Minimization of DFA

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More Pumping Lemma Examples

Prove that the language $L = \{a^n b^m \mid m > n\}$ is not regular.

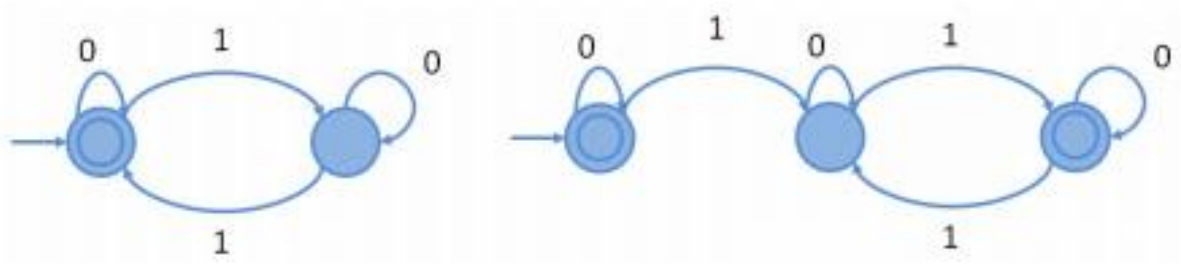
Minimization of DFA

DFA Minimization

Every DFA accepts a unique language.

- But in general, there may be many DFAs for a given language.
- In practice, we are interested in DFA with the minimal number of states.

These DFAs accept the same language.



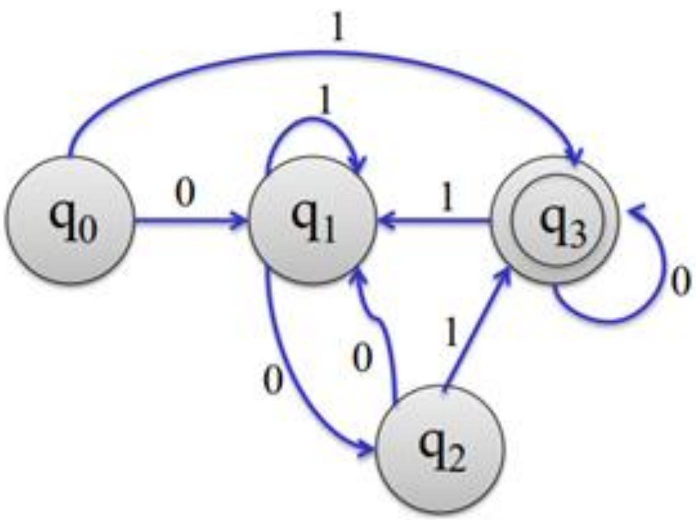
How to decide which states to Collapse?

Intuitively two states are mergeable if they behave similarly (in terms of language acceptance) for the same input string

Starting from respective states, with the same input string, either both lead to respective final states or none lead to respective final states

