Roll Number: _____

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

Makeup Mid Semester Test

B.E. (Sem: IV, VI,VIII)	Course Code: UMA035
	Course Name: Optimization Techniques
Time: 2 Hours, M. Marks: 25	Name of Coordinators: NK, RJD

Note: (1) **Attempt all the questions.**

(2) **Attempt all parts of a question at one place.**

1. (a) A firm manufactures three products A, B and C. Time to manufacture product A is twice that for B and thrice that for C and if the entire labour is engaged in making product A, 1600 units of this product can be produced. These products are to be produced in the ratio $3:4:5$. There is demand for at least $300, 250$ and 200 units of products A, B and C and the profit earned per unit is $90, 40$ and 30 respectively. Formulate the model as linear programming problem (LPP). [4 marks]

   (b) Find the optimal solution of following LPP by algebraic method:
   $Max\ Z = 3x_1 + 4x_2$
   Subject to $x_1 + x_2 \leq 450;\ 2x_1 + x_2 \leq 600;\ x_1, x_2 \geq 0.$
   [3 marks]

2. (a) Solve the following LPP using two phase method.
   $Max\ Z = 5x_1 - 4x_2 + 3x_3$
   Subject to $2x_1 + x_2 - 6x_3 = 20;\ 6x_1 + 5x_2 + 10x_3 \leq 76;\ 8x_1 - 3x_2 + 6x_3 \leq 50;\ x_1, x_2, x_3 \geq 0.$ [3 marks]

   (b) Solve the following LPP using graphical method and also write its standard form.
   $Max\ Z = -x_1 + 2x_2$
   Subject to $x_1 - x_2 \leq -1;\ -0.5x_1 + x_2 \leq 2;\ x_1, x_2 \geq 0.$ [2+1 marks]

   (c) Check graphically whether the set $\{(x, y) \in \Re^2 : xy \leq 1\}$ is convex or not. [2 marks]

3. (a) Consider the following LPP as primal problem:
   $Max\ Z = 5x_1 + 8x_2 + 10x_3$
   Subject to $x_1 + x_2 + 2x_3 \leq 120;\ 3x_1 - 2x_2 - x_3 \geq 90;\ 2x_1 + 4x_2 + 2x_3 = 100;\ x_1, x_2, x_3 \geq 0.$
   Write the dual of above LPP. Using complementary slackness theorem, find the optimal solution of dual and it is given that s_1(slack variable in first constraint), x_1 and x_3 are the basic variables in the primal optimal table. [3 marks]

   (b) State and prove Weak duality theorem. [3 marks]

4. Consider the following LPP:
   $Max\ Z = 5x_1 + 12x_2 + 4x_3$
   Subject to $x_1 + 2x_2 + x_3 \leq 5;\ 2x_1 - x_2 + 3x_3 = 2; x_1, x_2, x_3 \geq 0.$
   Introducing s_1(slack variable in the first constraint) and a_2(artificial variable in second constraint) as starting basic variables respectively, the optimal table of given LPP is as follows:

B.V.	x_1	x_2	x_3	s_1	a_2	Solution
$z_j - c_j$	0	0	$\frac{3}{5}$	$\frac{29}{5}$	$M - \frac{2}{5}$	$\frac{141}{5}$
x_2	0	1	$-\frac{1}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{8}{5}$
x_1	1	0	$\frac{7}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{9}{5}$

   (a) Using post optimality analysis, find optimal solution if R.H.S. is changed in the given LPP from $[5, 2]^T$ to $[3, 9]^T$. [2.5 marks]

   (b) Discuss the effect on the optimal solution if one more variable with column $[1, 1]^T$ and cost 3 is added in the above given LPP. [1.5 marks]

End of Question Paper