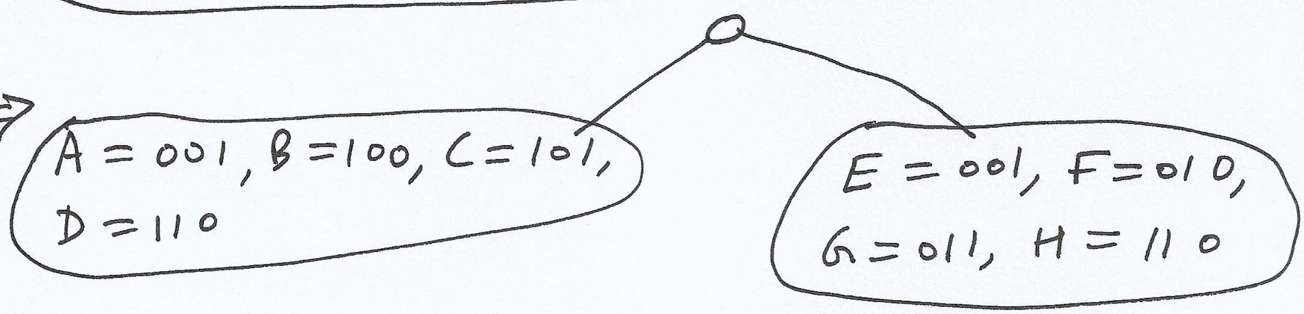
DAA Tutorial 4 Solution

2: (a) A = 0000, B = 0001, C = 001,
D = 01, E = 100, F = 101, G = 110, H = 111 (4)

(b) A = 0001, B = 0100, C = 0101, D = 0110, E = 1001, F = 1010,
G = 1011, H = 1110

⇒



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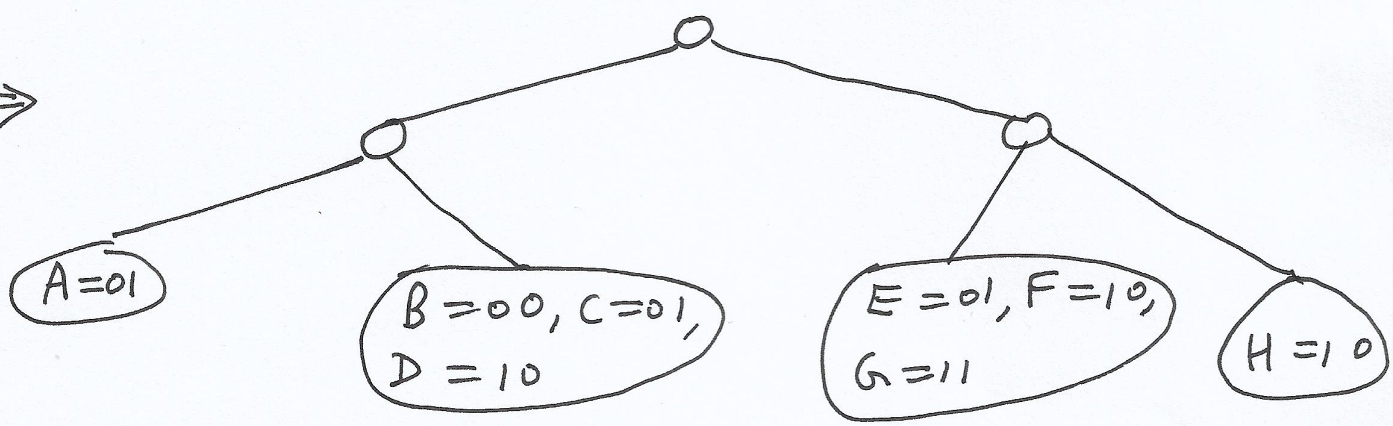