- Turns out to be a necessary and sufficient condition

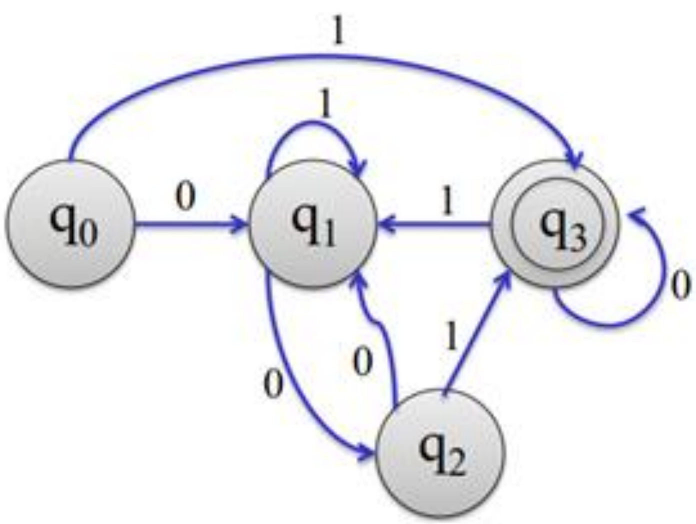
Example



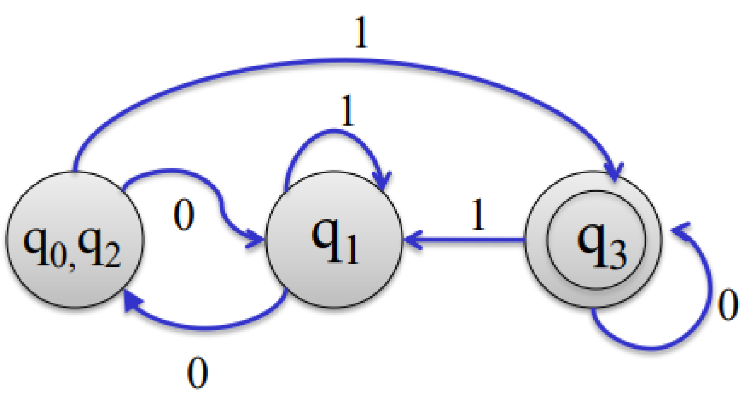
Here, states, q0 and q2 have the same behavior: – For each input alphabet, both go to the same state: with 1 to q3 and with 0 to q1 . Hence, q0 and q2 are equivalent.

However, states, q2 and q3 are not showing similar behavior: – since, q2 on 1 going to accept state q3 whereas, q3 on 1 going to non-accept state q1. Hence, q2 and q3 are not equivalent states.

Example



Non-Minimal DFA



Minimal DFA

State Equivalence

Two states are equivalent (also known as indistinguishable) if there is no string which leads from one to an accepting state and from the other to a non-accepting state.

Two states p and q of a DFA are called indistinguishable **($p \equiv q$)** if $\delta(p, w) \in F$ implies $\delta(q, w) \in F$, or $\delta(p, w) \notin F$ implies $\delta(q, w) \notin F$, for all $w \in \Sigma^*$

Distinguishable States

Two states p and q are not equivalent if there is a string w such that either δ (q,w) $\in$ F and δ(p,w) $\notin$F or δ (q,w) $\notin$ F and δ(p,w) $\in$ F

In other words, we can also say that p and q are separated by w.

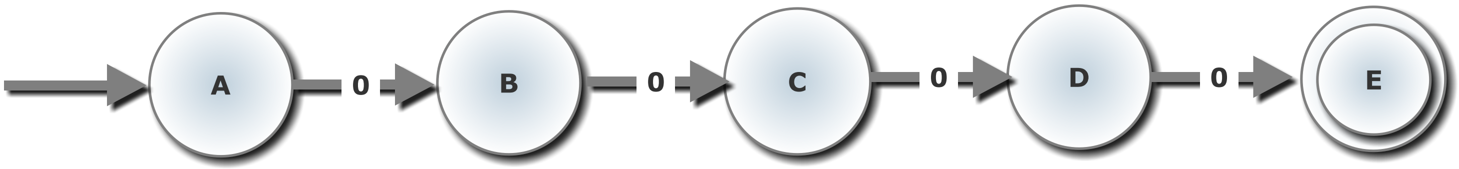
K-Equivalence

Two states p and q are k-equivalent (**$p \equiv_k q$**) if they have no separating string of length $\leq k$

Two K-equivalent states are always K-1, K-2, K-3,----0 equivalent.

- Two equivalent states are always 0-equivalent.
- Two final and two non-final states are always 0-equivalent but need not to be 1-equivalent.

Example



The states A and B are 2-equivalent because there is no string (of length 0,1,or 2) that separates them. Formally: $A \equiv_2 B$

But, states A and B are not 3-equivalent because there is a string, w=000, that separates them. $\delta(A,000)=D \notin F$ and $\delta(B,000)=E \in F$

Steps for Minimizing the DFA

Step 1

- Construct an 0-equivalence class by taking set of accept states in one set and set of non-accept states in another set.

