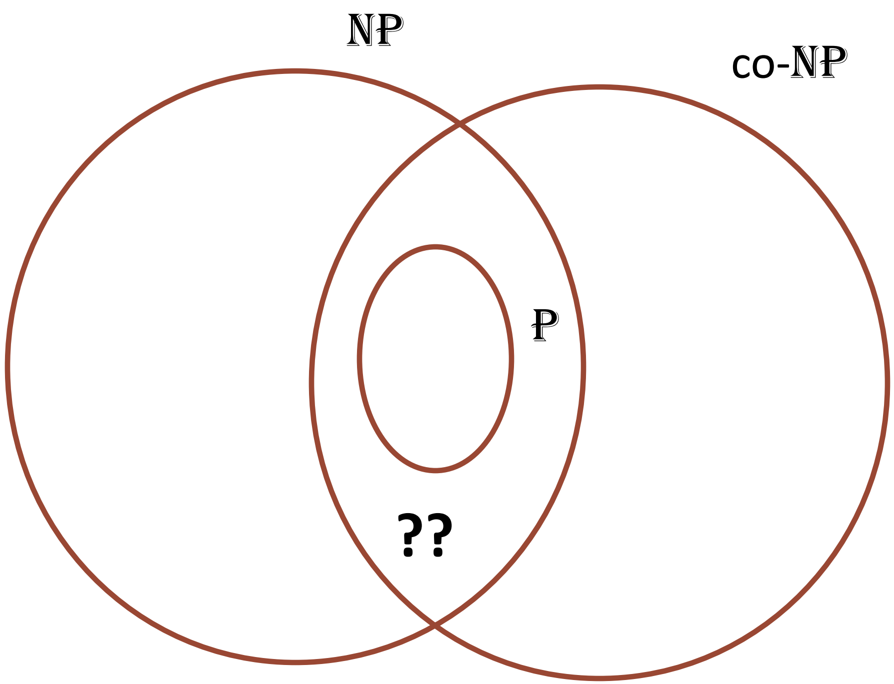
CS F364
Design & Analysis of Algorithms

Algorithms - Complexity

Complexity Classes: CURRENT STATUS

Complexity Classes and Relations among them



Claim:

$\mathbb{P} \subseteq \mathbb{NP} \cap$ co-$\mathbb{NP}$

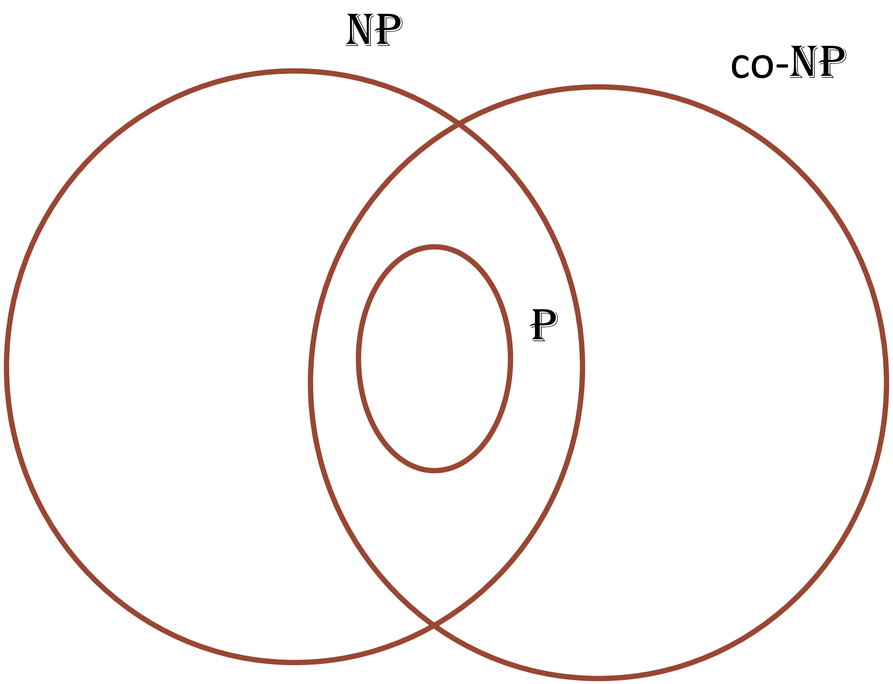
Proof:

- $\mathbb{P} \subseteq \mathbb{NP}$
- $\mathbb{P} =$ co-$\mathbb{P} \subseteq$ co-$\mathbb{NP}$

The converse: Is $\mathbb{NP} \cap$ co-$\mathbb{NP} \subseteq \mathbb{P}$?

