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Theory of Computation

CS F351

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Agenda

- Recursive Languages
- Recursive Functions
- Recursively Enumerable Languages
- Prove: “If L is recursive, then it is Recursively enumerable”
- Prove: “If L is recursive, then its complement L' is also recursive”.

- “M decides L”

Let Σ_0 the alphabet over which input string is defined. Thus we allow the TM to use extra symbols during the computation.

M decides a language if for any string $w \in \Sigma_0^*$ the following is true:

“If $w \in L$ then M accepts w ; and if w does not belong to L , then M reject w ”

Recursive Language / Turing decidable language

- A language L is recursive if there is a TM that “decides” it.

Recursive functions

- Let Σ_0 be $\Sigma - \{\Delta, \sqcup\}$.
- Suppose M halts on input w and from configuration $\triangleright \sqcup w$ the TM halts in configuration $\triangleright \sqcup y$, where $y \in \Sigma_0^*$. y is called the output of M on w and is denoted as $M(w)$.
- So $M(w)$ is defined only if M halts on input w .
- Let f be any function from Σ_0^* to Σ_0^* . TM M computes function f if for all $w \in \Sigma_0^*$, $M(w) = f(w)$.
- In other words, for all $w \in \Sigma_0^*$, M eventually halts on w and when it halts that tape contents are $\Delta \sqcup f(w)$.
- A function f is recursive if there is a TM that computes f .

"M semi-decides L"

TM M semidecide L if for any string $w \in \Sigma_0^*$ the following is true:

- $w \in L$ iff M halts on input w

Recursively Enumerable Languages

- A language L is recursively enumerable iff there is a TM that semi-decides it.

TM's that semi-decides languages are no algorithms

If a language is recursive, then it is recursively enumerable:

Proof:

Let $M = (K, \Sigma, \delta, s, \{y, n\})$ be a TM that decides L . We can define a machine M' that semi-decides L as follows:

$M' = (K, \Sigma, \delta', s, \{y\})$, where δ' contains additionally the transitions of the form:

$\delta'(n, a) = (n, a)$ for all $a \in \Sigma$ and n is not a halting state.

- A recursive language is Recursively Enumerable, but not vice versa.
- If L is recursive, then its complement L' is also recursive.
- Recursively Enumerable languages are not closed under complement.



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Thank You