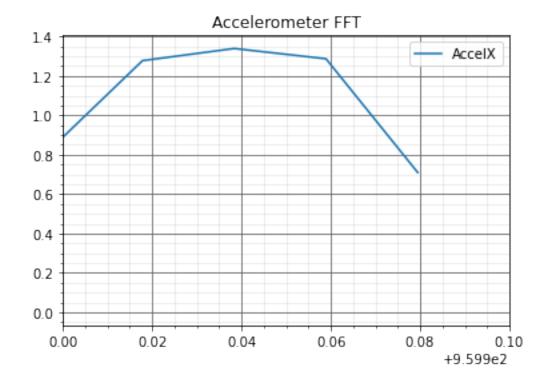
sensor fusion 4 1

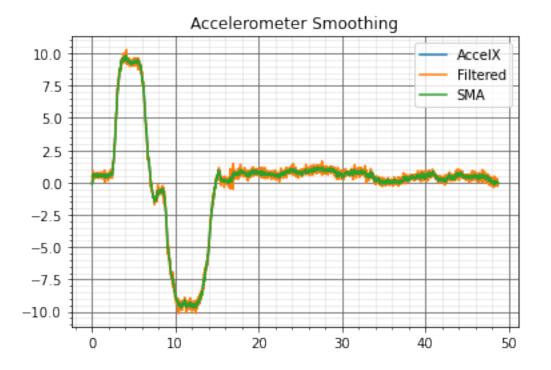
April 1, 2021

```
[227]: import math
                      import numpy as np
                      import pandas as pd
                      import matplotlib.pyplot as plt
                      TEST NAME = "euler angles 2"
                      dt = 1/960
                      GRAVITY = 9.80665
                      RAD_TO_DEG = 180 / math.pi
                      DEG_TO_RAD = math.pi / 180
[228]: # read test params from CSVP
                      csvp = open(f"data/{TEST_NAME}.csvp")
                      # create params array
                      params = np.array([eval(line) for line in csvp])
                      print(params)
                                1 3848
                                                                0 300
                                                                                                                             30 960
                                                                                                                                                            96
                                                                                                                                                                             16 2000
                                                                                                                                                                                                            92
                                                                                                                                                                                                                            92
                                                                                                                                                                                                                                            63
                                                                                               0
                             63
                                             14
                                                                             86
                                                                                                                               0
                                                                                                                                               0
                                                                                                                                                               0
                                                                                                                                                                               0
                                                                                                                                                                                                               0
                                                                                                                                                                                                                               0
                                                                                                                                                                                                                                               0
                                                             14
                                                                                            86
                                                                                                                                                                                              0
                                0
