Theory of Computation
CS F351

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Vishal Gupta
Department of Computer Science and Information Systems
Birla Institute of Technology and Science
Pilani Campus, Pilani

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Agenda: 1) Determinism and Parsing 2) Top Down Parsing

Determinism and Parsing

Parser is an Algorithm to determine whether a given string is in the language generated by a given CFG, and, if so, to construct the parse tree of the string.

A PDA is not of IMMEDIATE practical use in parsing because of its non-deterministic nature.

Q. Can we make every PDA deterministic ???

To facilitate making most PDA's deterministic, let us modify the acceptance convention.

" A language is said to be deterministic context free if it is recognized by a deterministic PDA that also has a extra capability of sensing the end of input string".

Let the last symbol by $ which is not in ∑.

Q. Let $L = a^* \cup \{a^n b^n : n \geq 1\}$. Case 1: Try making a deterministic PDA for L.

Case 2: Try making a deterministic PDA for L$.

“Heuristic” to prove that some CFL’s are not deterministic

“The class of deterministic CFL’s is closed under complement”

Q. Let L = {$a^n b^m c^p$: m, n, p ≥ 0 and m ≠ n or m ≠ p. Prove that this language is not deterministic CFL.

Types of PDA’s

- Given a deterministic CFG G, we can make following types of PDA’s for it:
  - Non-deterministic PDA.
  - Deterministic PDA which does not act as a parser.
  - Deterministic PDA which acts as a top down parser.
  - Deterministic PDA which acts as a bottom up parser.

Top Down Parsing: "Look-ahead" WHY and HOW

Let L = $\{a^n b^n : n \geq 0\}$. Let the corresponding Grammar be:

G = ({a, b, S} , {a, b} , R , S)

Where R contains: $S \rightarrow aSb \mid e$ [*Note: e is used for empty string*]

The corresponding PDA will be:

M = ({p, q} , {a, b} , {a, b, S} , Δ , p , {q})

Where Δ contains:

((p, e, e) , (q, S))
((q, e, S) , (q, aSb))
((q, e, S) , (q, e))
((q, a, a) , (q, e))
((q, b, b) , (q, e))

The transformed PDA will be:

M_1=({p, q, q_a, q_b, $q_\$$} , {a, b} , {a, b, S} , Δ_1 , p ,{$q_\$$})

Where Δ_1 contains:

((p, e, e) , (q, S))
((q, a, e) , (q_a, e))
((q_a, e, a), (q, e))
((q_a, e, S), (q_a, aSb))
((q, b, e), (q_b, e))
((q_b, e, b), (q, e))
((q_b, e, S), (q_b, e))
((q, \$, e), ($q_\$$, e))

