Pushdown Automaton

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Pushdown Automata (PDA)

CFG are specification mechanisms, i.e., a CFG generates a context-free language.

Pushdown-automata are for recognizing mechanisms, i.e., a PDA recognizes a context-free language.

- CFG are like regular expressions and PDA are like FA.

Pushdown Automata (PDA)

PDA are a new type of computation model.

- PDAs are like NFAs but have an extra component called a stack.

The stack provides additional memory beyond the finite amount available in the control

- The stack allows PDA to recognize some non-regular languages

PDA and CFG

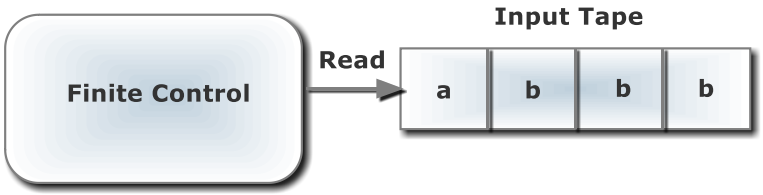
PDA are equivalent in specification power with CFG

This is useful because it gives us two options for proving that a language is context-free:

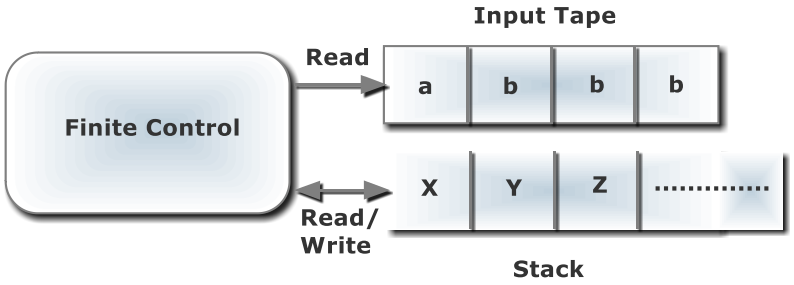
- Construct a CFG that generates the language
- Construct a PDA that recognizes the language

Schematic Representation of NFA and PDA

NFA



PDA



Pushdown Automata (PDA)

A PDA has an input tape, a finite set of states and an unbounded stack.

At any given point of time, the PDA

- is in some state p,
- reads some input symbol a_i
- can access the topmost few elements of the stack.

Pushdown Automata (PDA)

We allow the stack to have a different alphabet (usually denoted as Γ)

Given an input w the PDA at any given instance does the following

- Reads a_i from w where $a_i \in \Sigma \cup \in$
- Goes from state p to q
- Replaces the topmost element of the stack, X with Y where X, Y $\in \Gamma^*$

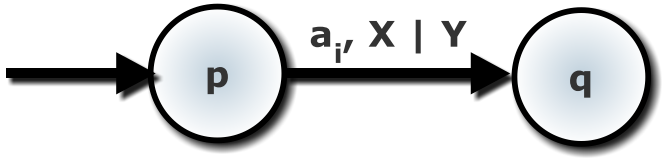
Pushdown Automata (PDA)

The PDA at any point of time

- can move from state p to q,
- can change the topmost element.

This will be represented as

- $(p, a_i, X) \rightarrow (q, Y)$



Pushdown Automata (PDA)

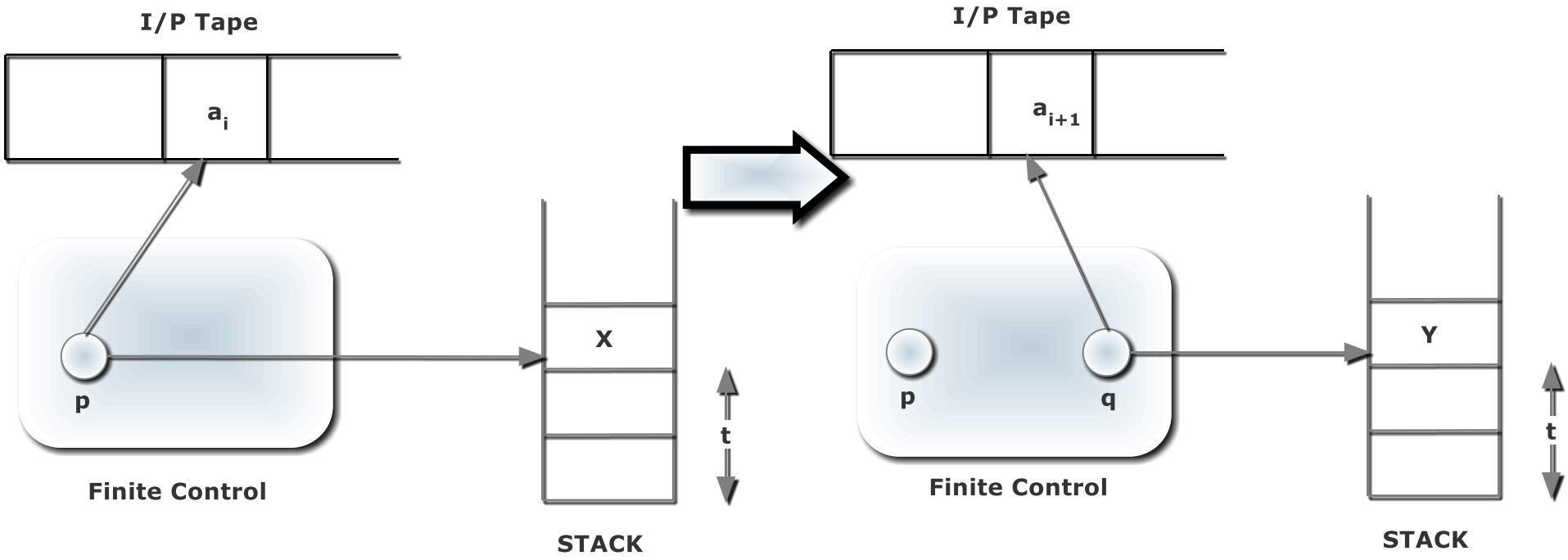
What does it mean for X to be ∈(Epsilon)?

