1. Solve the recurrence $T(n) = T(\sqrt{n}) + 1$. Give a θ bound.

   Let $n = 2^k$. Thus we have $T'(k) = T(2^k) = T(\sqrt{2^k}) + 1 = T'(k/2) + 1$. Thus $T'(k) = \log k$. Thus $T(n) = \log\log n$.

2. Consider the following functions: $n - \log n, \log\log n, 2^{\log^2 n}, \sqrt{n/\log n}, n^{\log n}$. Arrange them in order s.t. the ith function is $O()$ of the $i+1$th function.

   You may want to note that if $\lim_{n\to\infty} \frac{f(n)}{g(n)} < c$ for any constant c, then $f(n) = O(g(n))$. It is fine if you use this.

   $\log\log n, \sqrt{n/\log n}, n - \log n, 2^{\log^2 n} = n^{\log n}$

3. Consider an array $x[1..n]$. An element $x[i]$ is a local minimum if it is no larger than $x[i-1]$ (if any), as well as $x[i+1]$ (if any). Give an $O(\log n)$ algorithm to find a local minimum.

   Algorithm:

   (a) If $n < 10$ then check each element. The smallest number is certainly a local minimum, and this will be found.

   (b) If $x[n/2] < x[n/2+1]$, recurse on $x[1..n/2]$. If this call returns i s.t. $1 \leq i < n/2$ then it will be a local minimum by correctness of the recursion. If it returns $i = n/2$ then we will have $x[n/2-1] \geq x[n/2]$. But we already have $x[n/2] < x[n/2+1]$. Thus $x[i]$ is indeed a local minimum.

   (c) Else recurse on $x[n/2+1..n]$. The rest of the argument is similar.

4. The input to this problem are two sorted arrays, of length m, n respectively. All the $m+n$ values in the two arrays are distinct. Another input is an integer s, satisfying $s \leq m+n$. The goal is to find the sth smallest value from the $m+n$ values. Show that this can be done using at most $1 + \lceil \log s \rceil$ comparisons.

   Let the arrays be $A[1..m], B[1..n]$. Compare $A[\lceil s/2 \rceil], B[\lceil s/2 \rceil]$. Say $A[\lceil s/2 \rceil] < B[\lceil s/2 \rceil]$. Clearly $A[1..\lfloor s/2 \rfloor]$ are all smaller than the sth smallest. Thus for the next step we can drop these elements and look for the $s - \lfloor s/2 \rfloor$th element. The recursion will end when $s = 1$, whereupon we compare $A[1], B[1]$.