Experiment 4 - Helmholtz Resonator

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1 Aim

Measuring the acoustic response of Helmholtz resonators and finding the speed of sound through the slope of the graph of f^2 vs $1/V$

2 Apparatus

A beaker/bottle/glass, two smartphones (one to generate white noise and the other to record the sound), the Advanced Spectrum Analyzer app on one of the phones

3 Theory

Mass of air in the neck of the container,

$$m = AL\rho \tag{1}$$

where A = Area of cross-section of neck, L = length of neck, and ρ = density of gas.

Now, if the mass is pushed down by a distance x, the volume occupied by the remaining gas in the container is reduced by $V - Ax$, where V is the volume of the container excluding the neck.
Hence, pressure changes from P_0 (atmospheric pressure) to $P_0 + p$.

Now, these changes–compression–are fast. Hence, this is an adiabatic process.

Now, γ (or k) = $\frac{C_P}{C_V}$ (γ depends on whether the gas in monoatomic, diatomic, etc.)
Hence, $\gamma = \frac{f+2}{f}$
where f = degree of freedom of gas
For monoatomic gas ($f = 3$), $\gamma_1 = 1.67$
For diatomic gas ($f = 5$), $\gamma_2 = 1.4$

Now, since this is an adiabatic process,

$$0 = dQ = dU + dW = nC_v dT + PdV \tag{2}$$

Also, since $PV = nRT$ for an ideal gas,

$$nRdT = PdV + VdP \tag{3}$$

From (2) and (3), and from the fact that $C_P - C_V = R$, we have

$$(\frac{R + C_V}{C_V})\frac{dV}{V} = -\frac{dP}{P} \tag{4}$$

or,

$$ln(V^{\gamma}) = -lnP \tag{5}$$

Hence,

$$PV^{\gamma} = constant \tag{6}$$

Also,

$$\gamma(\frac{\delta V}{V}) = -\frac{\delta P}{P} \tag{7}$$

Now, since pressure changed from P_0 to $P_0 + p$ (where $p = \delta P$), we have

$$\frac{d^2x}{dt^2} = \frac{F}{m} = \frac{pA}{\rho AL} = -\frac{\gamma AxP_0}{V\rho L} \tag{8}$$

Hence,

$$\frac{\gamma\delta V}{V} = \frac{\gamma Ax}{V} = -\frac{\delta P}{P_0} \tag{9}$$

And hence,

$$\frac{d^2x}{dt^2} = -(\frac{\gamma AP_0}{V\rho L}) \cdot x \tag{10}$$

which is of the form $-\frac{k}{m}x$

Hence, frequency $f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{\rho VL}}$
Now, c = speed of sound = $\sqrt{\frac{\gamma RT}{M}}$, where M = molar mass of the gas.
Hence, we have

$$f = \frac{c}{2\pi} \cdot \sqrt{\frac{A}{V \cdot L}} \tag{11}$$

To be noted: $f \propto \frac{1}{\sqrt{V}}$

Now, a rigid cavity with an open neck can be modelled as a mass-spring system, where the cavity is the spring and the neck is the mass, the so-called "Helmholtz resonator". The frequency of this system is given by:

$$f = \frac{c}{2\pi} \cdot \sqrt{\frac{A}{V \cdot L'}} \tag{12}$$

where L' (which replaces L in equation (11)) is the effective length of the neck, A is the area of the neck, V is the volume of the cavity, and c is the speed of sound in the inner gas.

Because a little amount of mass of gas is moving outside the edges of the neck dragged by the gas inside the neck, the effective length of the neck L' is slightly greater than the physical length of the neck L. This end correction depends on the boundary conditions:
If the outer end of the container if flanged (i.e. if it has a protruding edge), $L' = L + 1.7a$
Else, if the outer edge of the container is unflanged, $L' = L + 1.4a$
where a is the radius of the opening.
Substituting $V = AH$ where H is the height of cylinder, we get:

$$f = \frac{c}{2\pi}\sqrt{\frac{1}{H \cdot L'}} \tag{13}$$

Or,

$$c = 2\pi f\sqrt{HL'} \tag{14}$$

If we plot a curve of f^2 vs $1/V$, we get

$$slope = \frac{c^2A}{4\pi^2(L')} \tag{15}$$

4 Procedure

1. Take a beaker/bottle/glass, fill it partially with water (colored water can be used to help with detecting water level more clearly) and keep it in a place where environmental noise is minimal.

