BT6270: Computational Neuroscience
FitzHugh-Nagumo model
Assignment 2

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Introduction

FitzHugh-Nagumo model is constructed by reducing the 4-variable Hodgkin Huxley model to a two-variable model by applying suitable assumptions. It is assumed that the time scales for gating variables m, h and n are not of the same order. The FitzHugh-Nagumo Model is a relaxation oscillator because, the system exhibits a characteristic excursion in phase space as the external stimulus I_{ext} exceeds a certain threshold value, before the variables v and w relax back to their rest values. This is a typical behaviour for spike generations (a short, nonlinear elevation of membrane voltage v, diminished over time by a slower, linear recovery variable w) in a neuron after stimulation by an external input current. We have the following two equations determining the system:

$dv/dt = f(v) - w + I_{ext}$

$dw/dt = b*v - r*w$

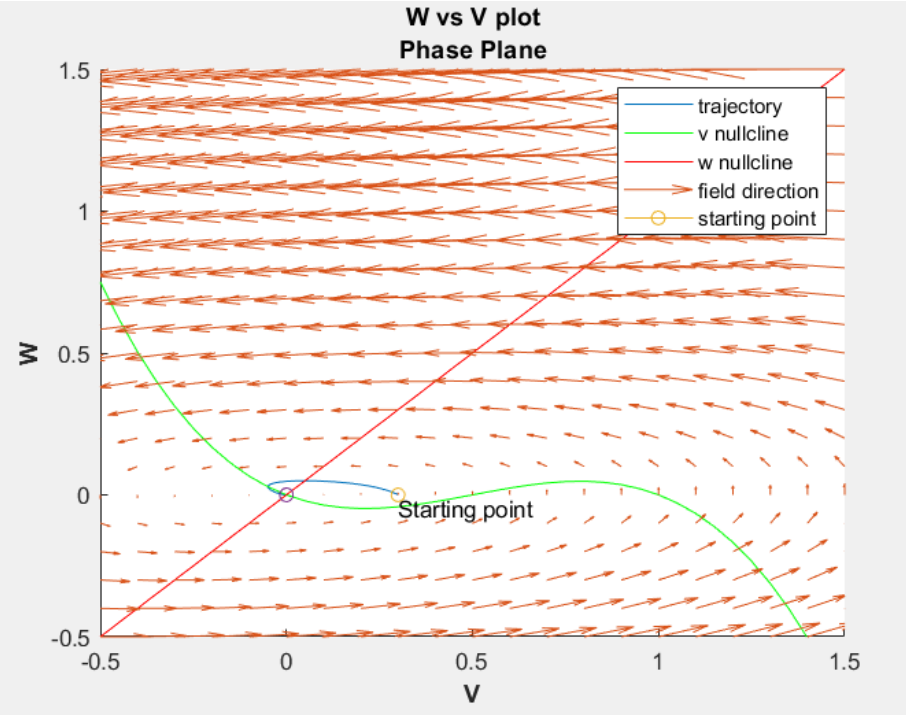
where, $f(v) = v*(a - v) *(v - 1)$

Here b and r are very small positive values.

Given a = 0.5 and b, r = 0.1

Case 1: Excitability (I_{ext}= 0)

