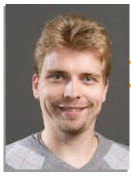

Computational Structures in Data Science

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Lecture 5: Recursion

Administrative Issues

- Where is Lecture 4? See Last slides.
- Labs are to help you learn the materials, so please make full use of them
- Materials for midterm go through March 4th Lecture.

Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
  - indexing
- Data structures
- Tuple assignment
- Call Expressions
- Function Definition Statement
- Conditional Statement
- Iteration:
  - data-driven (list comprehension)
  - control-driven (for statement)
  - while statement
- Higher Order Functions
  - Functions as Values
  - Functions with functions as argument
  - Assignment of function values
- Higher order function patterns
  - Map, Filter, Reduce
- Function factories – create and return functions

Today: Recursion

re·cur·sion
/riˈkərZHən/

noun MATHEMATICS LINGUISTICS

the repeated application of a recursive procedure or definition.
- a recursive definition.
  plural noun: **recursions**

re·cur·sive
/riˈkərsiv/

adjective

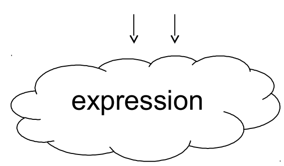
characterized by recurrence or repetition, in particular.
- MATHEMATICS LINGUISTICS
  relating to or involving the repeated application of a rule, definition, or procedure to successive results.
- COMPUTING
  relating to or involving a program or routine of which a part requires the application of the whole, so that its explicit interpretation requires in general many successive executions.

- Recursive function calls itself, directly or indirectly

Review: Functions

def <function name> (<argument list>) :

return




```
def concat(str1, str2):
  return str1+str2;

concat("Hello","World")
```

- Generalizes an expression or set of statements to apply to lots of instances of the problem
- A function should *do one thing well*

Review: Higher Order Functions

- Functions that operate on functions
- A function

```
def odd(x):
    return x%2

>>> odd(3)
1
```

- A function that takes a function arg

Why is this not 'odd' ?

```
def filter(fun, s):
    return [x for x in s if fun(x)]

>>> filter(odd, [0,1,2,3,4,5,6,7])
[1, 3, 5, 7]
```

Review Higher Order Functions (cont)

- A function that returns (makes) a function

```
def leq_maker(c):
    def leq(val):
        return val <= c
    return leq
```

```
>>> leq_maker(3)
<function leq_maker.<locals>.leq at 0x1019d8c80>

>>> leq_maker(3)(4)
False

>>> filter(leq_maker(3), [0,1,2,3,4,5,6,7])
[0, 1, 2, 3]
>>>
```


Review: One more example

- What does this function do?

```
def split_fun(p, s):
    """ Returns <you fill this in>."""
    return [i for i in s if p(i)], [i for i in s if not p(i)]
```

```
>>> split_fun(leq_maker(3), [0,1,2,3,4,5,6])
([0, 1, 2, 3], [4, 5, 6])
```


Function Review

- A function cannot…

  A) have a function as argument
  B) define a function within itself
  C) return a function
  D) call itself
  E) None of the above.

Solution:
E) A, B, C, D are all possible!

Function Review

- A Python function can…

  A) not return a value
  B) return different values for the same input
  C) halt the entire program
  D) change global variables
  E) All of the above.

Solution:
E) A, B, C, D are all possible!

Recall: Iteration

1. Initialize the "base" case of no iterations

2. Starting value

3. Ending value

```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

4. New loop variable value

Recursion Key concepts – by example

1. Test for simple "base" case

2. Solution in simple "base" case

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

3. Assume recusive solution to simpler problem

4. Transform soln of simpler problem into full soln

In words

- The sum of no numbers is zero
- The sum of 1^2 through n^2 is the
  - sum of 1^2 through $(n-1)^2$
  - plus n^2

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```


Why does it work

```
sum_of_squares(3)

# sum_of_squares(3) => sum_of_squares(2) + 3**2
#                   => sum_of_squares(1) + 2**2 + 3**2
#                   => sum_of_squares(0) + 1**2 + 2**2 + 3**2

#                   => 0 + 1**2 + 2**2 + 3**2 = 14
```


How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case
- Reason about correctness "by induction"
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

Questions

- In what order do we sum the squares ?
- How does this compare to iterative approach ?

