Theory of Computation
CS F351

Vishal Gupta

Department of Computer Science and Information Systems
Birla Institute of Technology and Science
Pilani Campus, Pilani

BITS Pilani

Pilani|Dubai|Goa|Hyderabad

Agenda

- Recursive Languages
- Recursive Functions
- Recursively Enumerable Languages
- Prove: "If L is recursive, then it is Recursively enumerable"
- Prove: "If L is recursive, then its complement L' is also recursive".

- "M decides L"

  Let $\sum_0$ the alphabet over which input string is defined. Thus we allow the TM to use extra symbols during the computation.

  M decides a language if for any string w ϵ $\sum_0$* the following is true:

"If w ϵ L then M accepts w; and if w does not belong to L, then M reject w"

Recursive Language / Turing decidable language

- A language L is recursive if there is a TM that "decides" it.

Recursive functions

- Let $\sum_0$ be $\sum$ - {Δ, ␣ }.
- Suppose M halts on input w and from configuration $\triangleright \underline{␣}\, w$ the TM halts in configuration $\triangleright \underline{␣}\, y$, where $y \in \sum_0^*$. y is called the output of M on w and is denoted as M(w).
- So M(w) is defined only if M halts on input w.
- Let f be any function from $\sum_0^*$ to $\sum_0^*$. TM M computes function f if for all $w \in \sum_0^*$, M(w) = f(w).
- In other words, for all $w \in \sum_0^*$, M eventually halts on w and when it halts that tape contents are $\Delta \underline{␣}\, f(w)$.
- A function f is recursive if there is a TM that computes f.

"M semi-decides L"

TM M semidecide L if for any string w ϵ $\sum_0$* the following is true:

- w ϵ L iff M halts on input w

Recursively Enumerable Languages

- A language L is recursively enumerable iff there is a TM that semi-decides it.

TM's that semi-decides languages are no algorithms

If a language is recursive, then it is recursively enumerable:

Proof:

Let M = (K, ∑, δ, s {y, n}) be a TM that decides L. We can define a machine M' that semi-decides L as follows:

M' = (K, ∑, δ', s {y}), where δ' contains additionally the transitions of the form:

δ' (n, a) = (n, a) for all a ϵ ∑ and n is not a halting state.

- A recursive language is Recursively Enumerable, but not vice versa.
- If L is recursive, then its complement L' is also recursive.
- Recursively Enumerable languages are not closed under complement.

innovate achieve lead

BITS Pilani

Pilani|Dubai|Goa|Hyderabad

Thank You