Mechanical Engineering Research; Vol. 9 No. 2; 2019
ISSN 1927-0607 E-ISSN 1927-0615
Published by Canadian Center of Science and Education

Contradictory Postulates of Singularity

Kyle Baker[1], Eryn Culton[1], Joshua Ten Eyck[1], Zachary Lewis[1] & Timothy Sands[2]

[1] Graduate School of Engineering and Applied Sciences, Naval Postgraduate School, Mongterey, USA

[2] Fu Foundation School of Engineering and Applied Science (CVN), Columbia University, New York, USA

Correspondence: Timothy Sands, Fu Foundation School of Engineering and Applied Science (CVN), Columbia University, New York, USA. E-mail: dr.timsands@caa.columbia.edu, Tel: 1-831-656-3954.

Received: December 19, 2019 Accepted: December 31, 2019 Online Published: January 15, 2020

doi:10.5539/mer.v9n2p28 URL: https://doi.org/10.5539/mer.v9n2p28

Abstract

Modification of rigid body angular momentum permits controlled rotational maneuvers, and one common momentum-exchange actuator contains challenging mathematical singularities that occur when the actuator geometrically aligns perpendicularly to the commanded torque direction. Substantial research has arisen toward singularity avoidance, singularity escape (when avoidance fails), and singularity penetration which permits safe flight through regions of singularity. The latter two in particular, singularity escape and penetration require mathematical calculations of singular and near-singular quantities (very large numbers) using constituent numbers that are sometimes very small. This dichotomy leads to interesting peculiarities in some specific geometries. This short communication critically evaluates three often spoke postulates for defining singularity and the axioms that accompany the postulates. Researchers using disparate postulates arrive at contradictory conclusions about singularities, and we examine these peculiarities, leading to a few conclusions. Singular conditions must never be declared in the abstract without consideration for the commanded maneuver (e.g. the claim “the CMG system is singular”). Seeking the true angular momentum capability at near-planar skew angles, this research concludes that performance prediction is difficult installations at low skew angles should be avoided whenever permissible to enhance abilities of mathematical calculations. It will be shown that maximum momentum performance is easily predicted at very high and very low skew angles, and performance will be shown to be lowest at mid-values of skew angle. Meanwhile, maximum singularity-free performance remains elusive at even modestly low skew-angles.

Keywords: actuators; attitude control; control theory; guidance, navigation, and control; system design; spacecraft dynamics

1. Introduction

Control moment gyroscope actuators accomplish the (designed) commanded change in angular momentum by using a steering law, and this steering law is the topic of considerable research in the literature (Agrawal, Kim, & Sands, 2017; Kim, Sands, & Agrawal, 2007; Lewis et al., 2019; Sands, Kim, & Agrawal, 2009; Sands, 2007; Sands, Kim, & Agrawal, 2006, 2007, 2012, 2016, 2018; Sands, Lu, Chu, & Cheng, 2018) particular in the case of control moment gyroscopes. The steering law contains mathematical singularities, i.e. instances where the correctly derived mathematical expression for the torque command to a gyro (or combination) contains zero(s) in the denominator derived from zero-valued sinusoidal functions that happen to appear in the denominator of the matrix inverse equation (i.e. the determinant). One way to express the steering laws for each gyro actuator is matrix expressions, and these singular instances express themselves as loss of rank in the system matrix, which is particularly troublesome since the steering law requires inversion of the matrix. The steering law matrix contains the gyro gimbal angles, and the skew angles representing the geometric installation of the gyros.

In the voluminous literature, several disparate definitions of singularity arise. Mathematically inclined authors define singularity as i*nstances where the steering law matrix is not invertible (i.e. the matrix has become rank deficient)*. Systems engineers often refer to singularity as instances with an *inability to produce an arbitrary torque*, while spacecraft attitude control engineers would more strictly define singularities as instances with an *inability to produce the commanded torque*. These later two interpretations avoid the strictly mathematical definition of the singular condition in favor of the physics-based illustration of one (or more) gyro that becomes perpendicularly aligned with the direction of torque and is therefore unable to generate the commanded torque. The mathematical connection is the expression of the gyro’s angular position vis-à-vis sines and cosines in the steering law.

