Designing a Neuro-Fuzzy Controller for a Ball and Plate System

Sameer Talwar, Shreshth Mehrotra, Shubham Jha, Vivek Sharma

***Abstract*—Many attempts have already been made to design a controller for the frequently studied ball and plate system. Over the years, classical approaches have molded into modern-day adaptive techniques of soft control. This paper presents one such technique of control by designing a neuro-fuzzy system to make the ball track a complex Lissajous trajectory. The results are compared with a classical PID controller. Finally, a combination of both controllers is deployed and the final results are compared.**

***Index Terms*—Neuro-Fuzzy controller, Classical PID controller, Lissajous trajectory tracking**

I. Introduction

In recent decades, Ball plate systems have been challenging in terms of controlling the position of the ball and tracking with the desired trajectory. The system equations are solved using the dynamic equations derived from the analysis of the non-linear model.

[1] makes use of a linear model with small angle approximations and assumes the derivative of the plate angle to be zero. A PID controller is used while taking into account noise and disturbances whereas [2] compares the result of classical PD controller and PD-tuned fuzzy controller concluding that the PD-tuned fuzzy controller produces better results which are proved in [3] where the gains of the PID controller are adjusted using fuzzy control.[4] compares classical PID controllers and fuzzy-tuned PID controllers.

In [5], a complete mathematical model of the ball and plate system without ignoring cross-coupling is used, leading to high-precision tracking performance. [6] uses a state observer to devise a state feedback controller for a linear system. Compensators are also used for higher frequency bandwidth operation which allows a higher frequency of disturbances or demanded trajectory and a lower tendency for damping and phase delay of the controlled variable.

[7] considers the system using the input-output feedback linearization method which is used to design the control law to globally asymptotically stabilize the defined output to zero. [8] considers the state feedback controller on the Luenberger observer. The linear model of the ball and plate and a linear observer is used. The choice of observer gains within the stability region has been difficult. In [9] ball and plate system is designed using the H∞ control theory, backstepping method, and Lyapunov stability theory.[10] uses a 4 DOF linear model implemented using an LQR controller which is simple in programming and guarantees the system converges asymptotically. A model-free adaptive dynamic programming approach is used in [11], it approximates an arbitrary reference trajectory so that it is compatible with the underlying control system. In [12] non-linear BPS is used with 2 DOF, TS fuzzy is used for state spacing and modeling and a full-state feedback controller is implemented using a linear quadratic regulator (LQR) whereas [13] also uses LQR based along with joint space controller for desired trajectory tracking.

[14] uses a backstepping and cascaded controller and results are compared. In cascaded a non-linear model is implemented using two controllers and independently for both x and y directions and similar procedures are used in the backstepping controllers along with Lyapunov stability theory.

The main disadvantage of PID is the selection of the best values for its parameters using traditional methods that do not achieve the best response. It takes time to settle and bypass the error between the desired and actual response. High starting overshoot, oscillations, and sensitivity to controller gains. The objective of tuning is to get the desired membership functions and the tuning of fuzzy is simplified using a neural network.

The Fuzzy controller demonstrates better performance compared to a traditional PID controller. In setpoint tracking, it proved to adapt better to changes in the reference. On the other hand, the Fuzzy controller presented a better response to changes in the dynamics of the system, as well as better disturbance rejection. The design of the Fuzzy logic proved to be more simplified than the design of the PID control since the former requires no mathematical model.

The adjustment of membership functions in fuzzy requires expert knowledge for the necessary information to determine the values of the system. In the case of fuzzy, there may not be a solid knowledge base for a specific case, that allows the best answer. Knowledge of numerous algorithms is required apart from trial and error.

Neural fuzzy combines the learning capacity of an artificial neural net with the computation of linguistic variables for FIS. These combined components can adjust the rule base of the fuzzy system and produce very less error.

A fuzzy system is sensitive to noise mainly for low amplitudes of the input signal. In neuro-fuzzy, there is less impact even with a lower signal-to-noise ratio. Whenever there is disturbance from the original path, there is less deviation from the original path in the case of neuro-fuzzy systems. The comparison of the responses obtained by simulations shows advantages in favor of the neuro-fuzzy controller, even considering the presence of noise generated in the sensors and the occurrence of eventual external disturbances.

