CSCI 420 - COMPUTER GRAPHICS
PROGRAMMING ASSIGNMENT 2:
SIMULATING A ROLLER COASTER

PROBLEM:

Derive the steps that lead to the physically realistic equation of updating u
(i.e. u_new = u_old + (dt)(sqrt(2g(h_max-h))/mag(dp/du))).

PROOF:

For a falling body, starting from rest ($v_{INITIAL} = 0$),

Distance $(h_{max} - h)$ travelled by the body in time t is given by

$$(h_{max} - h) = \frac{1}{2} g t^2 \qquad \text{———} \quad (1)$$

where, h_{max} is final height
h is initial height
t is duration of fall
g is acceleration due to gravity

Taking derivative on both sides w.r.t. time,

$$\frac{d(h_{max} - h)}{dt} = gt$$

$$\Rightarrow d(h_{max} - h) = dt \cdot gt \qquad \text{———} \quad (2)$$

where, $d(h_{max} - h)$ is distance traveled in single timestep dt.

Instantaneous velocity v of a falling object after elapsed time t is given by

$$v = gt \qquad \text{———} \quad (3)$$

where, v is instantaneous velocity
g is acceleration due to gravity
t is duration of fall

Putting value of t from (1) into (3),

$$v = g \sqrt{\frac{2(h_{max} - h)}{g}}$$

$$\Rightarrow v = \sqrt{2g(h_{max} - h)} \qquad \text{———} \quad (4)$$

From (2) and (3),

$d(h_{max} - h)$ i.e. distance traveled in single timestep dt $= dt \cdot v$

Putting value of v from (4),

$d(h_{max} - h)$ i.e. distance traveled in single timestep dt $= dt \cdot \sqrt{2g(h_{max} - h)}$

Now, we want to parameterize this distance by increasing u linearly.

So, we take steps depending on the magnitude of the tangent vector.

$\therefore$ Time taken to cover $(u_{new} - u_{old})$ in steps of 1,
is equal to the time taken to cover distance $d(h_{max} - h)$ with
parameterization speed of $\|dp/du\|$

$$\frac{u_{new} - u_{old}}{1} = \frac{dt \cdot \sqrt{2g(h_{max} - h)}}{\left\| \frac{dp}{du} \right\|}$$

[We're equating the times]

$$\therefore \boxed{u_{new} = u_{old} + \frac{dt \cdot \sqrt{2g(h_{max} - h)}}{\left\| \frac{dp}{du} \right\|}}$$

SHREYA BHAUMIK