Chapter-2 Complex Number

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7. If $1, a_1, a_2, a_3 \ldots . a_{n-1}$ are the n roots of unity, then show that $(1 - a_1)(1 - a_2)(1 - a_3) \ldots .(1 - a_{n-1}) = n$
(1984- 2 Marks).

8. Show that the area of the triangle on the Argand
diagram formed by the complex numbers
z, iz & $z + iz$ is $\frac{1}{2}|z|^2$.
(1986-2 Marks).

9. Let $\mathbf{Z_1}$=$10 + 6i$ and $\mathbf{Z_2}$=$4 + 6\iota$. if $\mathbf{Z}$ is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$
is $\frac{\pi}{4}$ then prove that $Z - 7 - 9i = 3\sqrt{2}$
(1990-4 Marks).

10. if iz^3 -z^2-z+ι = 0 then show that $|z| = 1$
(1995-5 Marks).

11. If $|Z| \le 1$,$|W| \le 1$,show that
$(Z - W)^2 \le (|Z| - |W|)^2 + (\arg Z - \arg W)^2$
(1995-5 Marks).

12. Find all non-zero complex numbers $\mathbf{Z}$
satisfying $\bar{Z}$=iZ^2
(1996-2 Marks).

13. Let $\mathbf{z}_1$ and $\mathbf{z}_2$ be roots of the equation
z^2+pz+q=0, where the coefficients $\mathbf{p}$ and $\mathbf{q}$
may be complex numbers. Let $\mathbf{A}$ and $\mathbf{B}$ represent
z_1 and z_2 in the complex plane . if$\angle\mathbf{ABC} = \alpha \neq 0$
and $\mathbf{OA}$=$\mathbf{OB}$,where $\mathbf{O}$ is the origin,prove that
p^2=$4q\cos^2\left(\frac{\alpha}{2}\right)$.
(1997-5 Marks)

14. For complex number $\mathbf{z}$ and $\mathbf{w}$,prove
that $|z|^2\, w$-$|w|^2\, z$=z- w if and only
if $z = w$ or $\bar{w} = 1$.
(1999-10 Marks).

15. Let a complex number α ,$\alpha \neq 1$, be a root of the equation z^{p+q}-z^p-z^q+1=0, where $\mathbf{p}$,$\mathbf{q}$ are the distinct primes. Show that either
1+α+α^2+...+α^{p-1} = 0 or 1+α+α^2+...+α^{q-1} = 0,
but not both together.
(2002-5 Marks)

16. If $\mathbf{z_1}$ and $\mathbf{z_2}$ are two complex number such that $|z_1 < 1 <| |z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1$.
(2003-2 Marks)

17. Prove that there exists no complex number $\mathbf{z}$ such that $|z|$¡$\frac{1}{3}$ and $\sum_{r=1}^{n} a_r\, z^r$ =1 where $|a_r|$¡2.
(2003-2 Marks)

18. Find the centre and radius of circle given by $\left|\frac{z-\alpha}{\beta}\right|$=$k$,$k \neq 1$ where,z=x+i,α=α_1+$i\alpha_2$,
β=β_1+$i\beta_2$.
(2004-2 Marks)

19. If one the vertices of the square
circumscribing the circle $|z - 1| = \sqrt{2}is2 + \sqrt{3}i$

find the other vertices of the square .
(2005-4 Marks)