2. Take a smartphone and generate a constant white noise with it (optionally, the noise can also be generated with the human mouth) and make sure that the noise is as close to the brimtop of the container as possible.

3. Take another smartphone and opening the Advanced Spectrum Analyzer app, analyze the resonance of the sound coming from the container.

4. Repeat the experiment for different water levels inside the container, and obtain the spectrum for those water levels.

5. Analyze the data

5 Observations, Calculations and Analysis (IMT2019084 - Shrey Tripathi)

5.1 Snapshots of apparatus


5.2 Procedure

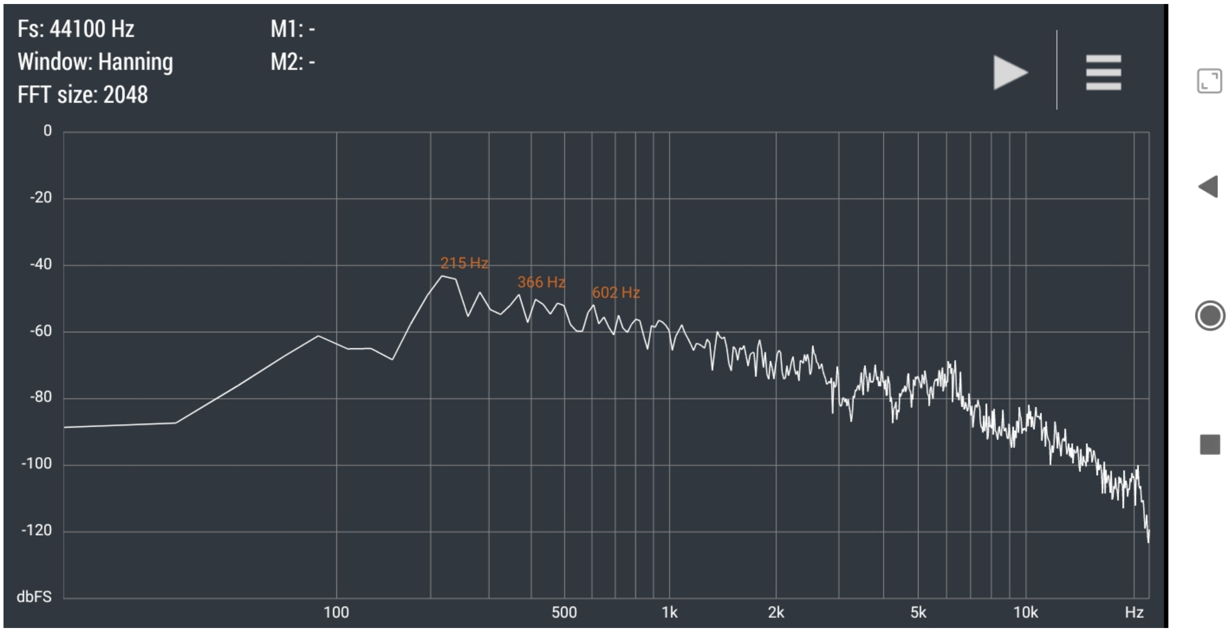
1. Fill the bottle upto different levels of water

2. Measure the resonance frequency using the Advanced Spectrum Analyzer app, and plot the graph of f^2 vs $1/V$, where V is the volume of empty space in the bottle.

