# lab7 2024 v1

## April 3, 2024

# 1 Lab 7: Predicting Social Mobility using Cross Validation and Random Forests

## 1.1 Methods/concepts: loops, steady states, random forests, cross validation

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#### LAB DESCRIPTION

This is the second lab on prediction policy questions. In this lab, you will predict upward mobility using decision trees and random forests. The measure of upward mobility that we will focus on is **Statistic 1: Absolute Mobility at the 25th Percentile** in each county (kfr\_pooled\_pooled\_p25). For more details on the variables included in these data, see Table 1.

The "training" dataset is a 50% random sample of all counties with at least 10,000 residents available from the Opportunity Atlas. You will use 121 community characteristics to predict the variable **kfr\_pooled\_pooled\_p25**. The other half of these data has been set aside as a "lock box" data set that you will use to evaluate your models. The training data set contains predictors for both the test and training sets to avoid data snooping, but only outcomes for the training set. The lock box data set contains only the outcomes for the test set.

In the R labs, we will start from starter scripts that can either be run on your computer or the Ec 50 Jupyter Hub.

## 1.2 QUESTIONS

1. Primer on for loops in the context of steady states. We will start with a review of the calculation from the Lecture where Professor Chetty introduced the concept of a steady state (Becker and Tomes 1979). This review will also give us an opportunity to walk through loops step by step. Chetty et al. (2020) report the following rank-rank regression pooling all races and genders:

$$Rank_{kids} = 33.31 + 0.351Rank_{parents}$$

Using the sample code, show that this model predicts convergence in incomes across racial groups. This result is unrealistic because racial disparities have persisted for many generations in the U.S. However, the model is incorrect: we know from Lab 2 and Lecture that children of different races experience very different rates of upward mobility across generations. In particular, Chetty et al. (2020) report the following rank-rank regression for Black children:

$$Rank_{kids} = 25.4 + 0.28 Rank_{parents}$$

and for Hispanic children:

