Non-HOMOGENEOUS LINEAR RECURRENCE RELATIONS

Find a particular solution of the nonhomogeneous linear recurrence $a_n = a_{n-1} + a_{n-2} + 2n$.

In this section, we discuss methods for solving inhomogeneous recurrence relations of the form of (1). The key idea in solving these relations is that a general solution for an inhomogeneous relation is made up of a general solution to the associated homogeneous relation [obtained by deleting the g(n) term] plus any one particular solution to the inhomogeneous relation.

Find a solution of the nonhomogeneous linear recurrence relation $a_n = a_{n-1} + 2a_{n-2} - 4$ satisfying the initial conditions $a_0 = 6$ and $a_1 = 7$.

Find a particular solution of the nonhomogeneous linear recurrence relation $a_n = a_{n-1} + 3a_{n-2} + a_{n-3} + 3^n$.

Find the general solution of $a_n = 2a_{n-1} + 2^n$.

Normally, when the nonhomogeneous part of a linear recurrence relation $g(n)$ is an exponential function kb^n, we can expect to obtain a particular solution that is also an exponential function. This is obviously not the case when b is a characteristic root of the associated homogeneous linear recurrence relation. However, in this case, it will still be possible to find a particular solution of a slightly different form. When b is a root of multiplicity m, we can show that there will always be a particular solution of the form $p(n) = cn^m b^n$.

Solve the recurrence relation $a_n = 3a_{n-1} - 4n + 3 \times 2^n$ to find its general solution. Also find the solution when $a_1 = 8$.