                                                0
                                                               0]
[229]: # load saved mag offsets (this approach isn't being used right now,
                      # since min/max averaged offsets are recalculated for each data set)
                      # read mag offsets from file
                      offsets_dat = open(f"data/mag_offsets.dat")
                      # create mag offsets array
                      mag_offsets = np.array([eval(line) for line in offsets_dat])
[230]: # read data from CSV
                      data = pd.read_csv(f"data/{TEST_NAME}.csv", names=["AccelX", "AccelY", uata = pd.read_csv(f"data/{TEST_NAME}.csv", names=["AccelX", uata = pd.read_csv", names=["A
                         →"AccelZ", "GyroX", "GyroY", "GyroZ", "MagX", "MagY", "MagZ"],
                         →index_col=False)
```

```
sample_rate = params[7]
# add time axis to data set
time = np.arange(0, len(data)/sample_rate, 1/sample_rate)
data.insert(0, "Time", time)
# sign data
data = data.applymap(lambda x: x-65535 if x > 32767 else x)
# apply accel sensitivity
acc_cols = ["AccelX", "AccelY", "AccelZ"]
acc_sens = params[9]
data[acc\_cols] = data[acc\_cols].applymap(lambda x: x * acc\_sens * GRAVITY / <math>_{\sqcup}
→32768)
# apply gyro sensitivity
gyro_cols = ["GyroX", "GyroY", "GyroZ"]
gyro_sens = params[10]
data[gyro_cols] = data[gyro_cols].applymap(lambda x: x * gyro_sens / 32768)
# apply mag sensitivity
mag_cols = ["MagX", "MagY", "MagZ"]
mag_sens = 4800
data[mag_cols] = data[mag_cols].applymap(lambda x: x * mag_sens / 8192)
# FIXME copy for debugging, remove later
original_mag_data = data[mag_cols]
# calculate offsets for each sensor (first 0.5s of data)
acc_offsets = data[acc_cols].head(480).mean()
gyro_offsets = data[gyro_cols].head(480).mean()
#mag_offsets = data[mag_cols].mean()
# min/max method
mag_offsets = (data[mag_cols].max() + data[mag_cols].min()) / 2
# apply offsets to each sensor (remove sensor bias)
# TODO: hold off on accel until actual IMU calibration is implemented
for i, axis in enumerate(gyro_cols):
    data[axis] = data[axis].map(lambda x: x - gyro_offsets[i])
for i, axis in enumerate(mag_cols):
    data[axis] = data[axis].map(lambda x: x - mag_offsets[i])
# create new mag dataframe by removing all NaNs
mag_data = data[mag_cols + ["Time"]].dropna()
```

```
# for some reason, the first mag data point is always erroneous, so remove it
       mag_data = mag_data.iloc[1:]
      print(mag_offsets)
             -222.070312
      MagX
      MagY
               67.675781
      MagZ
               -7.910156
      dtype: float64
[231]: def show_plot(title=""):
           '''Utility method to graph a plot with grid lines, a legend, and a title.'''
           plt.title(title)
           plt.grid(b=True, which='major', color='#666666', linestyle='-')
           plt.minorticks_on()
           plt.grid(b=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
           plt.legend()
           plt.show()
[232]: # FFT analysis of accelerometer
       # (not currently being used, experimental)
       import scipy.signal
       # calculate fft for AccelX
       fourierTransform = np.fft.fft(data["AccelX"])/len(data["AccelX"])
       tpCount = len(data["AccelX"])
       values = np.arange(tpCount)
       timePeriod = tpCount/sample_rate
       frequencies = values/timePeriod
       plt.plot(frequencies, abs(fourierTransform), label="AccelX")
       # display the plot
       plt.xlim(959.9,960)
       show_plot("Accelerometer FFT")
       # normalized cutoff frequency = cutoff frequency / (2 * sample rate)
       ORDER = 10
       # 959.93838386
       CUTOFF FREQ = 100
       NORM_CUTOFF_FREQ = CUTOFF_FREQ / (2 * 960)
       # Butterworth filter
       num_coeffs, denom_coeffs = scipy.signal.butter(ORDER, NORM_CUTOFF_FREQ)
       filtered_data = scipy.signal.lfilter(num_coeffs, denom_coeffs, data["AccelX"])
```

