DAA Tutorial 12 Solution

(10)

4: (a) A() can halt in

1st iteration ($n \leftarrow \{0,1,2,3\}$) with probability $= \frac{2}{4}$

2nd iteration ($n \leftarrow \{0,1,2,3,4\}$) with probability $= \frac{2}{4} \times \frac{2}{5}$

3rd iteration ($n \leftarrow \{0,1,2,3,4,5\}$) with probability $= \frac{2}{4} \times \frac{3}{5} \times \frac{2}{6}$

$\vdots$

i^{th} iteration ($n \leftarrow \{0,1,\cdots,i+2\}$) with probability $= \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \cdots \times \frac{i}{i+2} \times \frac{2}{i+3}$

$\vdots$

to

Probability of halting of A()

$$= \frac{2}{4} + \frac{2}{4} \times \frac{2}{5} + \frac{2}{4} \times \frac{3}{5} \times \frac{2}{6} + \cdots + \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \cdots \times \frac{i}{i+2} \times \frac{2}{i+3} + \cdots \quad (4)$$

$$= \frac{12}{2.3.4} + \frac{12}{3.4.5} + \frac{12}{4.5.6} + \cdots + \frac{12}{(i+1)(i+2)(i+3)} + \cdots \quad (2)$$

$$= 6\left[\frac{4-2}{2.3.4} + \frac{5-3}{3.4.5} + \frac{6-4}{4.5.6} + \cdots + \frac{(i+3)-(i+1)}{(i+1)(i+2)(i+3)} + \cdots\right]$$

$$= 6\left[\frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \frac{1}{4.5} - \frac{1}{5.6} + \cdots + \frac{1}{(i+1)(i+2)} - \frac{1}{(i+2)(i+3)} + \cdots\right]$$

$$= \frac{6}{2.3} = \boxed{1} \quad (4)$$

(6) Expected number of iterations of the while loop in A()

$$= \frac{2}{4} \times 1 + \frac{2}{4} \times \frac{2}{5} \times 2 + \frac{2}{4} \times \frac{3}{5} \times \frac{2}{6} \times 3 + \cdots + \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \cdots + \frac{i}{i+2} \times \frac{2}{i+3} \times i + \cdots \quad (4)$$

$$= \frac{12 \times 1}{2.3.4} + \frac{12 \times 2}{3.4.5} + \frac{12 \times 3}{4 \times 5 \times 6} + \cdots + \frac{12 \times i}{(i+1)(i+2)(i+3)} + \cdots \quad (2)$$

$$= 12\left[\frac{2+3-4}{2.3.4} + \frac{3+4-5}{3.4.5} + \frac{4+5-6}{4.5.6} + \cdots + \frac{(i+1)+(i+2)-(i+3)}{(i+1)(i+2)(i+3)} + \cdots\right]$$

$$= 12\left[\frac{1}{3.4} + \frac{1}{2.4} - \frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{3.5} - \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{4.6} - \frac{1}{4.5} + \cdots\right.$$

$$\left. + \frac{1}{(i+2)(i+3)} + \frac{1}{(i+1)(i+3)} - \frac{1}{(i+1)(i+2)} + \cdots\right]$$

$$= 6\left[-\frac{1}{1\cdot 3} + \frac{2}{2\cdot 4} + \frac{2}{3\cdot 5} + \frac{2}{4\cdot 6} + \cdots + \frac{2}{(i+1)(i+3)} + \cdots \right]$$ (2)

$$= -2 + 6\left[\frac{4-2}{2\cdot 4} + \frac{5-3}{3\cdot 5} + \frac{6-4}{4\cdot 6} + \cdots + \frac{(i+3)-(i+1)}{(i+1)(i+3)} + \cdots \right]$$

$$= -2 + 6\left[\frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \cdots + \frac{1}{i+1} - \frac{1}{i+3} + \cdots \right]$$

$$= -2 + 3 + 2 = \boxed{3}$$ (4)