CS F364
Design & Analysis of Algorithms

COMPLEXITY – OPTIMIZATION PROBLEMS

Approximation Algorithms

- Absolute Error and Absolute Approximation

- Example and Counter-example

Approximation Algorithms

- Given an optimization problem π, an algorithm A is said to be ***an approximation algorithm*** if it finds a feasible solution for any input instance
  - i.e. for any x in I_π A(x) is in F_π (x)

- Given an optimization problem π, for any input instance x and for any feasible solution y, ***the absolute error*** of y is defined as:
  - $D(x,y) = |m^*(x) - m(x,y)|$

- Given an optimization problem π, an algorithm A is said to be ***an absolute approximation algorithm*** if there exists a constant k such that
  - i.e. for any x in I_π $D(x, A(x)) <= k$

(Planar) Graph Coloring – Planar Graphs

- Euler's Theorem on Planar Graphs:
  - The smallest degree must be at most 5.
- Algorithm GC6P(G)
  - Find a vertex u with degree at most 5 // bound to exist
  - Remove u (and incident edges) to get G'
  - GC6P(G') // G' is planar
  - Choose a color for u that is different from all its neighbors.

- Approximation Algorithm GCP(G) // G = (V,E)
  - If E is empty, then each vertex gets the same color
  - Else if G is bipartite color it with 2 colors.
  - Else GC6P(G)

Approximation Algorithms

- GCP is an absolute approximation algorithm because
  - For any planar graph G, the absolute error
  - D(G, GC6P(G)) <= 6 – 3 = 3

- Is it possible to get an absolute approximation algorithm for all $\mathbb{NP}$-complete optimization problems?

Non-Existence of Absolute Approximation Algorithms

- Theorem:
  - Unless $\mathbb{P}=\mathbb{NP}$, no polynomial-time absolute approximation algorithm exists for 0,1 KNAPSACK
- Proof:
  - Given set of n items with profits p1,p2,…pn, and weights w1,w2,…wn and a bound B,
    - assume that there is an absolute approximation algorithm with error bound k
  - Create a new instance by multiplying all profits by k+1
    - The optimal solution will be preserved (Why?)
    - But the measure of any feasible solution will be a multiple of k+1
      - And the only feasible solution with absolute error bounded by k is the optimal solution.
  - This is a contradiction (unless P=NP).