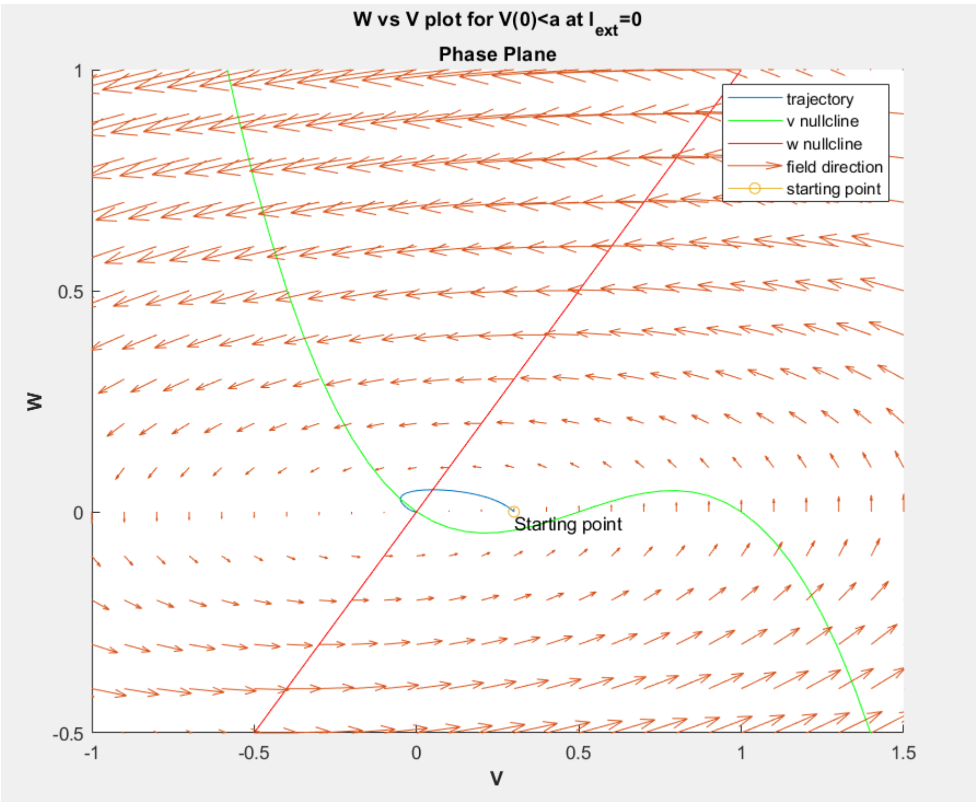
1. Phase plot



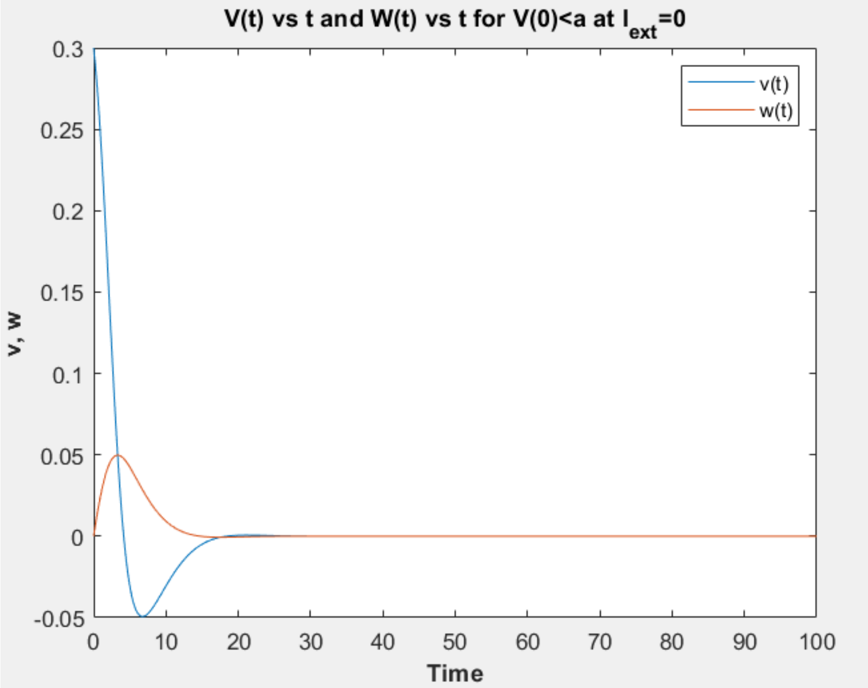
2.

a) V(0) < a and W (0)= 0

The plot below shows the phase trajectories and the null-clines for V(0) = 0.3 (< a), W(0) = 0 and I_{ext} = 0:

