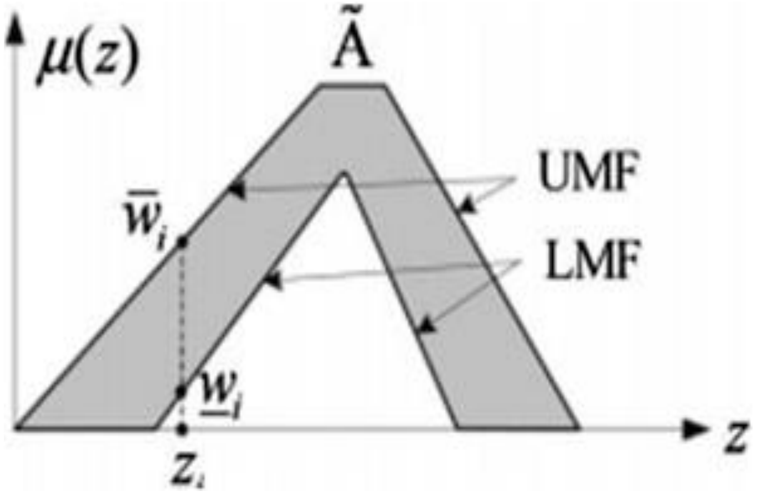
Type-2 Fuzzy Logic

✓ Membership values/functions are not crisp

✓ More vague/noisy environment



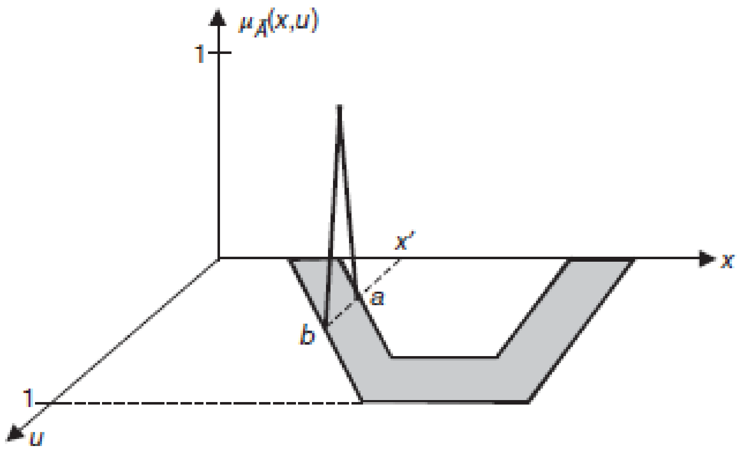
$$\mu_{\tilde{A}} = \left\{ \frac{0.7}{2} , \frac{0.4}{3} \right\}$$: T-1

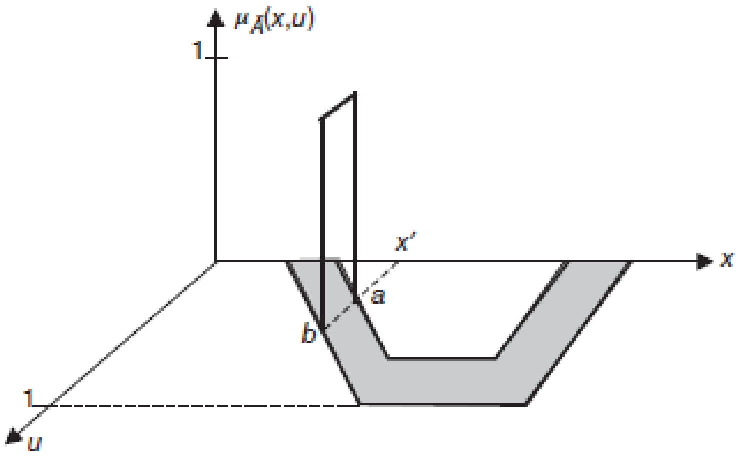
$$\mu_{\tilde{A}} = \left\{ \frac{\left\{\frac{0.7}{0.6}, \frac{1}{0.7}, \frac{0.7}{0.8}\right\}}{2} , \frac{\left\{\frac{0.3}{0.0}, \frac{0.7}{0.2}, \frac{1}{0.4}, \frac{0.7}{0.6}, \frac{0.3}{0.8}\right\}}{3} \right\}$$: GT-2

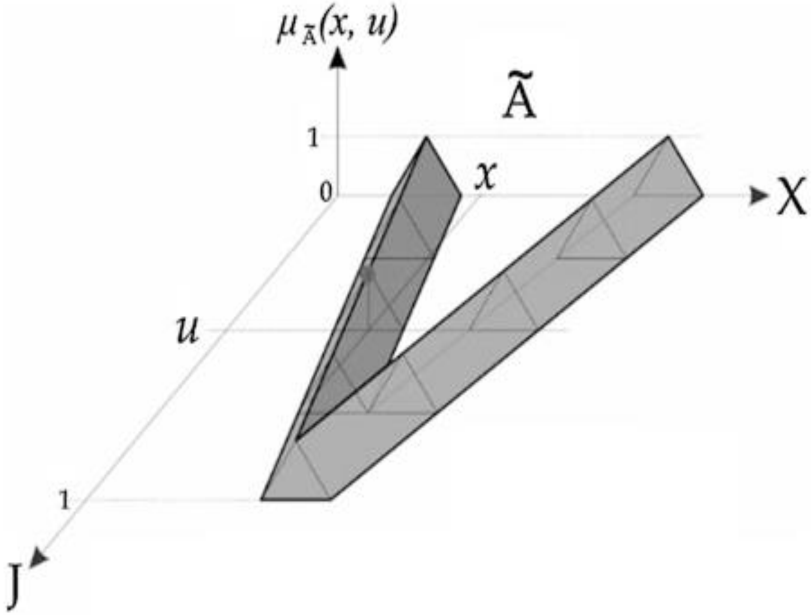
$$\mu_{\tilde{A}} = \left\{ \frac{\left\{\frac{1}{0.6}, \frac{1}{0.7}, \frac{1}{0.8}\right\}}{2} , \frac{\left\{\frac{1}{0.0}, \frac{1}{0.2}, \frac{1}{0.4}, \frac{1}{0.6}, \frac{1}{0.8}\right\}}{3} \right\}$$: IT-2

✓ Primary membership values

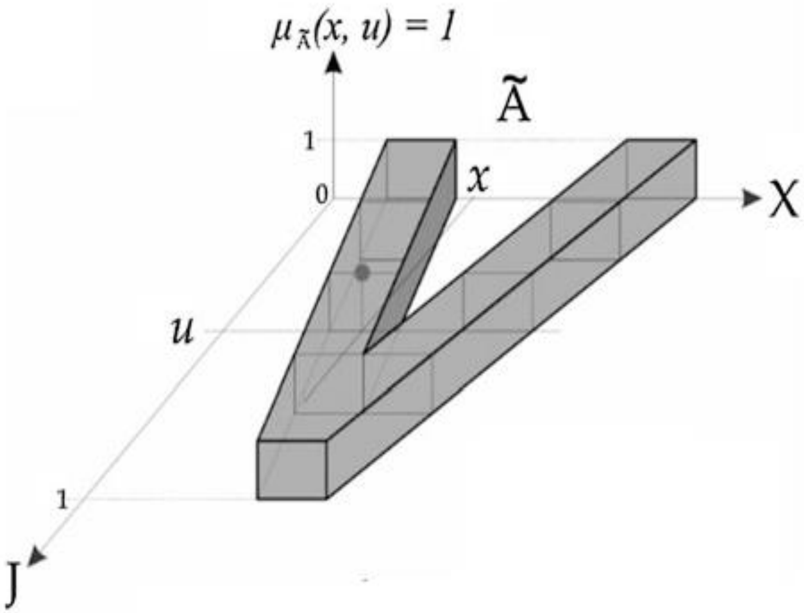
✓ Secondary membership values







T2FS

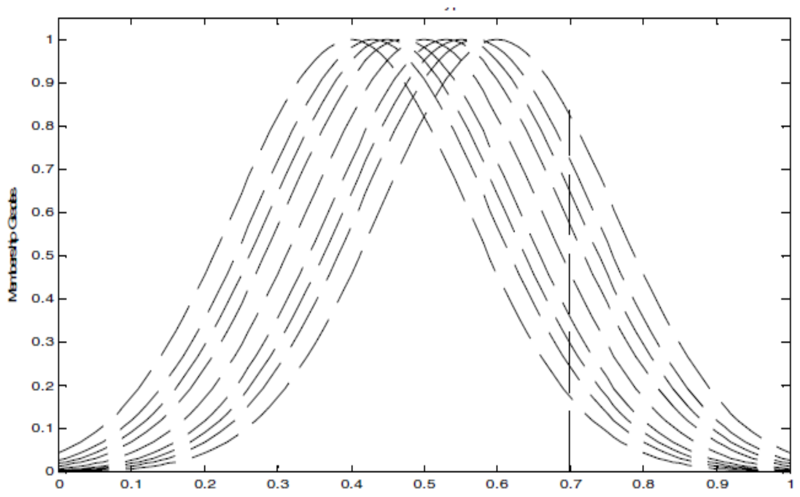


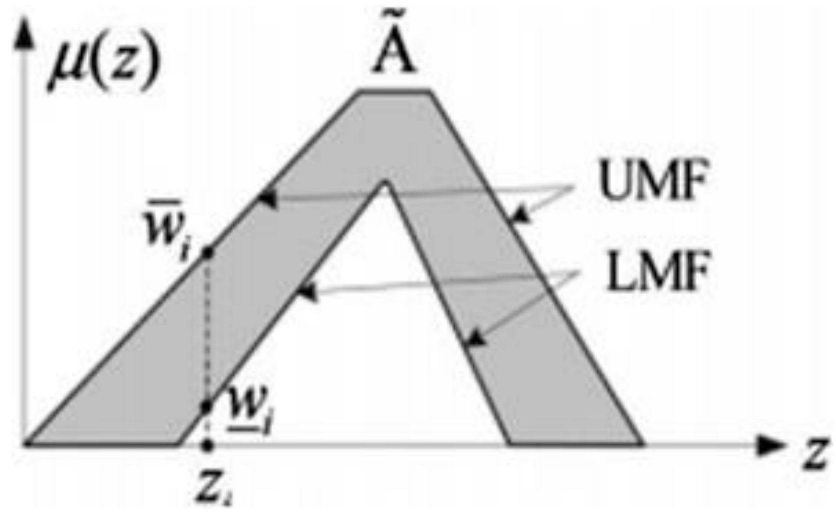
IT2FS

✓ Footprint Of Uncertainty (FOU)