Status of the converse: *Open*

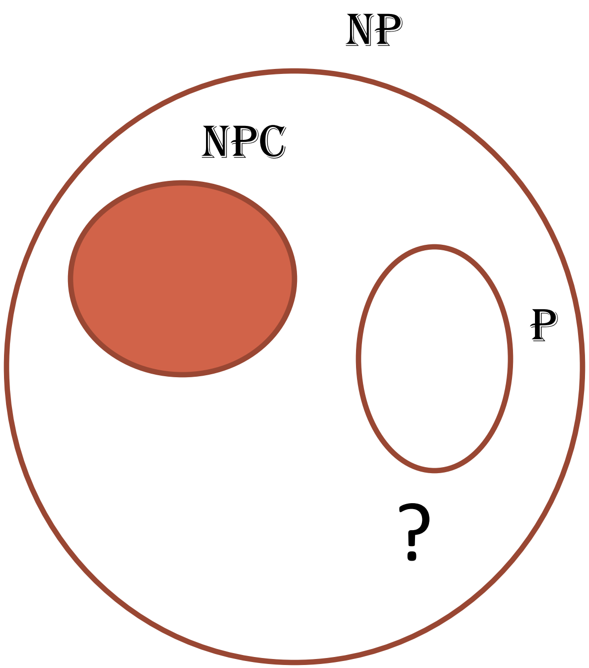
Complexity Classes and Relations among them [2]



Claim:
$\mathbb{P} = \mathbb{NP}$ ==> $\mathbb{NP}$ = co-$\mathbb{NP}$
but the converse need not be true.

Prove the above claim.

Relation Between $\mathbb{P}$ And $\mathbb{NPC}$



$\mathbb{NPC}$ refers to the set of $\mathbb{NP}$-complete problems

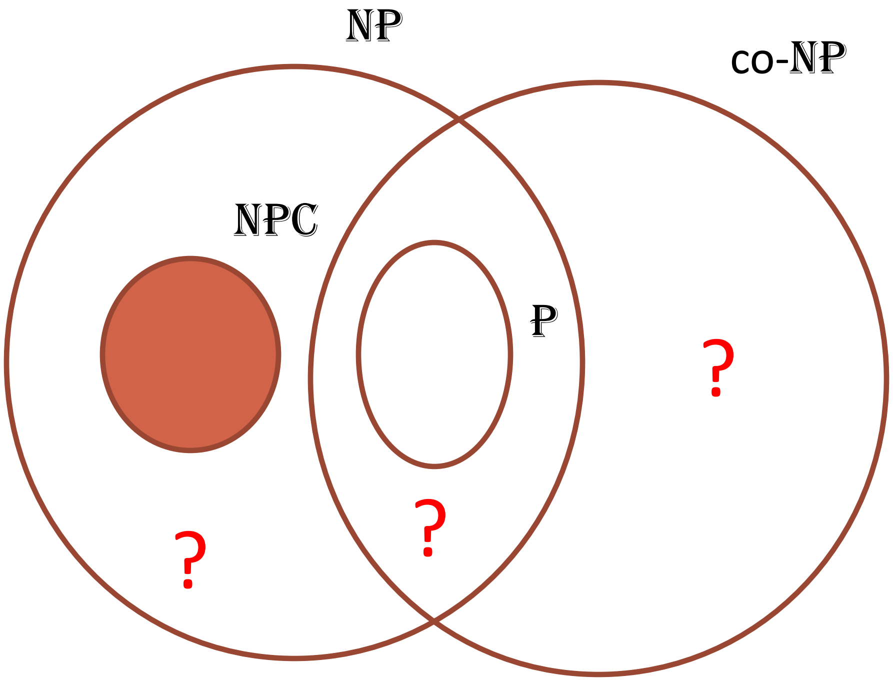
Claims:

1. If $\mathbb{P} = \mathbb{NP}$ then the three classes in the (adjacent) figure collapse into one.
   - Proof: *Trivial*

2. If $\mathbb{P}$!= $\mathbb{NP}$ then $\mathbb{P}$ and $\mathbb{NPC}$ are disjoint
   - Proof: *If an $\mathbb{NP}$-complete problem can be solved in polynomial time all problems in $\mathbb{NP}$ can be solved so*.

- Current Belief (*based on abundant evidence*): $\mathbb{P}$!= $\mathbb{NP}$

Complexity Classes – Missing Links



Question:

Are there problems that belong to these regions marked in red ?