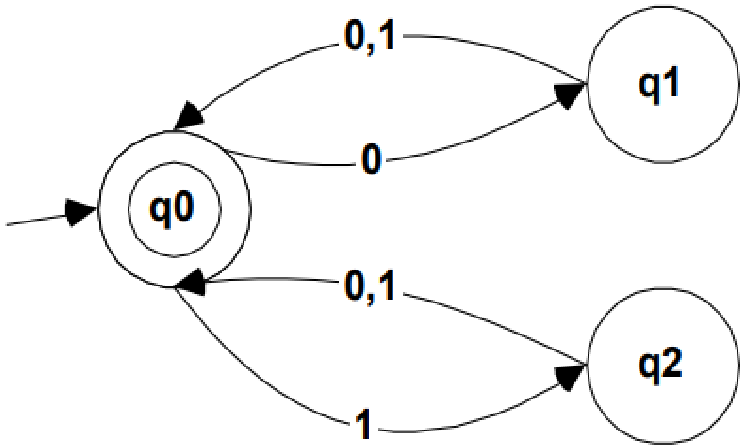
Step 2

- Repeat this process of calculating the equivalence classes until K-equivalent and K-1 equivalent classes are same.

Step 3

- Construct a DFA by taking all the sets of K-equivalence class as new states of minimal DFA.

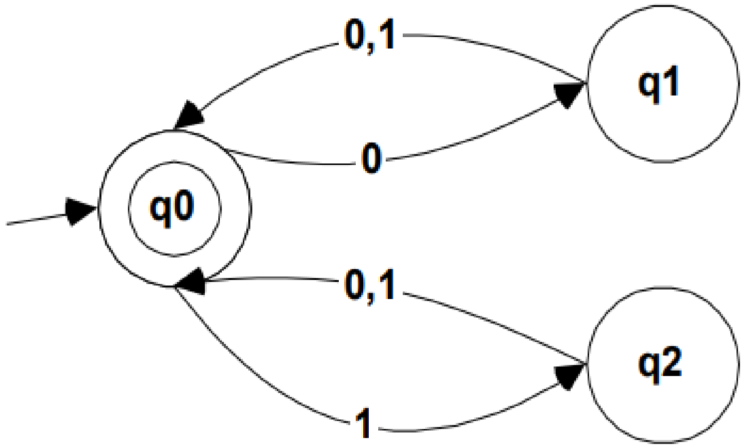
Example



For E_0, we have string of length 0 that separates between q0 and q1 ,q2 .
Hence, E_0 is composed of two equivalents classes: {q0},{q1,q2}.

E_0 = {{q0}, {q1,q2}}

Example (Continued……)

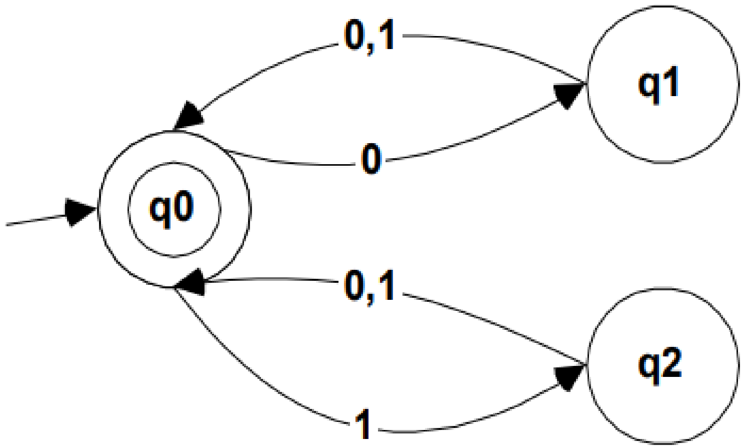


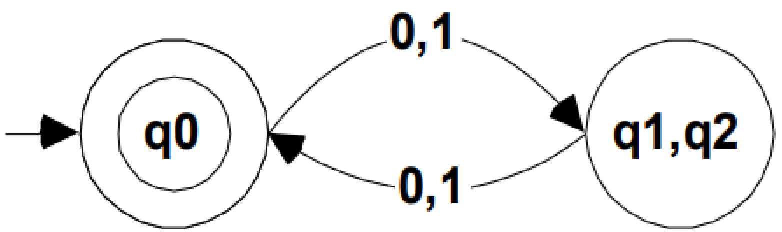
E_0 : {{q0}, {q1,q2}}

- The set with only one state can't be further divided into subclasses.
- But, the set with more than one state can be divided. So, lets check the set {q1 ,q2 }.
- For any string of length equal to 1 both of them lead to the same state q0.
- So, there is no separating word ($|w| \leq 1$) between q1 and q2 .
- That means that they are 1-equivalent.
- Hence: E_1 : {{q0}, {q1,q2}}

