Roll Number: ______________

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

Auxiliary Exam

B.E. (Sem: IV,VI,VIII)	Course Code: UMA035
	Course Name: Optimization Techniques
Time: 3 Hours, M. Marks: 100	Name of Coordinators: NK, RJD

Note: (1) **Attempt all the questions.**
(2) **Calculator without graphing mode and alphanumeric memory is permitted.**

1. (a) A company makes two kinds of leather belts. Belt A is a high quality belt, and belt B is of lower quality. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1500 per day. The supply of leather is sufficient for only 1000 belts per day (both A and B combined). Belt A requires a fancy buckle, and only 500 per day are available. There are only 800 buckles a day available for belt B. The profits in belt A and B are Rs. 3 and Rs. 2 per belt, respectively. Formulate the linear programming problem (LPP) to maximize the profit. [5marks]

   (b) Solve the following LPP using graphical method and also write its standard form.
   $Max\ Z = -x_1 + 4x_2$
   Subject to $3x_1 - x_2 \geq -3$; $-0.3x_1 + 1.2x_2 \leq 3$; $x_1, x_2 \geq 0$. [5+2 marks]

2. (a) Consider the LPP: $Min\ Z = x_1 - 3x_2 + 3x_3$
   Subject to $3x_1 - x_2 + 2x_3 \leq 7$; $-2x_1 - 4x_2 \leq 12$; $-4x_1 + 3x_2 + 8x_3 \leq 10$; $x_1, x_2, x_3 \geq 0$.
   Find the missing values from A to M if one of the simplex iteration of the given LPP is

B.V.	x_1	x_2	x_3	s_1	s_2	s_3	Solution
$z_j - c_j$	A	0	B	C	D	$\frac{-8}{5}$	E
x_1	1	F	$\frac{14}{5}$	G	0	$\frac{1}{5}$	H
s_2	0	I	$\frac{156}{5}$	J	1	$\frac{14}{5}$	K
x_2	0	L	$\frac{32}{5}$	M	0	$\frac{3}{5}$	$\frac{58}{5}$

[10 marks]

   (b) State and prove weak duality theorem of LPP. [5marks]

3. (a) Consider the following transportation cost matrix and apply Least Cost Entry method to the following transportation problem.

	Q_1	Q_2	Q_3	Demand
P_1	1	0	3	15
P_2	4	9	5	20
P_3	6	7	2	10
Supply	20	10	15	

[4 marks]

   (b) Find the alternate optimal solution of following transportation problem if one optimal solution is $x_{11} = 50$, $x_{12} = 50$, $x_{13} = 50$, $x_{23} = 150, x_{31} = 150, x_{42} = 150$.

	W_1	W_2	W_3	Supply
A	2	4	5	150
B	4	4	4	150
C	0	2	6	150
D	2	4	6	150
Demand	200	200	200	

[6 marks]

   (c) Five different jobs can be done on five different machines. The matrix below gives the cost in rupees of performing job J_i on machine M_j. Solve the given assignment problem by Hungarian method.

	M_1	M_2	M_3	M_4	M_5
J_1	10	5	9	18	11
J_2	13	9	6	12	14
J_3	3	2	4	4	5
J_4	18	9	12	17	15
J_5	11	6	14	19	10

[10 marks]

4. Solve the following integer programming problem using Branch & Bound technique (If in case all the variables are non-integer then solve the problem by considering the branches for the variable x_1 only).
Maximize $Z = 5x_1 + 4x_2$
Subject to $x_1 + x_2 \leq 5$; $10x_1 + 6x_2 \leq 45$; $x_1, x_2 \geq 0$; x_1, x_2 are integers. [8marks]

5. (a) Construct the network for the given dependency relationship [5 marks]

Activity	A	B	C	D	E	F
Predecessor	–	–	–	A, B	B	B, C

(b) The following table gives data for a project: [10 marks]

Activity	1-2	1-3	2-4	2-5	3-4	4-6	5-6	6-7
Normal time (days)	6	4	5	3	6	8	4	3
Normal Cost (Rs.)	60	60	50	45	90	80	40	45
Crash time (days)	4	2	3	1	4	4	2	2
Crash cost (Rs.)	100	200	150	65	200	300	100	80

(i) Find the normal cost for completing the project in normal duration.
(ii) Can the duration of project be reduced by one day? If yes, find the cost for completing the by reducing one day.

6. (a) Use Lagrange multiplier method to solve the following non-linear programming problem (NLPP). Does the solution maximize or minimize the objective function? [10marks]
Optimize $Z = 5x_1 + x_2 - (x_1 - x_2)^2$
Subject to $x_1 + x_2 = 4$; $x_1, x_2 \geq 0$.

