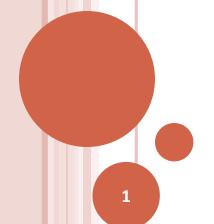
CS F364 Design & Analysis of Algorithms

ALGORITHM DESIGN - APPROXIMATION

Approximation Algorithms

- Approximation Class **APX**
- Limits of Relative Approximation and NPO APX



CLASS APX

- \bullet APX is the class of all NPO problems π such that,
 - for some constant r > 1, there exists a polynomial-time r-approximation algorithm for π
- Examples:
 - Bin Packing, Planar Graph Coloring, Cardinality Vertex Cover, Metric TSP are all in APX
 - o We have seen polynomial time r-approximate algorithms for each of these.
- o TSP is not in ♠₽X

GRAPH COLORING

- Consider Graph Coloring as an example:
 - 1. 3-coloring of planar graphs is NP-complete.
 - 2. But every planar graph can be colored with at most 4 colors.
- Thus, no r-approximate algorithm exists for planar graph coloring for r<4/3 unless P=NP.
 - As a generalization no r-approximate algorithm exists exists for graph coloring, for r<4/3 unless P=NP.
- As it turns out
 - (General) Graph Coloring, like TSP, is in №0 №PX

RELATIVE APPROXIMATION

- For a given NP-complete problem, what is the best approximation ration obtainable?
 - In practice, even 2 may not be a "good" ratio:
 - o an approximation ratio of 2 implies that the solution could be 100% worse than the optimal solution
 - So, can we obtain better than factor of 2 solutions?
 - o In particular, can we find algorithms with an approximation ratio of $1+\epsilon$ where ϵ can be arbitrarily small?
 - o Note that ϵ must be positive:
 - $\circ \epsilon = 0$ would imply an exact solution for a NP hard problem.