```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return n**2 + sum_of_squares(n-1)
```


Tail Recursion

- All the work happens on the way down the recursion
- On the way back up, just return

```
def sum_up_squares(i, n, accum):
    """Sum the squares from i to n in incr. order"""
    if i > n:
        Base Case
    else:
        Tail Recursive Case

>>> sum_up_squares(1,3,0)
14
```


Local variables

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

- Each call has its own "frame" of local variables
- What about globals?
- Let's see the environment diagrams (next lecture)

https://goo.gl/CiFaUJ

Iteration vs Recursion

For loop:

```
def sum(n):
  s=0
  for i in range(0,n+1):
    s=s+i
  return s
```


Iteration vs Recursion

While loop:

```
def sum(n):
  s=0
  i=0
  while i<n:
    i=i+1
    s=s+i
  return s
```

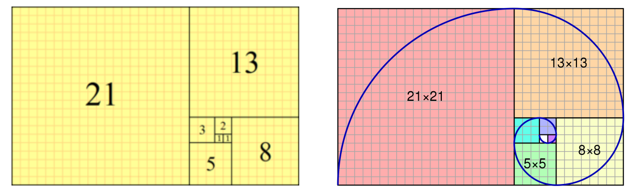

Iteration vs Recursion

Recursion:

```
def sum(n):
  if n==0:
      return 0
  return n+sum(n-1)
```


For Homework

```
fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
   where fibonacci(1) == fibonacci(0) == 1
```



Another Example

```
def first(s):
    """Return the first element in a sequence."""
    return s[0]
def rest(s):
    """Return all elements in a sequence after the first"""
    return s[1:]
```

indexing an element of a sequence

Slicing a sequence of elements

```
def min_r(s):
    """Return minimum value in a sequence."""
    if  Base Case
    else:
        Recursive Case
```

- **Recursion over sequence length, rather than number magnitude**

Visualize its behavior (print)

```
In [104]: def min_r(s):
              print('min_r:', s)
              if len(s) == 1:
                  return first(s)
              else:
                  result = min(first(s), min_r(rest(s)))
                  print('min_r:', s," => ", result)
                  return result
```

```
In [105]: min_r([3,4,2,5,11])

min_r: [3, 4, 2, 5, 11]
min_r: [4, 2, 5, 11]
min_r: [2, 5, 11]
min_r: [5, 11]
min_r: [11]
min_r: [5, 11]  =>  5
min_r: [2, 5, 11]  =>  2
min_r: [4, 2, 5, 11]  =>  2
min_r: [3, 4, 2, 5, 11]  =>  2
```

- **What about sum?**
- **Don't confuse print with return value**

Trust …

- The recursive "leap of faith" works as long as we hit the base case eventually

What happens if we don't?

Recursion

- Recursion is…

A) Less powerful than a for loop
B) As powerful as a for loop
C) As powerful as a while loop
D) More powerful than a while loop
E) Just different all together

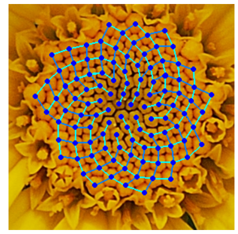
Solution:
C) Any recursion can be formulated as a while loop and any while loop can be formulated as a recursion (with a global variable).

Why Recursion?

- "After Abstraction, Recursion is probably the 2nd biggest idea in this course"
- "It's tremendously useful when the problem is self-similar"
- "It's no more powerful than iteration, but often leads to more concise & better code"
- "It's more 'mathematical'"
- "It embodies the beauty and joy of computing"
- …

Why Recursion? More Reasons

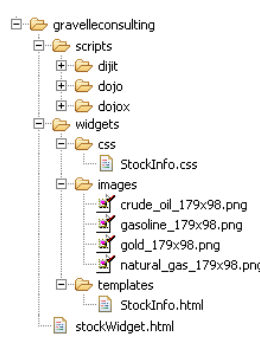
- Recursive structures exist (sometimes hidden) in nature and therefore in data!
- It's mentally and sometimes computationally more efficient to process recursive structures using recursion.



Recursion (unwanted)

Example I

List all items on your hard disk



- Files
- Folders contain
  - Files
  - Folders

Recursion!

List Files in Python

