

Chapter-2 Complex Number

AI24BTECH11032 Shreyansh Sonkar

- 7) If $1, a_1, a_2, a_3, \dots, a_{n-1}$ are the n roots of unity, then show that $(1 - a_1)(1 - a_2)(1 - a_3) \dots (1 - a_{n-1}) = n$ (1984- 2 Marks).
- 8) Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz & $z + iz$ is $\frac{1}{2} |z|^2$. (1986-2 Marks).
- 9) Let $Z_1 = 10 + 6i$ and $Z_2 = 4 + 6i$. If Z is any complex number such that the argument of $\frac{(Z - Z_1)}{(Z - Z_2)}$ is $\frac{\pi}{4}$ then prove that $Z - 7 - 9i = 3\sqrt{2}$. (1990-4 Marks).
- 10) If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$. (1995-5 Marks).
- 11) If $|Z| \leq 1, |W| \leq 1$, show that $(z - w)^2 \leq (|Z||W|)^2 + (\arg Z - \arg W)^2$. (1995-5 Marks).
- 12) Find all non-zero complex numbers Z satisfying $\bar{Z} = iZ^2$. (1996-2 Marks).
- 13) Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let **A** and **B** represent z_1 and z_2 in the complex plane. If $\angle ABC = \alpha \neq 0$ and $OA = OB$, where O is the origin, prove that $p^2 = 4q \cos^2\left(\frac{\alpha}{2}\right)$. (1997-5 Marks)
- 14) For complex number z and w , prove that $|z|^2 w - |w|^2 z = z - w$ if and only if $z = w$ or $\bar{w}e = 1$. (1999-10 Marks).
- 15) Let a complex number $\alpha, \alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$, where **p, q** are the distinct primes. Show that either $1 + \alpha + \alpha^2 + \dots + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + \dots + \alpha^{q-1} = 0$, but not both together. (2002-5 Marks)
- 16) If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that $\left| \frac{1 - z_1 \bar{z}_2}{z_1 - z_2} \right| < 1$. (2003-2 Marks)
- 17) Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^n a_r z^r = 1$ where $|a_r| < 2$. (2003-2 Marks)
- 18) Find the centre and radius of circle given by $\left| \frac{z - \alpha}{b} \right| = k, k \neq 1$ where, $z = x + i, \alpha = \alpha_1 + i\alpha_2, b = b_1 + ib_2$. (2004-2 Marks)
- 19) If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}i$ find the other vertices of the square. (2005-4 Marks)

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Let **A, B, C** be three sets of complex number as defined below

$$\mathbf{A} = \{z: |\operatorname{Im} z| \geq 1\}$$

$$\mathbf{B} = \{z: |z - 2 - i| = 3\}$$

$$\mathbf{C} = \{z: \operatorname{Re}((1 - i)z) = \sqrt{2}\}$$

- a) The number of element in the set $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ is. (2008)
 (a) 0 (b) 1 (c) 2 (d) ∞
- b) Let z be any point in $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between. (2008)
 i) 25 and 29 iii) 35 and 39
 ii) 30 and 34 iv) 40 and 44
- c) Let z be any point $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between. (2008)
 i) -6 and 3 iii) -6 and 6
 ii) -3 and 6 iv) -3 and 9