Optimization:Solved Examples

Ankush Jahagirdar

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1. Suppose the probability of a mutation at a single bit position is 0.1. a) Calculate the probability of a 10-bit string surviving mutation without change. b) Calculate the probability of a 20-bit string surviving mutation without change. c) Recalculate the survival probabilities for both 10-bit and 20-bit string when the mutation probability is 0.01.

   Solution:
   The chromosome does not change if none of its bits is changed. If p_m is the probability of mutation and L is the number of bits in the chromosome, probability that no bits are changed is $p_{NC} = (1 - p_m)^L$.
   Therefore,
   a) $p_{NC} = (1 - 0.1)^{10} = 0.349$
   b) $p_{NC} = (1 - 0.1)^{20} = 0.122$
   c) $p_{NC,10} = (1 - 0.01)^{10} = 0.904$
   $p_{NC,20} = (1 - 0.01)^{20} = 0.818$

2. Given the fitness function $f(x) = x^2$, calculate the probability of selecting the individuals $x = 1, x = 2, x = 3$, using Roulette Wheel selection. Calculate the probability of selecting the same individuals when the fitness function is $f_1(x) = f(x) + 10$.
   Comment on the *selection pressure*.

   Solution:

$$p(x=1) = \frac{f(x=1)}{f(x=1)+f(x=2)+f(x=3)} = \frac{1}{14} \tag{1}$$
$$p(x=2) = \frac{f(x=2)}{f(x=1)+f(x=2)+f(x=3)} = \frac{4}{14} \tag{2}$$
$$p(x=3) = \frac{f(x=3)}{f(x=1)+f(x=2)+f(x=3)} = \frac{9}{14} \tag{3}$$

   For transposed function

$$p(x=1) = \frac{f_1(x=1)}{f_1(x=1)+f_1(x=2)+f_1(x=3)} = \frac{11}{44} \tag{4}$$
$$p(x=2) = \frac{f_1(x=2)}{f_1(x=1)+f_1(x=2)+f_1(x=3)} = \frac{14}{44} \tag{5}$$
$$p(x=3) = \frac{f_1(x=3)}{f_1(x=1)+f_1(x=2)+f_1(x=3)} = \frac{19}{44} \tag{6}$$

   The selection pressure is lower in second case because the probability values are closer.

3. Six strings have the following fitness function values $5, 10, 15, 25, 50, 100$. Under Roulette Wheel selection, calculate the expected number of copies of each string in the mating pool if a constant population size , $n = 6$ is maintained.

   Solution:

$$\bar{f} = \frac{5 + 10 + 15 + 25 + 50 + 100}{6} = 34.166 \tag{7}$$

   Hence, the expected counts $(f_i/\bar{f})$ are $0.146, 0.292, 0.4391, 0.7318, 1.4637, 2.9274$ respectively.

4. In a 3-variable problem of optimization with genetic algorithm, the following variable bounds are specified:

$$0.1 \leq w \leq 0.9 \tag{8}$$

$$0.0009 \leq x \leq 0.007 \tag{9}$$

$$-7 \leq y \leq 12 \tag{10}$$

   Compute the minimum string length of any point $(w, x, y)^T$ coded in a binary string to achieve three-significant-digits accuracy in the solution.

   Solution:
   For w, three significant digits accuracy correspond to 0.001.
   Accuracy$= 0.001 = \dfrac{0.9 - 0.1}{2^l}$
   $l = 9.64 \approx 10$

   For x, three significant digits accuracy correspond to 0.000001.
   Accuracy$= 0.000001 = \dfrac{0.007 - 0.0009}{2^l}$
   $l \approx 11$

   For x, three significant digits accuracy correspond to 0.01.
   Accuracy$= 0.01 = \dfrac{12 - (-7)}{2^l}$
   $l \approx 17$

   Thus minimum string length would be 38.