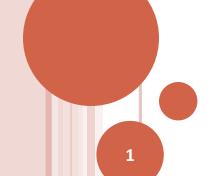
CS F364 Design & Analysis of Algorithms

# **COMPLEXITY – OPTIMIZATION PROBLEMS**

**Intractable Problems** 

- Optimization Problems - Examples



### **OPTIMIZATION PROBLEMS - CHARACTERIZATION**

- o Formal Description:
  - An optimization problem  $\pi$  is characterized by the quadruple  $(I_{\pi}, F_{\pi}, m_{\pi}, goal_{\pi})$ 
    - $oI_{\pi} = \{ x \mid x \text{ is an input instance of } \pi \}$
    - ${}_{\sigma}F_{\pi}(x) = \{ s \mid s \text{ is a feasible solution for } x, \text{ where } x \in I_{\pi} \}$
    - o  $m_{\pi}(x, y) = v$  where v is a quantitative measure of the "value" of the feasible solution  $y \in F_{\pi}(x)$  for  $x \in I_{\pi}$
    - o goal  $_{\pi} \in \{ \min, \max \}$

## **OPTIMIZATION PROBLEMS - EXAMPLES**

- o Min Vertex Cover
  - I = {G| G is an undirected graph}
  - F(G) = { S | S ⊆ V s.t. for any (u,w) in E:

u in S or v in S, where G = (U,V)

- m(G, S) = |S| where G in I and S in F(G)
- goal: min

#### o TSP

- I = {G | G is a weighted, completely-connected graph}
- F(G) = { (u1, u2, u3, ... un, u1) | G=(V,E,w), n = |V|, ui in V for any i in 1...n, ui <> uj for any i and j in 1...n }
- $m(G, P) = \Sigma_{e \text{ in } P} w(e)$  where G = (V,E,w), P in F(G)
- goal: min

# **OPTIMIZATION PROBLEMS**

- Optimal Solution:
  - The optimal solution for a given instance x of a problem  $\pi$  is characterized by:
    - oThe optimal measure:

$$om_{\pi} *(x) = goal_{\pi} \{ m_{\pi} (x, y) \mid y \text{ in } F_{\pi} (x) \}$$

- The optimal solution is characterized thus:
  - OPT  $_{\pi}(x) = y$  where y in  $F_{\pi}(x)$  and  $m_{\pi}^*(x) = m_{\pi}(x, y)$

### o Note:

- The objective for a given context may be to find
  - The optimal solution (*Constructive Version*)
  - The optimal measure i.e. the measure of the optimal solution (*Evaluative Version*)