DAA Tutorial 11 Solution

3: **Algo:** Sort the items in increasing order in $O(n \log n)$ time. Find the maximum number $\le B$. Let it be a_{i^*}. Find k such that $\sum_{i=1}^{k} a_i \le B$ and $\sum_{i=1}^{k+1} a_i > B$. Output maximum of $\sum_{i=1}^{k} a_i$ or a_{i^*} (assuming $a_1 \le a_2 \le \cdots \le a_n$ after sorting) [5]

A good upper bound for OPT is:

$$OPT \le \sum_{i=1}^{k} a_i + a_{k+1} \longrightarrow (1) \quad (5)$$

Consider 2 cases:

Case (1): a_{i^*} is the solution $\Rightarrow a_{i^*} \ge \sum_{i=1}^{k} a_i \longrightarrow (2)$

$$\Rightarrow \frac{OPT}{a_{i^*}} \le \frac{\sum_{i=1}^{k} a_i}{a_{i^*}} + \frac{a_{k+1}}{a_{i^*}} \le 1 + \frac{\sum_{i=1}^{k} a_i}{a_{i^*}} \le 2 \quad (5)$$

From (1), (2) and $a_{i^*} \ge a_j \ \forall j \in [1..n]$

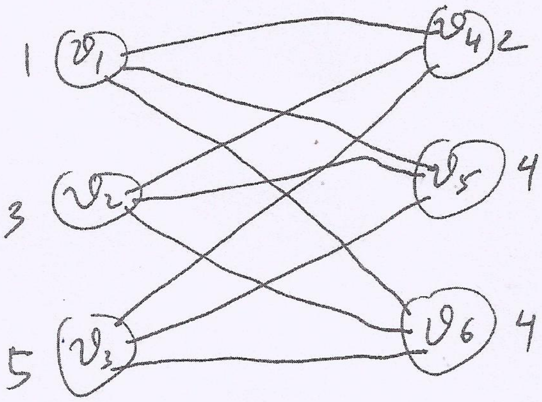
Case (2): $\sum_{i=1}^{k} a_i$ is the solution $\Rightarrow a_{i^*} \le \sum_{i=1}^{k} a_i \longrightarrow (3)$

$$\Rightarrow \frac{OPT}{\sum_{i=1}^{k} a_i} \le \frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} a_i} + \frac{a_{k+1}}{\sum_{i=1}^{k} a_i} \le 1 + \frac{a_{i^*}}{\sum_{i=1}^{k} a_i} \le 2 \quad [5]$$

From (1), (3) and $a_{i^*} \ge a_j \ \forall j \in [1..n]$.

$\Rightarrow$ approx ratio of our Algo is 2.

7[redacted]: The weighted Vertex Cover Problem is: 2a



(a) $E = \{e_{14}, e_{15}, e_{16}, e_{24}, e_{25}, e_{26}, e_{34}, e_{35}, e_{36}\}$ [redacted]

$S_1 = \{e_{14}, e_{15}, e_{16}\}, \; w(S_1) = 1$ $\longleftrightarrow x_1$

$S_2 = \{e_{24}, e_{25}, e_{26}\}, \; w(S_2) = 3$ $\longleftrightarrow x_2$

$S_3 = \{e_{34}, e_{35}, e_{36}\}, \; w(S_3) = 5$ $\longleftrightarrow x_3$

$S_4 = \{e_{14}, e_{24}, e_{34}\}, \; w(S_4) = 2$ $\longleftrightarrow x_4$

$S_5 = \{e_{15}, e_{25}, e_{35}\}, \; w(S_5) = 4$ $\longleftrightarrow x_5$

$S_6 = \{e_{16}, e_{26}, e_{36}\}, \; w(S_6) = 4$ $\longleftrightarrow x_6$

(2)

(3)

(b) ILP Formulation:

Minimize: $x_1 + 3x_2 + 5x_3 + 2x_4 + 4x_5 + 4x_6$

Subject to:

$x_1 + x_4 \geq 1 \quad \longleftrightarrow \quad e_{14} \times y_1$

$x_1 + x_5 \geq 1 \quad \longleftrightarrow \quad e_{15} \times y_2$

$x_1 + x_6 \geq 1 \quad \longleftrightarrow \quad e_{16} \times y_3$

$x_2 + x_4 \geq 1 \quad \longleftrightarrow \quad e_{24} \times y_4$

$x_2 + x_5 \geq 1 \quad \longleftrightarrow \quad e_{25} \times y_5$

$x_2 + x_6 \geq 1 \quad \longleftrightarrow \quad e_{26} \times y_6$

$x_3 + x_4 \geq 1 \quad \longleftrightarrow \quad e_{34} \times y_7$

$x_3 + x_5 \geq 1 \quad \longleftrightarrow \quad e_{35} \times y_8$

$x_3 + x_6 \geq 1 \quad \longleftrightarrow \quad e_{36} \times y_9$

$x_1, x_2, x_3, x_4, x_5, x_6 \in \{0, 1\}$ (2)

