Chapter-2 Complex Number

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7. If $1, a_1, a_2, a_3....a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1})$=n
(1984- 2 Marks).

8. Show that the area of the triangle on the Argand
diagram formed by the complex numbers
z, iz & $z+iz$ is $\frac{1}{2}|z|^2$.
(1986-2 Marks).

9. Let $Z_1 = 10+6\iota$ and $Z_2 = 4+6\iota$. if Z is any complex number such that the argument of $\frac{(Z-Z_1)}{(Z-Z_2)}$
is $\frac{\pi}{4}$ then prove that $Z-7-9\iota = 3\sqrt{2}$
(1990-4 Marks).

10. if $iz^3 - z^2 - z + \iota = 0$ then show that $|z| = 1$
(1995-5 Marks).

11. If $|Z| \le 1, |W| \le 1$,show that
$(Z-W)^2 \le (|Z|-|W|)^2 + (\arg Z - \arg W)^2$
(1995-5 Marks).

12. Find all non-zero complex numbers Z satisfying $\bar{Z} = \iota Z^2$
(1996-2 Marks).

13. Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent
z_1 and z_2 in the complex plane . if$\angle AOB = \alpha \neq 0$
and $OA = OB$,where O is the origin,prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$.
(1997-5 Marks)

14. For complex number z and w,prove that $|z|^2 w - |w|^2 z = z - w$ if and only if $z = wor\bar{w} = 1$.
(1999-10 Marks).

15. Let a complex number α ,$\alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$, where p,q are
the distinct primes. Show that either $1+\alpha+\alpha^2+...+\alpha^{p-1} = 0$ or $1+\alpha+\alpha^2+...+\alpha^{q-1} = 0$,
but not both together.
(2002-5 Marks)

16. If $z_1$andz_2 are two complex number such that $|z_1 < 1 < ||z_2|$ then prove that $|\frac{1-z_1\bar{z}_2}{z_1-z_2}| < 1$.
(2003-2 Marks)

17. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$.
(2003-2 Marks)

18. Find the centre and radius of circle given by $|\frac{z-\alpha}{z-\beta}| = k, k \neq 1$ where,$z = x+\iota, \alpha = \alpha_1 + \iota\alpha_2$, $\beta = \beta_1 + \iota\beta_2$.
(2004-2 Marks)

19. If one the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2+\sqrt{3}\iota$. find the other vertices of the square .
(2005-4 Marks)