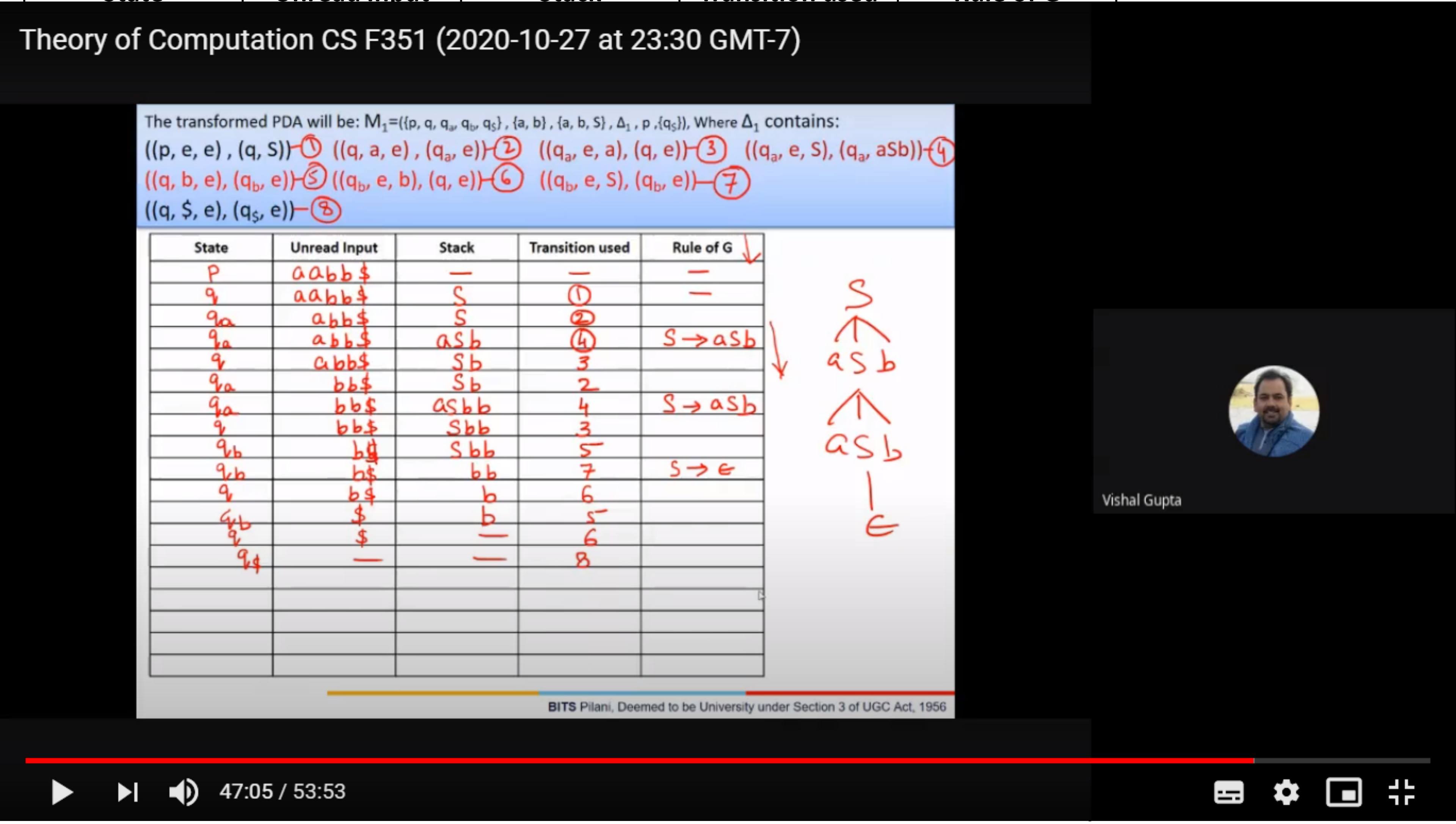
The transformed PDA will be: $M_1=(\{p, q, q_a, q_b, q_\$\}, \{a, b\}, \{a, b, S\}, \Delta_1, p, \{q_\$\})$, Where Δ_1 contains:

$((p, e, e), (q, S))$ $((q, a, e), (q_a, e))$ $((q_a, e, a), (q, e))$ $((q_a, e, S), (q_a, aSb))$

$((q, b, e), (q_b, e))$ $((q_b, e, b), (q, e))$ $((q_b, e, S), (q_b, e))$

$((q, \$, e), (q_\$, e))$

State	Unread Input	Stack	Transition used	Rule of G



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The transformed PDA will be: $M_1=(\{p, q, q_a, q_b, q_\$\}, \{a, b\}, \{a, b, S\}, \Delta_1, p, \{q_\$\})$, Where Δ_1 contains:

$((p, e, e), (q, S))$ – ① $((q, a, e), (q_a, e))$ – ② $((q_a, e, a), (q, e))$ – ③ $((q_a, e, S), (q_a, aSb))$ – ④

$((q, b, e), (q_b, e))$ – ⑤ $((q_b, e, b), (q, e))$ – ⑥ $((q_b, e, S), (q_b, e))$ – ⑦

$((q, \$, e), (q_\$, e))$ – ⑧

State	Unread Input	Stack	Transition used	Rule of G
p	aabb\$	—	—	—
q	aabb\$	S	①	—
q_a	abb\$	S	②	
q_a	abb\$	aSb	④	S → aSb
q	abb\$	Sb	3	
q_a	bb\$	Sb	2	
q_a	bb\$	aSbb	4	S → aSb
q	bb\$	Sbb	3	
q_b	b\$	Sbb	5	
q_b	b\$	bb	7	S → ∈
q	b\$	b	6	
q_b	\$	b	5	
q	\$	—	6	
$q_\$$	—	—	8	

