Pumping Lemma

Language for which you can construct FSM or Regular Expression are regular language.

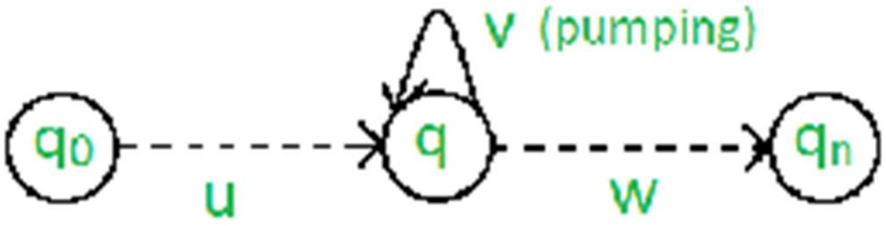
Language for which you can not construct FSM or Regular Expression are non regular language or irregular.

Ex of irregular language

– L1 = $\{a^n b^n$ | n≥ 0}
  L2 = {ww | w ∈ {0,1}*}
  L3 = {ww^R | w ∈ {0,1}*}
  L4 = $\{a^n b^{n+1}$ | n≥ 0}
  L5 = $\{a^i b^j$ | i > $j\}$
  L6 = {w | $n_a(w) = n_b(w)$ and w ∈ {a,b}*}

Pumping Lemma for Regular Language

- Pumping lemma is the theorem used to proof language is not regular
- For any regular language L, there exists an integer n, such that for all $x \in L$ with $|x| \geq n$, there exists $u, v, w \in \Sigma*$, such that $x = uvw$, and
  (1) $|uv| \leq n$
  (2) $|v| \geq 1$
  (3) for all $i \geq 0$: $uv^iw \in L$
- In simple terms, this means that if a string v is 'pumped', i.e., if v is inserted any number of times, the resultant string still remains in L.
- If there exists at least one string made from pumping which is not in L, then L is surely not regular.



Prove that L = $\{0^n 1^n \mid n \geq 0\}$ is irregular

Let us assume that L is regular, then by Pumping Lemma we have to prove it regular

- let x ∈ L and |x| ≥ n. So, by Pumping Lemma, there exists u, v, w such that

  (1) $|uv| \leq n$
  (2) $|v| \geq 1$
  (3) for all $i \geq 0$: $uv^iw \in L$

Let x = 0011 ∈ L,

divide x into u, v and w. If L is regular then $uv^iw \in L$ for all $i \geq 0$

let u = 0, v = 01, w = 1

$x = uv^iw = 0(01)^i1$

For i=0 , $0(01)^01 =$ 01 ∈ L

For i=2, $0(01)^21 =$ 0 0101 1 ∉ L

For i=3, $0(01)^31 =$ 0 010101 1 ∉ L

Language L is not satisfying the pumping lemma condition that means L is not regular or irregular

Prove that L = {ww| $w \in \{0,1\}^*$} is irregular

Let us assume that L is regular, then by Pumping Lemma we have to prove it regular

- let $x \in L$ and $|x| \geq n$. So, by Pumping Lemma, there exists u, v, w such that

  (1) $|uv| \leq n$
  (2) $|v| \geq 1$
  (3) for all $i \geq 0$: $uv^iw \in L$

Let x = 10 10 $\in$ L, divide x into u, v and w. If L is regular then $uv^iw \in L$ for all $i \geq 0$

let u = 1, v = 01, w = 0

$x = uv^iw = 1(01)^i0$

For i=0,	$1(01)^00 =$	$10 \notin L$
For i=2,	$1(01)^20=$	$1\ 01\ 01\ 0 \notin L$
For i=3,	$1(01)^30 =$	$1\ 010\ 101\ 0 \in L$

Language L is not satisfying the pumping lemma condition that means L is irregular

L= $\{a^p \mid$ p is prime number$\}$

$L = \{aa, aaa, aaaaa, \text{-----}\}$

let $n = 3$

$w = aaa$

$$\begin{array}{ccc} a & a & a \\ \downarrow & | & | \\ u & v & w \end{array}$$

$uv^iw \rightarrow i = 0, \; aa \in L$

$i = 1 = aaa \in L$

$i = 2 = aa^2a \notin L$, Not reg.

$L = \{ 0^{n^2} \mid n \geq 0 \}$ not reg.

$L = \{ \epsilon, 0, 0000, 000000000, \dots \}$

$n = 4$

$\underbrace{0}_{u}\underbrace{0}_{v}\underbrace{00}_{w}$

$i = 0$) $000 \notin L$

Application: (Explain the application of the Pumping Lemma to show a Language is Regular or Not)

The pumping lemma is extremely useful in proving that certain sets are non-regular. The general methodology followed during its applications is :

- Select a string z in the language L.
- Break the string z into x, y and z in accordance with the above conditions imposed by the pumping lemma.
- Now check if there is any contradiction to the pumping lemma for any value of i.