



## Theory of Computation CS F315

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Let L1 = {"M" "w" | M is a Turing Machine that accepts string w}. Answer the following.

- Is L1 a Recursive language?
- Is L1 a Recursively Enumerable language?
- Is complement of L1 a Recursively Enumerable language? Why?

"Decidable languages are closed under UNION operation". TRUE/FALSE.

What is wrong with the following proof for above theorem?

Let T1 = (K1,  $\Sigma$ 1,  $\delta$ 1, s1, {Y1, N1}) and T2 = (K2,  $\Sigma$ 2,  $\delta$ 2, s2, {Y2, N2}) be two single tape Turing Machines that decide languages L1 and L2, respectively. Y1 and Y2 are accepting halting states. Similarly, N1 and N2 are non-accepting halting states. Assuming both T1 and T2 starts in initial configuration, we need to design a single tape Turing Machine T = (K,  $\Sigma$ ,  $\delta$ , s, H) that decides L = L1 U L2.

Is the following construction of T correct?—Justify your answer briefly.

- $T = (K, \Sigma, \delta, s, H)$ , where
- $K = K1 \cup K2 \cup \{qn\}$ ,  $\sum = \sum 1 \cup \sum 2$ , S = S1,  $H = H1 \cup H2$
- $\forall x \in \Sigma 1, \forall y \in \Sigma 1 \{ \triangleright \},$
- $\delta = \delta 1 \cup \delta 2 \cup \{(N1, x)(qn, x)\} \cup \{(qn, y)(qn, \leftarrow)\} \cup \{(qn, p)(s2, \rightarrow)\}$
- [Note: Transition of the form (q, x)(p, y) means in state q if the symbol read is x, write y in place of x and move to state p.]

- Let L = {"M" | M is a Turing Machine which decides its respective language}. Give a high level description of Turing Machine U<sub>TM</sub> which takes "M" ∈ L (i.e. encoding of M) as input and outputs all halting states of M.
- Also, is L(U<sub>TM</sub>) decidable?



#### Problem

Let G1 be a CFG and string  $x \in L(G1)$ . Also, let G2 be a CFG such that  $L(G2) = L(G1) - \{x\}$ . Prove that (by giving an algorithm) computing grammar G2 from grammar G1 is decidable.

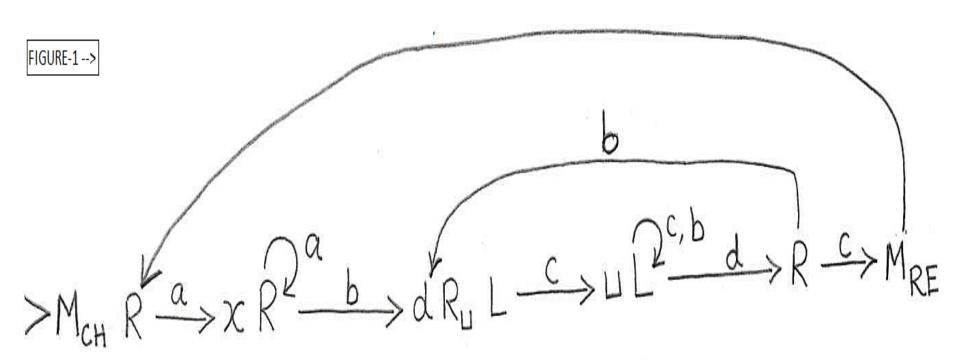


#### Problem

- Let G1 be a CFG and string x ∈ L(G1). Also, let G2 be a CFG such that L(G2) = L(G1) {x}. Prove that (by giving an algorithm) computing grammar G2 from grammar G1 is decidable.
- Let L = {G | G is a CFG and G generates at least 51 strings}. Prove that L is decidable. You can give an algorithm here as a high-level description of corresponding TM.

[<u>Hint</u>: Think about the outcome of the question just before this one.]

- Let L = {a<sup>i</sup> b<sup>j</sup> c<sup>k</sup> | i x j = k and i, j, k > 0}. Consider the single tape
   Turing Machine (FIGURE-1) to accept L. Its initial configuration is
   ► \_\_ wand final configuration is ► \_\_ w1, where w1 is any string. It uses machine M<sub>CH</sub> (which accepts if all a's are followed by all b's and all b's are followed by all c's; otherwise it rejects). Also it uses machine M<sub>RE</sub> to restore all b's (understand yourself why it is required). Attempt the following:
- Make TM's for  $M_{CH}$  and  $M_{RE}$  so that they are meaningful in deciding language L by TM in FIGURE-1.
- Modify FIGURE-1 so that it semi-decides the language L.
- Finally, modify FIGURE-1 so that it decides the language L.





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# Thank You