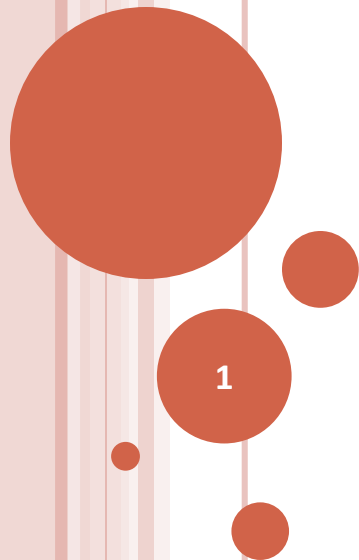


COMPLEXITY – OPTIMIZATION PROBLEMS

Approximation Algorithms

- Relative Approximation
- Example: Bin-Packing



RELATIVE APPROXIMATION

- Given an optimization problem π , for any input instance x and for any feasible solution y , the performance ratio of y is defined as:
 - $R(x,y) = \max(m(x,y)/m^*(x), m^*(x)/m(x,y))$
- Given an optimization problem π , an algorithm A is said to be an r -approximate algorithm if there exists a constant r such that
 - i.e. for any x in I_π $R(x,A(x)) \leq r$
- Example:
 - Greedy_Vertex_Cover is a 2-approximation algorithm

RELATIVE APPROXIMATION – EXAMPLE – BIN PACKING

○ Problem Definition: BIN PACKING

- Given a set of N items each with values S_1, S_2, \dots, S_n , distribute them into equal-sized bins such that the number of bins required is minimum.

○ Application:

- Memory allocation problem (e.g.):
 - Memory is available as fixed-size blocks (i.e. bins)
 - Memory requests come in different sizes

○ Note:

- As the bins are equal-sized, we can assume that they are of unit size and scale the values S_1, S_2, \dots, S_n accordingly.

RELATIVE APPROXIMATION — EXAMPLE — BIN PACKING [2]

○ Formal Problem Definition: BIN PACKING

- $I = \{ S \mid S \text{ is a finite multi-set of } n \text{ rational numbers in } (0,1] \}$
- $F(S) = \text{a partition } \{ B_1, B_2, \dots, B_k \} \text{ of } S$
s.t. $\sum_{a \in B_j} a \leq 1$ for each j
- $m(S, \{B_1, B_2, \dots, B_k\}) = k$
- goal = min

○ Definition: Partition

- $\{ B_1, B_2, \dots, B_k \}$ is said to be a partition of S if
- $\cup B_j = S$ and $\cap B_j = \{\}$

RELATIVE APPROXIMATION – EXAMPLE – BIN PACKING [3]

- Lower Bound (on the optimal solution):
 - Given input instance S , let $A = \sum_{a \in S} a$
 - Claim:
 - $m^*(x) \geq \text{ceil}(A)$
 - Proof:
 - Trivial (Perfect packing)
- Algorithm Next_Fit (S)
 1. $i=0$;
 2. for each item a in S :
 - if $((a + \sum_{b \in B_i} b) \leq 1)$ { assign a to B_i }
 - else { assign a to B_{i+1} ; $i = i+1$; }

RELATIVE APPROXIMATION – EXAMPLE – BIN PACKING [5]

○ Comments:

- The technique used by NEXT_FIT is referred to as *Sequencing*.
- NEXT_FIT is an online algorithm.

○ Theorem:

- NEXT_FIT is a polynomial time 2-approximate algorithm for BIN PACKING.
- Proof:
 - For each pair of consecutive bins
 - the sum of values assigned to these two bins is > 1
 - i.e. $\# \text{ bins used} / 2 \leq A$ where $A = \sum_{a \in S} a$
 - $m_{\text{NEXT_FIT}}(S) \leq 2 * \text{ceil}(A) \leq 2 * m^*(S)$

RELATIVE APPROXIMATION – EXAMPLE – BIN PACKING [4]

○ Claim:

- The approximation ratio of 2 is asymptotically tight for NEXT_FIT
- Proof:
 - Given any integer n , consider the instance with $4n$ items
$$S = \{ 1/2, 1/2n, 1/2, 1/2n, \dots, 1/2, 1/2n \}$$
 - Optimal solution would require: $n+1$ bins
 - NEXT_FIT would require: $2*n$ bins

○ Question:

- What is the weakness of NEXT_FIT ?