

$$\text{arr}[1] + \text{arr}[4] = 0 + 2 = 2$$

## Binary lifting

There is a tree of size  $N$ , rooted at 0, answer  $Q$  queries: given  $v$  &  $K$ , find  $k$ -th ancestor of  $v$ .

Bruteforce ( $Q \times N$ )

each query in Brute force  $\Rightarrow O(N)$

we want

$O(\log n)$

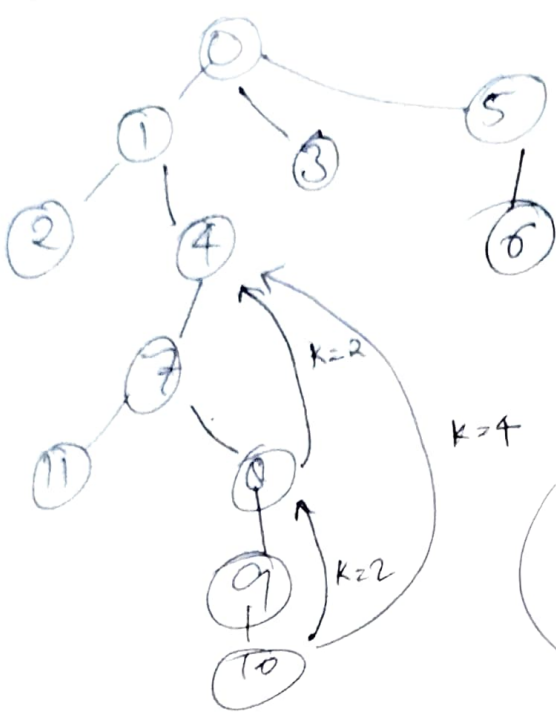
How to improve from  $O(N)$  to  $O(\log(N))$ ?

divide by 2

- BS
- D&C

Power of 2

- segment tree
- binary lifting
- sparse tables



int up[N][log];  
 $up[v][j] \Rightarrow 2^j$ -th ancestor of  $v$

```
for v = 0 ... N-1
    up[v][0] = up parent[v]
    for j = 1 ... log N - 1
        up[v][j] = up[up[v][j-1]][j-1]
        up[v][2] = up[up[v][1]][1]
        up[v][3] = up[up[v][2]][2]
        ...
```

```
for v = 0 ... N-1.
    up[v][0] = par[v]
    for j = 1 ... log N - 1:
        up[v][j] = up[up[v][j-1]][j-1]
```

T.C. & S.C  $\Rightarrow \mathcal{O}(N \log N)$

If  $parent[i] < i$ ; then above code is true

else do as: ① run dfs.  
 ② written below:

int up[N][log]:

$up[v][j] \dots 2^j$ -th ancestor of  $v$

for  $v = 0 \dots N-1$ :

$up[v][0] = parent[v]$

for  $j = 1 \dots \log N - 1$ :

for  $v = 0 \dots N-1$ :

$up[v][j] = up[up[v][j-1]][j-1]$

$E \Rightarrow 19 = 16 + 2 + 1 \Rightarrow x = up[4][0]$

$x = up[x][1]$

$x = up[x][4]$

How to avoid extra (over) binary lifting :- 8

Ex  $\Rightarrow$  Suppose the root of binary in given tree has the following binary lifting procedure.

Root = 0 ; assume Parent[root] = -1

up[0][0] = Parent[0]

now if we apply same logic as in binary lifting code then there will be a problem.

Problem  $\Rightarrow$  There is no ancestor for the root.

Solution  $\Rightarrow$  Parent[root] = 0 ; // Magical Line

Intuition  $\Rightarrow$  for above ex; root = 0  
up[0][0] = Parent[0] = 0

$$\text{up}[0][1] = \text{up}[\text{up}[0][1-1]][1-1] = \text{up}[0][0] = 0$$

$$\text{up}[0][2] = \text{up}[\text{up}[0][2-1]][2-1] = \text{up}[0][1] = 0$$

⋮

$$\text{up}[0][j-1] = \text{up}[\text{up}[0][j-2]][j-2] = \text{up}[0][j-2] = 0$$

$$\text{up}[0][j] = \text{up}[\text{up}[0][j-1]][j-1] = \text{up}[0][j-1] = 0$$

⋮

$$\text{up}[0][\log] = \text{up}[\text{up}[0][\log-1]][\log-1] = \text{up}[0][\log-1] = 0$$

Conclusion  $\Rightarrow$  Similarly, for all the nodes which have no possibility of binary lifting same logic will be applied.

