CS F364
Design & Analysis of Algorithms

Algorithm Design: Greedy Technique

Spanning Trees vs. Steiner Trees

STEINER TREES

- Recall: Given a computer network modeled as an graph:
  - Shortest paths minimize the cost of communication per source-destination pair.
  - Spanning trees form the (optimal) broadcast path
- Multicast Communication:
  - Within a network a subset of nodes are destinations.
  - How do you minimize the cumulative weight of the multicast path?
    - Observation: The subset may not form a connected component.

Steiner Trees

- Given G=(V,E,w) and a subset S of V, a minimal Steiner tree T is the tree of minimum total weight that connects all vertices in S.
  - Special cases:
    - $|S|=1$, $|S|=2$, $|S|=|V|$
  - When $2 < |S| < |V|$
    - A spanning tree including only nodes of S may not exist
    - Even if a spanning tree exists for S, the MST for S may not be a minimal Steiner tree.
      - Why?

STEINER TREES

- Vertices in V-S that are used in constructing a Steiner tree for S are referred to as Steiner vertices.
- No known polynomial time algorithm exists for solving the Minimal Steiner Tree problem:
    - Special case: a constant number of Steiner vertices are given.
        - i.e. Given G= (V,E,w), a subset S of V, a subset T of V-S, such that |T|=k, k is a constant,
        - find a tree of minimum total weight that connects all nodes in S but may include any vertex in S U T.

Minimum Steiner Trees

- Exercise:
  - Provide an intuitive explanation of why the Minimum Steiner Tree problem is harder to solve than the Minimum Spanning Tree problem.
  - Write an algorithm for the special case in the previous slide.
    - Analyze the algorithm for time complexity.