$$Rank_{kids} = 36.14 \ + \ 0.26 Rank_{parents}$$

Use a for loop to find the **steady state prediction** of the model for Black and Hispanic children.

```
[1]: # The following program starts with a primer on loops in the context of steady
     # states. You'll modify the example loop to study how differences in intergen-
       erational mobility by race and ethnicity affects inequality in the long run.
     #
     # The next part of the program shows how to implement cross validation and
     # random forests using data from the Opportunity Atlas. The example code_
      \hookrightarrow illustrates
     # 10-fold cross validation. You have to modify the code to instead implement
      \hookrightarrow 5 fold
     # cross validation. You also have to choose your own predictors. After
      ⇔running the
     # loop for cross validation, the example code shows how to plot the cross_
      \rightarrow validation
       RMSPE as a function of the tree depth.
      The next step is to use the cross validation graph to choose the optimal
      → tree depth
       and implement a decision tree with that chosen depth.
     #
       Then the code illustrates how to estimate two random forests (one with
      hand picked predictors and the other using all the predictors), generate
       predictions, and make variable importance plot for the large random forest.
       The last part of the code calculates the RMSPE for the three models using
       the training data (atlas training.dta) and the "lock box data"
      \hookrightarrow (atlas_lockbox.dta)
     #
     # The code may have some typos -- please be on the look out for them -- and to
     # receive credit for the lab you have to make edits to estimate your own
     # decision tree and random forests. These are simply examples of what you\square
      \rightarrow might
     # what to do in your analysis, but you are expected to make an effort to
```

```
understand what you are doing with the code.
#
 Inputs: atlas_training.dta and atlas_lockbox.dta (download from canvas)
            randomForest to estimate random forest models
#
            rpart library to estimate decision trees
#
            tidyverse library for data manipulations
            haven library to load stata data sets into R
#
 Outputs: figure1.png, figure2.png, figure3.png
# Question 1 example code
rm(list=ls()) # removes all objects from the environment
# Install packages (if necessary) and load required libraries
if (!require(haven)) install.packages("haven"); library(haven)
if (!require(randomForest)) install.packages("randomForest");
→library(randomForest)
if (!require(rpart)) install.packages("rpart"); library(rpart)
if (!require(tidyverse)) install.packages("tidyverse"); library(tidyverse)
#Set seed for cross validation and random forests
HUID <- 50505050 #Replace with your HUID
set.seed(HUID)
# Primer on loops in the context of steady states
# Chetty et al. (2020) report the following rank-rank regression pooling all
# races and genders: Rank \{kids\} = 33.31 + 0.351 * Rank \{parents\}
# The average income rank for white parents today is the $57.9$th percentile.
# We predict that the average white child will reach the following percentile
# when they are adults:
parents_rank <- 57.9
kids_rank <- 33.31 + 0.351 * parents_rank
kids rank
# For the next generation, the predicted rank for the (average) white grandchild
# is:
parents_rank = kids_rank
kids_rank = 33.31 + 0.351 * parents_rank
kids rank
# Now we may want to study the predictions from this model across many
```

```
# generations: the grandchilden, the great grandchildren, and the
# great great grand children, and so on. We can do this using a loop
generations <- seq(1,7,1) #This is for generations 1 through 7
parents_rank = 57.9 #This is the starting value for the parent's generation
# use a for loop to run the experiment
for(j in generations){
  #Calculate kid's predicted rank
 kids_rank <- 33.31 + 0.351 * parents_rank
  #Print the output to the console
 print(paste0("In generation ", j, ", parent_rank = ", parents_rank, ", __
 ⇔child_rank = ", kids_rank))
  #Set parent's rank equal to kids' rank so we are ready for the next iteration
 parents_rank <- kids_rank</pre>
}
# Notice that the model predicts that the average white child will reach the
# 51.3rd percentile in generation 7, which is lower than where the first
# generation started from (which you will recall was the $57.9$th percentile).
# This is a reduction of around 6 percentiles.
# Meanwhile, the average income rank for Black parents is the 32.7th percentile.
# If we apply the same rank-rank relationship between Black parents and Black
# children that we did above, our model predicts that the average Black child
# will reach the 44.9th percentile when they grow up, with further gains for
# future generations.
generations <- seq(1,7,1) #This is for generations 1 through 7
parents_rank = 32.7 #This is the starting value for the parent's generation
for(j in generations){
 kids_rank <- 33.31 + 0.351 * parents_rank
print(paste0("In generation ", j, ", parent_rank = ", parents_rank, ", u

→child_rank = ", kids_rank))
 parents_rank <- kids_rank</pre>
# As you can see, this model predicts substantial improvements in outcomes for
# Black children across generations, with average gains of 12 percentiles in a
# single generation. The model makes the unrealistic prediction of convergence
\# in outcomes across racial groups: By generation 7, both white and Black_{\sqcup}
⇔children
# are at the 51.3 percentile. This result is unrealistic because racial.
 \rightarrow disparities
```

```
# have persisted for many generations in the United States.
#
# However, the model is incorrect: we know from Lab 2 and Lecture that Black
# children and white children experience very different rates of upward_
 \hookrightarrow mobility
# across generations. In particular, Chetty et al. (2020) report the following
# rank-rank regression for Black children:
\# Rank_{kids} = 25.4 + 0.28 * Rank_{parents}
# The prediction implied by this rank-rank graph is very different than the
# previous calculations suggested. To see this, in the first generation this
# model predicts:
parents_rank = 32.7
kids_rank = 25.4 + 0.28 * parents_rank
kids rank
# In stark contrast to the predictions from the calculations earlier, here
# there is hardly any predicted improvement in the income rank for Black
 \hookrightarrow children
# in generation 1. To see what happens across generations, we can update our
# loop:
generations <- seq(1,7,1) #This is for generations 1 through 7
parents_rank = 32.7 #This is the starting value for the parent's generation
for(j in generations){
  kids_rank = 25.4 + 0.28 * parents_rank
  print(paste0("In generation ", j, ", parent_rank = ", parents_rank, ", __

child_rank = ", kids_rank))
  parents_rank <- kids_rank</pre>
}
Loading required package: haven
Loading required package: randomForest
randomForest 4.7-1.1
Type rfNews() to see new features/changes/bug fixes.
Loading required package: rpart
Loading required package: tidyverse
  Attaching core tidyverse packages
                                                  tidyverse
2.0.0
```

```
1.1.3
                      readr
                                 2.1.4
 dplyr
 forcats 1.0.0
                                 1.5.0
                       stringr
                                 3.2.1
          3.4.4
                      tibble
 ggplot2
 lubridate 1.9.3
                      tidyr
                                 1.3.0
 purrr
           1.0.2
 Conflicts
tidyverse conflicts()
 dplyr::combine() masks
randomForest::combine()
 dplyr::filter()
                   masks
stats::filter()
 dplyr::lag()
                   masks stats::lag()
 ggplot2::margin() masks
randomForest::margin()
 Use the conflicted package
(<http://conflicted.r-lib.org/>) to force all conflicts to
become errors
53.6329
52.1351479
[1] "In generation 1, parent_rank = 57.9, child_rank = 53.6329"
[1] "In generation 2, parent_rank = 53.6329, child_rank = 52.1351479"
[1] "In generation 3, parent_rank = 52.1351479, child_rank = 51.6094369129"
[1] "In generation 4, parent_rank = 51.6094369129, child_rank =
51.4249123564279"
[1] "In generation 5, parent_rank = 51.4249123564279, child_rank =
51.3601442371062"
[1] "In generation 6, parent_rank = 51.3601442371062, child_rank =
51.3374106272243"
[1] "In generation 7, parent rank = 51.3374106272243, child rank =
51.3294311301557"
[1] "In generation 1, parent_rank = 32.7, child_rank = 44.7877"
[1] "In generation 2, parent_rank = 44.7877, child_rank = 49.0304827"
[1] "In generation 3, parent_rank = 49.0304827, child_rank = 50.5196994277"
[1] "In generation 4, parent_rank = 50.5196994277, child_rank =
51.0424144991227"
[1] "In generation 5, parent_rank = 51.0424144991227, child_rank =
51.2258874891921"
[1] "In generation 6, parent_rank = 51.2258874891921, child_rank =
51.2902865087064"
[1] "In generation 7, parent_rank = 51.2902865087064, child_rank =
51.312890564556"
34.556
[1] "In generation 1, parent_rank = 32.7, child_rank = 34.556"
[1] "In generation 2, parent_rank = 34.556, child_rank = 35.07568"
[1] "In generation 3, parent_rank = 35.07568, child_rank = 35.2211904"
```

```
[1] "In generation 4, parent_rank = 35.2211904, child_rank = 35.261933312"
[1] "In generation 5, parent_rank = 35.261933312, child_rank = 35.27334132736"
[1] "In generation 6, parent_rank = 35.27334132736, child_rank = 35.2765355716608"
[1] "In generation 7, parent_rank = 35.2765355716608, child_rank = 35.277429960065"
```

```
[2]: # As you can see, the income rank hardly improves at all for the second and
    # third generation, and eventually stabilizes after four generations at the
     # 35.3rd percentile. The 35.3rd percentile is the steady state (or fixed point)
     # prediction for Black children, because the model predicts no further
     # improvement once average income reaches this level.
    #
    # In contrast, similar calculations would show that the 54.2nd percentile
    # is the steady state (or fixed point) of the model for white children.
     # These stark steady state gaps show that addressing racial disparities in
     # upward mobility is crucial for lessening racial disparities in the United
      \hookrightarrow States.
     # ## Trying it on your own
     # Chetty, Hendren, Jones, and Porter (2020) report the following estimates
     # for Hispanic children:
    # Rank_{kids} = 36.14 + 0.26 * Rank_{parents}
    # The average income rank for Hispanic parents is the 36.17th percentile.
     # Write your own for loop to study the predictions from the model for Hispanic
     # children over the next 7 generations.
     # Using the output from your for loop, what is the steady state prediction for
     # Hispanic children?
    ## YOUR QUESTION 1 CODE GOES HERE
    parents_rank = 36.14 #This is the starting value for the parent's generation
    for(j in generations){
      kids_rank = 36.14 + 0.26 * parents_rank
      print(paste0("In generation ", j, ", parent_rank = ", parents_rank, ", __
     parents_rank <- kids_rank
    }
```

```
[1] "In generation 1, parent_rank = 36.14, child_rank = 45.5364"
[1] "In generation 2, parent_rank = 45.5364, child_rank = 47.979464"
[1] "In generation 3, parent_rank = 47.979464, child_rank = 48.61466064"
[1] "In generation 4, parent_rank = 48.61466064, child_rank = 48.7798117664"
[1] "In generation 5, parent_rank = 48.7798117664, child_rank = 48.822751059264"
```

```
[1] "In generation 6, parent_rank = 48.822751059264, child_rank =
48.8339152754086"
[1] "In generation 7, parent_rank = 48.8339152754086, child_rank =
48.8368179716063"
```

#### Question 1 Answer

From this loop, we can see that the steady state prediction for Hispanic children is 48.8 percentile rank as this is the approximate value that the rank converges to after the 4th or 5th iteration of the loop.