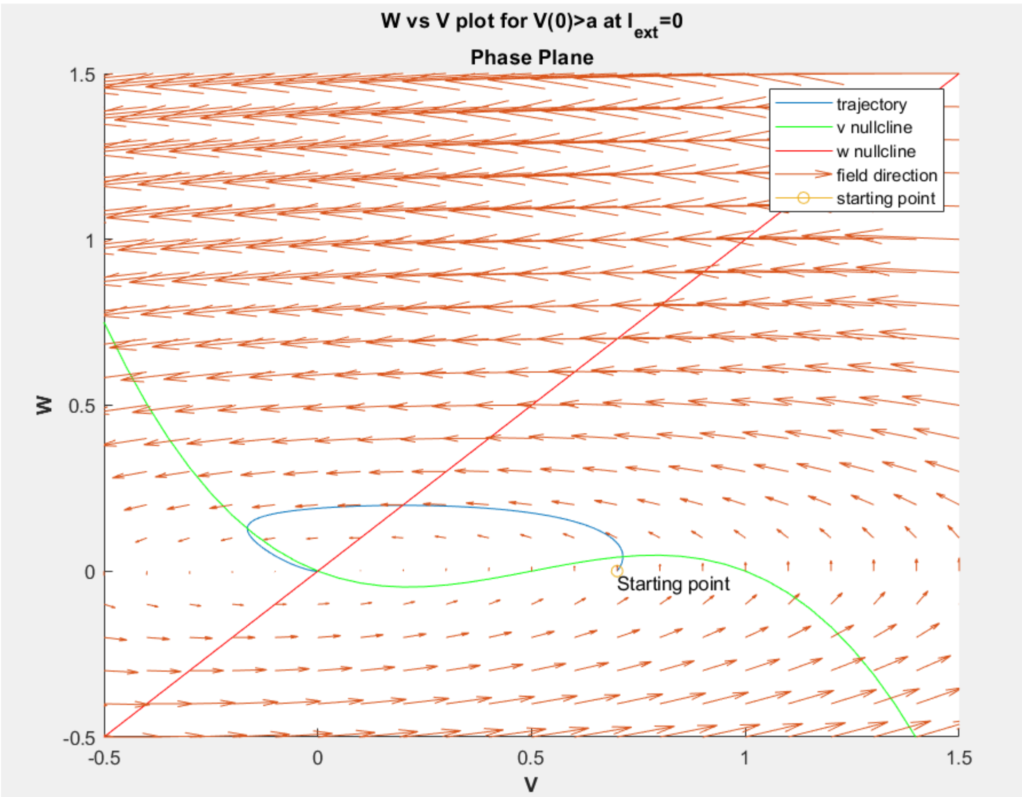


The plots below show the variation of V(t) and W(t) with time

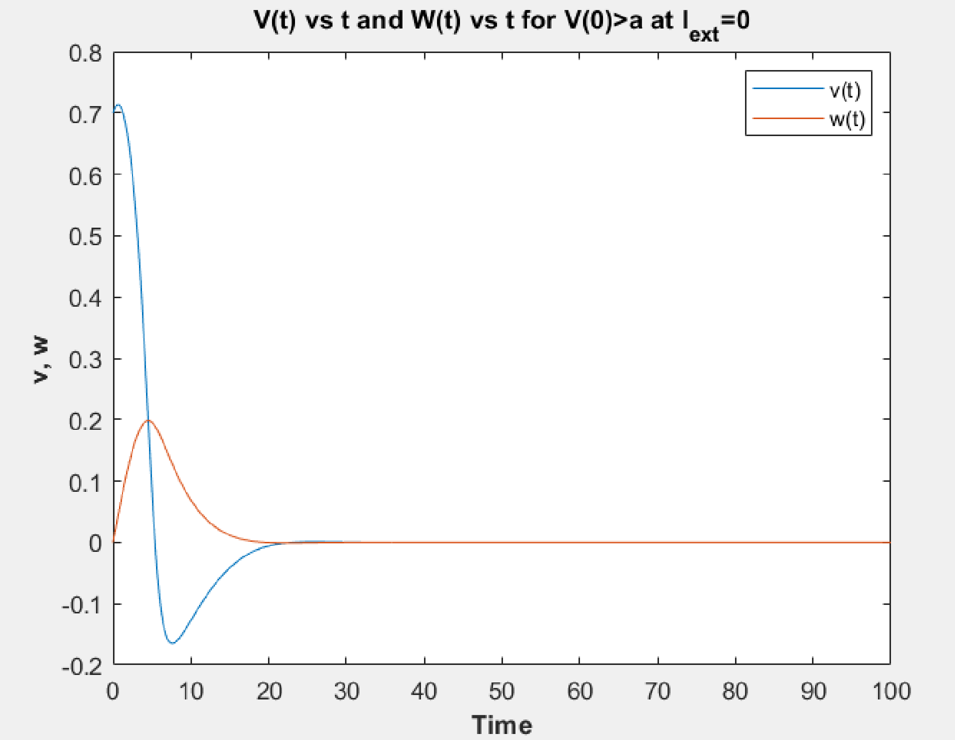


b) V(0) > a and W (0)= 0

The plot below shows the phase trajectories and the null-clines for V(0) = 0.7 (> a), W(0) = 0 and I_{ext} = 0:



The plots below show the variation of V(t) and W(t) with time



In both the cases the trajectory ends at fixed point (0,0) which is stable.

