Birla Institute of Technology and Science, Pilani
First Semester 2020 – 21
CS F351 (Theory of Computation)
Test-1 (Solution and Marking Scheme)
September 18, 2020

Q1. [4 Marks]. Over $\Sigma = \{b\}$, Let $L = \{b^{2n} \mid n \geq 0\}$. Using pumping lemma, consider the following proof to prove that L is not a regular language.

Proof: Suppose L is regular and p is the pumping length. Let string $w = b^{2p}$. We can see that $w \in L$ and $|w| > p$.

According to pumping lemma, w can be written as xyz (i.e. w = xyz) with required conditions (i.e. $|y| > 0$ and $|xy| \leq p$) such that $xy^iz \in L$, $\forall i \geq 0$.

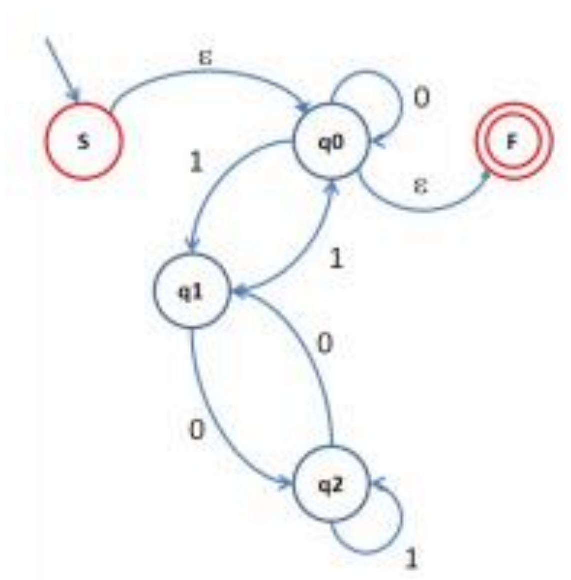
Let $x = \epsilon$, $y = b$, $z = b^{2p-1}$. Now, for i = 0, $xy^iz = \epsilon\, (b)^0\, b^{2p-1} = b^{2p-1} \notin L$. Therefore, L is not a regular language.

Is the above proof correct? Briefly, justify your answer.

Ans: The proof is not correct. First of all, the given language L is a regular language. Therefore, according to pumping lemma, for the chosen string w, there exists a partition of w with required conditions. Observe here that the quantifier is "there exists" and the given partition is just one such partition. So, for example, we can choose $x = \epsilon$, $y = b^{2x}$, and $z = b^{2p-2x}$. Now for every value of i>=0, xy^iz would belong to L.

[Marking Scheme: 1M for writing "The proof is not correct". 4M for justifying also.]

Q2**[4 Marks]**. Over $\Sigma = \{0, 1\}$ consider the following NFA. Find the corresponding regular expression after removing the states in the following order: q2, q1, and q0.



Ans: Removing state q2 would result in a self loop on q1 with: 01*0

Removing state q1 would result in a self loop on q0 with: 1(01*0)*1

Removing state q0 would give the RE as: (0 + 1(01*0)*1)*

In this problem you were supposed to give the above regular expression only because you were asked to follow certain algorithm. If your regular expression is different, you were not given marks even though the language accepted is same. Also, for some minor mistakes (parenthesis mismatch etc.) 1M was deducted.

Q3. [4 Marks]. Over $\Sigma = \{1, 2\}$, Let L1 = $\{1^n 2^m 1^{n+m} \mid n \geq 0, m \geq 0\}$. Observe the following proof to show that L1 is not a regular language.

Proof: Assume L1 is regular. We know that language L2 = $\{2^n 1^n \mid n \geq 0\}$ is not a regular language. Now, the following expression is TRUE:

L2 = L1 ⋂ L (2*1*), where L (2*1*) is the language of regular expression 2*1*.

Since regular languages are closed under intersection, L1 is not regular.

Is the above proof correct? If Yes, just write "Proof is Correct" as your answer. If No, what part of the above proof requires modification (just mention this modification).

Ans: The proof is not correct because L2 is not equal to L1 ⋂ L (2*1*). Rather, the following expression is True: L2 = (L1 ⋂ L (2*1*)) – (L(11*)).

Marking scheme: 1M for writing that the proof is not correct; and 3M for suggesting the correct modification. Here, you were supposed to give the modification in the proof, and not why proof is not correct.

