GAME THEORY

Introduction

Definition: The study of strategically interdependent behaviour.

-Strategic Interdependence: actions of two individuals affect their respective outcomes.

Reasons to study game theory-

The logic of strategically interdependent situations gets extremely complicated extremely fast.

Game theory allows us to quickly draw parallels from one situation to another.

The Prisoner's Dilemma and Strict Dominance

Two criminals are detained. The police suspect them of having conspired on a major crime but only have evidence of a minor crime. To charge them for the greater crime, they need to elicit a confession.

To do this, they lock the criminals in separate interrogation chambers and offer each of them the following deal: If both keep quiet, then they will each spend a minimal time in jail. If one confesses while the other keeps quiet, the police will let the rat go free while throwing the book at the silent one. If both confess, the testimony is no longer necessary, and so both will be charged for the larger crime—though the time in jail will not be as bad as getting the book thrown at them.

Takeaway Points

1. We solve the prisoner's dilemma using the strict dominance solution concept.
2. Strategy x strictly dominates strategy y for a player if x generates a greater payoff than y regardless of what the other players do.
3. Strict dominance does not allow for equal payoffs. For example, suppose playing x and y both generated a payoff of 2 for an opposing strategy. Then x does not strictly dominate y.
4. Rational players never play strictly dominated strategies. After all, why would you want to play a strategy that always does worse than something else?
5. In a prisoner's dilemma, confessing strictly dominates keeping quiet for both players. Thus, we expect both players to confess.
6. The outcome of the game is inefficient. Both players would be better off if they both kept quiet instead. However, each player's individual interest is to confess if they know that the other player will keep quiet. As such, silence is unsustainable.
7. The conclusions a game theory model produces are only as the assumptions built into it. Here, we assumed that players only wanted to minimize their own jail time. In practice, this might not be the case. For example, a mafia boss may kill a lieutenant who acts as an informant to the police. If we wanted to

know what might happen in that sort of scenario, we would need to change the payoffs to accommodate that additional strategic concern.
8.

Iterated Elimination of Strictly Dominated Strategies

If it is known the opponent has a strictly dominated strategy, one should reason that the opponent will never play that strategy. Internalizing that might change what one wants to do in the game.
This concept formalizes that idea, showing how to use strict dominance to simplify games. In fact, the logic can grow more complicated. It may be that after player-1 factors in player-2's strictly dominated strategy, one of player-1's strategies becomes strictly dominated. Then player-2 can reason that player-1 will not play something because he knows that player-1 can reason that he will not play something. Iterated elimination of strictly dominated strategies is the process that guides that thinking.
<u>Takeaway Points</u>

1. We may remove strictly dominated strategies from a game matrix entirely.
2. A reduced matrix will still give us all the necessary information we need to solve a game.
3. We may continue eliminating strictly dominated strategies from the reduced form, even if they were not strictly dominated in the original matrix. We call this process iterated elimination of strictly dominated strategies.
4. If a single set of strategies remains after eliminating all strictly dominated strategies, then we have a prediction for the game's outcome.
5. Iterated elimination of strictly dominated strategies cannot solve all games. We will have to broaden our solution concept if we want to make progress elsewhere.

The Stag Hunt and Pure Strategy Nash Equilibrium

Sometimes strict dominance takes us nowhere. Other times, we may make one or two inferences based on it but then get stuck. The next option is to look for Nash equilibrium. We use the stag hunt to introduce the concept of pure strategy Nash equilibrium (PSNE). In a pure strategy Nash equilibrium, all players take deterministic actions with no element of randomness.
<u>Takeaway Points</u>

1. Holding all other players' actions constant, a best response is the most profitable move a particular player can make.
2. A game is in Nash equilibrium when all players are playing best responses to what the other players are doing.
3. Put differently, a Nash equilibrium is a set of strategies, one for each player, such that no player has incentive to change his or her strategy given what the other players are doing.
4. Nash equilibria can be inefficient.

5. At least one Nash equilibrium exists for all finite games. This is known as Nash's Theorem. John Nash, the person that A Beautiful Mind is based on, first proved this, hence why his name is attached to both the theorem and the solution concept.
6. A game is finite if the number of players in the game is finite and the number of pure strategies each player has is finite. The stag hunt has two players, each of whom has two pure strategies. Therefore, it is a finite game.
7. There may or may not be a Nash equilibrium in infinite games.

What Is a Nash Equilibrium?