- Not all CFL's have deterministic PDA's that can be derived from the "*standard non-deterministic*" one via the look ahead idea.
  - For example: L = {w w^R | w ϵ (a U b)*}
- Even for certain deterministic CFL's look ahead of just one symbol may not be sufficient.
  - For example: S -> a S A | e

    A -> a b S | c
- But some languages have certain "always observable" anomalies because of which they cannot become parsers using the idea of look ahead. Let's see these ……

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow id$

$F \rightarrow id\ (E)$

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

((p, e, e), (q, E))

((q, e, E), (q, E + T))

((q, e, E), (q, T))

((q, e, T), (q, T*F))

((q, e, T), (q, F))

((q, e, F), (q, (E)))

((q, e, F), (q, id))

((q, e, F), (q, id(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Left Factoring

Left Factoring: Whenever rules of the form

$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots\ldots\ldots\ldots\ldots \mid \alpha\beta_n$ where $\alpha \neq e$ and $n \geq 2$

are present, then replace them by the rules:

$A \rightarrow \alpha A'$

$A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots\ldots\ldots\ldots\ldots \mid \beta_n$

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

E → E + T

E → T

T → T * F

T → F

F → (E)

F → id

F → id (E)

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

((p, e, e), (q, E))

((q, e, E), (q, E + T))

((q, e, E), (q, T))

((q, e, T), (q, T*F))

((q, e, T), (q, F))

((q, e, F), (q, (E)))

((q, e, F), (q, id))

((q, e, F), (q, id(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

E → E + T

E → T

T → T * F

T → F

F → (E)

~~F → id~~ F -> id A

~~F → id (E)~~ A -> e | (E)

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

((p, e, e), (q, E))

((q, e, E), (q, E + T))

((q, e, E), (q, T))

((q, e, T), (q, T*F))

((q, e, T), (q, F))

((q, e, F), (q, (E)))

~~((q, e, F), (q, id))~~ ((q, e, F),(q, idA))

~~((q, e, F), (q, id(E)))~~ ((q, e, A),(q, e))

((q, e, A)(q,(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

E → E + T

E → T

T → T * F

T → F

F → (E)

~~F → id~~ F -> id A

~~F → id (E)~~ A -> e | (E)

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

((p, e, e), (q, E))

((q, e, E), (q, E + T))

((q, e, E), (q, T))

((q, e, T), (q, T*F))

((q, e, T), (q, F))

((q, e, F), (q, (E)))

~~((q, e, F), (q, id))~~ ((q, e, F),(q, idA))

~~((q, e, F), (q, id(E)))~~ ((q, e, A),(q, e))

((q, e, A)(q,(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Left Factoring and Left Recursion

<u>Left Recursion</u>: Whenever rules of the form

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots\ldots \mid A\alpha_n \mid \beta_1 \mid \beta_2 \mid \ldots\ldots\ldots\ldots\ldots \mid \beta_m$

where n > 0

are present, then replace them by the rules:

$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots\ldots\ldots \mid \beta_m A'$

$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \alpha_n A' \mid e$

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

E → E + T

E → T

T → T * F

T → F

F → (E)

~~F → id~~ F -> id A

~~F → id (E)~~ A -> e | (E)

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

((p, e, e), (q, E))

((q, e, E), (q, E + T))

((q, e, E), (q, T))

((q, e, T), (q, T*F))

((q, e, T), (q, F))

((q, e, F), (q, (E)))

~~((q, e, F), (q, id))~~ ((q, e, F),(q, idA))

~~((q, e, F), (q, id(E)))~~ ((q, e, A),(q, e))

((q, e, A)(q,(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Ques. Can Look-ahead always work??

Consider the following grammar and corres. PDA:

Grammar: G = (V, ∑, R, E)

Where R is

~~E → E + T~~ E -> TE’

E’ -> +TE’ | e

~~E → T~~

~~T → T * F~~ T -> FT’

T’ -> *FT’ | e

~~T → F~~

F → (E)

~~F → id~~ F -> id A

~~F → id (E)~~ A -> e | (E)

PDA M = (Q, ∑, Γ, Δ, s, F)

Where Δ is

~~((p, e, e), (q, E))~~ ???

