

Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2021-2022, MATH F421: Combinatorial Mathematics

Quiz-II (Closed Book)

Time: 30 Minutes

Date: November 17, 2021

Max. Marks: 25

1. There is no partial marking. For MCQ, -1 mark for incorrect answer. Write down the answer in the provided space only.

Name:

ID:

Part-A

Q.1 For the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + 2^n, \quad a_0 = 2, a_1 = 4,$$

the generating function $g(x) = \frac{A}{1-2x} + \frac{Bx^2}{(1-2x)^C}$, where the respective values of A, B, C are [4]

i. (2, 3, 4)

ii. (1, 3, 3)

iii. (1, 3, 4)

iv. (2, 4, 3)

v. None of these

Q.2 Find a recurrence relation (with initial conditions) for the number of sequences of $0, 1, 2, \dots, 7$ of length n which do not have consecutive even numbers. [3]

$$a_n = 4a_{n-1} + 16a_{n-2}, \quad n \geq 2, \quad a_0 = 1, \quad a_1 = 8$$

Q.3 Write the solution of the recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2} + 2 \times 3^n, \quad a_0 = 1, a_1 = 0.$$

$$a_n = (n^2 - 2n + 1) 3^n \text{ and } (n-1)^2 3^n$$

Q.4 The number of k with $1 \leq k \leq 1000$ which are divisible by 2, 3, or 5 are

[4]

i. 735

ii. 266

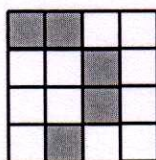
iii. 265

iv. 734

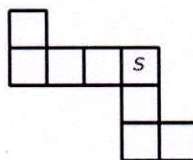
v. None of these

Q.5 Write the rook polynomial for the board in grey in Figure 1a (in the form $(a + bx + cx^2)(d + ex + fx^2)(g + hx + ix^2)$). [3]

$$(1 + 3x + x^2)(1 + 2x)$$



a



b

Figure 1:

Q.6 The rook polynomial for the board in Figure 1b is

[3]

i. $(1 + x)^2 + (1 + 4x + 2x^2)(1 + 3x + x^2)$

ii. $(1 + x)^2 + (1 + 5x + 2x^2)(1 + 4x + x^2)$

iii. $(1 + x)^2 + (1 + 4x + 3x^2)(1 + 3x + 2x^2)$

iv. $(1 + x)(1 + 2x) + (1 + 4x + 2x^2)(1 + 3x + x^2)$

v. None of these

Q.7 Using inclusion-exclusion principle, compute the number of permutations of the letters of the alphabet which do not have a vowel in its actual position? (i.e., a should not be in position 1, e should not be in position 5, and so on.) [4]

$$26! - {}^5C_1 25! + {}^5C_2 24! - {}^5C_3 23! + {}^5C_4 22! - {}^5C_5 21!$$

$$26! - 5 \cdot 25! + 10 \cdot 24! - 10 \cdot 23! + 5 \cdot 22! - 21!$$