DAA Tutorial 6 Solution

4: We define $f(b,i)$ to be the optimum solution for the numbers $(a_i, \ldots, a_n)$ with b as the lower bound for the selected numbers. We make a table for all n^2 possibilities of $f(b,i)$ and use the following recursion:

$f(b,i) = f(b,i+1)$ if $a_i < b$

otherwise

$f(b,i) = \max\{f(b,i+1), 1+f(a_i,i+1)\}$

$f(b,n) = 1$ if $a_n \geq b$

otherwise $f(b,n) = 0$ (5)

Our objective is to compute $f(\min\{a_i\}_{i=1}^{n}, 1)$.

LMS $(a_1, \ldots a_n)$

1. $(a'_1, \ldots a'_n) \leftarrow$ Sort $(a_1, \ldots, a_n)$ // Increasing order
2. Let $b[a'_1, \ldots, a'_n; 1..n]$ and $c[a'_1, \ldots a'_n; 1..n]$ be new tables
3. for $j = 1$ to n
4. if $a_n \geq a'_j$
5. $b[a'_j, n] = 1$
6. else
7. $b[a'_j, n] = 0$

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8.  for i = n-1 to 1
9.      for j = n to 1
10.         if a_i < a'_j
11.             b[a'_j, i] ← b[a'_j, i+1]
12.             c[a'_j, i] ← a'_j, i+1
13.         else
14.             b[a'_j, i] ← max{ b[a'_j, i+1],
                1 + b[a_i, i+1] }
15.             c[a'_j, i] ← (b, i+1) OR
            (a_i, i+1) // whichever gives maximum
            in (14)
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(5)

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16. Return b and c

    Print-LMS( a, b, c, i, j)
1.  if i = n
2.      if b[a'_j, i] = 1
3.          Print a_i
4.  else
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5. if $b[a'_j, i] > b[c[a'_j, i]]$

6. Print a_i

7. Print-LMS$(a, b, c, c[a'_j, i])$ (3)

Complexity of LMS is $\Theta(n^2)$ due to nested for loops in lines 8 and 9. Complexity of Print-LMS is $\Theta(n)$ because the recursive call in line 5 is called n times. (2)

b

$i \downarrow$	$a_i \downarrow$ \ $a'_j \rightarrow$	1	2	3	4	7	9
1	9	4	3	3	3	2	2
2	4	4	3	3	3	1	1
3	1	4	3	3	2	1	1
4	3	3	3	3	2	1	1
5	4	3	3	2	2	1	1
6	2	3	3	2	1	1	1
7	9	2	2	2	1	1	1
8	7	2	2	2	1	1	0
9	3	2	2	2	1	0	0
10	4	1	1	1	1	0	0

c

$i \downarrow$	$a_i \downarrow$ \ $a'_j \rightarrow$	1	2	3	4	7	9
1	9	1,2	2,2	3,2	4,2	9,2	9,2
2	4	1,3	2,3	3,3	4,3	7,3	9,3
3	1	1,4	2,4	3,4	4,4	7,4	9,4
4	3	1,5	2,5	3,5	4,5	7,5	9,5
5	4	1,6	2,6	3,6	4,6	7,6	9,6
6	2	2,7	2,7	3,7	4,7	7,7	9,7
7	9	1,8	2,8	3,8	4,8	7,8	9,8
8	7	1,9	2,9	3,9	4,9	7,9	9,9
9	3	3,10	3,10	3,10	4,10	7,10	9,10
10	4						

LMS = $\langle 1, 2, 3, 4 \rangle$ having length 4. (5)

2: Vertices are all pairs (i, j) such that $1 \le i \le j \le n$. (2)

Number of Vertices $= \sum_{i=1}^{n} \sum_{j=i}^{n} 1 = \sum_{i=1}^{n} (n-i+1)$ (2)

$= \sum_{i'=1}^{n} i' = \frac{n(n+1)}{2}$.

(i, j) has incoming edges from $(i, j+1), (i, j+2), \ldots, (i, n)$; and from $(1, j), (2, j), \ldots, (i-1, j)$. (4)

Total incoming edges to $(i,j) = (n-j)+(i-1)$

$= (n-1)+(i-j)$.