Example (Continued……)

Since, we are getting the same sets in both the K and K-1 equivalence classes, Stop

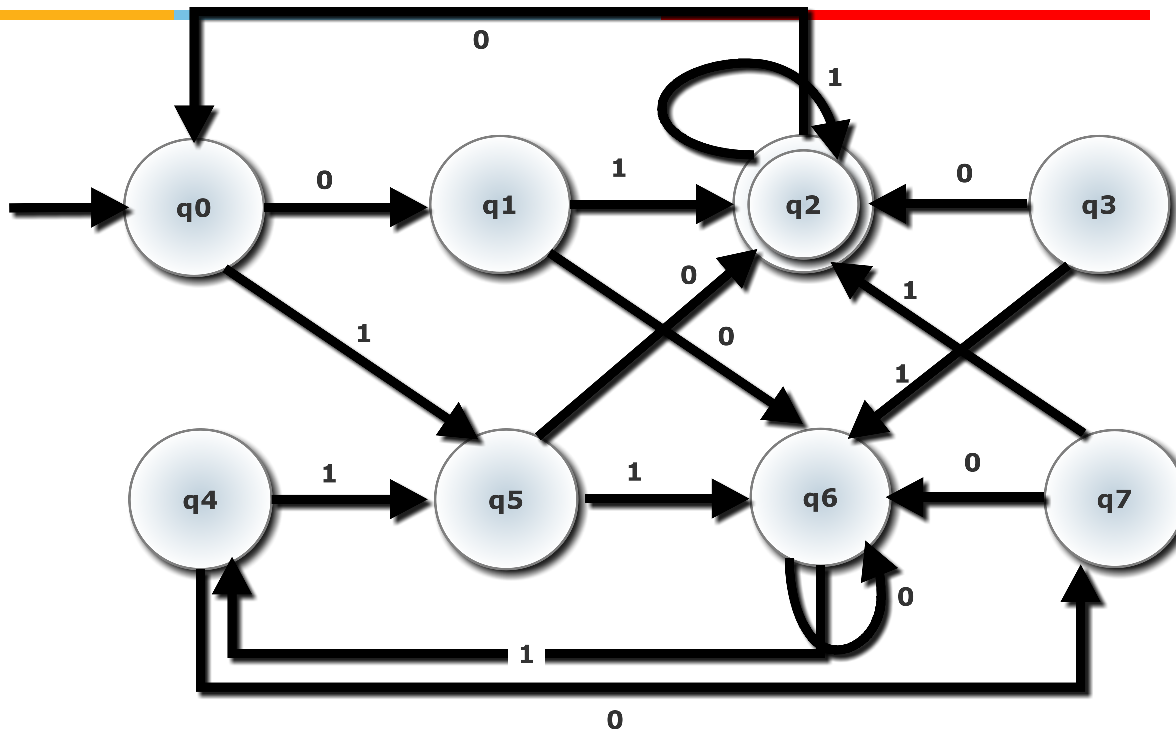




Minimal DFA

E_0 : {{q0}, {q1,q2}}
E_1 : {{q0}, {q1,q2}}

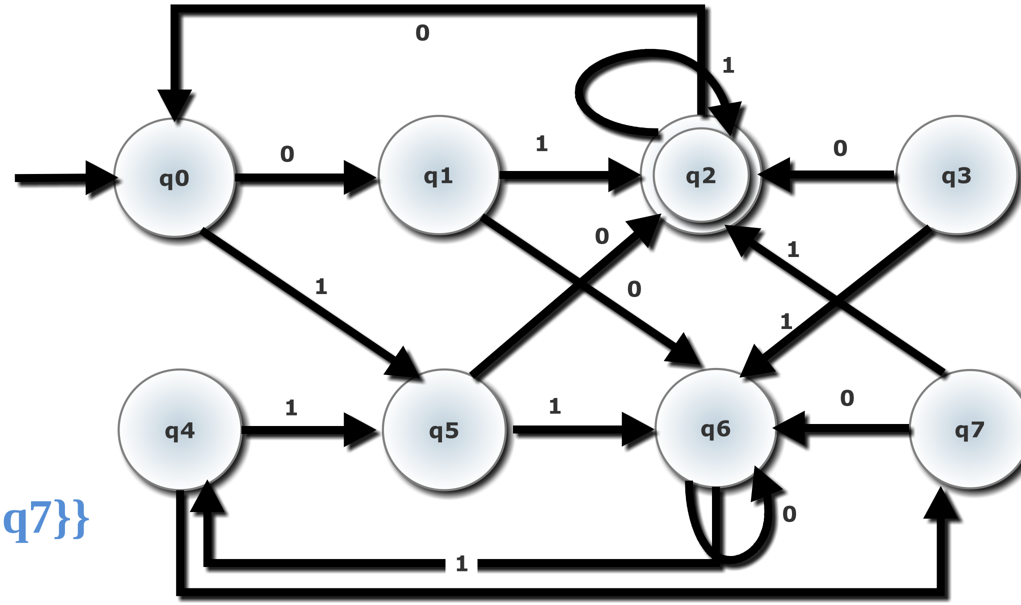
