

CS F364

Design & Analysis of Algorithms

23-02-2015

DYNAMIC PROGRAMMING

Graph Problems:

- **Transitive Closure**

GRAPHS – TRANSITIVE CLOSURE

- A graph $G = (V, E)$ captures a binary relation R on a set S :
 - i.e. $V = S$ and $E = R \subseteq S \times S$
 - i.e. the edge relation of G models R
- The transitive closure of R , denoted R^* is defined as:
 - $x R^* x$ for any x in S
 - $x R y \implies x R^* y$ for any x and y
 - $x R y$ and $y R z \implies x R^* z$ for any x, y , and z
- i.e. the path relation in G models R^*

GRAPHS – TRANSITIVE CLOSURE

- Question:

- How similar is this problem to All Pairs Shortest Paths (APSP)?
- Note :
 - This is not an optimization problem!

GRAPHS – TRANSITIVE CLOSURE

○ Exercise:

- Rewrite the recurrence for APSP to suit this problem:

$D[0,i,j]$

$= 0$
 $= w(v_i, v_j)$
 $= \text{INFINITY}$

if $i=j$;

if there is an edge (v_i, v_j) in E ;
otherwise

What will these
values be?

$D[k,i,j] = \min(D[k-1,i,j], D[k-1,i,k] + D[k-1,k,j])$

for $k > 0$

What will these
operators be?