Index

Name	Formula	E(X)	Var(X)
Binomial Distribution	$P(X=x) = {}^{n}C_{x}\, p^{x}(1-p)^{(n-x)}$	np	np(1 - p)
Geometric Distribution	P(X = x) = $(1\text{-}p)^{x-1}$ * p	1/p	$(1 – p)/p^2$
Negative binomial Distribution	$P(X=x) = \binom{x-1}{r-1} p^{r} q^{x-r}$ where q = 1 - p	r/p	$r(1 – p)/p^2$
Poisson Distribution	$P(X=x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$	λ	λ
Hypergeometric Distribution	$\mathrm{P}(X=k) = \dfrac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	$\dfrac{nm}{N}$	$\frac{nm}{N}\left(1-\frac{m}{N}\right)\left(\frac{N-n}{N-1}\right)$
Multinomial Distribution	$\mathrm{P}(X_1 = x_1 \text{ and } \ldots \text{ and } X_k = x_k)$ $= \begin{cases} \dfrac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}, & \text{when } \sum_{i=1}^{k} x_i = n \\ 0 & \text{otherwise,} \end{cases}$		
Normal Distribution	$f(X) = \dfrac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{(x-\mu)^2}{2\sigma^2}}$		
Lognormal Distribution	$f(x) = \left\{ \dfrac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\dfrac{(\ln x - \mu)^2}{2\sigma^2}\right],\ x \succ 0 \right.$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2/2}\,(e^{\sigma^2} - 1)$
Exponential Distribution	$f(x) = \begin{cases} \lambda e^{-\lambda x} & ,x \geq 0 \\ 0 & ,x < 0 \end{cases}$ CDF: $F(x) = \begin{cases} 1 - e^{-\lambda x} & ,x \geq 0 \\ 0 & ,x < 0 \end{cases}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$
Uniform distribution	$f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\dfrac{a+b}{2}$	$\dfrac{b-a}{\sqrt{12}}$
Gamma Distribution	Gamma function: $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt$ PDF of Gamma Distribution: $f(x) = \begin{cases} \dfrac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} & \text{at } x > 0, \\ 0 & \text{at } x \leq 0. \end{cases}$	$\dfrac{\alpha}{\lambda}$	$\dfrac{\alpha}{\lambda^2}.$
Weibull Distribution	$f(x) = \begin{cases} \dfrac{k}{\lambda}\left(\dfrac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\lambda\,\Gamma(1+1/k)$	$\lambda^2\left[\Gamma\left(1+\frac{2}{k}\right) - \left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right]$

Define $\tilde{n} = n + 4$, and $\tilde{p} = \dfrac{X+2}{\tilde{n}}$. Then a level $100(1-\alpha)\%$ confidence interval for p is

$$\tilde{p} \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \tag{5.5}$$

Define $\tilde{n} = n + 4$, and $\tilde{p} = \dfrac{X+2}{\tilde{n}}$. Then a level $100(1-\alpha)\%$ lower confidence bound for p is

$$\tilde{p} - z_{\alpha}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$$

and level $100(1-\alpha)\%$ upper confidence bound for p is

$$\tilde{p} + z_{\alpha}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$$

If the lower bound is less than 0, replace it with 0. If the upper bound is greater than 1, replace it with 1.

$$\overline{X} - \overline{Y} \pm z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$$

Let X be the number of successes in n_X independent Bernoulli trials with success probability p_X, and let Y be the number of successes in n_Y independent Bernoulli trials with success probability p_Y, so that $X \sim \text{Bin}(n_X,\ p_X)$ and $Y \sim \text{Bin}(n_Y,\ p_Y)$. Define $\tilde{n}_X = n_X + 2$, $\tilde{n}_Y = n_Y + 2$, $\tilde{p}_X = (X+1)/\tilde{n}_X$, and $\tilde{p}_Y = (Y+1)/\tilde{n}_Y$.