More Examples



For E_0, we have string of length 0 that separates between q2 and q0 ,q1, q3, q4, q5, q6, q7 .

Hence, E_0 : {{q2}, {q0 ,q1, q3, q4, q5, q6, q7}}.

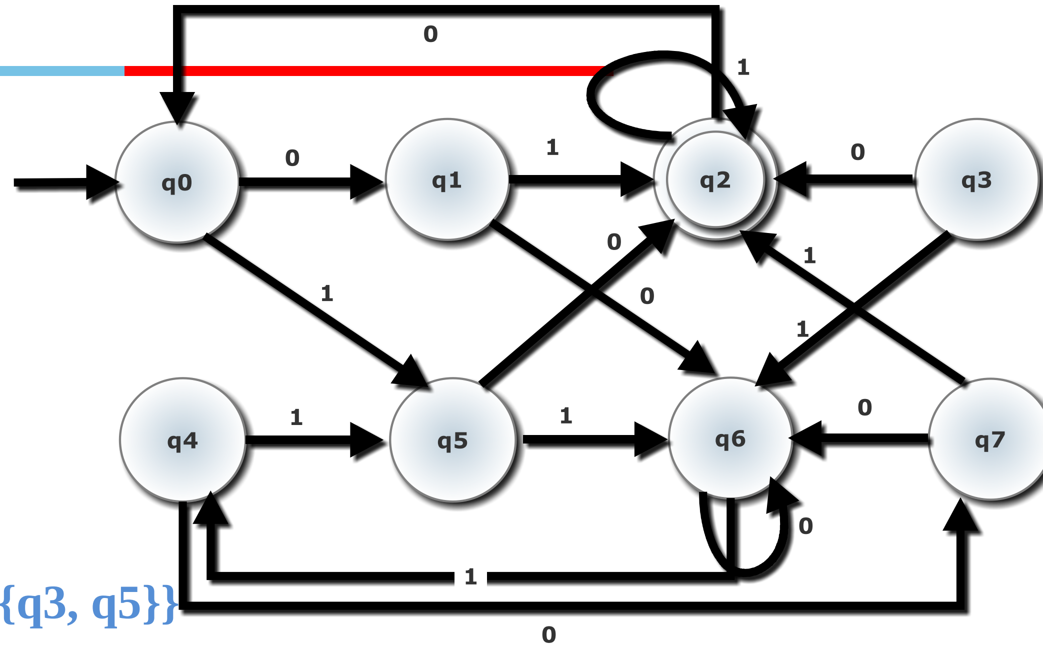
Example (Continued…..)



