

## Momentum Gradient Descent:

### Code:

```
def momentum_gradient_descent(gradient_func, initial_position,
learning_rate=0.01, momentum=0.9, num_iterations=100):
    position = initial_position
    velocity = 0

    for _ in range(num_iterations):
        gradient = gradient_func(position)
        velocity = momentum * velocity - learning_rate * gradient
        position += velocity

    return position

# Example usage:
def quadratic_function(x):
    return 2 * x - 4 # Gradient of the function 2x^2 - 4x

initial_position = 0 # Initial position of the optimization process
final_position_momentum = momentum_gradient_descent(quadratic_function,
initial_position)
print("Optimal solution using Momentum:", final_position_momentum)
```

### Output:

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Optimal solution using Momentum: 1.9915437725637428

## Stochastic Gradient Descent:

Code:

```
import random

def stochastic_gradient_descent(gradient_func, initial_position,
                               learning_rate=0.01, num_iterations=100):
    position = initial_position

    for _ in range(num_iterations):
        # Randomly select a data point (in this case, only one data point)
        random_data_point = random.uniform(-10, 10)
        gradient = gradient_func(random_data_point)
        position -= learning_rate * gradient

    return position

# Example usage:
def quadratic_function(x):
    return 2 * x - 4 # Gradient of the function  $2x^2 - 4x$ 

initial_position = 0 # Initial position of the optimization process
final_position_sgd = stochastic_gradient_descent(quadratic_function,
initial_position)
print("Optimal solution using Stochastic Gradient Descent:",
final_position_sgd)
```

Output:

```
Optimal solution using Stochastic Gradient Descent: 5.1390309991973966
```

## Nesterov Gradient Descent:

Code:

```
def nesterov_gradient_descent(gradient_func, initial_position,
                               learning_rate=0.01, momentum=0.9, num_iterations=100):
    position = initial_position
    velocity = 0

    for _ in range(num_iterations):
        # Compute the gradient at the intermediate position
        intermediate_position = position + momentum * velocity
```

```

    gradient = gradient_func(intermediate_position)

    # Update the velocity and position using the Nesterov update rule
    velocity = momentum * velocity - learning_rate * gradient
    position += velocity

    return position

# Example usage:
def quadratic_function(x):
    return 2 * x - 4 # Gradient of the function 2x^2 - 4x
initial_position = 0 # Initial position of the optimization process
final_position_nesterov = nesterov_gradient_descent(quadratic_function,
initial_position)
print("Optimal solution using Nesterov Gradient Descent:",
final_position_nesterov)

```

Output:

```
Optimal solution using Nesterov Gradient Descent: 1.9960756416676375
```