The bounded region obtained from union of all primary memberships

(i.e. region between UMF and LMF)





$$\mu_{\tilde{A}} = \left\{ \frac{\left\{ \frac{\mu_{11}}{u_{11}}, \frac{\mu_{12}}{u_{12}}, \ldots \frac{\mu_{1k}}{u_{1k}} \right\}}{x_1}, \frac{\left\{ \frac{\mu_{21}}{u_{21}}, \frac{\mu_{22}}{u_{22}}, \ldots \frac{\mu_{2k}}{u_{2k}} \right\}}{x_2}, \ldots \frac{\left\{ \frac{\mu_{N1}}{u_{N1}}, \frac{\mu_{N2}}{u_{N2}}, \ldots \frac{\mu_{Nk}}{u_{Nk}} \right\}}{x_N} \right\} \quad : \text{GT-2}$$

$$\mu_{\tilde{A}} = \left\{ \frac{\left\{ \frac{1}{u_{11}}, \frac{1}{u_{12}}, \ldots \frac{1}{u_{1k}} \right\}}{x_1}, \frac{\left\{ \frac{1}{u_{21}}, \frac{1}{u_{22}}, \ldots \frac{1}{u_{2k}} \right\}}{x_2}, \ldots \frac{\left\{ \frac{1}{u_{N1}}, \frac{1}{u_{N2}}, \ldots \frac{1}{u_{Nk}} \right\}}{x_N} \right\} \quad : \text{IT-2}$$

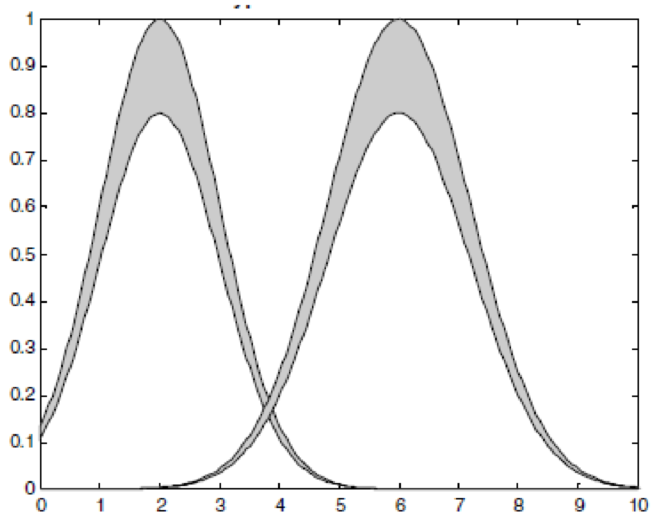
$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} \quad ; \; J_x \subseteq [0,1]$$

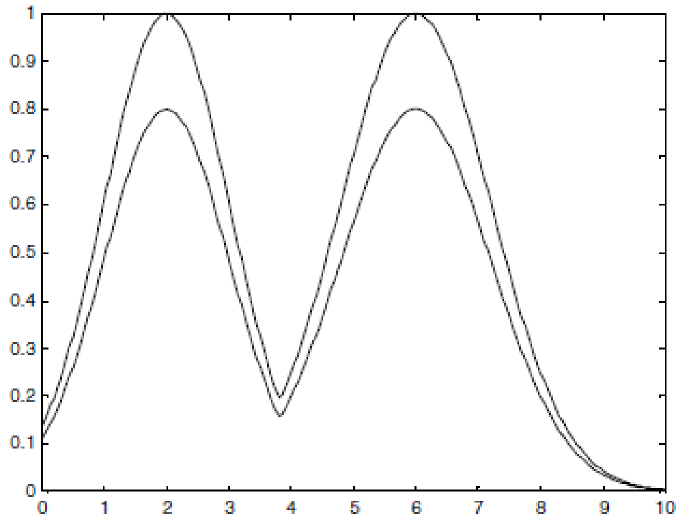
$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{1}{(x,u)} \quad ; \; J_x \subseteq [0,1]$$

Operations on IT-2 Fuzzy Sets:

1) Union:

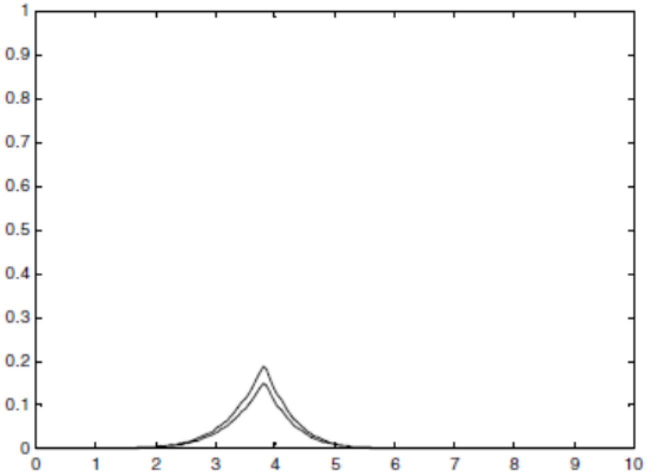
$$\widetilde{A} + \widetilde{B} = \frac{1}{\left[\underline{\mu}_{\widetilde{A}} + \underline{\mu}_{\widetilde{B}}\,,\ \overline{\mu}_{\widetilde{A}} + \overline{\mu}_{\tilde{B}}\right]}$$





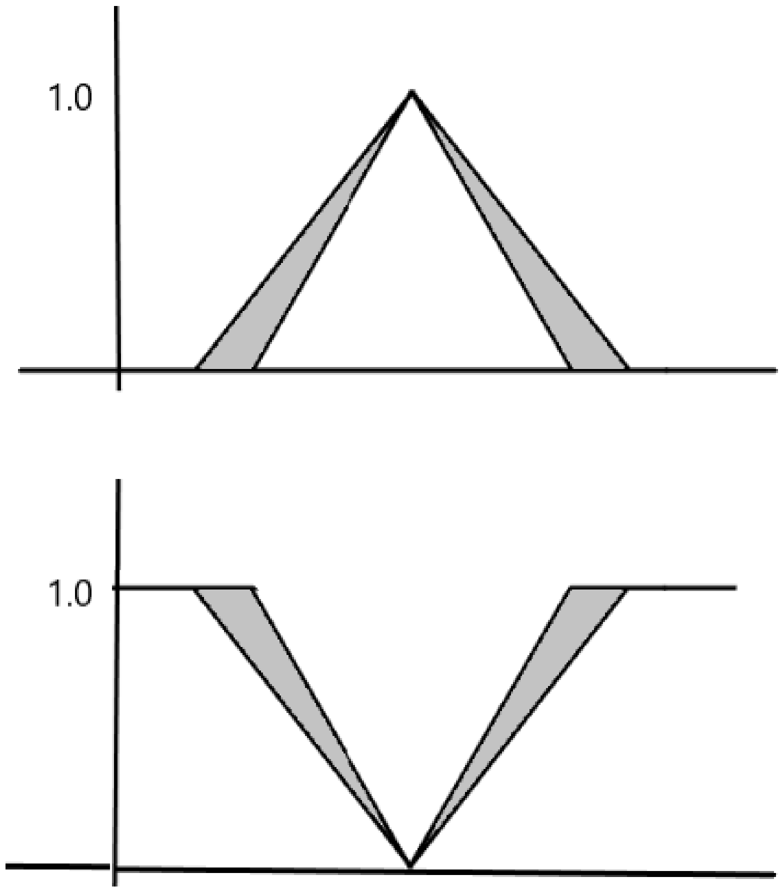
2) Intersection:

$$\widetilde{A}\,.\,\widetilde{B} = \frac{1}{\left[\underline{\mu}_{\widetilde{A}}.\,\underline{\mu}_{\widetilde{B}}\,,\ \overline{\mu}_{\widetilde{A}}.\,\overline{\mu}_{\tilde{B}}\right]}$$



3) Complement:

$$\overline{\tilde{A}} = \frac{1}{\left[1 - \overline{\mu}_{\tilde{A}}(x), 1 - \underline{\mu}_{\tilde{A}}(x)\right]} \quad ; \quad \forall\, x \in X$$