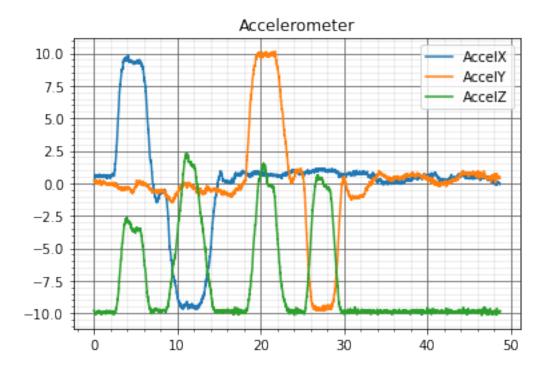


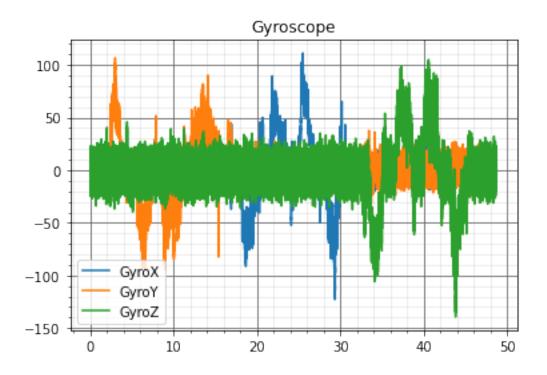


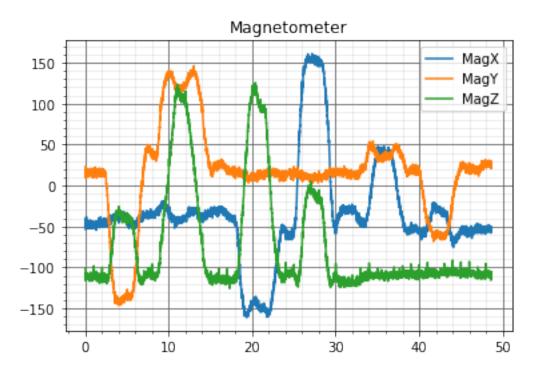
```
[233]: # plot acceleration
plt.plot(data["Time"], data["AccelX"], label="AccelX")
plt.plot(data["Time"], data["AccelY"], label="AccelY")
plt.plot(data["Time"], data["AccelZ"], label="AccelZ")
show_plot("Accelerometer")

# plot gyroscope
plt.plot(data["Time"], data["GyroX"], label="GyroX")
plt.plot(data["Time"], data["GyroY"], label="GyroY")
plt.plot(data["Time"], data["GyroZ"], label="GyroZ")
show_plot("Gyroscope")

# plot magnetometer
plt.plot(mag_data["Time"], mag_data["MagX"], label="MagX")
plt.plot(mag_data["Time"], mag_data["MagY"], label="MagY")
plt.plot(mag_data["Time"], mag_data["MagZ"], label="MagZ")
show_plot("Magnetometer")
```

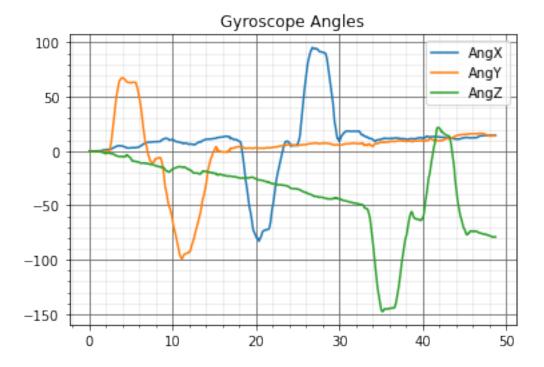






```
# calculate angles from gyroscope
ang_x = integrate.cumtrapz(y=data["GyroX"], x=data["Time"], initial=0)
ang_y = integrate.cumtrapz(y=data["GyroY"], x=data["Time"], initial=0)
ang_z = integrate.cumtrapz(y=data["GyroZ"], x=data["Time"], initial=0)

# plot gyroscope angles
plt.plot(data["Time"], ang_x, label="AngX")
plt.plot(data["Time"], ang_y, label="AngY")
plt.plot(data["Time"], ang_z, label="AngZ")
show_plot("Gyroscope Angles")
```



