Tutorial 10, Design and Analysis of Algorithms, 2019

1. In the `CLIQUE` problem we are given an undirected graph G and an integer K and have to decide whether there is a subset S of at most K vertices such that every two distinct vertices $u, v \in S$ have an edge between them (such a subset is called a *clique* of G). Prove that the `CLIQUE` problem is NP-Complete.

2. In the `VERTEX COVER` problem we are given an undirected graph G and an integer K and have to decide whether there is a subset S of at most K vertices such that for every edge $\overline{ij}$ of G, at least one of i or j is in S (such a subset is called a *vertex cover* of G). Prove that the `VERTEX COVER` problem is NP-Complete.

3. A *Hamiltonian path* in an undirected graph is a path that visits all vertices exactly once. Let `HAMPATH` denote the set of all undirected graphs that contain such a path. Prove that `HAMPATH` is NP-Complete.

4. Prove that the language `HAMCYCLE` of undirected graphs that contain Hamiltonian cycle (a simple cycle involving all the vertices without repetition) is NP-complete.

5. The problem `MATCH` is defined as follows: given a finite set S of strings of length n over the alphabet $\{0, 1, *\}$, determine if there exists a string w of length n over the alphabet $\{0, 1\}$ such that for every string $s \in S$, s and w have the same symbol in at least one position. For example, if $S = \{001*, *100, 10*0, 1010\}$, then $w = 0000$ is a solution. However, if $S = \{00, *1, 1*\}$, then there is no string w that "matches". Prove that the problem `MATCH` is NP-complete.

6. Study Kwek's polynomial-time algorithm for optimal search for rationals. Show how you will search $\frac{22}{7}$ using Kwek's algorithm.

7. Assuming that Π is an *NP Optimization* problem having the objective function rational valued, and given a polynomial-time algorithm $DCN(I, B)$ for solving the decision version of Π, prove that you can use $DCN(I, B)$ to construct a polynomial-time algorithm $OPT(I)$ for finding the optimal solution of $I \in D_{\Pi}$.