Birla Institute of Technology and Science, Pilani
First Semester 2020 – 21
CS F351 (Theory of Computation)
Test-2 (Solution and Marking Scheme)
October 16, 2020

Q1. [4 Marks] Consider the following four languages. Each of these falls into one of the following classes: *Regular Language* (RL), *Deterministic Context Free Language* (DCFL), Non-deterministic *Context Free language* (NDCFL), or *Not a CFL* (NCFL). Classify each of the language correctly.

[**Note**: *You have to select the most appropriate class (i.e. if a language is RL, then it is surely a CFL also; but your answer should be RL*).

a) Over $\sum = \{2, 3, 4\}$, $L = \{2^x 3^y 4^z \mid y < x \text{ and } y < z\}$
   L is not a CFL.
b) Over $\sum = \{2, 3, 4\}$, $L = \{4^*3^*2^*\} - \{4^k 3^k 2^k \mid k >= 0\}$
   L is a non-deterministic CFL.
c) Over $\sum = \{x, y\}$, $L = \{w_1w_2 \mid w_1, w_2 \in (x+y)^*, |w_1| = |w_2|, \text{ and } w_1 \neq w_2\}$
   L is non-deterministic CFL.
d) Over $\sum = \{0, 1\}$, $L = \{0^{3P+1} 1^{5p-2}, p \geq 1\}$
   L is a deterministic Context Free Language.

[Marking scheme: 1M for each correct answer.]

Q2. [6 Marks] (For this question, since you have to type the answer from keyboard, we are not using Greek letter symbols in this question. Rather we would use their corresponding English counterpart).

For some language L, let **sigma** be an input alphabet. Let there be any two strings w_1 and w_2 such that w_1 belongs to **sigma*** and w_2 also belongs to **sigma***. We write $w_1\#w_2$ if few symbols from w_1 (adjacent or non-adjacent) can be removed to get w_2. For example, over **sigma** = {a, b, c, d}, we can write adacba#ac, adacba#aaa , adacba#adacb; but we cannot write adacba#cab, adacba#caa.

On language L, define an operation $O(L) = \{w_2 \mid w_1 \text{ belongs to } L, \text{ and } w_1\#w_2\}$. We have to prove that CFL's are closed under O(L) operation.

Proof: Suppose L is a CFL. Then there exists a PDA M1 = (q1, sigma1, tau1, delta1, s1, F1) that generates L. The construction of PDA M2 = (q2, sigma1, tau1, delta2, s2, F2) that generates O(L) is as follows:

Complete the above proof by giving q2, delta2, s2 and, F2. Write your answer as follows:

q2 = __________, delta2 = ______________, s2 = _______________, F2 = _____________.

Q2 = q1, s2 = s1, F2 = F1,

delta2 = delta1 U {((q, epsilon, beta) (q', gamma)) for all ((q, a, beta) (q', gamma)) that belongs to delta1}.

Marking Scheme: 1M each for Q2, s2, and F2. 3M for delta2.

Q3. [4 Marks] Check whether or not the grammar G defined below accepts the language $L = \{ 0^i 1^j 2^k \mid i, j, k >= 1, j \geq i+k\}$. If *YES*, give all the strings of length 4 generated by G. If *NO*, give one string which is not accepted by G.

$$G = (\{S, X, Y\}, \{0, 1, 2\}, \{S \rightarrow 0X11Y2 \quad , \quad X \rightarrow 0X1|\varepsilon|1 \quad , \quad Y \rightarrow 1Y2|\varepsilon|1\}, S).$$

Sol: The given grammar does not accept the language L. For example, the string 0111112 cannot be generated by the given grammar.

The language of the grammar is: $0^n 1^{n+m} 2^m \cup 0^n 1^{n+m+1} 2^m \cup 0^n 1^{n+m+2} 2^m$

As an answer you were supposed to give one string which is not accepted by G.

[1M for writing NO, i.e. the given grammar does not generate L. 3M for the correct counter example in the form of one string.]

