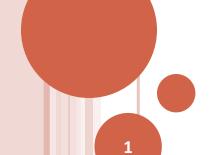
CS F364 Design & Analysis of Algorithms

COMPLEXITY – OPTIMIZATION PROBLEMS

Intractable Problems

- Optimization Problems Examples
- Complexity Classes PO and NPO



OPTIMIZATION PROBLEMS - CHARACTERIZATION

- o Formal Description:
 - An optimization problem π is characterized by the quadruple $(I_{\pi}, F_{\pi}, m_{\pi}, goal_{\pi})$
 - $ol_{\pi} = \{ x \mid x \text{ is an input instance of } \pi \}$
 - ${}_{\sigma}F_{\pi}(x) = \{ s \mid s \text{ is a feasible solution for } x, \text{ where } x \in I_{\pi} \}$
 - o $m_{\pi}(x, y) = v$ where v is a quantitative measure of the "value" of the feasible solution $y \in F_{\pi}(x)$ for $x \in I_{\pi}$
 - o goal $_{\pi} \in \{ \min, \max \} \}$

OPTIMIZATION PROBLEMS - EXAMPLES

- o Min Vertex Cover
 - I = {G| G is an undirected graph}
 - F(G) = { S | S ⊆ V s.t. for any (u,w) in E:

u in S or v in S, where G = (U,V)

- m(G, S) = |S| where G in I and S in F(G)
- goal: min

o TSP

- I = {G | G is a weighted, completely-connected graph}
- F(G) = { (u1, u2, u3, ... un, u1) | G=(V,E,w), n = |V|, ui in V for any i in 1...n, ui <> uj for any i and j in 1...n }
- $m(G, P) = \Sigma_{e \text{ in } P} w(e)$ where G = (V,E,w), P in F(G)
- goal: min

OPTIMIZATION PROBLEMS

- Optimal Solution:
 - The optimal solution for a given instance x of a problem π is characterized by:
 - oThe optimal measure:

$$om_{\pi} *(x) = goal_{\pi} \{ m_{\pi} (x, y) \mid y \text{ in } F_{\pi} (x) \}$$

- The optimal solution is characterized thus:
 - OPT $_{\pi}(x) = y$ where y in $F_{\pi}(x)$ and $m_{\pi}^*(x) = m_{\pi}(x, y)$

o Note:

- The objective for a given context may be to find
 - The optimal solution (*Constructive Version*)OR
 - The optimal measure i.e. the measure of the optimal solution (*Evaluative Version*)

CLASSES PO AND NPO

o ₽0

• An optimization problem π is in ${\Bbb P} 0$ if there exists an algorithm that solves π in polynomial time

o NPO:

- An optimization problem $\pi = (I, F, m, goal)$ belongs to the class NPO if
 - omembership I is decidable in polynomial time
 - othere exists a polynomial q such that
 - given x in I and for any y in F(x):
 - $|y| \le q(|x|)$ and
 - y in F(x) is <u>decidable in polynomial time</u>.
 - om is *computable in poly time*.
- Example:
 - Vertex Cover is in NPO