Then a level $100(1-\alpha)\%$ confidence interval for the difference $p_X - p_Y$ is

$$\tilde{p}_X - \tilde{p}_Y \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}_X(1-\tilde{p}_X)}{\tilde{n}_X} + \frac{\tilde{p}_Y(1-\tilde{p}_Y)}{\tilde{n}_Y}}$$

Compute $\widehat{p}_X = \dfrac{X}{n_X}$, $\widehat{p}_Y = \dfrac{Y}{n_Y}$, and $\widehat{p} = \dfrac{X+Y}{n_X+n_Y}$.

Compute the z-score: $z = \dfrac{\widehat{p}_X - \widehat{p}_Y}{\sqrt{\widehat{p}(1-\widehat{p})(1/n_X + 1/n_Y)}}$

The Chi-Square Goodness of fit test

$$\chi^2 = \sum_{i=1}^{k}\frac{(O_i - E_i)^2}{E_i}$$

The Chi-Square Test for Homogeneity, Independence

$E_{r,c} = (n_r * n_c) / n$

$$\chi^2 = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Or

$$r = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \overline{y})^2}}$$

$$\sum_{i=1}^{n} (x_i - \overline{x})^2 = \sum_{i=1}^{n} x_i^2 - n\overline{x}^2$$

$$\sum_{i=1}^{n} (y_i - \overline{y})^2 = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$$

$$\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) = \sum_{i=1}^{n} x_i y_i - n\overline{x}\,\overline{y}$$

Let X and Y be random variables with the bivariate normal distribution.
Let ρ denote the population correlation between X and Y.
Let $(x_1, y_1), \ldots, (x_n, y_n)$ be a random sample from the joint distribution of X and Y.
Let r be the sample correlation of the n points.
Then the quantity

$$W = \frac{1}{2} \ln \frac{1+r}{1-r} \tag{7.4}$$

is approximately normally distributed, with mean given by

$$\mu_W = \frac{1}{2} \ln \frac{1+\rho}{1-\rho} \tag{7.5}$$

and variance given by

$$\sigma_W^2 = \frac{1}{n-3} \tag{7.6}$$

- Find CI for μ_W as:

$$W \pm z_{\alpha/2} * \sigma$$

- Use upper and lower confidence bounds of μ_W to find CI for ρ using the following inequality:

$$\rho = \frac{e^{2\mu_W} - 1}{e^{2\mu_W} + 1}$$

For testing null hypotheses of the form

- $\rho = 0$,
- $\rho \leq 0$, or
- $\rho \geq 0$,

a simpler procedure is available. When $\rho = 0$, the quantity

$$U = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

has a Student's t distribution with $n - 2$ degrees of freedom.

Equation of least-squares line

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x$$

$\hat{\beta}_0$ and $\hat{\beta}_1$ are called least-squares coefficients, estimates of β_0 and β_1

Fitted value $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

Residual associated with the point (x_i, y_i)

$$e_i = y_i - \hat{y}_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)$$

The least-squares line is the line that minimizes the sum of the squared residuals, i.e.,

$\sum_{i=1}^{n} e_i^2$ is minimized

$$e_i = y_i - \hat{y}_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)$$

Therefore $\hat{\beta}_0$ and $\hat{\beta}_1$ are the quantities that minimize the sum

$$S = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

These quantities are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n} (x_i - \overline{x})^2}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

Intepreting slope in terms of r

$$\frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} = r\frac{s_y}{s_x} \quad \Rightarrow \quad \widehat{\beta}_1 = r\frac{s_y}{s_x}$$

Another form of least-squares line

Substituting $\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1\overline{x}$ in least-squares line

$$y_i = \widehat{\beta}_0 + \widehat{\beta}_1 x$$

we get,

$$\widehat{y} - \overline{y} = \widehat{\beta}_1(x - \overline{x})$$

$$\Rightarrow \quad \widehat{y} - \overline{y} = r\frac{s_y}{s_x}(x - \overline{x})$$

Goodness of fit

$\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$ the **error sum of squares**

$\sum_{i=1}^{n}(y_i - \overline{y})^2$ the **total sum of squares**.

