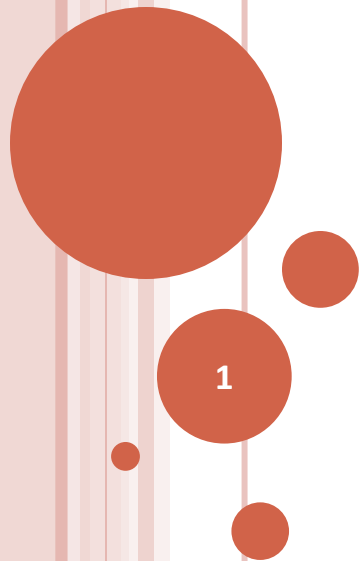


CS F364

Design & Analysis of Algorithms

## ALGORITHM DESIGN TECHNIQUES - GREEDY

### Greedy Algorithms - Example: (Fractional) Knapsack



# PROBLEM – (FRACTIONAL) KNAPSACK

- A thief wants to rob the grocery store:
  - Has a sack with max. capacity by weight:  $W$  kg.
  - Each item  $j$  (in store) is labeled with
    - Package Size :  $w_j$  kg and Price (of the package) : Rs.  $p_j$
- Assumption:
  - Any item can be taken in fractional quantity
  - All values ( $w_j$ ,  $p_j$ , and  $W$ ) are positive.
- Feasible Solution:
  - Fill the sack with maximum value (by price)
- Goal:
  - Maximize  $\sum p_i (x_i / w_i)$   
where  $0 \leq x_i \leq w_i$  for each  $i$  and  $\sum x_i \leq W$   
if  $x_i$  is the amount taken of item  $i$

# KNAPSACK – GREEDY ALGORITHM

- Algorithm KnapSack(S, W)
- // S Set of items; W capacity

Sort S by key  $v_j = p_j / w_j$

Initialize array X of size |S| with all 0s.

remW = W

while (remW > 0) {

    i = findMax(S);

    S = deleteMax(S,i)

    X[i] = min( $w_i$  , remW);

    remW = remW – X[i];

}

output X

# KNAPSACK – GREEDY CHOICE

- Knapsack satisfies Greedy Choice property:
  - Suppose there are items  $j$  and  $k$  such that
    - $x_k < w_k$ ,  $x_j > 0$ , and  $v_k < v_j$
  - Let
    - $y = \min ( w_k - x_k, x_j )$
  - Then
    - replace an amount  $y$  of item  $j$ , with same amount of item  $k$
    - and increase the value without increasing the weight!

# KNAPSACK – GREEDY ALGORITHM – TIME COMPLEXITY

○ Algorithm KnapSack(S, W) //S - list of items; W - capacity

1. Order S by key  $v_j = p_j / w_j$  ←  $O(n)$  where  $n = |S|$
2. Initialize array X of size  $|S|$  with all 0s. ←  $O(n)$
3.  $remW = W$
4. while ( $remW > 0$ ) {
5.    $i = \text{findMax}(S);$
6.    $S = \text{deleteMax}(S, i)$  ←  $O(\log(n))$
7.    $X[i] = \min(w_i, remW);$
8.    $remW = remW - X[i];$
9. } ←  $O(n)$

Assuming a  
Heap is used  
for storing  
keys ordered  
by unit price

**Time Complexity –  $O(n \cdot \log(n))$**

Question: Will there be an impact on performance if a sorted array is used for S instead of a heap?