2. Explain briefly how cross-validation helps us avoid the overfit problem.

```
[3]: # QUESTION 2 Code
```

## Question 2 Answer

Cross-validation is a statistical method used to evaluate the generalizability of a model by dividing the data into multiple parts, training the model on some parts and validating it on the others. This process helps in avoiding the overfitting problem by ensuring that the model performs well not just on the data it was trained on, but also on unseen data. By repeatedly training and testing the model on different subsets of data, cross-validation provides a more accurate measure of a model's predictive power and robustness, reducing the risk of it capturing noise as if it were a true pattern.

- 3. Modify the example code to implement **five-fold cross validation** to choose the depth of a decision tree that uses just two predictors. It is your choice of which two predictors, but they should not be the same as my two! Pick your own! The predictors are the variables P\_1 through P\_121 in the data, but their real names are included in the data dictionary available in Table 3 below.
  - 1. Plot the cross-validation pseudo out-of-sample root mean squared prediction error (CV RMSE) versus the depth of the tree.
  - 2. Using the graph that you produced, what tree depth is optimal?
  - 3. Now use the full training data set to estimate a tree of the depth you selected in the previous question. Visualize the tree. Which predictors are being used in the first several splits of the tree?
  - 4. Obtain predictions in the training sample.

```
#Create a training data frame with just predictors P_* and kfr_pooled_pooled_p25
training <- subset(atlas_training, training==1, vars)
training$kfr_pooled_pooled_p25 <-_u
atlas_training[atlas_training$training==1,]$kfr_pooled_pooled_p25
```

	geoid	place	pop	housing	$kfr\_pooled\_pooled\_p25$	test	training
A tibble: $6 \times 128$	<dbl $>$	<chr $>$	<dbl $>$	<dbl $>$	<dbl></dbl>	<dbl $>$	<dbl $>$
	1003	Baldwin County	187114	104061	38.88471	0	1
	1005	Barbour County	27321	11829	34.93856	0	1
	1007	Bibb County	22754	8981	36.33907	0	1
	1013	Butler County	20624	9964	35.72486	0	1
	1015	Calhoun County	117714	53289	36.18467	0	1
	1017	Chambers County	34145	17004	34.05749	0	1

 $\begin{array}{c} 1.\ 'P\_1'\ 2.\ 'P\_2'\ 3.\ 'P\_3'\ 4.\ 'P\_4'\ 5.\ 'P\_5'\ 6.\ 'P\_6'\ 7.\ 'P\_7'\ 8.\ 'P\_8'\ 9.\ 'P\_9'\ 10.\ 'P\_10'\ 11.\ 'P\_11'\\ 12.\ 'P\_12'\ 13.\ 'P\_13'\ 14.\ 'P\_14'\ 15.\ 'P\_15'\ 16.\ 'P\_16'\ 17.\ 'P\_17'\ 18.\ 'P\_18'\ 19.\ 'P\_19'\ 20.\ 'P\_20'\\ 21.\ 'P\_21'\ 22.\ 'P\_22'\ 23.\ 'P\_23'\ 24.\ 'P\_24'\ 25.\ 'P\_25'\ 26.\ 'P\_26'\ 27.\ 'P\_27'\ 28.\ 'P\_28'\ 29.\ 'P\_29'\\ 30.\ 'P\_30'\ 31.\ 'P\_31'\ 32.\ 'P\_32'\ 33.\ 'P\_33'\ 34.\ 'P\_34'\ 35.\ 'P\_35'\ 36.\ 'P\_36'\ 37.\ 'P\_37'\ 38.\ 'P\_38'\\ 39.\ 'P\_39'\ 40.\ 'P\_40'\ 41.\ 'P\_41'\ 42.\ 'P\_42'\ 43.\ 'P\_43'\ 44.\ 'P\_44'\ 45.\ 'P\_45'\ 46.\ 'P\_46'\ 47.\ 'P\_47'\\ 48.\ 'P\_48'\ 49.\ 'P\_49'\ 50.\ 'P\_50'\ 51.\ 'P\_51'\ 52.\ 'P\_52'\ 53.\ 'P\_53'\ 54.\ 'P\_54'\ 55.\ 'P\_55'\ 56.\ 'P\_56'\\ 57.\ 'P\_57'\ 58.\ 'P\_58'\ 59.\ 'P\_59'\ 60.\ 'P\_60'\ 61.\ 'P\_61'\ 62.\ 'P\_62'\ 63.\ 'P\_63'\ 64.\ 'P\_64'\ 65.\ 'P\_65'\\ 66.\ 'P\_66'\ 67.\ 'P\_67'\ 68.\ 'P\_69'\ 69'\ 70.\ 'P\_70'\ 71.\ 'P\_71'\ 72.\ 'P\_72'\ 73.\ 'P\_73'\ 74.\ 'P\_74'\\ 75.\ 'P\_75'\ 76.\ 'P\_76'\ 77.\ 'P\_77'\ 78.\ 'P\_78'\ 79.\ 'P\_79'\ 80.\ 'P\_80'\ 81.\ 'P\_81'\ 82.\ 'P\_82'\ 83.\ 'P\_83'\\ 84.\ 'P\_84'\ 85.\ 'P\_85'\ 86.\ 'P\_86'\ 87.\ 'P\_87'\ 88.\ 'P\_88'\ 89.\ 'P\_89'\ 90.\ 'P\_90'\ 91.\ 'P\_91'\ 92.\ 'P\_92'\\ 93.\ 'P\_93'\ 94.\ 'P\_94'\ 95.\ 'P\_95'\ 96.\ 'P\_96'\ 97.\ 'P\_97'\ 98.\ 'P\_98'\ 99.\ 'P\_99'\ 100.\ 'P\_100'\\ 101.\ 'P\_101'\ 102.\ 'P\_102'\ 103.\ 'P\_103'\ 104.\ 'P\_104'\ 105.\ 'P\_105'\ 106.\ 'P\_106'\ 107.\ 'P\_107'\\ 108.\ 'P\_108'\ 109.\ 'P\_109'\ 110.\ 'P\_110'\ 111.\ 'P\_111'\ 112.\ 'P\_112'\ 113.\ 'P\_113'\ 114.\ 'P\_111'\\ 115.\ 'P\_115'\ 116.\ 'P\_116'\ 117.\ 'P\_117'\ 118.\ 'P\_118'\ 119.\ 'P\_119'\ 120.\ 'P\_120'\ 121.\ 'P\_121'\\ \end{array}$ 

```
# First we need to set a few objects that will be used in the loop below
n <- nrow(training) # the number of observations
K <- 5 # the number of `folds'
B \leftarrow seq(1,20,1) #This is for tree depths for 1 to 20
# We'll create a copy of the training data that we'll use inside the loop
cv <- training
# Now we define the folds: create a vector of fold memberships (random order)
cv$foldid <- rep(1:K,each=ceiling(n/K))[sample(1:n)]</pre>
# Now we create an empty data frame of results
# In each iteration of the loop, we fill it in row by row
OOS <- data.frame(fold=rep(NA,K*length(B)),</pre>
                  squarederror=rep(NA,K*length(B)),
                  maxdepth=rep(NA,K*length(B) ))
# Start an object row = 0, that will be increased by 1 in each iteration of the
→loop
# We'll use that to fill in each row in the data frame
row <- 0
#Now we run a "double" loop: loop over tree depths 1 through B, loop over folds
\hookrightarrow 1 through K
#This part loops over tree depths
for(j in B){
  #This part loops over the folds
  for(k in 1:K){
    #This part increases the "row" by 1 in each iteration of the loop
    row <- row + 1
    #This part divides the data into all the folds but one for training
    #This part uses the k from the inner loop (looping over folds)
    cvtrain <- subset(cv, foldid != k) # train on all but fold `k'</pre>
    #This part sets the left out fold aside as a separate data frame
    #We'll use that to calculate the RMSPE
    #This part uses the k from the inner loop (looping over folds)
    cvfold <- subset(cv, foldid == k) # fold `k'</pre>
    # Now we fit a decision tree on all but fold `k'
    # You'll want to change the predictors from P_37 and P_56 to two others
    cvtree <- rpart(kfr_pooled_pooled_p25 ~ P_32 + P_64,</pre>
                    data=cvtrain, #Train data using all the folds but 1
```

```
maxdepth = c(j), #set the max depth equal to j from outer_
 → loop
                    cp=0)
    #Get predictions for the left out fold
    predfull <- predict(cvtree, newdata=cvfold) # Get predictions for fold `k'</pre>
    #Store the sum of squared errors using data in left out fold
    #It will be put in row 1 in the first iteration, row 2 in the next
 ⇔iteration, etc
    OOS$squarederror[row] <- sum((cvfold$kfr_pooled_pooled_p25 - predfull)^2) #_J
 → Calculate prediction errors for fold `k'
    #Store the depth that was used in the tree in this row of the data frame
    OOS$maxdepth[row] <- j # store the maxdepth</pre>
    #Store which fold was left out in this row of the data frame
    OOS$fold[row] <- k # store the fold
    #This bracket ends the inner loop (looping over folds)
 }
  #This bracket ends the outer loop (looping over tree depths)
}
#Summarize the results
OOS
summary(00S)
#Calculate the combined error across folds
ssr <- tapply(OOS$squarederror, OOS$maxdepth, sum)</pre>
ssr <- as.data.frame(ssr)</pre>
ssr$maxdepth <- seq(1,20,1)
ssr
#Calculate the CV RMSPE as a new variable inside the SSR data frame
ssr$rmse <- sqrt(ssr$ssr / nrow(training))</pre>
min_rmse_row <- ssr[which.min(ssr$rmse),]</pre>
#Draw a graph of the cross validation error rate versus the depth of the tree
ggplot(ssr, aes(x=maxdepth, y=rmse)) +
 geom_point() +
 geom line() +
 labs(y = "Cross Validation RMSPE", x = "Tree Depth") +
 geom_vline(xintercept = min_rmse_row$maxdepth, linetype="dashed", color = ___

¬"red") +
```