Q4 [4 Marks]. Let N = $(Q, \Sigma, \delta, q_0, F)$ be an NFA and $|Q| = m$. Assuming that N accepts at least one string, is the following statement True/False (with justification):

"N always accepts some string w, where $|w| \leq (m-1)$".

Ans: The statement is True. If a string of length m is accepted by an NFA with m states, atleast one state would repeat itself along the path. Now, if we remove this state, the string accepted would be of length (m-1).

Marking scheme: 1M for stating that the given statement is TRUE. 3M for correct and complete justification.

Q5 **[4 Marks**]. You know that regular languages are closed under intersection operation, i.e. if L1 and L2 are regular, then (L1 ⋂ L2) is always regular. Is vice versa also true? If Yes, prove it; else give a counter example.

Ans: Vice versa is not true. Counter example: Let L1 = {ab}, L2 = $\{a^n b^n \mid n>=0\}$. Now L1 ⋂ L2 = {ab}, which is a regular lang. But, L2 is not a regular language.

[Marking scheme: 1M for writing that the vice versa of the given statement is not True. 3M for for giving a correct counter example.]

Q6 [3Marks] How many DFAs are possible with two states (namely P and Q) over given input alphabet {0, 1} where P is always the start state?

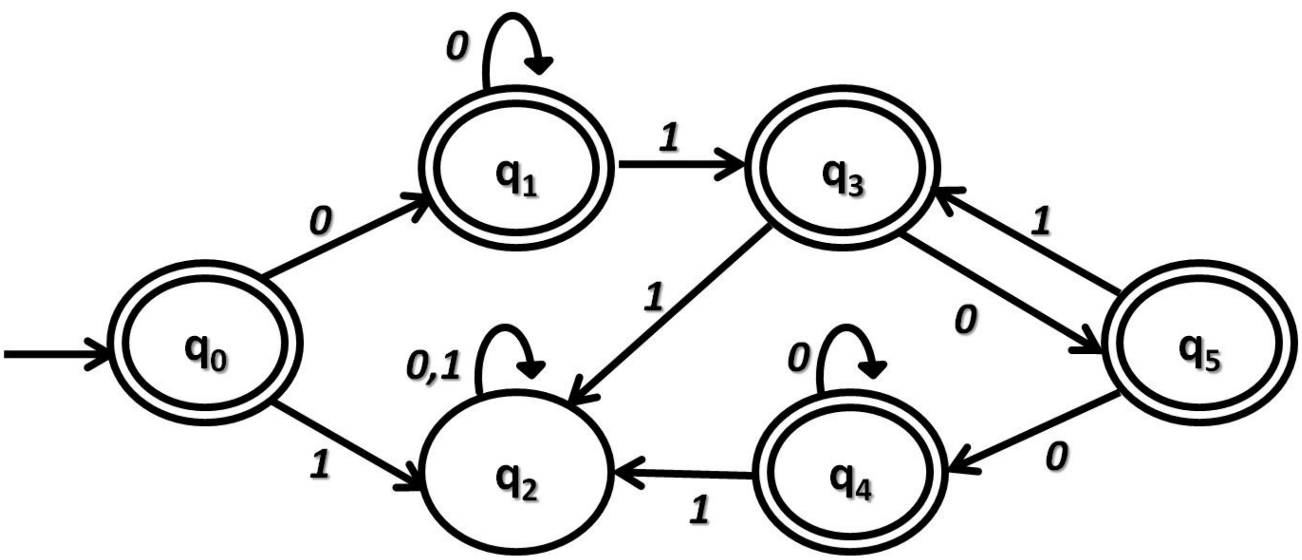
a) 16
b) 32
c) 64
d) None of the options given here.

[0/3Marks. No partial marks.]

Q7 **[3 Marks]**. Write the English explanation of accepted language from the following regular expression: (b + ∊) (aa*b)*a* . Consider Σ = {a, b}.

Ans: L = {w ∊ (a + b)* | w does not contain bb as a substring} Here there were no partial marks. The statement you wrote should have been complete. Also, few explained the language in terms of given regular expression itself (e.x. b or epsilon followed by a and then a*…….), which is wrong.

Q8 **[4 Marks]**. Consider the following DFA over Σ = {a, b}. Apply state equivalence mechanism on the DFA and find out the different equivalence classes in E_1 (i.e. 1-equivalent class).



E0 = {{q2}, {q0, q1, q3, q4, q5}}
E1 = {{q2}, {q0, q3, q4} {q1, q5}}
[0/4Marks. No partial marks.]