* periodic function.

if $\exists$ a λ s.t $f(x+\lambda) = f(x)$, $\forall x$.

Ex:- $\sin x \rightarrow$ P. $\longrightarrow$ $f(x+2\pi) = f(x)$ for $\lambda = 2\pi$

$f(x) = x \rightarrow$ N.P.

beacuse $f(x+\lambda) \neq f(x)$

$x+1 \neq x$. for $\lambda = 1$.

1) $\int_{-\pi}^{\pi} \sin^2 nx \cdot dx$.

$= \int_{-\pi}^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2nx$.

$\frac{1}{2}(2\pi) - \frac{1}{4n} \cancel{\sin 2xn}^{\to 0} \Big|_{-\pi}^{\pi}$

$\underline{\pi}$

* $\int_{-\pi}^{\pi} f(x) \cdot g(x) = 0$ if $f \neq g$

$\neq 0$ if $f = g$.

* $\{1, \sin x, \cos x, \sin 2x, \cos 2x \ldots\ldots\}$

$\downarrow$

Taking any two fncs and product will get 0 then it is called "orthogonal function".

* The fourier Series of a function $f(x)$ on $-\pi < x < \pi$ is.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

where, $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot dx$.

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \cdot dx$.

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \cdot dx$.

Euler's formula.

Ex:- $f(x) = x \quad ; -\pi < x < \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot dx = \frac{1}{2\pi}(\pi^2 - \pi^2) = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \cos nx \cdot dx$$

$$= \frac{1}{\pi}\left[\frac{x}{n}\sin nx \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\sin nx}{n} \right]$$

$$= \frac{-1}{\pi}\left[0 + \frac{1}{n^2}\cos nx \Big|_{-\pi}^{\pi} \right]$$

$$= \frac{-1}{\pi}\left[\frac{0}{n^2}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin nx \cdot dx$$

$\cos n\pi = (\cos(-n\pi))$

$$\left.\begin{aligned} &= \frac{1}{\pi}\left[-\frac{x}{n}\cos nx \Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \frac{\cos nx}{n} \right] \\ &= \frac{1}{\pi}\left[-\frac{x}{n}(0) + \frac{1}{n^2}\sin nx \Big|_{-\pi}^{\pi} \right] \\ &= \frac{1}{\pi}\left[\frac{1}{n^2}[2\sin n\pi]\right] \end{aligned}\right\} \times$$

Here limits should be changed.

$$= \frac{2}{n^2}\sin n\pi$$

$$\frac{1}{\pi}(2)\int_{0}^{\pi} x \cdot \sin nx \cdot dx$$

$$\frac{2}{\pi}\left[\frac{-x\cos nx}{n}\Big|_{0}^{\pi} + \int_{0}^{\pi} \frac{\cos nx}{n} \right]$$

$$\frac{2}{\pi}\left[-\frac{\pi}{n}\cos n\pi + \cancel{\frac{1}{n^2}\sin nx}^{\,0}\Big|_{0}^{\pi} \right]$$

$$= \frac{-2}{n}\cos n\pi \qquad \cos n\pi = (-1)^n$$

$f(x) = \frac{0}{2} + \sum_{n=1} 0 + \sum_{n=1} \left(-\frac{2}{n}\cos n\pi\right)(\sin nx)$

$= \sum_{n=1}^{\infty} -\frac{2}{n} \sin nx \, (-1)^n.$

$= -\frac{2}{1}(-1)\sin(1x) + \frac{-2}{2}(1)\sin 2x + \cdots$

$= 2\sin x - \sin 2x + \frac{2}{3}\sin 3x - \frac{1}{2}\sin 4x + \frac{2}{5}\sin 5x + \cdots$

2) $f(x) = x^2 \; ; \; -\pi < x < \pi$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 . dx = \frac{1}{3\pi}(\pi^3 + \pi^3) = \underline{\frac{2}{3}\pi^2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx . dx$

$= \frac{1}{\pi}\left[x^2 \frac{\sin nx}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2x}{n}\sin nx . dx \right]$

$= \frac{1}{\pi}\left[\frac{x^2}{n}[2\sin n\pi] - \frac{2\pi}{n}\left[-\frac{2}{n}\cos n\pi\right]\right]$

$= \frac{1}{\cancel{\pi}}\left[\frac{4\cancel{\pi}}{n^2}\cos n\pi\right] = \underline{\frac{4}{n^2}\cos n\pi}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx . dx$

$= \frac{1}{\pi}\left[\cancel{-\frac{x^2 \cos nx}{n}}^{\,0}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} 2x . \frac{\cos nx}{n}\right]$

$= \frac{1}{\pi}\left[-\frac{x^2}{n}\cancel{(2\cos n\pi)}^{\,0} + \frac{2}{n}(0)\right]$