Case 2: Limit Cycles ($I1<I_{ext}<I2$, I_{ext} = 0.6)

(Code to estimate I1 and I2 values attached)

I1 and I2 values:

To estimate threshold current values, I estimated the value of T(tau). The value of I_{ext} for which T changes it's sign from positive to negative and negative to positive are the threshold current values I1 and I2 respectively. The jump value for I_{ext} = 0.001 T is given by-

T = f'(v)-r

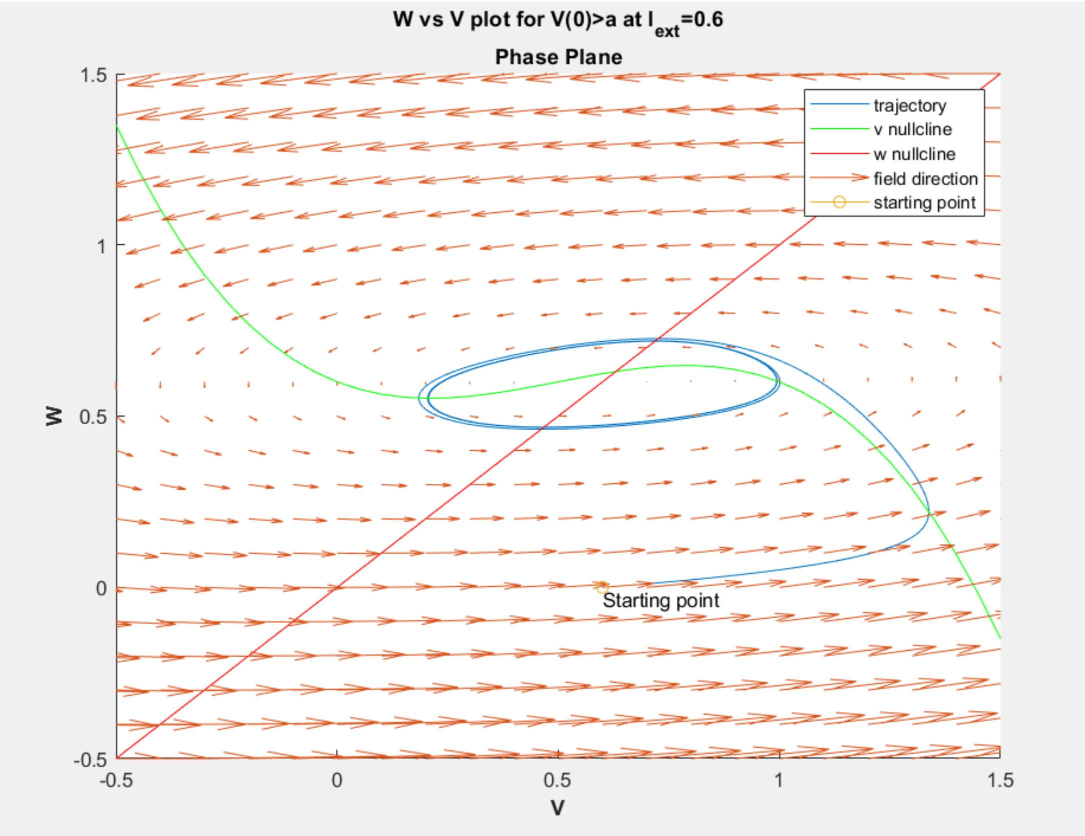
Where f'(v) = differentiation of v nullcline

r = 0.1

I1 = 0.3210 (any I_{ext} >I1 will show limit cycle behaviour and unstable fixed point)
I2 = 0.6790 (any I_{ext} <I1 will show limit cycle behaviour and unstable fixed point)