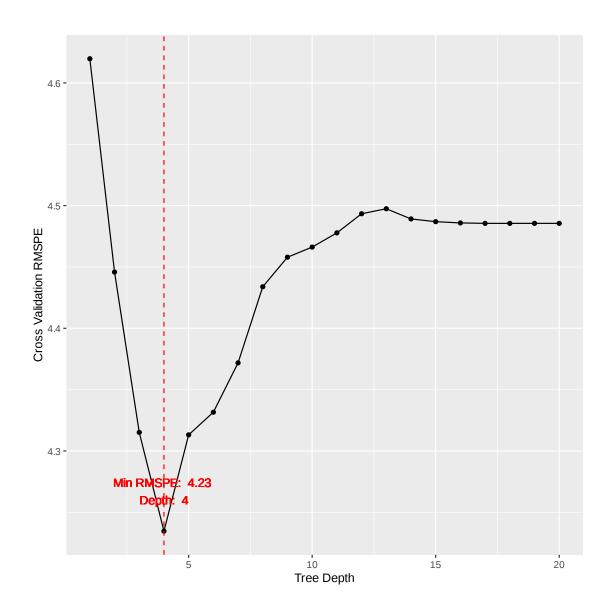
	fold	squarederror	$\max$ depth
	<int></int>	<dbl></dbl>	<dbl></dbl>
	1	5624.321	1
	2	5274.516	1
	3	6071.866	1
	4	5211.319	1
	5	4687.461	1
	1	5296.612	2
	2	4679.135	2
	3	5198.713	2
	4	4865.258	2
	5	4844.859	2
	1	4728.110	3
	2	4494.082	3
	3	5012.969	3
	4	4994.098	3
	5	4213.951	3
	1	4645.238	4
	2	4248.671	4
	3	4571.043	4
	4	4928.785	4
	5	4182.845	4
	1	4800.046	5
	2	4447.456	5
	3	4841.319	5
	4	4947.661	5
	5	4384.826	5
	1	4572.002	6
	2	4640.863	6
	3	5116.752	6
	4	4999.903	6
A data.frame: $100 \times 3$	5	4292.807	6
	1	5046.447	15
	2	5143.821	15
	3	5603.986	15
	4	5070.132	15
	5	4482.382	15
	1	5046.956	16
	2	5143.821	16
	3	5603.986	16
	4	5070.132	16
	5	4470.193	16
	1	5046.956	17
	2	5143.821	17
	3	5603.986	17
	4	5070.132	17
	5	4466.146	17
	1	5046.956	18
	2	5143.821	138
	3	5603.986	18
	4	5070.132	18
	<b>.</b>	1166 116	10

4466.146

fold	squarede	rror	maxo	depth
Min. :1	Min. :4	183	Min.	: 1.00
1st Qu.:2	1st Qu.:4	645	1st Qu	.: 5.75
Median :3	Median:5	047	Median	:10.50
Mean :3	Mean :49	973	Mean	:10.50
3rd Qu.:4	3rd Qu.:5	150	3rd Qu	:15.25
Max. :5	Max. :6	072	Max.	:20.00
		ssr	m	axdepth

		ssr	$\max$ depth
		<dbl></dbl>	<dbl $>$
	1	26869.48	1
	2	24884.58	2
	3	23443.21	3
	4	22576.58	4
	5	23421.31	5
	6	23622.33	6
	7	24062.95	7
	8	24750.76	8
A data.frame: $20 \times 2$	9	25020.36	9
A data.frame. 20 × 2	10	25113.11	10
	11	25243.42	11
	12	25419.55	12
	13	25465.71	13
	14	25371.89	14
	15	25346.77	15
	16	25335.09	16
	17	25331.04	17
	18	25331.04	18
	19	25331.04	19
	20	25331.04	20