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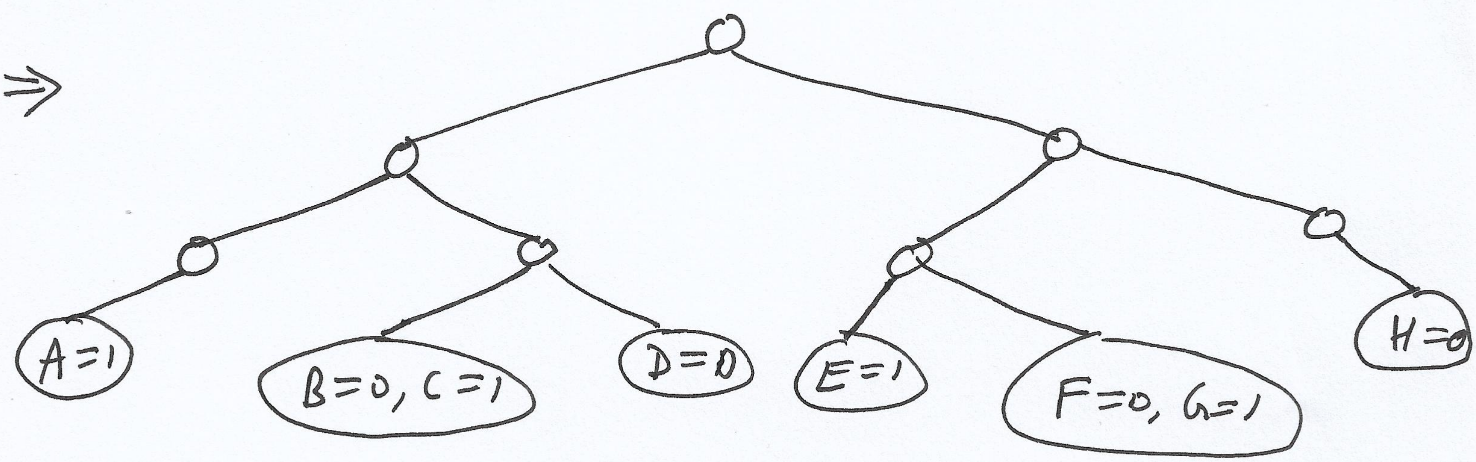
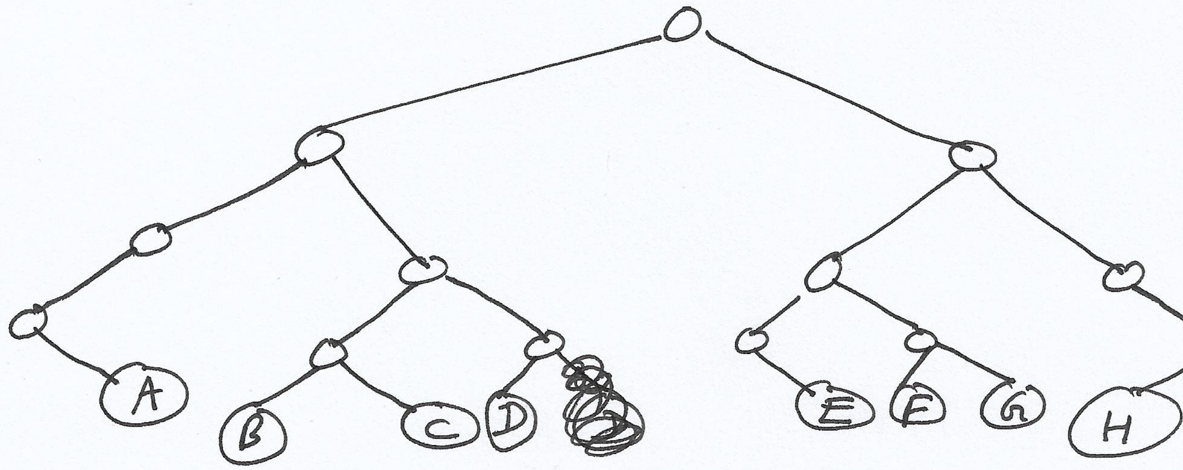


$\Rightarrow$



[4]

c $\left(f_H = \frac{21}{54}, f_G = \frac{13}{54}, f_F = \frac{8}{54}, f_E = \frac{5}{54}, f_D = \frac{3}{54}, f_C = \frac{2}{54}, f_B = \frac{1}{54}, f_A = \frac{1}{54}\right) \Rightarrow$

$\left(f_H = \frac{21}{54}, f_G = \frac{13}{54}, f_F = \frac{8}{54}, f_E = \frac{5}{54}, f_D = \frac{3}{54}, f_C = \frac{2}{54}, f_{BA} = \frac{2}{54}\right) \Rightarrow$

