CS F364
Design & Analysis of Algorithms

COMPLEXITY – OPTIMIZATION PROBLEMS

Intractable Problems

- Optimization Problems - Examples
- Complexity Classes **PO** and **NPO**

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Optimization Problems - Characterization

- Formal Description:
  - An optimization problem π is characterized by the quadruple (I_π, F_π, m_π, $goal_\pi$)
    - I_π = { x | x is an input instance of π }
    - $F_\pi(x)$ = { s | s is a feasible solution for x, where $x \in I_\pi$ }
    - $m_\pi(x, y)$ = v where v is a quantitative measure of the "value" of the feasible solution $y \in F_\pi(x)$ for $x \in I_\pi$
    - $goal_\pi \in$ { min, max }

Optimization Problems - Examples

- Min Vertex Cover
  - I = {G| G is an undirected graph}
  - F(G) = { S | S ⊆ V s.t. for any (u,w) in E:
    u in S or v in S, where G = (U,V)}
  - m(G, S) = |S| where G in I and S in F(G)
  - goal: min
- TSP
  - I = {G | G is a weighted, completely-connected graph}
  - F(G) = { (u1, u2, u3, … un, u1) | G=(V,E,w), n = |V|,
    ui in V for any i in 1..n,
    ui <> uj for any i and j in 1…n }
  - m(G, P) = $\Sigma_{e \text{ in } P}$ w(e) where G = (V,E,w), P in F(G)
  - goal: min

OPTIMIZATION PROBLEMS

- Optimal Solution:
  - The optimal solution for a given instance x of a problem π is characterized by:
    - The optimal measure:
      - $m_{\pi}{}^{*}(x) = goal_{\pi}\{m_{\pi}(x, y) \mid y \text{ in } F_{\pi}(x)\}$
  - The optimal solution is characterized thus:
    - $OPT_{\pi}(x) = y$ where y in $F_{\pi}(x)$ and $m_{\pi}{}^{*}(x) = m_{\pi}(x, y)$
- Note:
  - The objective – for a given context – may be to find
    - The optimal solution (***Constructive Version***) OR
    - The optimal measure i.e. the measure of the optimal solution (***Evaluative Version***)

Classes PO and NPO

- PO
  - An optimization problem π is in PO if there exists an algorithm that solves π in polynomial time
- NPO:
  - An optimization problem π = (I, F, m, goal) belongs to the class NPO if
    - membership I is *decidable in polynomial time*
    - there exists a polynomial q such that
      - given x in I and for any y in F(x):
      - |y| <= q(|x|) and
      - y in F(x) is *decidable in polynomial time*.
    - m is *computable in poly time*.
  - Example:
    - Vertex Cover is in NPO