 3.
(i) Non-Degenerate BFS solution (ii) Degenerate BFS solution **2**

(c) Define a convex set and show geometrically that the set given bellow is not a convex set.
$$\{(x_1, x_2): 0 < x_1^2 + x_2^2 \leq 4\}$$ **1.5**

Q2. A transport company ships truckloads of grain from three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation cost (in thousands of Rs) per truckload on the different routes are summarized in following table

		Mill				Supply
		1	2	3	4	
Silo	1	1	2	3	4	30
	2	7	6	2	5	50
	3	4	3	2	7	35
Demand		15	30	25	45	

(i) Determine the optimal cost shipping between silos and the mills (use Least Cost method for initial basic feasible solution)

(ii) Formulate the above (TP) as a linear programming problem. **7**

Q3(a) (i)A construction company has requested for subcontracts on five different projects. Five companies have responded, their bids amount (in thousands) are given in the following table. Use Hungarian method to determine minimum cost assignment of subcontracts to bidders, assuming that each bidder can receive only one contract. It is also assumed that I and III bidder are not eligible for III and IV project respectively.

		Projects			
		I	II	III	IV
Bidders	I	50	50	-	20
	II	70	40	20	30
	III	90	30	50	-
	IV	70	20	60	70

(ii) If an additional (fifth) bidder becomes available to participate at the respective cost of 60, 45, 30 and 80. Is it economical for construction company to replace the current four bidders with the new one? **5.5**

(b) Using Hessian matrix, decide the nature of (convex/concave) of the following function
$$f(X) = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$$ **1.5**

PTO

Q4(a). Consider the data of a project as shown in following table .

Activity	Normal time (in weeks)	Normal cost (in Rs.)	Crash time (in weeks)	Crash cost (in Rs.)
1-2	13	700	9	900
1-3	5	400	4	460
1-4	7	600	4	810
2-5	12	800	11	865
3-2	6	900	4	1130
3-4	5	1000	3	1180
4-5	9	1500	6	1800

(i) Draw a network diagram for this project and find the critical path, normal duration and normal cost of the project.

(ii) Find the most economical schedule if the project is to be completed in 23 days. **5**

(b) Construct the network by considering the following activity and their predecessor (s).

Activity	A	B	C	D	E	F	G
Predecessor(s)	-	-	A,B	B	D	C,E	D

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Q5(a). Solve the following nonlinear program by Lagrange's multiplier method.

$Max\ f(x) = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$, s/t $x_1 + x_2 + x_3 = 15$ $2x_1 - x_2 + 2x_3 = 20$, $x_1, x_2, x_3 \geq 0$

Also show that the above program is a convex program. **4.5**

(b) Consider the function $f = -x_1^2 - x_2^2 + x_1x_2 + x_1 + 2x_2$. Find the stationary point and classify whether this point is a point of maxima or minima. Is this point will be point of global optimum? If yes, justify your answer. **2.5**

Q6(a). Solve the following multi-objective linear programming problem.

$F(x) = \{(Max\ z = 2x_1 + 3x_2,\ Min\ z = x_1 - x_2,\ Max\ z = 3x_1 + 2x_2$, s/t $x_1 + x_2 \leq 1$, $x_1, x_2 \geq 0$. **3.5**

(b) Using Branch and bound algorithm, find the optimal solution of the following Integer LPP

$Minz = 3x_1 + 2.5x_2$ subject to $x_1 + 2x_2 \geq 20$, $3x_1 + 2x_2 \geq 50$, $x_1, x_2 \geq 0$ and integers. **3.5**