CS F364
Design & Analysis of Algorithms

Algorithm Design - Approximation

Approximation Algorithms

- Approximation Class APX
- Limits of Relative Approximation and NPO - APX

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CLASS APX

- APX is the class of all NPO problems π such that,
  - for some constant r > 1, there exists a polynomial-time r-approximation algorithm for π
- Examples:
  - Bin Packing, Planar Graph Coloring, Cardinality Vertex Cover, Metric TSP are all in APX
    - We have seen polynomial time r-approximate algorithms for each of these.
- TSP is not in APX

GRAPH COLORING

- Consider Graph Coloring as an example:
  1. ***3-coloring of planar graphs*** is NP-complete.
  2. But every planar graph can be colored with at most 4 colors.
- Thus, no ***r-approximate algorithm exists for planar graph coloring*** **for r<4/3** unless P=NP.
  - As a generalization no r-approximate algorithm exists exists for graph coloring, for r<4/3 unless P=NP.
- *As it turns out*
  - (General) Graph Coloring, like TSP, is in NPO – APX

Relative Approximation

- For a given $\mathbb{NP}$-complete problem, what is the best approximation ration obtainable?
  - In practice, even 2 may not be a “good” ratio:
    - an approximation ratio of 2 implies that the solution could be 100% worse than the optimal solution
  - So, can we obtain better than factor of 2 solutions?
    - In particular, can we find algorithms with an approximation ratio of $1+\epsilon$ where ϵ can be arbitrarily small?
    - Note that ϵ must be positive:
      - $\epsilon = 0$ would imply an exact solution for a $\mathbb{NP}$ hard problem.