Saving  $6.67 \times 6.67$  in image



```
[6]: # Question 3b sample code

### What is the optimal tree depth based on this graph?

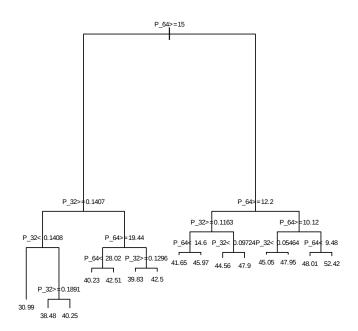
cv_optimal_depth = 4
```

```
#Visualize the fitted decision tree
plot(tree, margin = 0.2)
text(tree, cex = 0.5)

#Save figure
dev.copy(png,'figure2.png')
dev.off()
```

**png:** 3

**png:** 2



```
[8]: # Question 3d sample code

#Calculate predictions for all rows in training sample
y_train_predictions_tree <- predict(tree, newdata=training)</pre>
```

## Question 3 Answer

A. As seen in the plot, the out of sample RMSPE goes down with depth upto a certain point, after which the tree starts overfitting and the error for out of sample predictions shoots up again. B. Accordingly, the optimal depth is the point with the lowest out of sample RMSPE or 4, as labelled in the graph. C. The tree above is created from the full training data. In the first split of the tree, the decision is made based on the value of P\_64 (share of 18-64 persons without insurance). Visually, it seems that both P\_64 and P\_32 are used in roughly the same number of splits. D. Code to obtain predictions in training sample above.

4. Explain briefly how random forests improve upon decision trees using (i) bagging and (ii) input randomization.

```
[9]: # QUESTION 4 Code
```

#### Question 4 Answer

Bagging, or Bootstrap Aggregating, involves creating multiple decision trees on different subsets of the data (drawn with replacement), and then averaging their predictions. This reduces the variance and avoids overfitting, making the model more generalizable. Input randomization introduces further diversity by selecting a random subset of features for splitting at each node of a tree, thereby increasing the model's ability to capture complex patterns and interactions among features without being overly sensitive to noise. Together, these techniques make random forests a powerful and reliable predictive model.

5. Now implement a random forest with at least 1000 trees (bootstrap samples) using the same two predictors you selected for the decision tree. Obtain predictions in the training sample.

#### Call:

```
randomForest(formula = kfr_pooled_pooled_p25 ~ P_32 + P_64, data = training,

ntree = 1000, mtry = 2)

Type of random forest: regression

Number of trees: 1000

No. of variables tried at each split: 2

Mean of squared residuals: 18.50375

% Var explained: 30.74
```

## Question 5 Answer

Modified code above.

6. Next, implement a random forest with at least 1000 trees (bootstrap samples) using the full predictor set (consisting of the 121 predictors corresponding to variables P\_1 through P\_121 included in the training data). Obtain predictions in the training sample.

```
#Tuning parameters are ntree and mtry
#ntree is number of trees in your forest
#mtry is the number of predictors considered at each split (default is number)
of predictors divided by 3)
#Setting mtry<121 can help in small samples like this one.

mobilityforest #Review the Random Forest Results

#Generate predictions for training data
y_train_predictions_forest <- predict(mobilityforest, newdata=training,ustype="response")

### Try changing mtry to 20 (16.5% of predictors), 60 (50% of predictors), onusided and predictors)
```

#### Call:

```
randomForest(formula = kfr_pooled_pooled_p25 ~ ., data = training, ntree = \( \to 1000\), mtry = 60, importance = TRUE)

Type of random forest: regression

Number of trees: 1000

No. of variables tried at each split: 60

Mean of squared residuals: 4.747495

% Var explained: 82.23
```

### Question 6 Answer

Random Forest with mtry = 60 (50% of predictors) shown above.

7. Random forests typically result in improved accuracy over prediction using a single tree. Unfortunately, however, it can be difficult to interpret the resulting model. Recall from Lab 6 that one of the advantages of decision trees is the attractive and easily interpreted diagram that results.

One can obtain an overall summary of the importance of each predictor in a random forest by measuring how the mean squared error decreases when the predictor is used define tree splits. A large value indicates an important predictor.

Using the random forest from the previous question, which variables are the most important predictors using this metric? Refer to the data dictionary in Table 3 below to determine what these variables measure.

```
[12]: # QUESTION 7 Code

#-----
# What variables are the most important predictors?
#------
```

```
importance(mobilityforest)
varImpPlot(mobilityforest, type=1) #Plot the Random Forest Results

#type          is either 1 or 2, specifying the type of importance measure
#(1=mean decrease in accuracy, 2=mean decrease in node impurity)

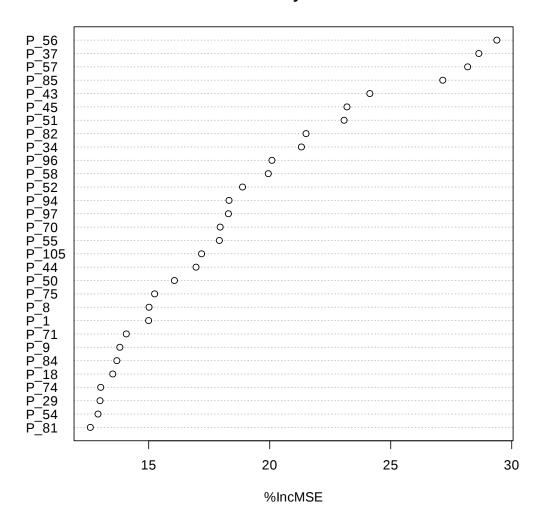
#Save figure
dev.copy(png,'figure3.png')
dev.off()
```