$\underline{= 0}$

$= \frac{\frac{2}{3}\pi^2}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos n\pi . \cos nx + \sum_{n=1}^{\infty} 0.$

$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos n\pi . \cos nx$

$x^2 = \frac{\pi^2}{3} - 4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \frac{4}{16}\cos 4x + \dots$

for $x=0$ $\quad \frac{\pi^2}{3} - 4\left[1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots\right]$

$\cancel{\frac{\pi^2}{3} - 4\left[1 - \frac{1}{4}\right.}$

$0 = \frac{\pi^2}{3} - 4\underbrace{\left[1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots\right]}_{K} \rightarrow (i)$

$\boxed{K = \frac{\pi^2}{12}}$

If $\cancel{x = \pi}$

$\cancel{\pi^2 = \frac{\pi^2}{3} + 4\left[(-1) + \frac{1}{4} + \frac{1}{9} - \frac{1}{16}\right.}$

$\cancel{0 = \frac{\pi^2}{6} + \dots}$

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{\pi^2}{6} \rightarrow 2.$

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$1 \cancel{+ 2} =$

If $x = \pi$

$\frac{2\pi^2}{3} = -4\left[\cos\pi - \frac{\cos 2\pi}{4} + \frac{1}{9}\cos 3\pi + \dots\right]$

$\frac{\pi^2}{6} = \left[1 + \frac{1}{4} + \frac{1}{9} + \dots\right] \rightarrow 2$

$1+2$

$= 1 + \frac{1}{9} + \frac{1}{25} + \dots = \left(\frac{\pi^2}{12} + \frac{\pi^2}{6}\right)\frac{1}{2}$

$\Rightarrow 1 + \frac{1}{9} + \frac{1}{25} + \dots = \frac{\pi^2}{8} \rightarrow 3$

* put $z = \frac{\pi x}{l}$. when $x = -l$, $z = -\pi$; $x = l$, $z = \pi$. $\quad dz = \frac{\pi}{l}\,dx$.

$x = \frac{zl}{\pi}$

$F(z)$ is periodic over $(-\pi, \pi)$ and having period 2π.

The fourier series of $F(z)$ is.

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nz + \sum_{n=1}^{\infty} b_n \sin nz.$$

where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\cdot dz$. $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\cos nz \cdot dz$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\cdot \sin nz \, dz$

$a_0 = \frac{1}{\pi}\int_{-l}^{l} F\left(\frac{\pi x}{l}\right)\cdot \frac{\pi}{l}\cdot dx$. $\underline{a_n} = \frac{1}{l}\int_{-l}^{l} F\left(\frac{\pi x}{l}\right)\cos\left(\frac{n\pi}{l}x\right)\cdot dx$

$a_0 = \frac{1}{l}\int_{-l}^{l} F\left(\frac{\pi x}{l}\right)\cdot dx$ $\underline{b_n} = \frac{1}{l}\int_{-l}^{l} F\left(\frac{\pi x}{l}\right)\sin\left(\frac{n\pi}{l}x\right)\cdot dx$

$\underline{a_0} = \frac{1}{l}\int_{-l}^{l} F\left(\frac{\pi x}{l}\right)\cdot dx$. $F\left(\frac{\pi x}{l}\right) = f(x)$

F. Series

* $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$

$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\cdot dx$

$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi}{l}x\right)\cdot dx$.

$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot \sin\left(\frac{n\pi}{l}\cdot x\right)\cdot dx$

* If $f(x)$ is even

$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$ where $b_n = 0$.

* If $f(x)$ is odd

$\cancel{\frac{a_0}{2}}^{0} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$ where $a_n = 0$.
$a_0 = 0$

Bernoulli's formula

$v_1 = \int v \cdot dx$.

$u' = \frac{du}{dx}$

$\int uv \cdot dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$.

$\int e^{ax} \cos bx \cdot dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$.

* $f(x) = x \quad (-l, l)$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \cdot dx$.

$= \frac{1}{l} \int_{-l}^{l} x \cdot dx = \frac{1}{2l} (l^2 - (-l)^2) = 0$

$b_n = \frac{1}{l} \int_{-l}^{l} x \cdot \sin n x \cdot dx$.

$= \frac{1}{l} \left[-x \cos n x \Big|_{-l}^{l} + l \frac{\sin n \frac{\pi x}{l}}{n\pi} \Big|_{-l}^{l} \right]$

$= \frac{1}{l} \left[-l \cos n l + l \cos n l + \frac{2l}{n\pi} \sin \frac{n l}{l} \right]$

$= \frac{l}{\pi} \times \frac{2}{n l} \sin n \pi x$.

$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \sin n \pi x$.

