Q-3 Maximum likelihood function

Part 1

Suppose we have a random sample $x_1, x_2 \ldots\ldots x_n$ for which the probability density function for each x_i is $f(x_i; q)$ then the joint probability mass density function of $x_1 \ldots\ldots x_n$ is $f(x_i; q) \times f(x_2; q) \ldots f(x_n; q) = \prod_{i=1}^{n} f(x_i; \theta)$

where q is the parameter.

$$L(q) = \prod_{i=1}^{n} f(x_i, q) \rightarrow \text{likelihood function}$$

$$L(q) = q^{x_1}(1-q)^{x_1} \times q^{x_2} \cdot (1-q)^{x_2} \ldots q^{x_{30}}(1-q)^{x_{30}}$$

$$L(q) = q^{\Sigma x_i}(1-p)^{n - \Sigma x_i}$$

where $x \rightarrow$ students who don't opt for the ticket

Part 2:

$$L(q) = q^{\sum x_i} (1-q)^{n-\sum x_i}$$

Now, we have to maximimise $L(q)$.
we'll take log on both sides and differentiate w.r.t. q to get the estimate of q
i.e. $\hat{q}$.

$$\log(L(q)) = \sum x_i \cdot \log(q) + (n-\sum x_i) \cdot \log(1-q)$$

$$\frac{\partial}{\partial q}(\log(L(q))) = \frac{\sum x_i}{q} - \frac{(n-\sum x_i)}{1-q}$$

$$\sum x_i (1-q) - q(n-\sum x_i) = 0$$

$$\sum x_i - \cancel{\sum x_i \cdot q} - n \cdot q + \cancel{q \sum x_i} = 0$$

$$\hat{q} = \frac{\sum x_i}{n}$$

Part 3

Now we have an estimate for

$\hat{q}$

$$\hat{q} = \frac{\sum x_i}{n}$$

$x =$ students who did not opt for ticket.

$$\boxed{\hat{q} = \frac{13}{30}}$$

Part 4:

we say a parameter, here $\hat{q}$ is an ubiased estimator of q, when

$$E(\hat{q}) = q$$

i.e. the expected value of $\hat{q} = q$

so lets say that there are N samples, and $X_1, X_2, \dots X_n$ are the random variable that denotes the proportion of 0's out of 30 students in multiple samples.

$$E(\hat{q}) = E\left(\frac{X_1 + X_2 + \dots X_N}{N}\right)$$

$$= \frac{1}{N} E(X_1 + \dots X_N).$$

Since, samples are independent....

$$= \frac{1}{N}\left[E(x_1) + E(x_2) \cdots + E(x_N)\right]$$

Now, since the samples are identically distributed

$$= \frac{1}{N}\left[q + q + \cdots q\right]$$

$$= \frac{1}{N} \cdot Nq$$

$$= q.$$

So, $E(\hat{q}) = q$ and hence $\hat{q}$ from MLE is the unbiased estimate of q.