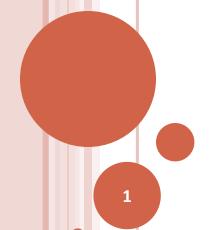
# CS F364 Design & Analysis of Algorithms

# **ALGORITHM DESIGN TECHNIQUES - GREEDY**

# **Greedy Algorithms**

- Greedy Choice and Optimal Substructure
- Limitation of Greedy Choice Example



## **OPTIMAL SUBSTRUCTURE**

- A problem exhibits optimal substructure if
  - an optimal solution to the problem contains optimal solutions for sub-problems:
    - o can be decomposed into optimal solutions for subproblems.
- Optimal substructure is necessary for greedy choice:
  - Otherwise local choice may not lead to global optimality
    i.e. the choice may not preserve optimality
  - Examples:
    - o Schedule for tasks  $T' = T \{j\}$  is part of the schedule for tasks T where j is the earliest starting task
    - o Schedule for tasks  $T' = T \{k\}$  is not necessarily part of the schedule for tasks T if k is not the earliest starting task

# GREEDY CHOICE, OPTIMAL SUBSTRUCTURE, AND INDUCTION

- Design of Greedy Algorithms can be viewed as a special case of Divide-And-Conquer:
  - where the problem of size N is divided into
    - a sub-problem of size 1 (or size k for some constant k) and
    - a sub-problem of size N-1 (or N-k as the case may be) where the latter is the same as the original problem.
- Greedy Choice
  - refers to making the right choice locally for sub-problem
    (i).
- Optimal substructure property is necessary to ensure that
  - optimal solution to sub-problem (ii) can be used as is if subproblem (ii) is part of the solution to the problem.

## **GREEDY CHOICE - LIMITATION**

- Greedy Choice does not always hold when Optimal Substructure Property holds.
- e.g. Consider the 0/1 KnapSack problem:
  - Optimal Substructure holds for 0/1 KnapSack:
    - oConsider the most valuable subset of items with weight at most W
    - olf we remove item j from this subset, the remaining subset must be the most valuable weighing at most  $W-w_{\rm j}$

## **GREEDY CHOICE - LIMITATION**

- Greedy Choice Property does not hold for 0/1 KnapSack:
  - Suppose we use <u>per unit profit</u> as the greedy choice
  - Consider three items:
    - 1. (10kg, Rs. 5,000). Value = Rs. 500 / kg
    - (20 kg, Rs. 8,000) Value = Rs. 400 / kg
    - 3. **(40kg, Rs. 9,000)** Value = Rs. 225 / kg
  - Let W = 60kg
    - o Any solution with item 1 is not optimal!
      - o i.e. ordering by unit weight is not useful

#### • Exercise:

- Generalize this argument (about remaining capacity) for any input.
- Similarly argue that other greedy choices (profit or weight) are also not useful.