PROJECT MANAGEMENT

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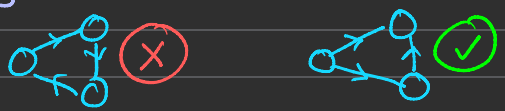
Construction of network :-

○ : represents a place where an activity starts or end

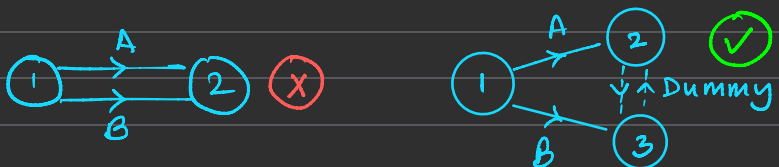
→ : Activity between nodes

Some Basic Rules of constructing network :-

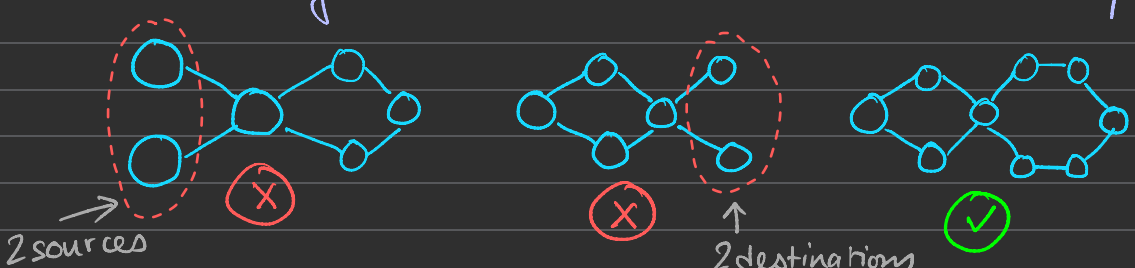
① Activity should not form a closed loop.



② Starting and ending nodes of 2 activities should not be same.



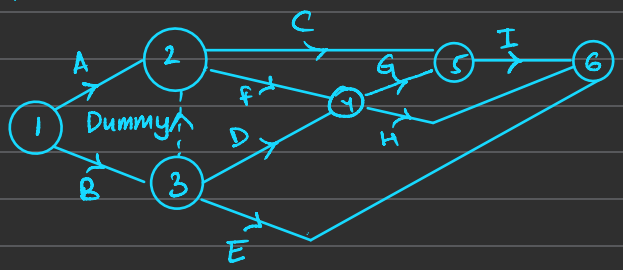
③ The origin and destination should be unique



Q. Construct a network by considering the following activities.

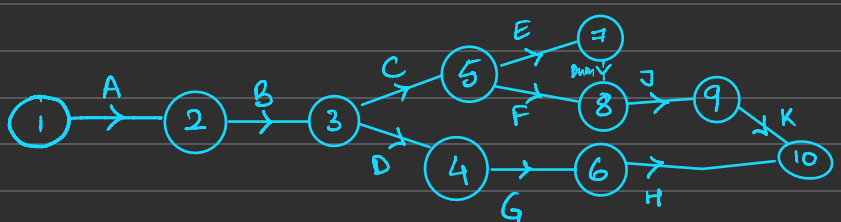
Activity	A	B	C	D & E	F	G & H	I
Predecessors	–	–	A & B	B	A & B	F & D	C & G

Ans :



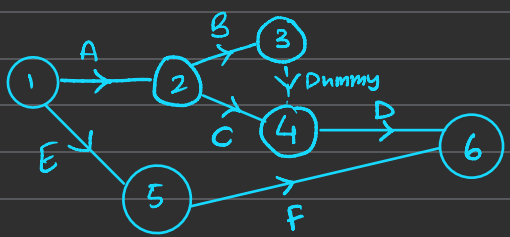
Q.

Activity	A	B	C & D	E & F	G	H	J	K
Predecessor	–	A	B	C	D	G	E & F	J



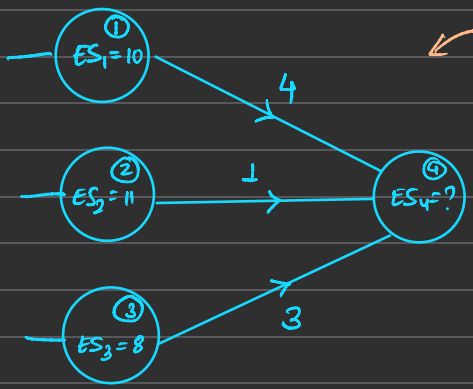
Q.

