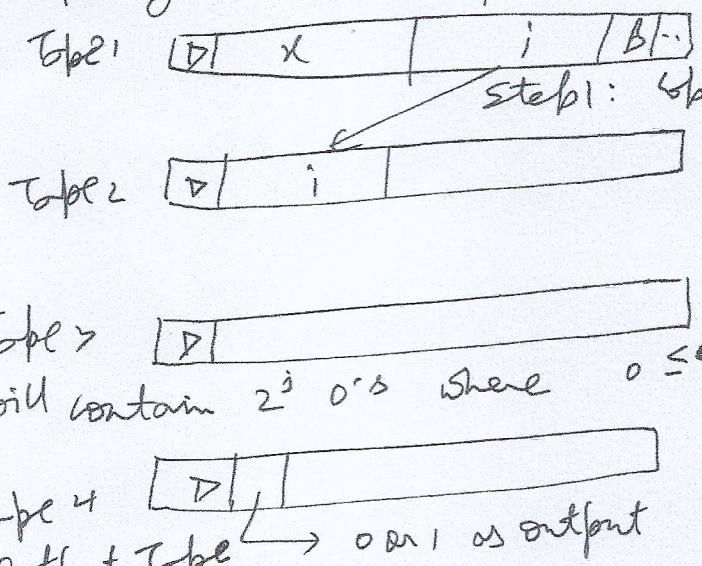
DAA Tutorial 8 Solution

(5) We can design a polynomial-time 4-Tape DTM for computing Lookup as follows:



Step 2: Start scanning from LSB of i on tape 2 (Rt to L direction)
For j^{th} LSB, write on Tape 3, 2^j 0's.

Step 3: Start scanning from left to right 2^j bits if j^{th} LSB of i is 1. If there is any overflow (scan goes beyond the bits of x) then output 0.

Step 4: Repeat the above two steps until all bits of i in tape 2 are scanned.

Step 5: output the current cell scanned on Tape 1 on Tape 4 (0 or 1).

All the above steps can be computed in polynomial time since in step 3 we are not going beyond x (total $O(|x|)$ steps) and also in step 2, computing 2^j only for j upto $2^{j-1} \le |x| < 2^j \Rightarrow 2^j = O(|x|)$. (20)

(2)

6 (a): use either BFS or DFS algorithm. (5)

6 (b): check all possible $\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$ possible triangles. (5)

6 (c): use BFS algo: Choose any $s \in V$ and color it Red. Color all the neighbors of s Blue. Color all the neighbors of "Blue" nodes as Red and repeat the process in BFS order. At the end if we have a valid 2-coloring of graph (each edge has vertices of different colors), then it is a bipartite graph, otherwise it is not a bipartite graph. (5)

6 (d): use either DFS or BFS to check whether the graph is connected or not. If it is connected, then verify that $|E| = |V| - 1$. (5)