(b) Solve the following NLPP using KKT conditions [10marks]
Maximize $Z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$
Subject to $2x_1 + x_2 \leq 5$; $x_1, x_2 \geq 0$.

7. (a) Find the stationary points and classify them for the function $F = 2 + 2x_1 + 3x_2 - x_1^2 - x_2^2$. [5marks]
(b) Use Fibonacci search technique to minimize $f(x) = x(x-2)$; $0 \leq x \leq 1.5$ with the interval of uncertainty $0.25L_0$. [5marks]

End of Question Paper

Roll No: ....

Thapar Institute of Engineering & Technology, Patiala
School of Mathematics

End Semester Examination (EST): (26 July, 2023)

Programme: **B.E.** | Course Code: **UMA035**
Semester: **Summer 2023-24** | Course Name: **Optimization Techniques**
Maximum Marks: **40**; | Duration: **3 Hrs**
Course coordinator: **Dr. Rajanish Kumar Rai**

Instructions: All questions are compulsory and preferably attempt them in the given sequence only.

Q1. (a) Find the optimal solution of given linear programming problem (LPP) by graphical method.

$$\text{Minimize } Z = 3x_1 + 2x_2,$$
$$\text{subject to } x_1 - x_2 \leq 1,\ x_1 + x_2 \geq 3,\ x_1, x_2 \geq 0.$$

(b) The optimal table of the integer programming problem (IPP):
Maximize $Z = x_1 + 2x_2$,
subject to $2x_2 \leq 7,\ x_1 + x_2 \leq 7,\ 2x_1 \leq 11,\ x_1, x_2 \geq 0$ and are integers.

Basic Variables	x_1	x_2	s_1	s_2	s_3	Solution
$Z_j - C_j$	0	0	1/2	1	0	21/2
x_2	0	1	1/2	0	0	7/2
x_1	1	0	-1/2	1	0	7/2
s_3	0	0	1	-2	1	4

Apply Gomory's cutting plane method to find the optimal solution of the given IPP.

(c) Write dual of the primal problem: Minimize $Z = x_1 + x_2 - x_3$,
subject to $x_1 - 3x_2 + 4x_3 = 5,\ x_1 - 2x_2 \leq 3,\ 2x_2 - x_3 \geq 4$,
$x_1 \geq 0,\ x_2 \leq 0,$ and x_3 is unrestricted. (3 + 5 + 2)

Q2. (a) For the following cost matrix for 4-workers and 4-jobs:

Jobs → Workers ↓	J_1	J_2	J_3	J_4
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30

(i) Find the optimal assignment by using Hungarian method.

(ii) Does this problem have more than one optimal solution ? If yes find alternate optimal solution.

(b) Consider a transportation problem (TP) with 3 sources and 4 destinations. The unit of cost of shipping is given by the following table:

Destinations → Sources ↓	D_1	D_2	D_3	D_4	Supply
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

(i) Find the initial basic feasible solution using Vogel approximation method.

(ii) Find the optimal solution to minimize the transportation cost by using modified distribution or u-v method.

(iii) Does this problem has more than one optimal solution ? If yes find one alternate optimal solution only.

(5 + 6)

Q3. (a) Draw a the network diagram and find the critical path on the basis of following data:

Activity	Duration (days)	Activity	Duration (days)
1-2	2	4-8	8
1-4	2	5-6	4
1-7	1	6-9	3
2-3	4	7-8	3
3-6	1	8-9	5
4-5	5	9-10	2

(b) Consider the following multi-objective linear programming problem (MLPP).
Maximize $x_1 + 9x_2$; Minimize $2x_1 - x_2$; Maximize $17.5x_1 + x_2$,
subject to $3x_1 + 4x_2 \leq 12$; $x_1, x_2 \geq 0$.
Find the efficient solutions and efficient frontier of the above MLPP. (5 + 3)

Q4. (a) Minimize the function $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2^2 + 1$, by using steepest descent method so that the error does not exceed by 0.5. The initial approximation is taken as $(1, 1)$.

(b) Determine the stationary point and classify them for the function

$$f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2 + 10.$$

Is it global optimal or not? If yes find the global optimal solution.

(c) Solve the following non-linear programming problem (NLPP) using Karush Kuhn Tucker (KKT) conditions.

$$\text{Minimize } Z = -\log x_1 - \log x_2,$$
$$\text{subject to } x_1 + x_2 \leq 2, \ x_1, x_2 \geq 0.$$

(4 + 3 + 4)

END OF QUESTION PAPER