Activity	A	B	C	D	E	F
Predecessor	–	A	A	B & C	–	E



Earlist Starting Time of an activity :- (ES or EST)

An activity at a node can start at any node only when all the activities reaching to the node will be completed.

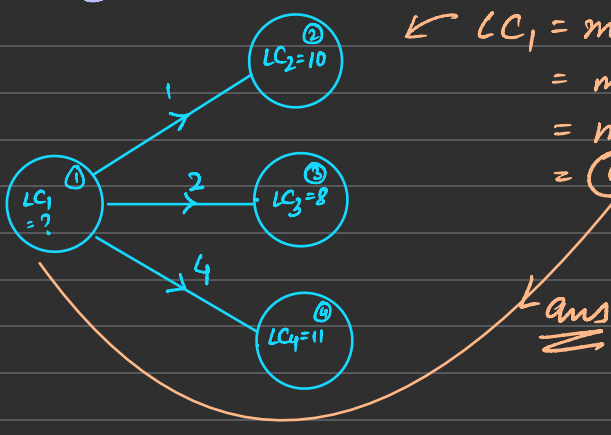


$ES_4 = \max\{ES_1 + t_{14}, ES_2 + t_{24}, ES_3 + t_{34}\}$
$= \max\{10+4, 11+1, 8+3\}$
$= \max\{14, 12, 11\}$
$= 14$

Ans

Latest Completion Time of an activity of node (LC, LCT) :-

This is the maximum starting time of all the activities of a node should be in such a manner that none of the activities should be late to reach its destination node.

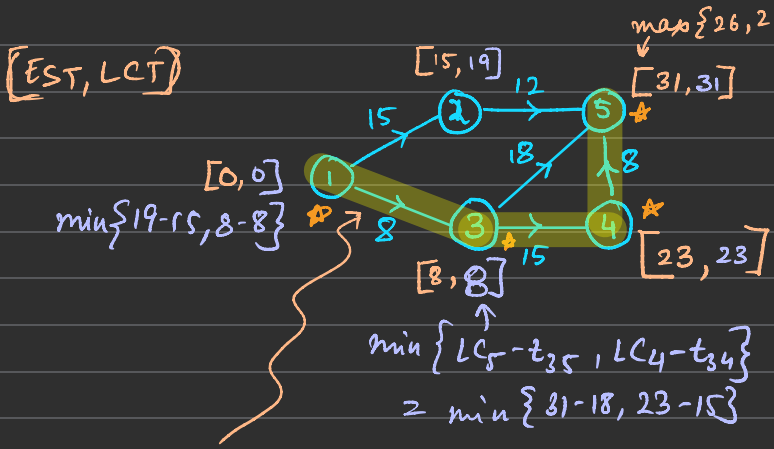


$LC_1 = \min\{LC_2 - t_{12}, LC_3 - t_{13}, LC_4 - t_{14}\}$
$= \min\{10-1, 8-2, 11-4\}$
$= \min\{9, 6, 7\}$
$= 6$

ans

Q. Calculate EST & LCT for the following network.

Activity (i,j)	(1,2)	(1,3)	(2,5)	(3,4)	(3,5)	(4,5)
Time	15	8	12	15	18	8



CRITICAL PATH METHOD

Critical Activity : The (i,j)th activity b/w node i & j is said to be critical if :-

$$ES_i = LC_i$$
$$ES_j = LC_j$$
$$ES_j - ES_i = LC_j - LC_i = t_{ij}$$

Critical Path : A path b/w the origin & destination, constructed by critical activities

Crash Duration (T_c) : The min time required to complete an activity

Crash Cost (C_c) : The cost required to complete an activity in min time (crash duration)