This paper is divided into 4 major sections hereafter. Section II. describes the dynamics and modeling of the ball-plate system using Euler-Lagrange equations. Section III. covers the design of a Fuzzy controller and transitions into the design of a Neuro-Fuzzy controller. Section IV. discusses the approach using a classical PID controller and then a combination of the two. The results are discussed in Section V. and conclusions are drawn.

II. Modeling

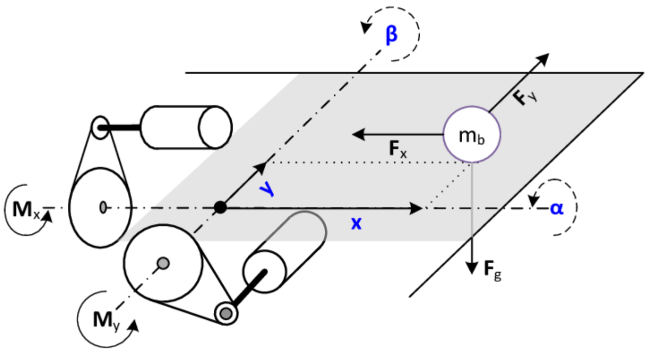

Fig. 1: Depiction of Ball and Plate system [15].

A. System Equations

We can use Lagrangian mechanics to derive the equations of motion of the system:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right)-\frac{\partial L}{\partial q_i}=Q_i \tag{1}$$

Where the Lagrangian is defined as:

$$T=T_b+T_p \tag{2}$$

$$T_b=\frac{1}{2}m_b\left(\dot{x}_b^2+\dot{y}_b^2\right)+\frac{1}{2}I_b\left(w_x^2+w_y^2\right) \tag{3}$$

Here, we can replace the values of w_x and w_y as $\dot{x}_b = w_x r_b$ and $\dot{y}_b = w_y r_b$.

$$T_p=\frac{1}{2}\left(I_b+I_p\right)\left(\dot{\alpha}^2+\dot{\beta}^2\right)+\frac{1}{2}m_b(x_b\dot{\alpha}+y_b\dot{\alpha})^2 \tag{4}$$

$$V=V_b=m_bgh=m_bg\left(x_b sin\alpha+y_b sin\beta\right) \tag{5}$$

$$L=T-V=T_b+T_p-V_b \tag{6}$$

Using equations 1-6 we can derive the dynamical equations of the system.Using the x and y positions and velocities as the state variables, we obtain the state-space representation as:

$$X=\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix}=\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}\quad;\ \dot{X}=\begin{bmatrix}\dot{x}\\ \frac{x_b\dot{\alpha}^2+y_b\dot{\alpha}\dot{\beta}-gsin\alpha}{C_1}\\ \dot{y}\\ \frac{y_b\dot{\beta}^2+x_b\dot{\alpha}\dot{\beta}-gsin\beta}{C_1}\end{bmatrix} \tag{7}$$

where, $C_1=1+\frac{I_b}{m_b r_b^2}\approx\frac{7}{5}$. We consider the approximation $\dot{\alpha}\approx\dot{\beta}\approx 0$ to linearize the system.The resulting linear system is then characterised by:

$$\dot{X}=AX+BU \tag{8}$$

$$A=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}B=\begin{bmatrix}0&1\\-7&0\\0&0\\0&-7\end{bmatrix}U=\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \tag{9}$$

III. Fuzzy Controller Design

A. Architecture

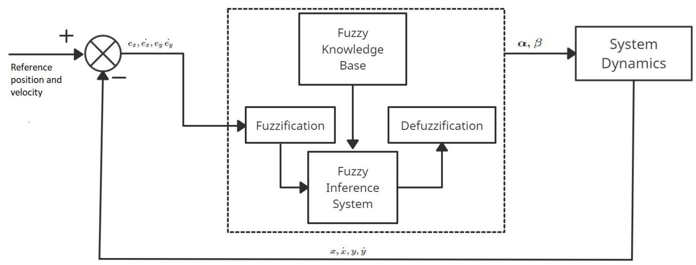

Fig. 2: Input membership function for error in position.

We have considered a fuzzy logic controller based on the Mamdani inference system. The position and velocity errors are used as inputs to the controller, which outputs the plate angles α and β to be fed to the system for tracking the required trajectory.

