# Computational Structures in Data Science

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# Lecture 5: Recursion

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http://inst.eecs.berkeley.edu/~cs88

# Administrative Issues

- Where is Lecture 4? See Last slides.
- Labs are to help you learn the materials, so please make full use of them
- Materials for midterm go through March 4th Lecture.

# Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
  - indexing
- Data structures
- Tuple assignment
- Call Expressions
- Function Definition Statement
- Conditional Statement

- Iteration:
  - data-driven (list comprehension)
  - control-driven (for statement)
  - while statement
- Higher Order Functions
  - Functions as Values
  - Functions with functions as argument
  - Assignment of function values
- Higher order function patterns
  - Map, Filter, Reduce
- Function factories – create and return functions

# Today: Recursion

re·cur·sion

/riˈkərZHən/

*noun* MATHEMATICS LINGUISTICS

the repeated application of a recursive procedure or definition.

- a recursive definition.
  plural noun: **recursions**

re·cur·sive

/riˈkərsiv/

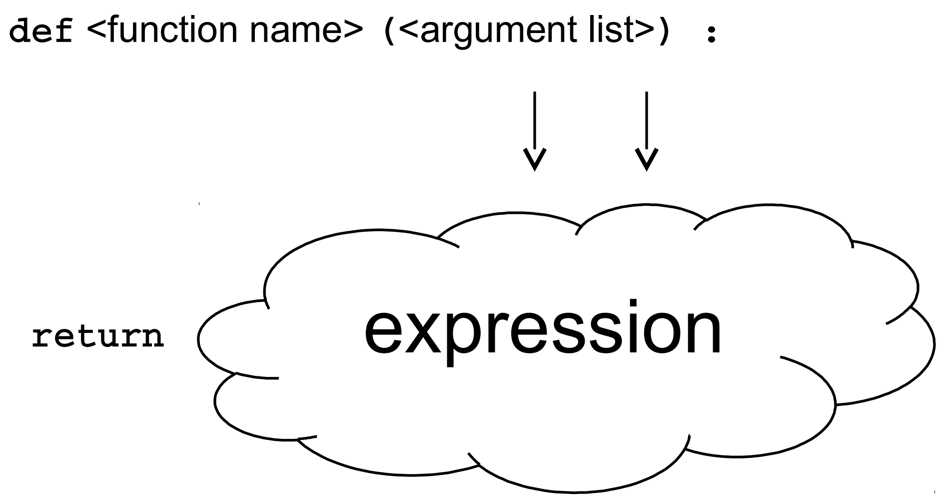
*adjective*

characterized by recurrence or repetition, in particular.

- MATHEMATICS LINGUISTICS
  relating to or involving the repeated application of a rule, definition, or procedure to successive results.
- COMPUTING
  relating to or involving a program or routine of which a part requires the application of the whole, so that its explicit interpretation requires in general many successive executions.

- **Recursive function calls itself, directly or indirectly**

# Review: Functions




```
def concat(str1, str2):
   return str1+str2;

concat("Hello","World")
```

- **Generalizes an expression or set of statements to apply to lots of instances of the problem**
- **A function should *do one thing well***

# Review: Higher Order Functions

- **Functions that operate on functions**
- **A function**

```
def odd(x):
    return x%2

>>> odd(3)
1
```

Why is this not ‘odd’ ?

- **A function that takes a function arg**

```
def filter(fun, s):
    return [x for x in s if fun(x)]

>>> filter(odd, [0,1,2,3,4,5,6,7])
[1, 3, 5, 7]
```

# Review Higher Order Functions (cont)

- **A function that returns (makes) a function**

```
def leq_maker(c):
    def leq(val):
        return val <= c
    return leq
```

```
>>> leq_maker(3)
<function leq_maker.<locals>.leq at 0x1019d8c80>

>>> leq_maker(3)(4)
False

>>> filter(leq_maker(3), [0,1,2,3,4,5,6,7])
[0, 1, 2, 3]
>>>
```