Normal Duration & Normal Cost : The time specified by a person to complete an activity is Normal duration (T_N) and the corresponding cost for normal duration (C_N)

$$T_N \geq T_C$$
$$C_N \leq C_C$$

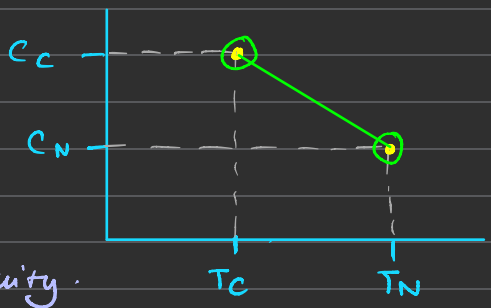
Crash limit of an activity :- The maximum time that can be reduced from an activity

$$\text{Crash limit} = \text{Normal duration } (T_N) - \text{Crash duration } (T_c)$$

Slope of an activity :

Slope of Activity : $\left|\frac{C_N - C_C}{T_N - T_C}\right|$

↑ = The cost required to reduce one unit time from the activity.



Free float of Non-Critical Activities (i,j):

$FF_N = ES_j - ES_i - t_{ij}$

↑ Non critical activities May have a $FF = 0$

$ff_{(critical)} = 0$ (always)

↑ A critical activity always have a free float of '0'.

☆ Critical Activities always have $ff = 0$. But $FF = 0$ does not necessarily mean that activity is critical.

Free float Limit : min {non-zero ff}

Compression Limit : min {ff limit, Crash limit}

Q Consider the following data.

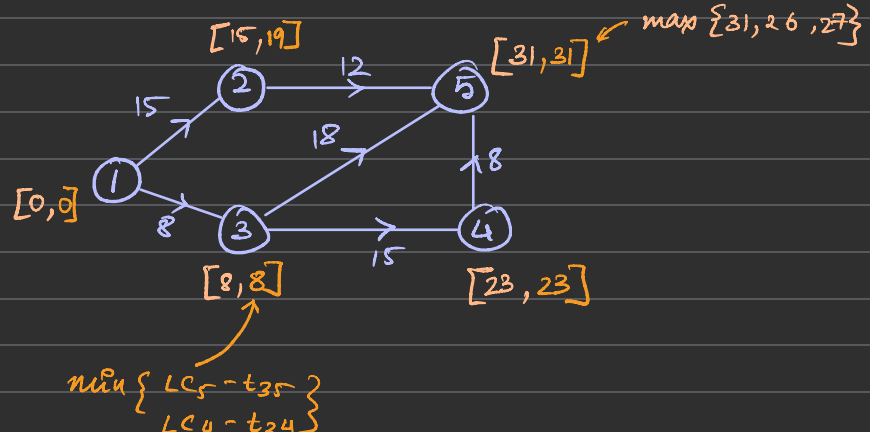
Activity	T_N	C_N	T_C	C_C
(1, 2)	15	600	12	1200
(1, 3)	8	700	5	1600
(2, 5)	12	750	6	1500
(3, 4)	15	650	12	1400
(3, 5)	18	700	3	1450
(4, 5)	8	500	5	950

find the cost for completing the project

(I) in normal duration
(II) in 29 days
(III) in 28 days
(IV) in 23 days

(V) find the crash cost for completing the project (i.e, the cost to complete the project in min time)

Ans



(i) The project will complete in 31 days

Cost = 600 + 700 + 750 + 650 + 700 + 500

= ₹ 3900

(ii) To complete the project in 29 days :–

∵ we have to find compression limit

① To reduce the no. of days from the project, we can reduce days only from the critical activities :–

Critical activities :– (1, 3), (3,4), (4,5)

∴ Path :– 1 – 3 – 4 – 5.

② To reduce the days from critical activities, select the critical activity whose slope is minimum.

Slope of : $(1,3) = \left|\frac{C_N - C_C}{T_N - T_C}\right| = \left|\frac{700-1600}{8-5}\right| = 300$

$(3,4) = 250$

$(4,5) = 150$

Min {slopes} = 150 which is corresponding to (4,5)

∴ we will reduce the no. of days from (4,5)

Now, to reduce :–

Compression limit (4,5) = min { Crash limit, ff limit }

= min {3, 4} = ③

Crash limit (4,5) = Normal (or current) duration – crash dur.

