CS F364
Design & Analysis of Algorithms

ALGORITHMS - COMPLEXITY

Complexity Classes

- **Nondeterministic Time Complexity Classes**
- **Class NP**

COMPLEXITY CLASSES - NTIME

- Recall

  TIME(f(n)) = { π | ∃ algorithm A: A solves π in O(f(n)) time}

- Define

  NTIME(f(n)) =

  { π | ∃ non-deterministic algorithm A: A solves π in O(f(n)) time}

- Alternatively,

  NTIME(f(n)) =

  { π | ∃ algorithm A: A verifies a certificate for π in O(f(n)) time}

- Given π ∈ NTIME(f(n)),
  - What can you infer about a certificate for π ?

Complexity Classes - $\mathbb{NP}$

- Recall $\mathbb{P}$ = { π | π is a decision problem that can be solved by a polynomial time algorithm }
  - i.e. $\mathbb{P} = U_{k \in \mathbb{N}}$ TIME(n^k)

- Define $\mathbb{NP}$ = {π | π is a decision problem that can be solved by a non-deterministic polynomial time algorithm }
  - i.e. $\mathbb{NP}$ = {π | π is a decision problem for which a certificate can be verified by a polynomial time algorithm}
  - i.e. $\mathbb{NP} = U_{k \in \mathbb{N}}$ NTIME(n^k)

Complexity Classes – $\mathbb{NP}$ – Example Problems

- Argue that the following problem is in $\mathbb{NP}$.

  i.e.

  - discover / define a certificate and calculate its length,
  - propose an algorithm for verifying the certificate, and
  - argue that the algorithm executes in worst case polynomial time.

  COMPOSITES:

  - Is a given positive integer composite (i.e. not prime)?

Complexity Classes – $\mathbb{NP}$ – Example Problems

- Argue that the following problems are in $\mathbb{NP}$.
  - Graph Isomorphism (**ISO)**:
    - Given graphs G1=(V1,E1) and G2=(V2,E2), is there a 1-1 onto function f from V1 to V2 such that there is an edge (u11, u12) in E1 if and only if there is an edge (f(u1),f(u2)) in E2.
  - Traveling Sales Person **(TSP)**:
    - Given a set of cities and distances of roads connecting each other, is there a tour starting and ending in the same city but visiting all other cities exactly once such that the total distance covered is <= K?
  - **0,1 Knapsack**