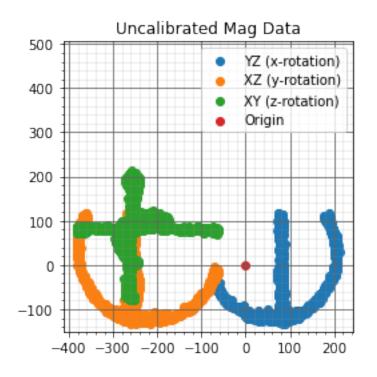
```
[235]: # least squares ellipse fitting
       # from https://www.hep.princeton.edu//mumu/target/Yan/ellipse_fit.pdf
       # (*) denotes tensor product,
       # @ denotes matrix multiplication
       def ellipse_fit(x_data, y_data):
           # create x and y vectors
           x = np.array(x_data)
           y = np.array(y_data)
           \# X = [x (*) x, x (*) y, y (*) y, x, y]
           \# (transpose to account for x and y not being column vectors)
           X = \text{np.array}([\text{np.multiply}(x, x), \text{np.multiply}(x, y), \text{np.multiply}(y, y), x,_{\bot})
        →y]).T
           # create n-dimensional column vector filled with 1s
           ones = np.c_[[1] * len(x)]
           # calculate coefficients [A,B,C,D,E] where:
           \# Ax^2 + Bxy + Cy^2 + Dx + Ey = 1
           beta = (np.linalg.inv(X.T @ X) @ X.T) @ ones
           return beta
       # fit data to 3D ellipsoid
```

```
# from: https://teslabs.com/articles/magnetometer-calibration/
def ellipsoid_fit(s):
   # D (samples)
   D = np.array([s[0]**2., s[1]**2., s[2]**2.,
                   2.*s[1]*s[2], 2.*s[0]*s[2], 2.*s[0]*s[1],
                   2.*s[0], 2.*s[1], 2.*s[2], np.ones_like(s[0])]
   # S, S_11, S_12, S_21, S_22 (eq. 11)
   S = np.dot(D, D.T)
   S_11 = S[:6,:6]
   S 12 = S[:6,6:]
   S_21 = S[6:,:6]
   S_22 = S[6:,6:]
   # C (Eq. 8, k=4)
   C = np.array([[-1, 1, 1, 0, 0, 0],
                   [1, -1, 1, 0, 0, 0],
                   [1, 1, -1, 0, 0, 0],
                   [0, 0, 0, -4, 0, 0],
                    [0, 0, 0, 0, -4, 0],
                   [0, 0, 0, 0, -4]])
   # v_1 (eq. 15, solution)
   E = np.dot(np.linalg.inv(C),
               S_11 - np.dot(S_12, np.dot(np.linalg.inv(S_22), S_21)))
   E_w, E_v = np.linalg.eig(E)
   v_1 = E_v[:, np.argmax(E_w)]
   if v_1[0] < 0: v_1 = -v_1
   # v_2 (eq. 13, solution)
   v_2 = np.dot(np.dot(-np.linalg.inv(S_22), S_21), v_1)
   # quadric-form parameters
   M = np.array([[v_1[0], v_1[3], v_1[4]],
                    [v_1[3], v_1[1], v_1[5]],
                    [v_1[4], v_1[5], v_1[2]])
   n = np.array([[v_2[0]]],
                    [v_2[1]],
                    [v 2[2]]])
   d = v_2[3]
   return M, n, d
```

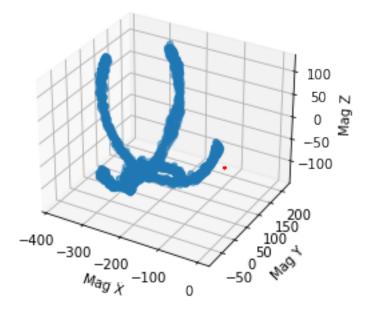
```
[236]: # magnetometer calibration techniques
from scipy.linalg import sqrtm
```