= 8 – 5

= ③

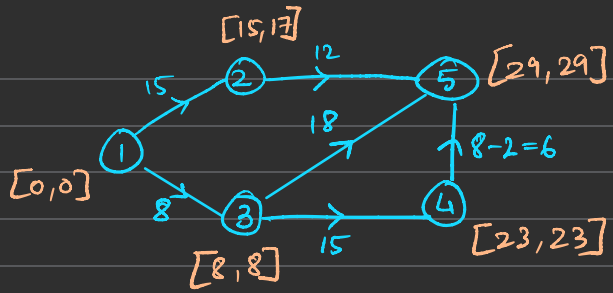
ff (1,2) = $ES_2 - ES_1 - t_{12} = 0$

(2,5) = $ES_5 - ES_2 - t_{25} = 4$

(3,5) = $ES_5 - ES_3 - t_{35} = 5$

ff limit = min (non zero ff) = min {4,5} = ④

* we can reduce 3 days from (4,5) but as per condition, we have to reduce only 2 days from (4,5) to complete project in 29 days.



Now the project will complete in 29 days

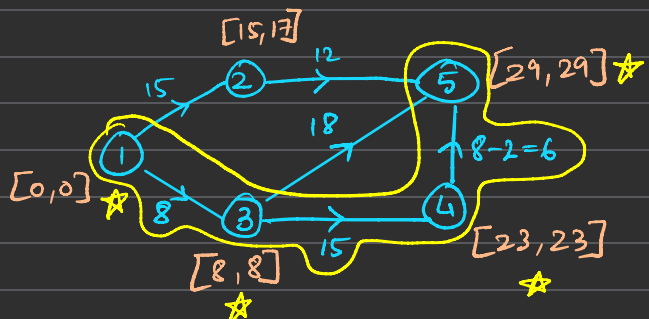
Cost = Normal cost + No. of days * Slope (4,5)

= 3900 + 2 * 150

= ₹ 4200

(iii) To complete the project in 28 days

↳ Reduce days only from the critical activity.



CA: (1,3) & (3,4) & (4,5)

CP: 1 - 3 - 4 - 5

Slope: (1,3) = 300
(3,4) = 250
(4,5) = 150

} min = (4,5) = 150 ↖ so we reduce days from here.

Compression limit = min { Crash limit, Ef limit }

= min {1, 2} = (1)

Crash limit (4,5) = $T_N - T_C$ = 6 - 5 = (1)

ff (1,2) : $ES_2 - ES_1 - t_{12} = 0$

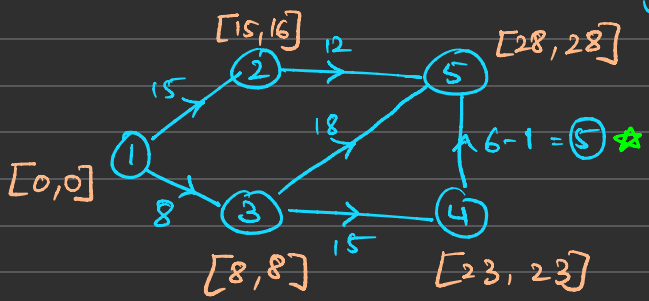
(2,5) : $ES_5 - ES_2 - t_{25} = 2$

(3,5) : $ES_5 - ES_3 - t_{35} = 3$

ff limit = min { non-zero ff } = min {2,3} = (2)

Thus Compression limit = no. of days we want to reduce.

∴ We will reduce '1' day from (4,5)



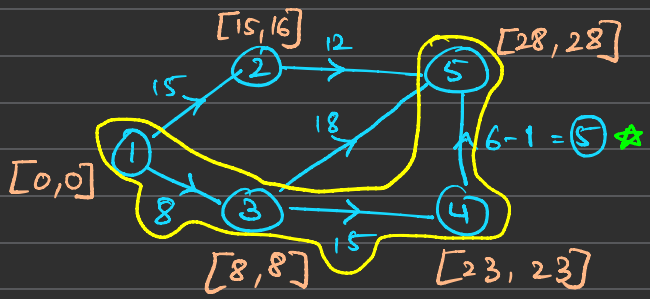
← add star as this activity has been reduced to its compr. limit

∴ The project will complete in 28 days &

cost = 4200 + 1 * 150

= ₹ 4350

(iv) for completing 23 days we have to reduce 5 days



← we can reduce only from critical activities.

