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Theory of Computation
CS F351

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Lecture 21

Closure properties of CFL

- **Closure properties** consider operations on CFL that are guaranteed to produce a CFL

- The CFL's are ______ under *union*, *concatenation*, *Kleene star*, *reversal*.

- CFL's are ______ under *intersection*, *complementation*, and *set-difference*.

- But the intersection of a CFL and a regular language is always a ______??

Set Union

CFL's are closed under union operation.

Proof by Construction:

Input

- CFG $G_1 = (V_1, \Sigma, R_1, S_1)$
- CFG $G_2 = (V_2, \Sigma, R_2, S_2)$

Output

- CFG $G_3 = (V_3, \Sigma, R_3, S_3)$
  - $V_3 = V_1 \cup V_2 \cup \{S\}$
    - Variable renaming to insure no names shared between V_1 and V_2
    - S is a new symbol not in V_1 or V_2 or Σ
  - $S_3 = S$
  - $R_3 = ?$

Set Concatenation

CFL's are closed under concatenation operation.

Proof by Construction:

Input

- CFG $G_1 = (V_1, \Sigma, R_1, S_1)$
- CFG $G_2 = (V_2, \Sigma, R_2, S_2)$

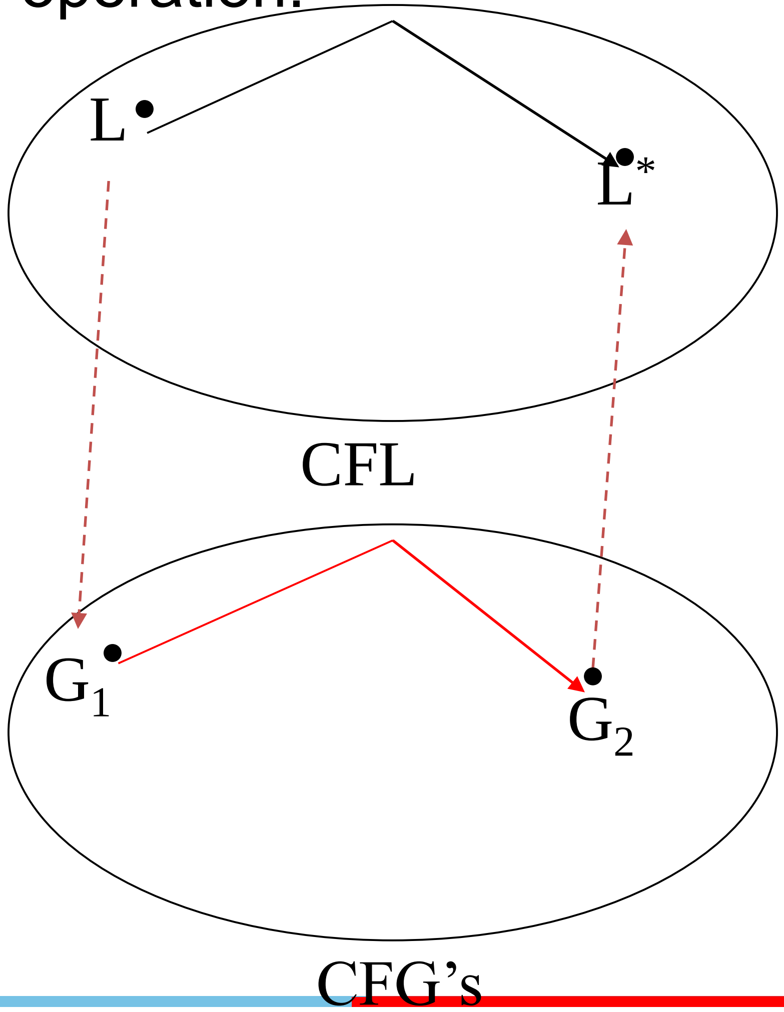
Output

- CFG $G_3 = (V_3, \Sigma, R_3, S_3)$
  - $V_3 = V_1 \cup V_2 \cup \{S\}$
    - Variable renaming to insure no names shared between V_1 and V_2
    - S is a new symbol not in V_1 or V_2 or Σ
  - $S_3 = S$
  - $R_3 = ?$

Kleene Star

CFL’s are closed under Kleene star operation.