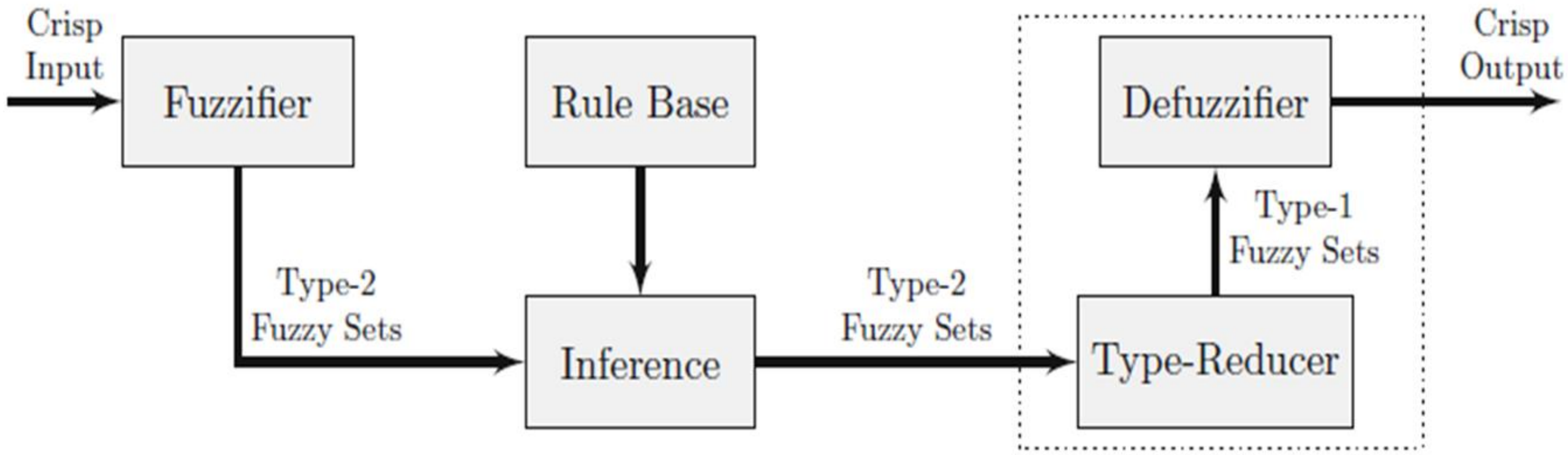


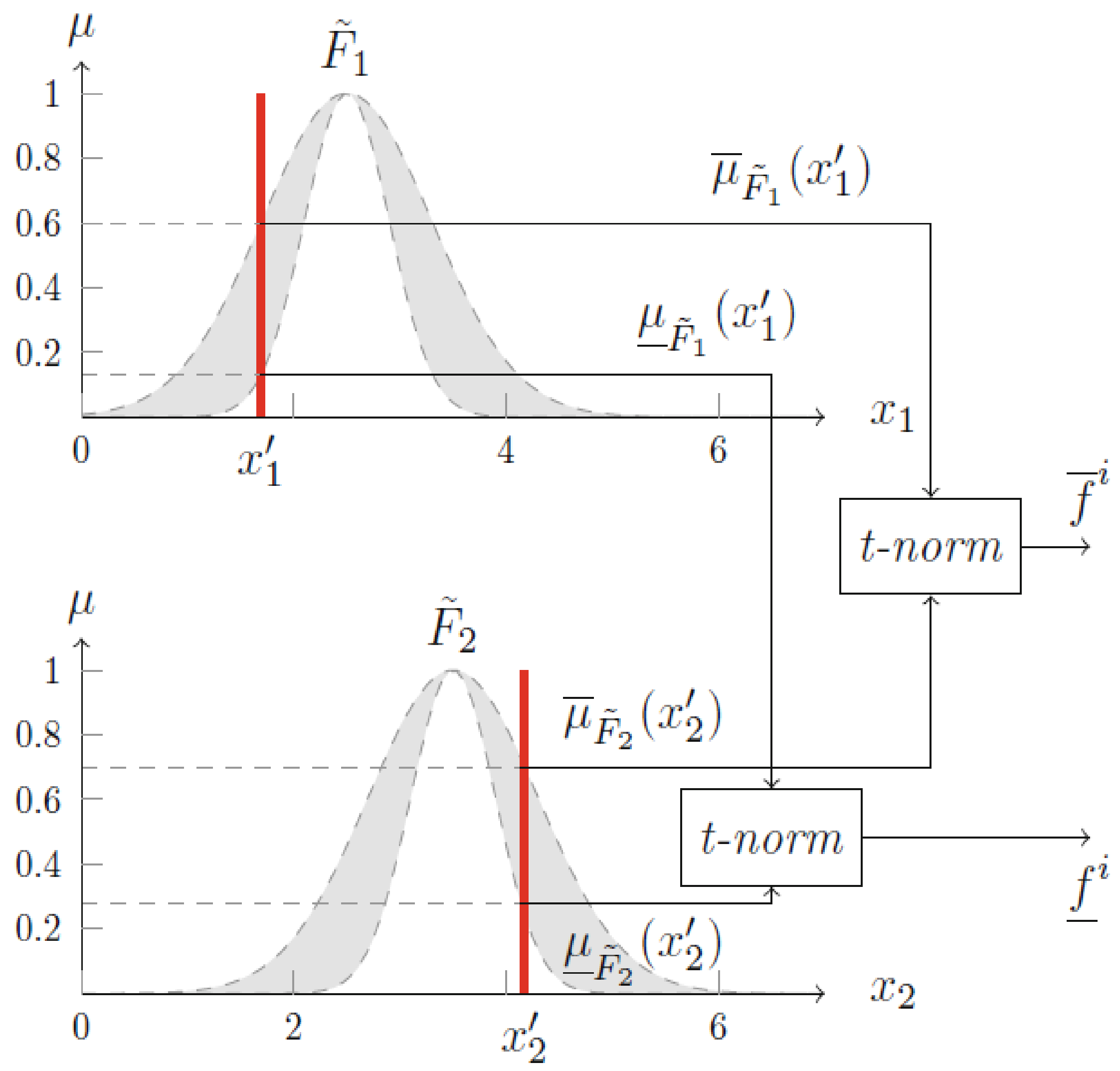
Ex. Let two fuzzy sets be given by

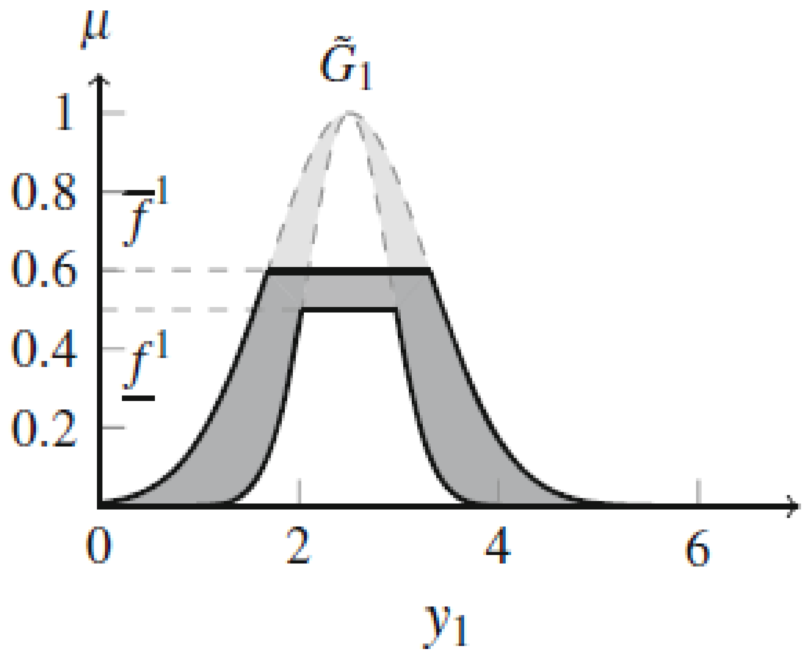
$$\widetilde{\mathrm{A}} = \left\{\frac{\{0.6\ ,\ 0.7\}}{1} + \frac{\{0.2\ ,\ 0.4\}}{3}\right\} \quad \text{and} \quad \widetilde{\mathrm{B}} = \left\{\frac{\{0.5\ ,0.8\}}{1} + \frac{\{0.3\ ,0.9\}}{3}\right\}$$

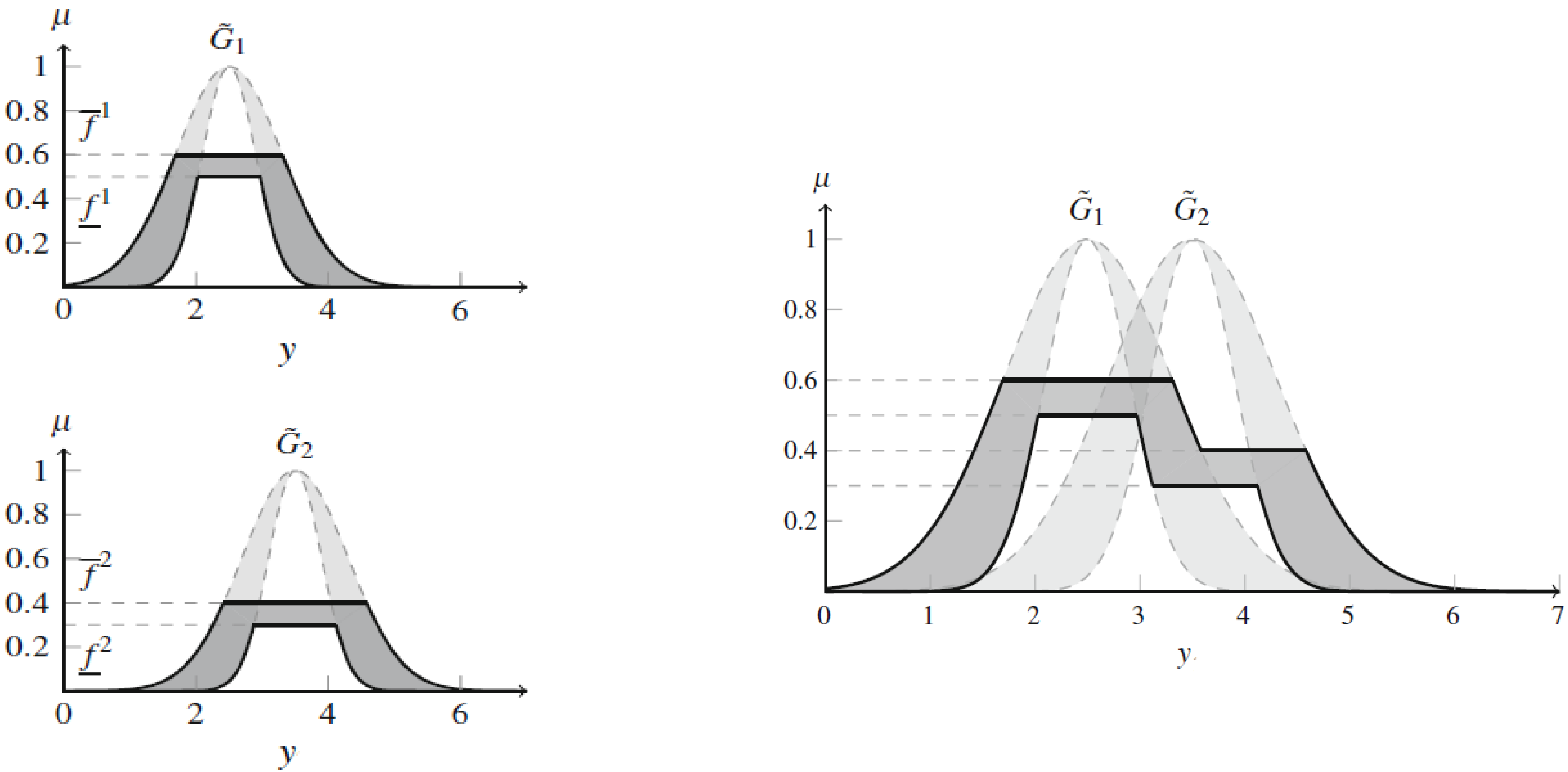
Compute $\widetilde{\mathrm{A}} + \widetilde{\mathrm{B}}$