$x = \frac{2}{l} \left[\frac{\sin \pi x}{1} + \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} + \ldots \right]$

$$b_n = \frac{1}{l}\int_{-l}^{l} x\cdot\sin\left(\frac{n\pi}{l}x\right)\cdot dx.$$

$$= \frac{1}{l}\left[\frac{-x\cdot\cos\frac{n\pi}{l}x}{\frac{n\pi}{l}}\Bigg|_{-l}^{l} + \int_{-l}^{l}\frac{\cos\frac{n\pi}{l}x}{\frac{n\pi}{l}}\,dx\right]$$

$$= \frac{1}{n\pi}\left[-\left[l\cos n\pi - (-l\cos n\pi)\right] + \cancel{\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}}\Bigg|_{-l}^{l}\,dx\right]$$

$$= \frac{1}{n\pi}\left[-2l\cos n\pi l\right]$$

$$= -\frac{2l}{n\pi}(-1)^n$$

$$x = \sum_{n=1}^{\infty} -\frac{2l}{n\pi}(-1)^n\cdot\sin\frac{n\pi}{l}x.$$

$$x = \left[+\frac{2l}{\pi}\sin\frac{\pi}{l} - \frac{2l}{2\pi}\sin\frac{2\pi}{l} + \frac{2l}{3\pi}\sin\frac{3\pi}{l}\right]$$

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* **Theorem** :- If $f(x)$ is periodic and piecewise smooth over $(-c, c)$, then the F.S of a function converges to $f(x)$ for each continuity and to $\frac{f(x+0)+f(x-0)}{2}$ at each point of jump discontinuity

1) Find F.S. of

$$f(x) = \begin{cases} -\pi & -\pi \le x < 0 \\ x & 0 < x \le \pi \end{cases}$$

$$\frac{f(0^+)+f(0^-)}{2} = -\pi/2$$

at $x=0$ $\cancel{\frac{f(0+0)-f(0-0)}{2}}$

$f(0^-) = f(0-0) = -\pi$

$$f(0+0) = \lim_{h\to 0^+} f(h) = 0.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot dx$$

$$\frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cdot dx + \int_{0}^{\pi} f(x)\cdot dx\right]$$

$$= \frac{1}{\pi}\left[\cancel{f(0)} \int_{-\pi}^{0} -\pi\cdot dx + \int_{0}^{\pi} x\cdot dx\right]$$

$$= \frac{1}{\pi}\left[-\pi(\pi) + \frac{\pi^2}{2}\right] = -\frac{\pi}{2} \,//$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\cdot dx.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\cos nx\cdot dx + \int_{0}^{\pi} x\cdot\cos nx\cdot dx\right]$$

$$= 0 \pm \frac{1}{\pi}\cancel{[} \qquad \int_{0}^{\pi} (\pi - x)\cos \cancel{n(\pi} - x)$$

$$= \frac{1}{\pi}\left[\int_{0}^{\pi} \pi\cos nx\cdot dx + \frac{x\sin nx}{n}\Big|_0^{\pi} + \frac{\cos nx}{n^2}\Big|_0^{\pi}\right]$$

$$\frac{\pi\sin nx}{n}$$

$$= \frac{1}{\pi}\left[\cancel{\pi(-1)^n - \pi}^{\,0} + \frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right]$$

$$= \cancel{\frac{1}{\pi}\left[(-1)^n\left(\pi + \frac{1}{n^2}\right) - 1\left(\pi + \frac{1}{n^2}\right)\right]}$$

$$= \cancel{\frac{1}{\pi}\left(\pi + \frac{1}{n^2}\right)\left[(-1)^n - 1\right]}$$

$$\frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\cdot dx.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\sin nx + \int_{0}^{\pi} x\sin nx\cdot dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi\cos nx}{n}\Big|_{-\pi}^{0} + \frac{-x\cos nx}{n}\Big|_{0}^{\pi} + \frac{x}{n^2}\sin nx\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\pi - \frac{\pi}{n}(-1)^n - \frac{\pi}{n}(-1)^n\right]$$

$$= \left[1 - \frac{2}{n}(-1)^n\right]$$

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Half ranged sine series

for $g(x)$ is odd fnc.

$$g(x) = \begin{cases} -f(-x) & -l \le x \le 0 \\ f(x) & 0 \le x \le l \end{cases}$$

$$g(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

where $b_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot\sin\frac{n\pi x}{l}$

$$g(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}, \quad \text{where } b_n = \frac{2}{l}\int_{0}^{l} f(x)\cdot\sin\frac{n\pi x}{l}$$

Half ranged cosine series

Now, $g(x)$ is even fnc.

$$g(x) = \begin{cases} f(x) & x\in[0,l] \\ f(-x) & x\in[-l,0] \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{l}.$$

where, $a_0 = \frac{2}{l}\int_{0}^{l} f(x)\cdot dx.$

$$a_n = \frac{2}{l}\int_{0}^{l} f(x)\cos\frac{n\pi x}{l}\cdot dx.$$

Ex:- Find half range sine & cosine of $f(x)=x$ in $[0, l]$.

