CS F364: Design & Analysis of Algorithms

Lecture-01: Introduction



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Logistics: (CS F364) Design & Analysis of Algorithms

- T Th S (11-12PM) 5101@BITS-Pilani
- Jointly to be taught by
 Dr. Abhishek Mishra (IC) and Dr. Kamlesh Tiwari.
- Grading
 - Tutorial Quiz (32%) 4 of 8% each, Open Book
 - Mid Semester Exam (28%) Open Book
 - Comprehensive Exam (40%) Open Book

Learn algorithm design techniques like Divide and Conquer, Greedy, Dynamic Programming, Approximation Algorithms, and Randomized Algorithms. Explore topics like Computational Complexity *etc*.

Books:

- [1] T.H. Cormen, C.E. Leiserson, R.L. Rivest, C. Stein, Introduction to Algorithms, 3rd Edition, PHI, 2012.
- [2] S. Arora, B. Barak, Computational Complexity: A Modern Approach, Cambridge University Press, 2009
- [3] J.Kleinberg, E. Tardos, Algorithm Design, Pearson, 2013.



Computational Problems



¹Complexity is a function

- Computational Problems
- Algorithms: input, output, definiteness, finiteness, effectiveness
- Pseudo code



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- Pseudo code
- Input size
- Analysis
 - Kind of resources¹: time, space, number of gates ...
 - Cases: Best, Worst and Average



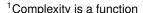
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- Correctness: initialize well, maintain invariance and terminate



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 - ► Kind of resources¹: time, space, number of gates ...
 - Cases: Best, Worst and Average
- Correctness: initialize well, maintain invariance and terminate
- Order of growth: O, o, θ , ω , Ω zoo
- Insertion and Merge sort





Insertion Sort

Incremental algorithm paradigm:

Algorithm 1: INSERTION-SORT (A)

```
for j = 2 to A.length do

key = A[j]

i = j - 1

while i > 0 and A[i] > key do

A[i + 1] = A[i]

i = i - 1

A[i + 1] = key
```

Analyse Insertion Sort

INSERTION-SORT (A)		cost	times
1	for $j = 2$ to A.length	<i>C</i> ₁	n
2	key = A[j]	c_2	<i>n</i> − 1
3	i = j - 1	<i>c</i> ₃	<i>n</i> − 1
4	while $i > 0$ and $A[i] > key$	<i>C</i> ₄	$\sum_{i=2}^{n} t_i$
5	A[i+1] = A[i]		$\sum_{j=2}^{n}(t_j-1)$
6	i = i - 1	<i>c</i> ₆	$\sum_{j=2}^{n} (t_j - 1)$
7	A[i+1] = key	C ₇	<i>n</i> – 1

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6	i = i - 1	<i>c</i> ₆	$\sum_{j=2}^{n}(t_j-1)$
7	A[i+1] = key	C ₇	<u>n – 1</u>

- Best case T(n) = O(n)
- Worst case $T(n) = O(n^2)$
- Average ?



Divide and conquer paradigm: Divide, Conquer and Combine

$$T(n) = \left\{ egin{array}{ll} \Theta(1) & ext{if } n \leq c \ aT(n/b) + D(n) + C(n) & ext{otherwise} \end{array}
ight.$$

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```
MERGE-SORT (A,p,r)

1 if p < r
2 q = \lfloor (p+r)/2 \rfloor
3 MERGE-SORT (A,p,q)
4 MERGE-SORT (A,q+1,r)
5 MERGE (A,p,q,r)
```

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```
MERGE(A, p, q, r)
  n_1 = q - p + 1
2 n_2 = r - q
3 let L[1...n_1 + 1] and R[1...n_2 + 1]
   be new arrays
4 for i = 1 to n_1
  L[i] = A[p+i-1]
6 for j = 1 to n_2
  R[j] = A[q+j]
8 \quad L[n_1+1] = \infty
  R[n_2+1]=\infty
  i = 1
  for k = p to r
13
  if L[i] \leq R[j]
A[k] = L[i]
i = i + 1
16 else A[k] = R[j]
j = j + 1
```

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   be new arrays
 4 for i = 1 to n_1
 5 	 L[i] = A[p+i-1]
 6 for j = 1 to n_2
 7 	 R[j] = A[q+j]
 8 \quad L[n_1+1] = \infty
 9 R[n_2 + 1] = \infty
  i = 1
12 for k = p to r
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  if L[i] \leq R[j]
A[k] = L[i]
i = i + 1
16 else A[k] = R[j]
j = j + 1
```

Average case $T(n) = O(n \log n)$. Best and Worst?

Thank You!

Thank you very much for your attention! (Reference²)

Queries ?

²[1] Book - Introduction to Algorithm, By THOMAS H. CORMEN, CHARLES E. LEISERSON, RONALD L. RIVEST, CLIFFORD STEIN