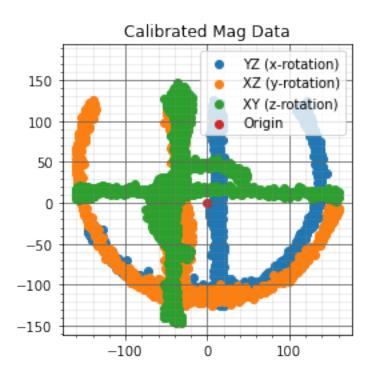
```
def draw_mag_sphere(x_data, y_data, z_data):
   ax.scatter(x_data, y_data, z_data)
   limits = np.array([getattr(ax, f'get_{axis}lim')() for axis in 'xyz'])
   ax.set_box_aspect(np.ptp(limits, axis=1))
   ax.set_xlabel("Mag X")
   ax.set_ylabel("Mag Y")
   ax.set_zlabel("Mag Z")
def draw_sphere(r=1, c=(0,0,0)):
   Draws a wireframe sphere in a 3D plot.
   By default, this function draws a unit sphere
    (a sphere with a radius of 1 and centered at the origin).
   u = np.linspace(0, np.pi, 30)
   v = np.linspace(0, 2 * np.pi, 30)
   x = np.outer(np.sin(u), np.sin(v))
   y = np.outer(np.sin(u), np.cos(v))
   z = np.outer(np.cos(u), np.ones_like(v))
   ax.plot_wireframe(c[0]+r*x, c[1]+r*y, c[2]+r*z, color="r", alpha=0.25)
# plot x-axis rotation (MagY, MagZ)
plt.scatter(original_mag_data["MagY"], original_mag_data["MagZ"], label="YZ_U
# plot y-axis rotation (MagX, MagZ)
plt.scatter(original_mag_data["MagX"], original_mag_data["MagZ"], label="XZ_L
# plot z-axis rotation (MagX, MagY)
plt.scatter(original_mag_data["MagX"], original_mag_data["MagY"], label="XY_U"
# plot origin (0,0)
plt.scatter([0], [0], label="Origin")
# display the plot
plt.axis("square")
show_plot("Uncalibrated Mag Data")
fig = plt.figure()
ax = fig.add_subplot(projection="3d")
# 3D plot of original mag data
draw_mag_sphere(original_mag_data["MagX"], original_mag_data["MagY"],__
→original_mag_data["MagZ"])
```

```
draw_sphere()
plt.title("Uncalibrated Mag Data")
plt.show()
# plot x-axis rotation (MagY, MagZ)
plt.scatter(mag_data["MagY"], mag_data["MagZ"], label="YZ (x-rotation)")
# plot y-axis rotation (MagX, MagZ)
plt.scatter(mag_data["MagX"], mag_data["MagZ"], label="XZ (y-rotation)")
# plot z-axis rotation (MagX, MagY)
plt.scatter(mag_data["MagX"], mag_data["MagY"], label="XY (z-rotation)")
# plot origin (0,0)
plt.scatter([0], [0], label="Origin")
# display the plot
plt.axis("square")
show_plot("Calibrated Mag Data")
fig = plt.figure()
ax = fig.add_subplot(projection="3d")
# 3D plot of mag data
draw_mag_sphere(mag_data["MagX"], mag_data["MagY"], mag_data["MagZ"])
draw_sphere()
plt.title("Calibrated Mag Data")
plt.show()
```