# Review: One more example

- **What does this function do?**

```
def split_fun(p, s):
    """ Returns <you fill this in>."""
    return [i for i in s if p(i)], [i for i in s if not p(i)]
```

```
>>> split_fun(leq_maker(3), [0,1,2,3,4,5,6])
([0, 1, 2, 3], [4, 5, 6])
```

# Function Review

- **A function cannot…**

  **A) have a function as argument**
  **B) define a function within itself**
  **C) return a function**
  **D) call itself**
  **E) None of the above.**


**Solution:**
**E) A, B, C, D are all possible!**

# Function Review

- **A Python function can…**

  **A) not return a value**
  **B) return different values for the same input**
  **C) halt the entire program**
  **D) change global variables**
  **E) All of the above.**


**Solution:**
**E) A, B, C, D are all possible!**

# Recall: Iteration

1. Initialize the “base” case of no iterations

2. Starting value

3. Ending value

```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

4. New loop variable value

# Recursion Key concepts – by example

1. Test for simple “base” case

2. Solution in simple “base” case

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

3. Assume recusive solution to simpler problem

4. Transform soln of simpler problem into full soln

# In words

- The sum of no numbers is zero
- The sum of $1^2$ through $n^2$ is the
  - sum of $1^2$ through $(n-1)^2$
  - plus $n^2$

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

# Why does it work

```
sum_of_squares(3)

# sum_of_squares(3) => sum_of_squares(2) + 3**2
#                   => sum_of_squares(1) + 2**2 + 3**2
#                   => sum_of_squares(0) + 1**2 + 2**2 + 3**2

#                   => 0 + 1**2 + 2**2 + 3**2 = 14
```

# How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case

- Reason about correctness “by induction”
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

# Questions

- In what order do we sum the squares ?
- How does this compare to iterative approach ?

```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return n**2 + sum_of_squares(n-1)
```

# Tail Recursion

- **All the work happens on the way down the recursion**
- **On the way back up, just return**

```
def sum_up_squares(i, n, accum):
    """Sum the squares from i to n in incr. order"""
    if i > n:
        Base Case
    else:
        Tail Recursive Case


>>> sum_up_squares(1,3,0)
14
```

# Local variables

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

- **Each call has its own "frame" of local variables**
- **What about globals?**
- **Let's see the environment diagrams (next lecture)**

https://goo.gl/CiFaUJ

# Iteration vs Recursion

**For loop:**

```
def sum(n):
   s=0
   for i in range(0,n+1):
      s=s+i
   return s
```

# Iteration vs Recursion

**While loop:**

```
def sum(n):
    s=0
    i=0
    while i<n:
        i=i+1
        s=s+i
    return s
```

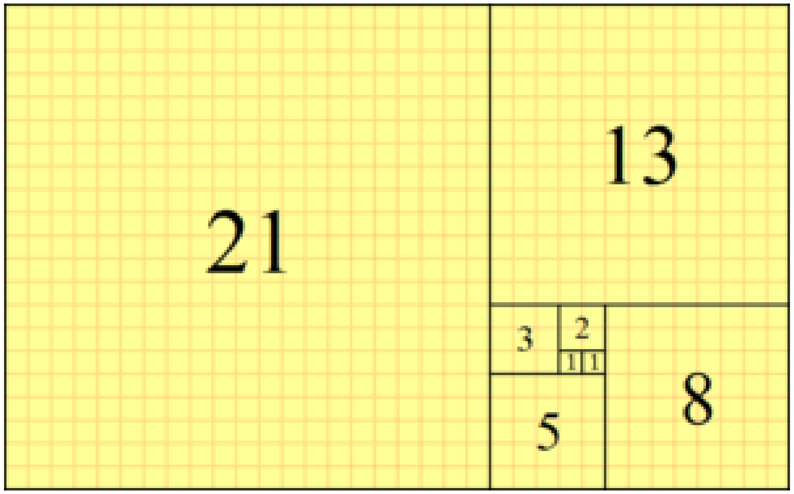
# Iteration vs Recursion

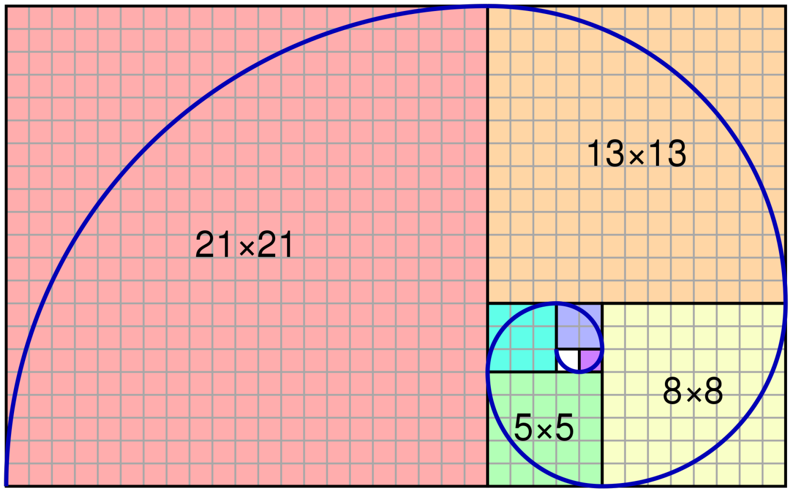
**Recursion:**