These disparate definitions of singularities give rise to seemingly conflicting analysis throughout the literature (Agrawal, Kim, & Sands, 2017; Kim, Sands, & Agrawal, 2007; Sands, Kim, & Agrawal, 2009; Sands, 2007; Sands, Kim, & Agrawal, 2006, 2007, 2012, 2016, 2018; Sands, Lu, Chu, & Cheng, 2018) where investigations of low skew angles are ubiquitously notorious. This short communication illustrates typical analysis techniques leading to disparate results by deriving each definition for singularity and illustrating the results for side-by-side comparison. It will be shown that high skew angles lead to matching results, while low skew angles lead to widely varying results. The generic explanation is believable if the reader considers how well computers can use variables valued near zero to calculate something divided by zero, an extraordinarily large number. This generic explanation is illustrated easily by performing identical analyses in several iterations of computational parameters (e.g. integration methods, integration step sizes, and node spacing where nodes are points of analysis in discretized versions of the continuous mathematical relationships.

2. Method

This study will be limited to non-redundant, constant speed, single gimbaled control moment gyroscopes. Well established methods are described first to elaborate typical skewed arrays of gyros (ubiquitous "benchmark" geometry (Sands, Kim, & Agrawal, 2006)), inverse steering laws, and the gyro system matrix (referred to as [A] in the literature). We introduce the three competing definitions of singularity and elaborate them in terms of the benchmark geometry, emphasizing analytic approaches to determining singularity-free conditions. Next are introduced new discrete numerical methods of determining singularity. The new methods prove dramatically inferior to the normal analytical approaches, but the strength of the new numerical methods is their exposure of the peculiarities of near-planar momentum generation when low skew angles predominate, and illustration that these peculiarities exist in analytic methods of analysis as well *emphasizing the importance of having the correct paradigm for strictly defining singularity*.

2.1 Non-redundant, constant speed, single-gimbaled control moment gyroscopes (SGCMG)

The torque vector direction (τ_{CMG}) of rotation for a gyro is also the axis around which the spacecraft maneuver is produced by that particular gyro, albeit in an opposite direction. The torque and the orthogonal relationship between the gyro's gimbal rotation (δ) and angular momentum h_i. These relationships provide intuition behind singularity generation. Gimballing rotates the angular momentum vectors $h_i \forall i = 1,2,3$ in planes perpendicular to the gimbal axis, and these planes contain the respective torque vectors in a right-hand arrangement: angular momentum, torque, and gimbal axis. Notice in figure (1) all directions defined by the angular momentum vector are orthogonal to the commanded torque direction, so any torque command that aligns with a gyros current angular momentum vector is impossible for fixed-speed gyros (a mathematically singular condition results).

The change in angular momentum is the product of the gimbal rates and a matrix containing gimbal angles and skew angles referred to in the literature as [A]. The relationship must be inverted, since the control u is known (defined as the negative of the desired spacecraft torque: $\dot{H}_{inertial} \equiv \dot{H}$), and the current known gimbal positions define the [A] matrix, while the necessary gimbal commands $\dot{\delta}$ are unknown. The [A] matrix is the spatial gradient matrix of angular momentum (normalized by one gyro's value of angular momentum) normalized by one gyro's momentum capability (i.e. 1h, 2h, and 3h). One method of expressing a matrix inverse is particularly useful to highlight the connection between disparate understandings of singular matrix inverse (to be defined in the next section of this short communication). One method of finding a matrix inverse involves using the matrix cofactor in the numerator with the matrix determinant in the denominator. Any instance where the determinant equals zero results in the need to calculate a quantity divided by zero. The physical interpretation follows. Instances where the combination of gimbal and skew angles (in the [A] matrix) align such that the net gyro commanded torque direction aligns with its orthogonal angular momentum vector, that gyro is physically unable to generate torque in the orthogonal direction, and that alignment mathematically expresses itself as a loss of matrix rank resulting in singular matrix inverse. These simultaneous physical and mathematical expressions of singular conditions have given rise to adoption of disparate definitions of gyro singularity.