- Pushing Y onto the stack.

What does it mean for Y to be ∈(Epsilon)?

- Popping out X from the stack.

Pushdown Automata (PDA)



Pushdown Automata

Each transition

- is based on the current input symbol and the top of the stack
- optionally pops the top of the stack, and
- optionally pushes new symbols onto the stack

Initially, the stack holds a special symbol z0 that indicates the bottom of the stack.

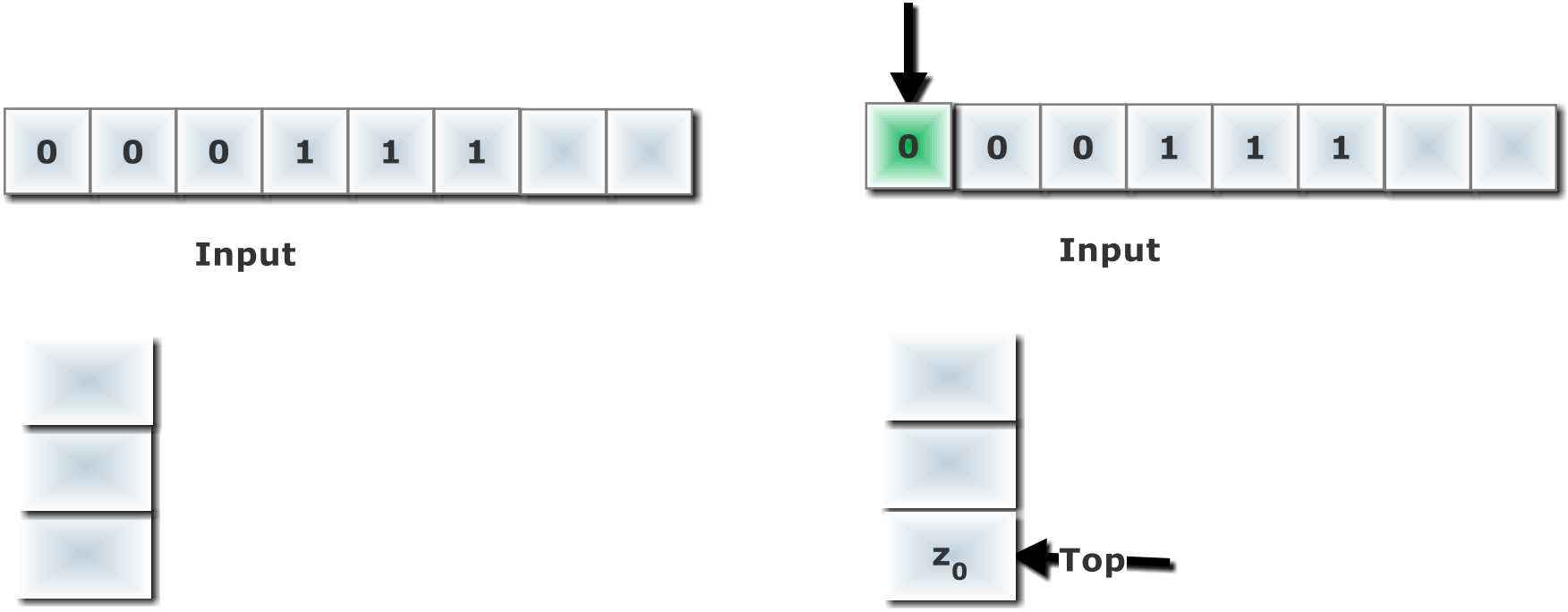
Example

Design a PDA for the following language

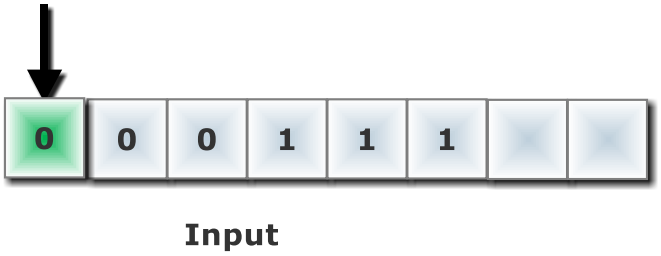
$$L = \{0^n 1^n \mid n \geq 1\}$$

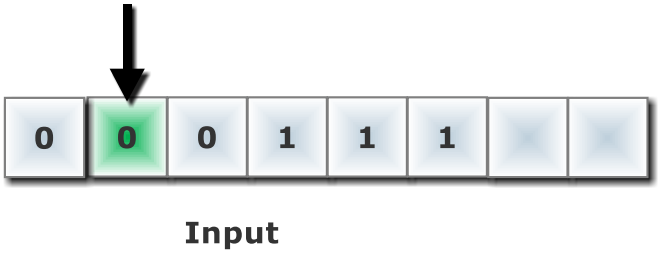
Example

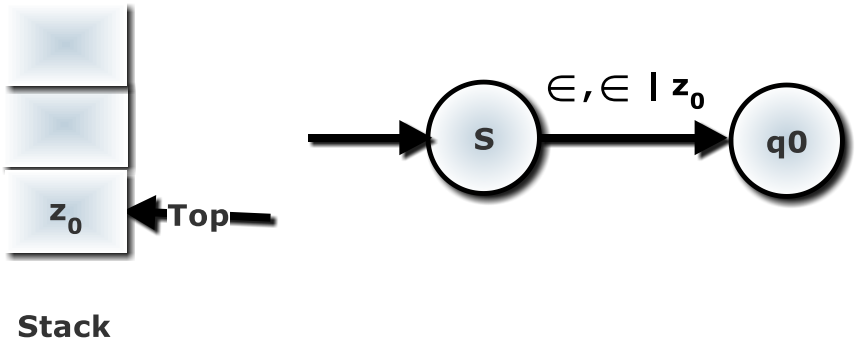
Design a PDA for the following language L ={0^n1^n| n ≥ 1}