```
def sum(n):
   if n==0:
        return 0
   return n+sum(n-1)
```

# For Homework

```
fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
   where fibonacci(1) == fibonacci(0) == 1
```



# Another Example

indexing an element of a sequence

```
def first(s):
    """Return the first element in a sequence."""
    return s[0]
def rest(s):
    """Return all elements in a sequence after the first"""
    return s[1:]
```

Slicing a sequence of elements

```
def min_r(s):
    """Return minimum value in a sequence."""
    if
```

Base Case

```
    else:
```

Recursive Case

- **Recursion over sequence length, rather than number magnitude**

# Visualize its behavior (print)

```
In [104]: def min_r(s):
              print('min_r:', s)
              if len(s) == 1:
                  return first(s)
              else:
                  result = min(first(s), min_r(rest(s)))
                  print('min_r:', s," => ", result)
                  return result
```

```
In [105]: min_r([3,4,2,5,11])
```

```
min_r: [3, 4, 2, 5, 11]
min_r: [4, 2, 5, 11]
min_r: [2, 5, 11]
min_r: [5, 11]
min_r: [11]
min_r: [5, 11]  =>  5
min_r: [2, 5, 11]  =>  2
min_r: [4, 2, 5, 11]  =>  2
min_r: [3, 4, 2, 5, 11]  =>  2
```

- **What about sum?**
- **Don't confuse print with return value**

# Trust …

- The recursive "leap of faith" works as long as we hit the base case eventually

**What happens if we don't?**

# Recursion

- Recursion is…

A) Less powerful than a for loop
B) As powerful as a for loop
C) As powerful as a while loop
D) More powerful than a while loop
E) Just different all together


**Solution:**
**C) Any recursion can be formulated as a while loop and any while loop can be formulated as a recursion (with a global variable).**

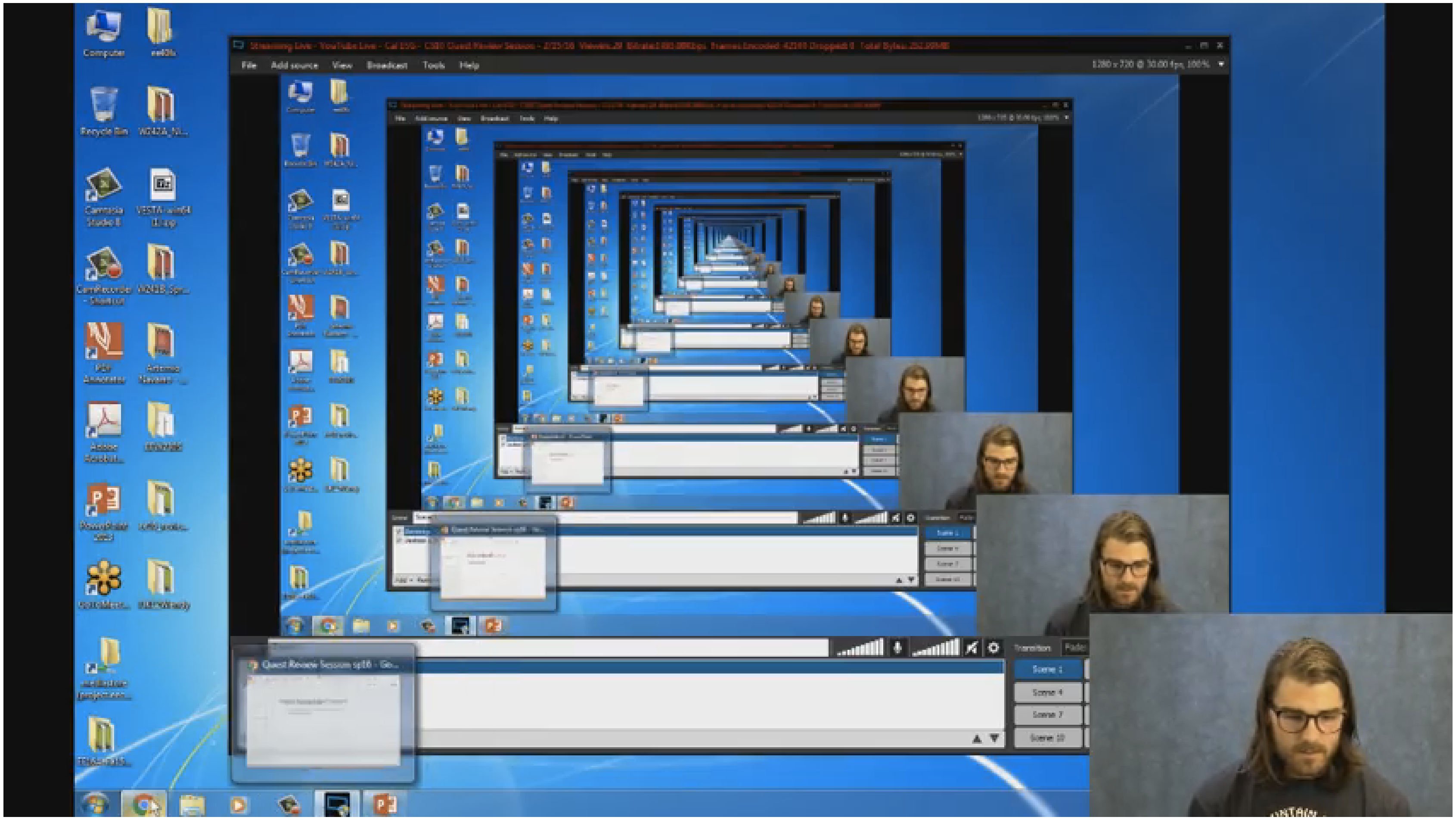