Sol:- H.R.S

$$b_n = \sum_{n=1}^{\infty} \frac{2}{l}\left[\int_0^l x \cdot \sin\frac{n\pi x}{l}\right] \cdot \sin\frac{n\pi x}{l} \cdot dx.$$

$$\sum_{n=1}^{\infty} \frac{-2l}{n\pi}(-1)^n \sin\frac{n\pi x}{l}$$

$$\downarrow$$

$$\frac{-2x \cos\frac{n\pi x}{l}}{\frac{n\pi}{l} \times l} + \int \frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}} \Rightarrow +\frac{2l}{\pi}$$

$$\frac{-2xl}{n\pi l} \cos\frac{n\pi x}{l}\Big|_0^l + \left(\frac{l}{n\pi}\right)^2 \sin\frac{n\pi x}{l}\Big|_0^l$$

$$\frac{-2l}{n\pi}(-1)^n + \cancel{\left(\frac{l}{n\pi}\right)^2 [0]} \qquad \cancel{\sum_{n=1}^{\infty} \left(\frac{2l}{n\pi}\right)(-1)^n = \sum_{n=1}}$$

H.R.C

$$\frac{a_0}{2} = \frac{1}{2} \cdot \frac{2}{l} \int_0^l x \cdot dx = \frac{1}{2} l.$$

$$a_n = \frac{2}{l} \int_0^l x \cdot \cos\frac{n\pi x}{l} \cdot dx.$$

$$= \frac{2x}{l}\left[\frac{l}{n\pi} \sin\frac{n\pi x}{l}\right]_0^l + \frac{2}{l}\left(\frac{l}{n\pi}\right)^2 \cos\frac{n\pi x}{l}\Big|_0^l.$$

$$= \frac{2l}{(n\pi)^2}\left[\cos n\pi - 1\right]$$

$$b_n = \sum_{n=1}^{\infty} \frac{-2l}{n\pi}(-1)^n \sin\frac{n\pi x}{l}$$

$$b_1 = \frac{2l}{\pi} \quad b_2 = \frac{-2l}{2\pi} \quad b_3 = \frac{2l}{3\pi} \ldots \qquad \boxed{l = \pi}$$

$$f(x) = l \times \frac{2}{\pi}\left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\right]$$

$$x = 2\left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \dots\right]$$

take $x = \frac{\pi}{2}$ $\qquad \frac{\pi}{4} = \left[1 - \frac{1}{3} + \frac{1}{5} + \dots\right]$ //.

for,

$$a_n = \frac{2l}{(n\pi)^2}[\cos n\pi - 1].$$

$l \in [0, \pi]$.

$$\sum_{n=1}^{\infty} \frac{2l}{(n\pi)^2}[\cos n\pi - 1] \cdot \cos \frac{n\pi x}{l}$$

$l = \pi$

$$\sum_{n=1}^{\infty} \frac{2}{n^2\pi}[\cos n\pi - 1] \cos nx.$$

$$\sum_{n=1}^{\infty} \frac{2}{n^2\pi}[(-1)^n - 1] \cos nx. \qquad a_0 = \frac{\pi}{2}$$

$$a_1 = \frac{-4}{\pi} \quad a_2 = 0 \quad a_3 = \frac{-4}{3^2\pi} \dots$$

$$x = \frac{\pi}{2} + \left[-\frac{4}{\pi}\cos x - \frac{4}{3^2\pi}\cos 3x + \dots\right.$$

$$x = \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right]$$

$\boxed{x = \pi}$

$$\pi - \frac{\pi}{2} = +\frac{4}{\pi}\left[1 + \frac{1}{9} + \frac{1}{25} + \dots\right.$$

$$\frac{8\pi}{2} = \frac{4}{\pi}\left[1 + \frac{1}{9} + \frac{1}{25} + \dots\right.$$

$$\frac{8\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25}$$ //

* Find halfrange sine series of $f(x) = x(\pi - x)$ on $(0,\pi)$ & deduct.