2.2. Postulates of SGCMG singularity.

In the voluminous literature, several disparate definitions of singularity arise. Mathematically inclined authors define singularity as instances where the steering law matrix is not invertible (i.e. the matrix has become rank deficient). Systems engineers often refer to singularity as instances with an inability to produce an arbitrary torque, while attitude control engineers would more strictly define singularities as instances with an inability to produce the commanded torque. These later two interpretations avoid the strictly mathematical definition of the singular condition in favor of the physics-based illustration of one (or more) gyro that becomes perpendicularly aligned with the direction of torque and is therefore unable to generate the commanded torque. The mathematical connection is the expression of the gyro's angular position vis-à-vis sines and cosines in the steering law expressed in equations (1)-(4).

Postulate 1. Singularities occur in instances of impossibility to generate *an arbitrary torque*

Axiom 1. Instances when det[A]=0, angular momentum generation is impossible

Postulate 2. Singularities occur in instances of impossibility to generate *the commanded torque*

Axiom 2. Instances when det[A]=0, angular momentum generation is impossible when the commanded torque aligns with a singular direction. Thus, angular momentum generation is possible (up to 2h) in instances where a single gyro is aligned in a singular direction while the other two gyros are not.

Postulate 3. Singularities occur in *instances of rank-deficient [A] matrix*

Axiom 3. Instances when det[A]=0, angular momentum generation is impossible

Another method of graphic display of singular conditions predominant in the literature is the plot of *maximum singularity free momentum capability*. This method normally examines each of the analytic expressions of singular conditions that result in singular matrix inversion, which comes from det[A]=0, the problematic condition. Next, for any iterated skew angle, the minimum singularity free value from the analytic expressions is plotted to reveal a usable inner zone of the momentum space that contains no singularities. Attitude control engineers can safely design maneuvers in this singularity-free zone without fear of loss of attitude control. *The case of skew angle equal to zero highlights the difference in three Postulates of gyro singularity*. Often neglected in plots is the case of β=0, yet it is one of the equations that necessarily leads to a rank-deficient [A] matrix.

In accordance with *Postulate 1*, the first singular case represented in equation (6) implies that angular momentum generation is impossible per *Axiom 1*. Consider near-planar momentum generation where nearly all skew angles are set to near-zero degrees. All three gyros would operate in plane aligned with zero skew angle (the "CMG Platform" plane depicted in figure (2). Thus, very high (maximal) torque could be produced by all three CMGs if the commanded torque lies in this plane. This invalidates Postulate 1, thus we must not claim that momentum generation is impossible when det[A]=0. This statement is untrue.

In accordance with *Postulate 2*, both the commanded torque direction and the condition of the [A] matrix are involved in defining singularity, thus we avoid ever saying the CMG system is singular, without mentioning the commanded direction.

In accordance with *Postulate 3*, all instances when det[A]=0 are singular. The accompanying Axiom is identical to that of *Postulate 1* and has the same fallacy. Exactly as the case of equation (6) disproved *Postulate 1*, it also disproves *Postulate 3*.

Any geometric installation with low skew angles lead to near-planar momentum generation (contemplatable when looking at figures 1 and 2b), and necessitates singular and near-singular computations (very large numbers) using constituent numbers that are very small (near zero), thus bringing about peculiarities of near-planar momentum generation. This one mathematical condition ($\beta_i = 0$) implies impossibility to generate arbitrary torques, implying *Postulate 1* and *Postulate 3* coincide, yet *Postulate 2* implies that any torque commanded in the plane could be achieved with as much as two gyros worth of angular momentum, referred to in the literature as 2h normalized angular momentum. Thus, according to *Postulate 2* the symmetric condition β=0 is not necessarily singular, while *Postulates 1 and 3* assert singularity.

