Chapter-2 Complex Number

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7. If $1,a_1,a_2,a_3....a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1}) = n$
   (1984- 2 Marks).

8. Show that the area of the triangle on the Argand
   diagram formed by the complex numbers
   $z, \iota z$ & $z+\iota z$ is $\frac{1}{2}|z|^2$.
   (1986-2 Marks).

9. Let $\mathbf{Z_1}$=$10+6\iota$ and $\mathbf{Z_2}$=$4+6\iota$. if $\mathbf{Z}$ is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$
   is $\frac{\pi}{4}$ then prove that $Z-7-9\iota$ =3 $\sqrt{2}$
   (1990-4 Marks).

10. if iz^3 -z^2-z+ι = 0 then show that $|z|$ =1
    (1995-5 Marks).

11. If $|Z| \leq 1$,$|W| \leq 1$,show that
    $(Z-W)^2 \leq (|Z|-|W|)^2 + (\arg Z - \arg W)^2$
    (1995-5 Marks).

12. Find all non-zero complex numbers $\mathbf{Z}$
    satisfying $\bar{Z}$=ιZ^2
    (1996-2 Marks).

13. Let $\mathbf{z}_1$ and $\mathbf{z}_2$ be roots of the equation
    z^2+pz+q=0, where the coefficients **p** and **q**
    may be complex numbers. Let **A** and **B** represent
    z_1 and z_2 in the complex plane . if$\angle$**ABC** = $\alpha \neq 0$
    and **OA**=**OB**,where **O** is the origin,prove that
    p^2=$4q\cos^2\left(\frac{\alpha}{2}\right)$.
    (1997-5 Marks)

14. For complex number **z** and **w**,prove
    that $|z|^2\, w$-$|w|^2\, z$=z- w if and only
    if z =w or $\bar{w}$ = 1.
    (1999-10 Marks).

15. Let a complex number α ,$\alpha \neq 1$, be a root of the equation z^{p+q}-z^p-z^q+1=0, where **p**,**q** are the distinct primes. Show that either
    1+α+α^2+...+α^{p-1} = 0 or 1+α+α^2+...+α^{q-1} = 0,
    but not both together.
    (2002-5 Marks)

16. If $\mathbf{z_1}$ and $\mathbf{z_2}$ are two complex number such that $|z_1 < 1 <| |z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1$.
    (2003-2 Marks)

17. Prove that there exists no complex number **z** such that $|z|$¡$\frac{1}{3}$ and $\sum_{r=1}^{n}\, a_r\, z^r$ =1 where $|a_r|$¡2.
    (2003-2 Marks)

18. Find the centre and radius of circle given by $\left|\frac{z-\alpha}{\beta}\right|$=$k$,$k \neq 1$ where,z=x+ι,α=α_1+$\iota\alpha_2$,
    β=β_1+$\iota\beta_2$.
    (2004-2 Marks)

19. If one the vertices of the square
    circumscribing the circle $|z-1| = \sqrt{2} is 2 + \sqrt{3}\iota$
    find the other vertices of the square .
    (2005-4 Marks)