~~((q, e, E), (q, E + T))~~ ???

~~((q, e, E), (q, T))~~ ???

~~((q, e, T), (q, T*F))~~ ???

~~((q, e, T), (q, F))~~ ???

((q, e, F), (q, (E)))

~~((q, e, F), (q, id))~~ ((q, e, F),(q, idA))

~~((q, e, F), (q, id(E)))~~ ((q, e, A),(q, e))

((q, e, A)(q,(E)))

((q, a, a), (q, e)) for all a ϵ ∑

Deterministic PDA

Thus we can construct the following deterministic PDA M4 that accepts the language L(G)$:

$$M_4 = (K, \Sigma \cup \{\$\}, V', \Delta, p, \{q_\$\}),$$

$$K = \{p, q, q_{\text{id}}, q_+, q_*, q_), q_(, q_\$\},$$

Where delta is given as →→

$((p, e, e), (q, E))$

$((q, a, e), (q_a, e))$ for each $a \in \Sigma \cup \{\$\}$

$((q_a, e, a), (q, e))$ for each $a \in \Sigma$

$((q_a, e, E), (q_a, TE'))$ for each $a \in \Sigma \cup \{\$\}$

$((q_+, e, E'), (q_+, +TE'))$

$((q_a, e, E'), (q_a, e))$ for each $a \in \{), \$\}$

$((q_a, e, T), (q_a, FT'))$ for each $a \in \Sigma \cup \{\$\}$

$((q_*, e, T'), (q_*, *FT'))$

$((q_a, e, T'), (q_a, e))$ for each $a \in \{+,), \$\}$

$((q_(, e, F), (q_(, (E)))$

$((q_{\text{id}}, e, F), (q_{\text{id}}, \text{id}A))$

$((q_(, e, A), (q_(, (E)))$

$((q_a, e, A), (q_a, e))$ for each $a \in \{+, *,), \$\}$

E -> TE'
E' -> +TE' | e
T -> FT'
T' -> *FT' | e
F → (E)
F -> id A
A -> e | (E)

Step	State	Unread Input	Stack	Rule of G'
0	p	id $*$ (id)\$	e	
1	q	id $*$ (id)\$	E	
2	q_{id}	$*$(id)\$	E	
3	q_{id}	$*$(id)\$	TE'	1
4	q_{id}	$*$(id)\$	$FT'E'$	4
5	q_{id}	$*$(id)\$	id$AT'E'$	8
6	q	$*$(id)\$	$AT'E'$	
7	q_*	(id)\$	$AT'E'$	
8	q_*	(id)\$	$T'E'$	9
9	q_*	(id)\$	$*FT'E'$	5
10	q	(id)\$	$FT'E'$	
11	$q_($	id)\$	$FT'E'$	
12	$q_($	id)\$	$(E)T'E'$	7
13	q	id)\$	$E)T'E'$	
14	q_{id}	)\$	$E)T'E'$	
15	q_{id}	)\$	$TE')T'E'$	1
16	q_{id}	)\$	$FT'E')T'E'$	4
17	q_{id}	)\$	id$AT'E')T'E'$	8
18	q	)\$	$AT'E')T'E'$	
19	$q_)$	\$	$AT'E')T'E'$	
20	$q_)$	\$	$T'E')T'E'$	10
21	$q_)$	\$	$E')T'E'$	6
22	$q_)$	\$	$)T'E'$	3
23	q	\$	$T'E'$	6
24	$q_\$$	e	$T'E'$	
25	$q_\$$	e	E'	6
26	$q_\$$	e	e	3

(1) $E \rightarrow TE'$
(2) $E' \rightarrow +TE'$
(3) $E' \rightarrow e$
(4) $T \rightarrow FT'$
(5) $T' \rightarrow *FT'$
(6) $T' \rightarrow e$
(7) $F \rightarrow (E)$
(8) $F \rightarrow \text{id}A$
(9) $A \rightarrow e$
(10) $A \rightarrow (E)$

Next : ……..

Bottom Up Parsing

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Thank You