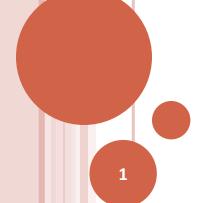
CS F364 Design & Analysis of Algorithms

ALGORITHMS - COMPLEXITY

Complexity Classes

- NP-Completeness Via Reductions
- Reduction Techniques:
 - Component Design
 - Example: Vertex Cover



PROBLEM: VERTEX-COVER

- Openition: Vertex Cover of a graph
 - Give an undirected graph G= (V,E), a subset S of V, is said to be a vertex cover of G
 - oif for each edge $(u, v) \in E$, either $u \in S$ or $v \in S$
- VERTEX-COVER:
 - Given an undirected graph G and a positive integer k, find whether there is a vertex cover of size at most k.
- VERTEX-COVER is №-complete:
 - Proof:
 - 1. VERTEX-COVER is in NP
 - 2. VERTEX-COVER is hard NP-hard

VERTEX-COVER IS IN NP

- VERTEX-COVER is in NP
 - Proof:
 - oInput G=(V,E) and k
 - o Certificate: a set $S \subseteq V$, such that $|S| \le k$
 - o Verification Algorithm O(|E|*log|V|) time:
 - For each edge (u,v) in E, verify u or v is in S.

VERTEX-COVER IS NP-HARD

- Reduction:
 - Given a Boolean expression B in 3CNF (i.e. with exactly 3 literals in each clause), construct a graph G and an integer k as follows:
 - o For each of the variables x_i occurring in B, add to G:
 - two vertices labeled x_i and x_i and an edge (x_i, x_i)
 - For each clause $C_i = (I1 \lor I2 \lor I3)$ in B, add a triangle to G:
 - three new vertices, say v_{i1} , v_{i2} , and v_{i3} and edges connecting them to each other.
 - o For each clause $C_i = (I1 \lor I2 \lor I3)$ connect the variable and the triangle in G:
 - oi.e. add edges to G: $(I1, v_{i1})$, $(I2, v_{i2})$, and $(I3, v_{i3})$
 - oLet k = n+2m where n is # variables in B, and m is # clauses in B.

[2]

o 3SAT ≾ VERTEX-COVER

• Reduction: [contd...]

• Given the graph G and integer k constructed from B as above we claim:

- If there is a vertex cover, of size at most k, for G then it must be of size exactly k
- Construction of G and k takes polynomial time
- If there is a satisfying assignment for B, then there is a vertex cover of size k for G.
- If there is a vertex cover of size at most k for G, then there is a satisfying assignment for B.
- Summary claim:
 - o G has a vertex cover of size at most k iff B is satisfiable
 - i.e. there is a poly-time reduction from 3SAT to VERTEX-COVER

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REDUCTION TECHNIQUES - COMPONENT DESIGN

- The reduction used for hardness of VERTEX-COVER is an instance of a reduction technique called "Component Design":
 - Central Idea:
 - o Use constituents of the target problem to
 - design "components" that can be combined to "realize" instances of the known hard problem.

REDUCTION TECHNIQUES – COMPONENT DESIGN [2]

- Component Design Example
 - Target problem:
 - VERTEX-COVER
 - Known hard problem:
 - o3SAT
 - Components:
 - Selection of vertices, testing each edge is covered
 - Realization:
 - oeach variable or its negative literal would be satisfied,
 - o at least one literal in each clause would be satisfied.

• Exercise:

• Identify the application of this technique in the hardness proof for CIRCUIT-SAT.