3. Results

This section provides a concise and precise description of the experimental results, their interpretation as compared to the predominant literature, as well as the experimental conclusions that can be drawn.

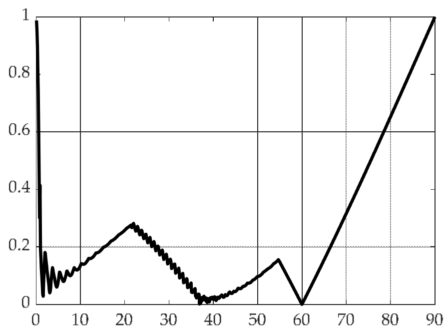
3.1 Analytics results for maximal singularity-free momentum

The analytical results already in the literature indicate consistency at skew angles above 25 degrees, but inconsistency at lower, near-planar skew angles. The appendix in Sands, Kim, and Agrawal (2007) graphically depicts the singularity hypersurfaces for symmetric skew angles iterated for every five degrees revealing the singularities coalesce as skew angle increases towards ninety degrees. This justifies the consistency in the literature, in that fewer unique singularities exist as the skew angle is iterated higher and higher amidst the hypothesis that inconsistencies in the literature arise from singularities being missed (skipped over) in discretization. The literature's inconsistency is the key evaluation in this research, and the newly introduced numerical method illustrates the difficulty by exasperating the weakness of discretized analysis of continuous analytic expressions. The key discovery is that computers have a difficult time dealing with singular values (a quantity divided by zero…necessary very large numbers), and the difficulty is compounded when the constituent calculations are using numbers that are near-zero. Figures in the literature ubiquitously ignore the case of zero degree skew angle, but nonetheless indicates a sloped-increase to near 0.4 with decreasing skew angle. Meanwhile Sands (2007) illustrates a curved decrease to zero at zero skew angle (accepting the definition that the determinant is zero at zero skew angle, and ignoring the definition of singularity that refers to inability to produce toque or alternatively inability to produce the commanded/desired torque), Postulate 1 and Postulate 2 respectively.

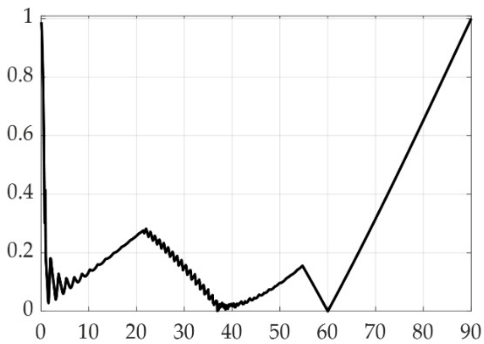
Sands, Kim, and Agrawal (2006) where near-zero skew angles are neglected is consistent with the interpretation in Postulate 3. On the other hand, Sands (2007) is consistent with the interpretation in Postulate 2 necessitating zero angular momentum generation when the symmetric skew angles are zero. These two works were generated by the same author using different integration schemes, but went through peer review in two different publications, highlighting the presence of at least these two disparate interpretations of singularity. Seeking to elaborate the disparate interpretations, new research is presented here where the analytic expressions for det [A] were not used. Instead, inspired by the singularity mapping of the heuristic numerical approach, the three-dimensional momentum space was discretized at iterated intervals, and momentum generation was only calculated at these discrete points and then subsequently minimized and plotted.

3.2 Numeric results for maximal singularity-free momentum

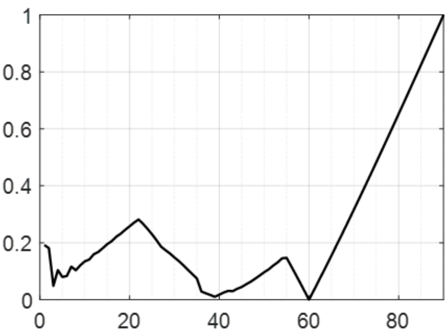
Using the heuristic numerical approach, the discretized (at iterated intervals) three-dimensional momentum space produced nodes to calculate momentum generation subsequently minimized and plotted in Figures 1 and 2.



