Chapter-2 Complex Number

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7) If 1,a_1,a_2,a_3....a_{n-1} are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1}) = n$ (1984- 2 Marks).

8) Show that the area of the triangle on the Argand diagram formed by the complex numbers
z, iz & $z+iz$ is $\frac{1}{2}|z|^2$. (1986-2 Marks).

9) Let Z_1=$10+6i$ and Z_2=$4+6i$. if Z is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$ is $\frac{\pi}{4}$ then prove that $Z-7-9i$ =$3\sqrt{2}$. (1990-4 Marks).

10) if iz^3 -z^2-z+ι = 0 then show that $|z|$ =1. (1995-5 Marks).

11) If $|Z| \leq 1$,$|W| \leq 1$,show that $(Z-W)^2 \leq (|Z|-|W|)^2 + (\arg Z - \arg W)^2$. (1995-5 Marks).

12) Find all non-zero complex numbers Z satisfying $\bar{Z}$=iZ^2 . (1996-2 Marks).

13) Let z_1 and z_2 be roots of the equation z^2+pz+q=0, where the coefficients p and q may be complex numbers. Let **A** and **B** represent z_1 and z_2 in the complex plane . if$\angle ABC$ = $\alpha \neq 0$ and OA=OB,where$O$ is the origin,prove that p^2=$4q\cos^2\left(\frac{\alpha}{2}\right)$. (1997-5 Marks)

14) For complex number z and w,prove that $|z|^2$ w-$|w|^2$ z=z- w if and only if z =w or $\bar{w}e$ = 1. (1999-10 Marks).

15) Let a complex number α ,$\alpha \neq 1$, be a root of the equation z^{p+q}-z^p-z^q+1=0, where **p**,**q** are the distinct primes. Show that either $1+\alpha+\alpha^2+...+\alpha^{p-1} = 0$ or $1+\alpha+\alpha^2+...+\alpha^{q-1} = 0$,but not both together. (2002-5 Marks)

16) If z_1 and z_2 are two complex number such that $|z_1 < 1 <||z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1$.. (2003-2 Marks)

17) Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r\ z^r$ =1 where $|a_r|$¡2. (2003-2 Marks)

18) Find the centre and radius of circle given by $\left|\frac{z-\alpha}{\beta}\right|$=$k$,$k \neq 1$ where,$z = x+i$,α=α_1+$i\alpha_2$, β=β_1+$i\beta_2$. (2004-2 Marks)

19) If one the vertices of the square
circumscribing the circle $|z-1| = \sqrt{2}$ is $2+\sqrt{3}i$ find the other vertices of the square

. (2005-4 Marks)

PASSAGE – 1

Let **A**,**B**,**C** be three sets of complex number as defined below

$\mathbf{A} = \{z: |\operatorname{Im} z \geq 1\}$

$\mathbf{B} = \{z: |z - 2 - i| = 3\}$

$\mathbf{C} = \{z: \operatorname{Re}((1 - i)z) = \sqrt{2}\}$

a) The number of element in the set $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ is. (2008) (a) 0 (b) 1 (c) 2 (d) ∞

b) Let **z** be any point in $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between. (2008)

i) 25 and 29
iii) 35 and 39
ii) 30 and 34
iv) 40 and 44

c) Let z be any point **A B C** and let **w** be any point satisfying $|w - 2 - i| < 3$.Then, $|z| - |w| + 3$ lies between. (2008)

i) -6 and 3
iii) -6 and 6
ii) -3 and 6
iv) -3 and 9