6.4 EXPONENTIAL GENERATING FUNCTIONS

The generating functions used in the previous sections to model selection with repetition problems are called **ordinary generating functions.**

In this section we discuss **exponential generating functions**.
They are used to model and solve problems involving arrangements with repetition.

Consider the problem of finding the number of different words (arrangements) of four letters when the letters are chosen from an unlimited supply of ***a***s, ***b***s and ***c***s, and the word must contain at least two ***a***s.

What is the generating function and corresponding equation for the given problem.

An **exponential generating function** $g(x)$ for a_r, the number of arrangements with r objects, is a function with the power series expansion

$$g(x) = a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots + a_r \frac{x^r}{r!} + \cdots$$

We build exponential generating functions in the same way that we build ordinary generating functions,

However, now each power x^r is divided by r!.

As an example, let us consider the four-letter word problem with at least two as.

We claim that the exponential generating function for the number of r-letter words formed from an unlimited number of as, bs, and cs containing at least two as is

Find the exponential generating function for a_r, the number of different arrangements of r objects chosen from four different types of objects with each type of object appearing at least two and no more than five times.

Find the generating function for a_r , the number of r arrangements without repetition of n objects. Hence, find a_r

Find the generating function for the number of ways to place r (distinct) people into three different rooms with at least one person in each room. Repeat with an even number of people in each room.

Using generating function, find the number of different arrangements of r objects chosen from unlimited supplies of n types of objects.

Using generating function, find the number of sequences of length 8 that can be formed using 1,2, or 3 a's; 2,3, or 4 b's; and 0,2, or 4 c's.

Using exponential generating function, find the number of ways to place 25 people into three rooms with at least one person in each room.

Using exponential generating function, find the number of r-digit quaternary sequences (whose digits are 0, 1, 2, and 3) with an even number of 0s and an odd number of 1s.

Find the number of ways to distribute n different objects to five different boxes if

a) An even number of objects are distributed to box 5.

b) A positive even number of objects are distributed to box 5.