Chapter-2 Complex Number

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7. If $1,a_1,a_2,a_3....a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1})=n$
(1984- 2 Marks).

8. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz & $z+iz$ is $\frac{1}{2}|z|^2$.
(1986-2 Marks).

9. Let $\mathbf{Z_1}$=$10+6i$ and $\mathbf{Z_2}$=$4+6\iota$. if $\mathbf{Z}$ is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$ is $\frac{\pi}{4}$ then prove that $Z-7-9i$ =$3\sqrt{2}$
(1990-4 Marks).

10. if iz^3 -z^2-z+ι = 0 then show that $|z|$ =1
(1995-5 Marks).

11. If $|Z|\leq 1$,$|W|\leq 1$,show that $(Z-W)^2 \leq (|Z|-|W|)^2+(\arg Z-\arg W)^2$
(1995-5 Marks).

12. Find all non-zero complex numbers $\mathbf{Z}$ satisfying $\bar{Z}$=iZ^2
(1996-2 Marks).

13. Let $\mathbf{z}_1$ and $\mathbf{z}_2$ be roots of the equation z^2+pz+q=0, where the coefficients **p** and **q** may be complex numbers. Let **A** and **B** represent z_1 and z_2 in the complex plane . if$\angle$**ABC** $=\alpha\neq 0$ and **OA**=**OB**,where **O** is the origin,prove that p^2=$4q\cos^2\left(\frac{\alpha}{2}\right)$.
(1997-5 Marks)

14. For complex number **z** and **w**,prove that $|z|^2$ w-$|w|^2$ z=z- w if and only if z =w or $\bar{w}$ = 1.
(1999-10 Marks).

15. Let a complex number α ,$\alpha\neq 1$, be a root of the equation z^{p+q}-z^p-z^q+1=0, where **p**,**q** are the distinct primes. Show that either $1+\alpha+\alpha^2+...+\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+...+\alpha^{q-1}=0$, but not both together.
(2002-5 Marks)

16. If $\mathbf{z_1}$ and $\mathbf{z_2}$ are two complex number such that $|z_1<1<||z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$.
(2003-2 Marks)

17. Prove that there exists no complex number **z** such that $|z|$¡$\frac{1}{3}$ and $\sum_{r=1}^{n}$ a_r z^r =1 where $|a_r|$¡2.
(2003-2 Marks)

18. Find the centre and radius of circle given by $\left|\frac{z-\alpha}{\beta}\right|$=$k$,$k\neq 1$ where,z=x+i,α=α_1+$i\alpha_2$, β=β_1+$i\beta_2$.
(2004-2 Marks)

19. If one the vertices of the square circumscribing the circle $|z-1|=\sqrt{2}is2+\sqrt{3}i$

find the other vertices of the square .
(2005-4 Marks)

$PASSAGE - 1$

Let **A**,**B**,**C** be three sets of complex number as defined below

A=$\{z : Imz \geq 1\}$

B=$\{z : |z - 2 - \iota| = 3\}$

C=$\{z : Re\,((1 - \iota)\,z) = \sqrt{2}\}$

a) The number of element in the set$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ is (2008)

(a) 0 (b) 1 (c) 2 (d) ∞

b) Let z be any point in$A \cap B \cap C$
Then,$|z + 1 - \iota|^2 + |z - 5 - \iota|^2$ lies between (2008)

i) 25 and 29
ii) 30 and 34
iii) 35 and 39
iv) 40 and 44

c) Let z be any point A B C and let w be any point satisfying $|w - 2 - \iota| < 3$.Then, $|z| - |w| + 3$ lies between (2008)

i) -6 and 3
ii) -3 and 6
iii) -6 and 6
iv) -3 and 9