Reasoning with IT-2 Fuzzy Systems:

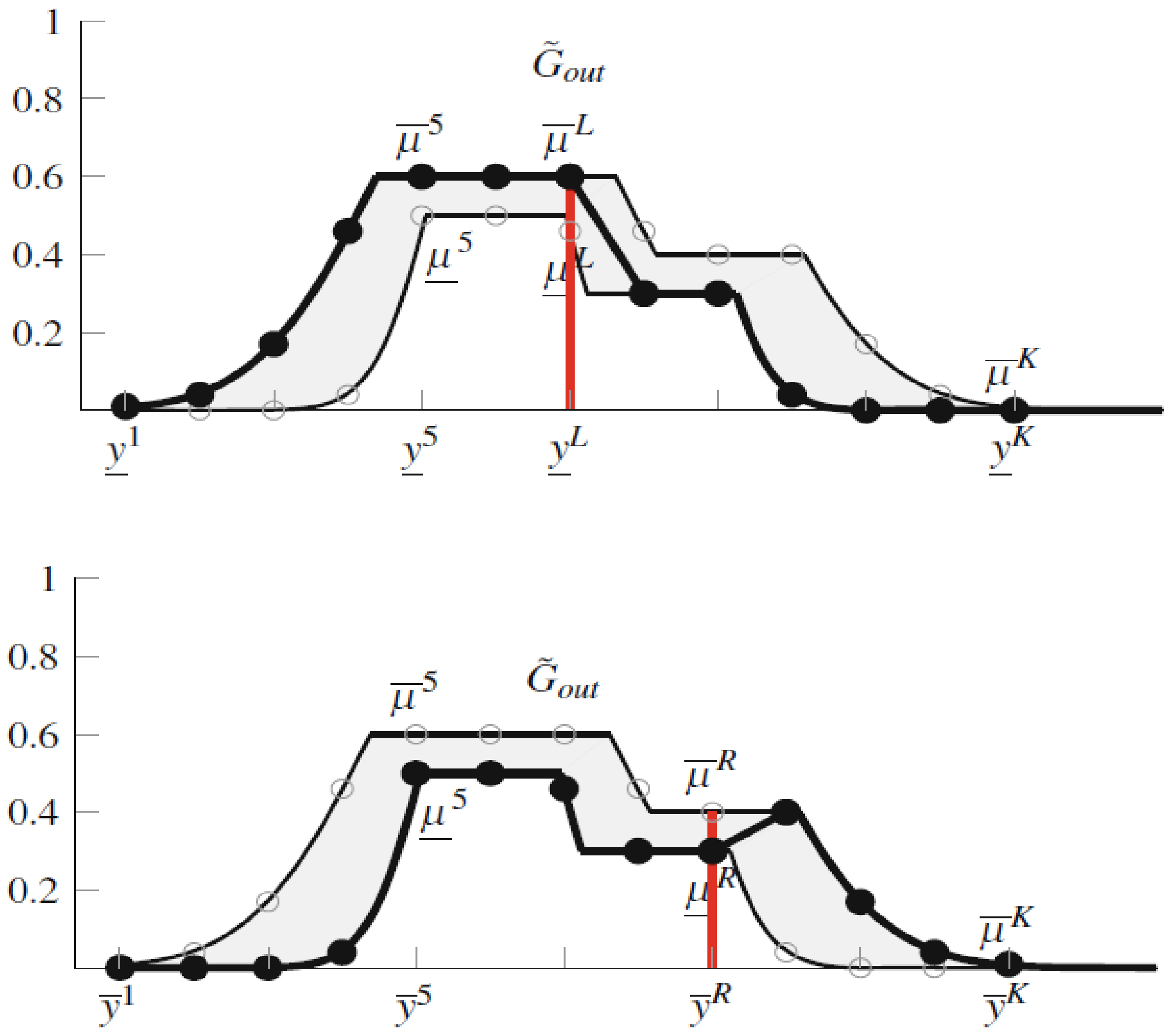



μ
$\tilde{F}_1$
1
0.8
0.6
0.4
0.2
$\overline{\mu}_{\tilde{F}_1}(x_1')$
$\underline{\mu}_{\tilde{F}_1}(x_1')$
x_1
0
x_1'
4
6
t-norm
$\overline{f}^i$
μ
$\tilde{F}_2$
1
0.8
0.6
0.4
0.2
$\overline{\mu}_{\tilde{F}_2}(x_2')$
t-norm
$\underline{f}^i$
$\underline{\mu}_{\tilde{F}_2}(x_2')$
x_2
0
2
x_2'
6


μ
$\tilde{G}_1$
1
0.8
0.6
0.4
0.2
$\overline{f}^1$
$\underline{f}^1$
0
2
4
6
y_1

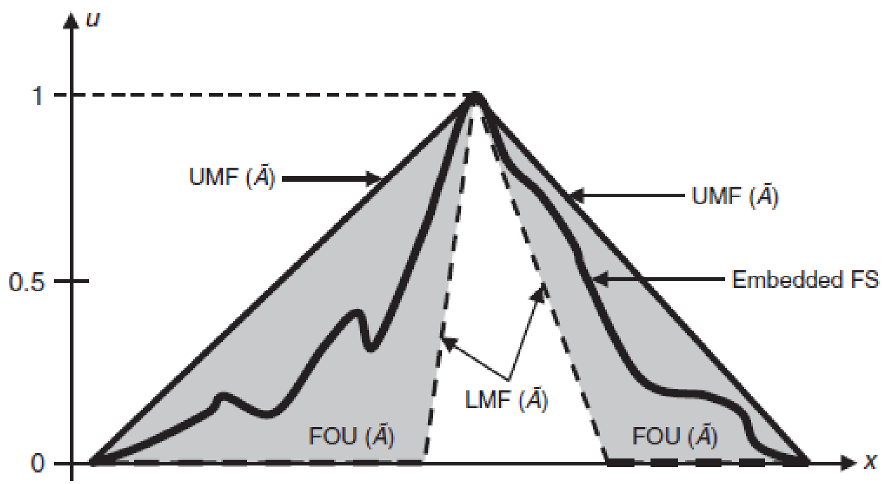


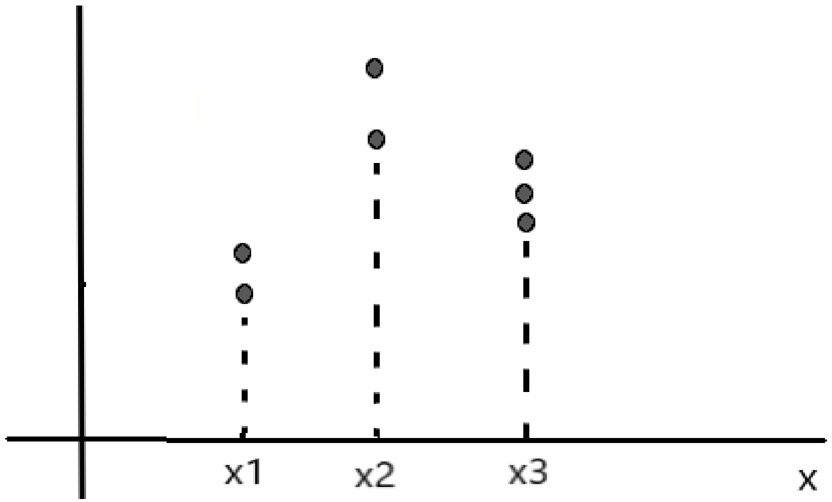
μ
$\tilde{G}_1$
1
0.8
$\overline{f}^1$
0.6
0.4
$\underline{f}^1$
0.2
0
2
4
6
y
μ
$\tilde{G}_2$
1
0.8
0.6
$\overline{f}^2$
0.4
0.2
$\underline{f}^2$
0
2
4
6
y
μ
$\tilde{G}_1$
$\tilde{G}_2$
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
y

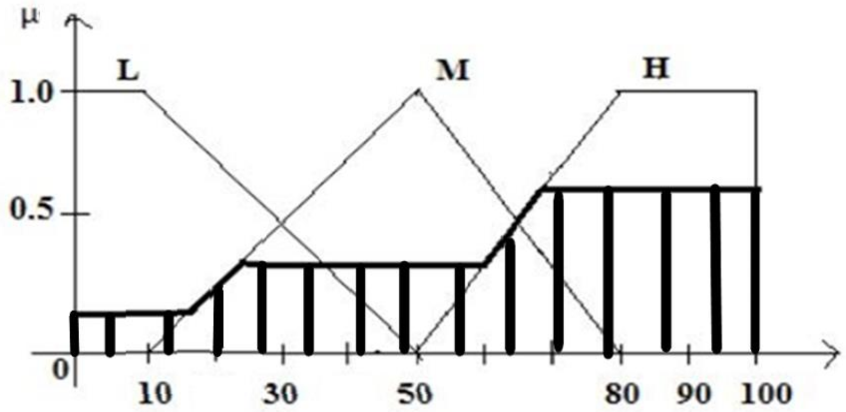


$$y^* = \frac{\underline{y}^L + \overline{y}^R}{2}$$

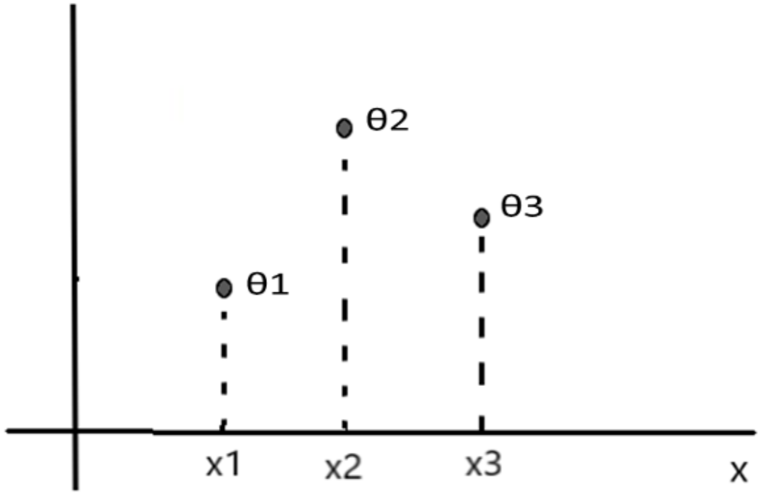
Type Reduction:

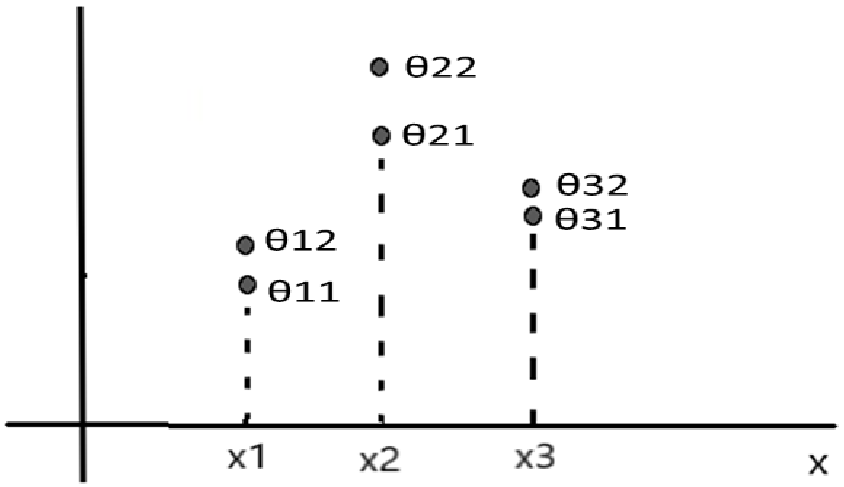






$$n_A = \prod_{i=1}^{N} M_i$$: Total no. of embedded fuzzy sets





$$x^* = \frac{\sum_{i=1}^{N} x_i\theta_i}{\sum_{i=1}^{N} \theta_i}$$

$$= \frac{x_1\theta_1 + x_2\theta_2 + x_3\theta_3}{\theta_1 + \theta_2 + \theta_3}$$

$$c_l, c_r = \min, \max \Bigg[\frac{x_1\theta_{11} + x_2\theta_{21} + x_3\theta_{31}}{\theta_{11} + \theta_{21} + \theta_{31}}, \quad \frac{x_1\theta_{12} + x_2\theta_{21} + x_3\theta_{31}}{\theta_{12} + \theta_{21} + \theta_{31}},$$

$$\frac{x_1\theta_{11} + x_2\theta_{22} + x_3\theta_{31}}{\theta_{11} + \theta_{22} + \theta_{31}}, \quad \frac{x_1\theta_{12} + x_2\theta_{22} + x_3\theta_{31}}{\theta_{12} + \theta_{22} + \theta_{31}},$$

$$\frac{x_1\theta_{11} + x_2\theta_{21} + x_3\theta_{32}}{\theta_{11} + \theta_{21} + \theta_{32}}, \quad \frac{x_1\theta_{12} + x_2\theta_{21} + x_3\theta_{32}}{\theta_{12} + \theta_{21} + \theta_{32}},$$

$$\frac{x_1\theta_{11} + x_2\theta_{22} + x_3\theta_{32}}{\theta_{11} + \theta_{22} + \theta_{32}}, \quad \frac{x_1\theta_{12} + x_2\theta_{22} + x_3\theta_{32}}{\theta_{12} + \theta_{22} + \theta_{32}} \Bigg]$$

Karnik-Mendel (KM) Algorithm (2001):

- ✓ Iterative way of type reduction
- ✓ Approximate solution
- ✓ EKM (Enhanced Karnik-Mendel) Algorithm

Centre of Sets (COS) Type Reduction:

- ✓ Similar to Centre of Sums defuzzification

  (i.e. outputs of all fired rules are not ORed)

Neuro-Fuzzy-GA Hybrid Systems

Why Hybridization?

- The three members have strengths in different contexts
- Objective is to combine their strengths
- And overcome their weaknesses
- Improved performance over wider ranges of variables
- Higher complexity

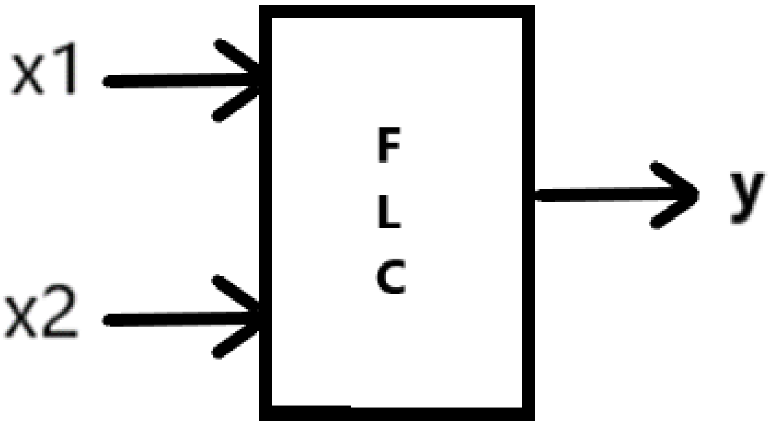
GA based Tuning of Fuzzy Systems

- Success of an FLS heavily depends on how appropriately the membership functions and the rule base have been defined

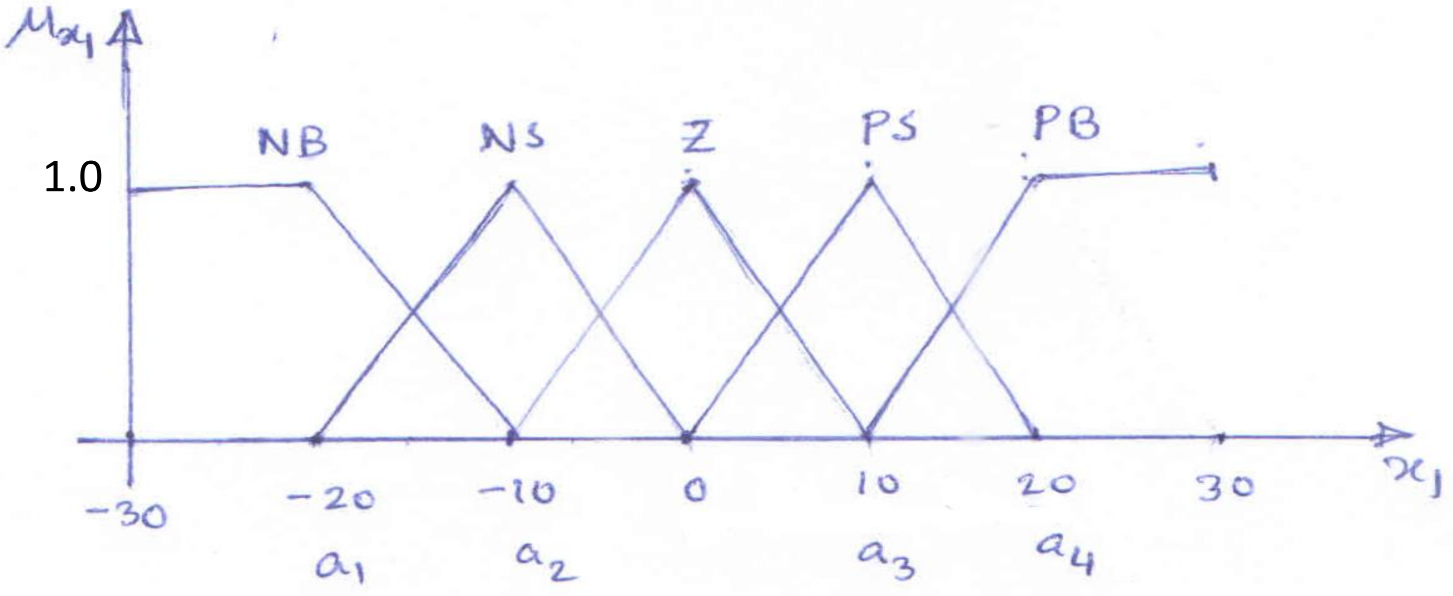
- They are designed from domain expertise/experience
- May be manageable for a few input-output variables
- Difficult and less accurate when the variables and/or fuzzy parameters grow
- An optimizer helps in arriving at an optimal setting of the parameters

