Curves and Surfaces

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- Displays of 3-dimensional curved lines and surfaces can be generated from an input set of mathematical functions defining the objects or from a set of user-specified data points.
- When functions are specified, a package can project the defining equations for a curve to the display plane and plot pixel positions along the path of the projected function.
- For surfaces, a functional description is often tessellated to produce a polygon-mesh approximation to the surface.
- This is done with triangular polygon patches to ensure that all vertices of any polygon are in one plane.

- Polygons specified with four or more vertices may not have all vertices in a single plane. Examples of display surfaces generated from functional descriptions include the quadrics and the superquadrics.
- When a set of discrete coordinate points is used to specify an object shape, a functional description is obtained that best fits the designated points according to the constraints of the application.
- Spline representations are examples of this class of curves and surfaces.
- These methods are commonly used to design new object shapes, to digitize drawings and to describe animation paths.

- Curve-fitting methods are also used to display graphs of data values by fitting specified curve functions to the discrete data set, using regression techniques such as the least-squares method.
- Curves and surface equations can be expressed in either a parametric or a nonparametric form.

Quadric Surfaces

- Frequently used class of objects, which are described with second-degree equations.
- They include spheres, ellipsoids, tori, paraboloids, and hyperboloids.
- The spheres and ellipsoids are common elements of graphics scenes, and they are often available in graphics packages as primitives from which more complex objects can be constructed.

Sphere

- In Cartesian coordinates, a spherical surface with radius r centered on the coordinate origin is defined as the set of points(x,y,z) that satisfy the equation

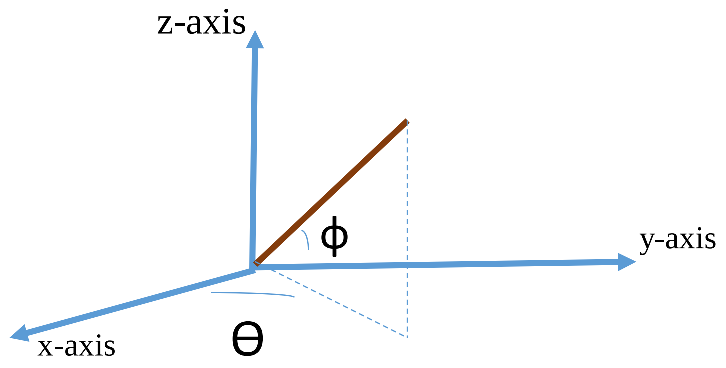
$$x^2 + y^2 + z^2 = r^2$$

The spherical surface in parametric form, using latitiute and longitude angles

$x = r\cos\phi\cos\Theta$ $\quad$ $-\pi/2 \le \phi \le \pi/2$

$y = r\cos\phi\sin\Theta$ $\quad$ $-\pi \le \Theta \le \pi$

$z = r\sin\phi$



Ellipsoid

- This surface can be described as an extension of a spherical surface, where radii in three mutually perpendicular directions can have different values.
- The Cartesian representation for points over the surface of an ellipsoid centered on the origin is

$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

- A parametric representation for the ellipsoid in terms of the latitutde angle ϕ and the longitude angle Θ is:

$$x = r_x \cos\phi \cos\Theta \qquad -\pi/2 \le \phi \le \pi/2$$

$$y = r_y \cos\phi \sin\Theta \qquad -\pi \le \Theta \le \pi$$

$$z = r_z \sin\phi$$

Torus

- A torus is a doughnut-shaped object. It can be generated by rotating a circle or other conic about a specified axis.
- The Cartesian representation for points over the surface of a torus can be written in the form:

$$\left[r - \sqrt{\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2}\right]^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

where r is any given offset value.

- A parametric representation for the torus are similar to those for an ellipse, except that angle ϕ extends over 360 degree. Using latitutde and longitude angles ϕ and Θ, the torus surface is described as the set of points that satisfy

$$x = r_x(r + \cos\phi)\cos\Theta \qquad -\pi \leq \phi \leq \pi$$

$$y = r_y(r + \cos\phi)\sin\Theta \qquad -\pi \leq \Theta \leq \pi$$

$$z = r_z \sin\phi$$

Superquadrics Surfaces

- This class of objects is a generalization of the quadric representations.
- They are formed by incorporating additional parameters into the quadric equations to provide increased flexibility for adjusting object shapes.
- The number of additional parameters used is equal to the dimension of the object: one parameter for curves and two parameters for surfaces.