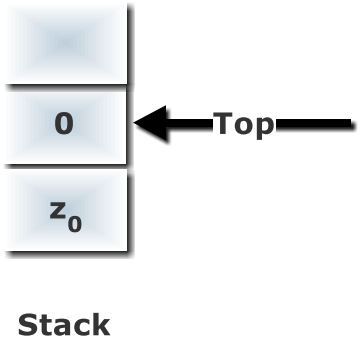


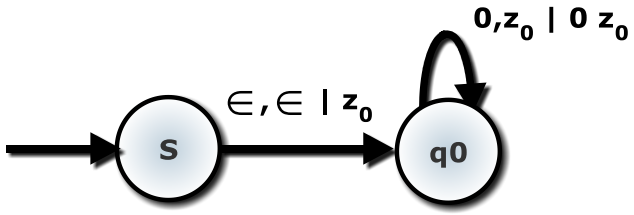
Example


0 0 0 1 1 1
Input

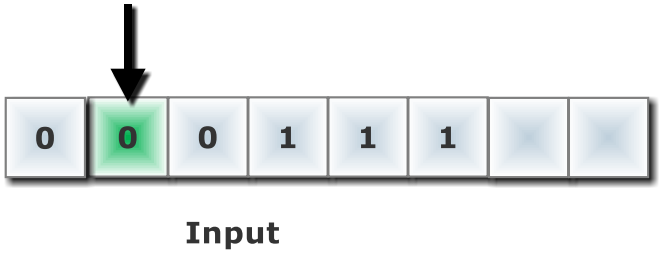

0 0 0 1 1 1
Input

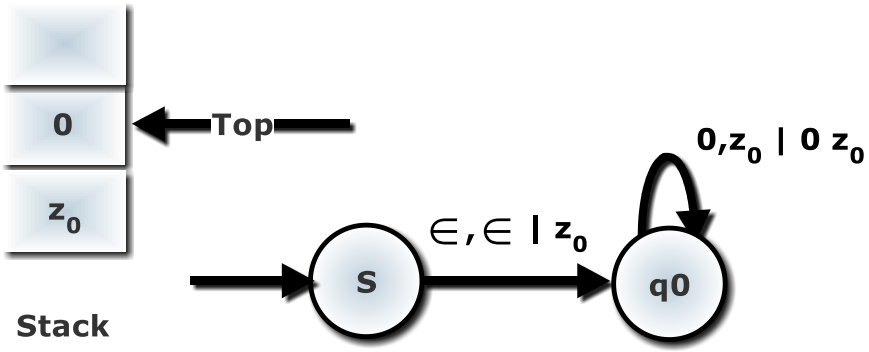

z_0
Top
S
$\in, \in \mid z_0$
q0
Stack

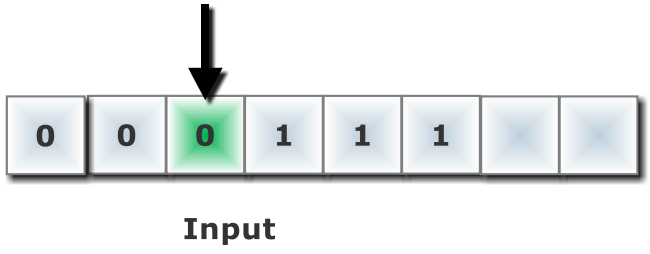

0
z_0
Top
Stack

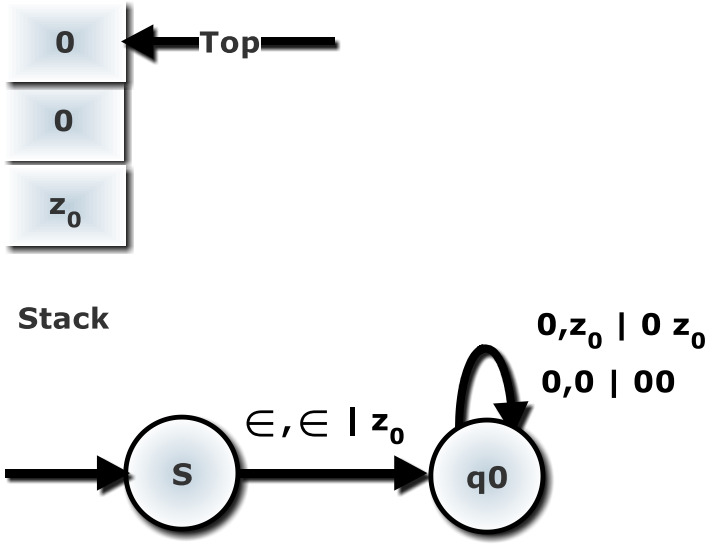

