Chapter-2 Complex Number

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7) If $1, a_1, a_2, a_3 \cdots a_{n-1}$ are the n roots of unity,then show that $(1-a_1)(1-a_2)(1-a_3)\cdots(1-a_{n-1}) = n$ (1984- 2 Marks).

8) Show that the area of the triangle on the Argand diagram formed by the complex numbers
z, iz & $z+iz$ is $\frac{1}{2}|z|^2$. (1986-2 Marks).

9) Let $Z_1 = 10+6i$ and $Z_2 = 4+6i$.if Z is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$ is $\frac{\pi}{4}$ then prove that $Z-7-9i = 3\sqrt{2}$. (1990-4 Marks).

10) if $iz^3+z^2-z+i=0$ then show that $|z|=1$. (1995-5 Marks).

11) If $|Z| \leq 1$,$|W| \leq 1$,show that $(z-w)^2 \leq (|Z|\,|W|)^2 + (\arg Z - \arg W)^2$. (1995-5 Marks).

12) Find all non-zero complex numbers Z satisfying $\bar{Z} = iZ^2$. (1996-2 Marks).

13) Let z_1 and z_2 be roots of the equation $z^2+pz+q=0$,where the coefficients p and q may be complex numbers.Let **A** and **B** represent z_1 and z_2 in the complex plane.if$\angle ABC = \alpha \neq 0$ and $OA = OB$,where$O$ is the origin,prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$. (1997-5 Marks)

14) For complex number z and w,prove that $|z|^2 w - |w|^2 z = z - w$ if and only if $z = w$ or $\bar{w}e = 1$. (1999-10 Marks).

15) Let a complex number α,$\alpha \neq 1$,be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$,where **p**,**q** are the distinct primes.Show that either $1+\alpha+\alpha^2+\cdots+\alpha^{p-1} = 0$ or $1+\alpha+\alpha^2+\cdots+\alpha^{q-1} = 0$,but not both together. (2002-5 Marks)

16) If z_1 and z_2 are two complex number such that $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1$. (2003-2 Marks)

17) Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where$|a_r| < 2$. (2003-2 Marks)

18) Find the centre and radius of circle given by $\left|\frac{z-\alpha}{b}\right| = k$,$k \neq 1$ where,$z = x+i$,$\alpha = \alpha_1 + i\alpha_2$,$b = b_1 + ib_2$. (2004-2 Marks)

19) If one the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2+\sqrt{3}i$ find the other vertices of the square . (2005-4 Marks)

$PASSAGE - 1$

Let **A**,**B**,**C** be three sets of complex number as defined below
$\mathbf{A} = \{z{:}|\ \mathrm{Im}\, z \geq 1\}$
$\mathbf{B} = \{z{:}\ |z-2-i| = 3\}$
$\mathbf{C} = \{z : \mathrm{Re}\,((1-i)z) = \sqrt{2}\}$

a) The number of element in the set $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ is. (2008)
(a) 0 (b) 1 (c) 2 (d) ∞

b) Let **z** be any point in $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ Then, $|z+1-i|^2 + |z-5-i|^2$ lies between. (2008)

i) 25 and 29
ii) 30 and 34
iii) 35 and 39
iv) 40 and 44

c) Let z be any point **A B C** and let w be any point satisfying $|w-2-i| < 3$.Then, $|z| - |w| + 3$ lies between. (2008)

i) -6 and 3
ii) -3 and 6
iii) -6 and 6
iv) -3 and 9