Gate MA-2010

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40) Consider the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, 0 < \text{x} < \pi, \text{t} > 0$, with $\text{u}(0, \text{t}) = \text{u}(\pi, \text{t}) = 0, \text{u}(\text{x}, 0) = \sin x$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$. Then $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

a) 2 b) 1 c) 0 d) -1

41) Let $I = \int_C \frac{e^y}{x} dx + (e^y \ln x + x) dy$, where C is the positively oriented boundary of the region enclosed by $y = 1 + x^2, y = 2, x = \frac{1}{2}$. Then the value of I is

a) $\frac{1}{8}$ b) $\frac{5}{24}$ c) $\frac{7}{24}$ d) $\frac{3}{8}$

42) Let $\{f_n\}$ be a sequence of real valued differentiable functions on $[a, b]$ such that int $f_n(x) \to f(x)$ as $n \to \infty$ for every $x \in [a, b]$ and for some Riemann-integrable function $f : [a, b] \to R$ Consider the statements

$$P_1 : \{f_n\} \text{converges uniformly}$$
$$P_2 : \{f_n'\} \text{converges uniformly}$$
$$P_3 : \int_n^b f_n(x) dx \to \int_n^b f(x) dx$$
$$P_4 : \text{f is differentiable}$$

Then which one of the following need NOT be true

a) P_1 implies P_1 b) P_2 implies P_1 c) P_2 implies P_4 d) P_3 implies P_1

43) Let $f_n(x) = \frac{x^n}{1+x}$ and $g_n(x) = \frac{x^n}{1+nx}$ for $x \in [0, 1]$ and $n \in N$. Then on the interval $[0, 1]$.

a) both $\{f_n\}$ and $\{g_n\}$ converge uniformly
b) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly
c) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly
d) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

44) consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{x^n}{n}$. Then

a) both converge on $(-1, 1]$
b) both converge on $[-1, 1)$
c) exactly one of them converges on $(-1, 1]$

d) none of them converges on $[-1, 1)$

45) Let X=N be equipped with the topology generated by the basis consisting of sets $A_n = (n, n+1, n+2\cdots), n \in$ N. Then X is

a) Compact and connected
b) Hausdorff and connected
c) Hausdorff and compact
d) Neither Compact nor connected

46) Four weightless rods form a rhombus PQRS with smooth hinges at the joints. Another weightless rod joins the midpoints E and F of PQ and PS respectively. The system is suspended from P and a weight 2W is attached to R. If the angle between the rods PQ and PS is 2θ. then the thrust in the rod EF is

a) W $\tan\theta$
b) 2W $\tan\theta$
c) W $\cot\theta$
d) 4W $\tan\theta$

47) For a continuous function f(t), $0 \leq r \leq 1$ the integral equation y$(t) = f(t) + 3\int_0^1$ ts $y(s)\,ds$ has

a) a unique solution if $\int_0^1 sf(s)\,ds \neq 0$
b) no solution if $\int_0^1 sf(s)\,ds = 0$
c) infinitely many solution if $\int_0^1 sf(s)\,ds = 0$
d) infinitely many solution if $\int_0^1 sf(s)\,ds \neq 0$

Common Data Question

Common Data for Question 48 and 49:

Let X and Y be continuous random variables with the joint probability density function f$(x, y) = \begin{cases} ae^{-zy}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$

48) The value of a is

a) 4
b) 2
c) 0
d) 0.5

49) the value of of E$(X \mid Y = 2)$ is

a) 4
b) 3
c) 2
d) 1

Common Data for Question 50 and 51:

Let X=N×Q with the subspace topology of the usual topology on R^2 and P=$\left\{\left(n, \frac{1}{n}\right) : n \in \text{N}\right\}$.

50) In the space X,

a) P is closed but is not open
b) P is open but is not closed
c) P is both open and closed
d) P is neither open but nor closed

51) The boundary of P and X is

a) an empty set
b) a singleton set
c) P
d) X

Linked Answer Question

Statement for linked Answer Questions 52 and 53:
For a differentiable function f(x), the integral $\int_0^h f(x)\,dx$ is approximated by the formula $h\left[a_0 f(0) + a_1 f(h)\right] + h^2\left[b_0 f'(0) + b_1 f'(h)\right]$, which is exact for all polynomials of degree at most 3.

52) The value of a_1 and b_1 respectively are

a) $\frac{1}{2}$ and $-\frac{1}{12}$
b) $-\frac{1}{12}$ and $\frac{1}{2}$
c) $\frac{1}{2}$ and $\frac{1}{12}$
d) $\frac{1}{12}$ and $-\frac{1}{2}$

53) The values of a_0 and b_0 respectively are

a) $\frac{1}{2}$ and $\frac{1}{2}$
b) $\frac{1}{12}$ and $-\frac{1}{12}$
c) $\frac{1}{2}$ and $\frac{1}{12}$
d) $\frac{1}{2}$ and $-\frac{1}{12}$