JEE Mains PYQ 04/04/2024 Shift-1

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16) Let the point, on the line passing through the points $P(1,-2,3)$ and $Q(5,-4,7)$, farther from the origin and at a distance of 9 units from the point P, be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to :

a) 155
b) 150
c) 160
d) 165

17) A square is inscribed in the circle $x^2+y^2-10x-6y+30=0$. One side of this square is parallel to $y=x+3$. If (x_i,y_i) are the vertices of the square , then $\sum\left(x_i^2+y_i^2\right)$ is equal to:

a) 148
b) 156
c) 160
d) 152

18) If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha,\beta]$ then $3\alpha+10\beta$ is equal to:

a) 97
b) 100
c) 95
d) 98

19) Let $f(x)=x^5+2e^{\frac{x}{4}}$ for all $x\in R$. Consider function $g(x)$ such that $(gof)(x)=x$ for all $x\in R$. Then the value of $8g'(2)$ is :

a) 16
b) 4
c) 8
d) 2

20) Let $\alpha\in(0,\infty)$ and $A=\begin{bmatrix}1&2&\alpha\\1&0&1\\0&1&2\end{bmatrix}$. If $\det\left(adj\left(2A-A^T\right).adj\left(A-2A^T\right)\right)=2^8$, then $(det(A))^2$ is equal to:

a) 1
b) 49
c) 16
d) 36

21) If $\lim_{x\to 1} \frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}} = \frac{m\sqrt{5}}{n(2n)^{\frac{2}{3}}}$, where $gcd\,(m,n) = 1$, then $8m + 12n$ is equal to.

22) In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $m+n$ is equal to

23) Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4sinx$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2} + 10\right)$ is equal to.

24) If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k \left[x^2\right] dx = \alpha - \sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to.

25) Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3. If the point(x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l,then $e^4 + l^4$ is equal to.

26) let $a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \cdots$, $b = 1 + \frac{^1C_0+^1C_1}{1!} + \frac{^2C_0+^2C_1+^2C_2}{2!} + \frac{^3C_0+^3C_1+^3C_2+^3C_3}{3!} + \cdots$ Then $\frac{2b}{a^2}$ is equal to .

27) Let A be a 3×3 matrix of non-negative real elements such that $A \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = 3 \cdot \begin{bmatrix}1\\1\end{bmatrix}$ Then the maximum value of $det\,(A)$ is.

28) Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to.

29) Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i}+2\hat{j}-7\hat{k}$, $\overrightarrow{BC} = a\hat{i}+b\hat{j}+c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, $d > 0$. Then The square of the length of the largest side of the triangle ABC is .

30) If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x\cos x}dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where $a, b \in N$, then $a + b$ is equal to.