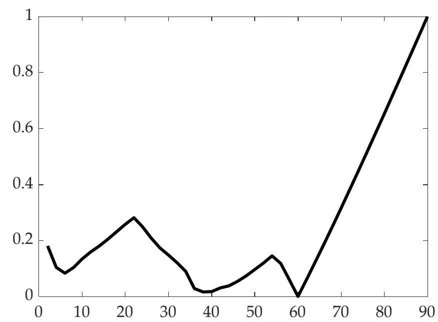
(a) Numerical solution skew angle on abscissa, singularity-free momentum magnitude on ordinant where numerical analysis uses an isotropic discretized grid of nodes spaced *0.1 degrees*. Notice zero skew angle is nearly captured.



(b) Numerical solution skew angle on abscissa, singularity-free momentum magnitude on ordinant where numerical analysis uses an isotropic discretized grid of nodes spaced *0.25 degrees*. Notice zero skew angle is nearly captured.

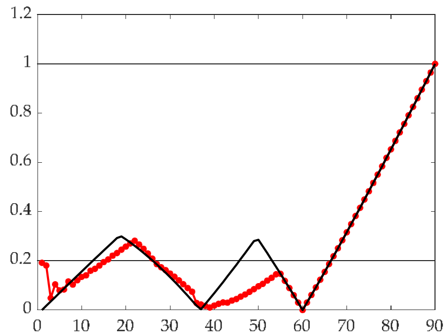


(c) Numerical solution skew angle on abscissa, singularity-free momentum magnitude on ordinant where numerical analysis uses an isotropic discretized grid of nodes spaced *1 degree*. Notice zero skew angle is not captured.

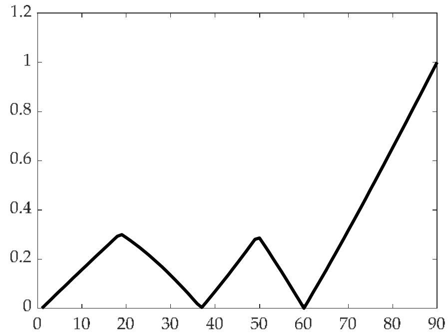


(d) Numerical solution skew angle on abscissa, singularity-free momentum magnitude on ordinant where numerical analysis uses an isotropic discretized grid of nodes spaced *2 degrees*. Notice zero skew angle is not captured.

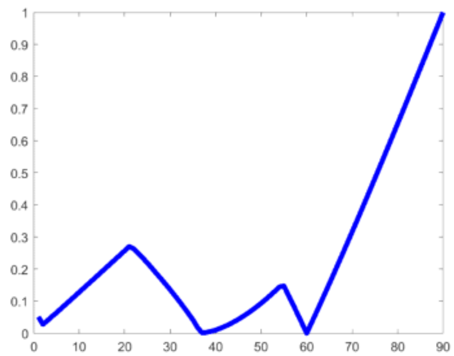
Figure 1. Numerical solution approaches: skew angle on the abscissa with singularity-free momentum magnitude on the ordinant for various integration schemes and discretization.



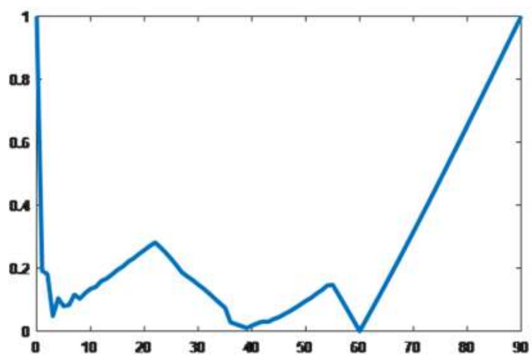
(a) Analytic and numerical solutions numerically calculated and presented with skew angle on abscissa, singularity-free momentum magnitude on ordinant where a discretized grid of nodes spaced 1 degree is used. Notice zero skew angle is not captured.