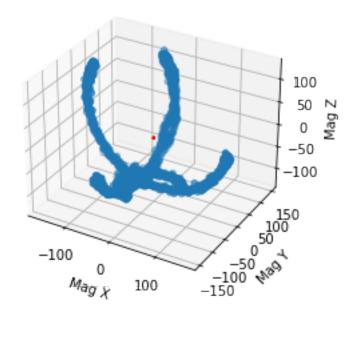


Uncalibrated Mag Data



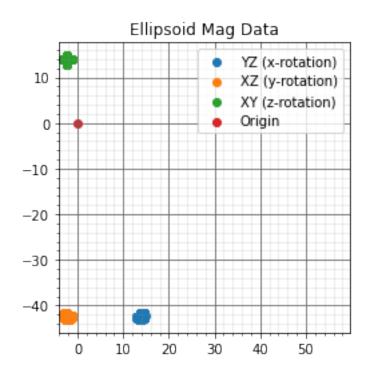


Calibrated Mag Data

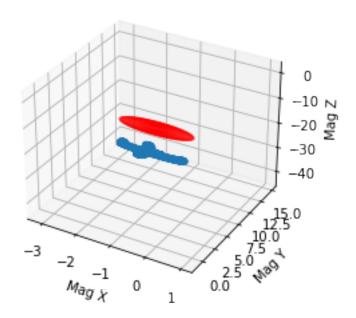


[237]: # ellipsoid fit magnetometer calibration (NOT CURRENTLY IN USE)
(from https://teslabs.com/articles/magnetometer-calibration/)

```
# calculate ellipsoid for data
s = np.array(mag_data[mag_cols]).T
M, n, d = ellipsoid_fit(s)
# calculate calibration parameters for:
\# h_m = A @ h + b where h = A^-1 @ (h_m - b)
M_1 = np.linalg.inv(M)
b = -np.dot(M 1, n)
A_1 = \text{np.real}(1 / \text{np.sqrt}(\text{np.dot}(\text{n.T}, \text{np.dot}(\text{M}_1, \text{n})) - d) * \text{sqrtm}(\text{M}))
# calculate h = A^-1 @ (h_m - b)
def calibrate_mag(row):
    res = A_1 @ np.c_[row] - b
    return res.flatten().tolist()
# calibrate magnetometer
ell_data[mag_cols] = mag_data[mag_cols].apply(calibrate_mag, axis=1,_u
→result_type='expand')
# plot x-axis rotation (MaqY, MaqZ)
plt.scatter(ell_data["MagY"], ell_data["MagZ"], label="YZ (x-rotation)")
# plot y-axis rotation (MagX, MagZ)
plt.scatter(ell_data["MagX"], ell_data["MagZ"], label="XZ (y-rotation)")
# plot z-axis rotation (MagX, MagY)
plt.scatter(ell_data["MagX"], ell_data["MagY"], label="XY (z-rotation)")
# plot origin (0,0)
plt.scatter([0], [0], label="Origin")
# display the plot
plt.axis("square")
show_plot("Ellipsoid Mag Data")
fig = plt.figure()
ax = fig.add_subplot(projection="3d")
# 3D plot of mag data
draw_mag_sphere(ell_data["MagX"], ell_data["MagY"], ell_data["MagZ"])
draw_sphere()
plt.title("Ellipsoid Mag Data")
plt.show()
```



Ellipsoid Mag Data



[238]: # bounding sphere test (EXPERIMENTAL)

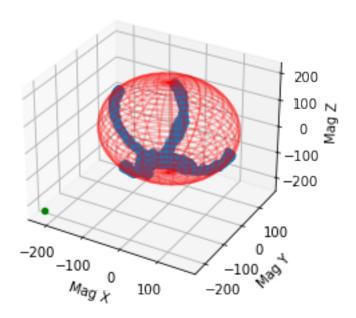
```
import miniball

# calculate 100% bounding sphere
C, r2 = miniball.get_bounding_ball(mag_data[mag_cols].to_numpy())

fig = plt.figure()
ax = fig.add_subplot(projection="3d")

# draw 100% bounding sphere
draw_mag_sphere(mag_data["MagX"], mag_data["MagY"], mag_data["MagZ"])
draw_sphere(math.sqrt(r2), C)
ax.scatter(mag_offsets[0], mag_offsets[0], mag_offsets[0], color="g")
plt.title("Bounding Sphere")
plt.show()
```