```
def listfiles(directory):
    content = [os.path.join(directory, x) for x in os.listdir(directory)]

    dirs = sorted([x for x in content if os.path.isdir(x)])
    files = sorted([x for x in content if os.path.isfile(x)])

    for d in dirs:
   print d
   listfiles(d)

    for f in files:
   print f
```

Iterative version about twice as much code and much harder to think about.

Answers for the Wandering Mind (Holiday Edition)

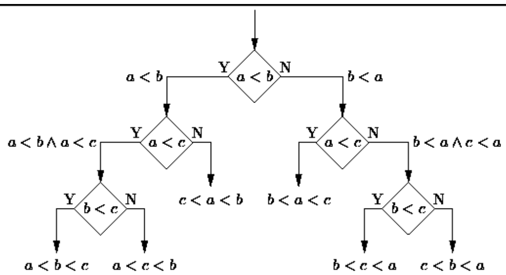
- **How many answers can be maximally responded to by 20 questions (how much data do I need on my game device)?**

Assume a number of answer possibilities b. This gives b^{20} possible answer paths.
In below device: $b=4$ ("unknown", "no", "yes", "sometimes") and
$4^{20}=1,099,511,627,776$.

Even if each questions and each answer was only *1* byte long, the device would have to have ***peta bytes*** of memory.

Example II

Sort the numbers in a list.



Hidden recursive structure: Decision tree!

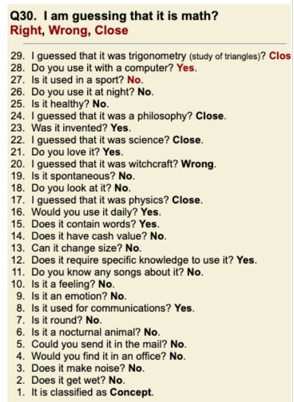
Answers for the Wandering Mind (Holiday Edition)

- **How can a 20-questions game get away with less?**

Different answers lead to the same path (redundancy). For example, making b effectively *2 (instead of 4)* results in only 2^{20}= *1,048,576* concepts. The 20-volume Oxford English Dictionary only describes *171,476* ($<4^9$) words. Typically, in our every-day life we deal with about *2000-4000* concepts ($<4^6$).

- **How can you make a 20 questions game fail?**

Pick a new concept "data science" or chose a random(!) one from the dictionary!



Q30. I am guessing that it is math?
Right, Wrong, Close

29. I guessed that it was trigonometry (study of triangles)? **Clos**
28. Do you use it with a computer? **Yes**.
27. Is it used in a sport? **No**.
26. Do you use it at night? **No**.
25. Is it healthy? **No**.
24. I guessed that it was a philosophy? **Close**.
23. Was it invented? **Yes**.
22. I guessed that it was science? **Close**.
21. Do you love it? **Yes**.
20. I guessed that it was witchcraft? **Wrong**.
19. Is it spontaneous? **No**.
18. Do you look at it? **No**.
17. I guessed that it was physics? **Close**.
16. Would you use it daily? **Yes**.
15. Does it contain words? **Yes**.
14. Does it have cash value? **No**.
13. Can it change size? **No**.
12. Does it require specific knowledge to use it? **Yes**.
11. Do you know any songs about it? **No**.
10. Is it a feeling? **No**.
9. Is it an emotion? **No**.
8. Is it used for communications? **Yes**.
7. Is it round? **No**.
6. Is it a nocturnal animal? **No**.
5. Could you send it in the mail? **No**.
4. Would you find it in an office? **No**.
3. Does it make noise? **No**.
2. Does it get wet? **No**.
1. It is classified as **Concept**.

Computational Concepts Toolbox

- **Data type: values, literals, operations,**
  - **e.g., int, float, string**
- **Expressions, Call expression**
- **Variables**
- **Assignment Statement**
- **Sequences: tuple, list**
  - **indexing**
- **Data structures**
- **Tuple assignment**
- **Call Expressions**
- **Function Definition Statement**
- **Conditional Statement**

- **Iteration:**
  - **data-driven (list comprehension)**
  - **control-driven (for statement)**
  - **while statement**
- **Higher Order Functions**
  - **Functions as Values**
  - **Functions with functions as argument**
  - **Assignment of function values**
- **Recursion**

Thoughts for the Wandering Mind

The computer choses a random element *x* of the list generated by `range(0,n)`. What is the smallest amount of iteration/recursion steps the best ever algorithms needs to guess *x?*

How would the algorithm look like?