5.3 Observations and Calculations (for air)

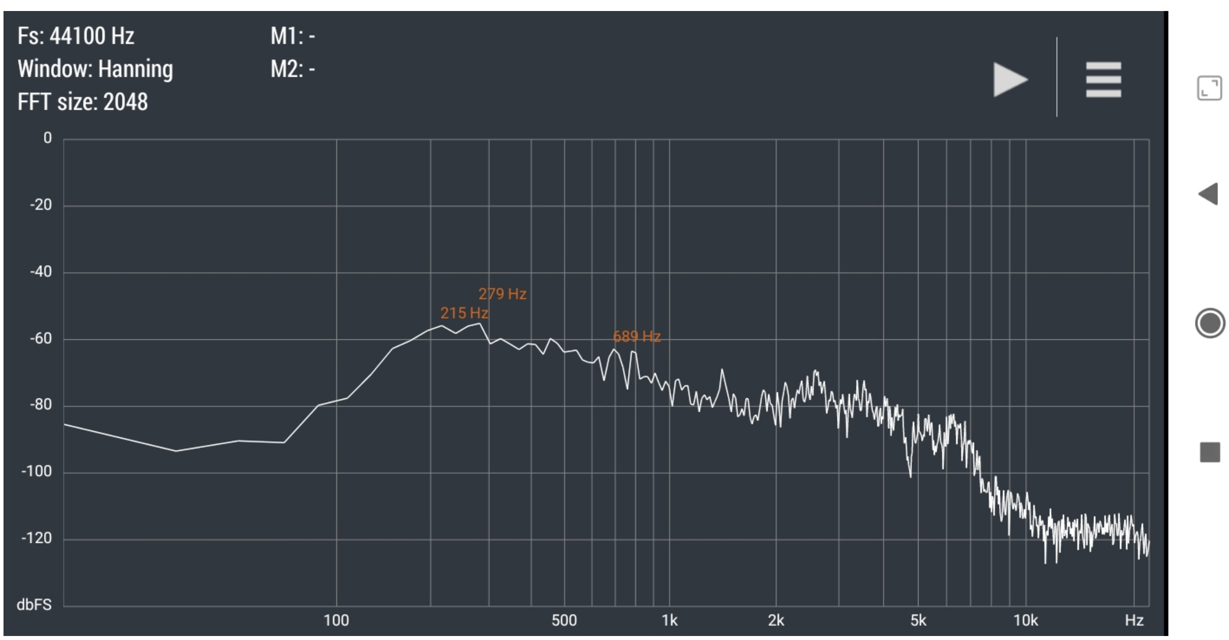
First of all, we observe that the length of the neck of the bottle is $L = 3cm$, and the length (height) of the bottle excluding the neck is $H = 25cm$. Also, the radius of the neck of the bottle is $r = 1.2cm$, and the radius of the cavity of the bottle is $R = 3.5cm$

Now, filling the bottle upto height $h = 14cm$, we get the following spectrum:



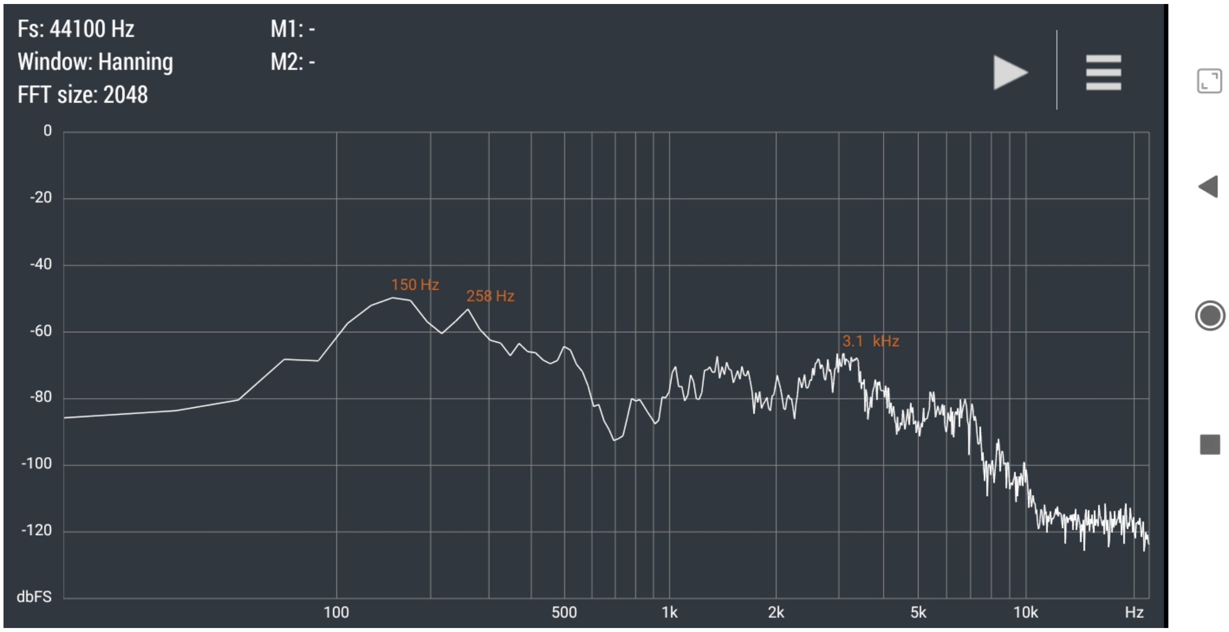
So, in this case, we observe that $f = f_1 = 215Hz$.
And, volume, $V_1 = \pi R^2(H-h) = \pi \cdot (3.5cm)^2 \cdot (25cm - 14cm) = 423.33cm^3$

