KLE Technological University
Creating Value, Leveraging Knowledge

DEPARTMENT OF MATHEMATICS

Mathematics Lab Assessment No. 1

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Question	1. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Coffee at 95°C temperature is poured into a cup in a room at a temperature of 20°C. (a) Write a differential equation that expresses Newton's Law of Cooling for this particular situation. What is the initial condition? (b) Prepare a graphical plot of the solution to this initial-value problem. Execute a Python program to compare (i) Cooling rate for coffee at initial temperature of 95°C decremented by 5°C till 70°C. (ii) Warming of ice-cream at -15°C placed in the room at 20°C
Solution	i) Identify the parameters and mathematical concept temperature is dependent time is independent surrounding temperature constant
	ii) Solve analytically

Newton's law of Cooling states that the rate of cooling of an object is proportional to the temp. difference b/w the object and its surroundings.
∴ temp. of Coffee, $T(t)$, satisfies the D.E.

$$\frac{dT}{dt} = -k\left[T(t) - T_{room}\right] \leftarrow \text{D.E.}$$

$$T(0) = 95^\circ C \; ; \; T_{room} = 20^\circ C$$

By separating the variable

$$\int_{T_0}^{T} \frac{dT}{T(t) - T_{room}} = \int_0^t -k\,dt$$

$$\ln\left[T(t) - T_{room}\right] = \left[-kt\right].$$

General soln

$$\therefore \; T(t) = Ce^{-kT} + T_{room} \rightarrow (1)$$

$$\therefore \; T(t) - T_{room} = Ce^{-kt} \rightarrow (2)$$

In order to solve for constant C & k we need additional BC.

Ex. 1: $T(0) = 95^\circ C$; $T_{room} = 20^\circ C$; $T(10\,min) = 45^\circ c$

Eqn (1) gives

$$95 = Ce^{-k(0)} + 20; \quad C = 95 - 20 = 75$$

$$\therefore 45 = 75e^{-k(10)} + 20; \quad \text{i.e} \quad \frac{45-20}{75} = e^{-10k}$$

$$k = 0.1098\,min^{-1}$$

$$\text{P.I} = T(t) = 75e^{-0.1098t} + 20.$$

iii) GeoGebra Screenshot / Program Execution

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```
import numpy as np
import matplotlib.pyplot as plt
k=0.0715
T0=95
Ts=20
t0=0
tmax=20
h=1
i=0
t=np.zeros(tmax+1)
T=np.zeros(tmax+1)
t[0]=t0
T[0]=T0
def f(T,t):
   return (-k*(T-Ts))
for i in range(tmax):
   k1=h*f(T[i],t[i])
   k2=h*f(T[i]+0.5*k1,t[i]+0.5*h)
   k3=h*f(T[i]+0.5*k2,t[i]+0.5*h)
   k4=h*f(T[i]+k3,t[i]+h)
   T[i+1]=T[i]+(1/6)*(k1+2*k2+2*k3+k4)
   t[i+1]=t[i]+h
print (t,T)
#time to sip coffee of temperature
temp=[75,60]
for tem in temp:
  timetosip = None
  for i in range (len(T)):
      if T[i]<=tem:
        timetosip=t[i]
        break
  if timetosip is not None:
      print('time is',timetosip)
  else:
      print('time is',timetosip)
plt.plot(t,T)
```

iv) Results and analysis from the graph

```
[ 0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 13. 14. 15. 16. 17.
 18. 19. 20.] [95.         89.82472197 85.00655732 80.52086388 76.34469992 72.45670676
 68.83699953 65.46706552 62.32966946 59.40876535 56.68941445 54.15770884
 51.80070031 49.6063341  47.5633873  45.6614114  43.89067888 42.24213348
 40.70734382 39.27846033 37.9481751 ]
time is 5.0
time is 9.0
[<matplotlib.lines.Line2D at 0x7e4164b7b790>]
```