(b) Analytic solution numerically calculated and presented with skew angle on abscissa, singularity-free momentum magnitude on ordinant Notice zero skew angle is now captured.



(c) Analytic solution reinvestigated with skew angle on abscissa, singularity-free momentum magnitude on ordinant. Notice zero skew angle is still not captured.



(d) Increased focus on very small (near zero) skew angles (near planar momentum generation) with symmetric skew angle on the abscissa and singularity-free angular momentum on the ordinate.

Figure 2. Analytic solution comparison: skew angle on abscissa, singularity-free momentum.

The goal (not reached) of the discrete approach was to reproduce results akin the analytic approach. While the goal was not explicated reached, the method highlighted the weakness of digital computation by computer and laid bare the glaring ease of contradictory results. In the newly executed discrete numerical approach inspired by the heuristic method, we see in Figure 1 the discretized approach to analysis misses (skips over) some singular conditions (and seems to find others) for each iterated case of node spacing for discrete analysis. While this fallacy is instinctive, there are some non-instinctive attributes. The plotted data has “chattering” attributes to be expected of discrete calculations where some singular surfaces are missed at some nodes yet not missed at neighboring nodes. Despite this attribute, the generation of momentum at high skew angles very closely matches the analytic approach in the literature. To see this, compare the results at skew angles of 60-90 degrees in references Sands (2007) and Sands, Kim, and Agrawal (2018). On the other hand, closely examine the results at skew angles lower than 60 degrees. These results indicate a consistency in incorrectly identifying minimum singular conditions for momentum generation. This consistency is theorized to reside in the consistency of numerical approaches (i.e. consistency of integration methods, integration step size, consistency of node spacing within each iteration etc.). With this inspiration, closely examine both the original analytical results and the new numerical results at low skew angles, where computation is ubiquitously challenging due to the need to perform calculations of singular momentum using very low variable values. One aspect to highlight is the different values of singularity-free momentum generation. Another is the nature of singularity-free momentum generation found very close to zero skew angle when small discretization and integration step-size is used. Contemplate the three Postulates introduced earlier in this short communication. As the symmetric skew angle approaches zero, Postulate 1 and Postulate 3 indicate no angular momentum generation is possible singularity-free, while Postulate 2 permits one single gyro to be singular yet torque production remains possible (up to 2h) if maneuvers are not commanded out-of-plane.

Inspired by the illumination of the computational difficultly, we expand the approach to utilize numerical computation of the analytic expressions of matrix inverse to produce non-instinctive results. Figure 2 iterates various integration parameters and discretization. Figure 2a profoundly illustrates the weakness of using too large of discretization, while Figure 2b depicts a comparison of increasingly small discretization to be elaborated in the next section of this short communication. This is very instinct to anyone who has encountered the tradeoff between faster computations (and less accurate results) at larger discretization. Again notice in figures 2a and 2b, both produce accurate analysis at high skew angles (where "accurate" implies close resemblance to the nominal literature).

Figures 1 and 2 uses modestly small discretization such that the results nearly appear continuous, yet even so…the results at low values of skew angles are not consistent with the analytical approaches in the literature. These two figures, in particular reveal the inappropriateness of the new computational method, but nonetheless illustrates the peculiarities of near-planar momentum generation that exacerbate the differences in statements espoused by engineers and scientists (presented here as Postulates) who hold each of the stated Postulates as correct.

3.3. Peculiarities of near-planar and non near-planar minimum singularity-free momentum

Statistical evaluation by means and standard deviations of numerical error for iterated skew angles listed in table 1 clearly illustrate numerical results are more accurate at high values of skew angle, yet diverge for low skew angles where near-planar momentum generation is attempted. This clearly illustrates the consistency in the literature for high skew angles and inconsistency for low values of skew angles. It is noteworthy to mention that whilst 56.73 degrees was taken as the benchmark decades ago, box and roof configurations with 90 degree skew angles currently dominate.

