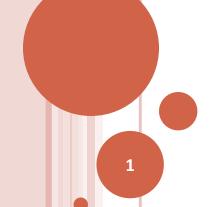
CS F364 Design & Analysis of Algorithms

Analysis - Problems - Reductions

Analysis of Problems

- Lower Bounds and Reduction: Example



PROBLEM - CONVEX HULL

- The Convex Hull problem in two dimensions is defined as follows:
 - Given a set S of points $\langle p_1, p_2, ... p_m \rangle$ in 2-d space, find the smallest convex polygon that encloses all p_i in S.
 - A polygon encloses a point if the point is inside or on (one of) the edges of the polygon

REDUCTION - EXAMPLE

- Example:
- Understanding CONVEX_HULL:
 - CONVEX_HULL requires points in 2-D space as input.
 - The output of CONVEX_HULL is a list of vertices of a polygon

SORT ≾_{O(M)} CONVEX_HULL

- o Intuition:
 - CONVEX_HULL requires points in 2-D space as input
 - oi.e. we need a mapping from points in 1-D space to 2-D space:
 - owe need a function f so that we can take the input to SORT a list of values $\langle v_1, v_2, ..., v_n \rangle$ and map it to a list of the form $\langle (v_1, f(v_1)), (v_2, f(v_2)), ..., (v_n, f(v_n)) \rangle$
 - The output of CONVEX_HULL is a list of vertices of a polygon

SORT ≾_{O(M)} CONVEX_HULL

- o Intuition:
 - CONVEX_HULL requires points in 2-D space as input
 - The output of CONVEX_HULL is a list of vertices of a polygon
 - i.e. we need to ensure that
 - othe output contains all the original points
 - and points can be extracted in order without too much additional cost

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$SORT \preceq_{O(M)} CONVEX_HULL$

- This forms a sorting algorithm:
 - Map the input list of points $L = \langle v_1, v_2, ..., v_n \rangle$ to $L' = \langle (v_1, v_1^2), (v_2, v_2^2), ..., (v_n, v_n^2) \rangle$
 - Apply a CONVEX_HULL algorithm on L' to get a permutation P of these points (as vertices of the polygon):
 - find p_i in P such that if $p_i = (v, v^2)$, then v is the minimum in L
 - Then output the points in the order p_i , $p_{(i+1)\%n}$, ... $p_{(i+n-1)\%n}$
- Exercise:
 - Argue that the output is in sorted order.