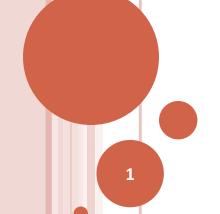
CS F364 Design & Analysis of Algorithms

ALGORITHM DESIGN TECHNIQUES

Kinds of Problems -

Optimization Problems - Example



CHARACTERIZING PROBLEMS

• One way of <u>characterizing and classifying problems</u> is by nature of the <u>the output to be computed</u> of a problem:

Category	Output	Examples
Decision	Is p(i) for input i ?	??
Search	Select one or more items i such that p(i) given a collection of items	??
Function	Compute f(i) on input I	??
Enumeration	Enumerate all j such that p(i,j) given input i	??
Counting	Count the number of j such that p(i,j) given input i	??
Optimization	Optimize m(i,j) such that p(i,j) given input i	??

OPTIMIZATION PROBLEMS - CHARACTERIZATION

o Formal Description:

An optimization problem π is characterized by the quadruple (I_π, F_π, m_π, goal_π)
oI_π = {x | x is an input instance of π}
oF_π(x) = {s | s is a feasible solution for x ∈ I_π}
o m_π(x, y) = v where v is a quantitative measure of the "value" of the feasible solution

i.e. $y \in F_{\pi}(x)$ for $x \in I_{\pi}$

 \circ goal $_{\pi} \in \{ \min, \max \} \}$

PROBLEM - 0/1 KNAPSACK

o Given:

- A sack with maximum capacity (by weight): B kg.
- Set S of items j (in store), each labeled with
 - o Weight : w_i (< B)</pre>
 - o Price: p_i

• Assumptions:

- An item is either taken (in full) or not
- All values (w_i, p_i, and B) are <u>positive</u>.

o Goal:

- Fill the sack with maximum value (by price)
 - oi.e. Find **T** subset of **S**, such that
 - $\circ \Sigma_{i \text{ in T}} \mathbf{p}_{i}$ is maximum and $\Sigma_{i \text{ in T}} \mathbf{w}_{i} \leq \mathbf{B}$

KNAPSACK - BRUTE-FORCE ALGORITHM

- Algorithm KnapSack(S, B) // S Set of items; B-capacity
 - 1. Find all the subsets of S say T1, T2, ...,Tk
 - Let the <u>cumulative weights</u> be **w(T1)**, **w(T2)**, ..., **w(Tk)** and the <u>cumulative prices</u> be **p(T1)**, **p(T2)**,...,**p(Tk)** respectively.
 - 2. return (Tm,v) such that
 v == max { p(Ti) | w(Ti) <= B and 1<i<=k } == p(Tm)</pre>

What is the time complexity of this algorithm?