Superellipse

- We obtain a Cartesian representation for a superellipse from the corresponding equation for an ellipse by allowing the exponent on the x and y terms to be variable.

$$\left(\frac{x}{r_x}\right)^{2/s} + \left(\frac{y}{r_y}\right)^{2/s} = 1$$

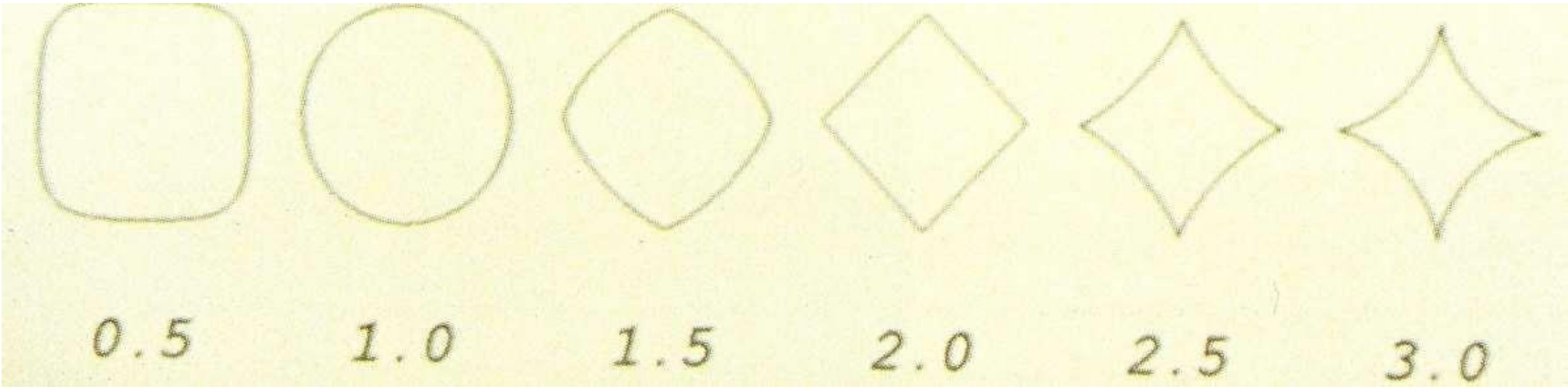
where parameter s can be assigned any real value.

When s=1, we get an ordinary ellipse

Corresponding parametric equations for the superellipse can be expressed as

$$x = r_x \cos^s \theta \qquad -\pi \le \theta \le \pi$$

$$y = r_y \sin^s \theta \qquad -\pi \le \theta \le \pi$$



Superellipses plotted with different values for parameter s and with $r_x = r_y$

Superellipsoid

A Cartesian representation for a superellipsoid is obtained from the equation for an ellipsoid by incorporating two exponent parameters:

$$\left[\left(\frac{x}{r_x}\right)^{2/s_2} + \left(\frac{y}{r_y}\right)^{2/s_2}\right]^{s_2/s_1} + \left(\frac{z}{r_z}\right)^{2/s_1} = 1$$

For s_1=s_2= 1, we have an ordinary ellipsoid.

