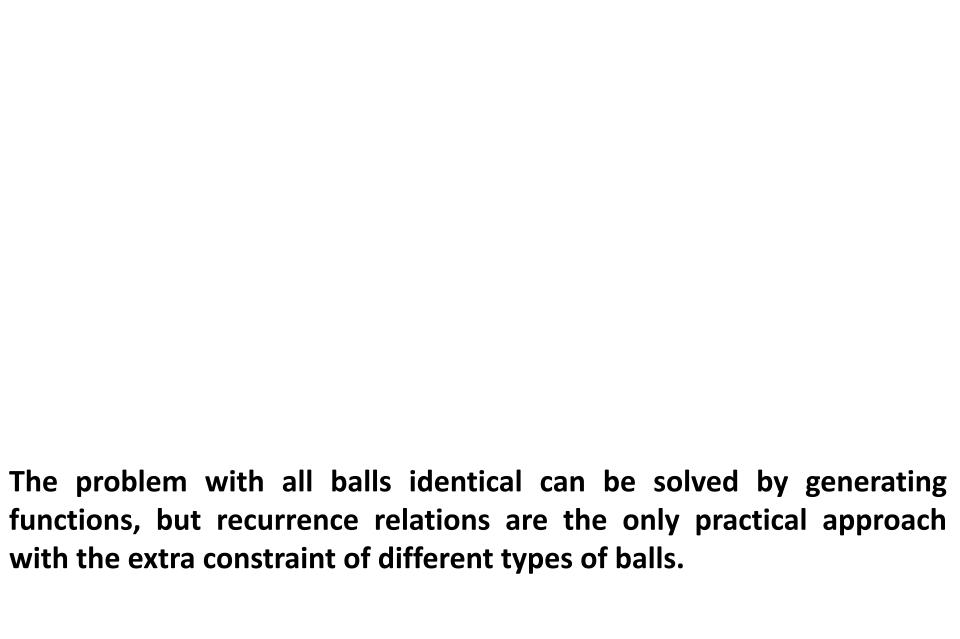
7.1 RECURRENCE RELATION MODELS

We now consider more complex recurrence relations involving two variable relations.

Let $a_{n,k}$ denote the number of ways to select a subset of k objects from a set of n distinct objects. Find a recurrence relation for $a_{n,k}$.

Find a recurrence relation for the ways to distribute n identical balls into k distinct boxes with between two and four balls in each box. Repeat the problem with balls of three colors.



Find recurrence relations for

- (a) The number of n-digit ternary sequences with an even number of 0s
- (b) The number of n-digit ternary sequences with an even number of 0s and an even number of 1s

(b) The number of n-digit ternary sequences with an even number of 0s and an even number of 1s

We will need simultaneous 3 recurrence relations:

 a_n : the number of n-digit ternary sequences with even 0s and even 1s; b_n : the number of n-digit ternary sequences with even 0s and odd 1s; c_n : the number of n-digit sequences with odd 0s and even 1s.

Why not for odd 0s and odd 1s.

Observe that $3^n - a_n - b_n - c_n$ is the number of n-digit ternary sequences with odd 0s and odd 1s.

- a_n is obtained either by having a
- 1 for the first digit followed by (n -1)-digit sequence with even 0s and odd 1s,
- or a 0 followed by (n-1)-digit sequence with odd 0s and even 1s, or a 2 followed by an (n-1)-digit sequence with even 0s and even 1s.

Thus
$$a_n = b_{n-1} + c_{n-1} + a_{n-1}$$
.

But from where we get b_{n-1} and c_{n-1}

Similar analyses yield

$$b_{n} = a_{n-1} + (3^{n-1} - a_{n-1} - b_{n-1} - c_{n-1}) + b_{n-1} = 3^{n-1} - c_{n-1}$$
 and
$$c_{n} = a_{n-1} + (3^{n-1} - a_{n-1} - b_{n-1} - c_{n-1}) + c_{n-1} = 3^{n-1} - b_{n-1}$$

The initial conditions are $a_1 = b_1 = c_1 = 1$.

To recursively compute values for a_n , we must simultaneously compute b_n and c_n .