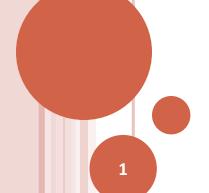
## CS F364 Design & Analysis of Algorithms

## **ALGORITHMS - COMPLEXITY**

## **Complexity Classes**

- NP-Completeness Via Reductions
- Reduction Techniques:
  - Component Design
    - Example: Vertex Cover



## PROBLEM: VERTEX-COVER

- Openition: Vertex Cover of a graph
  - Give an undirected graph G= (V,E), a subset S of V, is said to be a vertex cover of G
    - oif for each edge  $(u, v) \in E$ , either  $u \in S$  or  $v \in S$
- VERTEX-COVER:
  - Given an undirected graph G and a positive integer k, find whether there is a vertex cover of size at most k.
- VERTEX-COVER is №-complete:
  - Proof:
    - 1. VERTEX-COVER is in NP
    - 2. VERTEX-COVER is hard NP-hard

### VERTEX-COVER IS IN NP

- VERTEX-COVER is in NP
  - Proof:
    - oInput G=(V,E) and k
    - o Certificate: a set  $S \subseteq V$ , such that  $|S| \le k$
    - o Verification Algorithm O(|E|\*log|V|) time:
      - For each edge (u,v) in E, verify u or v is in S.

#### VERTEX-COVER IS NP-HARD

- Reduction:
  - Given a Boolean expression B in 3CNF (i.e. with exactly 3 literals in each clause), construct a graph G and an integer k as follows:
    - o For each of the variables  $x_i$  occurring in B, add to G:
      - two vertices labeled  $x_i$  and  $x_i$  and an edge  $(x_i, x_i)$
    - For each clause  $C_i = (I1 \lor I2 \lor I3)$  in B, add a triangle to G:
      - three new vertices, say  $v_{i1}$ ,  $v_{i2}$ , and  $v_{i3}$  and edges connecting them to each other.
    - o For each clause  $C_i = (I1 \lor I2 \lor I3)$  connect the variable and the triangle in G:
      - oi.e. add edges to G:  $(I1, v_{i1})$ ,  $(I2, v_{i2})$ , and  $(I3, v_{i3})$
    - oLet k = n+2m where n is # variables in B, and m is # clauses in B.

[2]

o 3SAT ≾ VERTEX-COVER

• Reduction: [contd...]

• Given the graph G and integer k constructed from B as above we claim:

- If there is a vertex cover, of size at most k, for G then it must be of size exactly k
- Construction of G and k takes polynomial time
- If there is a satisfying assignment for B, then there is a vertex cover of size k for G.
- If there is a vertex cover of size at most k for G, then there is a satisfying assignment for B.
- Summary claim:
  - o G has a vertex cover of size at most k iff B is satisfiable
  - i.e. there is a poly-time reduction from 3SAT to VERTEX-COVER

4/13/2016

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# REDUCTION TECHNIQUES - COMPONENT DESIGN

- The reduction used for hardness of VERTEX-COVER is an instance of a reduction technique called "Component Design":
  - Central Idea:
    - o Use constituents of the target problem to
      - design "components" that can be combined to "realize" instances of the known hard problem.

# REDUCTION TECHNIQUES – COMPONENT DESIGN [2]

- Component Design Example
  - Target problem:
    - VERTEX-COVER
  - Known hard problem:
    - o3SAT
  - Components:
    - Selection of vertices, testing each edge is covered
  - Realization:
    - oeach variable or its negative literal would be satisfied,
    - o at least one literal in each clause would be satisfied.

#### • Exercise:

• Identify the application of this technique in the hardness proof for CIRCUIT-SAT.