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27) Let $X_i = 1, 2, \cdots, n$, be *i.i.d*. random variables from a normal distribution with mean 1 and variance 4. Let $S_n = X_1^2 + X_2^2 + \cdots + X_n^2$. If $Var(S_n)$ denotes the variance of S_n, then the value of

$$\lim_{n\to\infty}\left(\frac{Var(S_n)}{n} - \left(\frac{E(S_n)}{n}\right)^2\right)$$

(in integer) is equal to ___________.

28) At a telephone exchange, telephone calls arrive independently at an average rate of 1 call per minute, and the number of telephone calls follows a Poisson distribution. Five time intervals, each of duration 2 minutes, are chosen at random. Let p denote the probability that in each of the five time intervals at most 1 call arrives at the telephone exchange. Then $e^{10}p$ (in integer) is equal to ___________.

29) Let X be a random variable with the probability density function

$$f(x) = \begin{cases} c(x - [x]), & 0 < x < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

where c is a constant and $[x]$ denotes the greatest integer less than or equal to x. If $A = \left[\frac{1}{2}, 2\right]$ then $P(x \in A)$ (rounded off to two decimal places) is equal to ___________.

30) Let X and Y be two random variables such that the moment generating function of X is $M(t)$ and the moment generating function of Y is

$$H(t) = \left(\frac{3}{4}e^{2t} + \frac{1}{4}\right)M(t),$$

where $t \in (-h, h), h > 0$. If the mean and the variance of X are $\frac{1}{2}$ and $\frac{1}{4}$ respectively, then the variance of Y (in integer) is equal to ___________.

31) Let $X_i = 1, 2, \cdots n$, be i.i.d random variables with the probability density function

$$f_X(x) = \begin{cases} \frac{1}{\sqrt{2}T\left(\frac{1}{6}\right)} x^{-\frac{5}{6}} e^{-\frac{x}{8}}, & 0 < x < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

where $T(\cdot)$ denotes the gamma function. Also ,let $\bar{X}_n = \frac{1}{n}(X_1 + X_1 + \cdots + X_n)$. If $\sqrt{n}\left(\bar{X}_n\left(3 - \bar{X}_n\right) - \frac{20}{9}\right)$ converges to $V\left(0, \sigma^2\right)$ in distribution,then σ^2 ((rounded off to two decimal places) is equal to ___________.

32) Consider a Poisson process $\{X(t), t \geq o\}$ The probability mass function of X(t) is given by

$$f(t) = \frac{e^{-4t}(4t)^n}{n!}, n = 0, 1, 2, \cdots$$

if C(t_1, t_2) is the covariance function of the Poisson process, then the value of C$(5, 3)$ (in integer) is equal to ___________.

33) A random sample of size 4 is taken from the distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If the observed sample values are $6, 5, 3, 6$, then the method of moments estimate (in integer) of the parameter θ, based on these observations, is ___________.

34) A company sometimes stops payments of quarterly dividends. If the company pays the quarterly dividend, the probability that the next one will be paid is 0.7. If the company stops the quarterly dividend, the probability that the next quarterly dividend will not be paid is 0.5. Then the probability ((rounded off to two decimal places) that the company will not pay quarterly dividend in the long run is___________.

35) Let $X_1, X_2, \cdots, X_8$, be a random sample taken from a distribution with the probability density function

$$f_X(x) = \begin{cases} \frac{x}{8}, & 0 < x < 4, \\ 0, & \text{elsewhere.} \end{cases}$$

Let $F_8(x)$ be the empirical distribution function of the sample. If α is the variance of $F_8(2)$, then 128α (in integer) is equal to ___________.

36) Let M be a 3×3 real symmetric matrix with eigenvalues $-1, 1, 2$ and the corresponding unit eigenvectors u,v,w, respectively . Let x and y be two vectors in $\mathbb{R}^3$ such that

$$Mx = u + 2(v + w) \text{ and } M^2 y = u - (v + 2w)$$

Considering the usual inner product in $\mathbb{R}^3$, the value of $[x + y]^2$, where $[x + y]$ is the length of the vector $x + y$, is

a) 1.25
b) 0.25
c) 0.75
d) 1

37) Consider the following infinite series:

$$S_1 := \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2 + 4} \text{ and } S_2 := \sum_{n=0}^{\infty} (-1)^n \left(\sqrt{n^2 + 1} - n\right).$$

Which of the above series is/are conditionally convergent?

a) S_1 only
b) S_2 only
c) Both S_1 only S_2
d) Neither S_1 nor S_2

38) Let $(3,6)^T, (4,4)^T, (5,7)^T$, and $(3,6)^T$ be four independent observations from a bivariate normal distribution with the mean vector μ and the covariance matrix $\sum$. Let $\hat{\mu}$ and $\hat{\sum}$ be the maximum likelihood estimates of μ and $\sum$, respectively, based on these observations. Then $\hat{\sum}\hat{\mu}$ is equal to

a) $\begin{pmatrix} 3.5 \\ 10 \end{pmatrix}$
b) $\begin{pmatrix} 7.5 \\ 4 \end{pmatrix}$
c) $\begin{pmatrix} 4 \\ 13.5 \end{pmatrix}$
d) $\begin{pmatrix} 10 \\ 3.5 \end{pmatrix}$

39) Let X=$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ follow $N_3(\mu, \sum)$ with $\mu = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$ and $\sum = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 2 & \alpha \\ 1 & \alpha & 2 \end{bmatrix}$ where $\alpha \in \mathbb{R}$. Suppose that the partial correlation coefficient between X_2 and X_3, keeping X_1 fixed ,is $\frac{5}{7}$ then α is equal to

a) 1
b) $\frac{3}{2}$
c) 2
d) $\frac{1}{2}$