Table 1. Analysis of numerical error for iterated skew angles[1]

	1°-38°	**39°-60°**	**61°-90°**	**Total**
Mean, σ	0.033	0.084	$3.69*10^{-5}$	0.053
Standard deviation, μ	0.039	0.071	$6.86*10^{-5}$	0.035

[1] Notice this results is exactly the opposite of what the benchmark solution (Wie, 2008) would indicate.

4. Discussion

When control moment gyroscopes geometrically align perpendicularly to the commanded torque direction, well known singularities in the angular momentum space arise. Substantial research has aligned along three principle disciplines of study: singularity avoidance, singularity escape (when avoidance fails), and singularity penetration which permits safe flight through regions of singularity. Singularity escape and penetration require mathematical calculations of singular and near-singular quantities (very large numbers) using constituent numbers that are sometimes very small. This dichotomy leads to interesting peculiarities in some specific geometries, and this short communication analyzed these peculiarities to the conclusion that geometric installations of gyros at low skew angles is highly undesirable due to these mathematical peculiarities. Install gyros at high skew angles to not only increase singularity-free performance, but also increase confidence in mathematical analyses absent of peculiarities associated with near-planar momentum generation. Utilization of high values of skew angles simultaneously bestows high maximum (saturation) capability, high singularity-free capability, and well-conditioned mathematical computations.

This short communication presented three disparate paradigms for the definition of singularity from the substantial literature arising from perspectives in math, physics, and systems engineering respectively, and these three definitions explain discrepancies seen in the literature, and illustration of these discrepancies is a key novelty in this article. Disparate definitions of singularities give rise to seemingly conflicting analysis throughout the literature where investigations of low skew angles are ubiquitously notorious. The conclusions are confirmed by identical analyses with several iterations of computational parameters (e.g. integration methods, integration step sizes, and node spacing where nodes are points of analysis in discretized versions of the continuous mathematical relationships. Singularities occur in instances of impossibility to generate the commanded torque, with emphasis added to "the commanded", highlighting a key feature of the definition of singularity includes alignment of the angular momentum vector with the commanded torque. Thus, angular momentum generation is possible (up to 2h for the systems studied here) in instances where a single gyro is aligned in a singular direction while the other two gyros are not. This short communication restricted analysis to symmetric installations of gyros with identical skew angles. Sequel research will investigate the performance of many (from an infinite combinations) asymmetric skew angle geometries following the recent results published by Lewis, et al.

Acknowledgments

Patent 9,567,112 February 14, 2017 by the author illustrates methods of singularity avoidance without null motion and singularity penetration in the event trajectories enter singularities (Sands, Kim, & Agrawal, 2012). Authorship has been limited to those who have contributed substantially to the manuscript. Conceptualization, T.S.; methodology, T.S.; software, T.S., E.C., J.T.E., Z.L.; validation, T.S.; formal analysis, T.S., E.C., J.T.E., Z.L.; investigation, T.S., E.C., J.T.E., Z.L.; writing—original draft preparation, T.S.; writing—review and editing, T.S.; visualization, E.C., J.T.E., Z.L.; supervision, T.S.; funding acquisition, T.S., please turn to the CRediT taxonomy for the term explanation. Authorship has been limited to those who have contributed substantially to the work reported. The education that lead to this self-funded research was funded by the U.S. Strategic Command's distance learning education program (Bittick & Sands, 2019; Sands, Camacho, & Mihalik, 2018) in response to an increased need for critical thinking in the nuclear enterprise in a period of global uncertainty (Nakatani & Sands, 2018; Sands & Mihalik, 2016; Sands, Mihalik, & Camacho, 2018). The APC was funded by corresponding author. The authors acknowledge professors Jae Jun Kim and Brij Agrawal for support given which is not covered by the author contribution or funding sections including considerable technical support and donations in kind (e.g., equipment used for experimental validation). Furthermore, special thanks to professor Marcello Romano for permitting T.S. to teach his course while on sabbatical, providing the venue for this research.

References

Agrawal, B. N., Kim, J. J., & Sands, T. A. (2017). *U.S. Patent No. 9,567,112*. Washington, DC: U.S. Patent and Trademark Office.