$0, z_0 \mid 0\ z_0$
$\in, \in \mid z_0$
S
q0

Example

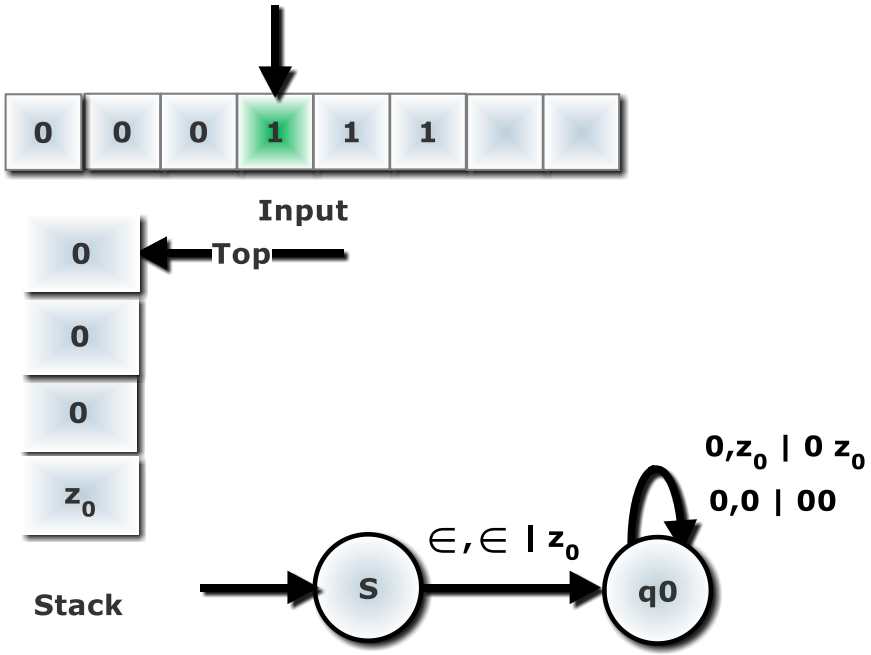

0 0 0 1 1 1
Input

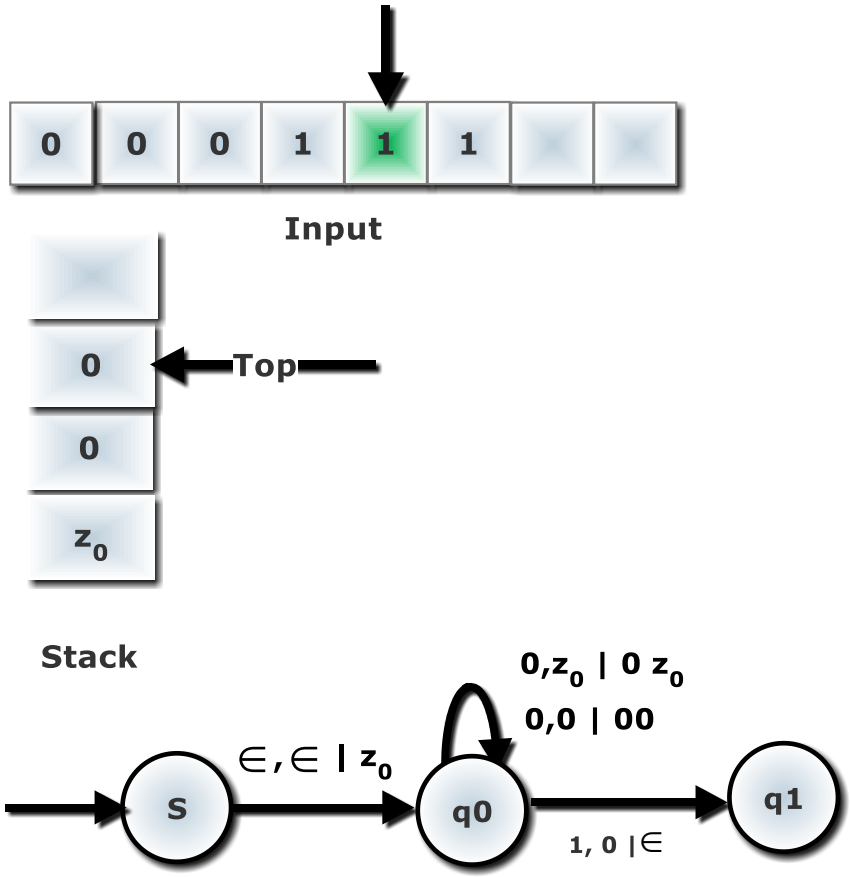

0
Top
z_0
Stack
S
q0
$\in, \in \mid z_0$
$0, z_0 \mid 0\ z_0$


0 0 0 1 1 1
Input

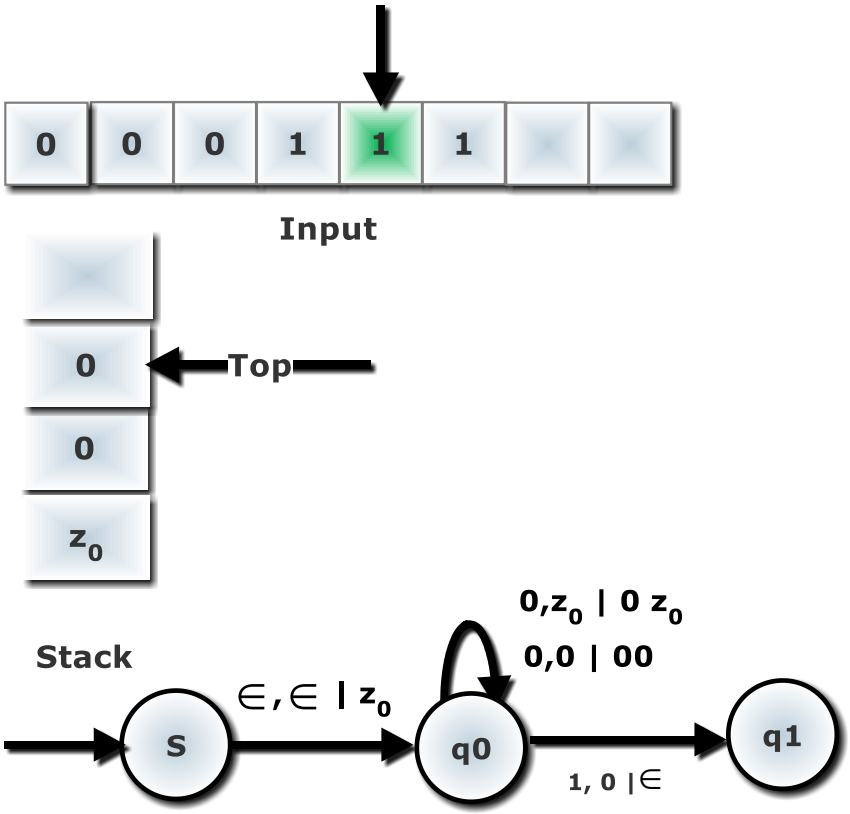

0
Top
0
z_0
Stack
S
q0
$\in, \in \mid z_0$