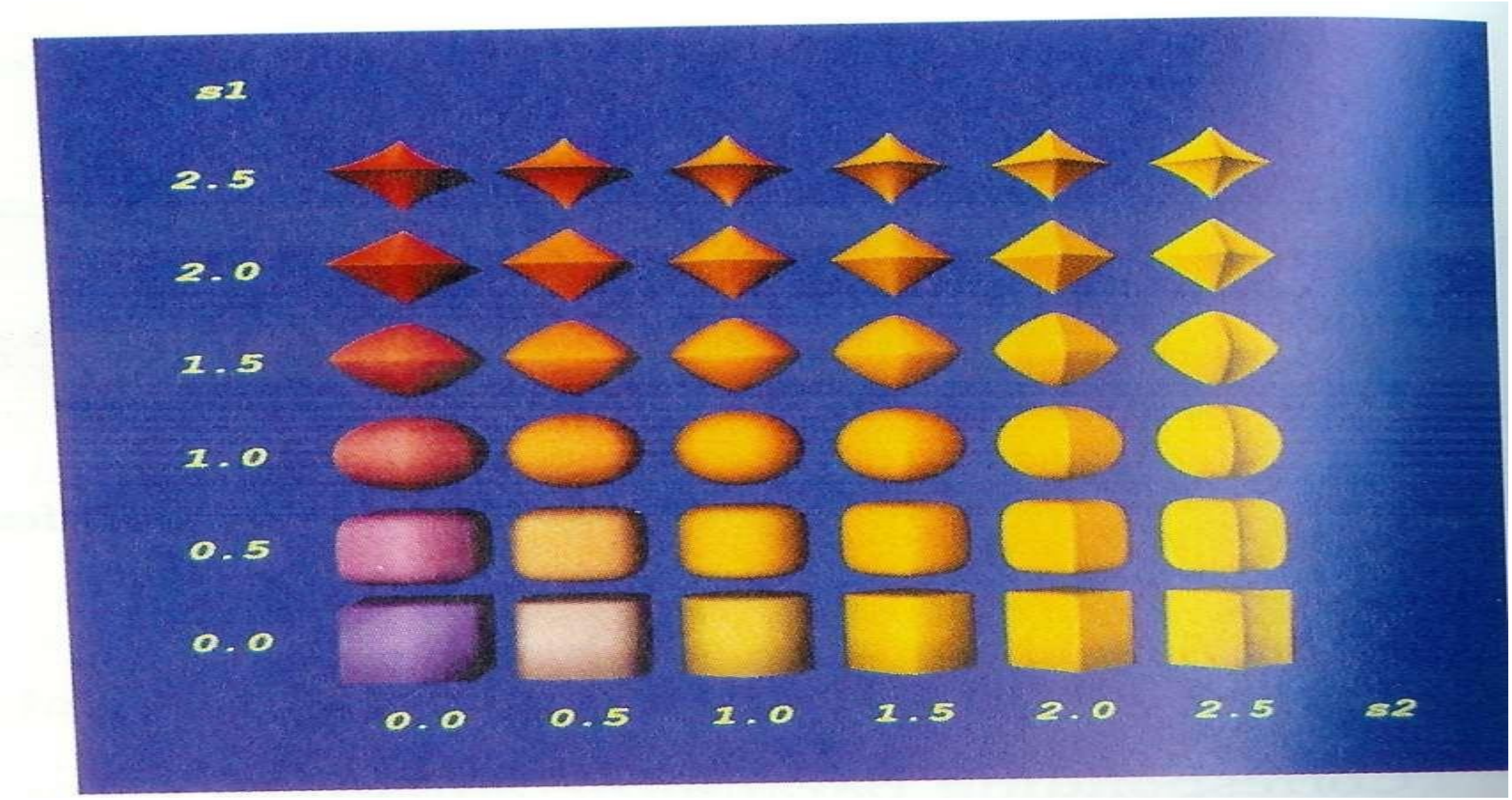
The corresponding parametric representation for the superellipsoid as:

$$x = r_x \cos^{s_1}\phi \cos^{s_2}\Theta \qquad -\pi/2 \leq \phi \leq \pi/2$$

$$y = r_y \cos^{s_1}\phi \sin^{s_2}\Theta \qquad -\pi \leq \Theta \leq \pi$$

$$z = r_z \sin^{s_1}\phi$$

These and other superquadric shapes can be combined to create more complex structures, such as furniture, threaded bolts and other hardware.