$\sum_{i=1}^{n}(y_i - \overline{y})^2 - \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$ the **regression sum of squares.**

Total sum of squares = Regression sum of squares + Error sum of squares

$$r^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}}$$

Estimate of error variance

$$s^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-2} = \frac{\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{n-2}$$

or

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{n-2} = \frac{(1-r^2)\sum_{i=1}^{n}(y_i - \overline{y})^2}{n-2}$$

In the linear model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, under assumptions 1 through 4, the observations $y_1, \ldots, y_n$ are independent random variables that follow the normal distribution. The mean and variance of y_i are given by

$$\mu_{y_i} = \beta_0 + \beta_1 x_i$$

$$\sigma_{y_i}^2 = \sigma^2$$

The slope β_1 represents the change in the mean of y associated with an increase of one unit in the value of x.

Under assumptions 1 through 4 (page 540),

- The quantities $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are normally distributed random variables.
- The means of $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are the true values β_0 and β_1, respectively.
- The standard deviations of $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are estimated with

$$s_{\widehat{\beta}_0} = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}} \qquad (7.36)$$

$\widehat{\beta}_0$ ~ N(β 0 , $s_{\widehat{\beta}_0}$)

and

$\widehat{\beta}_1$ ~ N(β 1 , $s_{\widehat{\beta}_1}$)

$$s_{\widehat{\beta}_1} = \frac{s}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}} \qquad (7.37)$$

where $s = \sqrt{\frac{(1 - r^2)\sum_{i=1}^{n}(y_i - \overline{y})^2}{n - 2}}$ is an estimate of the error standard deviation σ.

Level $100(1 - \alpha)\%$ confidence intervals for β_0 and β_1 are given by

$$\widehat{\beta}_0 \pm t_{n-2,\alpha/2} \cdot s_{\widehat{\beta}_0} \qquad \widehat{\beta}_1 \pm t_{n-2,\alpha/2} \cdot s_{\widehat{\beta}_1}$$

where

$$s_{\widehat{\beta}_0} = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}} \qquad s_{\widehat{\beta}_1} = \frac{s}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}}$$

Standard Normal Probabilities

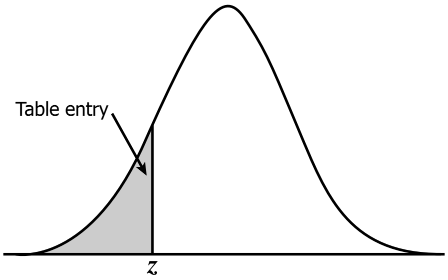


Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

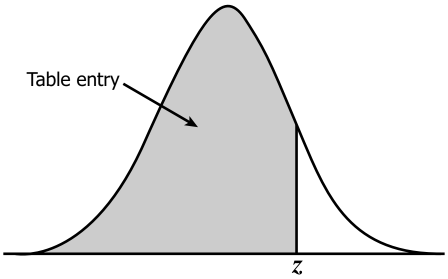
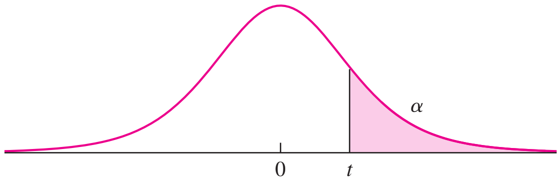


Table entry for z is the area under the standard normal curve to the left of z.

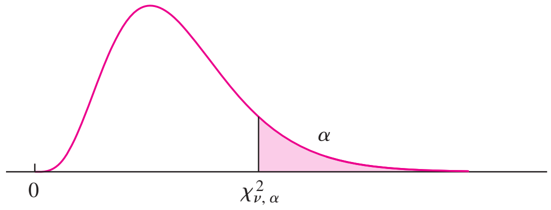
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A.3 Upper percentage points for the Student's t distribution



ν	α 0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

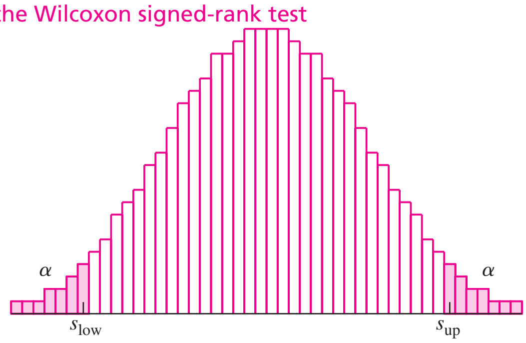
TABLE A.7 Upper percentage points for the χ^2 distribution



	α									
ν	**0.995**	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.893	62.883
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

For $\nu > 40$, $\chi^2_{\nu,\alpha} \approx 0.5(z_\alpha + \sqrt{2\nu - 1})$.