$0, z_0 \mid 0\ z_0$
$0,0 \mid 00$

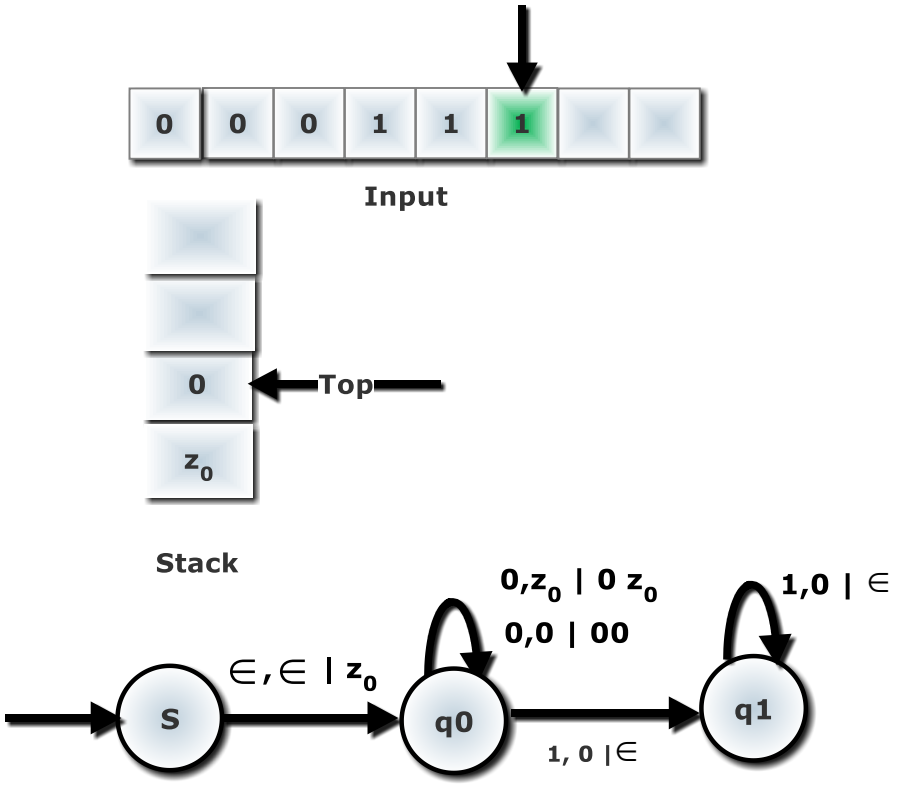
Example


0 0 0 1 1 1
Input
0
0
0
z_0
Top
Stack
S
q0
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00

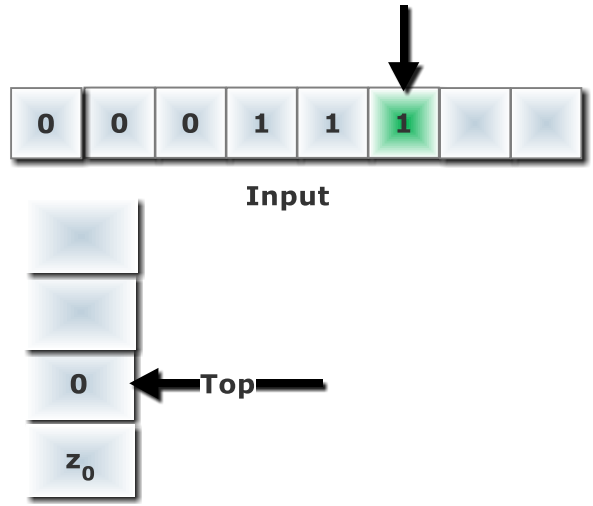

0 0 0 1 1 1
Input
0
0
z_0
Top
Stack
S
q0
q1
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00
1, 0 | ∈

