CS F364

Design & Analysis of Algorithms

ALGORITHMS - COMPLEXITY

Complexity Classes:

- Complement Class
- Closure Property

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Complexity Classes - Complements

- Complement of a (Decision) Problem:
  - Given a decision problem π its complement π' is defined as follows:
    - $\pi(x) = 1$ if and only if $\pi'(x) = 0$
- Complement (Complexity) Class:
  - Given a complexity class $\mathbb{C}$ its complement class co-$\mathbb{C}$ is defined as follows:
    - co-$\mathbb{C} = \{ \pi \mid \pi' \text{ is in } \mathbb{C} \}$

Complexity Classes – Complements and Closure

- Recall
  - $\mathbb{P}$ = { π | π is a decision problem that can be solved by a polynomial time algorithm }
  - i.e. $\mathbb{P} = \bigcup_{k \in \mathbb{N}} TIME(n^k)$
- Then we can define co-$\mathbb{P}$ as :

  the class of problems whose complements are in $\mathbb{P}$
- Question:
  - Is $\mathbb{P}$ closed under *complementation*?
    - i.e. Is $\mathbb{P}$ = co-$\mathbb{P}$?
- Answer:
  - Yes. *(By definition of decision problems, any algorithm that solves a problem solves its complement as well.)*

Complexity Classes – Complements and Closure [2]

- Recall :
  - $\mathbb{NP}$ = {π | π is a decision problem that can be solved by a non-deterministic polynomial time algorithm }
- Then we can define co-$\mathbb{NP}$
  - as the class of problems whose complements are in $\mathbb{NP}$
- Examples:
  - Is **Validity** (of Boolean formulas) in co-$\mathbb{NP}$?
  - Is **Graph Isomorphism** in co-$\mathbb{NP}$?
- Exercise:
  - Argue that **Validity** is (co-$\mathbb{NP}$)-complete.

Complexity Classes – Complements and Closure [2]

- Question:
  - Is $\mathbb{NP}$ closed under *complementation*?
    - i.e. Is $\mathbb{NP}$ = co-$\mathbb{NP}$?
- Answer:
  - Ability to (efficiently) verify a positive certificate does not imply the ability to (efficiently) verify a negative certificate (or vice versa)
    - i.e. Let π be a decision problem. Assume that there exists a polynomial time algorithm A to verify a certificate y for an instance x of π
      - This does not imply that there exists a polynomial time algorithm A' to verify a certificate y' for an instance x' of *the complement of π*