PRINCIPLE COMPONENTS ANALYSIS

x_1	2.5	0.5	2.2	1.9	3.1	2.3	2.0	1.0	1.5	1.1
x_2	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	1.6	0.9

$n = 10$

$\bar{x}_1 = 1.81 \quad \bar{x}_2 = 1.\cancel{85}91$

For covariance matrix:

$\sigma_{xx} = (0.4761 + 1.7161 + 0.1521 + 0.0081 + 1.6641 + 0.2401 + 0.0361 + 0.6561 + 0.0961 + 0.5041)/10$

$= 0.5549$

$\sigma_{xy} = \sigma_{yx} = (0.3381 + 1.5851 + 0.3861 + 0.0261 + 1.4061 + 0.3871 - 0.0589 + 0.6561 + 0.0961 + 0.7171)/10$

$= 0.5539$

$\sigma_{yy} = (0.2401 + 1.2641 + 0.9801 + 0.0841 + 1.1881 + 0.6241 + 0.0961 + 0.6561 + 0.0961 + 1.0201)$

$= 0.6449$

Covariance matrix: $\begin{bmatrix} 0.5549 & 0.5539 \\ 0.5539 & 0.6449 \end{bmatrix}$

For eigen values: $\begin{vmatrix} 0.5549-\lambda & 0.5539 \\ 0.5599 & 0.6449-\lambda \end{vmatrix} = 0 \Rightarrow$

$(0.5549-\lambda)(0.6449-\lambda) = 0.5539^2 = 0.30680521$

$\Rightarrow 0.35785501 - 1.1998\lambda + \lambda^2 = 0.30680521$

$\Rightarrow \lambda = \dfrac{1.1998 \pm \sqrt{1.1998^2 - 4(0.05510498}}{2} = \dfrac{1.1998 \pm 1.11149882}{2}$

$\Rightarrow \lambda_1 = \dfrac{0.88650118}{2} \quad \Rightarrow \lambda_2 = \dfrac{2.31129882}{2}$

$= 0.44327059 \quad = 1.15624941$

λ_2 is chosen as it is a ~~stoge~~ stronger value.

$\begin{bmatrix} -0.6007 & 0.5539 \\ 0.5539 & -0.5107 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} -0.6007v_1 + 0.5539v_2 = 0 \\ 0.5539v_1 - 0.5107v_2 = 0 \end{matrix}$

$\Rightarrow \boxed{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$