Bounding Sphere



```
[239]: # ellipse fitting test (z-axis)
# (NOT IN USE)

# plot z-axis rotation (MagX, MagY)
plt.scatter(mag_data["MagX"], mag_data["MagY"], label="XY (z-rotation)")
plt.plot()

# calculate coefficients for ellipse fitting
A,B,C,D,E = ellipse_fit(mag_data["MagX"], mag_data["MagY"])
```

```
x = np.linspace(-500, 500, 1000)
y = np.linspace(-500, 500, 1000)
x, y = np.meshgrid(x, y)

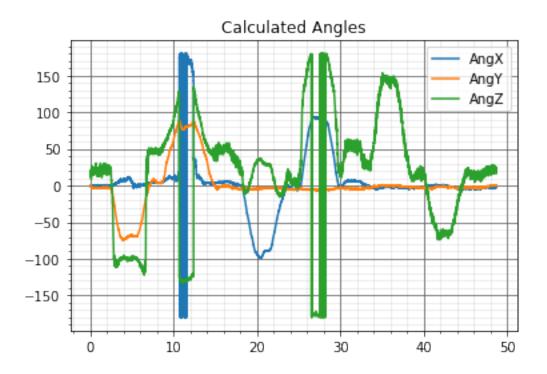
plt.contour(x, y, A*x**2 + B*x*y + C*y**2 + D*x + E*y, [0])

# display the plot
plt.axis("square")
show_plot("Magnetometer Ellipse Fitting")
```



```
# TODO: added 2 negatives to acc ang x, doesn't work without it: why?
acc_ang_x = np.arctan2(-data["AccelY"], -np.sign(data["AccelZ"]) * np.
acc ang y = np.arctan2(-data["AccelX"], np.sqrt(data["AccelY"]**2 +11

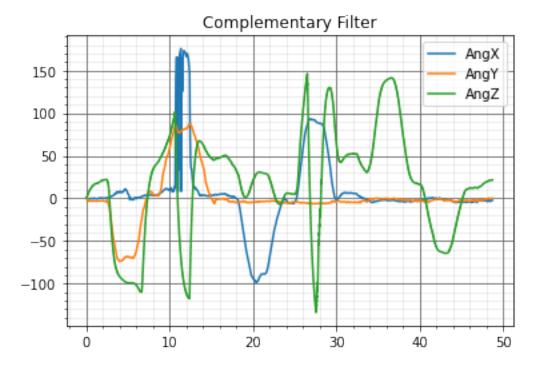
data["AccelZ"]**2))
# calculate magnetometer + accelerometer parameters
M x = mag_data["MagX"] * np.cos(acc_ang_y) + mag_data["MagZ"] * np.
→sin(acc_ang_y)
M_y = mag_data["MagX"] * np.sin(acc_ang_x) * np.sin(acc_ang_y) +__
→mag_data["MagY"] * np.cos(acc_ang_x) - mag_data["MagZ"] * np.sin(acc_ang_x)_
→* np.cos(acc_ang_y)
# remove NaNs from mag params
M_x = M_x[\sim np.isnan(M_x)]
M_y = M_y[\sim np.isnan(M_y)]
# calculate z-angle (yaw) from accelerometer & magnetometer
mag_ang_z = np.arctan2(M_y, -M_x)
# convert all angles to degrees
acc ang x *= RAD TO DEG;
acc_ang_y *= RAD_TO_DEG;
mag_ang_z *= RAD_TO_DEG;
# plot accelerometer angles
plt.plot(data["Time"], acc_ang_x, label="AngX")
plt.plot(data["Time"], acc_ang_y, label="AngY")
# plot mag+accel z-axis angle
plt.plot(mag_data["Time"], mag_ang_z, label="AngZ")
# display the plot
show_plot("Calculated Angles")
```



```
[241]: # complementary filter
       HP weight = 0.98
       LP_weight = 0.02
       # create empty array w/ 3 axes
       # set first elements to 0 (will be removed)
       cf_ang = [[0],[0],[0]]
       # group all axes of calculated angles together
       gyro_ang = np.array([data["GyroX"].to_numpy(), data["GyroY"].to_numpy(),_

→data["GyroZ"].to_numpy()])
       calc_ang = np.array([acc_ang_x, acc_ang_y, mag_ang_z], dtype=object)
       # pair the calculated arrays for each axis together and loop
       for i, (gyro_arr, calc_arr) in enumerate(zip(gyro_ang, calc_ang)):
           # pair the gyro & calc samples together and loop
           for j, (gyro, calc) in enumerate(zip(gyro_arr, calc_arr)):
               cf_ang_prev = cf_ang[i][-1]
               cf_ang_samp = HP_weight * (cf_ang_prev + gyro * dt) + LP_weight * calc
               cf_ang[i].append(cf_ang_samp)
               # if this is the magnetometer axis,
```

```
# repeat sample 9x to ensure it lines up with accel/gyro
        if i == 2:
            for _ in range(9):
                cf_ang[i].append(cf_ang_samp)
# since accel/gyro can have additional samples past 10 x mag,
# repeat the last sample to line up mag axis w/ accel/gyro
num_extra = len(cf_ang[0]) - len(cf_ang[2])
for _ in range(num_extra):
   cf_ang[2].append(cf_ang[2][-1])
# remove initial O values
for axis in cf_ang: del axis[0]
plt.plot(data["Time"], cf_ang[0], label="AngX")
plt.plot(data["Time"], cf_ang[1], label="AngY")
plt.plot(data["Time"], cf_ang[2], label="AngZ")
# display the plot
show_plot("Complementary Filter")
```



```
[242]: import csv

# save angle data as CSV
```

```
with open("out.csv", "w", newline="") as csvfile:

    # define CSV writer
    writer = csv.writer(csvfile, delimiter=",", quotechar='"', quoting=csv.

QUOTE_MINIMAL)

# loop over each row of (x,y,z),
    # and write each line to CSV
for row in zip(*cf_ang):
    writer.writerow(row)
```