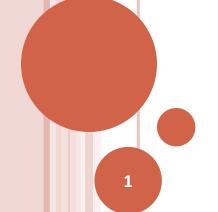
CS F364 Design & Analysis of Algorithms

ALGORITHM DESIGN TECHNIQUES - GREEDY

Greedy Algorithms

- Greedy Choice and Optimal Substructure
- Limitation of Greedy Choice Example



OPTIMAL SUBSTRUCTURE

- A problem exhibits optimal substructure if
 - an optimal solution to the problem contains optimal solutions for sub-problems:
 - o can be decomposed into optimal solutions for subproblems.
- Optimal substructure is necessary for greedy choice:
 - Otherwise local choice may not lead to global optimality
 i.e. the choice may not preserve optimality
 - Examples:
 - o Schedule for tasks $T' = T \{j\}$ is part of the schedule for tasks T where j is the earliest starting task
 - o Schedule for tasks $T' = T \{k\}$ is not necessarily part of the schedule for tasks T if k is not the earliest starting task

GREEDY CHOICE, OPTIMAL SUBSTRUCTURE, AND INDUCTION

- Design of Greedy Algorithms can be viewed as a special case of Divide-And-Conquer:
 - where the problem of size N is divided into
 - a sub-problem of size 1 (or size k for some constant k) and
 - ii. a sub-problem of size N-1 (or N-k as the case may be)
 - where the latter is the same as the original problem.
- Greedy Choice
 - refers to making the right choice locally for sub-problem
 (i).
- Optimal substructure property is necessary to ensure that
 - optimal solution to sub-problem (ii) can be used as is if subproblem (ii) is part of the solution to the problem.

GREEDY CHOICE - LIMITATION

- Greedy Choice does not always hold when Optimal Substructure Property holds.
- e.g. Consider the 0/1 KnapSack problem:
 - Optimal Substructure holds for 0/1 KnapSack:
 - Consider the most valuable subset of items with weight at most W
 - olf we remove item j from this subset, the remaining subset must be the most valuable weighing at most $W-w_{\rm j}$

GREEDY CHOICE - LIMITATION

- Greedy Choice Property does not hold for 0/1 KnapSack:
 - Suppose we use <u>per unit profit</u> as the greedy choice
 - Consider three items:
 - 1. (10kg, Rs. 5,000). Value = Rs. 500 / kg
 - (20 kg, Rs. 8,000) Value = Rs. 400 / kg
 - 3. **(40kg, Rs. 9,000)** Value = Rs. 225 / kg
 - Let W = 60kg
 - o Any solution with item 1 is not optimal!
 - o i.e. ordering by unit weight is not useful

• Exercise:

- Generalize this argument (about remaining capacity) for any input.
- Similarly argue that other greedy choices (profit or weight) are also not useful.