We set Iext = 0.6

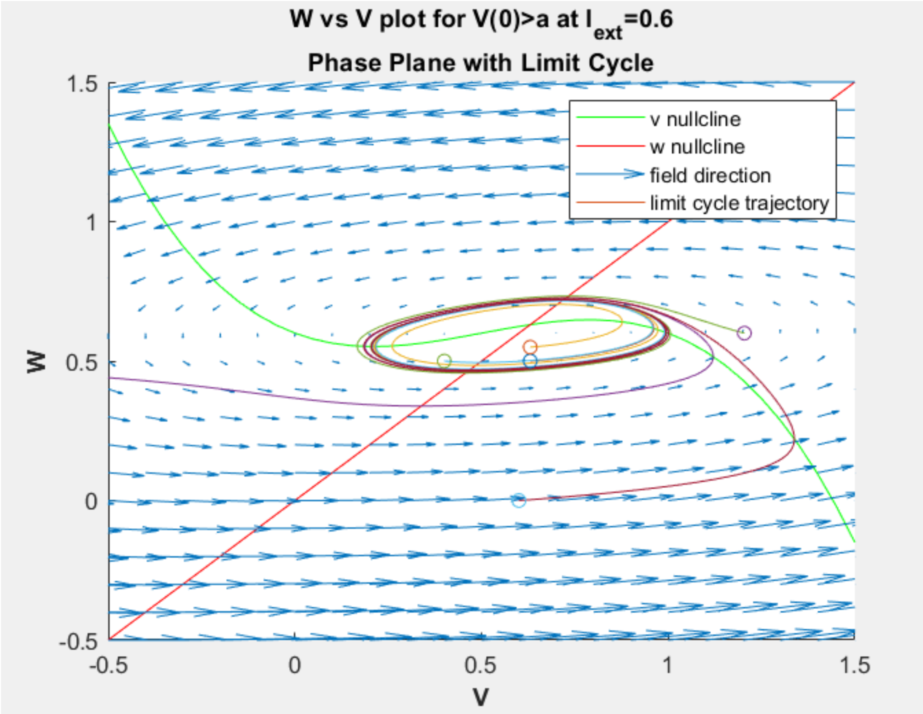
a) The plot below shows the phase trajectories as well as the null-clines



b) Show that the fixed point is unstable i.e., for a small perturbation there is a no return to the fixed point (show the trajectory on the phase plane) – also show limit cycle on the phase plane

The fixed point for $I_{ext} = 0.6$ is (v,w) = (0.6304, 0.6304).

The following figure shows the limit cycle on phase plane. The fixed point is unstable fixed point as even a small perturbation from fixed point leads to complete deflection from the fixed point.



Fixed point is (0.6304, 0.6304)

