Design and Analysis of Algorithms (CS F364) Mid Sem Exam (2019), Second Semester

There are 3 questions in all and total marks is $10 + 20 + 20 = 50$. This is an **open book exam**. Only hard copies of textbooks, reference books, and lecture notes are allowed. No electronic instruments (calculator, mobile phone, tablet, laptop etc.) are allowed. Show all computation steps for solving any problem, and write complete algorithms with time complexity derivations. Time: 90 minutes.

1. Solve the following instance of the *Fractional Knapsack Problem*, by applying the *Greedy Algorithm*:
   There are 7 items with profit of the items given by $(p_1, p_2, p_3, p_4, p_5, p_6, p_7) = (20, 10, 30, 14, 12, 36, 6)$, weight of the items given by $(w_1, w_2, w_3, w_4, w_5, w_6, w_7) = (2, 3, 5, 7, 1, 4, 1)$, and the knapsack capacity given by $W = 15$.

2. A *Vertex Cover* of a graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ that includes at least one endpoint of every edge in E. Give a linear-time *Dynamic Programming* algorithm for the following problem:
   Input: An undirected tree $T = (V, E)$.
   Output: A smallest vertex cover of T.
   Show the working of your algorithm for the following input (figure 1):

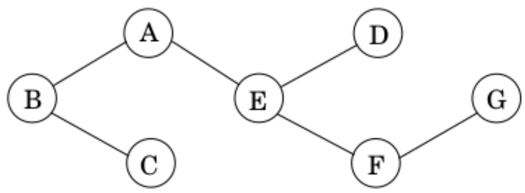

Figure 1: Figure for problem 2.

3. We formulate the *Load Balancing Problem* as follows:
   We are given a set of m machines $M_1, ..., M_m$, and a set of n jobs; each job j has a processing time t_j. We seek to assign each job to one of the machines so that the loads placed on all machines are as "balanced" as possible. More concretely, in any assignment of jobs to machines, we can let $A(i)$ denote the set of jobs assigned to machine M_i; under this assignment, machine M_i needs to work for a total time of $T_i = \Sigma_{j \in A(i)} t_j$, and we declare this to be the *load* on machine M_i. We seek to minimize a quantity known as the *makespan*; it is simply the maximum load on any machine, $T = \max_i T_i$. Give a decision version of the Load Balancing Problem, and prove that it is NP-Complete.