Design and Analysis of Algorithms (CS F364) Quiz 2 (2018)
(Open Book)

There are 2 questions in this quiz with total marks $20 = 10 + 10$. Time: 40 minutes. Only hard copies of textbooks, reference books, and lecture notes are allowed. No electronic instruments (calculator, mobile phone, tablet, laptop etc.) are allowed. Please write your answer in the question paper in the space provided.
Name:
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Signature:

1. Let $L_e = \{0,1\}^*0$, the language of all binary strings ending in 0. Consider the following DTM accepting L_e: The alphabet is given by $\Gamma = \{\rhd, B, 0, 1\}$. Here $\rhd$ is the start symbol, and B is the blank symbol. The set of states is given by $Q = \{q_0, q_h, q_1, q_2\}$. q_0 is the start state, and q_h is the final state. The transition function is defined by: $\delta(q_0, \rhd) = (q_1, \rhd, R)$, $\delta(q_1, 0) = (q_1, 0, R)$, $\delta(q_1, 1) = (q_1, 1, R)$, $\delta(q_1, B) = (q_2, B, L)$, $\delta(q_2, 0) = (q_h, 0, S)$. We assign binary code to alphabet symbols as follows: $\rhd = 00$, $B = 11$, $0 = 01$, $1 = 10$. We assign binary code to states as follows: $q_0 = 00$, $q_h = 11$, $q_1 = 01$, $q_2 = 10$. We define the i'th snapshot of computation as the binary encoding of the pair (q_i, a_i) such that in i'th step, the TM head is scanning the symbol a_i, and it is in state q_i. For example, $Z_1 = (q_0, \rhd) = 0000$. A CNF formula is written corresponding to input 01 to the TM, using the polynomial-time reduction $L \leq_p SAT$ for any language L in `NP` (as discussed in the previous class: Z_{i+1} is a function of Z_i, $Z_{f(i+1)}$, and $g(i+1)$, where $g(i)$ is the head position at step i, and $f(i)$ is the largest number of step j such that in j'th step the head was at the same position as the i'th step, where $j < i$). Let X_{11}, X_{12}, X_{21}, and X_{22} be the variables corresponding to the binary encoding of an input of length 2. The given TM runs for 6 steps on input 01. Let Z_{ij} be the variable corresponding to j'th bit of Z_i ($1 \leq i \leq 6, 1 \leq j \leq 4$). Below is given a CNF formula that encodes the computation of the given TM on input 01 using the given reduction. Some variables are missing in the formula (Y_k for $1 \leq k \leq 10$). You have to write the correct variable (out of only Z_{ij} or $\overline{Z_{ij}}$ for $1 \leq i \leq 6, 1 \leq j \leq 4$) corresponding to each Y_k ($1 \leq k \leq 10$) in correct order according to the given reduction. Let the function $\Phi(u_1, u_2, u_3, u_4, u_5, u_6)$ be defined as a 6CNF formula having 59 clauses in all possible combinations of the 6 literals (either a variable u_l or its complement $\overline{u_l}$ for $1 \leq l \leq 6$) except for the following 5 clauses: $(u_1 \vee u_2 \vee u_3 \vee u_4 \vee u_5 \vee \overline{u_6})$, $(u_1 \vee \overline{u_2} \vee u_3 \vee \overline{u_4} \vee u_5 \vee \overline{u_6})$, $(u_1 \vee \overline{u_2} \vee \overline{u_3} \vee u_4 \vee u_5 \vee \overline{u_6})$, $(u_1 \vee \overline{u_2} \vee \overline{u_3} \vee \overline{u_4} \vee \overline{u_5} \vee u_6)$, and $(\overline{u_1} \vee u_2 \vee u_3 \vee \overline{u_4} \vee \overline{u_5} \vee \overline{u_6})$. Think how Φ is generated from the transition function. Let the function $\Psi(v_1, v_2, v_3, v_4)$ be defined as a 2CNF formula: $\Psi(v_1, v_2, v_3, v_4) = (v_1 \vee \overline{v_3}) \wedge (\overline{v_1} \vee v_3) \wedge (v_2 \vee \overline{v_4}) \wedge (\overline{v_2} \vee v_4)$. Think what is the role of Ψ. The CNF formula is given as:
$(\overline{X_{11}}) \wedge (X_{12}) \wedge (X_{21}) \wedge (\overline{X_{22}}) \wedge (\overline{Z_{11}}) \wedge (\overline{Z_{12}}) \wedge (\overline{Z_{13}}) \wedge (\overline{Z_{14}}) \wedge \Phi(Z_{11}, Z_{12}, Z_{13}, Z_{14}, Z_{21}, Z_{22}) \wedge$
$(Y_1) \wedge (Y_2) \wedge \Phi(Z_{21}, Z_{22}, Z_{23}, Z_{24}, Z_{31}, Z_{32}) \wedge (Y_3) \wedge (Y_4) \wedge \Phi(Z_{31}, Z_{32}, Z_{33}, Z_{34}, Z_{41}, Z_{42}) \wedge$
$(Y_5) \wedge (Y_6) \wedge \Phi(Z_{41}, Z_{42}, Z_{43}, Z_{44}, Z_{51}, Z_{52}) \wedge \Psi(Z_{53}, Z_{54}, Y_7, Y_8) \wedge \Phi(Z_{51}, Z_{52}, Z_{53}, Z_{54}, Z_{61}, Z_{62}) \wedge$
$\Psi(Z_{63}, Z_{64}, Y_9, Y_{10}) \wedge (Z_{61}) \wedge (Z_{62})$. The above formula follows the sequence: input is correct, TM starts correctly, computation is according to the transition function of the TM, and finally TM halts correctly. Write your answers here:

$Y_1 =$ .... $Y_2 =$ ....
$Y_3 =$ .... $Y_4 =$ ....
$Y_5 =$ .... $Y_6 =$ ....
$Y_7 =$ .... $Y_8 =$ ....
$Y_9 =$ .... $Y_{10} =$ ....

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2. Consider the flow graph in Figure 1. Capacities of the edges are shown along the edges.

    (a) Using the Ford-Fulkerson algorithm, find the maximum s-t flow, showing all the steps involved. What is the value of max s-t flow? [5]

    (b) Using your solution in (a), find the minimum s-t cut, giving an explanation for the conversion. What is the value of min s-t cut? [5]

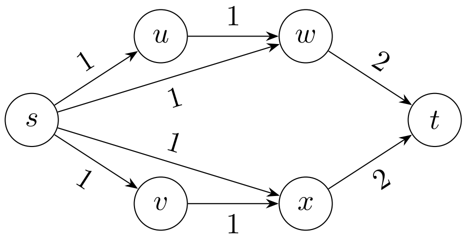

Figure 1: Flow graph for question 2.