		%IncMSE	IncNodePurity
	P_1	14.996716	197.33550
	P_2	11.731866	89.10111
	P_3	10.649786	105.86876
	P_4	10.871882	106.48611
	P_5	11.891062	187.91943
	P_6	11.533372	131.07387
	P_7	11.937764	161.48765
	P_8	15.015651	215.45977
	P_9	13.807028	182.19695
	P_10	10.129496	110.21672
	P_11	7.069986	45.09324
	P_12	9.248953	69.45720
	P_13	3.024961	33.63679
	P_14	6.962237	71.57946
	P_15	9.864219	70.26290
	P_16	2.755587	47.55528
	P_17	9.809135	86.88104
	P_18	13.515204	114.72705
	P_19	7.292563	67.64377
	P_20	5.585463	48.94728
	P_21	4.939702	57.35775
	P_22	2.509362	43.88483
	P_23	6.231628	64.46041
	P_24	9.972091	83.87208
	P_25	8.630618	99.41561
	P_26	4.600764	38.26911
	P_27 P_28	2.796103 6.996938	60.03872
	P 29	12.989977	52.18413 507.06177
	P 30	11.998202	149.69852
A matrix: $121 \times 2$ of type dbl	100	11.550202	140.00002
	P_92	3.0260163	40.719620
	P 93	7.7861138	100.800474
	P 94	18.3189091	274.813780
	P 95	10.2073310	106.898849
	P_96	20.0921848	1784.413342
	P_97	18.2935647	197.527935
	P_98	3.6940300	7.403184
	P_99	5.4872325	27.393566
	P_100	-0.6146927	3.982722
	P_101	-0.1544891	2.151949
	P_102	-0.1076852	2.070993
	P_103	1.0032107	10.035314
	P_104	2.0820152	37.618459
	P_105	17.1845041	470.911115
	P_106	0.8553893	4.338622
	P_107	-1.2237806	1.490381
	P_108	2.1860349	9.584670
	P_109	4.9066970	35.035898
	P_110	0.9151437	1.228150
	P_111	5.6019019	20.654596
	P_112	0.5472770	10.741587

**png:** 3

**png:** 2

## mobilityforest



## Question 7 Answer

According to this plot, the most important predictors are (in order) P\_56, P\_37, P\_57 and P\_85 (followed by a gap). The interpretation of these variables as from the table below is:

P\_56 - Mentally Unhealthy Days per Month (Persons 18 Years and Over)

 $P_37$  - Share black 2000

P\_57 - Percent of Adults That Report Fair or Poor Health (Persons 18 Years and Over)

P\_85 - % Total: Roman Catholic

8. Calculate and compare the root mean squared prediction error for your three models in the **training sample**. Which model does the best?

```
\begin{array}{c} \text{RMSPE} & \text{method} \\ & < \text{dbl} > & < \text{chr} > \\ \text{A data.frame: } 3 \times 2 & \hline{3.9710639} & \text{Tree} \\ & 2.1104591 & \text{Small RF} \\ & 0.8673262 & \text{Large RF} \end{array}
```

## Question 8 Answer

As seen below, a large RF has the lowest RMSPE and therefore performs the best, with a significant improvement over tree and small RF.

9. Now turn to the lock box data set atlas\_lockbox.dta. These data contain a variable called kfr\_actual which is the "truth:" the actual value of kfr\_pooled\_pooled\_p25 for all the counties in the sample, including the 50% of the data in the lock box sample. Calculate predictions from your models and use kfr\_actual to calculate the root mean squared prediction error for the test sample. Which model did the best?

```
#Merge with truth to evaluate predictions.
atlas <- left_join(atlas_test, atlas_training , by="geoid")
#Separate test data set as a separate data frame
test <- subset(atlas, training==0)</pre>
#Get predictions for test data
y_test_predictions_tree <- predict(tree, newdata=test)</pre>
y_test_predictions_smallforest <- predict(smallforest, newdata=test,_</pre>
 ⇔type="response")
y_test_predictions_forest <- predict(mobilityforest, newdata=test,_
 ⇔type="response")
#Calculate RMSPE for test data
p <- 3
OOS_RMSPE <- matrix(0, p, 1)</pre>
OOS_RMSPE[1] <- sqrt(mean((test$kfr_actual - y_test_predictions_tree)^2, na.
 →rm=TRUE))
OOS_RMSPE[2] <- sqrt(mean((test$kfr_actual - y_test_predictions_smallforest)^2,_
 →na.rm=TRUE))
OOS_RMSPE[3] <- sqrt(mean((test\$kfr_actual - y_test_predictions_forest)^2, na.
 →rm=TRUE))
# Display table of results
data.frame(OOS_RMSPE, method = c("Tree", "Small RF", "Large RF"))
```

	OOS_RMSPE	method
	<dbl></dbl>	<chr $>$
A data.frame: $3 \times 2$	4.128169	Tree
	4.277144	Small RF
	2.209853	Large RF

## Question 9 Answer

Here as well the large RF has the lowest RMSPE in the out of sample predictions, however the gap is somewhat smaller that in the case of training data.

10. Create an annotated/commented do-file, .ipynb Jupyter Notebook, or .R file that can replicate all your analyses above. This will be the final code that you submit on Gradescope. The motivation for using do-files and .R files is described on page 4, which has been adapted from training materials used by Innovations for Poverty Action (IPA) and the Abdul Latif Jameel Poverty Action Lab (J-PAL).

#### Final Submission Checklist for Lab 7

If you're working with R

If you're working with Stata

Lab 7 Write-Up:

PDF of your answers. For graphs, you must save them as images (e.g., .png files) and insert them into the document.

Lab 7 Code:

.R script file, well-annotated replicating all your analyses;OR

.ipynb file and a .PDF version of this file.

Lab 7 Write-Up:

PDF of your answers. For graphs, you must save them as images (e.g., .png files) and insert them into the document.

Lab 7 Code:

do-file, well-annotated replicating all your analyses; AND

log-file, not a .smcl file, with the log showing the output generated by your final do-file.

## If you're working with an .ipynb notebook

It is likely that your .ipynb file will be greater than 1 MB in size. Therefore, for this assignment please submit both your well-annotated .ipynb file and a .PDF version of this file. The notebook should replicate all your analyses for Lab 5 (with enough comments that a principal investigator on a research project would be able to follow and understand what each step of the code is doing).

## 1.3 How to submit your assignment

**Step 1** Access the lab assignment under the

"Assignments" tab on Canvas

Step 2 Access Gradescope from Canvas

Step 3 Access the lab assignment on

Gradescope

Step 4 Upload your files Check What files to

submit to confirm what files you need to submit.

Step 5 What you'll see after submitting your

lab assignment

Step 6 Check your submitted files

Step 7 You'll receive an email confirmation as

well

### 1.4 What files to submit

If you're using Python Notebook to write your R code, and a document editor to write your answers
If you're using a Python Notebook to write your R code AND to write your answers

## 1.5 WHAT ARE DO-FILES AND .R FILES AND WHY DO WE NEED ONE?

Let's imagine the following situation - you just found out you have to present your results to a partner- all the averages you produced and comparisons you made. Suppose you also found out that the data you had used to produce all these results was not completely clean, and have only just fixed it. You now have incorrect numbers and need to re-do everything.

How would you go about it? Would you reproduce everything you did for Lab 1 from scratch? Can you do it? How long would it take you to do? Just re-typing all those commands into Stata or R in order and checking them would take an hour.

