NFA / DFA $\longrightarrow$ RE

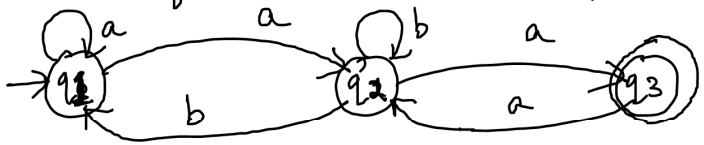
Use Arden's Theorem

If P & Q are two REs and P does not contain ϵ

then $R = Q + RP$ has ● the unique solution

● $R = QP^*$

Q// Find the RE for the strings accepted the following NFA.



Ans: The given FA contains no ϵ-moves and a single initial state.
Therefore, we can apply the Arden's theorem based approach.
Now, the equations corresponding to the three states are:

$$q_1 = q_1 a + q_2 b + \epsilon \quad \text{——} (1)$$

$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{——} (2)$$

$$q_3 = q_2 a \quad \text{——} (3)$$

Applying (3) in (2), we get

$$q_2 = q_1 a + q_2 b + q_2 aa$$

$$\Rightarrow \underbrace{q_2}_{R} = \underbrace{q_1}_{Q} a + \underbrace{q_2}_{R} \underbrace{(b + aa)}_{P}$$

$$\Rightarrow q_2 = q_1 a (b + aa)^* \quad \text{——} (4) \text{ (Applying Arden's Theorem)} -$$

Subtituting ④ in ①, we get

$$q_1 = q_1 a + q_1 a(b+aa)^* b + \epsilon$$

$$\Rightarrow q_1 = \underbrace{\epsilon}_{Q} + q_1 \underbrace{(a + a(b+aa)^* b)}_{P} \quad \text{—— ⑤}$$

Applying Arden's theorem to ⑤, we get

$$q_1 = \epsilon \cdot (a+a(b+aa)^* b)^*$$

$$\Rightarrow q_1 = (a+a(b+aa)^* b)^*$$

Therefore, $q_2 = q_1 a(b+aa)^*$

$$\Rightarrow q_2 = (a+a(b+aa)^* b)^* a(b+aa)^*$$

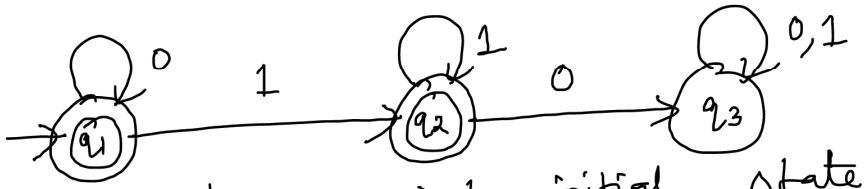
Now, $q_3 = q_2 a$

$$\Rightarrow q_3 = (a+a(b+aa)^* b)^* a(b+aa)^* a$$

Since, q_3 is the final state,

$\boxed{(a+a(b+aa)^* b)^* a(b+aa)^* a}$ is the RE for the given FA.

Q// find RE for the following DFA



Ans The given DFA has a single initial state & no ϵ moves.

Therefore, we can apply Arden's theorem based rule.

The equations of states are

$$q_1 = q_1 0 + \epsilon \quad \text{—①}$$

$$q_2 = q_1 1 + q_2 1 \quad \text{—②}$$

$$q_3 = q_2 0 + q_3 0 + q_3 1 \quad \text{—③}$$

Apply Arden's theorem to ①, we get

$$q_1 = \epsilon 0^* = 0^* \quad \text{—④} \qquad \because R = q_1,\ Q = \epsilon \ \&\ P = 0$$

Now applying ④ in ②, we get

$$q_2 = 0^* 1 + q_2 1$$

$$\Rightarrow q_2 = (0^* 1)1^* \quad \text{—⑤ (By Arden's Theorem)}$$

Since, q_1 & q_2 are final states, the RE for the given DFA is

$$q_1 + q_2 = 0^* + 0^*11^*$$
$$= 0^*(\epsilon + 11^*)$$
$$= 0^*1^* \quad \text{(Ans)}$$