Sol:- H.R.S.

$$b_n = \frac{2}{l}\int_0^l x(\pi - x)\cdot \sin\frac{n\pi x}{l}\,dx$$

$$\cancel{\frac{2}{l}\pi} \quad -\frac{2l}{n\pi}(-1)^n - \frac{2}{l}\int_0^l x^2 \sin\frac{n\pi x}{l}\,dx$$

$$-\frac{2l}{n\pi}(-1)^n - \frac{2}{l}\left[-x^2\frac{l}{n\pi}\cos\frac{n\pi x}{l}\Big|_0^l + \frac{l}{n\pi}\int_0^l 2x\cos\frac{n\pi x}{l}\right]$$

$$-\frac{2l}{n\pi}(-1)^n - \frac{2}{l}\left[-\frac{l^3}{n\pi}(-1)^n + \frac{2l}{n\pi}\left[\frac{l^2}{2n\pi}[\cos n\pi - 1]\right]\right]$$

$$\Rightarrow -\frac{2l}{n\pi}(-1)^n + \frac{2l^2}{n\pi}(-1)^n - \frac{2L^2}{(n\pi)^2}[\cancel{\cos}\,)^n - 1].$$

$l = \pi$.

$$b_n = \frac{-2}{n}(-1)^n + \frac{2\pi}{n}(-1)^n - \frac{2}{n^2}[(-1)^n - 1]$$

$$\frac{2}{n}(\pi - 1)(-1)^n.$$

$$b_1 = \cancel{\frac{2}{\pi} - 2\pi + 4} \quad 2(\pi - 1) + 4$$

$$b_2 = \frac{2}{2}(\pi - 1)$$

$$b_3 = \frac{2}{3}(\pi - 1) + \frac{4}{3^2}$$

$$[2(\pi - 1) + 4]$$

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Find a_0, a_n, b_n.

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ \sin x & 0 \le x \le \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_0^\pi \sin x \cdot dx = -\frac{1}{\pi}[-1-1] = \frac{2}{\pi}$$

$$b_n = \frac{1}{\pi}\int_0^\pi \sin x \cdot \sin nx \cdot dx.$$

$$b_1 = \frac{1}{\pi}\int_0^\pi \sin^2 x \cdot dx = \frac{1}{2\pi}\int_0^\pi \cos(1-n)x - \cos(1+n)x \cdot dx$$

$$b_1 = \frac{1}{2\pi}\left[\pi - \frac{\sin 2x}{2}\Big|_0^\pi\right] \qquad \frac{\sin(1-n)x}{2\pi(1-n)}\Big|_0^\pi - \frac{\sin(1+n)x}{(1+n)2\pi}\Big|_0^\pi$$

$$b_1 = \frac{1}{2\pi}[\pi] = \frac{1}{2}$$

$\frac{1}{2}\left(\frac{1}{2}(\pi)\right) - \frac{1}{2}\sin$

$\frac{1}{2} - \frac{\cos 2x}{2}$

$$b_1 = -\frac{\sin 2x}{4\pi}\Big|_0^\pi$$

$$a_n = \frac{1}{\pi}\int_0^\pi \sin x \cos nx \cdot dx.$$

$$= \frac{1}{2\pi}\left[\sin(n+1)x + \sin(1-n)x \cdot dx.\right.$$

$$= -\frac{\cos(n+1)x}{2\pi}\Big|_0^\pi + -\frac{\cos(n-1)x}{2\pi}\Big|_0^\pi$$

$$= -\frac{1}{2\pi}\left[\frac{\cos(n+1)\pi}{n+1} - \frac{1}{n+1}\right] - \frac{1}{2\pi}\left[\frac{\cos(n-1)\pi}{n-1} - \frac{1}{n-1}\right]$$

$$= -\frac{1}{2\pi}\left[\frac{\cos(n+1)\pi}{n+1} - \frac{\cos(n-1)\pi}{n-1}\right] - \left[\frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= \cancel{-\frac{1}{2\pi}\left[2\sin(n\pi) - 2\right]} \qquad -\left[\frac{2n}{n^2-1}\right]$$

$$-\frac{1}{2\pi}\left[\frac{(-1)(-2)}{n^2-1} - \frac{2n}{n^2-1}\right] = \frac{1}{\pi}$$

$$-\frac{1}{2\pi}\left[\frac{(-1)^{n+1}}{n+1} - \frac{(-1)^{n+1}\pi}{n-1} - \right.$$

$$= -\frac{1}{2\pi}\left[\frac{2n}{n^2-1}\right]\left[(-1)^{n+1}\right.$$

$$a_n = \frac{-1}{2\pi}\left[\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{1-n}}{1-n} - \frac{2}{1-n^2}\right]$$

$$a_1 = \frac{1}{2\pi}\int_0^\pi 2\sin x\cos x \cdot dx$$

$$= \frac{1}{2\pi} - \frac{\cos 2x}{2}\Big|_0^\pi$$

$$= \frac{-1}{4\pi}[1-1] = 0$$

* a_n may not work in all cases.