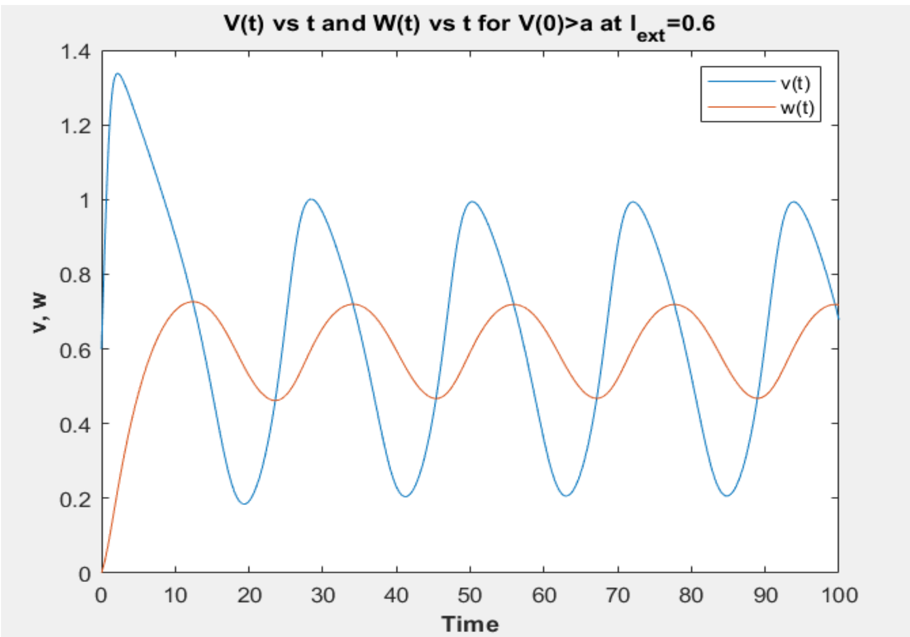
Trace = f'(fixed point)-r = 0.0990

And, delta = f'(fixed point)*(-r) + b = 0.0801

As both Trace>0 and delta>0, the fixed point is unstable fixed point

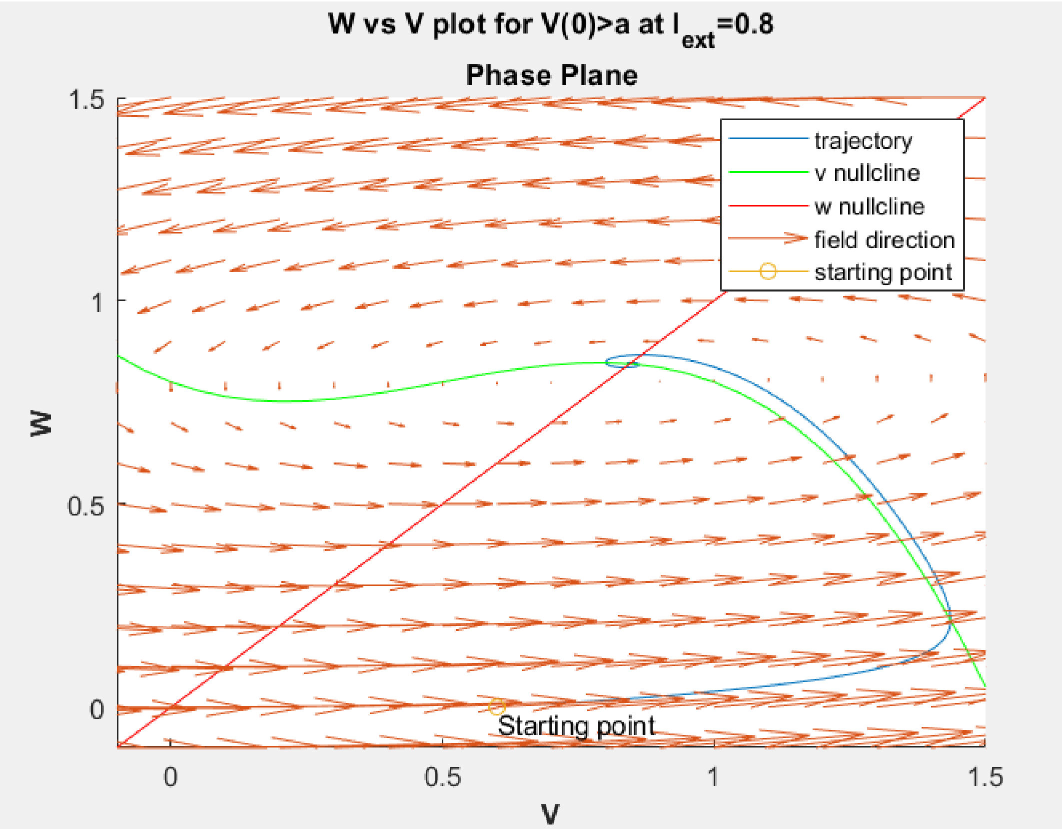
Looking at limit cycle, as shown a small perturbation from fixed point there is no return to fixed point. Hence the fixed point is Unstable.

