Design and Analysis of Algorithms (CS F364) Mid Sem Exam (2018)

There are 6 questions in all and total marks is $10 + 10 + 10 + 10 + 10 + 10 = 60$. Please show all steps in computations or proofs. This is a **closed book exam**. Calculators are not allowed. Time: 90 minutes.

1. Find 6789×9876 using *Karatsuba's Divide and Conquer Multiplication Algorithm* showing the computations in a divide and conquer graph.

2. Find *all possible* shortest salseman tours and cost using the *Dynamic Programming Algorithm* for the following instance of the *Traveling Salseman Problem* for a directed K_4 graph having the following adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 0 & 5 & 6 \\ 3 & 4 & 0 & 5 \\ 2 & 3 & 4 & 0 \end{bmatrix}$$

3. Using the *Dynamic Programming Algorithm*, solve the following instance of the *Matrix Chain Multiplication Problem:*
$< A_{4\times 3}, B_{3\times 2}, C_{2\times 1}, D_{1\times 5}, E_{5\times 6}, F_{6\times 7} >$

4. Prove that the number of different binary trees with n nodes is

$$\frac{1}{n+1}\binom{2n}{n}$$

5. There exists a $O(n)$-time deterministic algorithm (M) for finding median of n given numbers. Using this algorithm as a subroutine, design a $O(n)$-time deterministic algorithm for solving the fractional knapsack problem (items are $(I_i)_{i=1}^{n}$, weight of items are $(w_i)_{i=1}^{n}$, profit of items are $(p_i)_{i=1}^{n}$, and knapsack capacity is W), and also prove its time complexity.

6. The owners of an independently operated gas station are faced with the following situation. They have a large underground tank in which they store gas; the tank can hold up to L gallons at one time. Ordering gas is quite expensive, so they want to order relatively rarely. For each order, they need to pay a fixed price P for delivery in addition to the cost of the gas ordered. However, it costs c to store a gallon of gas for an extra day (l gallons remaining at the end of a day will give extra cost of lc), so ordering too much ahead increases the storage cost. They are planning to close for a week in the winter, and they want their tank to be empty by the time they close. Luckily, based on years of experience, they have accurate projections for how much gas they will need each day until this point in time. Assume that there are n days left until they close, and they need g_i ($g_i \leq L$) gallons of gas for each of the days $i = 1, ..., n$. Assume that the tank is empty at the end of day 0. Give a *Dynamic Programming Algorithm* to decide on which days they should place orders, and how much to order so as to minimize their total cost. Also find the time complexity of the algorithm.