B. Fuzzification

5 linguistic variables: Large Negative (LN), Negative (N), Zero (Z), Positive (P), and Large Positive (LP) were used for all 4 inputs to the fuzzy controller. Gaussian membership functions were used for fuzzification.

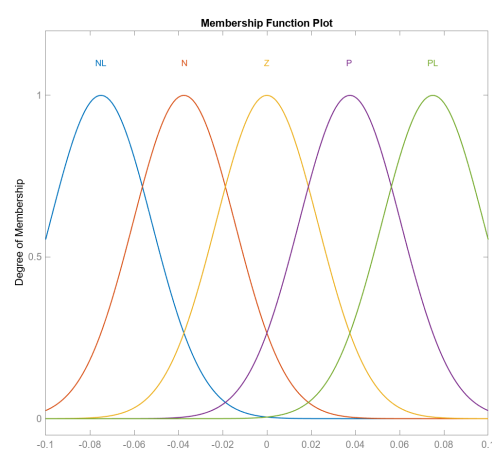

Fig. 3: Input membership function for error in position.

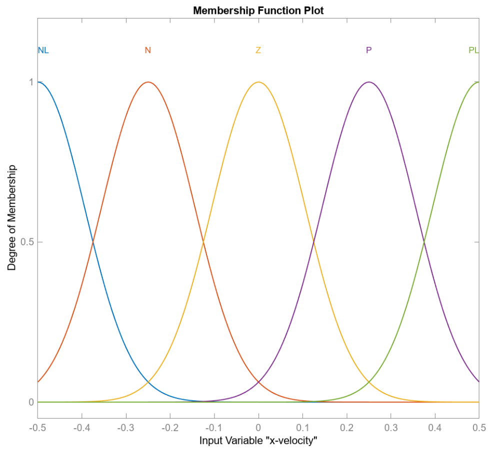

Fig. 4: Input membership function for error in velocity

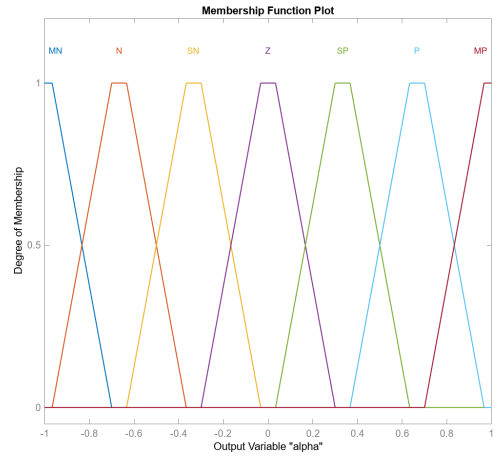

Fig. 6: Output membership function for plate angle

C. Rule Base

We created a rule base consisting of 50 rules, based on intuition. We are treating the x and y control independently: α controls the x position and velocity, and β controls the y position and velocity. So we get 25 rules for each control input, based on the 5×5 combinations possible from 5 linguistic variables.

Error (velocity) → Error (position) ↓	NL	N	Z	P	PL
NL	MN	MN	N	SN	Z
N	MN	N	SN	Z	SP
Z	N	SN	Z	SP	P
P	SN	Z	SP	P	MP
PL	Z	SP	P	MP	MP

Fig. 5: Fuzzy Rule Base Table

D. Output Membership Functions,Inference and Defuzzification

The trapezoidal membership Function was used as the output membership function, and we considered 7 linguistic variables for both angles. Major Negative (MN), Negative (N), Small Negative (SN), Zero (Z), Small Positive (SP), Positive (P), Major Positive (MP). Min composition was used, which was then followed by the centroid method for defuzzification, which gave the 2 control inputs α and β.