TABLE A.5 Critical points for the Wilcoxon signed-rank test



n	s_{low}	s_{up}	α
4	1	9	0.1250
	0	10	0.0625
5	3	12	0.1562
	2	13	0.0938
	1	14	0.0625
	0	15	0.0312
6	4	17	0.1094
	3	18	0.0781
	2	19	0.0469
	1	20	0.0312
	0	21	0.0156
7	6	22	0.1094
	5	23	0.0781
	4	24	0.0547
	3	25	0.0391
	2	26	0.0234
	1	27	0.0156
	0	28	0.0078
8	9	27	0.1250
	8	28	0.0977
	6	30	0.0547
	5	31	0.0391
	4	32	0.0273
	3	33	0.0195
	2	34	0.0117
	1	35	0.0078
	0	36	0.0039
9	11	34	0.1016
	10	35	0.0820
	9	36	0.0645
	8	37	0.0488
	6	39	0.0273
	5	40	0.0195
	4	41	0.0137
	3	42	0.0098
	2	43	0.0059
	1	44	0.0039

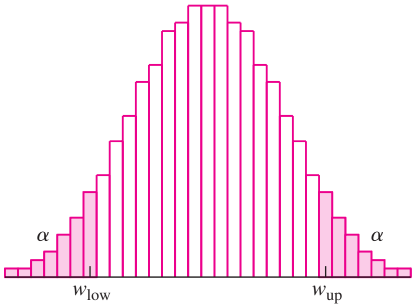
n	s_{low}	s_{up}	α
10	15	40	0.1162
	14	41	0.0967
	11	44	0.0527
	10	45	0.0420
	9	46	0.0322
	8	47	0.0244
	6	49	0.0137
	5	50	0.0098
	4	51	0.0068
	3	52	0.0049
11	18	48	0.1030
	17	49	0.0874
	14	52	0.0508
	13	53	0.0415
	11	55	0.0269
	10	56	0.0210
	8	58	0.0122
	7	59	0.0093
	6	60	0.0068
	5	61	0.0049
12	22	56	0.1018
	21	57	0.0881
	18	60	0.0549
	17	61	0.0461
	14	64	0.0261
	13	65	0.0212
	10	68	0.0105
	9	69	0.0081
	8	70	0.0061
	7	71	0.0046
13	27	64	0.1082
	26	65	0.0955
	22	69	0.0549
	21	70	0.0471
	18	73	0.0287
	17	74	0.0239
	13	78	0.0107

n	s_{low}	s_{up}	α
	12	79	0.0085
	10	81	0.0052
	9	82	0.0040
14	32	73	0.1083
	31	74	0.0969
	26	79	0.0520
	25	80	0.0453
	22	83	0.0290
	21	84	0.0247
	16	89	0.0101
	15	90	0.0083
	13	92	0.0054
	12	93	0.0043
15	37	83	0.1039
	36	84	0.0938
	31	89	0.0535
	30	90	0.0473
	26	94	0.0277
	25	95	0.0240
	20	100	0.0108
	19	101	0.0090
	16	104	0.0051
	15	105	0.0042
16	43	93	0.1057
	42	94	0.0964
	36	100	0.0523
	35	101	0.0467
	30	106	0.0253
	29	107	0.0222
	24	112	0.0107
	23	113	0.0091
	20	116	0.0055
	19	117	0.0046
17	49	104	0.1034
	48	105	0.0950
	42	111	0.0544
	41	112	0.0492

n	s_{low}	s_{up}	α
	35	118	0.0253
	34	119	0.0224
	28	125	0.0101
	27	126	0.0087
	24	129	0.0055
	23	130	0.0047
18	56	115	0.1061
	55	116	0.0982
	48	123	0.0542
	47	124	0.0494
	41	130	0.0269
	40	131	0.0241
	33	138	0.0104
	32	139	0.0091
	28	143	0.0052
	27	144	0.0045
19	63	127	0.1051
	62	128	0.0978
	54	136	0.0521
	53	137	0.0478
	47	143	0.0273
	46	144	0.0247
	38	152	0.0102
	37	153	0.0090
	33	157	0.0054
	32	158	0.0047
20	70	140	0.1012
	69	141	0.0947
	61	149	0.0527
	60	150	0.0487
	53	157	0.0266
	52	158	0.0242
	44	166	0.0107
	43	167	0.0096
	38	172	0.0053
	37	173	0.0047