$$a_2 = \frac{-1}{2\pi}\left[\frac{-1}{3} + \frac{1}{+1} - \frac{2}{1-4}\right] = \left[\frac{1}{3}+1\right] = \frac{-2}{3\pi}$$

$$a_3 = \frac{-1}{2\pi}\left[\frac{1}{4} - \frac{1}{2} + \frac{1}{4}\right] = 0$$

$$a_4 = \frac{-1}{2\pi}\left[\frac{-1}{5} + \frac{1}{3} + \frac{2}{15}\right] = \frac{-1}{2\pi}\left[\frac{-15+7\times 6}{90 \quad 15\times 6}\right] = \frac{-3}{20\pi}$$

$$\cancel{= \frac{-1+5}{15}} \quad \frac{-2}{15\pi} = \frac{-2}{3\cdot 5\pi}$$

$$a_{odd} = 0.$$

$$a_{n\,even} = \frac{-2}{\pi}\left[\frac{1}{(n+1)(n+3)(n+5)+\ldots}\right]$$

$$f(x) = \frac{a_0}{2} + a_2\cos 2x + a_4\cos 4x + \ldots + b_1\sin x.$$

put $x=0$.

$$\sin 0 = \frac{1}{\pi}\cancel{a_0} + a_2 + a_4 + \ldots \rightarrow \frac{-1}{\pi} = \frac{-2}{\pi}\left[\frac{1}{3} + \frac{1}{3\cdot 5} + \frac{1}{3\cdot 5\cdot 7} + \ldots\right]$$

$$\cancel{\frac{-1}{\pi} = \frac{-2}{\pi}\sum_{n=2}^{n=\infty}\frac{1}{(n+1)(n+3)(n+5)}\ldots}$$

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{3\cdot 5} + \frac{1}{3\cdot 5\cdot 7} + \frac{1}{3\cdot 5\cdot 7\cdot 9}$$

$$\frac{1}{2} = 1 - \frac{1}{3} - \frac{1}{3\cdot 5} - \frac{1}{3\cdot 5\cdot 7} \ldots$$

put $x = \frac{\pi}{2}$

$$1 = \frac{1}{\pi} + \frac{2}{\pi}\left[\frac{1}{3} - \frac{1}{3\cdot5} + \frac{1}{3\cdot5\cdot7} - \cdots\right] + \frac{1}{2}$$

$$\left(\frac{1}{2} - \frac{1}{\pi}\right)\frac{\pi}{2} = \left[\frac{1}{3} - \frac{1}{3\cdot5} + \cdots\cdots\right]$$

$$\frac{\pi}{4} - \frac{1}{2} = \frac{1}{3} - \frac{1}{3\cdot5} + \frac{1}{3\cdot5\cdot7} - \cdots$$

* find half range sine series of

$$f(x) = \sin x \quad 0 \le x \le \pi$$

$$= \cancel{\frac{2}{\pi}\int_0^{\pi}} \quad b_n = \frac{2}{l}\int_0^{l} \sin x \cdot \sin\frac{n\pi x}{l}$$

$$b_n = \frac{1}{l}\int_0^{l} \cos\left(1 - \frac{n\pi}{l}\right)x - \cos\left(1 + \frac{n\pi}{l}\right)x$$

for $n = 1$

$$b_1 = \frac{2}{l}\int_0^{l} \cos\left(1 - \frac{\pi}{l}\right)x - \cos\left(1 + \frac{\pi}{l}\right)x$$

$$b_1 = \frac{1}{l}\left[\frac{+\sin\left(1 - \frac{\pi}{l}\right)x}{1 - \frac{\pi}{l}}\Bigg|_0^{l} - \frac{\sin\left(1 + \frac{\pi}{l}\right)x}{1 + \frac{\pi}{l}}\Bigg|_0^{l}\right]$$

$$b_1 = \frac{1}{l}\left[\frac{\sin\left(1 - \frac{\pi}{l}\right)l}{1 - \frac{\pi}{l}} - \frac{\sin\left(1 + \frac{\pi}{l}\right)l}{l}\right]$$

$l = \pi$

$$b_1 = \frac{1}{\pi}\left[\cancel{\sin 0}^{\,0} - \frac{\sin 2\pi}{\pi}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} (1 - \cos 2x)\cdot dx$$

$$b_n = \frac{1}{\pi}\left[\pi\right] - \frac{1}{2\pi}\left[\cancel{\sin 2x}^{\,0}\right] = 1$$

$M_g f = gf$

M_g is a Linear Transformation.

$D(f) = f'$ - Linear

Differentiated fnc. $I(f) = \int_a^b f(x) \cdot dx$.

$C : [a,b] \to R$.

composition of any two Linear transformation is again a L.T.

