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Thapar Institute of Engineering & Technology, Patiala

Department of Mathematics

Mid Semester Examination (March 14, 2024)

Course Name: Optimization Techniques M. Marks: 30

Course Code: UMA035 Time: 2 Hrs

Faculty: Dr. Meenakshi Rana, Dr. Amit Kumar, Dr. Vikas Sharma, Dr. Sanjeev Kumar, Dr Navdeep Kailey, Dr. Pankaj Narula, Dr. Jolly Puri, Dr. Mamta Gulati, Dr. Farhan Musanna, Dr. Rajesh Dhayal, Dr. Tina Verma, Dr. Rajnish Rai

NOTE : All questions are compulsory and write answers of subparts of each question together only. In following questions 's'-variable stand for slack/surplus.

1. (a) Let the function $f(x_1, x_2) = (-3x_1 + x_2)$ be minimized over the intersection of region $y \geq |x|$ and the solid triangular region ABC with A:(-1,0), B:(2,0), C:(0,1). Draw the feasible region and formulate this problem as a Linear Programming Problem (LPP). **(3)**

   (b) State and prove Fundamental Theorem of Linear Programming Problem (LPP) for a Maximization LPP with bounded region. **(3)**

   (c) Prove that the feasible region of an LPP is convex set. **(1.5)**

2. (a) Solve the given LPP using Simplex method: Max $z = 3x_1 + 5x_2 + 4x_3$ **(3.5)**

$$2x_1 + 3x_2 \leq 8, \quad 2x_1 + 5x_2 \leq 10, \quad 3x_1 + 2x_2 + 4x_3 \leq 15, \quad x_1, x_2, x_3 \geq 0$$

   (b) The given LPP: Min $z = 4x_1 + 8x_2 + 3x_3$

$$x_1 + x_2 \geq 2, \quad 2x_1 + x_3 \geq 5, \quad x_1, x_2, x_3 \geq 0.$$

   is solved by Two Phase Method, and an optimal/last table of Phase I is given below, where a_1, a_2 are artificial variables.

B.V.	x_1	x_2	x_3	s_1	s_2	a_1	a_2	Solution (X_B)
$z_j - c_j \rightarrow$	0	0	0	0	0	-1	-1	0
x_1	1	0	1/2	0	-1/2	0	1/2	5/2
s_1	0	-1	1/2	1	-1/2	-1	1/2	1/2

   Give the following answers:

   (i) Using this table, go to Phase II, and find an optimal solution of given LPP. **(1)**

   (ii) Write Dual of given LPP and using the complementary slackness conditions, find an optimal solution of the dual. **(2+1)**

3. (a) Consider the following LPP with objective function as Min $z = 2x_1 - x_2$

$$2x_1 + 3x_2 + 5x_3 \leq 8, \quad x_1 + 2x_2 \leq 4, \quad x_1, x_2, x_3 \geq 0$$

   Construct the simplex table corresponding to the corner point $(x_1, x_2, x_3) = (0, 2, 0)$ (Do not use row-operations/simplex iterations for construction.) **(4.5)**

   (b) Does the simplex table constructed in above question (Q3(a)) is optimal? If yes, find all possible alternate optimal solutions. **(3)**

4. (a) Solve the Integer Programming Problem (IPP) using Branch & Bound method. **(3)**

$$\text{Min } z = -3x_1 + 2x_2 \text{ subject to } x_1 - 2x_2 \leq 5, \quad 2x_1 + x_2 \leq 3, \quad x_1, x_2 \geq 0, \quad \textit{and integers}.$$

   (b) Consider the problem, Max $z = 2x_1 - x_2$ subject to $x_1 + x_2 \leq 2, \quad x_1 - x_2 \leq 1, \quad x_1, x_2 \geq 0$. An optimal table of the above LPP is given as:

B.V.	x_1	x_2	s_1	s_2	Solution (X_B)
$z_j - c_j \rightarrow$	0	0	1/2	3/2	5/2
x_2	0	1	1/2	-1/2	1/2
x_1	1	0	1/2	1/2	3/2

   Use Sensitivity analysis to answer the following:

   (i) Let right hand side of constraints $(2, 1)^T$ is changed to $(2, b)^T$, find range of b so that feasibility is not disturbed. **(2)**

   (ii) Assuming x_1, x_2 as integer variables, construct the Gomory Constraint for variable x_1 and find an optimal integer solution. **(2.5)**