Design and Analysis of Algorithms (CS F364) Comprehensive Exam (2019), Second Semester

There are 4 questions in all and total marks is $10 + 20 + (15 + 5) + 20 = 70$. This is an **open book exam**. Only hard copies of textbooks, reference books, and lecture notes are allowed. No electronic instruments (calculator, mobile phone, tablet, laptop etc.) are allowed. Show all computation steps for solving any problem, and write complete algorithms with time complexity derivations. Time: 180 minutes.

1. Find all possible $s - t$ cuts and their capacity in the flow graph of figure 1. What is the minimum $s - t$ cut and what is its capacity?

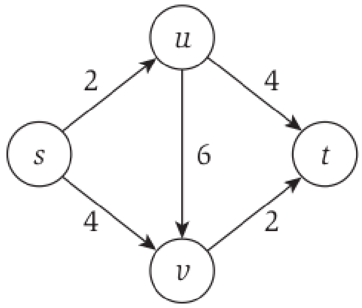

Figure 1: Flow graph for question 1.

2. Design an efficient *Fully Polynomial Time Approximation Scheme (FPTAS)* $(A_\epsilon)_{\epsilon \in (0,1)}$ for the $0 - 1$ Knapsack problem such that the algorithm $A_\epsilon(I)$ runs in time $O\left(\frac{|I|^2}{\epsilon}\right)$ and

$$\frac{A_\epsilon(I)}{OPT(I)} \geq 1 - \epsilon$$

3. We identify a Las Vegas algorithm A together with an input x with the probability distribution p on its running time $T_A(x)$. Thus p is a probability distribution over $\mathbb{N}$ and $p(t)$ denotes the probability that $A(x)$ stops after exactly t units of time. We consider the problem of minimizing the expected running time of $A(x)$ using strategies $S(t) = (t, t, t, \ldots)$ which simulate $A(x)$ as follows:
(1) Run $A(x)$ for t units of time
(2) If ($A(x)$ completes its execution in (1))
(3) then return
(4) else restart $A(x)$ from the beginning (using an independent sequence of random bits) and goto (1)
Let $l(t)$ be the expected running time of $S(t)$ when applied to $A(x)$ with probability distribution p. Let $q(t) = \sum_{t' \leq t} p(t')$ be the cumulative distribution function of p.

   (a) Prove that

   $$l(t) = \frac{t - \sum_{t' < t} q(t')}{q(t)}$$

   (b) Let $p(t) = 2^{-t}$. Find the optimal strategy $S(t^*)$ so that $l(t^*)$ is minimized. What is $l(t^*)$?

4. By making use of efficient algorithms find the last three (least significant) digits in the decimal expansion of

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