DAA Tutorial 5 Solution

6: Structure of the optimal Solution:

If (i, j) is the optimal solution, then the following three conditions are satisfied:

(1) if $k < i$, then $p(k) \geq p(i)$.

If this is not the case, then (k, j) will give a better solution: $p(j) - p(k) > p(j) - p(i)$.

(2) If $k > j$, then $p(k) \leq p(j)$.

If this is not the case, then (i, k) will give a better solution: $p(k) - p(i) > p(j) - p(i)$

(3) If $i < k < j$, then $p(i) \leq p(k) \leq p(j)$.

If this is not the case, then (i, k) will give a better solution if $p(k) > p(j)$:
$p(k) - p(i) > p(j) - p(i)$;
and (k, j) will give a better solution if $p(k) < p(i)$: $p(j) - p(k) > p(j) - p(i)$.

Let $opt(1, n)$ denote the solution of the original problem. From (1) and (2) we get:

$opt(1, n) = opt(k_1, k_2)$ where k_1 is such that $p(k_1)$ is minimum in $[1 .. k_1]$ and k_2 is such that $p(k_2)$ is maximum in $[k_2 .. n]$. (5)

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Profit ( (p)_1^n )
1   i ← 1
2   j ← 1
3   for k1 = 2 to n-1 do
4         if (p(k1) < p(i))
5               i ← k1
6   for k2 = n-1 to i+1 do
7         if (p(k2) > p(j))
8               j ← k2
9   if ( p(j) - p(i) ≤ 0 )
10        return " No Profit"
11  else
12        return " Profit = p(j) - p(i) at (i,j)"
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(5)

complexity of the above algorithm is $\Theta(n)$ due to the two for loops at lines 3 and 6 respectively.

(2)

(83, 18, 34, 2, 71, 26, 44, 46, 48, 75)

$\Rightarrow$ (18, 34, 2, 71, 26, 44, 46, 48, 75)

$\Rightarrow$ (2, 71, 26, 44, 46, 48, 75).

Now there is no more k_1 and k_2

$\Rightarrow$ max profit $= 75 - 2 = 73$ at $(4, 10)$ (4)

(83, 75, 71, 48, 46, 44, 34, 26, 18, 2)

$\Rightarrow$ (75, 71, 48, 46, 44, 34, 26, 18, 2)

$\Rightarrow$ (71, 48, 46, 44, 34, 26, 18, 2)

$\Rightarrow$ (48, 46, 44, 34, 26, 18, 2)

$\Rightarrow$ (46, 44, 34, 26, 18, 2)

$\Rightarrow$ (44, 34, 26, 18, 2)

$\Rightarrow$ (34, 26, 18, 2)

$\Rightarrow$ (26, 18, 2)

$\Rightarrow$ (18, 2)

$\Rightarrow$ No profit as $2 - 18 = -16 < 0$ (4)