An important feature of any good research project is that the results should be reproducible. For Stata and R the easiest way to do this is to create a text file that lists all your commands in order, so anyone can re-run all your Stata or R work on a project anytime. Such text files that are produced within Stata or linked to Stata are called do-files, because they have an extension .do (like intro\_exercise.do). Similarly, in R, these files are called .R files because they have an extension of .R. These files feed commands directly into Stata or R without you having to type or copy them into the command window.

An added bonus is that having do-files and .R files makes it very easy to fix your typos, re-order commands, and create more complicated chains of commands that wouldn't work otherwise. You can now quickly reproduce your work, correct it, adjust it, and build on it.

Finally, do-files and .R files make it possible for multiple people to work on a project, which is necessary for collaborating with others or when you hand off a project to someone else.

## 1.6 DATA DESCRIPTION, FILE: atlas training.dta [training data]

The data consist of all 2,518 counties with at least 10,000 residents available from the Opportunity Atlas. For n=1,259 counties in the "test" portion of the data, the outcome variable is set to missing. These observations are a 50% random sample of all counties with at least 10,000 residents available from the Opportunity Atlas. For more details on the construction of the variables included in this data set, please see Chetty, Raj, John Friedman, Nathaniel Hendren, Maggie R. Jones, and Sonya R. Porter. 2018. "The Opportunity Atlas: Mapping the Childhood Roots of Social Mobility." NBER Working Paper No. 25147.

#### TABLE 1

Training I	Data
------------	------

Variable

Definition

Obs.

- (1)
- (2)
- (3)

geoid

County FIPS code

pop

County Population from DataCommons

2,518

housing

Total number of housing units from Census

2,518

kfr\_pooled\_pooled\_p25

Statistic 1: Absolute Mobility at the 25th Percentile

(missing for n = 1,259 counties in the test data, non-missing for the other n = 1,259 counties)

1,259

test

1 = Observation is in test data set (outcome variable is missing)

0 = Observation is in training data (outcome variable is non-missing)

2,518

training

1 = Observation is in training data set (outcome variable is non-missing)

0 = Observation is in the test data (outcome variable is missing)

2,518

P 1 through P 121

Predictors taken from the Opportunity Insights' county characteristics file and various other sources 2,518

*Note:* Full list of definitions of  $P_1$  through  $P_1$ 21 is in Table 3.

# 1.7 DATA DESCRIPTION, FILE: atlas\_lockbox.dta [Lock box data]

The data consist of all 2,518 counties with at least 10,000 residents available from the Opportunity Atlas. For n=1,259 counties in the "test" portion of the data, the outcome variable is set to missing. These observations are a 50% random sample of all counties with at least 10,000 residents available from the Opportunity Atlas. For more details on the construction of the variables included in this data set, please see Chetty, Raj, John Friedman, Nathaniel Hendren, Maggie R. Jones, and Sonya R. Porter. 2018. "The Opportunity Atlas: Mapping the Childhood Roots of Social Mobility." NBER Working Paper No. 25147.

## TABLE 2

Lock Box Data

Variable	Definition	Obs.
(1)	(2)	(3)
$kfr\_actual$	Actual value for $kfr\_pooled\_pooled\_p25$ for all 2,518 counties with at	2,518
	least 10,000 residents	
geoid	County FIPS code	2,518

# 1.8 DATA DESCRIPTION, FILE: atlas\_training.dta [training data] TABLE 3

Complete List of All Predictor Variables in Training Data

Variable Description Obs. (1)(2)(3)1 geoid County FIPS code 2,518 2 pop County Population from DataCommons 2,518 3 housing Total number of housing units from Census 2,5184 kfr\_pooled\_pooled\_p25

```
Statistic 1 Absolute Mobility at the 25th Percentile
1,259
5
test
1 = Observation is in test data set (outcome variable is missing)
0 = Observation is in training data (outcome variable is non-missing)
2,518
6
training
1 = Observation is in training data set (outcome variable is non-missing)
0 = Observation is in the test data (outcome variable is missing)
2,518
7
P_1
Bankruptcies per 1000 adults in 2008
2,518
8
P 2
Bankruptcies per 1000 adults in 2009
2,518
9
P_3
Bankruptcies per 1000 adults in 2010
2,518
10
P_4
Bankruptcies per 1000 adults in 2011
2,518
11
P_5
Bankruptcies per 1000 adults in 2012
```

```
12
P 6
Bankruptcies per 1000 adults in 2013
2,518
13
P_7
Bankruptcies per 1000 adults in 2014
2,518
14
P_8
Bankruptcies per 1000 adults in 2015
2,518
15
P_9
Bankruptcies per 1000 adults in 2016
2,518
16
P_10
\% of Individuals Earning <138\% of the FPL without Insurance in 2013
2,518
17
P_11
\% of Individuals Earning 138%-400% of the FPL without Insurance in 2013
2,518
18
P_12
Total Violent and Property Crimes Rate
2,518
19
P_13
Total Violent Crimes Rate: Murder Rate
```

20 P\_14 Total Violent Crimes Rate: Rape Rate 2,518 21 P\_15 Total Violent Crimes Rate: Robbery Rate 2,518 22  $P_16$ Total Violent Crimes Rate: Aggravated Assault Rate 2,518 23 P\_17 Total Property Crimes Rate 2,51824  $P_18$ Total Property Crimes Rate: Burglary Rate 2,518 25  $P_{19}$ Total Property Crimes Rate: Larceny Rate 2,51826  $P_{20}$ Total Property Crimes Rate: Motor Vehicle Theft Rate 2,518 27 P\_21

Total Violent and Property Crime Arrests Rate

28 P 22 Total Violent and Property Crime Arrests Rate: Violent Crime Arrests Rate 2,518 29  $P_{23}$ Total Violent and Property Crime Arrests Rate: Property Crime Arrests Rate 2,518 30  $P_{24}$ Mean Household Income 2000 2,518 31  $P_25$ Average Commute Time of Working Adults in 2000 2,518 32  $P_26$ Fraction of Residents w/ a College Degree or More in 2000  $2,\!518$ 33  $P_27$ Fraction of Residents w/ a College Degree or More in 2006-2010 ACS 2,518 33  $P_{28}$ Share of Population Born Outside the U.S. in 2006-2010 ACS 2,518 34 P 29 Median Household Income in 2016

35 P\_30 Median Household Income in 1990 2,518 36  $P_31$ Share Below Poverty Line 2006-2010 ACS 2,518 37 P\_32 Share Below Poverty Line 20002,518 38 P\_33 Share Below Poverty Line 1990 2,51839  $P_34$ Share black 2010  $2,\!518$ 40 P\_35 Share hisp 20102,51841 P\_36 Share asian 20102,518

