CS 161 HW 5
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1a) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

$Smoke$	$Fire$	$(Smoke \Rightarrow Fire)$	$(\neg Smoke \Rightarrow \neg Fire)$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

ANSWER: Neither

1b) $(Smoke \Rightarrow Fire) \Rightarrow \big((Smoke \vee Heat) \Rightarrow Fire\big)$

$Smoke$	$Fire$	$Heat$	$(Smoke \Rightarrow Fire)$	$\big((Smoke \vee Heat) \Rightarrow Fire\big)$	$(Smoke \Rightarrow Fire) \Rightarrow \big((Smoke \vee Heat) \Rightarrow Fire\big)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

ANSWER: Neither

1c) $\big((Smoke \wedge Heat) \Rightarrow Fire\big) \Leftrightarrow \big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big)$

$Smoke$	$Fire$	$Heat$	$\big((Smoke \wedge Heat) \Rightarrow Fire\big) \Leftrightarrow \big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

ANSWER: Valid

2a)

$(i)\ Mythical \Rightarrow Immortal$
$(ii)\ \neg Mythical \Rightarrow \neg Immortal \wedge Mammal$
$(iii)\ Immortal \vee Mammal \Rightarrow Horned$
$(iv)\ Horned \Rightarrow Magical$

2b)

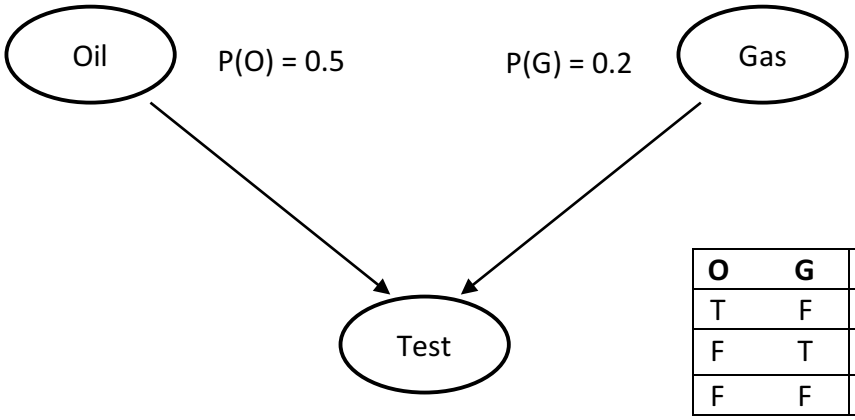
$(1)\ (\neg Mythical \vee Immortal)$
$(2)\ (Mythical \vee \neg Immortal)$
$(3)\ (Mythical \vee Mammal)$
$(4)\ (\neg Immortal \vee Horned)$
$(5)\ (\neg Mammal \vee Horned)$
$(6)\ (\neg Horned \vee Magical)$

2c)

$(7)\ (Immortal \vee Mammal)$	1&6
$(8)\ (\neg Mythical \vee Horned)$	1&4
$(9)\ (Mythical \vee Horned)$	3&5
$(10)\ Horned$	8&9
$(11)\ (\neg Immortal \vee Magical)$	4&6
$(12)\ (\neg Mammal \vee Magical)$	5&6
$(13)\ (Mythical \vee Magical)$	3&12
$(14)\ Magical$	6&10

We cannot prove that the unicorn is Mythical.

3a)



O	G	T	P(T|O,G)
T	F	T	0.9
F	T	T	0.3
F	F	T	0.1

3b)

$N = \neg(O \vee G)$
$P(O) = 0.5$
$P(G) = 0.2$
$P(N) = 0.3$
$P(T|O) = 0.9$
$P(T|G) = 0.3$
$P(T|N) = 0.1$

$$P(T) = P(T|O)P(O) + P(T|G)P(G) + P(T|N)P(N)$$
$$= (0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)$$
$$= 0.54$$

$$P(O|T) = \frac{P(T|O)P(O)}{P(T)} = \frac{(0.9)(0.5)}{0.54} = 0.833$$

4a) $P(A, B, C, D, E, F, G, H)$
$$= P(A) * P(B) * P(C|A) * P(D|A, B) * P(E|B) * P(F|C, D) * P(G|F) * P(H|F, E)$$

4b) $P(A, B, C, D, E, F, G, H)$
$$= f_1(A) * f_2(B) * f_3(A, C) * f_4(A, B, D) * f_5(B, E) * f_6(C, D, F) * f_7(F, G) * f_8(E, F, H)$$