E. Neuro-Fuzzy Controller Design

We now consider the design of a neuro-fuzzy controller for trajectory tracking.The neuro-fuzzy controller is more adaptive than a simple fuzzy controller, as it contains parameters which can be learned easily depending on the trajectory. We consider a 2 layer neural network with a tansigmoid nonlinearity at the output layer. Based on the outputs of the gaussian membership functions, we have created a weight vector W. For control in the x direction, the elements of W are the combinations

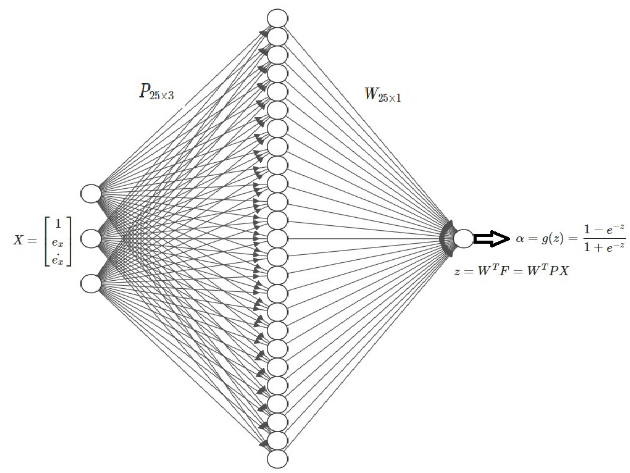

Fig. 7: Neuro Fuzzy Architecture

(product) of the two membership functions for x position (μ) and velocity(ϕ). The elements are then normalized to restrict their range to $[0, 1]$.

$$w_{5(j-1)+k} = \mu_j \phi_k \bar{w_i} = \frac{w_i}{\Sigma w_i} \tag{10}$$

The control input alpha, for the control in x direction is then defined as:

$$z = \Sigma_i w_i f_i = W' F \tag{11}$$

$$\alpha = g(z) = \frac{1 - e^{-x}}{1 + e^{-x}} \tag{12}$$

Where the matrices W and F are defined as:

$$W = \begin{bmatrix} \bar{w_i} \\ \bar{w_2} \\ .. \\ \bar{w_{25}} \end{bmatrix} ; F = \begin{bmatrix} f_1 \\ f_2 \\ .. \\ f_{25} \end{bmatrix} = PX \tag{13}$$

Where the matrix X contains the inputs to the fuzzy logic controller responsible for control in the x direction (position and velocity error), and the matrix P is the parameter matrix:

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ .. & .. & .. \\ p_{25,1} & p_{25,2} & p_{25,3} \end{bmatrix}; X = \begin{bmatrix} 1 \\ e_x \\ \dot{e_x} \end{bmatrix} \tag{14}$$

We need to adjust the values of this parameter matrix using backpropagation, as part of the training phase. We define a suitable loss function based on the squared error (SE):

$$L(x, \dot{x}) = \frac{1}{2}(x_d - x)^2 + \frac{1}{2}(\dot{x_d} - \dot{x})^2 \tag{15}$$

The parameters are then updated using gradient descent with momentum, using a suitable learning rate and momentum parameter:

$$dw_t = -\eta \frac{\partial L}{\partial p_{ij}} \tag{16}$$

$$W_{t+1} = W_t + \gamma dw_t + (1 - \gamma) dw_{t-1} \tag{17}$$

where t denotes the time-step. Similar expressions for β are used for control in the y direction.It must be reiterated that control in the x and y directions are being treated separately with no coupling.

IV. Design of Neuro-Fuzzy-PID Controller

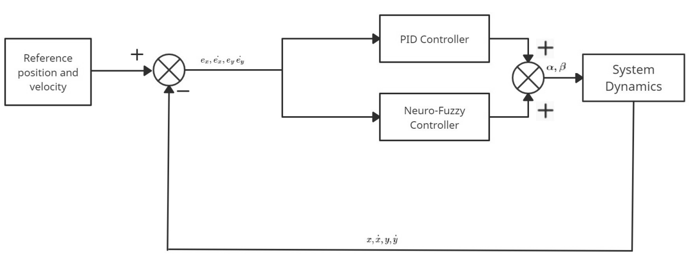

Fig. 8: System Diagram for combined controller

The Neuro-Fuzzy controller discussed in the previous section still yields a slight steady-state error.A PI controller was used in parallel with the Neuro-Fuzzy controller to drive the steady state error to 0. The control inputs from both controllers ($\alpha_{nf}, \alpha_{pid}$) are weighted using a parameter k:

$$\alpha_{pid} = K_p e_x + K_i \int_0^t e_x dt \tag{18}$$

$$\alpha = k\alpha_{nf} + (2 - k)\alpha_{pid} \tag{19}$$

V. Results Comparison and Discussion

A. Fuzzy Controller

Fig 9 and 10 show the response of the system when a fuzzy logic controller is used. The ball settles quickly on the trajectory path, without significant steady state error.The control inputs are of the order 0.1 radians, with the maximum value of 0.32 at the initial instant.This means the system can be controlled without significant control effort.

