Chapter-2 Complex Number

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7) If $1, a_1, a_2, a_3 .... a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1}) = n$
(1984- 2 Marks).

8) Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz & $z+iz$ is $\frac{1}{2}|z|^2$.
(1986-2 Marks).

9) Let $\mathbf{Z_1}$=$10+6i$ and $\mathbf{Z_2}$=$4+6i$. if $\mathbf{Z}$ is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$ is $\frac{\pi}{4}$ then prove that $Z-7-9i$ $=3\sqrt{2}$
(1990-4 Marks).

10) if iz^3 -z^2-z+ι = 0 then show that $|z|$ =1
(1995-5 Marks).

11) If $|Z| \leq 1$,$|W| \leq 1$,show that $(Z-W)^2 \leq (|Z|-|W|)^2 + (\arg Z - \arg W)^2$
(1995-5 Marks).

12) Find all non-zero complex numbers $\mathbf{Z}$ satisfying $\bar{Z}$=iZ^2
(1996-2 Marks).

13) Let $\mathbf{z_1}$ and $\mathbf{z_2}$ be roots of the equation z^2+pz+q=0, where the coefficients **p** and **q** may be complex numbers. Let **A** and **B** represent z_1 and z_2 in the complex plane . if$\angle$**ABC** $= \alpha \neq 0$ and **OA**=**OB**,where **O** is the origin,prove that p^2=$4q\cos^2\left(\frac{\alpha}{2}\right)$.
(1997-5 Marks)

14) For complex number **z** and **w**,prove that $|z|^2\, w$-$|w|^2\, z$=z- w if and only if z =w or $\bar{w}$ = 1.
(1999-10 Marks).

15) Let a complex number α ,$\alpha \neq 1$, be a root of the equation z^{p+q}-z^p-z^q+1=0, where **p**,**q** are the distinct primes. Show that either $1+\alpha+\alpha^2+...+\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+...+\alpha^{q-1}=0$, but not both together.
(2002-5 Marks)

16) If $\mathbf{z_1}$ and $\mathbf{z_2}$ are two complex number such that $|z_1 < 1 <| \, |z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1$.
(2003-2 Marks)

17) Prove that there exists no complex number **z** such that $|z|$¡$\frac{1}{3}$ and $\sum_{r=1}^{n}$ a_r z^r =1 where $|a_r|$¡2.
(2003-2 Marks)

18) Find the centre and radius of circle given by $\left|\frac{z-\alpha}{\beta}\right|$=$k$,$k \neq 1$ where,z=x+i,α=α_1+$i\alpha_2$, β=β_1+$i\beta_2$.
(2004-2 Marks)

19) If one the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2+\sqrt{3}i$ find the other vertices of the square .
(2005-4 Marks)

PASSAGE – 1

Let **A**,**B**,**C** be three sets of complex number as defined below

A = {z:| Im $z \geq 1$}

B={z: $|z-2-i|=3$}

C = {z: Re $((1-i)z) = \sqrt{2}$}

a) The number of element in the set$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ is

(2008)

(a) 0 (b) 1 (c) 2 (d) ∞

b) Let **z** be any point in$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$
Then,$|z+1-i|^2 + |z-5-i|^2$ lies between

(2008)

i) 25 and 29
ii) 30 and 34
iii) 35 and 39
iv) 40 and 44

c) Let **z** be any point **A B C** and let **w** be any point satisfying $|w-2-i| < 3$.Then, $|z| - |w| + 3$ lies between

(2008)

i) -6 and 3
ii) -3 and 6
iii) -6 and 6
iv) -3 and 9