$P(E, F, G, H) =$
$\sum_A \sum_B \sum_C \sum_D \left(f_1(A) * f_2(B) * f_3(A, C) * f_4(A, B, D) * f_5(B, E) * f_6(C, D, F) * f_7(F, G) * f_8(E, F, H)\right)$
$= \sum_A \sum_B \sum_C \sum_D f_9(A, B, C, D, E, F, G, H) = \sum_A \sum_B \sum_C f_{10}(A, B, C, E, F, G, H)$
$= \sum_A \sum_B f_{11}(A, B, E, F, G, H) = \sum_A f_{12}(A, E, F, G, H)$
$= f_{13}(E, F, G, H)$

4c) $P(A, \neg B, C, D, \neg E, F, \neg G) =$
$(0.2) * (0.3) * P(C|A) * (0.6) * (0.1) * P(F|C, D) * P(\neg G|F) * P(H|F, \neg E)$
$= (0.0036) * P(C|A) * P(F|C, D) * P(\neg G|F) * P(H|F, \neg E)$

4d) A and B are independent so you can multiply as follows
$P(\neg A, B) = P(\neg A) * P(B) = (0.8)(0.7) = 0.56$

A and E are independent so we know that $P(\neg E|A) = P(\neg E)$
E is dependent on B, so $P(\neg E) = P(\neg E, B) + P(\neg E, \neg B)$
$= (0.9)(0.7) + (0.1)(0.3) = 0.66$

4e) A is conditionally independent of all nodes
B is conditionally independent of all nodes
C is conditionally independent of all nodes except for A
D is conditionally independent of all nodes except for A and B
E is conditionally independent of all nodes except for B
F is conditionally independent of all nodes except for C and D
G is conditionally independent of all nodes except for F
H is conditionally independent of all nodes except for E and F

4f) The Markov blanket for D is $\{A, B, C, F\}$

4g)

A	B	D	$f_4(A,B,D)$	B	E	$f_5(B,E)$	A	B	D	E	$f_{14}(A,B,D,E)$
T	T	T	0.5	T	T	0.1	T	T	T	T	0.5*0.1 = 0.05
T	T	F	0.5	T	F	0.9	T	T	T	F	0.5*0.9 = 0.45
T	F	T	0.6	F	T	0.9	T	T	F	T	0.5*0.1 = 0.05
T	F	F	0.4	F	F	0.1	T	T	F	F	0.5*0.9 = 0.45
F	T	T	0.1				T	F	T	T	0.6*0.9 = 0.54
F	T	F	0.9				T	F	T	F	0.6*0.1 = 0.06
F	F	T	0.8				T	F	F	T	0.4*0.9 = 0.36
F	F	F	0.2				T	F	F	F	0.4*0.1 = 0.04
							F	T	T	T	0.1*0.1 = 0.01
							F	T	T	F	0.1*0.9 = 0.09
							F	T	F	T	0.9*0.1 = 0.09
							F	T	F	F	0.9*0.9 = 0.81
							F	F	T	T	0.8*0.9 = 0.72
							F	F	T	F	0.8*0.1 = 0.08
							F	F	F	T	0.2*0.9 = 0.18
							F	F	F	F	0.2*0.1 = 0.02

4h) $f_{15}(A,B,E) = \sum_D f_{14}(A,B,D,E) = f_{14}(A,B,D,E) + f_{14}(A,B,\neg D,E)$

A	B	E	$f_{15}(A,B,E)$
T	T	T	0.05+0.05 = 0.1
T	T	F	0.45+0.45 = 0.9
T	F	T	0.54+0.36 = 0.9
T	F	F	0.06+0.04 = 0.1
F	T	T	0.01+0.09 = 0.1
F	T	F	0.09+0.81 = 0.9
F	F	T	0.18+0.72 = 0.9
F	F	F	0.08+0.02 = 0.1