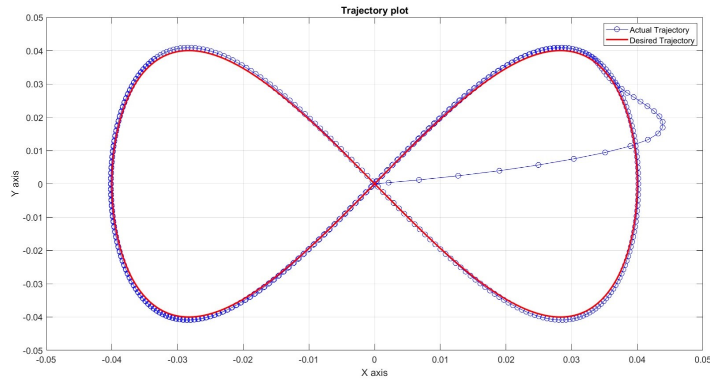

Fig. 9: Actual and desired trajectory of the ball using Fuzzy controller.

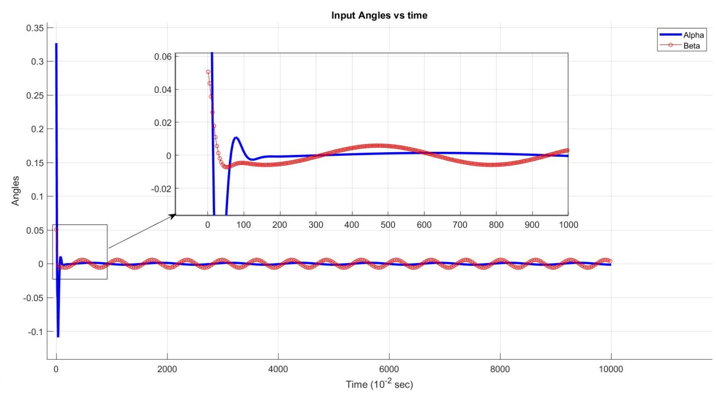

Fig. 10: Input angles to plates versus time in Fuzzy Controller

B. Neuro-Fuzzy Controller

As seen in figure 11, there is significant steady state error. The control input required has been decreased, and is now of the order 0.005 radians, with a maximum value of 0.03 radians.

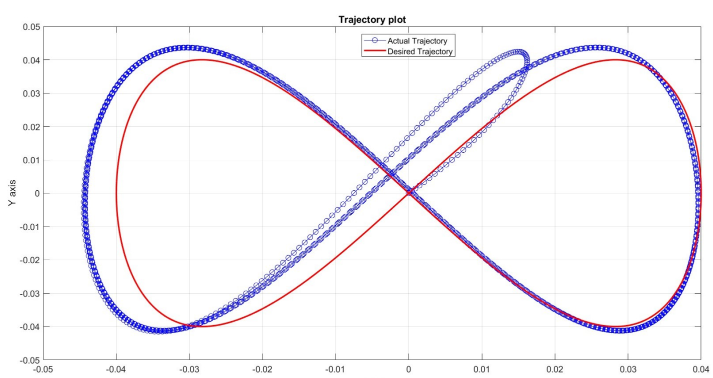

Fig. 11: Actual and desired trajectory of the ball using Neuro-Fuzzy controller.

C. Classical PID Controller

Figure 13 shows the output when a PID controller is used. Both the settling time and the steady state error have been

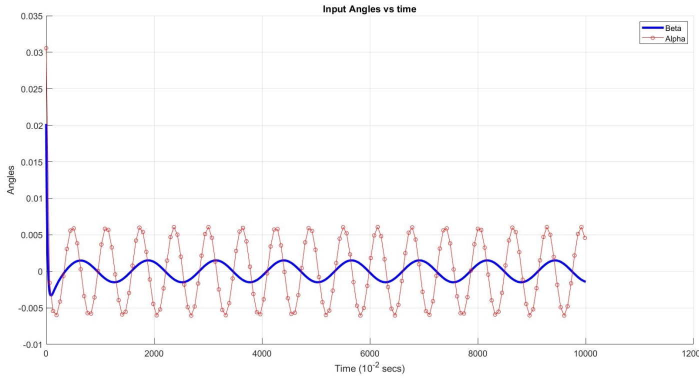

Fig. 12: Input angles to plates versus time in Neuro-Fuzzy Controller

reduced significantly, but at the cost of increased requirement of control input, as depicted by figure 14.