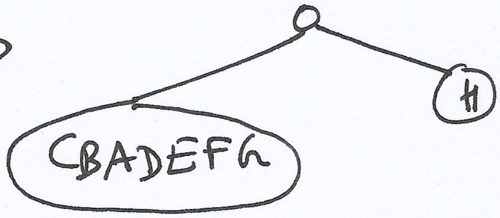
$\left(f_H = \frac{21}{54}, f_G = \frac{13}{54}, f_F = \frac{8}{54}, f_E = \frac{5}{54}, f_{CBA} = \frac{4}{54}, f_D = \frac{3}{54}\right)$

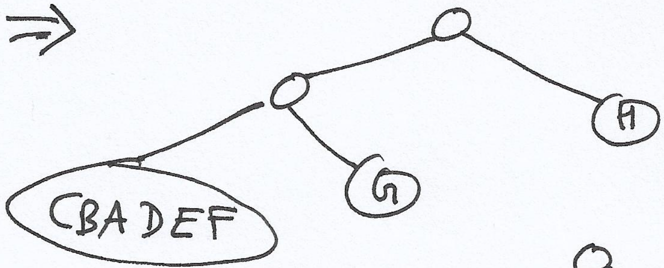
$\Rightarrow \left(f_H = \frac{21}{54}, f_G = \frac{13}{54}, f_F = \frac{8}{54}, f_{CBAD} = \frac{7}{54}, f_E = \frac{5}{54}\right)$

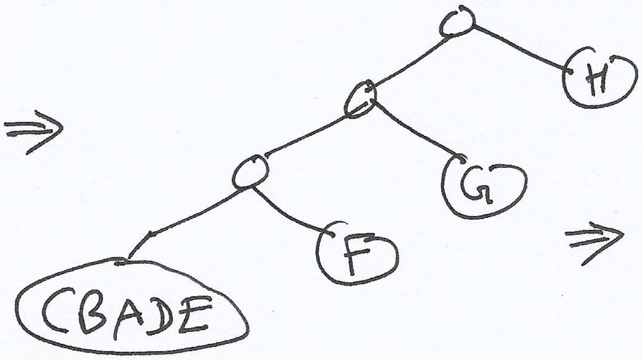
$\Rightarrow \left(f_H = \frac{21}{54}, f_G = \frac{13}{54}, f_{CBADE} = \frac{12}{54}, f_F = \frac{8}{54}\right)$

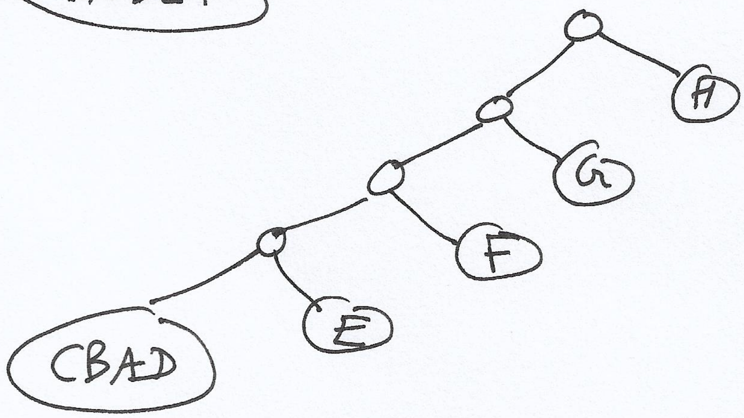
$\Rightarrow \left(f_H = \frac{21}{54}, f_{CBADEF} = \frac{20}{54}, f_G = \frac{13}{54}\right)$

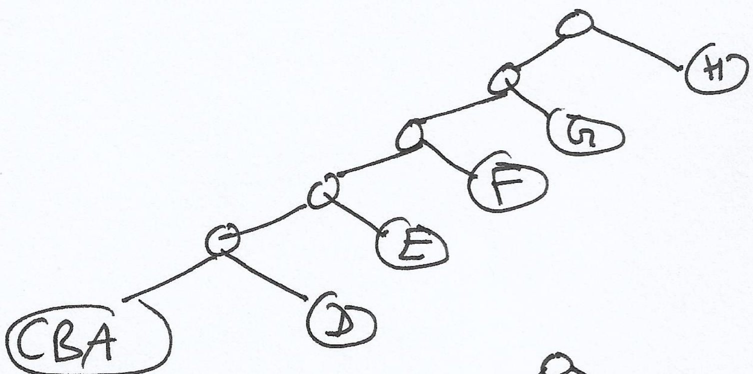
$\Rightarrow \left(f_{CBADEFG} = \frac{33}{54}, f_H = \frac{21}{54}\right)$ (3)

