CS F364: Design & Analysis of Algorithm

08 Matroids and Greedy method

Dr. Kamlesh Tiwari
Assistant Professor, Department of CSIS,
BITS Pilani, Pilani Campus, Rajasthan-333031 INDIA

Feb 05, 2021 ONLINE (Campus @ BITS-Pilani Jan-May 2021)

http://ktiwari.in/algo

Matroids

Theory for some situations in which the greedy yields optimal solutions

- **Matroids:** ordered pair $M = (S, I)$ satisfying the following
  1. S is a finite set
  2. **Hereditary property:** I is a nonempty family of subsets of S, called the independent subsets of S, such that $B \in I$ and $A \subseteq B$, then $A \in I$. (Question: is ϕ a member of I? Y)
  3. **Exchange property:** If $A \in I$, $B \in I$, and $|A| < |B|$ then there exists some element $x \in B - A$ such that $A \cup \{x\} \in I$.

Graphic Matroid $M_G = (S_G, I_G)$

Defined in terms of a given undirected graph $G = (V, E)$
1. $S = E$
2. If A is a subset of E, then $A \in I_G$ if and only of A is acyclic. So (V, A) is a forest

For $G = (V, E)$ a undirected graph $M_G = (S_G, I_G)$ is a matroid

Proof:
- S_G is finite
- Hereditary property: subset of a forest is a forest
- Exchange property:
  Number of trees in a forest (V, E_f) is $|V| - |E_f|$
  For $A, B \in I$ if $|A| < |B|$ then B has fewer trees
  $\rightarrow$ Consider and edge $x \in B$ that links two trees in A

Extention: x extends A
Maximal independent set: set that can not be extended

All maximal independent set have same size

Suppose to the contrary that A is a maximal independent subset of M and there exists another larger maximal independent subset B of M

- Then due to exchange property $\exists x \in B - A$ so that A could be extended
- so A is not maximal independent set
- **Contradiction.**

Weighted matroid

- $M = (S, I)$ is weighted if it is associated with weight function $w(x)$ for all $x \in S$
- $w(A)$ is defined as
$$w(A) = \sum_{x \in A} w(x)$$
- One example of w be the weight of the edge
- $w'(e) = w_0 - w(e)$

Minimum Spanning Tree Problem

- Subset of the edges that connects all of the vertices together and has minimum total length
- It is like finding maximal independent set in M_G

Algorithm 1: Greedy(M, w)

```
A = φ
sort M.S in decreasing order of weight w
for x ∈ M.s take in order do
    if A ∪ {x} ∈ M.I then
        A = A ∪ {x}
return A
```

Thank You!

Thank you very much for your attention! (Reference[1])

Queries ?

[1] [1] Book - *Introduction to Algorithm*, By THOMAS H. CORMEN, CHARLES E. LEISERSON, RONALD L. RIVEST, CLIFFORD STEIN