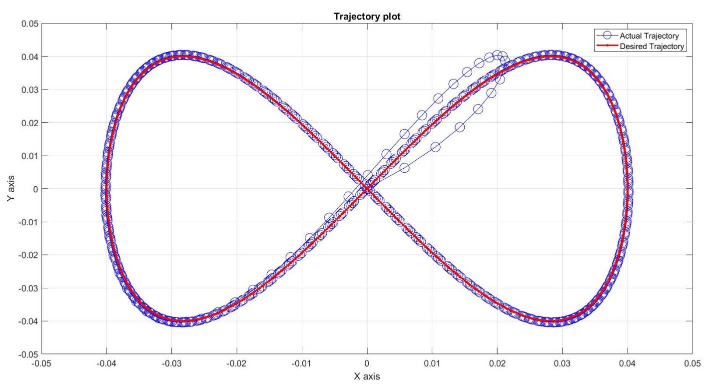

Fig. 13: Actual and desired trajectory of the ball using PID controller.

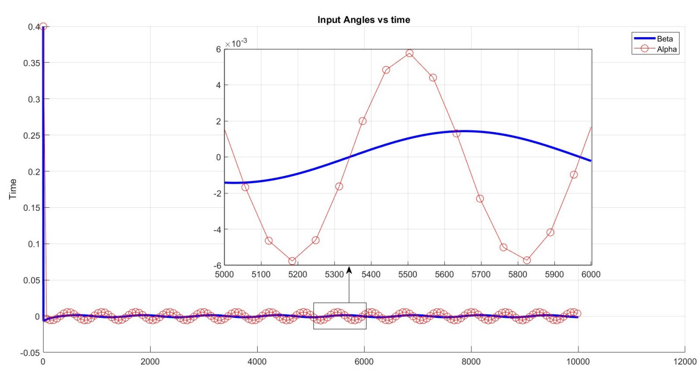

Fig. 14: Input angles to plates versus time in PID Controller

D. Combined NeuroFuzzy and PID Controller

The PID controller combined with the Neuro-Fuzzy controller helps in driving the steady-state error to zero as shown in figure 15. This controller combines the aggressive control offered by the PID controller with the robustness of a neural

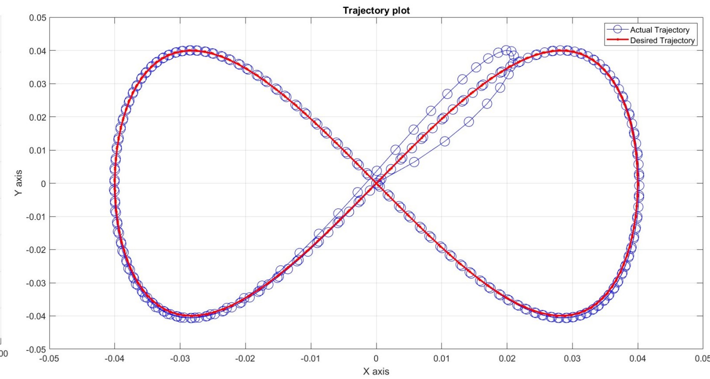

Fig. 15: Actual and desired trajectory of the ball using Neuro-Fuzzy-PID controller.

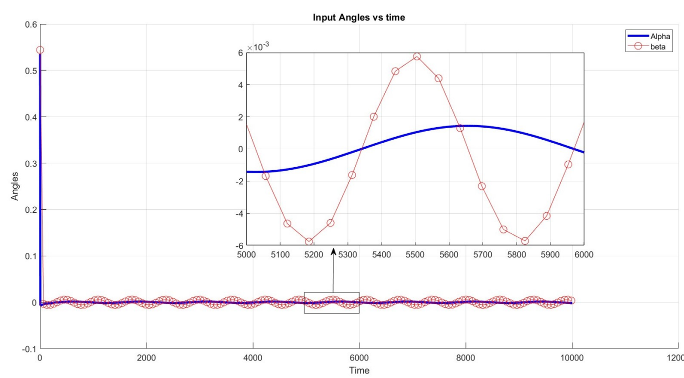

Fig. 16: Input angles to plates versus time in Neuro-Fuzzy-PID Controller

network based controller. To further validate the robustness of the Neuro-Fuzzy combined with PID controller as compared to a simple PID controller, we tested both controllers on another, more complex Lissajous trajectory. The results have been tabulated in figure 19.