E_0 : {{q2}, {q0 ,q1, q3, q4, q5, q6, q7}}

- For any string of length equal to 1, the states {q0, q4, q6} over Σ={0,1} can lead to combination of {NF,NF}. Hence, they are 1-equivalent states.
- On the other hand, the states {q1, q7} over Σ={0,1} can lead to combination of {NF,F}for all strings of length 1.
- Similarly, the states {q3, q5} over Σ={0,1} can lead to combination of {F,NF}for all strings of length 1.
- That means that they are 1-equivalent.
- Hence: E_1 : {{q2}, {q0, q4, q6}, {q1, q7}, {q3, q5}}

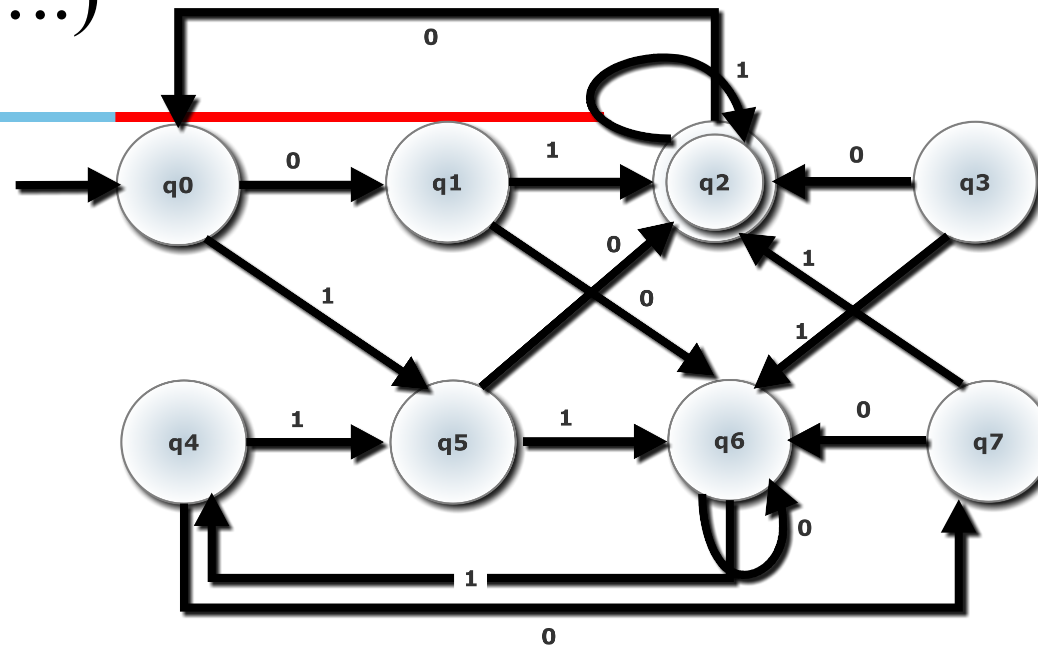
Example (Continued…..)



E_1 : {{q2}, {q0, q4, q6}, {q1, q7}, {q3, q5}}

- For any string of length equal to 2, the states {q0, q4, q6} can be further partitioned into {q0, q4} and {q6} since, **δ(q0,01) = q2 ∈ F, δ(q4,01) = q2 ∈ F. However, δ(q6,01) = q4 ∈ NF**
- On the other hand, the states {q1, q7} cannot be further partitioned since **δ(q1,00) = q6 ∈ NF, δ(q7,00) = q6 ∈ NF.**
- Similarly, the states {q3, q5} cannot be further partitioned since **δ(q3,01) = q2 ∈ F, δ(q5,01) = q2 ∈ F**. That means that they are 2-equivalent.
- Hence: **E_2 : {{q2}, {q0, q4}, {q6}, {q1, q7}, {q3, q5}}**

