Formal Definition

Definition. A Turing machine is a 5-tuple $(K, \Sigma, \delta, s, H)$, where:

- K is a finite set of states
- Σ is a set of symbols (the alphabet). It contains the blank symbol ($\sqcup$), and the left end symbol ($\triangleright$). It does not contain $\leftarrow$ and $\rightarrow$
- $s \in K$ is the initial state.
- H is a subset of K, and is the set of halting states.
- δ, the transition function, is a function from
  $(K-H) \times \Sigma$ to $K \times (\Sigma \cup \{\rightarrow, \leftarrow\})$, such that,
  - For all $q \in K-H$, if $\delta(q, \triangleright) = (p, b)$, then $b = \rightarrow$
  - For all $q \in K-H$ and $a \in \Sigma$, if $\delta(q, a) = (p, b)$ then $b \neq \triangleright$

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Configuration of a TM

- Configuration of a TM is a member of:

  $K \times \triangleright\Sigma^* \times (\Sigma^* (\Sigma - \{\sqcup\}) \cup \{e\})$
- In other words, configuration specifies the tape contents, state, and position of head. For e.g., following are all configurations:
- $(q, \triangleright a, aba) \rightarrow \triangleright \underline{a}aba$
- $(h, \triangleright \sqcup \sqcup \sqcup, \sqcup a) \rightarrow \triangleright \sqcup \sqcup \underline{\sqcup} \sqcup a$
- $(q, \triangleright \sqcup a \sqcup \sqcup, e) \rightarrow \triangleright \sqcup a \sqcup \underline{\sqcup}$

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My First Turing Machine

Construct a Turing machine such that:

- **input**: a substring over the alphabet Σ ={a,b}, the substring contains at least 1 character b
- the header is pointing to the first cell in tape
- When the Turing machine halts, the header must be pointing to the first occurrence of b in the input

State	Input x	δ (q, x)
q_0	⊔	$(q_0, \rightarrow)$
q_0	a	$(q_0, \rightarrow)$
q_0	b	$(h, \rightarrow)$

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Example

Design a TM for the language L = {w w^R | w ∈ (a + b)*}

State	Input x	δ (q, x)
q0	⊔	$(q_1, \rightarrow)$
q_1	a	$(q_2, ⊔)$
q_2	⊔	$(q_4, \rightarrow)$
q_4	a	$(q_4, \rightarrow)$
q_4	b	$(q_4, \rightarrow)$
q_4	⊔	$(q_5, \leftarrow)$
q_5	a	$(q_5, ⊔)$
q_5	⊔	$(q_6, \leftarrow)$
q_6	a	$(q_6, \leftarrow)$
q_6	b	$(q_6, \leftarrow)$
q_6	⊔	$(q_1, \rightarrow)$
q_1	⊔	$(h, \rightarrow)$

State	Input x	δ (q, x)
q_1	b	$(q_3, ⊔)$

When to halt ?

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