Superellipsoids plotted with different values for parameter s_1 and s_2 and with $r_x = r_y = r_z$

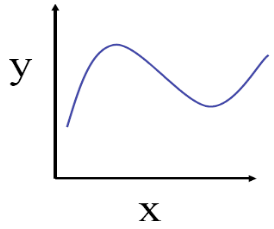
Representation of curves and surfaces

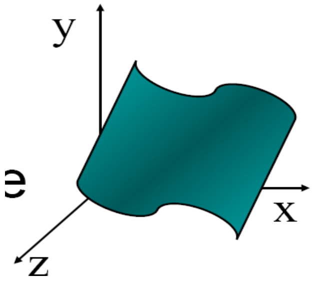
- **Explicit Representation**

  The explicit form of a curve in two dimensions gives the value of one variable, the dependent variable, in terms of the other,

  the independent variable

  In *x*, *y space*, we might write $\boldsymbol{y = f(x)}$





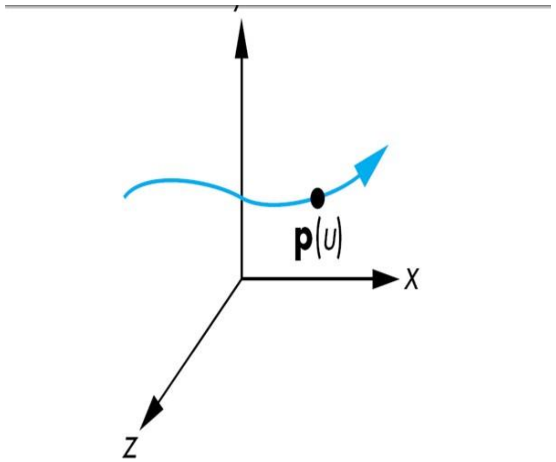
  The surface represented by an equation of the form $\boldsymbol{z = f(x, y)}$

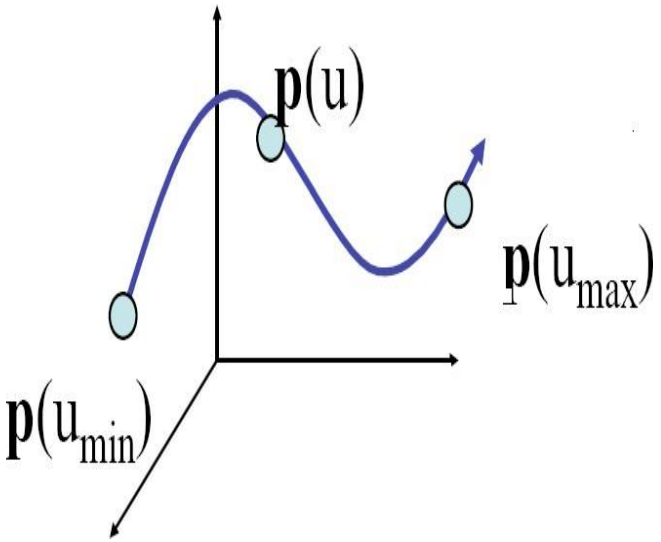
- **Implicit Representations**

  - In two dimensions, an implicit curve can be represented by the equation $f(x, y) = 0$
  - The implicit form is less coordinate-system dependent than is the explicit form.
  - In three dimensions, the implicit form $f(x, y, z) = 0$
  - Curves in three dimensions are not as easily represented in implicit form.
  - We can represent a curve as the intersection, if it exists, of the two surfaces: $f(x, y, z) = 0, \; g(x, y, z) = 0$

- **Parametric Form**
  - The parametric form of a curve expresses the value of each spatial variable for points on the curve in terms of an independent variable, ***u****, the* parameter. In three dimensions, we have three explicit functions:

$$x = x(u)\ ,\ y = y(u)\ ,\ z = z(u)$$





- One of the advantages of the parametric form is that it is the same in two and three dimensions. In the former case, we simply drop the equation for *z*.

- Parametric surfaces require two parameters. We can describe a surface by three equations of the form *:*

$$x = x(u, v)\ ,\ y = y(u, v)\ ,\ z = z(u, v)$$

Summary

- Discussed about the curves and surfaces in 3 D.
- It also discussed about different curved surfaces and their representation methods.