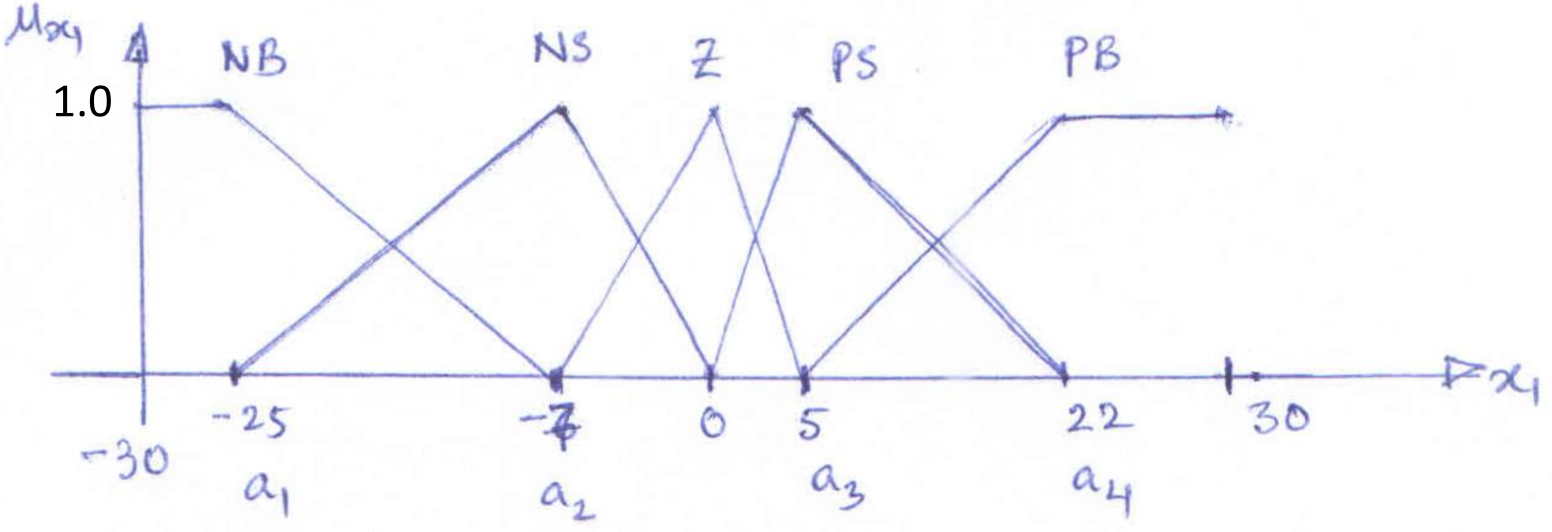
- Because of large number of parameters, traditional optimization tools are nearly inapplicable
- Requires some input-output data to be acquired along with the qualitative understanding of the problem
- Membership functions, scaling factors and rule base are tuned

Tuning of Membership Functions:



S.No.	x1	x2	y
1.	5.0	2.5	10.1
2.	3.5	-4.5	-8.2
⋮	⋮	⋮	⋮
T			





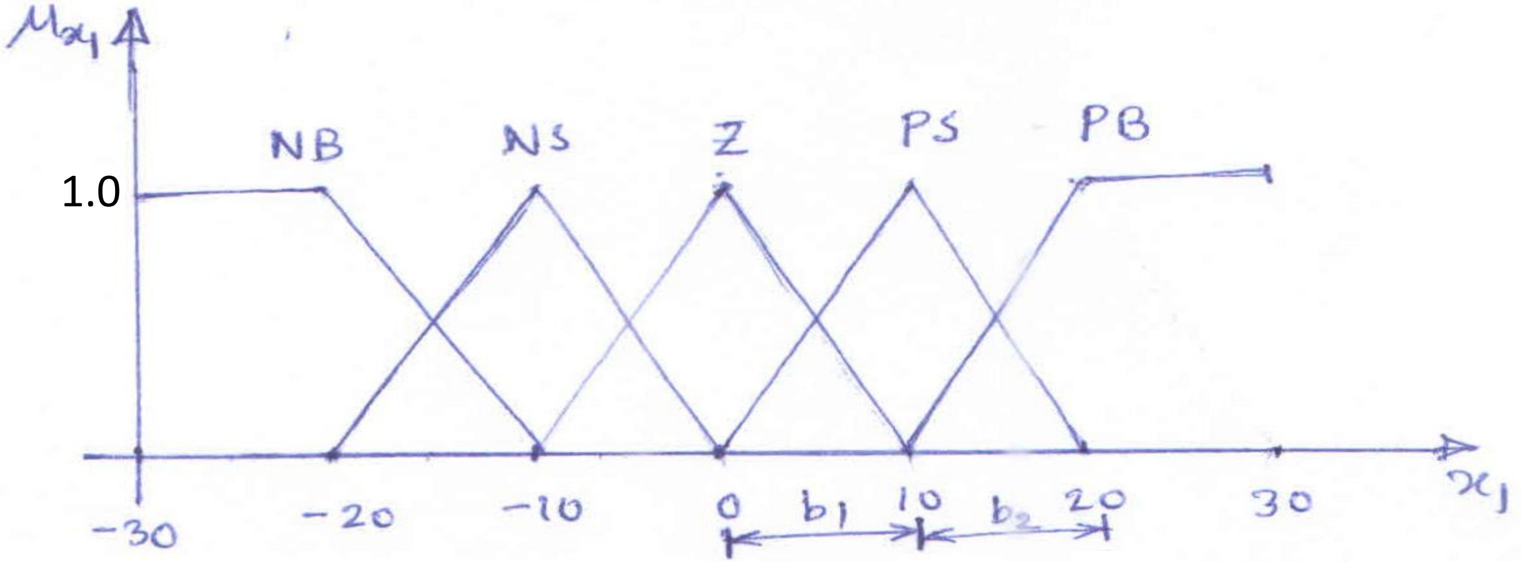
Let ,

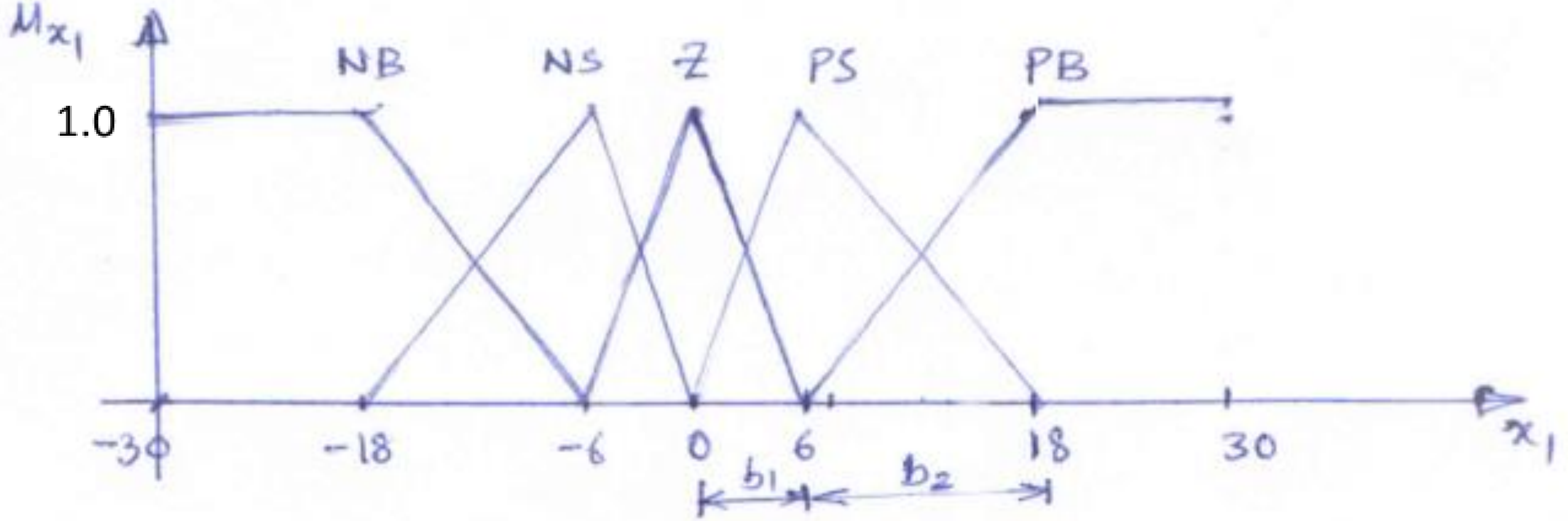
$a_1 \in \; [-25, -15]$

$a_2 \in \; [-15, -5]$

$a_3 \in \; [5, 15]$

$a_4 \in \; [15, 25]$

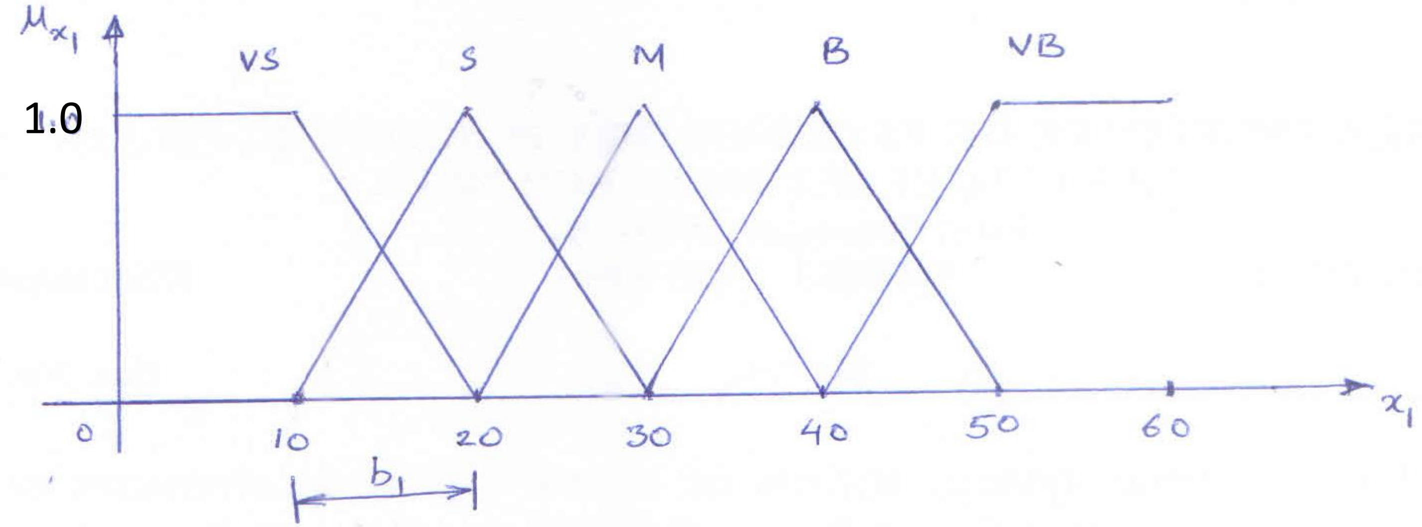




Let ,

$b_1 \in \; [5, 15]$

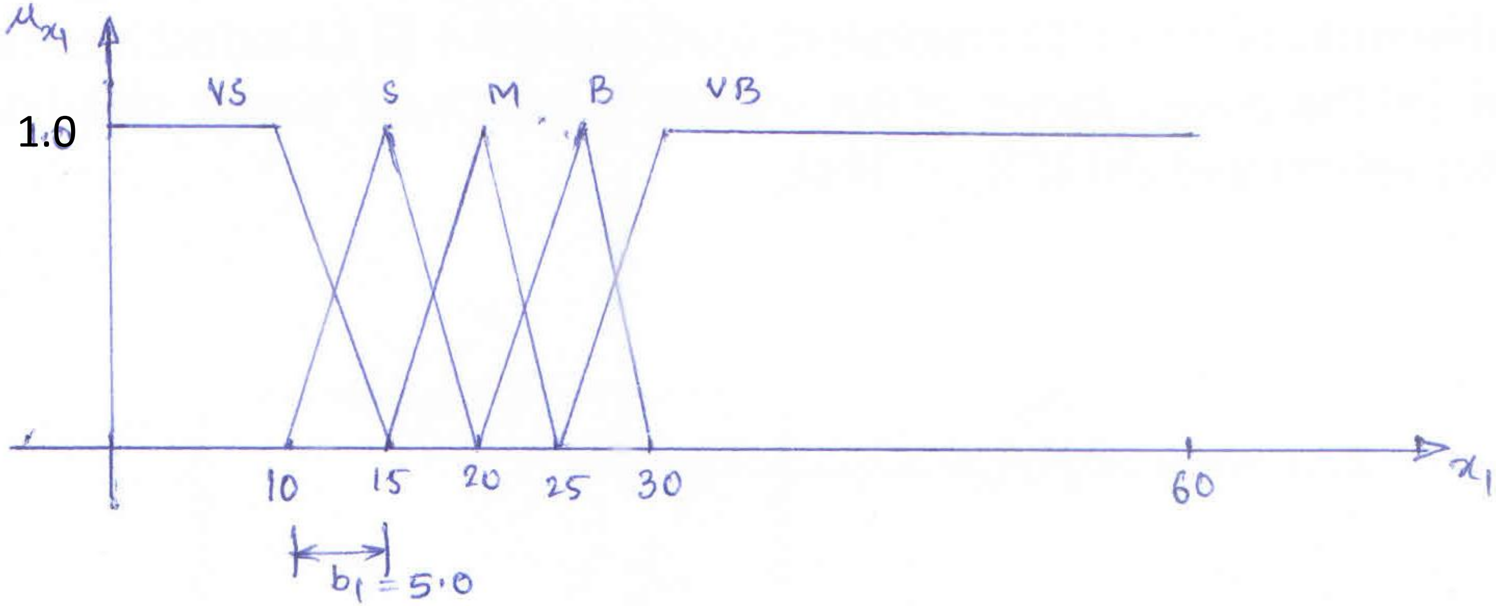
$b_2 \in \; [5, 15]$



μ_{x_1}
VS
S
M
B
VB
1.0
0
10
20
30
40
50
60
x_1
b_1

Let ,

$b_1 \in \ [5, 15]$



μ_{x_1}
VS
S
M
B
VB
1.0
10
15
20
25
30
60
x_1
$b_1 = 5.0$

Rule Base:

x1 \ x2	**NB**	**NS**	**Z**	**PS**	**PB**
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB

S.No.	x1	x2	**y**
1.	5.0	2.5	10.1
2.	3.5	-4.5	-8.2
⋮	⋮	⋮	⋮
T			

(Data Base)

$b_1 \in \ [5, 15]$

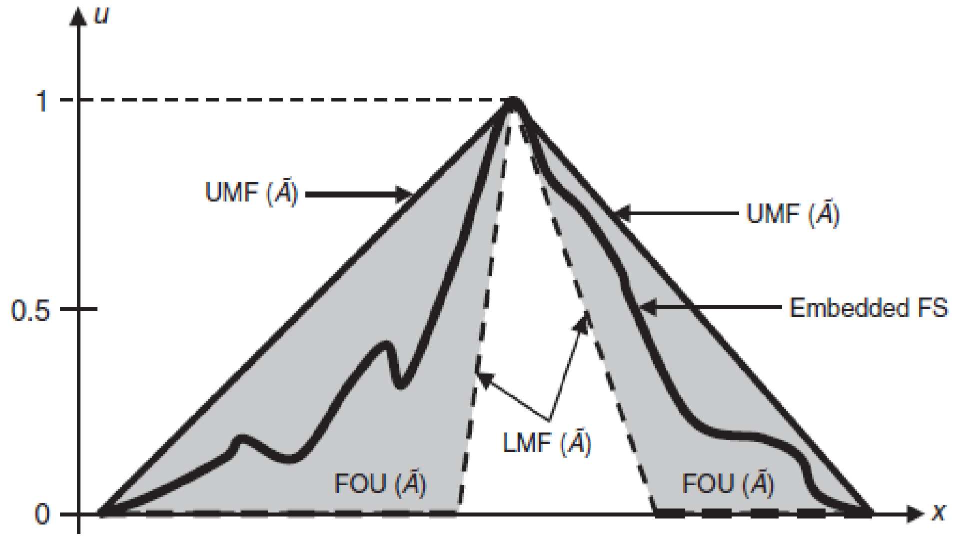
(6 parameters b1, b2, b3, b4, b5, b6 are tuned)

$b_2 \in \ [5, 15]$

Initial Population:

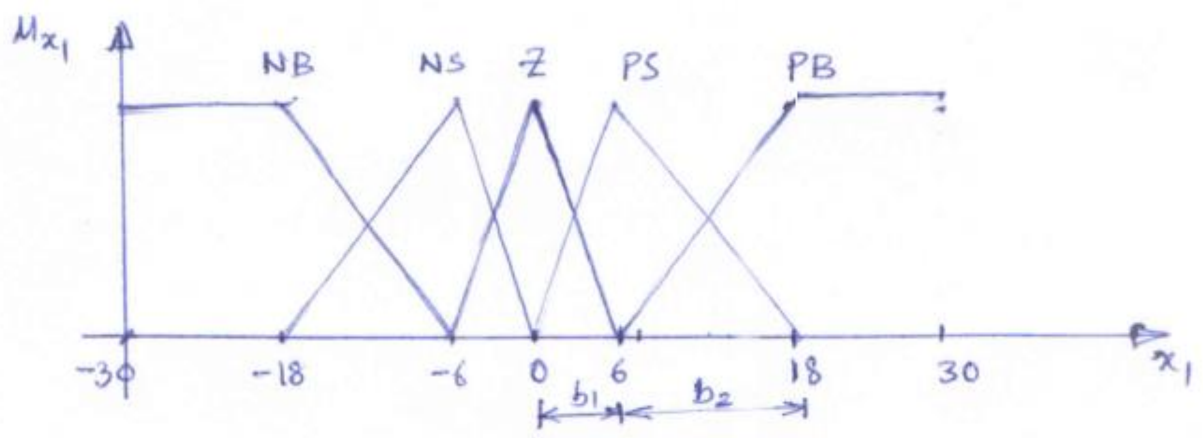
S. No.	GA String (36 bit)	Error	Fitnes
1.	010011 111000 . . . 001010 **b1 b2 . . . b6**	$\overline{e}_1 = \frac{1}{T} \sum \lvert e_1 \rvert$	$f_1 = (\overline{e}_1)^{-1}$
⋮	⋮	⋮	⋮
N			

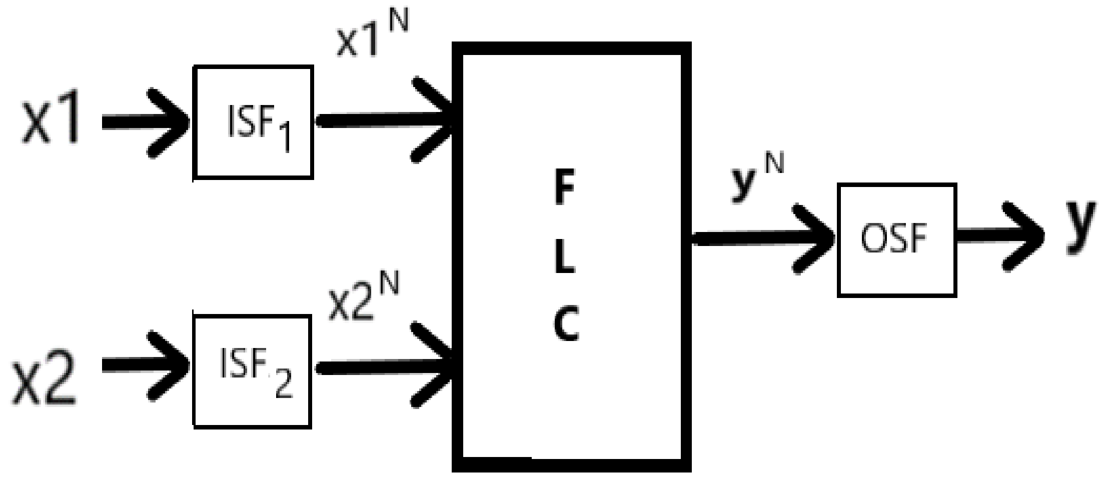
Tuning of Interval Type-2 Fuzzy Models:



- ✓ Similar to the Type-1 case, LMF and UMF can be tuned

Tuning of Scaling Factors:





- ✓ 6+3 = 9 tunable parameters
- ✓ 54 bit GA strings/chromosomes

Reduction of Rule Base:

- ✓ Some rules may be redundant
- ✓ They may be removed taking help from the database

Rule Base:

x1 \ x2	**NB**	**NS**	**Z**	**PS**	**PB**
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB

S.No.	x1	x2	y
1.	5.0	2.5	10.1
2.	3.5	-4.5	-8.2
⋮	⋮	⋮	⋮
T			

(Data Base)