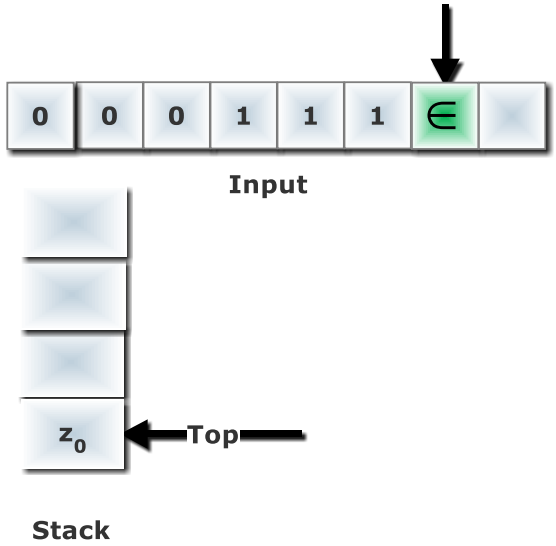
Example

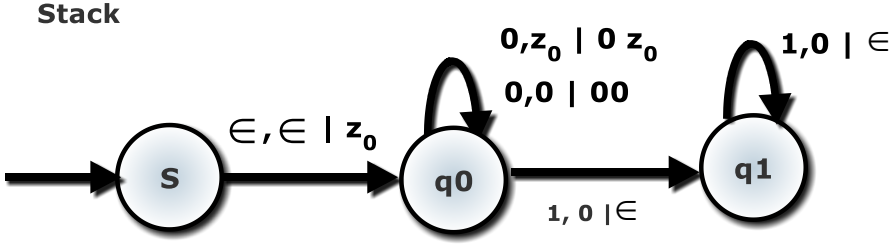

0 0 0 1 1 1
Input
0
Top
0
z_0
Stack
$0,z_0 \mid 0\ z_0$
$0,0 \mid 00$
$\in,\in \mid z_0$
S
q0
q1
$1, 0 \mid \in$

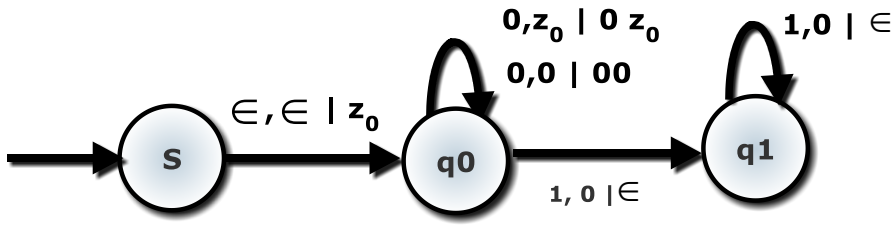

0 0 0 1 1 1
Input
0
Top
z_0
Stack
$0,z_0 \mid 0\ z_0$
$0,0 \mid 00$
$1,0 \mid \in$
$\in,\in \mid z_0$
S
q0
q1
$1, 0 \mid \in$