E. Conclusion

Considering performance aspects as well as robustness, sensitivity to change in system and environment dynamics and noise, it is evident that the Neuro-Fuzzy with PID controller provides the best results in tracking a complex trajectory.

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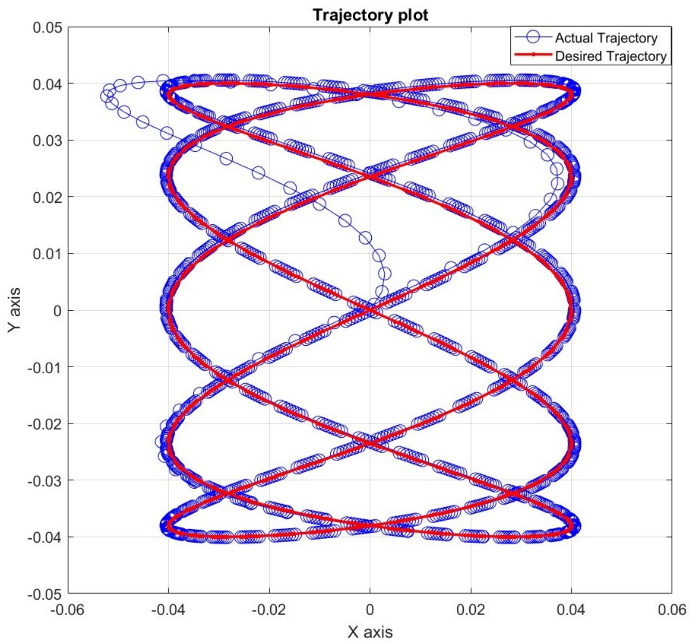

Fig. 17: Actual and desired complex trajectory of the ball using Neuro-Fuzzy-PID controller.

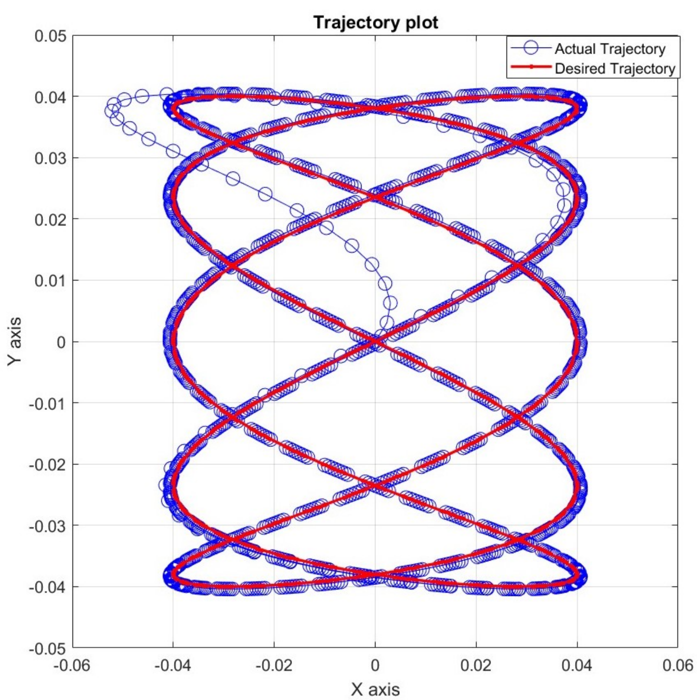

Fig. 18: Actual and desired complex trajectory of the ball using PID controller.

Controller	RMS error (position)	RMS error (velocity)
Fuzzy	0.0012	0.0054
Neuro Fuzzy	0.0050	0.0042
PID	0.0020	0.0021
Neuro Fuzzy + PID	0.0020	0.0020
PID (Complex Trajectory)	0.0022	0.0027
Neuro Fuzzy + PID (Complex Trajectory)	0.0021	0.0024

Fig. 19: Table for RMS error in various controllers.

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