JEE Mains PYQ 27/07/2021 Shift-1

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16) Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then the value of $\lim_{x\to 2} \frac{x^2 f(2) - 4f(x)}{x-2}$ is equal to :

a) 4 b) 8 c) 16 d) 12

17) Let P and Q be two distinct points on a circle which has centre at $C(2, 3)$ and which passes through origin O .If OC is perpendicular to both the line segment CP and CQ then the set $\{P, Q\}$ is equal to

a) $\{(4, 0), (0, 6)\}$
b) $\left\{\left(2 + 2\sqrt{2}, 3 - \sqrt{5}\right), \left(2 - 2\sqrt{2}, 3 + \sqrt{5}\right)\right\}$
c) $\left\{\left(2 + 2\sqrt{2}, 3 + \sqrt{5}\right), \left(2 - 2\sqrt{2}, 3 - \sqrt{5}\right)\right\}$
d) $\{(-1, 5), (5, 1)\}$

18) let α , β be two roots of the equation $x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$. Then $\alpha^8 + \beta^8$ is equal to

a) 10 b) 100 c) 50 d) 160

19) The probability that a randomly selected 2 – digit number belongs to the set $\{n \in \mathbf{N} : (2^n - 2)$ is a multiple of $3\}$ is equal to

a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

20) Let

$$A = \left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\right\},$$
$$B = \left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\right\} \text{ and}$$
$$C = \left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\right\},$$

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

a) $\frac{3+\sqrt{10}}{2}$
b) $\frac{2+\sqrt{10}}{2}$
c) $\frac{3+2\sqrt{10}}{2}$
d) $1 + \sqrt{5}$

21) For real number α and β , consider the following system of linear equation: $x+y-z = 2, x + 2y + \alpha z = 1, 2x - y + z = \beta$. If the system has infinite solution ,then $\alpha + \beta$ is equal to

22) Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b}$ and $\mathbf{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to

23) if $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

24) Let the domain of the function $f(x) = \log_4 \left(\log_5 \left(log_3 \left(18x - x^2 - 77\right)\right)\right)$ be (a, b). Then the value of the integral $\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a+b-x))}\, dx$ is equal to

25) Let

$$f(x) = \begin{bmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{bmatrix}, \quad x \in [0, \pi]$$

Then the maximum value of $f(x)$ is equal to

26) Let $F : [3, 5] \to \mathbf{R}$ be a twice differentiable function on $(3, 5)$ such that

$$F(x) = F(x) = e^{-x} \int_3^x (3t^2 + 2t + 4F'(t))\, dt$$

If $F'(4) = \frac{\alpha e^\beta - 224}{(e^\beta - 4)^2}$, then $\alpha + \beta$ is equal to.

27) Let a plane P pass through the point $(3, 7, -7)$ and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d,then d^2 is equal to .

28) Let $S = \{1, 3, 4, 5, 6, 7\}$. Then the number of possible function $f : \mathbb{S} \to \mathbb{S}$ such that $f(m.n) = f(m).f(n)$ for every $m, n \in S$ and $m.n \in S$ is equal to .

29) If $y = y(x), y \in [0, \frac{\pi}{2})$ is the solution of the differential equation

$$\sec(y)\frac{d}{dx}(y) - \sin(x + y) - \sin(x - y) = 0$$

then $5y'\left(\frac{\pi}{2}\right)$ is equal to.

30) Let $f : [0, 3] \in \mathbf{R}$ be defined by

$$f(x) = min\{x - [x], 1 + [x] - x\}$$

where $[x]$ is the greatest integer less than or equal to x.Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous , and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable .Then the sum of number of element in P and Q is equal to.