papergrid
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$E\left(\sum_{i=1}^{n} |a_i - i|\right)$

$= \sum_{i=1}^{n} E\left(|a_i - i|\right)$

For $E|a_i - i|$
when $a_i - i > 0$ $\therefore$ position changes

$(i-1) + (i-2) \dots + 1$

Similarly for $a_i - i < 0$ have the same expression

$(i-1) + (i-2) \dots + 1$

$\therefore E\left(|a_i - i|\right) = \dfrac{(i-1) + (i-2) + \dots 1 + (i-1)(i-2) \dots 1}{n}$

$(i-1) \dots + 1 = \dfrac{i(i-1)}{2}$

$\therefore E\left(|a_i - i|\right) = \dfrac{2\, i(i-1)}{2n}$

$\sum E\left(|a_i - i|\right) = \sum_{i=1}^{n} \dfrac{i(i-1)}{n}$

$= \dfrac{1}{n}\left(\sum i^2 - \sum i\right)$

$$= \frac{1}{n}\left(\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right)$$

$$= (n+1)\left(\frac{2n+1}{6} - \frac{1}{2}\right)$$

$$= \frac{(n+1)(2n-2)}{6}$$

$$= \frac{n^2-1}{3} \quad //$$