CS F364
Design & Analysis of Algorithms

ALGORITHMS - COMPLEXITY

Complexity Classes > NP-Completeness Via Reductions

- Reduction Techniques:
  - Local Replacement
  - Examples: CLIQUE, SUBSET-SUM

PROBLEM: CLIQUE

- Definition: Clique of a graph:
  - A clique in a graph G=(V,E) is a subset C of V, such that for each u and v in C, (u,v) is in E.
- CLIQUE: Given a graph G and a positive integer k, find whether there is a clique of size at least k in G.
- CLIQUE is in $\mathbb{NP}$
  - Proof:
    - What would be the certificate?
    - Can it be verified in polynomial time?

CLIQUE IS $\mathbb{NP}$-HARD

- Proof: VERTEX-COVER $\precsim$ CLIQUE
- Reduction:
  - Given the instance (G, k) of VERTEX-COVER construct the instance (G^c, k') of CLIQUE as follows:
    - Let G=(V,E), |V|=n
    - G^c is the complement graph of G
      - i.e G^c = (V, E') where (u,v) is in E' iff (u,v) is not in E.
    - k' = n – k
  - Claim:
    - There is a vertex cover of size at most k for G iff there is a clique of size at least k for G^c
    - The reduction (shown above) takes polynomial time.

Reduction Technique: Local Replacement

- The reduction used in proving hardness of CLIQUE is known as ***Local Replacement:***
  - Divide the known hard problem and the target problem instances into basic units and
  - convert basic units (of one problem) locally into basic units (of the other)
- Exercise:
  - Explain how local replacement is used in reducing CNF-SAT to 3-SAT

SET-COVER

- Definition: ***Set Cover of a collection of sets***
  - Give a collection of sets S1, S2, … Sm a subcollection Si1, Si2, …Sk for k<=m, is a set cover of the collection if
    - $\cup_{j=1 \text{ to } m} S_j = \cup_{j=1 \text{ to } k} Si_j$
- SET-COVER:
  - Given a collection of m sets S1, S2, … Sm and a positive integer k, find whether there is a set cover of size at most k.
- VERTEX-COVER $\precsim$ SET-COVER
- Proof: Omitted.

$\mathbb{NP}$-Completeness Via Reductions: SUBSET-SUM

- SUBSET-SUM:
  - Given a set S of positive integers { s1, s2, …, sn} and a positive integer k,
  - find whether there is a subset T of S, such that the sum of the elements in T is k
- SUBSET-SUM is $\mathbb{NP}$-hard
  - VERTEX-COVER $\precsim$ SUBSET-SUM
  - Proof: (Omitted) By Local Replacement