Tutorial 4

Sol 1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3, \quad b = 2, \quad f(n) = n^2$

$\because$ a & b are constant and $f(n)$ is the function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

case 3 is applied here

$T(n) = \Theta(n^2)$

Sol 2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$\because$ a & b are const. & $f(n)$ is the function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2$

$= 2$

$\therefore n^c = n^2$

$n^c = f(n)$

$\therefore$ case 2 is applied

$T(n) = \Theta(n^2 \log n)$

Sol3. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

a & b are constant & $f(n)$ is +ve function

∴ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$n^c = n^0 = 1$

$f(n) > n^c$

∴ case 3 is applied

$T(n) = \theta(2^n)$

Sol4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, \quad b = 2, \quad f(n) = n^n$

∴ a is not constant, its value depend on n

∴ Master's theorem is not applicable here

Sol5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

∴ a & b are constant & $f(n)$ is +ve function

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

∴ $f(n) < n^c$

Case 1 is applied here

$T(n) = \theta(n^2)$

Sol 6. $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2, \quad b = 2$

$f(n) = n\log n$

$\therefore$ a, b are constant & $f(n)$ is +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore f(n) > n^c$

case 3 is applied

$T(n) = \theta(n\log n)$

Sol 7. $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a = 2, \quad b = 2, \quad f(n) = n/\log n$

$\therefore$ a & b are constant & $f(n)$ is +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore$ non-polynomial difference b/w $f(n)$ & n^c

$\therefore$ Master's theorem is not applicable

Sol 8 $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$\therefore$ a & b are constant & $f(n)$ is +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applied

$T(n) = \theta(n^{0.5+\epsilon})$

Sol 9. $T(n) = 0.5\, T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5, \quad b = 2 \quad f(n) = \frac{1}{n}$

$\because a < 1$

Master's theorem is not applicable.

Sol 10. $T(n) = 16\, T\left(\frac{n}{4}\right) + n!$

$a = 16, \quad b = 4 \quad f(n) = n!$

$\because$ a & b are const. & $f(n)$ is +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2 \log_4 4$

$= 2$

$n^c = n^2$

$\therefore f(n) > n^c$

case 3 is applied

$T(n) = \theta(n!)$

Sol 11. $T(n) = 4\, T\left(\frac{n}{2}\right) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

a & b are constant & $f(n)$ is +ve

$\therefore c = \log_b a \quad \log_2 4 = 2$

$n^c = n^2$

$f(n) < n^c$

Case 1 is applied

$T(n) = \theta(n^2)$

Sol 12 $T(n) = T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

$\therefore$ a is not constant

$\therefore$ Master's theorem is not applicable

Sol 13- $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$, $b = 2$, $f(n) = n$

a & b are constant & $f(n)$ is +ve

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 0.158$

$n^c = n^{0.158}$

$f(n) < n^c$

case 1 is applied

$T(n) = \theta(n^{1.58})$

Sol 14 $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

a & b are constant & $f(n)$ is +ve

Master's theorem is applied

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) < n^c$

case 1 is applied

$T(n) = \theta(n)$

Sol 16

$$T(n) = 4T\left(\frac{n}{2}\right) + c \cdot n$$

$a = 4, \quad b = 2, \quad f(n) = c \cdot n$

∵ a & b are constant & $f(n)$ is +ve

Master's theorem is applied

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) < n^c$$

$$T(n) = \theta(n^2)$$

Sol 17:

$$T(n) = 3T\left(\frac{n}{4}\right) + n \log n$$

$a = 3, \quad b = 4, \quad f(n) = n \log n$

a & b = constant, $f(n)$ is +ve

Master's theorem is applied

$$c = \log_b a = \log_4 3 = 0.79$$

$$n^c = n^{0.79}$$

$$f(n) > n^c$$

$$T(n) = \theta(n \log n)$$

Sol 17:

$$T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$$

$a = 3, \quad b = 3 \quad f(n) = \frac{n}{2}$

$$c = \log_b a = \log_3 3 = 1$$

$$n^c = n^1 = n$$

$$f(n) = n^c$$

$$T(n) = n \log n$$

Sol 18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 6, \quad b = 3, \quad f(n) = n^2 \log n$

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$f(n) > n^c$

case 3 :-

$T(n) = \theta(n^2 \log n)$

Sol 19. $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 4, \quad b = 2, \quad f(n) = n/\log n$

$c = \log_b a = \log_2 4 = 2$

$f(n) < n^c$

$T(n) = \theta(n^2)$

Sol 20. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

∴ a & b are ~~not~~ constant but function is -ve Master's theorem is not applied.

Sol 21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$f(n) > n^c$

$T(n) = \theta(n^2)$

Date
Page

Q.22 $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$\therefore$ $f(n)$ is not regular function

$\therefore$ master's theorem is ~~does not~~ applicable.