Gate PH-2013

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40) A proton is confined to a cubic box, whose sides have length $10^{-12}m$. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27}kg and Planck's constant is 6.63×10^{-34}Js.

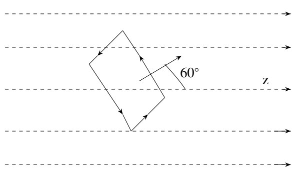
    a) 1.1×10^{-17}J b) 3.3×10^{-17}J c) 9.9×10^{-17}J d) 6.6×10^{-17}J

41) For the function $f(\text{z}) = \frac{16\mathbf{z}}{(z+3)(z-1)^2}$ the residue at the pole z = 1 is (your answer should be an integer)

42) 2 The degenerate eigenvalue of the matrix
$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$ **is** (your answer should be an integer)

43) Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \rightarrow \mu^- + \nu_\mu^-$ in the pion rest frame $m_\pi = 139.6\frac{MeV}{c^2}, m_\mu = 105.7\frac{MeV}{c^2}, m_\nu \approx 0$ The energy (in MeV) of the emitted neutrino, to the nearest integer is

44) In a constant magnetic field of 0.6 Tesla along the z direction, find the value of the path integral $\oint \overrightarrow{A} \cdot \overrightarrow{dl}$ in the units of on $\left(\text{Tesla } m^2\right)$ a square loop of side length $\left(\frac{1}{\sqrt{2}}\right)$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in the figure. The answer should be up to two decimal places.



45) A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z then what is the expectation value, up to one decimal place, of the operator$10\sigma_z + 5\sigma_x$? Here,symbols have their usual meanings.

46) Consider the wave function A$e^{ikr}\left(\frac{r_0}{r}\right)$ where A is the normalization constant. For r=$2r_0$ the magnitude of probability current density up to two decimal places, in units of $\frac{A^2\hbar k}{m}$ is

47) An n-channel junction field effect transistor has $5mA$ source to drain current at shorted gate (I_{DSS}) and $5V$ pinch off voltage (V_P). Calculate the drain current in mA for a gate-source voltage (V_{GS}) of $-2.5V$. The answer should be up to two decimal places.

Common Data Questions

Common Data for Questions 48 and 49: There are four energy levels E, 2E, 3E

and 4E (where$E > 0$). The canonical partition function of two particles is , if these particles are

48) two identical fermions

a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

c) $\left(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}\right)^2$

d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

49) two distinguishable particles

a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

c) $\left(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}\right)^2$

d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Common Data for Questions 50 and 51: To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by

$$\epsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

, where ϵ is a small quantity

50) The ground state eigenvector of the unperturbed Hamiltonian is

a) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ b) $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$ c) $(0, 0, 1)$ d) $(1, 0, 0)$

51) A pair of eigenvalues of the perturbed Hamiltonian, using first order perturbation theory, is

a) $3 + 2\epsilon + 7 + 2\epsilon$ b) $3 + 2\epsilon, 2 + \epsilon$ c) 3,7+2ϵ d) 3,2+2ϵ

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53: In the Schmidt model of nuclear magnetic moments, we have, $\vec{\mu} = \frac{e\hbar}{2Mc}\left(g_l \vec{l} + g_s \vec{S}\right)$ where the symbols have their usual meaning

52) For the case $j = l + \frac{1}{2}$ where J is the total angular momentum, the expectation value of $\vec{S} \cdot \vec{J}$ in the nuclear ground state is equal to,

a) $\frac{(J-1)}{2}$ b) $\frac{(J+1)}{2}$ c) $\frac{J}{2}$ d) $\frac{-J}{2}$

53) For the O^{17} nucleus $(A = 17, Z = 8)$ the effective magnetic moment is given by, $\vec{\mu} = \frac{e\hbar}{2Mc} g \vec{J}$ where is equal to, ($g_s = 5.59$ for proton and -3.83 for neutron)

a) 1.12 b) 0.77 c) -1.28 d) 1.28