# Why Recursion?

- "After Abstraction, Recursion is probably the 2nd biggest idea in this course"
- "It's tremendously useful when the problem is self-similar"
- "It's no more powerful than iteration, but often leads to more concise & better code"
- "It's more 'mathematical'"
- "It embodies the beauty and joy of computing"
- …

# Why Recursion? More Reasons

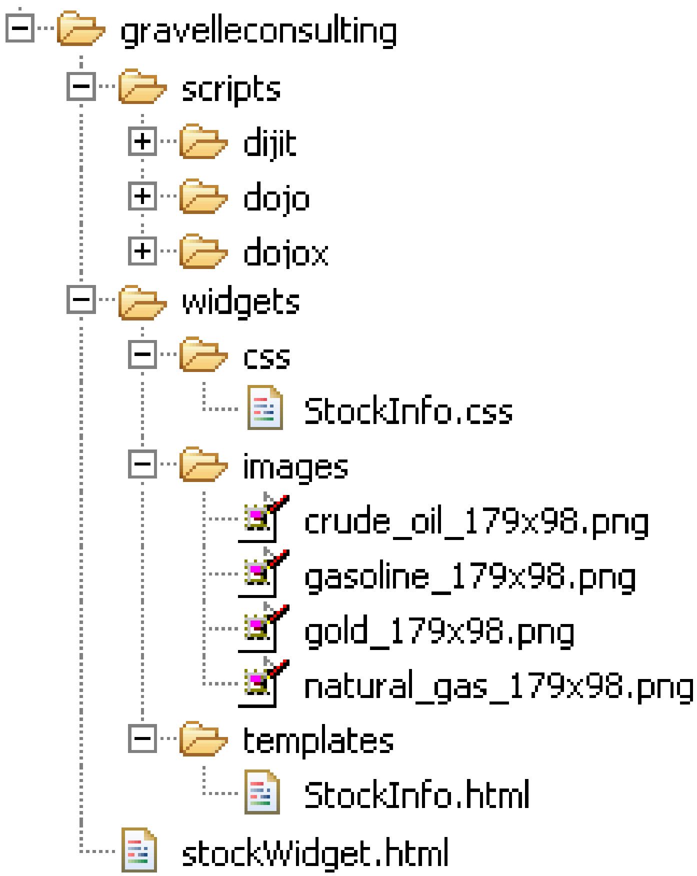
- Recursive structures exist (sometimes hidden) in nature and therefore in data!
- It's mentally and sometimes computationally more efficient to process recursive structures using recursion.

# Recursion (unwanted)



# Example I

## List all items on your hard disk




- **Files**
- **Folders contain**
  - **Files**
  - **Folders**

Recursion!

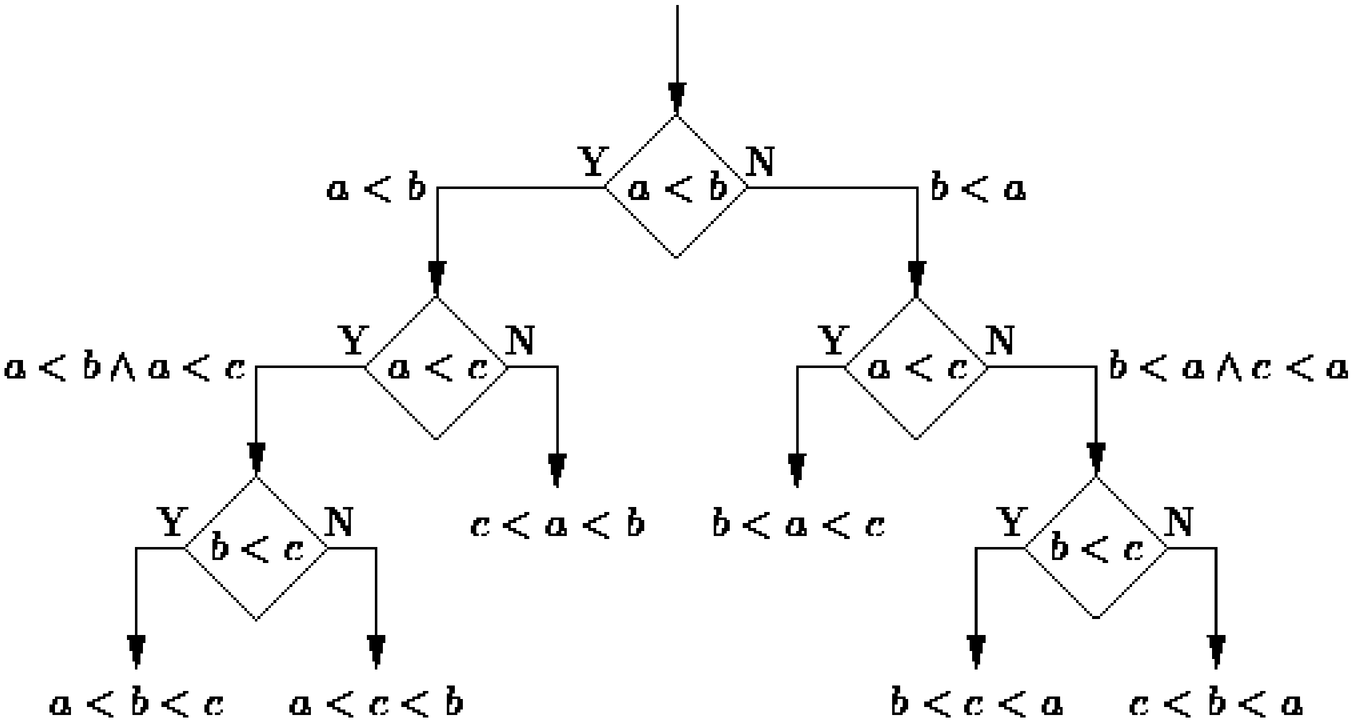
# List Files in Python

