Roll Number: ______________

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

Mid Semester Test (13-03-2023)

B.E/B.Tech | UMA035: Optimization Techniques

Time: 2 hours, M. Marks: 25 | Coordinators: NK, RJD

Note: All questions are compulsory. Attempt all parts of a question at one place.

Q1. (a) The AGMC has two plants, each of which produces and supplies two products: milk and butter. Each plant can work up to 16 hrs a day. In plant A, it takes 3 hrs to prepare and pack 1000 liters of milk and 1 hr to prepare and pack 100 kg of butter. In plant B, these figures are 2 hrs and 1.5 hrs. In plant A (B) it cost Rs. 15000(18000) to prepare and pack 1000 liter of milk and Rs. 28000(26000) for 100 kg of butter daily. The AGMC is obliged to produce daily at least 10000 liters of milk and 800 kg of butter. Formulate the problem as a Linear Programming Problem(LPP). [4 marks]

(b) State and prove Fundamental theorem of Linear Programming Problem (LPP) for maximization case only. [2.5 marks]

Q2. (a) Find the optimal solution of given LPP by graphical method. Also find the optimal solution by simplex method and show the correspondence of basic feasible solution with corner points of feasible region.

Max $z = 6x_1 - 2x_2$, Subject to $2x_1 - x_2 \le 2, x_1 \le 4, x_1, x_2 \ge 0.$ [3.5 marks]

(b) Find the optimal solution of LPP (if exists) by using Big M method.

Max $z = 3x_1 + 2x_2 + x_3$

Subject to $2x_1 + x_2 + x_3 = 12,\ 3x_1 + 4x_3 = 11,\ x_1$ is unrestricted and $x_2, x_3 \ge 0$. [3 marks]

(c) Consider the following LPP as primal problem: Max $z = 5x_1 + 12x_2 + 4x_3$ subject to $x_1 + 2x_2 + x_3 \le 5,\ 2x_1 - x_2 + 3x_3 = 2,\ 2x_1 + x_2 \ge 3,\ x_1, x_2, x_3 \ge 0$. Write the dual of above LPP. Using complementary slackness conditions, find the optimal solution of dual and it is given that s_3 (surplus variable in third constraint), x_1 and x_2 are the basic variables in the primal optimal table with nonzero value. [3.5 marks]

Q3. (a) Prove that intersection of two convex set is a convex set. [2 marks]

(b) Consider the following LPP: Max $z = -x_1 + 2x_2 - x_3$

subject to $x_1 + 2x_2 - 2x_3 \le 4,\ x_1 - x_3 \le 3,\ 2x_1 - x_2 + 2x_3 \le 2, x_1, x_2, x_3 \ge 0$.

The optimal table of given LPP is as follows (where s_1, s_2 and s_3 are slack variables corresponding to first, second and third constraint respectively.):

Basic Variable	x_1	x_2	x_3	s_1	s_2	s_3	Solution
$z_j - c_j$	9/2	0	0	3/2	0	1	8
x_2	3	1	0	1	0	1	6
s_2	7/2	0	0	1/2	1	1	7
x_3	5/2	0	1	1/2	0	1	4

(i) Using sensitivity analysis, find the optimal solution and optimal value if a new constraint $x_1 + x_2 = 4$ is added in the given LPP. [2 marks]

(ii) Within what range the coefficient of x_1 in the objective function varies without affecting the optimality. [1.5 marks]

(c) The optimal Simplex table of LPP is given below (x_1, x_2 are decision variables and s_1, s_2 and s_3 are slack variables corresponding to first, second and third constraint respectively). Find the Linear Programming Problem. [3 marks]

Basic Variable	x_1	x_2	s_1	s_2	s_3	Solution
$z_j - c_j$	0	0	1	0	0	9
x_1	1	0	1	0	0	2
s_2	0	0	1	1	$-1/2$	1/2
x_2	0	1	-1	0	1/2	3/2