Gate MA-2024

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53) Which of the following is/are eigenvalue(s) of the Sturm-Liouville problem

$$y^n + \lambda y = 0, 0 \le x \le \pi,$$
$$y(0) = y\prime(0)$$
$$y(\pi) = y\prime(\pi)?$$

a) $\lambda = 1$
b) $\lambda = 2$
c) $\lambda = 3$
d) $\lambda = 4$

54) Let $f : \mathbb{R}^2 \to R$ be a function such that

$$f(x, y) = \begin{cases} \left(1 - \cos\frac{x^2}{y^2}\right)\sqrt{x^2 + y^2}, \text{ If y } \neq 0, x \in \mathbb{R} \\ 0, \text{ otherwise} \end{cases}$$

Which of the following is/are correct?

a) f is continuous at $(0, 0)$, but not differentiable at $(0, 0)$
b) f is differentiable at $(0, 0)$
c) All the directional derivatives of f at $(0, 0)$ exist and they are equal to zero
d) Both the partial derivatives of f at $(0, 0)$ exist and they are equal to zero

55) For an integer n, let $f_n(x) = xe^{-nx}$ where $x \in [0, 1]$. Let S $:= \{f_n : n \ge 1\}$. Consider the matric space $(C([0, 1]), d)$ where

$$d(f, g) = \sup_{x\in(0,1)}\{|f(x) - g(x)|\}, f, g \in C([0, 1]).$$

Which of the following statement(s) is/are true?

a) S is an equi-continuous family of continuous functions
b) S is closed in $(C([0, 1]), d)$
c) S is bounded in $(C([0, 1]), d)$
d) S is compact in $(C([0, 1]), d)$

56) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be an $\mathbb{R}$-linear transformation such that 1 and 2 are the only eigenvalues of T Suppose the dimensions of Kernel (T $- I_4$) and Range$(T - 2I_4)$ are1and 2, respectively. Which of the following is/are possible (upper triangular) Jordan canonical form(s) of T?

a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

57) Let $L^2([-1,1])$ denote the space of all real-valued Lebesgue square-integrable functions on $[-1,1]$, with the usual norm $\|\cdot\|$. Let P_I be the subspace of $L^2([-1,1])$ consisting of all the polynomials of degree at most 1. Let $f \in L^2([-1,1])$ be such that $\|f\|^2 = \frac{18}{5}$, $\int_{-1}^{1} f(x)\,dx = 2$, and $\int_{-1}^{1} f(x)\,dx = 0$. Then

$$\inf_{g \in P_1} \|f - g\|^2 = \_\_\_\_\_\_\_\_\_\_\_\_$$

(Round off to TWO decimal places)

58) The maximum value of $f(x,y,z) = 10x + 6y - 8z$ subject to the constraints

$$5x - 2y + 6z \leq 20$$
$$10x + 4y - 6z \leq 30$$
$$x, y, a \geq,$$

is equal to ____________(Round off to TWO decimal places)

59) Let K⊆C be the field extension of $\mathbb{Q}$ Q obtained by adjoining all the roots of the polynomial equation $\left(X^2 - 2\right)\left(X^3 - 3\right) = 0$ The number of distinct fields F such that $\mathbb{Q} \subseteq F \subseteq K$ is equal to ____________(answer in integer)

60) Let H be the subset of S_3 consisting of all $\sigma \in S_3$ such that

$$\text{Trace } (A_1 A_2 A_3) = \text{ Trace } (A_{\sigma(1)} A_{\sigma(2)} A_{\sigma(3)})$$

for all $A_1, A_2, A_3 \in M_2(C)$.The number of elements in H is equal to ____________(answer in integer)

61) Let r: $[0,1] \to \mathbb{R}^2$ be a continuously differentiable path from $(0,2)$ to $(3,0)$ and let F: $\mathbb{R}^2 \to \mathbb{R}^2$ be defined $\mathrm{F}(x,y) = (1-2y, 1-2x)$. The line integral of F along r

$$\int F.dr$$

is equal to ____________ (round off to TWO decimal places)

62) Let $u = u(x,t)$ be the solution of the initial value problem

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, x \in \mathbb{R}, t > 0,$$
$$u(x,0) = 0, x \in \mathbb{R},$$
$$\frac{\partial u}{\partial t}(x,0) = \begin{cases} x^4(1-x)^4, & \text{If } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

If $\alpha = \inf\{t > 0 : u(2,t) > 0\}$, then α is equal to ____________ (round off to TWO decimal places)

63) The boundary value problem

$$x^2 y\prime\prime = 2xy\prime + 2y = 0, 1 \le x \le 2, \quad (63.1)$$
$$y(1) - y\prime(1) = 1, \quad (63.2)$$
$$y(2) = ky\prime(2) = 4, \quad (63.3)$$
$$(63.4)$$

has infinitely many distinct solutions when k is equal to ____________ (round off to TWO decimal places)

64) The global maximum of $\mathrm{f}(x,y) = \left(x^2 + y^2 e^{2-x-y}\right)$ on $\left\{(x,y) \in \mathbb{R}^2 : x \ge 0, y \ge 0\right\}$ is equal to ____________ (round off to TWO decimal places)

65) Let $k \in \mathbb{R}$ and D= $\{(r,\theta) : 0 < r < 2, 0 < \theta < \pi\}$. Let $u(r,\theta)$ be the solution of the following boundary value problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, (r,\theta) \in D,$$
$$u(r,0) = u(r,\pi) = 0, 0 \le r \le 2,$$
$$u(2,\theta) = k\sin(2\theta), 0 < \theta < \pi$$

If u$\left(1, \frac{\pi}{4}\right)$ = 2, then the value of k is equal to ____________ (round off to TWO decimal places)