$I \circ M_{k(p,x)}$ = is a L.T.

$$(I \circ M_k)f = g(M_k f)$$
$$= g(kf)$$

$$T(f(x)) = \int_a^b k(p.x) \cdot f(x) \cdot dx.$$

$a = 0$; $b = \infty$, $k(P, x) = e^{-px}$

$L[f(x)] = \int_0^\infty e^{-px} f(x) \cdot dx$ - Laplace Transformation.

$a = -\infty$; $b = \infty$ $k(p, x) = e^{-ipx}$.

$F[f(x)] = \int_{-\infty}^{\infty} e^{-ipx} \cdot dx$ - Fourier Series

If $a = -\infty$; $b = \infty$ $k(p, x) = \frac{1}{\sqrt{2\pi}} e^{ipx}$.

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\pm ipx} \cdot f(x)$$

$$= \frac{1}{\sqrt{2\pi}} \int_0 \cancel{\cos px} \cdot f(x) \cdot dx$$
e^{ipx}

$F_s\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sin px \cdot f(x)\, dx \rightarrow a=0;\ b=\infty$

$\downarrow$ $\qquad K(p,x) = \sqrt{\frac{2}{\pi}} \sin px$

Fourier Sine Transform

2) $F_c\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos px \cdot f(x) \cdot dx \rightarrow a=0;\ b=\infty$

$\downarrow$ $\qquad k(p,x) = \sqrt{\frac{2}{\pi}} \cos px$

Fourier Cosine Transform

3) If $L[f(x)] = F(p)$, then

$L[f(x)\, e^{ax}] = F(p-a)$;- $p > a$.

$F_s\{f(x)\} = \int_0^{\infty} e^{i(p-a)x} \cdot f(x) \cdot dx$

Eg:- Find $F\{1\}$; $F_c\{1\}$; $F_s\{1\}$

$F\{1\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} \cdot 1 \cdot dx = \frac{1}{2\sqrt{2\pi}} \frac{e^{ipx}}{ip}\Big|_{-\infty}^{\infty}$

$= \frac{1}{\sqrt{2\pi}} [0-\infty] = \infty$

$F_s\{1\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sin px \cdot dx$

$= -\sqrt{\frac{2}{\pi}} \times \frac{1}{p} \cos px \Big|_0^{\infty} = -\sqrt{\frac{2}{\pi}} \frac{1}{p} [\cos\infty - 1]$

$F_c\{1\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos px \cdot dx$

$= \sqrt{\frac{2}{\pi}} \frac{\sin px}{p} \Big|_0^{\infty} = \sqrt{\frac{2}{\pi}} \frac{1}{p} [\sin\infty]$

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$$\mathbb{F}\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx = F(s).$$

$$\mathbb{F}_s\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} f(x) \sin sx\, dx = F_s(s)$$

$$\mathbb{F}_s\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} f(x) \cos sx\, dx = F_c(s).$$

Then,

$$\mathbb{F}^{-1}\{F(s)\} = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds.$$

$$F_s^{-1}\{F_s(s)\} = f(x) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} F_s(s) \sin sx\, ds.$$

$$F_c^{-1}\{F_c(s)\} = f(x) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} F_c(s) \cos sx\, ds.$$

<u>properties</u>

$\mathbb{F}_1$, $\mathbb{F}^{-1}$, $\mathbb{F}_c$, $\mathbb{F}_c^{-1}$, $\mathbb{F}_s$, $\mathbb{F}_s^{-1}$ are Linear.

1) shifting

$$\mathbb{F}\{e^{iax} f(x)\} = F(s+a)$$

<u>proof</u>:-

$$e^{iax} f(x) = \mathbb{F}^{-1}(F(s+a))$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s+a) \cdot e^{+i(s+a)x} \cdot ds.$$

$$= \frac{e^{iax}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{i(s+a)x} \cdot ds.$$

$$= e^{iax} f(x).$$

2) <u>Change of Scale</u>

$$\mathbb{F}\{f(ax)\} = \frac{1}{|a|} F(s/a)$$

$$\mathbb{F}_s\{f(ax)\} = \frac{1}{a} F_s(s/a)$$

$$\mathbb{F}_c\{f(ax)\} = \frac{1}{a} F_c(s/a)$$

3) $$\mathbb{F}\{x^n f(x)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathbb{F}_s\{-f(x)\} = -\frac{d}{ds} F_c(s).$$