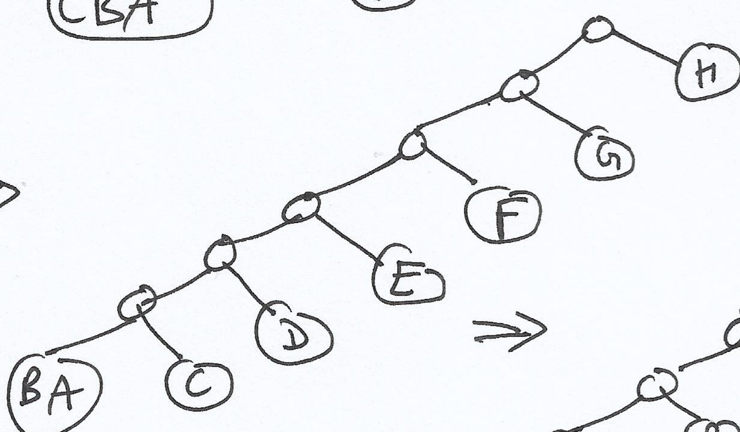
⇒ 

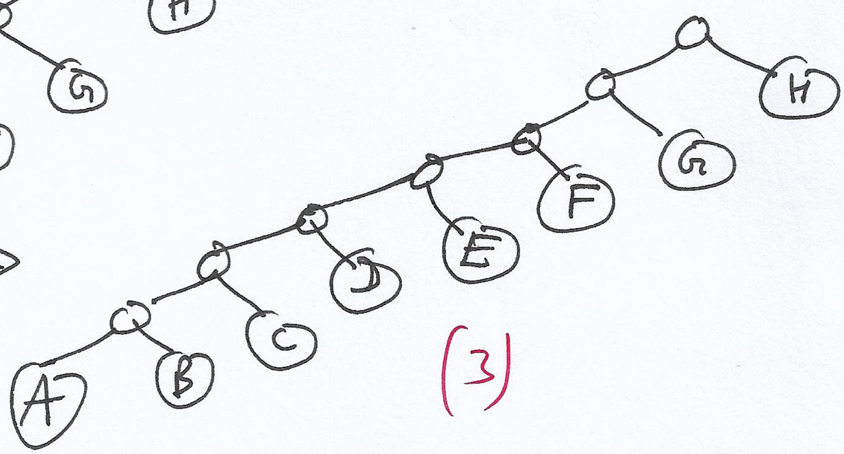
⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

(3)

Optimal Prefix Code: A = 0000000, B = 0000001, C = 000001, D = 00001, E = 0001, F = 001, G = 01, H = 1 (2) d

$$ABL = (1\times7+1\times7+2\times6+3\times5+5\times4+3\times8+2\times13+21\times1)/54 = \frac{132}{54} \approx 2.44$$ (4)

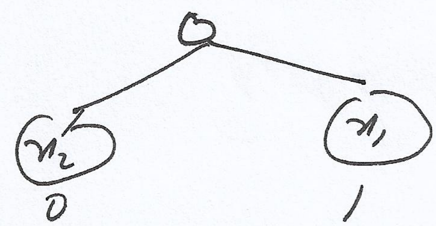
4: a is same as 2 c. (8)

b For $x_1 = F_1, x_2 = F_2, \ldots, x_{n-1} = F_{n-1}, x_n = F_n$, the code will be: $x_1 = 1^{n-1}$; $x_2 = 1^{n-2}0, \ldots,$

$x_i = 1^{n-i}0, \ldots, x_{n-1} = 10, x_n = 0.$

Proof by induction : Base case : $n=2$:

$x_1 = F_1 = 1, \; x_2 = F_2 = 1$



Code is binary $x_1 = 1$

$x_2 = 0$ (2)

Assuming Induction hypothesis to be true for upto n. Consider the first $n+1$ fibonacci numbers:

$x_1 = F_1, \; x_2 = F_2, \ldots, x_n = F_n, \; x_{n+1} = F_{n+1}$

$(x_{n+1} = F_{n+1},\ x_n = F_n, \ldots, x_3 = F_3,\ x_2 = F_2,\ x_1 = F_1)$

$\Rightarrow (x_{n+1} = F_{n+1},\ x_n = F_n, \ldots, x_3 = F_3,\ x_2 x_1 = F_1 + F_2)$