Nash equilibrium is the most important solution concept in game theory.It is a set of strategies, one for each player, such that no player has incentive to change his or her strategy given what the other players are doing. Stated like this, Nash equilibrium does not have a clear conceptual application.
This is deceiving. In fact, Nash equilibrium has a basic underlying interpretation.
<u>Takeaway Points</u>
1. Another way to think of a Nash equilibrium is as a law that no one would want to break even in the absence of an effective police force.
2. Suppose that the police do not exist.
3. Imagine that the government passes a law.
4. The required behaviors of people that the law outlines is a Nash equilibrium if everyone still wants to abide by it.

Example: Suppose two cars are sitting perpendicular from each other at a stoplight. The light is green for one of them and red for the other. If the red light car goes, it will cause a crash. If the green light does not go, it is just wasting time. Thus, everyone is happy to follow the law as stated, even if there is no police to ticket the drivers for breaking it. Thus, following the rules of the stoplight is a Nash equilibrium.

Best Responses and Safety in Numbers

We know that a Nash equilibrium is a set of strategies, one for each player, such that no player has incentive to change his or her strategy given what the other players are doing. In order to find Nash equilibria in complicated games with many strategies, We are introduced with a simple algorithm to find all of a game's pure strategy Nash equilibria. All it requires is some time to go through and mark all of a player's best responses.
<u>Takeaway Points</u>
1. Review: A player's best response is the strategy (or strategies) that generate the greatest payoff for him or her given what the other players are doing.
2. In larger games, it may prove helpful to mark best responses with asterisks (*) in the payoff matrix.

3. Best responses allow for indifference. For example, if the best payoff a player can earn in response to a particular opposing strategy is 0, then all instances of 0 receive the asterisk.
4. After doing this for all strategies, if all the payoffs in a particular cell have an asterisk next to them, then that strategy profile is a pure strategy Nash equilibrium.
5. Any outcome that does not have asterisks for all of its payoffs are not equilibria.

Matching Pennies and Mixed Strategy Nash Equilibrium

We have till now seen how to find pure strategy Nash equilibria. Sometimes the methods will not find any. However, Nash's Theorem says that all finite games have at least one Nash equilibrium. What are we missing?
The answer is mixed strategy Nash equilibrium. When a player selects a mix strategy, he or she randomizes among two or more pure strategies. We use matching pennies to introduce the concept of mixed strategy Nash equilibrium.
<u>Takeaway Points</u>

1. If the game is finite and there is no pure strategy Nash equilibrium, then there must be a mixed strategy Nash equilibrium.
2. In a mixed strategy Nash equilibrium, at least one of the players plays multiple strategies with positive probability.
3. This mixed strategy leaves the opponent indifferent to playing his pure strategies. (When there are more than two strategies, this gets a little more complicated—it may be that the mixed strategy leaves the other player indifferent between playing two of his strategies and strictly worse off playing a third.)
4. The mixed strategy that makes the opponent indifferent is not always obvious. We need to develop an algorithm to do that, which is the subject of the next lecture.

The Mixed Strategy Algorithm

We now look at the algorithm we use to solve for mixed strategy Nash equilibrium in simple 2×2 games.
<u>Takeaway Points</u>

1. If there is a mixed strategy Nash equilibrium, it usually is not immediately obvious.
2. However, there is a straightforward algorithm that lets you calculate mixed strategy Nash equilibria. We will employ it frequently.

3. The algorithm involves setting the payoffs for a player's two pure strategies equal to each other and solving for the mixed strategy of the other player that makes this equation true.
4. The mixed strategy algorithm does not involve any fancy mathematics. All one needs is basic knowledge of algebra. However, the process requires that the analyst be careful in putting the right values in the right places.
5. The trickiest part of the logic of mixed strategy Nash equilibrium is that we must use one player's payoffs to solve for the player's strategy. This is because the point of my mixed strategy is to make you indifferent, and vice versa.

How NOT to Write a Mixed Strategy Nash Equilibrium

Here we see why we should not use decimals to express mixed strategy Nash equilibria. Use fractions instead!

Takeaway Points

1. Fractions are more precise than decimals. For example, 1/3 is not equal to .33.
2. Even slight differences like 1/3 versus .33 mean that a player has a profitable deviation, and thus the "equilibrium" you have is not an equilibrium at all.
3. Be safe and express everything as a fraction.

Battle of the Sexes

Here we see how to solve the battle of the sexes.

A man and a woman want to get together for a date this evening. There are only two forms of entertainment in town: a ballet and a fight. The woman wants to see the ballet. The man wants to see the fight. But if they wind up in different locations, they will both have to head back home unhappy. What are the rational ways to resolve this dilemma?

Takeaway Points

1. Battle of the Sexes has three equilibria: two in pure strategies and one in mixed strategies.
2. The mixed one is worse than either of the pure strategy equilibria for both players. We will see this when we learn how to calculate payoffs.
3. safe and express everything as a fraction.