Probabilistic Turing Machine (PTM) is a TM with two transition functions δ_0, δ_1. To execute a PTM M on an input x we choose in each step with probability $1/2$ to apply the transition function δ_0 and with probability $1/2$ to apply δ_1. This choice is made independently of all previous choices. The machine only outputs 1 (Accept) or 0 (Reject). We denote by $M(x)$ the random variable corresponding to the value M writes at the end of this process. For a function $T: N \to N$, we say that M runs in $T(n)$-time if for any input x, M halts on x within $T(|x|)$ steps regardless of the random choices it makes.

The complexity classes BPTIME and BPP:

For a language $L \subseteq \{0,1\}^*$ and $x \in \{0,1\}^*$, we define $L(x) = 1$ if $x \in L$ and $L(x) = 0$ otherwise. For $T: N \to N$ and $L \subseteq \{0,1\}^*$ we say that a PTM decides L in time $T(n)$ if for every $x \in \{0,1\}^*$, M halts in $T(|x|)$ steps regardless of its random choices, and $\Pr[M(x) = L(x)] \geq 2/3$. We let $BPTIME(T(n))$ be the class of languages decided by PTMs in $O(T(n))$ time and define $BPP = \bigcup_c BPTIME(n^c)$.

The complexity classes RTime(T(n)) and RP :

RTime(T(n)) contains every language L for which there is a PTM M running in $T(n)$ time such that

$$x \in L \Rightarrow \Pr[M(x) = 1] \geq 2/3$$
$$x \notin L \Rightarrow \Pr[M(x) = 1] = 0$$

We define $RP = \cup_{c>0} RTime(n^c)$.

The complexity class coRP is defined as

$$coRP = \{ L \mid \bar{L} \in RP \} \Leftrightarrow$$

$$x \in L \Rightarrow \Pr[M(x) = 0] = 0$$

$$x \notin L \Rightarrow \Pr[M(x) = 0] \geq \frac{2}{3}$$

The complexity classes ZTime(T(n)) and ZPP :

The class ZTime(T(n)) contains all the languages L for which there is an expected-time $O(T(n))$ PTM M such that for every input x, whenever M halts on x, the output $M(x)$ it produces is exactly $L(x)$. We define $ZPP = \cup_{c>0} ZTime(n^c)$.

Relation between Randomized complexity classes :

1. $ZPP = RP \cap coRP$
2. $RP \subseteq BPP$
3. $coRP \subseteq BPP$
4. $RP \subseteq NP$
5. $P \subseteq BPP$
6. $P \subseteq RP$
7. $P \subseteq co\text{-}RP$
8. $P \subseteq ZPP$