Q4. [4 Marks] Over sigma = {4, 3}, suppose $L_1 = \{4^n 3^n \mid n \neq 100, \text{ and } n \geq 0\}$. See the following proof to prove that L_1 is a CFL. Complete the proof.

To Prove: The given language L_1 is a CFL.
Proof: Let $L_2 = \{4^{100} 3^{100}\}$. We know that L_2 (as well as its complement) is a Regular Language.

Let $L_3 = \{4^n 3^n \mid n \geq 0\}$. We know that L_3 is a CFL.

…… Complete the Proof using L_2 and L_3.

Sol: L1 = complement(L2) $\cap$ L3

Since CFL $\cap$ RL is a CFL, L1 is a CFL.

[Marking Scheme: 0/4 M.]

Q5. [4 Marks] Consider the following Language L_1 over $\Sigma = \{0, 1, 2\}$

$$L_1 = \{\# \; 2 \; \#^R \; 2 \; \# \mid \# \in (0 + 1)^*\}$$

a) Is L_1 Context-Free Language?
b) Write L_1 as the intersection of two context-free languages (say L_2 and L_3). You need to write L_2 and L_3 in a generalized manner (just like L_1).
[Note: If you want to write x^n, use ^ sign and write x^n.]

[Sol: L1 is not a CFL.

Marking Scheme: 1M for writing that L1 is not a CFL. 3M for giving two languages whose intersection if L1.]

Q6. [2 Marks] Consider the following two different CFG's whose production rules as shown below. In both the grammars, upper case letters are non-terminal symbols, lower case letters are terminal symbols, and S is the start symbol. Which of the these can be simulated by a DFA, and which cannot?

S-> aSa | bSb | aSb | bSa | ε
S -> a | aSa | aSb | bSa | bSb
[Sol: First one is a Regular Language and second one is a CFL (but not regular).

1M for each correct answer]

Q7. [2 Marks] Consider the following two different CFG's whose production rules as shown below. In both the grammars, upper case letters are non-terminal symbols, lower case letters are terminal symbols, and S is the start symbol. Which of the these is/are ambiguous grammars?

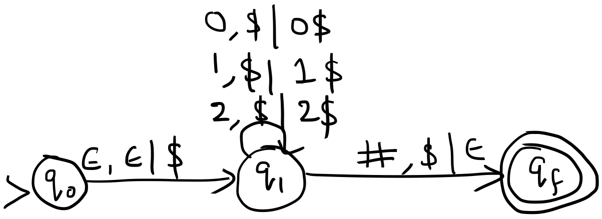
S-> aSa | bSb | aSb | bSa | ε
S -> a | aSa | aSb | bSa | bSb
[Sol: Both the grammars are non-ambiguous. 1M for each.]

Q8. [4 Marks] Over ∑ = {0, 1, 2} and \$ as bottom of the stack, Let L = {w ∊ ∑* | number of 0's in w = number of 1's + number of 2's}. Consider the following ***deterministic*** PDA
M = ({q0, q1, qf} , {0, 1, 2} , {0, 1, 2, \$} , Δ , q0 , qf), where Δ is defined as follows:

(q0, ε, ε) -> (q1, \$)
(q1, 0, \$) -> (q1, 0\$)
(q1, 1, \$) -> (q1, 1\$)
(q1, 2, \$) -> (q1, 2\$)
(q1, #, \$) -> (qf, ε)

Few transitions are missing in the given PDA. Write these missing transitions. You do not require to add any new state.

[**Note**: While writing your answer you can use e to denote epsilon.]



0, 0 | 00
0, 1 | ε
0, 2 | ε

1, 0 | ε
1, 1 | 11
1, 2 | 12

2, 1 | 21
2, 2 | 22
2, 0 | ε

[Marking Scheme: There are three (logical) sets of transitions as shown. Partial marks were given set-wise, and not transition-wise. That is, if any set is wrong, 1.5 M were deducted.]