DAA Tutorial 9 Solution

1: (a) The verifier TM $V(x, u)$ will take input a graph $G = (V, E) = x$, and the certificate u is the 2-coloring of the vertices. $|u| = |V| = O(|x|)$. (1)

In $|V| + |E|$ time, the verifier TM $V(x, u)$ will check that all edges have vertices with different colors. If this is the case, then accept, otherwise reject. (1) Running time of $V(x, u) = |V| + |E| = O(|x|)$. (1)

$x \in 2COL \Rightarrow \exists$ a 2-coloring u of $G \Rightarrow V(x, u) = 1$ (1)

$x \notin 2COL \Rightarrow$ NO 2-coloring exists $\Rightarrow V(x, u) = 0 \;\forall u$ (1)

$\Rightarrow 2COL \in NP$.

(b) Same as (a) above, except that now the certificate u is a 3-coloring of vertices. (5)

(c) The verifier TM $V(x, u)$ will take input a graph $x = G = (V, E)$, and the certificate u

$= \{ (v_1 \rightsquigarrow v_2) \mid v_1 \sim v_2$ is a path from v_1 to v_2, $v_1, v_2 \in V$, $v_1 \neq v_2 \}$

$$|u| = O\left(\frac{|V|(|V|-1)}{2}|V|\right) = O(|V|^3) = O(|x|^3)$$ (1)

(2)

In $O(|x|^3)$ time, the verifier TM $V(x,u)$ will verify that there is a path between all pairs of vertices. If this is the case, then accept, otherwise reject (1) Running time of $V(x,u)$ $= O(|x|^3)$ (1) $x \in$ connected $\Rightarrow v_1 \sim v_2 \ \forall (v_1, v_2) \in V \times V \Rightarrow V(x,u) = 1$ for u encoding the paths (1)

$x \notin$ connected $\Rightarrow \exists (v_1, v_2)$ such that $v_1 \nsim v_2$
$\Rightarrow V(x,u) = 0 \quad \forall u$ (1)

(d) <u>2col $\in$ P</u>: use the BFS algorithm: choose any $s \in V$ and color it Red. Color all the neighbors of s blue. Color all the neighbors of "Blue" nodes Red and repeat the process in BFS order. At the end, if there is no conflict in coloring then accept, otherwise reject. Running of BFS algorithm is $O(|V| + |E|) = O(|x|)$. (2)

<u>connected $\in$ P</u>: use either the BFS or the DFS algorithm. If all the vertices are visited then accept, otherwise reject. Complexity $= O(|V| + |E|) = O(|x|)$ (2)

<u>3col $\in$ NP-complete</u>, therefore, under the assumption of $P \neq NP$, 3col $\notin$ P. (1)

5: (a) Linear time reduction f_1 for $L_1 \leq_p L_1 \oplus L_2$ (3)

is given by $f_1(w) = w0$. (2)

$w \in L_1 \Leftrightarrow w0 \in L_1\{0\} \Leftrightarrow f_1(w) \in L_1 \oplus L_2$ (2)

because f_1 will match $w \in L_1$ with $w0 \in L_1\{0\}$. (1)

Linear time reduction f_2 for $L_2 \leq_p L_1 \oplus L_2$ is given by $f_2(w) = w1$ (2)

$w \in L_2 \Leftrightarrow w1 \in L_2\{1\} \Leftrightarrow f_2(w) \in L_1 \oplus L_2$ (2) because f_2 will match $w \in L_2$ with $w1 \in L_2\{1\}$. (1)

(b) Let the polynomial time reduction for $L_1 \leq_p L_2$ be g_1, and let the polynomial time reduction for $L_2 \leq_p L$ be g_2. Then the polynomial time reduction g_3 for $L_1 \oplus L_2 \leq_p L$ is given by:

$g_3(w0) = g_1(w)$

and $g_3(w1) = g_2(w)$. (4)

$w0 \in L_1 \oplus L_2 \Leftrightarrow w0 \in L_1\{0\} \Leftrightarrow w \in L_1 \Leftrightarrow g_1(w) \in L$ (3)

$w1 \in L_1 \oplus L_2 \Leftrightarrow w1 \in L_2\{1\} \Leftrightarrow w \in L_2 \Leftrightarrow g_2(w) \in L$. (3)