```
def listfiles(directory):
    content = [os.path.join(directory, x) for x in os.listdir(directory)]

    dirs = sorted([x for x in content if os.path.isdir(x)])
    files = sorted([x for x in content if os.path.isfile(x)])

    for d in dirs:
   print d
   listfiles(d)

    for f in files:
   print f
```

**Iterative version about twice as much code and much harder to think about.**

# Example II

## Sort the numbers in a list.




**Hidden recursive structure: Decision tree!**

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- Recursion

# Answers for the Wandering Mind (Holiday Edition)

- **How many answers can be maximally responded to by 20 questions (how much data do I need on my game device)?**

Assume a number of answer possibilities *b*. This gives $b^{20}$ possible answer paths.
In below device: $b=4$ (“unknown”, “no”, “yes”, “sometimes”) and
$4^{20}=1{,}099{,}511{,}627{,}776$.

Even if each questions and each answer was only *1* byte long, the device would have to have ***peta bytes*** of memory.

# Answers for the Wandering Mind (Holiday Edition)

- **How can a 20-questions game get away with less?**

Different answers lead to the same path (redundancy). For example, making *b* effectively *2 (instead of 4)* results in only $2^{20} = 1,048,576$ concepts. The 20-volume Oxford English Dictionary only describes *171,476* ($<4^9$) words. Typically, in our every-day life we deal with about *2000-4000* concepts ($<4^6$).

- **How can you make a 20 questions game fail?**

Pick a new concept "data science"

or chose a random(!) one from the dictionary!

**Q30. I am guessing that it is math?**
**Right**, **Wrong**, **Close**

29. I guessed that it was trigonometry (study of triangles)? **Clos**
28. Do you use it with a computer? **Yes**.
27. Is it used in a sport? **No**.
26. Do you use it at night? **No**.
25. Is it healthy? **No**.
24. I guessed that it was a philosophy? **Close**.
23. Was it invented? **Yes**.
22. I guessed that it was science? **Close**.
21. Do you love it? **Yes**.
20. I guessed that it was witchcraft? **Wrong**.
19. Is it spontaneous? **No**.
18. Do you look at it? **No**.
17. I guessed that it was physics? **Close**.
16. Would you use it daily? **Yes**.
15. Does it contain words? **Yes**.
14. Does it have cash value? **No**.
13. Can it change size? **No**.
12. Does it require specific knowledge to use it? **Yes**.
11. Do you know any songs about it? **No**.
10. Is it a feeling? **No**.
9. Is it an emotion? **No**.
8. Is it used for communications? **Yes**.
7. Is it round? **No**.
6. Is it a nocturnal animal? **No**.
5. Could you send it in the mail? **No**.
4. Would you find it in an office? **No**.
3. Does it make noise? **No**.
2. Does it get wet? **No**.
1. It is classified as **Concept**.

# Thoughts for the Wandering Mind

**The computer choses a random element *x* of the list generated by `range(0,n)`. What is the smallest amount of iteration/recursion steps the best ever algorithms needs to guess *x*?**

**How would the algorithm look like?**