- Let L be an arbitrary CFL
- Let G_1 be a CFG s.t. $L(G_1) = L$
  - G_1 exists by definition of L_1 in CFL
- Construct CFG G_2 from CFG G_1
- Argue $L(G_2) = L^*$
- There exists CFG G_2 s.t. $L(G_2) = L^*$
- L^* is a CFL



Kleene Star

CFL's are closed under Kleene star operation.

Proof by Construction:

Input

- CFG $G_1 = (V_1, \Sigma, R_1, S_1)$

Output

- CFG $G_2 = (V_2, \Sigma, R_2, S_2)$
  - $V_2 = V_1$ union {S}
    - S is a new symbol not in V_1 or Σ
  - $S_2 = S$
  - $P_2 = P_1$ U __________

Kleene Star

Suppose CFG $G_1 = (\{S\}, \{a, b\}, \{S \rightarrow aa \mid ab \mid ba \mid bb\}, S)$

CFG G_2 such that $L(G_2) = L(G_1^*)$ is:

Transpose Operation

- For and $x \in \Sigma^*$ and $a \in \Sigma$, $(xa)^T = a(x)^T$
- For example, $(aaabab)^T$ = babaaa
- CFL's are closed under transpose operation.

Let G = (V, Σ, R, S) be a CFG. Then the grammar for G^T is (V, Σ, R^T, S), where the production rules of R^T are constructed by reversing the symbols on LHS and RHS of every production in R.

Example: $S \rightarrow aTb \mid b \mid ab$

$T \rightarrow Ta \mid b$

Intersection of a CFL and RL

Is intersection of a CFL and RL a RL ?

Suppose:

- L is a CFL and corresponding PDA M1 = ($Q1, \sum, \boldsymbol{\tau}1, \boldsymbol{\Delta}1, s1, F1$)
- R is a RL, and corresponding DFA M = ($Q2, \sum, \delta, s2, F2$)

<u>Construction of PDA</u> <u>M</u> =

$(Q, \sum, \boldsymbol{\tau}, \boldsymbol{\Delta}, s, F)$, *s.t.* $L(M) = L(M1) \cap L(M2)$

$Q = Q1 \times Q2$

$\boldsymbol{\tau} = \boldsymbol{\tau}1$

$s = (s1, s2)$

$F = F1 \times F2$

Intersection of a CFL and RL

Suppose:

- L is a CFL and corresponding PDA M1 = (Q1, ∑, **τ**1, **Δ**1, s1, F1)
- R is a RL, and corresponding DFA M = (Q2, ∑, δ, s2, F2)

<u>Construction of PDA</u> <u>M</u> =

(Q, ∑, **τ**, **Δ**, s, F), *s.t. L(M) = L(M1)* $\cap$ *L(M2)*

Q = Q1 x Q2

τ = **τ**1

s = (s1, s2)

F = F1 x F2

Where **Δ** is defined as:

- For each transition of PDA (q1, a, β) (p1, γ) and for each state q2 $\in$ Q2, add the following transition to **Δ**:

- For each transition of PDA (q1, ε, β) (p1, γ) and for each state q2 $\in$ Q2, add the following transition to **Δ**:

Intersection Operation

Are CFL’s closed under intersection operation?

Let $L1 = \{a^n\ b^n\ c^m \mid m, n \geq 0\}$

$L2 = \{a^n\ b^m\ c^m \mid m, n \geq 0\}$

$L1 \cap L2 = ??$

Complement Operation

Are CFL’s closed under complement operation?

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Thanks !