Participatory Democracy

The goal in participatory democracy is to get more people involved in government decisions, especially at the local level. Work-to-date has focused mostly on budgeting decisions, like which capital expenditures to prioritize. For example, residents might be asked whether they'd rather see money spent on improving parks, schools, or public housing. This forces voters to grapple with the types of trade-offs faced by the government. *Participatory budgeting* is getting increasingly popular—currently, 31 of New York City's 51 local districts use it every year.

1. k-Approval Voting

The systems currently in place typically use "k-approval voting" — each voter is told the overall budget and a list of project descriptions with costs, and the voter picks their k favorite projects, with no ordering between them.

k-Approval Voting
1. Each voter votes for at most k projects. 2. Sort the projects in decreasing order of number of votes. 3. Fund the maximal prefix such that the total cost is at most the budget B.

A drawback with k-approval voting is that voters need not take into account projects' costs, which results in more expensive projects being overrepresented. For example, suppose that the budget is 1 million USD, that k = 1, and that all voters have identical values for the three possible projects:

Project Number	Value (per voter)	Project Cost
1	4	1 million
2	3	500K
3	2	500K

It is clear that the socially optimal thing to do is to fund the second and third projects, garnering value 5 per voter. It's not clear that voters will vote in this way, however. Given that a voter can only vote for one project, it is possible (even likely) that she will vote for the single project for which she has the most value. This would result in everybody voting for the first project, a suboptimal result. Thus the outcome of straightforward voting in the k-approval scheme need not be Pareto optimal. Approval voting is also not strategyproof.

2. Knapsack Voting

We next take a glimpse into the current state-of-the-art, and describe one current prototype for a replacement: knapsack voting. The idea is to allow a voter to approve any number of projects, as long as their total cost is at most the budget.

Knapsack Voting

1. Each voter votes for a subset Si of projects for which the total cost is at most the budget B.
2. Sort the projects in decreasing order of number of votes.
3. Fund the maximal prefix such that the total cost is at most the budget B.

For the proofs, we also allow the final project considered—the most popular one that can't be fully funded—to be partially funded, so that the entire budget of B is spent. It may or may not be realistic to partially fund projects, but partial funding and insistence on spending all of one's budget are both common in practice. The intuition behind knapsack voting is that it forces voters to account for project costs— voting for more expensive projects decreases the number of projects that you can vote for— and hence should result in a better choice of projects. In the three-project example in the preceding section, straightforward knapsack voting results in the optimal choice (with the second and third projects getting funded).

3. Properties of Knapsack Voting

We'll strive for both strategyproofness and Pareto optimality guarantees, and this requires making fairly specific assumptions (otherwise impossibility results kick in). The two assumptions are:
1. Voter i has some set of projects S ∗ i that she wants to fund, with the total cost of S^*_i at most the budget B.
2. Voter i wants as much money as possible to be spent on the projects in S^*_i. Thus the utility of voter i is

$$\sum_{j \in s^* i} \text{[money spent on project j]}$$

The definition implies that if a project (of S^*_i) is partially funded, then the utility earned is prorated accordingly. The most unrealistic aspect of this utility model is the extreme assumption that a voter i has absolutely no value for any project outside its preferred set S ∗ i . Under the assumptions, knapsack voting has several good properties.

Proposition A. *With voter utilities as in (ii), knapsack voting is strategyproof, meaning that a player always maximizes her utility by voting for her true set S^*_i .*
Intuitively, misreporting your preferred set can only transfer funds from projects you do want to projects that you don't want.

Proposition B. *With voter utilities as in (ii), and assuming that voters report their true sets, knapsack voting results in a Pareto optimal choice of projects.*
Intuitively, this follows from the greedy nature of the way projects get funded, from most popular to least popular.

Proposition C. *Knapsack voting is the maximum likelihood estimator for a seminatural generative model of votes.*

The setup is reminiscent of the Mallows model for ranked lists. The model here is: there is a ground truth set S* of “correct” projects, and the probability of seeing a given vote S from a voter decays exponentially with the cost of the projects in S*\S, that is, with the amount of funds not spent on S*. Proposition C then states that knapsack voting is the maximum likelihood solution for this Mallows-type model.