LP Relaxation:

Just change the last constraint to:

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (2)

Dual LP: Multiplying the constraints by corresponding y_i ($1 \leq i \leq 9$, $y_i \geq 0$) and adding:

$$(y_1+y_2+y_3)x_1 + (y_4+y_5+y_6)x_2 + (y_7+y_8+y_9)x_3 +$$
$$(y_1+y_4+y_7)x_4 + (y_2+y_5+y_8)x_5 + (y_3+y_6+y_9)x_6$$
$$\geq y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9$$

We have to find best possible lower bound for Primal LP $\Rightarrow$

Dual LP:

Maximize: $y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9$

subject to:

$$y_1+y_2+y_3 \leq 1 \longrightarrow (1) \longleftrightarrow S_1$$
$$y_4+y_5+y_6 \leq 3 \longrightarrow (2) \longleftrightarrow S_2$$
$$y_7+y_8+y_9 \leq 5 \longrightarrow (3) \longleftrightarrow S_3$$
$$y_1+y_4+y_7 \leq 2 \longrightarrow (4) \longleftrightarrow S_4$$
$$y_2+y_5+y_8 \leq 4 \longrightarrow (5) \longleftrightarrow S_5$$
$$y_3+y_6+y_9 \leq 4 \longrightarrow (6) \longleftrightarrow S_6$$
$$y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9 \geq 0$$

(2)

Primal-Dual Algorithm: We start with the initial dual feasible solution $y_1=y_2=y_3=y_4=y_5=y_6=y_7=y_8=y_9=0$.

Step 1: e_{14} is not covered $\Rightarrow y_1 = 1$

$\Rightarrow$ (1) is tight $\Rightarrow$ we take the subset $\boxed{S_1}$

$\Rightarrow$ e_{14}, e_{15}, e_{16} are covered

$\Rightarrow$ $(1,0,0,0,0,0,0,0,0)$

[redacted] 5

Step 2: e_{24} is not covered $\Rightarrow y_4 = 1$

$\Rightarrow$ (4) is tight $\Rightarrow$ we take the subset $\boxed{S_4}$

$\Rightarrow e_{14}, e_{15}, e_{16}, e_{24}, e_{34}$ are covered

$\Rightarrow (1, 0, 0, 1, 0, 0, 0, 0, 0)$

Step 3: e_{25} is not covered $\Rightarrow y_5 = 2$

$\Rightarrow$ (2) is tight $\Rightarrow$ we take the subset $\boxed{S_2}$

$\Rightarrow e_{14}, e_{15}, e_{16}, e_{24}, e_{34}, e_{25}, e_{26}$ are covered.

$\Rightarrow (1, 0, 0, 1, 2, 0, 0, 0, 0)$

Step 4: e_{35} is not covered $\Rightarrow y_8 = 2$

$\Rightarrow$ (5) is tight $\Rightarrow$ we take the subset $\boxed{S_5}$

$\Rightarrow e_{14}, e_{15}, e_{16}, e_{24}, e_{34}, e_{25}, e_{26}, e_{35}$ are covered

$\Rightarrow (1, 0, 0, 1, 2, 0, 0, 2, 0)$

Step 5: e_{36} is not covered $\Rightarrow y_9 = 3$

$\Rightarrow$ (3) is tight $\Rightarrow$ we take the subset $\boxed{S_3}$

$\Rightarrow e_{14}, e_{15}, e_{16}, e_{24}, e_{34}, e_{25}, e_{26}, e_{35}, e_{36}$ are covered

$\Rightarrow (1, 0, 0, 1, 2, 0, 0, 2, 3)$

All elements are covered $\Rightarrow$ Set Cover $= \boxed{\{S_1, S_2, S_3, S_4, S_5\}}$

Covering weight $= 1+3+5+2+4 = \boxed{15}$ (10)

(c) Corresponding Vertex Cover

$= \boxed{\{v_1, v_2, v_3, v_4, v_5\}}$ [redacted]

(2)