For $n > 20$, compute $z = \frac{S_+ - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}$ and use the z table (Table A.2).

TABLE A.6 Critical points for the Wilcoxon rank-sum test



m	n	w_{low}	w_{up}	α
2	5	4	12	0.0952
		3	13	0.0476
	6	4	14	0.0714
		3	15	0.0357
	7	4	16	0.0556
		3	17	0.0278
	8	5	17	0.0889
		4	18	0.0444
		3	19	0.0222
3	4	7	17	0.0571
		6	18	0.0286
	5	8	19	0.0714
		7	20	0.0357
		6	21	0.0179
	6	9	21	0.0833
		8	22	0.0476
		7	23	0.0238
	7	9	24	0.0583
		8	25	0.0333
		7	26	0.0167
		6	27	0.0083
	8	10	26	0.0667
		9	27	0.0424
		8	28	0.0242
		7	29	0.0121
		6	30	0.0061
4	4	12	24	0.0571
		11	25	0.0286
		10	26	0.0143
	5	13	27	0.0556
		12	28	0.0317
		11	29	0.0159
		10	30	0.0079
	6	14	30	0.0571
		13	31	0.0333
		12	32	0.0190
		11	33	0.0095
		10	34	0.0048
	7	15	33	0.0545
		14	34	0.0364
		13	35	0.0212
		12	36	0.0121
		11	37	0.0061
		10	38	0.0030
	8	16	36	0.0545
		15	37	0.0364
		14	38	0.0242
		13	39	0.0141
		12	40	0.0081
		11	41	0.0040
5	5	20	35	0.0754
		19	36	0.0476
		18	37	0.0278
		17	38	0.0159
		16	39	0.0079
		15	40	0.0040
	6	21	39	0.0628
		20	40	0.0411
		19	41	0.0260
		18	42	0.0152
		17	43	0.0087
		16	44	0.0043
	7	22	43	0.0530
		21	44	0.0366
		20	45	0.0240
		19	46	0.0152
		18	47	0.0088
		17	48	0.0051
		16	49	0.0025
	8	24	46	0.0637
		23	47	0.0466
		22	48	0.0326
		21	49	0.0225
		20	50	0.0148
		19	51	0.0093
		18	52	0.0054
		17	53	0.0031
6	6	29	49	0.0660
		28	50	0.0465
		27	51	0.0325
		26	52	0.0206
		25	53	0.0130
		24	54	0.0076
		23	55	0.0043
	7	30	54	0.0507
		29	55	0.0367
		28	56	0.0256
		27	57	0.0175
		26	58	0.0111
		25	59	0.0070
		24	60	0.0041
	8	32	58	0.0539
		31	59	0.0406
		30	60	0.0296
		29	61	0.0213
		28	62	0.0147
		27	63	0.0100
		26	64	0.0063
		25	65	0.0040
7	7	40	65	0.0641
		39	66	0.0487
		37	68	0.0265
		36	69	0.0189
		35	70	0.0131
		34	71	0.0087
		33	72	0.0055
		32	73	0.0035
	8	42	70	0.0603
		41	71	0.0469
		39	73	0.0270
		38	74	0.0200
		36	76	0.0103
		35	77	0.0070
		34	78	0.0047
8	8	52	84	0.0524
		51	85	0.0415
		50	86	0.0325
		49	87	0.0249
		46	90	0.0103
		45	91	0.0074
		44	92	0.0052
		43	93	0.0035

When m and n are both greater than 8, compute $z = \frac{W - m(m+n+1)/2}{\sqrt{mn(m+n+1)/12}}$ and use the z table (Table A.2).