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Theory of Computation
CS F351

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Lecture: Pumping Lemma for CFL’s

Pumping Lemma for CFL

- Pumping lemma for regular languages told us that if there was a string long enough to cause a cycle in the DFA for the language, then we could “pump” the cycle and discover an infinite sequence of strings that had to be in the language.
- For CFL’s the situation is a little more complicated.
- We can always find two pieces of any sufficiently long string to “pump” together.
  - This means: if we repeat each of the two pieces the same number of times, we get another string in the language.

Statement of Pumping Lemma for CFL

Let G = (V, ∑, R, S) be a CFG. Then any string $w \in L(G)$ of length greater than $\Phi(G)^{|V|}$ (i.e. there exists a constant p such that |w| >= p) can be rewritten as w = u v x y z such that

1. $|vxy| \leq p$.
2. $|vy| > 0$.
3. For all $i \geq 0$, uv^ixy^iz is in L(G).

Example

Prove that L = $\{a^n b^n c^n \mid n \geq 0\}$ is not a CFL.

Proof: Suppose L is a CFL and let p be the pumping length. Let w = $a^p b^p c^p$. Then, according to pumping lemma, w can be written as u v x y z, with the required conditions.

Case1: v and y contain only one type of alphabet symbol, i.e. v or y does not contain both a's and b's, or both b's and c's.

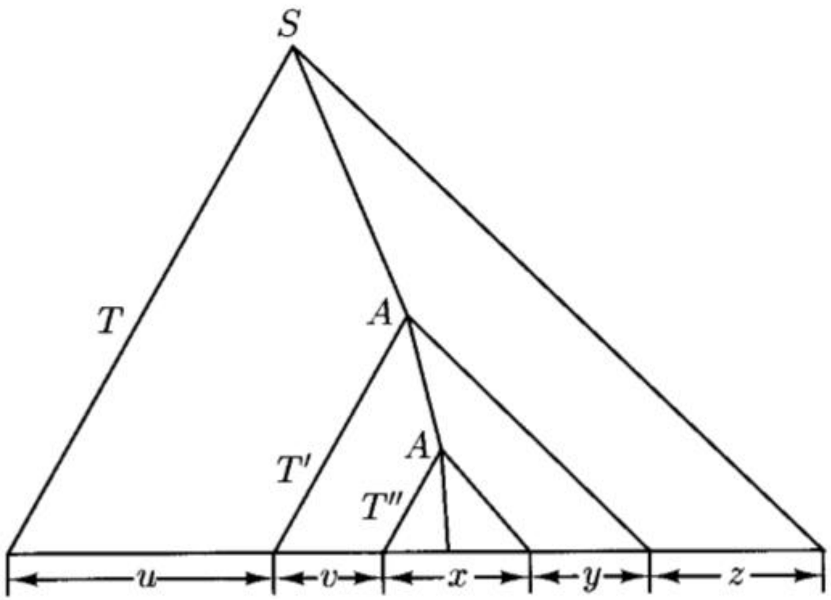
Case 2: v and y contain more that one type of symbol.

Proof of Pumping Lemma for CFL's

Proof:

Let w be a string, and let T be the parse tree with root labelled S and with yield w that has smallest number of leaves among all parse trees with the same root and yield.

- Since, T's yield is > $\Phi(G)^{|V|}$, T has a path of length atleast (|V| + 1) nodes.
- This path has atleast |V| + 2 nodes, with only one labelled as a terminal, and the remaining non-terminals.
- This means that number of nodes in the path are greater than number of non-terminals. That is atleast one non-terminal will repeat itself along the path.



Example

Prove that $L = \{a^i\ b^j\ c^k \mid 0 \le i \le j \le k\}$ is not a CFL.

Examples (Try these)

Using Pumping lemma, prove that the following languages are non CFL's.

$L1 = \{a^n \mid n >= 1 \text{ is a prime number}\}$

$L2 = \{a^{n^2} \mid n >= 0\}$

$L3 = \{www \mid w \in (a+b)^*\}$

$L4 = \{w \in (a+b+c)^* \mid w \text{ has equal number of a's, b's, and c's}\}$

Next ….

Determinism and Parsing …..