Gate MA-2019

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40) Consider the ordered square I_0^2, the set $[0,1] \times [0,1]$ with the dictionary order topology. Let the general element of I_0^2, be denoted by $x \times y$, where x,y$\in$ $[0,1]$. Then the closure of the subset

$$S = \left\{ x \times \frac{3}{4} : 0 < a < x < b < 1 \right\} \text{ in } I_0^2$$

a) $S \bigcup ((a,b] \times \{0\}) \bigcup ([a,b) \times \{1\})$
b) $S \bigcup ([a,b) \times \{0\}) \bigcup ((a,b] \times \{1\})$
c) $S \bigcup ((a,b) \times \{0\}) \bigcup ((a,b) \times \{1\})$
d) $S \bigcup ((a,b] \times \{0\})$

41) Let P_2 be the vector space of all polynomials of degree at most 2 over $\mathbf{R}$ (the set of real numbers). Let a linear transformation $T : P_2 \to P_2$ be defined by

$$T\left(a + bx + cx^2\right) = (a+b) + (b-c)x + (a+c)x^2$$

consider the following statements:

I. The null space of T is $\left\{a\left(-1 + x + x^2\right) : a \in \mathbb{R}\right\}$.
II. The range space of T is spanned by the set $\left\{1 + x^2, 1 + x\right\}$.
III. $T(T(1+x)) = 1 + x^2$.
IV. If M is the matrix representation of T with respect to the standard basis $\left\{1, x, x^2\right\}$ of P_2, then the trace of the matrix M is 3.

Which of the above statement are TRUE?

a) I and II only
b) I ,III and IV only
c) I ,II and IV only
d) II and IV only

42) Let T_1 and T_2 be two topologies defined on $\mathbb{N}$ (the set of all natural number), where T_1 is the topology generated by B= $\{\{2n-1, 2n\} : n \in \mathbb{N}\}$ and T_2 is the discrete topology on $\mathbb{N}$. Consider the following statements:

I. IN $(\mathbb{N}, T_1)$, every infinite subset has a limit point.
II. The function f:$(\mathbb{N}, T_1) \in (\mathbb{N}, T_2)$ is defined by

$$f(n) = \begin{cases} \frac{n}{2}, \text{if n is even} \\ \frac{n+1}{2}, \text{if n is odd} \end{cases}$$

is a continuous function

which of the above statement is/are TRUE?

a) both I and II
b) I only
c) II only
d) Neither I or II

43) Let $1 \leq p < q < \infty$ Consider the following statements:
I $\ell^p \subset \ell^q$
II $L^p\,[0,1] \subset L^q\,[0,1]$,
where $\ell^p = \{(x_1, x_2, \dots) : x_i \in \mathbb{R}, \sum_{i=1}^{\infty} |x_i|^p < \infty\}$ and
$L^p = \left\{f : [0,1] \to \mathbb{R} : \text{ f is } \mu - \text{measurable}, \int_{[0,1]} |f|^p \, d\mu < \infty, \text{ where } \mu \text{ is the Lebesgue measure } \right\}$

($\mathbb{R}$ is the set of all real number)
Which of the above statements is/are TRUE?

a) both I and II
b) I only
c) II only
d) Neither I or II

44) Consider the differential equation

$$t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = 0, t > 0, y\,(0+) = 1, \left(\frac{dy}{dt}\right)_{t=0+} = 0.$$

If Y(s) is the Laplace transform of Y(t), then the value of Y(1) is___________ (round off to 2 places of decimal).
(Here, the inverse trigonometric functions assume principal values only)

45) Let R be the in region in the xy-plane bounded by the curve $y = x^2, y = 4x^2, xy = 1$ and $xy = 5$.
Then the value of the integral $\int \int_R \frac{y^2}{x}$dydx is equal to ___________.

46) Let V be the vector space of all 3×3 matrices with complex entries over the real field.If

$$W_1 = \left\{A \in V : A = A^T\right\} \text{ and } W_2 = \{A \in V : \text{ trace of } A = 0\},$$

then the dimension of $W_1 + W_2$ is equal to ____________.
$(A)^T$ denote the conjugate transpose of A

47) The number of elements of order 15 in the additive group $Z_{10} \times Z_{10}$ is ______. (Z_{10} denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n).

48) Consider the following cost matrix of assigning four jobs to four persons:

Jobs

Persons	J_1	J_2	J_3	J_4
P_1	5	8	6	10
P_2	2	5	4	8
P_3	6	7	6	9
P_4	6	9	8	10

Then the minimum cost of the assignment problem subject to the constraint that job J_4 is assigned to person P_2 is ___________.

49) Let $y : [-1, 1] \to \mathbb{R}$ with $y(1) = 1$ satisfy the Legendre differential equation

$$\left(1 - x^2\right) \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0 \text{ for } |x| < 1.$$

Then the value of $\int_{-1}^{1} y(x)\left(x + x^2\right)\mathrm{d}x$ is equal to ___________(round off to 2 places of decimal).

50) Let $\mathbb{Z}_{125}$ be the ring of integer modulo 125 under the operations of addition modulo 125 and multiplication modulo 125. if m is the number if maximal ideals of $\mathbb{Z}_{125}$ and n is the number of non-units of $\mathbb{Z}_{125}$, then $m + n$ is equal to ___________.

51) The maximum value of the error term of the composite Trapezoidal rule when it is used to evaluate the definite integral

$$\int_{0.2}^{1.4} (\sin x - \log_e x)\, dx$$

with 12 sub-intervals of equal length, is equal to ___________. (round off to 3 places of decimal)

52) By the Simplex method, the optimal table of the linear programming problem:

$$\text{Maximize} Z = \alpha x_1 + 3x_2$$
$$\text{subject to } \beta x_1 + x_2 + x_3 = 8,$$
$$2x_1 + x_2 + x_4 = \gamma, x_1, x_2, x_3, x_4 \geq 0,$$

where α,β,γ are real constant is

$c_j \rightarrow$	α	3	0	0	
Basic variable	x_1	x_2	x_3	x_4	Solution
x_2	1	0	2	-1	6
x_1	0	1	-1	1	2
$z_j - c_j$	0	0	2	1	-

Then the value of $\alpha + \beta + \gamma$ is ___________