

### Question 1:

For a  $d$ -ary heap the `delMax()` function has a time complexity of  $O(\log N) * d$  in the worst case. The  $d$  is the number of children allowed by each parent, the  $N$  is the total number of elements in the array. In binary heap sort, where there are 2 children, the time complexity for worst case is  $O(\log N)$  which implies that for a heap sort with  $d$  number of children there would be a coefficient added to the time complexity for binary heap.

```
public int delMax() {
    int max = 0;
    int maxKey = 0;
    int last = heapSize - 1;
    for(int i = 0; i < last; i++) {
        if(a[i] > max) {
            max = a[i];
            maxKey = i;
        }
    }
    exch(a, last, maxKey);
    heapSize--;
    sink(maxKey);
    a[last] = 0;
    return max;
}

private void sink(int k) {
    int childSet = (k * child) + 1;
    if(childSet < heapSize) {
        int max = childSet;
        int counter = childSet + child;
        if(counter > heapSize) {
            counter = heapSize - 1;
        }
        for(int i = childSet; i < counter; i++) {
            if (a[i] > a[max]) {
                max = i;
            }
        }
        exch(a, max, k);
        sink(max);
    }
}
```

So the running time of deleting a heap with  $N$  number of nodes is equal to the height of the heap. The height of a heap is equal to  $\log(N + 1)$  approximately. So for `delMax()` function for a  $d$ -ary heap it would be  $d * \text{the height of the heap}$ .

For a d-ary heap the sort function has a time complexity of  $O(\log N) * d$  in the worst case as well where the  $N$  is the total number of elements in the array. For the binary heap the worst case is  $O(\log N)$ . Since the sort function also depends on the height of the heap the time complexity for the `delMax()` function and the `daryHeapsort()` is the same.

```
public int [] daryHeapsort() {  
    int [] arraySort = new int [length];  
    for(int i = 0; i < length; i++) {  
        arraySort[i] = a[i];  
    }  
    int size = heapSize - 1;  
  
    while(size > 2) {  
        exch(arraySort, 0, size--);  
        print(arraySort);  
        sinkSorted(arraySort, 0, size);  
    }  
  
    for (int i = 1; i < child; i++) {  
        if (arraySort[0] > arraySort[i]) {  
            exch(arraySort, 0, i);  
        }  
    }  
  
    return arraySort;  
}
```