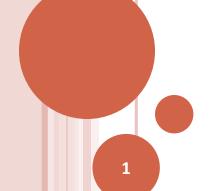
## CS F364 Design & Analysis of Algorithms

## **COMPLEXITY – OPTIMIZATION PROBLEMS**

### **Approximation Algorithms**

- Absolute Error and Absolute Approximation
- Example and Counter-example



## **APPROXIMATION ALGORITHMS**

- Given an optimization problem  $\pi$ , an algorithm A is said to be an approximation algorithm if it finds a feasible solution for any input instance
  - i.e. for any x in  $I_{\pi}$  A(x) is in  $F_{\pi}$  (x)
- Given an optimization problem  $\pi$ , for any input instance x and for any feasible solution y, **the absolute error** of y is defined as:
  - $D(x,y) = |m^*(x) m(x,y)|$
- Given an optimization problem  $\pi$ , an algorithm A is said to be an absolute approximation algorithm if there exists a constant k such that
  - i.e. for any x in  $I_{\pi}$  D(x, A(x)) <= k

# (Planar) Graph Coloring — Planar Graphs

- Euler's Theorem on Planar Graphs:
  - The smallest degree must be at most 5.
- Algorithm GC6P(G)
  - Find a vertex u with degree at most 5 // bound to exist
  - Remove u (and incident edges) to get G'
  - GC6P(G') // G' is planar
  - Choose a color for u that is different from all its neighbors.
- O Approximation Algorithm GCP(G) // G = (V,E)
  - If E is empty, then each vertex gets the same color
  - Else if G is bipartite color it with 2 colors.
  - Else GC6P(G)

## **APPROXIMATION ALGORITHMS**

- GCP is an absolute approximation algorithm because
  - For any planar graph G, the absolute error
  - $D(G, GC6P(G)) \le 6 3 = 3$
- Is it possible to get an absolute approximation algorithm for all NP-complete optimization problems?

## Non-Existence of Absolute Approximation Algorithms

#### • Theorem:

 Unless P=NP, no polynomial-time absolute approximation algorithm exists for 0,1 KNAPSACK

### • Proof:

- Given set of n items with profits p1,p2,...pn, and weights w1,w2,...wn and a bound B,
  - o assume that there is an absolute approximation algorithm with error bound k
- Create a new instance by multiplying all profits by k+1
  - o The optimal solution will be preserved (Why?)
  - o But the measure of any feasible solution will be a multiple of k+1
    - And the only feasible solution with absolute error bounded by k is the optimal solution.
- This is a contradiction (unless P=NP).