42

 $P_{37}$ 

2,518

Share black 2000

43 P 38 Share white 2000 2,518 44 P\_39 Share hisp 2000 2,518 45  $P_{40}$ Share asian 2000 2,518 46P\_41 Average School District Level Standardized Test Scores in 3rd Grade in 2013 2,518 47  $P_42$ Average Rent for Two-Bedroom Apartment in 2015 2,518 48  $P_43$ Share of Single-Headed Households with Children 2006-2010 ACS  $\,$ 2,518 49  $P_{44}$ Share of Single-Headed Households with Children 1990 2,518 50  $P_{45}$ Share of Single-Headed Households with Children 2000

51 P 46 Share of Working Adults w/ Commute Time of 15 Minutes Or Less in 2006-2010 ACS 2,518 52  $P_{47}$ Employment Rate 2000 2,518 53  $P_{48}$ Census Form Rate Return Rate 2010 2,518 54P\_49 Log wage growth for HS Grad., 2005-2014 2,51855 P 50 Share of People who are not white 2010 2,518 56  $P_51$ Population Density (per square mile) in 2010 2,518 57  $P_52$ Population Density (per square mile) in 2000 2,518 58  $P_53$ Average Annual Job Growth Rate 2004-2013

```
59
P 54
Job Density (in square miles) in 2013
2,518
60
P_55
Physically Unhealthy Days per Month (Persons 18 Years and Over)
2,518
61
P_{-}56
Mentally Unhealthy Days per Month (Persons 18 Years and Over)
2,518
62
P_57
Percent of Adults That Report Fair or Poor Health (Persons 18 Years and Over)
2,518
63
P 58
Percent of Low Birthweight Births (<2.5kg)
2,518
64
P_{59}
Primary Care Physicians (PCP) Rate per 100,000 Population
2,518
65
P_{-60}
Mental Health Providers (MHP) Rate per 100,000 Population
2,518
66
P_61
Dentists Rate per 100,000 Population
```

```
67
P 62
Health Care Costs Price-adjusted Medicare Reimbursements
2,518
68
P_{-63}
Percent of Persons Without Insurance (Population Under 19 Years, 2013 est.)
2,518
69
P_64
Percent of Persons Without Insurance (Population 18 to 64 Years, 2013 est.)
2,518
70
P_{-}65
Percent of Persons Without Insurance (Population Under 65 Years, 2013 est.)
2,518
71
P 66
Premature Age-adjusted Mortality Rate per 100,000 Population
2,518
72
P_67
Drug Poisoning Mortality Rate per 100,000 Population
2,518
73
P_{68}
Percent Diabetics (Adults)
2,518
74
P 69
Percent of Diabetic Medicare Enrollees Receiving Hba1c Test
2,518
```

75 P 70 Diabetic Medicare Enrolless (Out of Total Medicare Enrolles) 2,518 76 P\_71 Teen Births Rate per 100,000 Population (Females 15 to 19 Years) 2,518 77  $P_72$ Chlamydia Cases Rate per 100,000 Population 2,518 78 P\_73 HIV Prevalence Rate per 100,000 Population 2,518 79  $P_74$ Percent Current Smokers (Persons 18 Years and Over) 2,518 80  $P_{-75}$ Percent Drinking Adults (Persons 18 Years and Over) 2,51881 P\_76 Percent of Persons with Limited Access to Healthy Foods 2,518 82  $P_77$ Percent of Persons with Access to Exercise Opportunities

83 P\_78 Percent Obese Persons (20 Years and Over) 2,518 84 P\_79 Percent Percent Physically Inactive Persons (20 Years and Over) 2,518 85  $P_80$ Percent of Children Eligible for Free Lunch (Persons < 18 Years) 2,518 86 P\_81 Food Environment Index 2,51887  $P_82$ % Total: Evangelical Protestant 2,518 88  $P_83$ % Total: Mainline Protestant 2,51889 P\_84 % Total: Historically Black Protestant 2,518 90  $P_85$ % Total: Roman Catholic

91

P\_86

% Total: Jewish Congregations

2,518

92

P\_87

% Total: Latter-day Saint (Mormon)

2,518

93

P\_88

% Total: Islamic

2,518

94

P\_89

% Total: Hindu

2,518

95

P\_90

% Total: Buddhist

2,518

96

P\_91

% Total: Orthodox Christian

2,518

97

P\_92

% Total: Jehovah's Witnesses

2,518

98

 $P_93$ 

% Total: Other

```
99
P 94
\% Total: Evangelical Protestant Member Count
2,518
100
P_{95}
% Total: Mainline Protestant Member Count
2,518
101
P_{96}
\% Total: Historically Black Protestant Member Count
2,518
102
P_97
\% Total: Roman Catholic Member Count
2,518
103
P 98
\% Total: Jewish Member Count
2,518
104
P_{99}
% Total: Latter-day Saint (Mormon) Member Count
2,518
105
P_100
\% Total: Islamic Member Count
```

106

 $P_101$ 

2,518

% Total: Hindu Member Count

```
107
P 102
% Total: Buddhist Member Count
2,518
108
P_103
\% Total: Orthodox Christian Member Count
2,518
109
P_{104}
\% Total: Jehovah's Witnesses Member Count
2,518
110
P_105
\% Total: Other Member Count
2,518
111
P_106
\% Total Evangelical Protestant: Advent Christian Church
2,518
112
P_107
\% Total Evangelical Protestant: Adventists - Other
2,518
113
P_108
\% Total Evangelical Protestant: Church of God General Conference
2,518
114
P_{109}
% Total Evangelical Protestant: Seventh Day Adventists
```

```
115
P 110
% Total Evangelical Protestant: Seventh Day Church of God
2,518
116
P_111
% Total Evangelical Protestant: American Baptist Association
2,518
117
P_112
\% Total Evangelical Protestant: Baptist General Conference
2,518
118
P_113
\% Total Evangelical Protestant: Baptist - Other
2,518
119
P 114
\% Total Evangelical Protestant: Baptist Bible Fellowship
2,518
120
P_115
% Total Evangelical Protestant: Baptist Missionary Association of America
2,518
121
P_116
% Total Evangelical Protestant: Cooperative Baptist Fellowship
2,518
122
P_117
\% Total Evangelical Protestant: Independent Baptist Churches
```

	123
	P_118
	% Total Evangelical Protestant: Conservative Baptist Association
	2,518
	124
	P_119
	% Total Evangelical Protestant: Free Will Baptists
	2,518
	125
	P_120
	% Total Evangelical Protestant: General Assoc. of Regular Baptists
	2,518
	126
	P_121
	% Total Evangelical Protestant: Assoc. of General Baptists
	2,518
]:	