$$\mathbb{F}_c\{-f(x)\} = \frac{d}{ds} F_s(s).$$

<u>Theorem</u>:- $\mathbb{F}\{f'(x)\} = -isF(s)$

$$\mathbb{F}_s\{f'(x)\} = -s\,F_c(s)$$

$$\mathbb{F}_c\{f'(x)\} = s\,F_s(s) - \sqrt{\frac{2}{\pi}}\,f(0).$$

$$(f*g)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\,g(t-x)\,dt.$$

<u>Th</u>:- $\mathbb{F}(f*g) = F(s)\cdot G(s)$

(or) $\mathbb{F}^{-1}\{F(s)G(s)\} = (f*g)(x)$

<u>Ex</u>:- Find F.T of $f(x) = \begin{cases} 0 & x \le a \\ 1 & a < x < b \\ 0 & x \ge b. \end{cases}$

$$\int_{-\infty}^{a} + \frac{1}{\sqrt{2\pi}} \int_{a}^{b} e^{isx}\,dx + \int_{b}^{\infty} 0$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is}\right]_a^b$$

$$= \frac{1}{is\sqrt{2\pi}} \left[e^{isb} - e^{isa}\right].$$

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} \cdot dx$$

$$= \frac{1}{}$$

1) $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| \geq a \end{cases}$

$-a < x < a$

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} f(x) \cdot e^{isx} \cdot dx$$

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{isx} \cdot dx = \frac{1}{\sqrt{2\pi}}$$

2) $F(s) = \frac{1}{\sqrt{2\pi}}\left(\frac{2\sin as}{s}\right) \rightarrow$ inverse fnc

$$f(x) = F^{-1}(F(s)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} \cdot ds$$

$$= \left(\frac{1}{\sqrt{2\pi}}\right) \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \frac{\sin as}{s} \cdot \underset{\text{I}}{e^{-isx}} \cdot ds$$

$\cos sx - i \sin sx$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \cancel{\frac{\sin as}{s} e^{-isx}} \int_{-\infty}^{\infty} \left[\frac{\sin as \cdot \cos sx}{s} - \cancel{\frac{i \sin as \sin sx}{s}}^{0}\right] \cdot ds$$

$$f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as \cos sx}{s} \cdot ds = \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as \cos sx}{s} \cdot ds = \cancel{\frac{\pi}{2} f(x)}$$

$$\cancel{= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin(a+x)s}{s} + \frac{\sin(a-x)s}{s} ds}$$

$$\cancel{= \frac{1}{\pi} \int_{0}^{\infty} \frac{\sin(a+x)s}{s} ds + \frac{1}{\pi} \int_{0}^{\infty} \frac{\sin(a-x)s}{s} ds}$$

$$= \cancel{\frac{2}{\pi} \int_{0}^{\infty}}$$ put $x = 0$

$$\frac{\pi}{2} f(x) = \int_{0}^{\infty} \frac{\sin as \cos x}{s} \cdot ds$$

If $x=0$

$$\frac{\pi}{2} f(0) = \int_0^\infty \frac{\sin as}{s} \, ds.$$

$$\frac{\pi}{2}(1) = \int_0^\infty \frac{\sin as}{s} \, ds.$$

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Partial Differential equations

1) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

$$2\frac{\partial z}{\partial x} = \frac{2x}{a^2}$$

$$\frac{\partial z}{\partial y} = \frac{y}{b^2}$$

$$2z = \frac{x\partial z}{\partial x} + \frac{y\partial z}{\partial y}$$

$$2z = px + qy$$

2) $z = (x+y)\phi(x^2-y^2)$.

$$\frac{\partial z}{\partial x} = y\phi'(x^2-y^2)(2x) + \phi(x^2-y^2)$$

$$\frac{\partial z}{\partial y} = x\phi'(x^2-y^2)(-2y) + \phi(x^2-y^2)$$

$$p+q = \phi(x^2-y^2)$$

$$z = (x+y)(p+q)$$

$$\frac{\partial z}{\partial x} = \phi(x^2-y^2) + (x+y)\phi'(x^2-y^2)2x$$

$$\frac{\partial z}{\partial y} = \phi(x^2-y^2) + (x+y)\phi'(x^2-y^2)(-2y)$$

$$yp + xq = (y+x)\phi(x^2-y^2)$$

$$\frac{yp+xq}{y+x}(x+y) = z.$$

$$z = yp + xq$$

3) $p = x$.

$$\frac{\partial z}{\partial x} = x$$

$$z = \frac{x^2}{2} + \phi(y).$$

4) $\frac{\partial^2 z}{\partial x \partial y} + 9x^2y^2 - \frac{1}{2}\cos(2x-y) = 0$ $\longrightarrow$ Direct Integration.

$$\frac{\partial z}{\partial y} + 3x^2y^3 - \frac{1}{2}\sin(2x-y) = 0 + \phi(x)$$

$$z + x^3y^3 - \cos(2x-y) + \int\phi(x) + g(y) = 0$$

$$z + x^3y^3 - \cos(2x-y) + F(x) + g(y) = 0$$