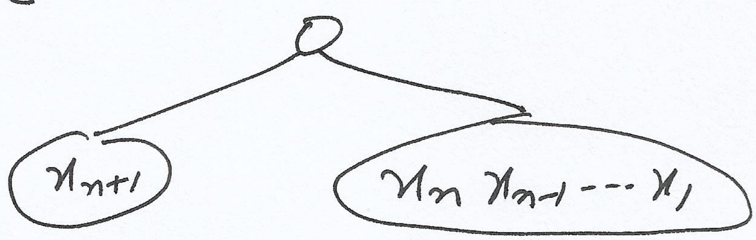
$\Rightarrow (x_{n+1} = F_{n+1},\ x_n = F_n, \ldots, x_5 = F_5\ x_3 x_2 x_1 = F_1 + F_2 + F_3,$
$x_4 = F_4)$

$\Rightarrow (x_{n+1} = F_{n+1}, x_n = F_n, \ldots, x_6 = F_6,\ x_4 x_3 x_2 x_1 =$
$F_1 + F_2 + F_3 + F_4,\ x_5 = F_5)$

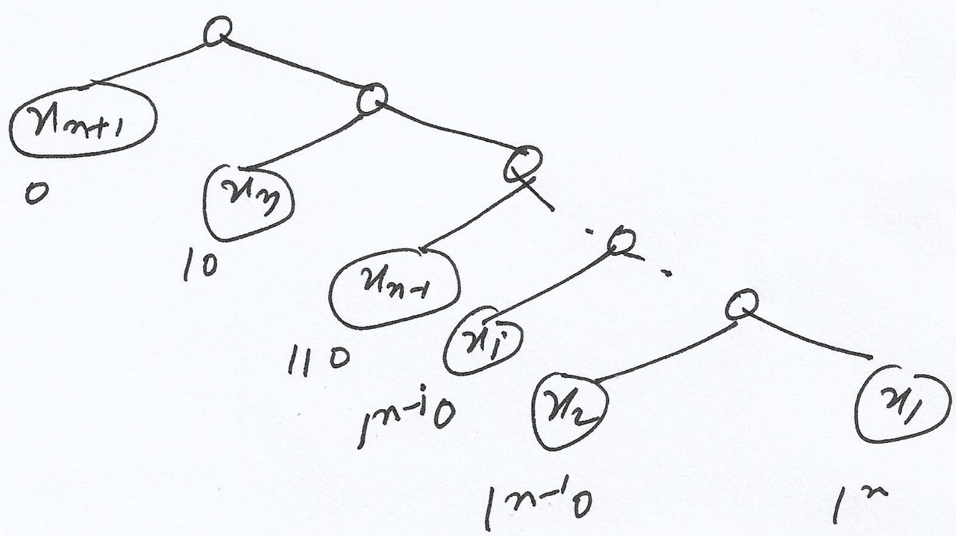
$\vdots$

$\Rightarrow (x_n x_{n-1} \cdots x_1 = F_1 + F_2 + \cdots + F_n,\ x_{n+1} = F_{n+1})$

$\Rightarrow$



Applying Induction Hypothesis On the right subtree we get:



(4)

Here we have used the following result:

For $n \geq 2$

$$F_{n+1} \leq S_n = F_1 + F_2 + \cdots + F_n < F_{n+2}$$

<u>Proof by induction</u> : 1 $F_{n+1} \leq S_n$

<u>Base case $n=2$</u> : $F_3 = 2 = S_2 = F_1 + F_2 = 1+1$ is true.

Assuming the above to be true for up to n.

$$S_{n+1} = F_{n+1} + S_n \geq F_{n+1} + F_{n+1} > F_{n+1} + F_n$$
$$= F_{n+2} \quad (3)$$

<u>Proof by induction</u> : 2 $S_n < F_{n+2}$

<u>Base case $n=2$</u> : $S_2 = F_1 + F_2 = 1+1 = 2 < F_4 = 3$ is true.

Assuming the above to be true for up to n.

$$S_{n+1} = F_{n+1} + S_n < F_{n+1} + F_{n+2} = F_{n+3}. \quad (3)$$