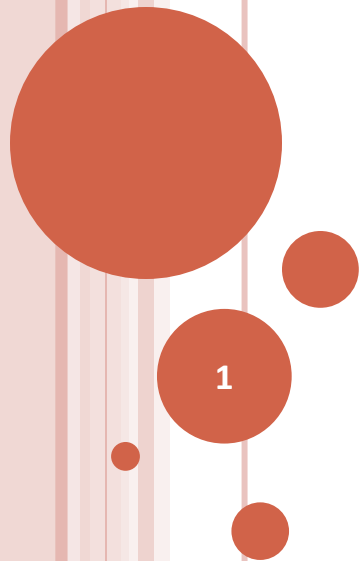


CS F364

Design & Analysis of Algorithms

## ALGORITHM DESIGN: GREEDY TECHNIQUE

### Spanning Trees vs. Steiner Trees



# STEINER TREES

- Recall: Given a computer network modeled as an graph:
  - Shortest paths minimize the cost of communication per source-destination pair.
  - Spanning trees form the (optimal) broadcast path
- Multicast Communication:
  - Within a network a subset of nodes are destinations.
  - How do you minimize the cumulative weight of the multicast path?
    - Observation: The subset may not form a connected component.

# STEINER TREES

- Given  $G=(V,E,w)$  and a subset  $S$  of  $V$ , a minimal Steiner tree  $T$  is the tree of minimum total weight that connects all vertices in  $S$ .
  - Special cases:
    - $|S|=1$ ,  $|S|=2$ ,  $|S|=|V|$
  - When  $2 < |S| < |V|$ 
    - A spanning tree including only nodes of  $S$  may not exist
    - Even if a spanning tree exists for  $S$ , the MST for  $S$  may not be a minimal Steiner tree.
      - Why?

# STEINER TREES

- Vertices in  $V-S$  that are used in constructing a Steiner tree for  $S$  are referred to as Steiner vertices.
- No known polynomial time algorithm exists for solving the Minimal Steiner Tree problem:
  - Special case: a constant number of Steiner vertices are given.
    - i.e. Given  $G = (V, E, w)$ , a subset  $S$  of  $V$ , a subset  $T$  of  $V-S$ , such that  $|T|=k$ ,  $k$  is a constant,
    - find a tree of minimum total weight that connects all nodes in  $S$  but may include any vertex in  $S \cup T$ .

# MINIMUM STEINER TREES

## ○ Exercise:

- Provide an intuitive explanation of why the Minimum Steiner Tree problem is harder to solve than the Minimum Spanning Tree problem.
- Write an algorithm for the special case in the previous slide.
  - Analyze the algorithm for time complexity.