We now reduce the water level upto a height $h = 11cm$, and we get the following spectrum:



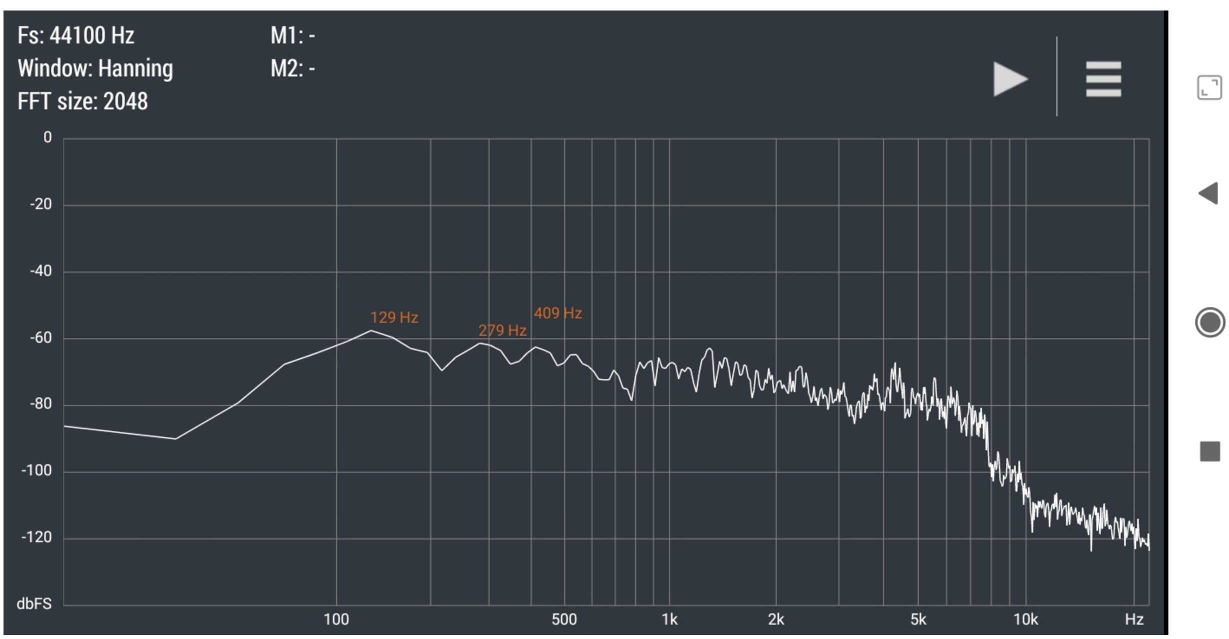
In this case, we observe that $f = f_2 = 215Hz$, but taking errors into consideration, we have $f = f_2 = 200Hz$.
And, volume, $V_2 = \pi R^2(H-h) = \pi \cdot (3.5cm)^2 \cdot (25cm - 11cm) = 538.78cm^3$

The water level is now reduced upto a height $h = 5cm$, and we get the following spectrum:



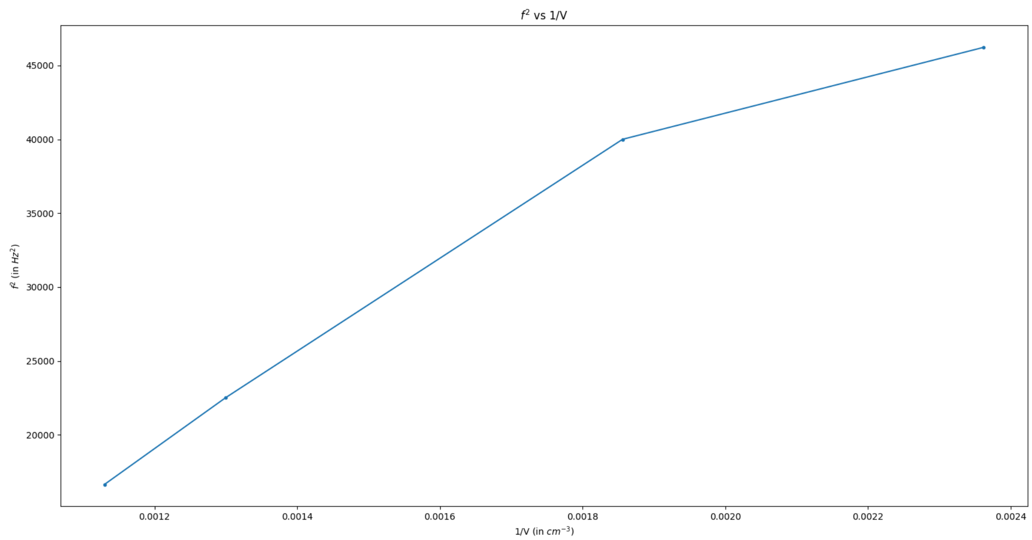
In this case, we observe that $f = f_3 = 150Hz$.
And, volume, $V_3 = \pi R^2(H-h) = \pi \cdot (3.5cm)^2 \cdot (25cm - 5cm) = 769.70cm^3$

And finally, the water level is reduced upto a height $h = 2cm$, and we get the following spectrum:



In this case, we observe that $f = f_4 = 129Hz$.
And, volume, $V_4 = \pi R^2(H-h) = \pi \cdot (3.5cm)^2 \cdot (25cm - 2cm) = 885.14cm^3$

Now, plotting the graph of f^2 vs $1/V$, we get the following curve:



We can see that the plot is almost linear, with slope, $m = 22997315.644 Hz^2 cm^3$

Also, from equation (15), we have:

$$c = 2\pi\sqrt{\frac{mL'}{A}} \tag{16}$$

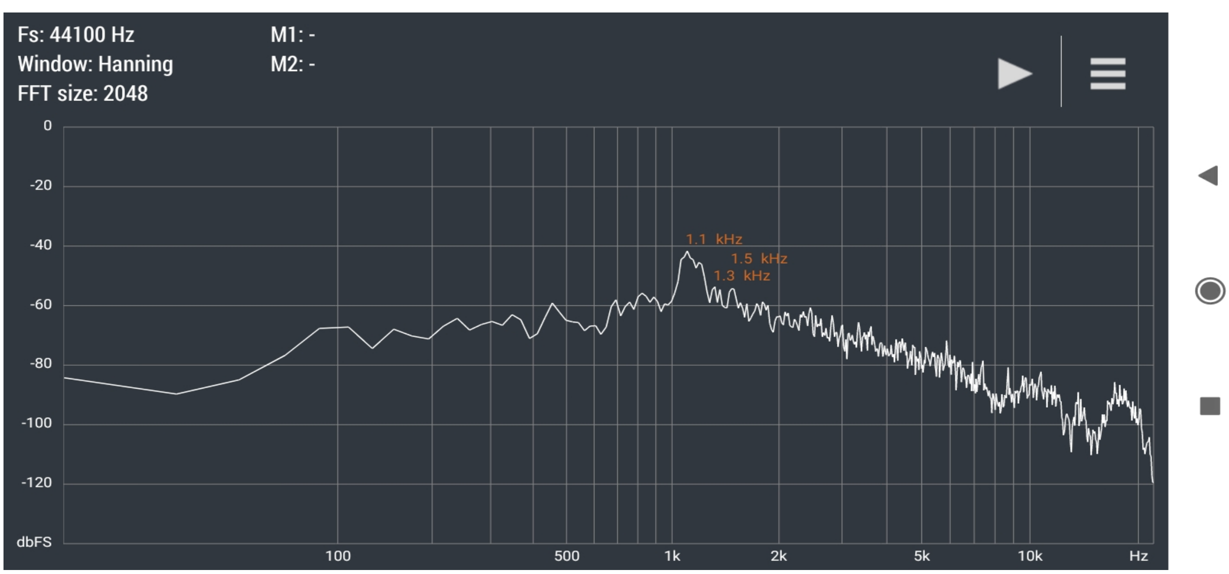
Putting values of $m = 22997315.644 Hz^2 cm^3, L' = L + 1.4a = 3cm + 1.4(1.2cm) = 4.68cm$, and $A = \pi(1.2cm)^2 = 4.52cm^2$, we get:

$$c = 2\pi\sqrt{\frac{22997315.644 Hz^2 cm^3 \cdot 4.68cm}{4.52cm^2}} = 306.6\ m/s \tag{17}$$

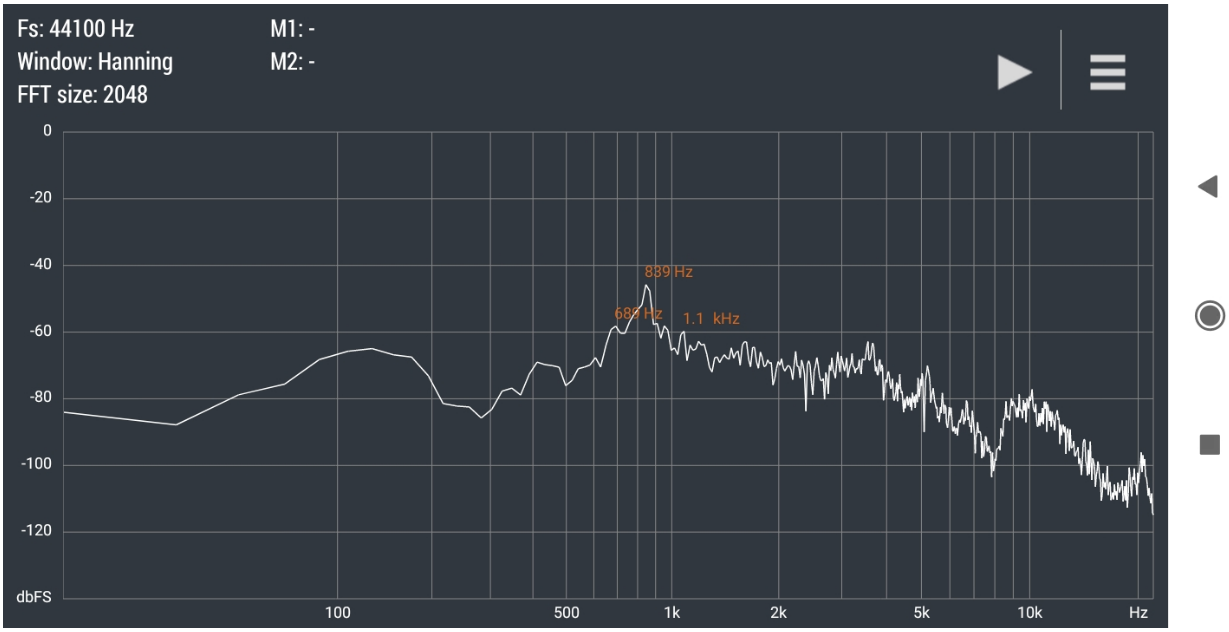
5.4 Observations and Calculations (for water vapour)

In this case, we obtain the following spectra:

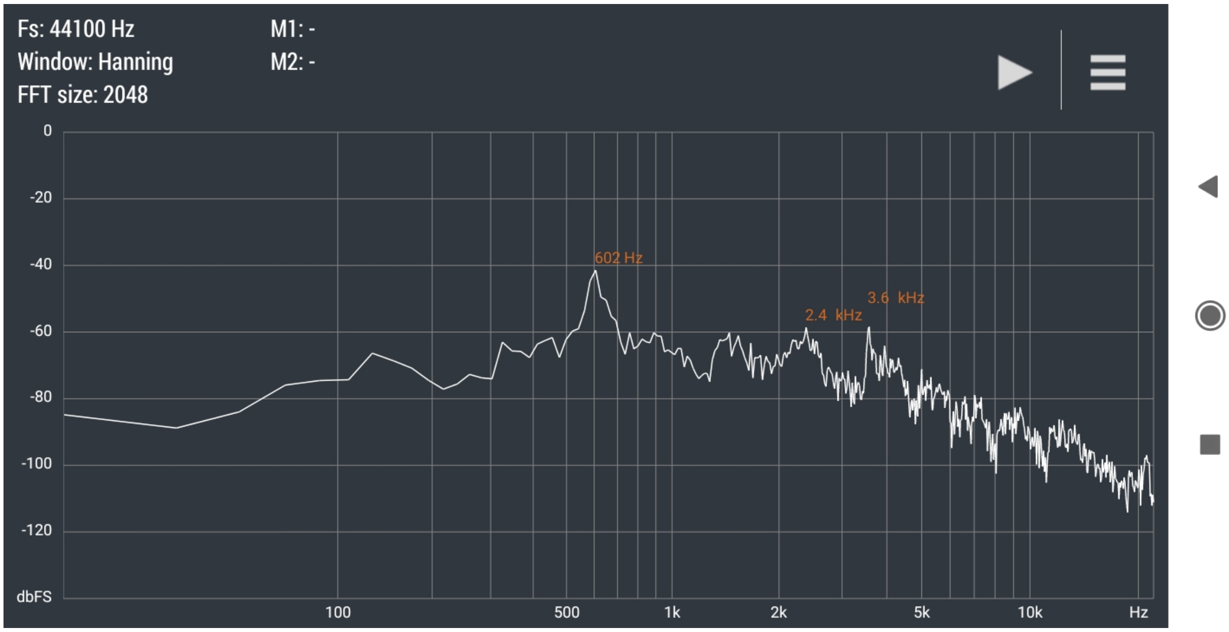
For $h = 14cm$:



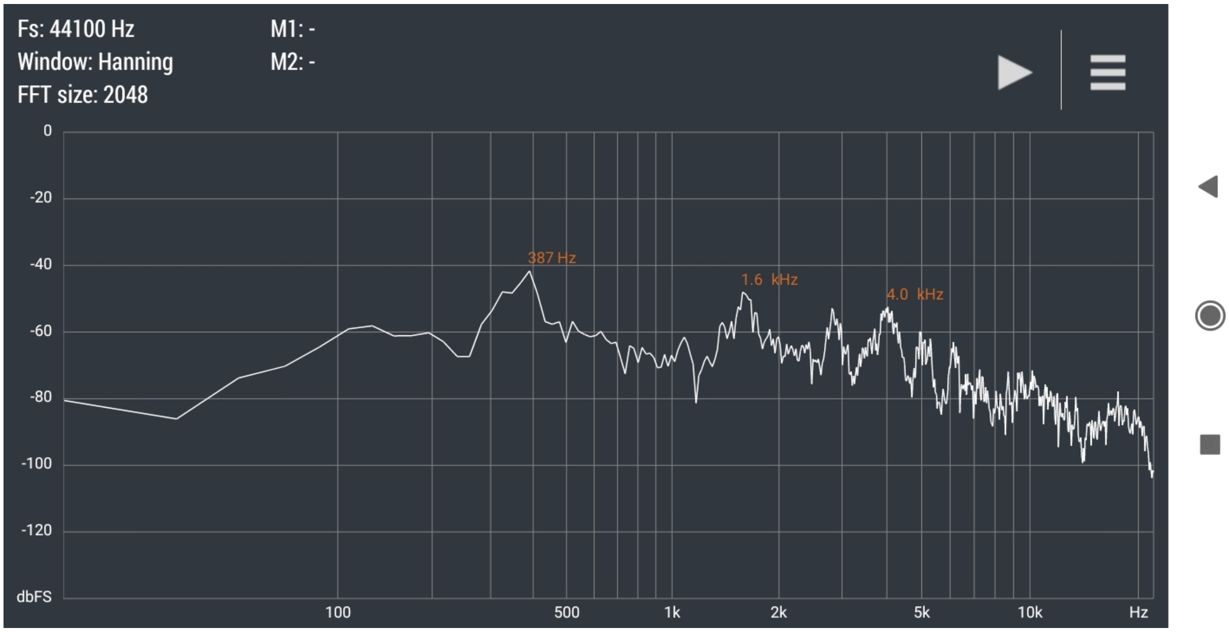
For $h = 11cm$:



For $h = 8cm$:



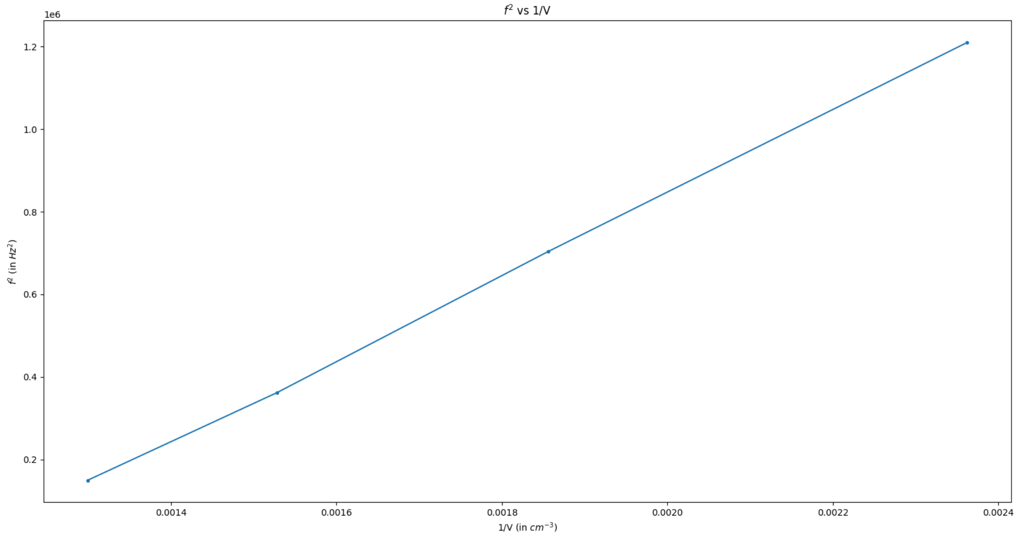
For $h = 5cm$:



Hence, we observe that:

- For $h_1 = 14cm$, $V_1 = 423.33cm^3$, $f_1 = 1100Hz$,
- For $h_2 = 11cm$, $V_2 = 538.78cm^3$, $f_2 = 839Hz$,
- For $h_3 = 8cm$, $V_3 = 654.24cm^3$, $f_3 = 602Hz$, and
- For $h_4 = 5cm$, $V_4 = 769.70cm^3$, $f_4 = 387Hz$.

Now, plotting the graph of f^2 vs $1/V$, we get the following curve:



We can see that the plot is almost linear, with slope, $m = 50358223.797Hz^2cm^3$

Again, from equation (15), we have:

$$c = 2\pi\sqrt{\frac{mL'}{A}} \tag{18}$$

Putting values of $m = 50358223.797Hz^2cm^3, L' = 4.68cm$, and $A = 4.52cm^2$, we get:

$$c = 2\pi\sqrt{\frac{50358223.797Hz^2cm^3 \cdot 4.68cm}{4.52cm^2}} = 453.7\ m/s \tag{19}$$

5.5 Possibilities of error

1. Slight error in reading the water level due to parallax errors
2. Inaccuracies in the sound that is going into the container, since the position of source may differ for different volumes
3. Existence of environmental noise may cause inaccuracies in the Spectrum Analyzer
4. Breathing may also have an unwanted effect on the Spectrum Analyzer

5.6 Conclusions

1. We observe that speed of sound in air, $c_{air} = 306.6\ m/s$ (taking errors into consideration)
2. Speed of sound in water vapours is $c_{vapour} = 453.7\ m/s$ (taking errors into consideration), which is considerably larger than that in air.