The plots below show the variation of V(t) and W(t) with time



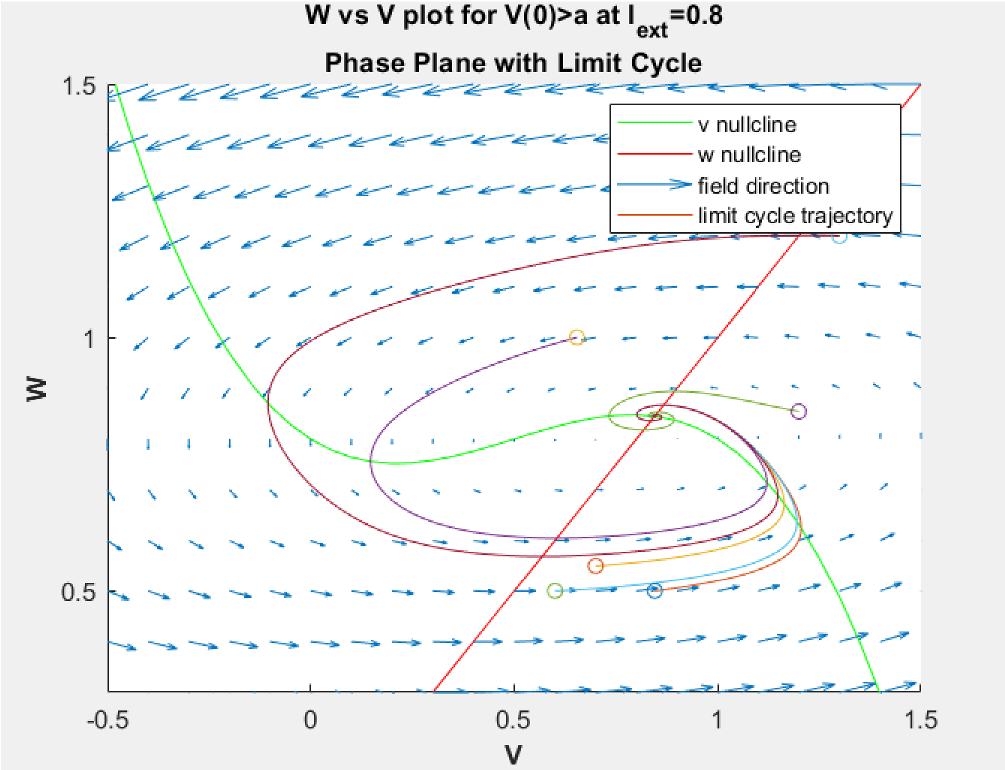
Case 3: Depolarization (I_{ext}>I2, I_{ext} = 0.8)

a) Draw a Phase Plot for some sample value of Iext



b) Show that the fixed point is stable i.e., for a small perturbation there is a return to the fixed point (show the trajectory on the phase plane)

The following figure shows the limit cycle on phase plane for I greater than I2 (relaxed state). The fixed point is a stable fixed point any perturbation from fixed point will return to this point.



Fixed point is (0.8452, 0.8452)

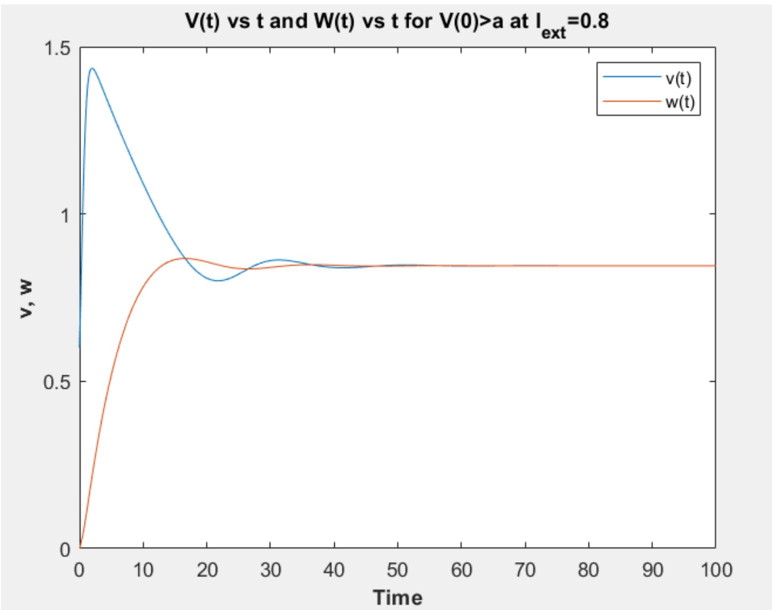
Trace = f'(fixed point)-r = -0.2075

And, delta = f'(fixed point)*(-r) + b = 0.1107

As Trace<0 and delta>0, the fixed point is a stable fixed point.