Example

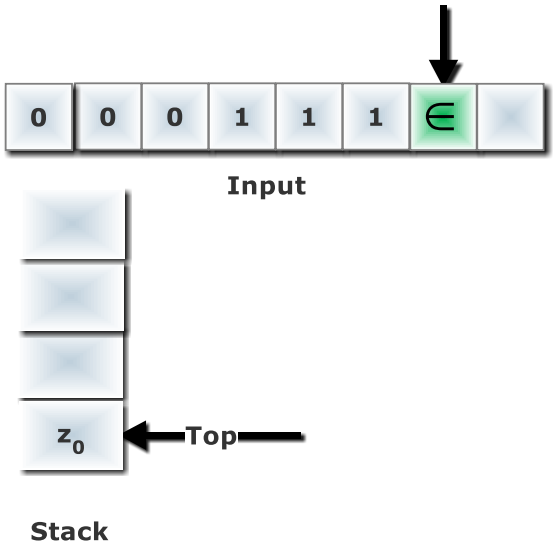

0 0 0 1 1 1
Input
0
Top
z_0
Stack

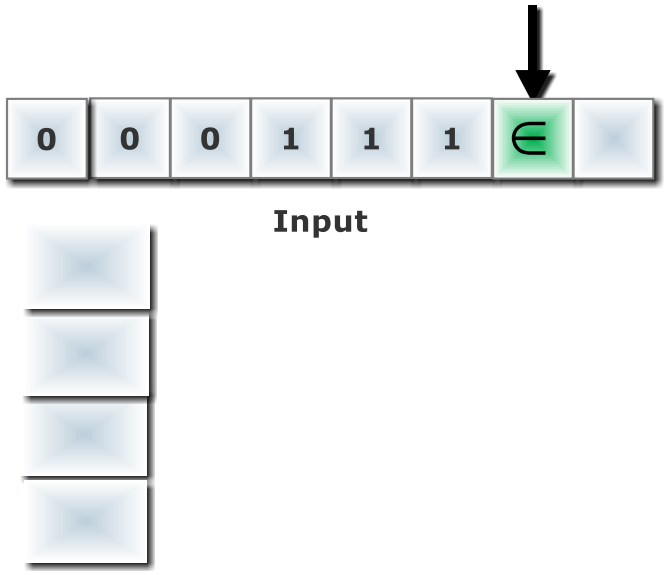

0 0 0 1 1 1 ∈
Input
z_0
Top
Stack

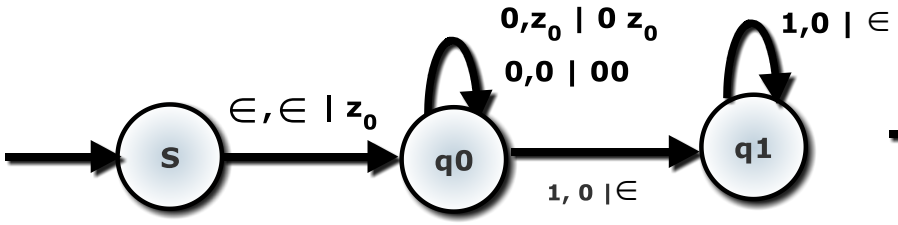

S
q0
q1
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00
1,0 | ∈
1, 0 | ∈

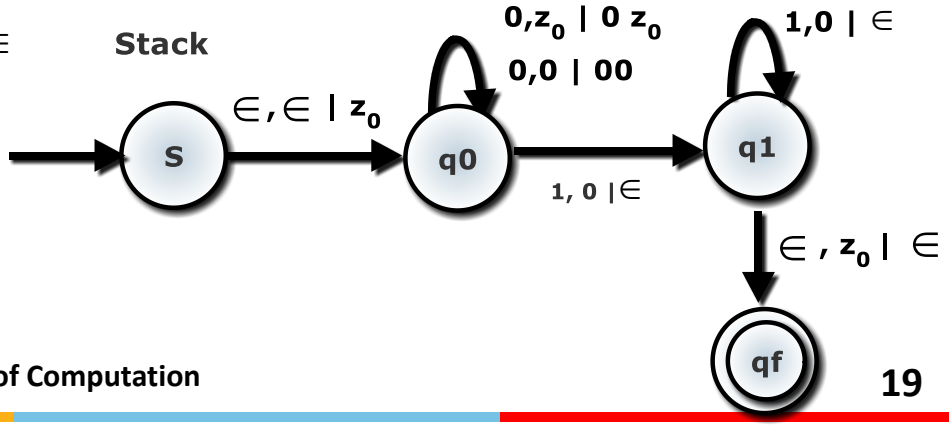

S
q0
q1
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00
1,0 | ∈
1, 0 | ∈

Example


0 0 0 1 1 1 ∈
Input
z_0
Top
Stack

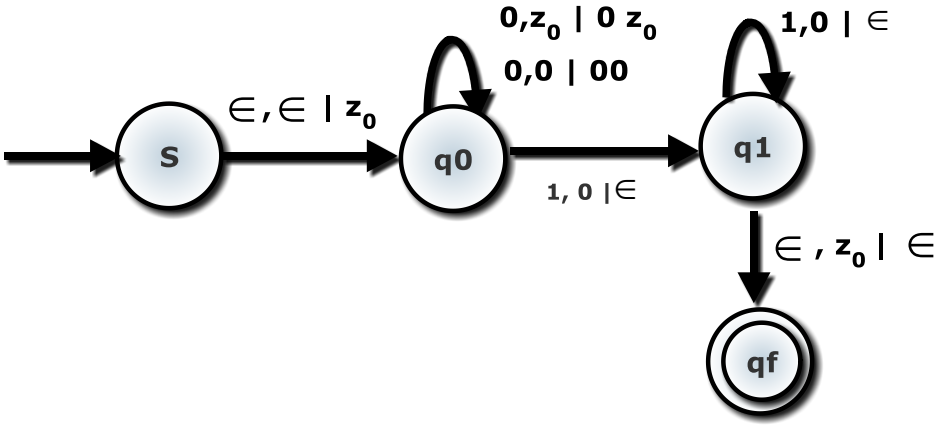

0 0 0 1 1 1 ∈
Input
Stack


S
q0
q1
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00
1,0 | ∈
1, 0 | ∈


Stack
S
q0
q1
qf
∈, ∈ | z_0
0,z_0 | 0 z_0
0,0 | 00
1,0 | ∈
1, 0 | ∈
∈ , z_0 | ∈

Notations for PDA

Transition Diagram Notation for PDA



Transition Function Notation for PDA

$$\delta(S, \in, \in) = (q0, z_0)$$

$$\delta(q0, 0, z_0) = (q0, 0z_0)$$

$$\delta(q0, 0, 0) = (q0, 00)$$

$$\delta(q0, 1, 0) = (q1, \in)$$

$$\delta(q1, 1, 0) = (q1, \in)$$

$$\delta(q1, \in, z_0) = (qf, \in)$$