Total number of edges $= \sum_{i=1}^{n} \sum_{j=i}^{n} [(n-1)+(i-j)]$

$= \sum_{i=1}^{n} \left[(n-1)(n-i+1) + (n-i+1)i - \left(\frac{n(n+1)}{2} - \frac{(i-1)i}{2} \right) \right]$

$= n(n-1)(n+1) - \frac{(n-1)n(n+1)}{2} + \frac{(n+1)n(n+1)}{2}$

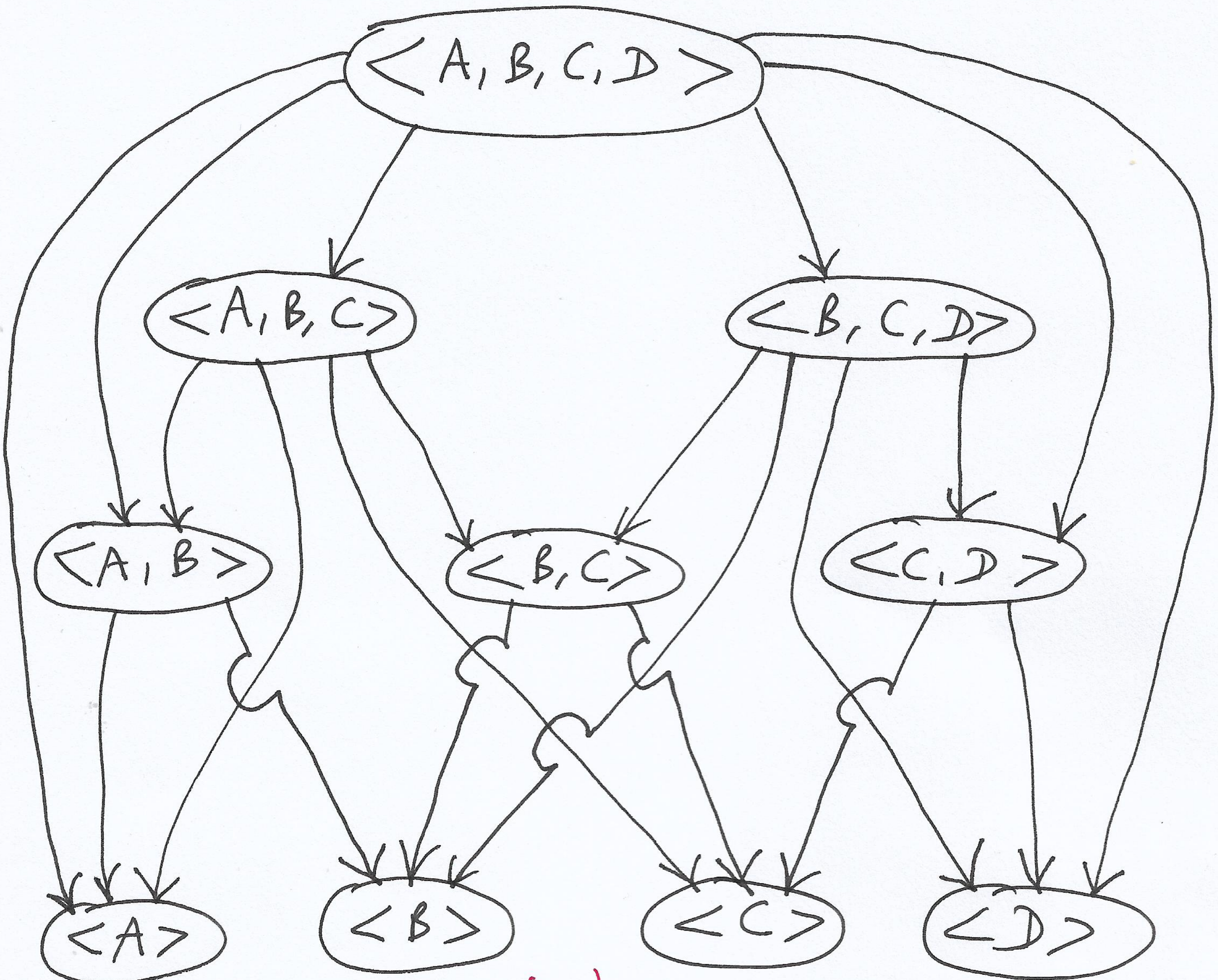
$- \frac{n(n+1)(2n+1)}{6} - \frac{n^2(n+1)}{2} + \frac{n(n+1)(2n+1)}{12}$

$- \frac{n(n+1)}{4} = \frac{n(n+1)}{12} [6n-6+6n+6-2n-1$

$-6n-3] = \frac{n(n+1)(4n-4)}{12} = \frac{n(n+1)(n-1)}{3}$

(4)

Subproblem Graph for <A, B, C, D>



(8)