CA : (1,3), (3,4) & (4,5)

CP : (1-3-4-5)

Slopes: (1,3) = 300
(3,4) = 250 ← min
(4,5) = 150 ← This is minimum but we can not reduce no' of days from (4,5) as it has reached its crash duration.

Compression limit of (3,4) = min { crash limit (3,4), ff limit }
= min { 3, 1 } = (1)

↑ so in this case the Comp. limit < Reducing no' of days

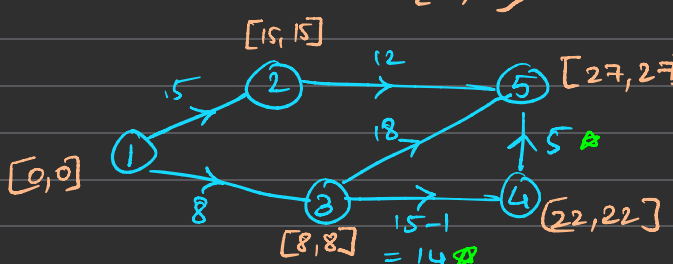
Crash limit (3,4) = $T_N - T_C$ = 15 - 12 = 3

ff: (1,2) = 0
(2,3) = 1
(3,5) = 2

ff limit = min { non-zero ff }
= min {1,2} = 1

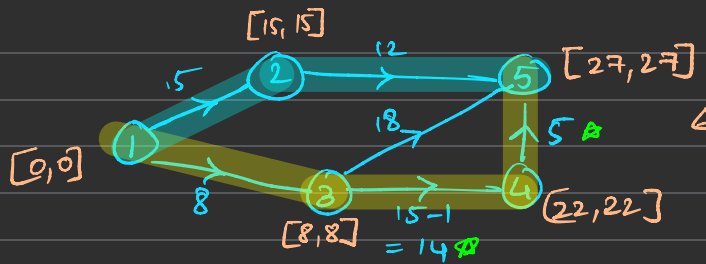
1 < 5

↓ make graph as contains from there



← This project will complete in 27 days
Cost: 4350 + 1*250
= ₹4600

We can not stop here as we have to reduce 4 more days from critical activities.



← 2 critical paths.

CA_1 : (1,3), (3,4), & (4,5) CA_2 : (1,2) & (2,5)

CP_1 : 1-3-4-5 CP_2 : 1-2-5

★ we will reduce days from the critical activity of each critical path.

1-3-4-5

Slope : (1,3) = 300

(3,4) = 250 ✓ ← min

(4,5) = 150 ✗

↑ max compression reached

1-2-5

Slope : (1,2) = 200

(2,5) = 125 ✓ ← min.

↑ we will reduce days from (3,4) & (2,5)

Compression limit = min { Crash limit, ff limit }

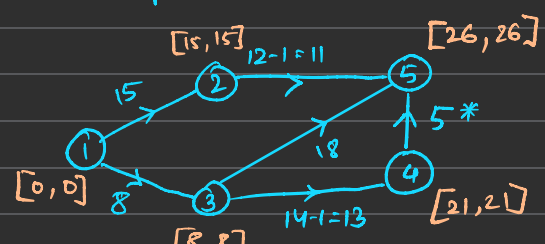
= min { 2, 6, 1 } = 1

↑ This means we can reduce only 1 day from (3,4) & (2,5)

Crash limit (3,4) = 14 − 12 = 2

Crash limit (2,5) = 12 − 6 = 6

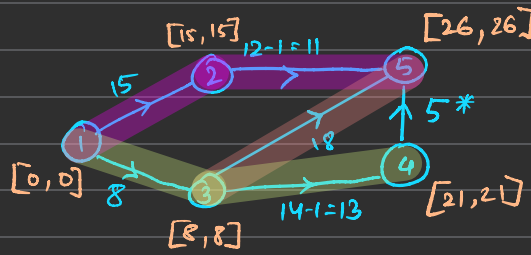
ff of (3,5) = 27 − 18 − 1 = 1



→ Thus the project will complete in 26 days.