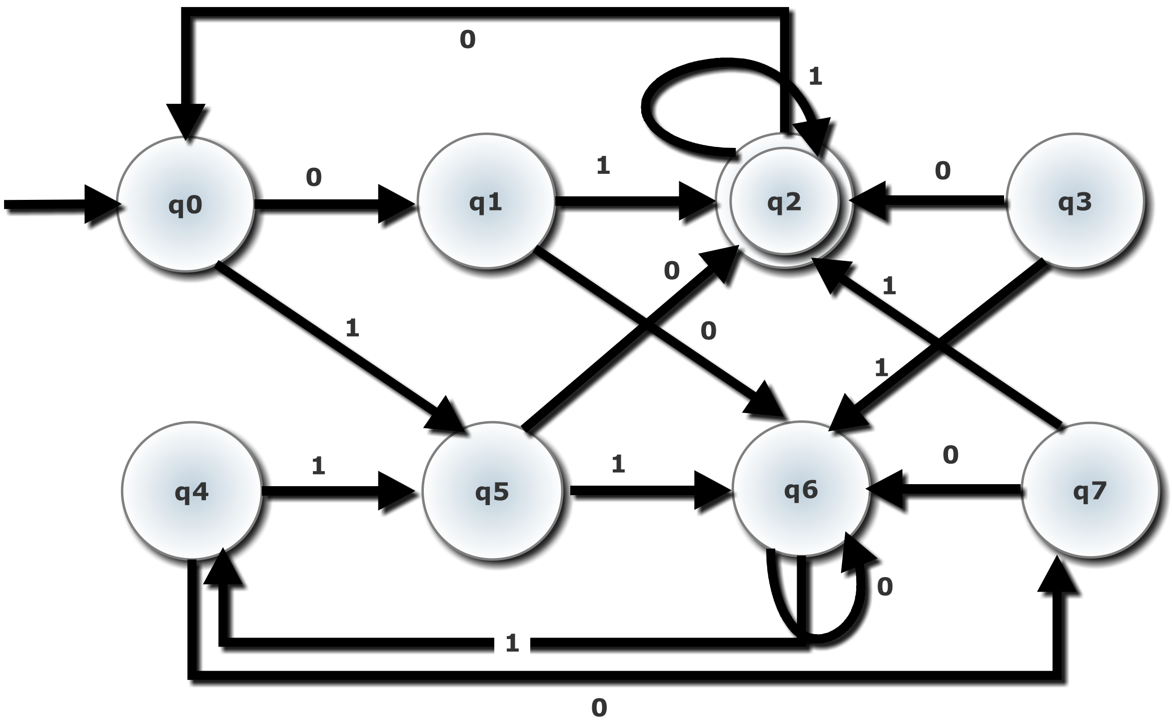
Example (Continued…..)



- E_2 : {q2}, {q0, q4}, {q6}, {q1, q7}, {q3, q5}

- For any string of length equal to 3, the states {q0, q4} cannot be further partitioned since, $\delta(q0,011) = q2 \in F$, $\delta(q4,011) = q2 \in F$.
- On the other hand, the states {q1, q7} cannot be further partitioned since $\delta(q1,000) = q6 \in NF$, $\delta(q7,000) = q6 \in NF$.
- Similarly, the states {q3, q5} cannot be further partitioned since $\delta(q3,011) = q2 \in F$, $\delta(q5,011) = q2 \in F$. That means that they are 3-equivalent.
- Hence: E_3 : {{q2}, {q0, q4}, {q6}, {q1, q7}, {q3, q5}}

Example (Continued…..)



I/P Symbol / STATE	0	1
→$[q_0q_4]$	$[q_1q_7]$	$[q_3q_5]$
$[q_1q_7]$	q_6	q_2
$[q_3q_5]$	q_2	q_6
q_2	$[q_0q_4]$	q_2
q_6	q_6	$[q_0q_4]$

E_0 : {{q2}, {q0 ,q1, q3, q4, q5, q6, q7}}
E_1 : {{q2}, {q0, q4, q6}, {q1, q7}, {q3, q5}}
E_2 : {{q2}, {q0, q4}, {q6}, {q1, q7}, {q3, q5}}
E_3 : {{q2}, {q0, q4}, {q6}, {q1, q7}, {q3, q5}}

Transition Table Notation for Minimal DFA