- ✓ Let ‘0’ imply absence and ‘1’ imply presence of a particular rule
- ✓ Hence a 25 bit long string will represent the entire rule base

Initial Population:

S. No.	GA String (61 bit)	Error	Fitnes
1.	010011 111000 ... 001010 $\underbrace{10110\ 10001\ 01111\ 00110\ 11100}_{\text{RB}}$ **b1** **b2** **...** **b6**	$\overline{e}_1 = \frac{1}{T} \sum \lvert e_1 \rvert$	$f_1 = (\overline{e}_1 + P_1)^{-1}$
⋮	⋮	⋮	⋮
N			

$P_1 =$ Number of rules present (a penalty term to the fitness value)

Rule Base Corresponding to String#1:

x1 \ x2	**NB**	**NS**	**Z**	**PS**	**PB**
NB	NB	-	NS	NS	-
NS	NB	-	-	-	PS
Z	-	NS	Z	PS	PS
PS	-	-	PS	PS	-
PB	Z	PS	PS	-	-

Generation of Rule Base:

- ✓ Designer does not have enough intuition to design the initial rule base
- ✓ GA can design the rule base that will fit the input-output data set

Let NB = 000

NS = 001

Z = 010

PS = 011

PB = 100

- ✓ Three bits are assigned to the consequent part of each rule
- ✓ Hence a 75 bit string for the consequent parts of the entire rule base

S. No.	GA String (136 bit)	Error	Fitnes
1.	$\underbrace{010011}_{b1}\ \underbrace{111000}_{b2}\ \underbrace{\ldots}_{\ldots}\ \underbrace{001010}_{b6}\ \underbrace{10110\ 10001\ 01111\ 00110\ 11100}_{RB}\ \underbrace{001}_{con\ R1}\ \underbrace{100}\ \ldots\ \underbrace{011}_{con\ R25}$	⋮	⋮
⋮	⋮		
N			

NB = 000

NS = 001

Z = 010

PS = 011

PB = 100

Some Observations:

- ✓ Too long GA strings
- ✓ Switch to Real Coded GA

✓ Mixed Real and Integer Valued Optimization problem

NB = 000 = 1

NS = 001 = 2

Z = 010 = 3

PS = 011 = 4

PB = 100 = 5

S. No.	GA String	Error	Fitnes
1.	12.9 (**b1**) 5.5 (**b2**) . . . 7.1 (**b6**) $\underbrace{\text{10110 10001 01111 00110 11100}}_{\text{RB}}$ $\underbrace{2}_{\text{con R1}}$ $\underbrace{5}$ . . . $\underbrace{4}_{\text{con R25}}$	⋮	⋮
⋮	⋮		
N			

- ✓ After going through the GA operations, we may not arrive at a valid consequent
- ✓ Hence number of output fuzzy sets may be increased to 8 or decreased to 4
- ✓ But neither will be symmetrical
- ✓ May be made symmetrical by dropping the Zero fuzzy set
- ✓ However, may lead to complexity in forming some Rules

S. No.	GA String (136 bit)	Error	Fitnes
1.	010011 111000 . . . 001010 10110 10001 01111 00110 11100 001 100 . . . 011 **b1** **b2** **. . .** **b6** RB con R1 con R25	⋮	⋮
⋮	⋮		
N			

Michigan Approach:

- ✓ Each Rule is represented by a GA string
- ✓ Population size = Number of rules
- ✓ Hence RB is represented by the whole population

R1: 1 1 1
R2: 1 2 1
R3: 1 3 2
.
.
R25: 5 5 5

x1 \ x2	**NB**	**NS**	**Z**	**PS**	**PB**
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB

NB = 1

NS = 2

Z = 3

PS = 4

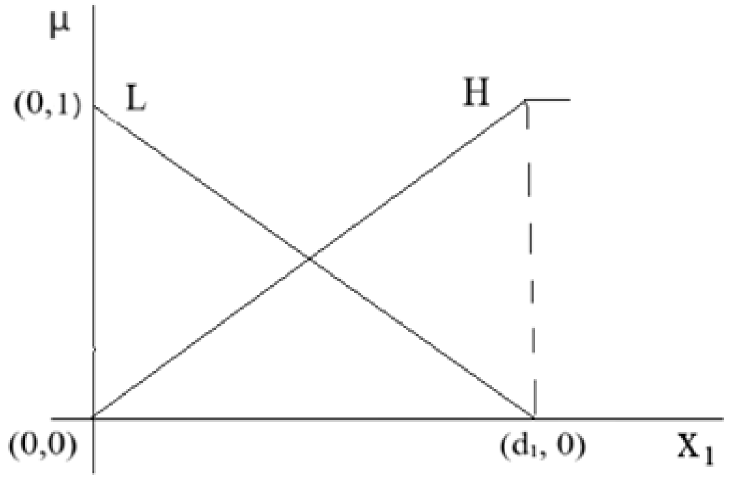
PB = 5

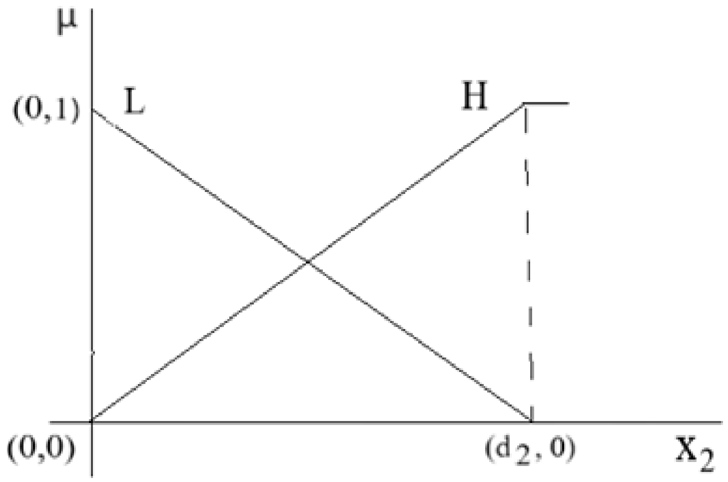
Pittsburgh Approach:

- ✓ Entire RB is represented by a single GA string

Numerical Example:

The inputs of a two input, single output system are described by the following membership functions where L and H imply Low and High respectively and $d_1, d_2 \in [5, 10]$:





Training set:

S. No.	X_1	X_2	Y
1.	2.5	7.5	2.0

The output is given by a zero order Sugeno model. The output corresponding to the i^{th} rule is given by

$$y^{(i)} = a_i \text{ , (i=1,2,3,4) where } a_i \in [1, 4]$$

The input and output fuzzification parameters (i.e. d_1, d_2, a_i) are to be optimized using the standard PSO algorithm to match the given training data.

Assume a random initial population of size two. Given that the initial velocities are zero and w=0.9, c_1=c_2=2.0, r_1=0.2, r_2=0.4, show one iteration to update the population.

Answer:

$$\underline{x}_1 = \begin{bmatrix} 6.0 \\ 9.0 \\ 2.0 \\ 3.5 \\ 3.0 \\ 1.5 \end{bmatrix} \longrightarrow \begin{bmatrix} 6.0 \\ 9.0 \\ 2.0 \\ 3.5 \\ 3.0 \\ 1.5 \end{bmatrix} \qquad \underline{x}_2 = \begin{bmatrix} 8.0 \\ 7.0 \\ 2.4 \\ 3.8 \\ 1.2 \\ 1.5 \end{bmatrix} \longrightarrow \begin{bmatrix} 6.4 \\ 8.6 \\ 2.08 \\ 3.56 \\ 2.66 \\ 1.50 \end{bmatrix}$$

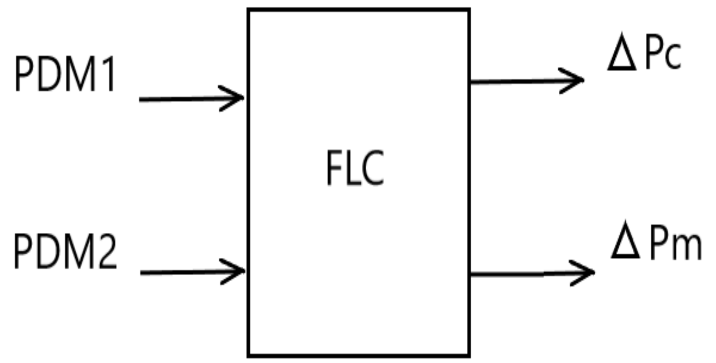
Fuzzy based Tuning of GA Parameters:

- ✓ Usually GA parameters $\mathrm{p_c}$, $\mathrm{p_m}$ and N are set beforehand
- ✓ They are chosen intuitively
- ✓ An FLS may be brought in to make the search more effective

  (i.e. to avoid premature convergence)
- ✓ Two diversity measures PDM1 and PDM2 are defined in Dynamic Parametric GA (DPGA)

✓ $PDM_1 = \frac{\bar{f}}{f_{best}} \in [0,1]$

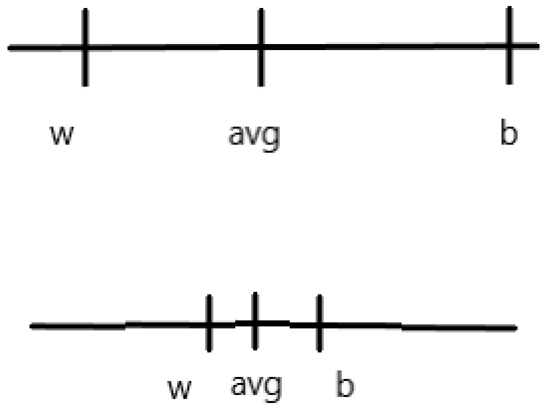
$PDM_2 = \frac{f_{worst}}{\bar{f}} \in [0,1]$



Typical Rules:

If PDM1 is High Then ΔPc is Positive

If PDM1 is Low Then ΔPc is Negative



If PDM2 is High Then ΔPm is Positive

If PDM2 is Low Then ΔPm is Negative