Cost = 4600 + 1*Slope(3,4) + 1*Slope(2,5) = ₹4975

To reduce 3 more days
↳ Reduce days from critical path.



1-3-4-5	1-3-5	1-2-5
Slope (1,3) = 300 ⟷ common	(1,3) = 300	(1,2) = 200
(3,4) = 250 (min)	(3,5) = 150 (min)	(2,5) = 125 (min)
(4,5) = 150 ⊗		

* (1,3) is common to path 1-3-4-5 & 1-3-5, so instead of reducing from (3,4) & (3,5), the time from common activity (1,3) can be reduced if.

Slope (1,3) < Slope (3,4) + Slope (3,5)
300 < 250 + 150 ✓

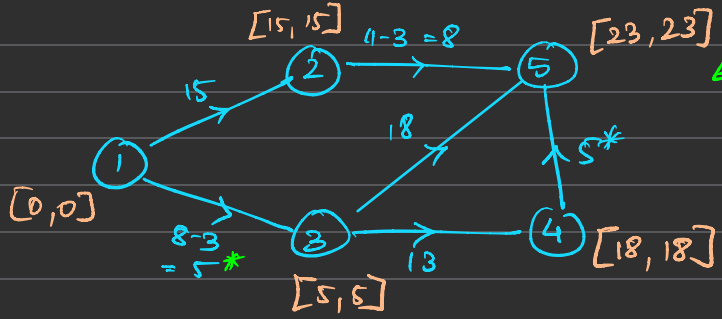
∴ We will reduce days from (1,3) for 1-3-4-5 & 1-3-5

Compression lim = min (crash lim, ff limit) = min {3, 4}
= 3

Crash limit:
(1,3) = 8 - 5 = 3
(2,5) = 11 - 6 = 4

ff limit:
Does not exist ∵ there are no non-critical activities

∴ We will reduce no. of days as '3' from (1,3) & (2,5)



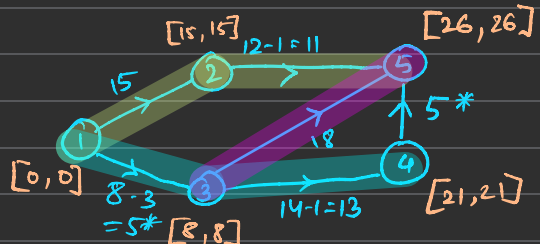
← Thus the project will complete in 23 days

Cost = 4975 + 3*S(1,3) + 3*S(2,5)

= 4975 + 3*300 + 3*125

= ₹ 6250

(v) We have to complete project in min time.



Compression limit = min { Crash lim, ff limit } (ff limit ×)

= min {1, 5, 2} = 1

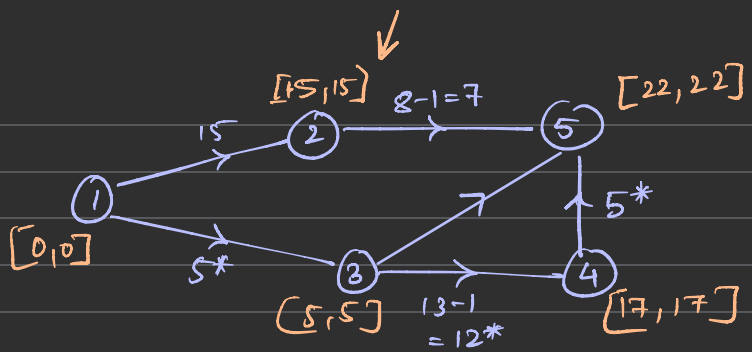
Crash lim (3, 4) = 13 - 12 = 1

(3, 5) = 18 - 13 = 5

(2, 5) = 8 - 6 = 2

☆ Now we have to reduce the no. of days from each critical path

New circuit.



The project will complete in 22 days
& cost = 6250 + 1*[(3,4) + (3,5) + (2,5)]
= 6250 + [250+125+150] = ₹ 6775

We repeat this process & stop when our critical path is (1-3-4-5).

↳ we stop because all the activities in this path have reached their Crash duration. If any such CP occurs then we can not reduce anything from them anymore because they have reached their critical limit.