Bittick, L., & Sands, T. (2019). Political Rhetoric or Policy Shift: A Contextual Analysis of the Pivot to Asia. *Journal of Social Sciences*, *15*(1), 92-125. https://doi.org/10.3844/jssp.2019.92.125

Kim, J. J., Sands, T., & Agrawal, B. N. (2007). Acquisition, tracking, and pointing technology development for bifocal relay mirror spacecraft. In S. L. Chodos & W. E. Thompson (Eds.), *SPIE Proceedings, Acquisition, Tracking, Pointing, and Laser Systems Technologies XXI* (p. 656907). SPIE. https://doi.org/10.1117/12.720694

Lewis, Z., Ten Eyck, J., Baker, K., Culton, E., Lang, J., & Sands, T. (2019). Non-symmetric gyroscope skewed pyramids. *Aerospace*, *6*(9), 98.

Mihalik, R., Camacho, H., & Sands, T. (2017). Continuum of learning: Combining education, training and experiences. *Education*, *8*, 9-13.

Nakatani, S., & Sands, T. (2018). Eliminating the existential threat from North Korea. *Sci. Technol*, *8*, 11-16. https://doi.org/10.5923/j.scit.20180801.02

Sands, T. A. (2007). *Fine pointing of military spacecraft* (Doctoral dissertation). Naval Postgraduate School, Monterey.

Sands, T. A., Kim, J. J., & Agrawal, B. (2006, December - 2006, December). 2H Singularity-Free Momentum Generation with Non-Redundant Single Gimbaled Control Moment Gyroscopes. In *Proceedings of the 45th IEEE Conference on Decision and Control* (pp. 1551-1556). IEEE. https://doi.org/10.1109/CDC.2006.377310

Sands, T. A., Kim, J. J., & Agrawal, B. (2009, March - 2009, March). Control moment gyroscope singularity reduction via decoupled control. In *IEEE Southeastcon 2009* (pp. 388-391). IEEE. https://doi.org/10.1109/SECON.2009.5174111

Sands, T. A., Kim, J. J., & Agrawal, B. N. (2012). Nonredundant Single-Gimbaled Control Moment Gyroscopes. *Journal of Guidance, Control, and Dynamics*, *35*(2), 578-587. https://doi.org/10.2514/1.53538

Sands, T., & Mihalik, R. (2016). Outcomes of the 2010 and 2015 nonproliferation treaty review conferences. *World J. Soc. Sci. Humanities*, *2*, 46-51.

Sands, T., Camacho, H., & Mihalik, R. (2017). Education in nuclear deterrence and assurance. *J. Def. Manag*, *7*, 166.

Sands, T., Camacho, H., & Mihalik, R. (2018). Nuclear Posture Review: Kahn Vs. Schelling…and Perry. *Journal of Social Sciences*, *14*(1), 145-154. https://doi.org/10.3844/jssp.2018.145.154

Sands, T., Kim, J. J., & Agrawal, B. N. (2007). Spacecraft fine tracking pointing using adaptive control. In *Proceedings of 52th International Astronautical Congress*.

Sands, T., Kim, J., & Agrawal, B. (2016). Experiments in control of rotational mechanics. *Int. J. Autom. Control Intell. Syst*, *2*, 9-22.

Sands, T., Kim, J., & Agrawal, B. (2018). Singularity penetration with unit delay (SPUD). *Mathematics*, *6*(2), 23. doi: 10.3390/math6020023

Sands, T., Lu, D., Chu, J., & Cheng, B. (2018). Developments in angular momentum exchange. *Int. J. Aero. Sci*, *6*, 1-6. https://doi.org/10.5923/j.aerospace.20180601.01

Sands, T., Mihalik, R., & Camacho, H. (2018). Theoretical Context of the Nuclear Posture Review. *Journal of Social Sciences*, *14*, 124-128.

Wie, B. (2008). *Space vehicle dynamics and control*. American Institute of Aeronautics and Astronautics.