SIMPLE ARRANGEMENTS AND SELECTIONS

Permutation and Combination

A **permutation** of n distinct objects is an **ordered arrangement** **(without repeatition)** of the n objects.

An ***r*-permutation** of n distinct objects is an arrangement using r of the n objects.

An ***r*-combination** of n distinct objects is an **unordered selection** **(without repeatition)**, or *subset*, of r out of the n objects.

We use $P(n, r)$ and $C(n, r)$ to denote the number of r-permutations and r-combinations, respectively, of a set of n objects.

How to derive mathematical expressions for $P(n, r)$ and $C(n, r)$

Computing $P(n, r)$

From the multiplication principle we obtain

$$P(n,2) = n(n-1), \qquad P(n,3) = n(n-1)(n-2),$$
$$P(n,n) = n(n-1)(n-2) \times \cdots \times 3 \times 2 \times 1$$

In enumerating all permutations of n objects, we have n choices for the first position in the arrangement, $n-1$ choices (the $n-1$ remaining objects) for the second position, …, and finally one choice for the last position. Using the notation $n! = n(n-1)(n-2)\cdots \times 3 \times 2 \times 1$ ($n!$ is said "n factorial"), we have the formulas

$$P(n,n) = n!$$

and

$$P(n,r) = n(n-1)(n-2) \times \cdots \times [n-(r-1)] = \frac{n!}{(n-r)!}$$

Computing $C(n, r)$ using $P(n, r)$

Our formula for $P(n, r)$ can be used to derive a formula for $C(n, r)$. All r-permutations of n objects can be generated by first picking any r-combination of the n objects and then arranging these r objects in any order. Thus $P(n, r) = C(n, r) \times P(r, r)$, and solving for $C(n, r)$ we have

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!/(n-r)!}{r!} = \frac{n!}{r!(n-r)!}$$

Q. How many ways are there to arrange the seven letters in the word SYSTEMS? In how many of these arrangements, do the three Ss appear consecutively?

How many different 8-digit binary sequences are there with six 1s and two 0s?

Permutation or Combination?

Here order is not important

A committee of k people is to be chosen from a set of seven women and four men. How many ways are there to form the committee if

(a) The committee consists of three women and two men?

(b) The committee can be any positive size but must have equal numbers of women and men?

(c) The committee has four people and one of them must be Mr. Baggins?

(d) The committee has four people and at least two are women?

(e) The committee has four people, two of each sex, and Mr. and Mrs. Baggins cannot both be on the committee?

Permutation or Combination?

In all cases, order is not required

a. A committee of k people is to be chosen from a set of seven women and four men.
How many ways are there to form the committee if **the committee consists of three women and two men?**

b. A committee of k people is to be chosen from a set of seven women and four men. How many ways are there to form the committee of any positive size with equal number of women and men.

c. A committee of k people is to be chosen from a set of seven women and four men.
How many ways are there to form the committee if the committee has four people and one of them must be Mr. Baggins?

d. How many ways are there to form the committee if the committee has four people and at least two are women?

e. A committee of k people is to be chosen from a set of seven women and four men.
How many ways are there to form the committee if the committee has four people, two of each sex, and Mr. and Mrs. Baggins cannot both be on the committee?

How many sub-cases are required.

A complementary approach.

We can consider all $C(7, 2)\times C(4, 2)$ 2-women–2-men committees and then subtract the forbidden committees that contain both Bagginses.

The forbidden committees are formed by picking one more woman and one more man to join Mr. and Mrs. Baggins—done in $C(6, 1)\times C(3, 1)$ ways.

Answer: $21\times6-6\times3 = 108$.

How to grapple with the two constraints simultaneously

How many arrangements of the seven letters in the word SYSTEMS have the E occurring somewhere before the M?

C(7, 2) * P(5, 2)
C(7, 2) * C(5, 2)
P(7, 2) * P(5, 2)
C(7, 2) + P(5, 2)
P(7, 2) + P(5, 2)
C(7, 2) + C(5, 2)

We start by picking which of the two out of the seven positions in an arrangement are where the E and M will go (Permutation or Combination)
:C(7, 2) = 21 ways (**how, why**)
(**count it by fixing the position of E and moving the position of M 6+5+4+3+2+1**).

Here there is an order first E and then M but we don't use permutation because ME is not allowed. In a bigger picture, there is no ordering.

Now we fill in the five other positions in the arrangement by picking a position for the Y and the T: (Permutation or Combination)

$P(5, 2) = 5 \times 4 = 20$ ways
and then putting the three Ss in the three remaining positions.
The answer is thus $21 \times 20 = 420$.

Hw. A manufacturing plant produces ovens. At the last stage, an inspector marks the ovens A (acceptable) or U (unacceptable). How many different sequences of 15 As and Us are possible in which the third U appears as the twelfth letter in the sequence?

Ans. 440 sequences.