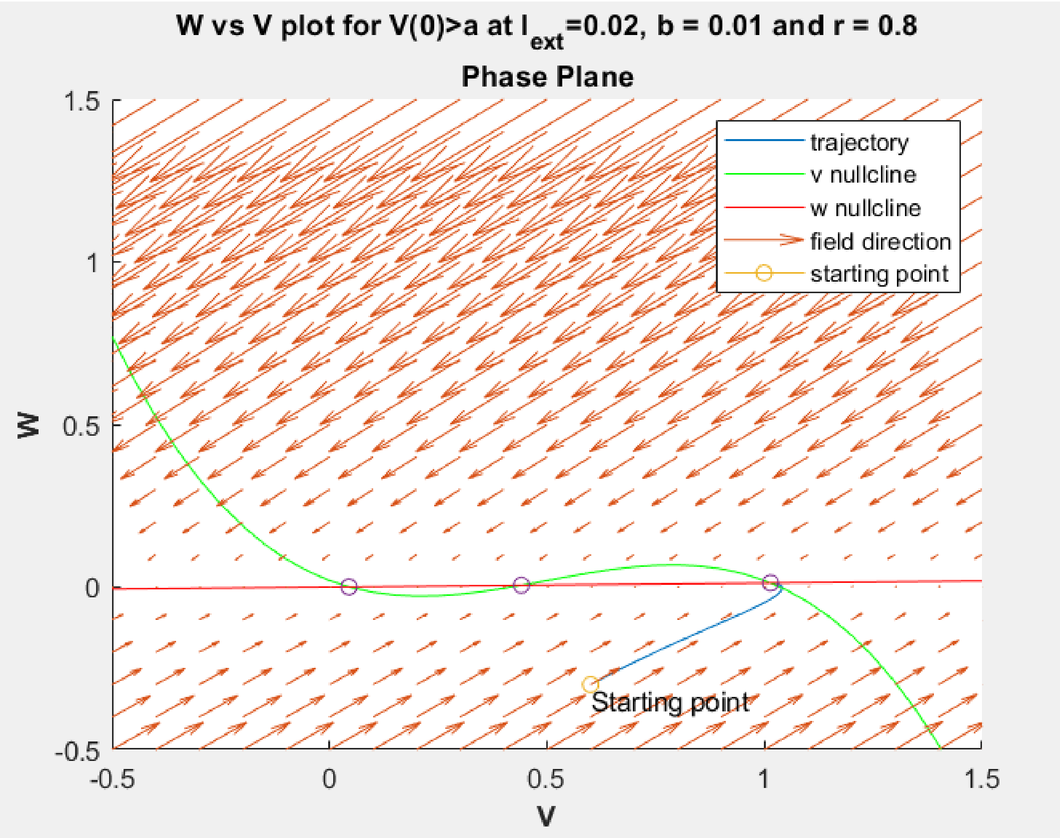
Also, by looking at limit cycle, a small perturbation from fixed point will return to the fixed point. Hence the fixed point is Stable.

Plot V(t) vs t and W(t) vs t



Case 4: Bistability (a=0.5, b=0.01, r=0.8)

a) Phase Plot



b) Stability of Fixed Points

Fixed points (FP): (V,W)

a. FP1 = (1.0141, 0.0127)
   f'(fp) = -0.5429 < 0
   Trace = f'(fp)-r = -0.5429-0.8 = -1.3429 <0
   Determinant (delta) = 0.4443 >0
   Since delta>0 and Trace<0, the fixed point is stable point. Any small perturbation around this point brings it back to that point.

b. FP2 = (0.4413, 0.0055)
   f'(fp) = 0.2397 > 0
   Trace = f'(fp)-r = 0.2397-0.8 = -0.5603 < 0
   Determinant (delta) = -0.1817 < 0
   Since delta<0 the fixed point is saddle point irrespective of Trace value. Any small perturbation around repels the point and take it to FP1 or FP3

c. FP3 = (0.0447, 0.0006)
   f'(fp) = -0.3719<0
   Trace = f'(fp)-r = -0.3719-0.8 = -1.1719<0

Determinant (delta) = 0.3075>0
Since delta>0 and Trace<0, the fixed point is stable point. Any small perturbation around this point brings it back to that point.

c) Plot V(t) vs t and W(t) vs t