Neural Networks & Fuzzy Logic (BITS F312)

1. Introduction (Prof. Bhanot)
2. Artificial Neural Networks (Prof. Bhanot)

65%

3. Nontraditional Optimization (Myself)
4. Fuzzy Logic (Myself)

35%

Lecture Plan:

Module1: Nontraditional Optimization (6 Lectures)

Module2: Fuzzy Logic (9 Lectures)

Books:

1. Engineering Optimization by S.S. Rao
2. Evolutionary Optimization Algorithms by Dan Simon
3. Soft Computing by D.K. Pratihar / Computational Intelligence by Nazmul Siddique
4. Soft Computing by Jang, Sun and Mizutani
5. Introduction to Type-2 Fuzzy Logic Control by Mendel et al.
6. Fuzzy Control by Hao Ying

Topics to be Covered:

- ✓ Fuzzy Logic Basics (3 L)
- ✓ Type-2 Fuzzy Logic (0.5 L)
- ✓ Neuro-Fuzzy-GA Hybrid Systems (2 L)
- ✓ Fuzzy PID Control (2 L)
- ✓ Fuzzy Clustering (2 L)

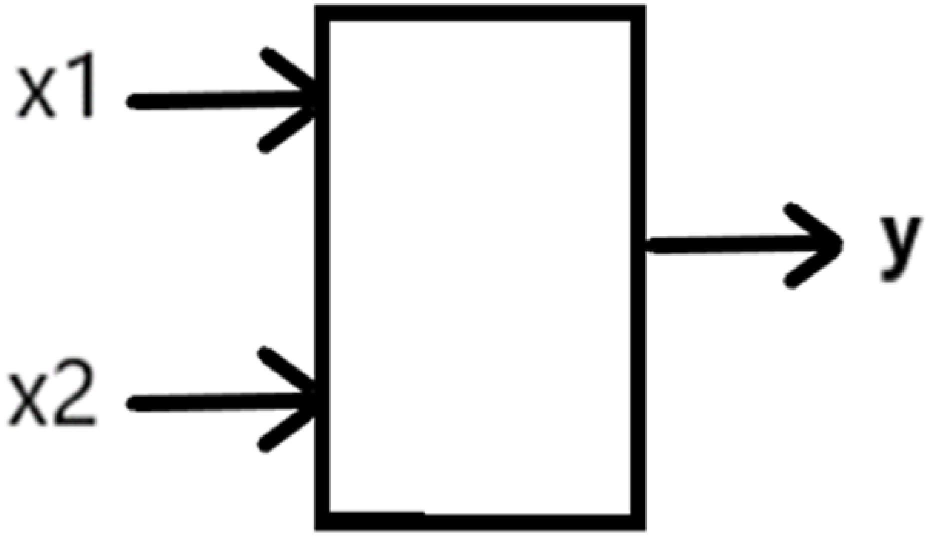
Fuzzy Logic Basics

What is Fuzzy Logic?

- FUZZY : not clear or precise; blurred; vague
- FUZZY LOGIC: A form of knowledge representation in vague linguistic terms

  (for phenomena which are difficult to describe mathematically)
- A generalization of conventional (Boolean) logic to handle the concept of partial truth i.e. truth values between “completely true” (membership value 1.0) and “completely false” (membership value 0.0)
- Age less than 40 years → YOUNG

  Age more than 40 years → OLD
- All boundaries are Blurred!



- First Principle approach
- Data based approach
- Linguistic approach

- A class of students (E.G. M.Sc. Students)
- The universe of discourse: X

No ambiguity

1 0 1 1 0 1 1

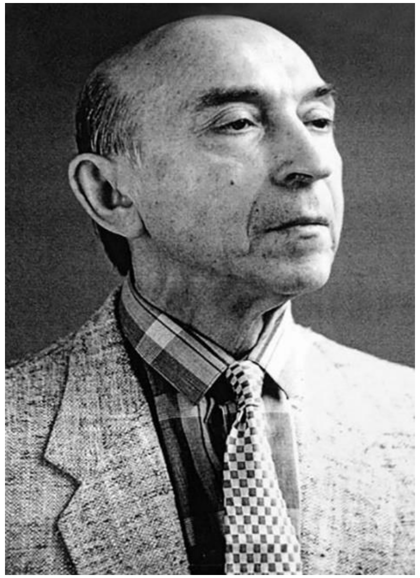
- “Who does have a driver’s licence?”
- A subset of X = A (Crisp) Set

Ambiguous

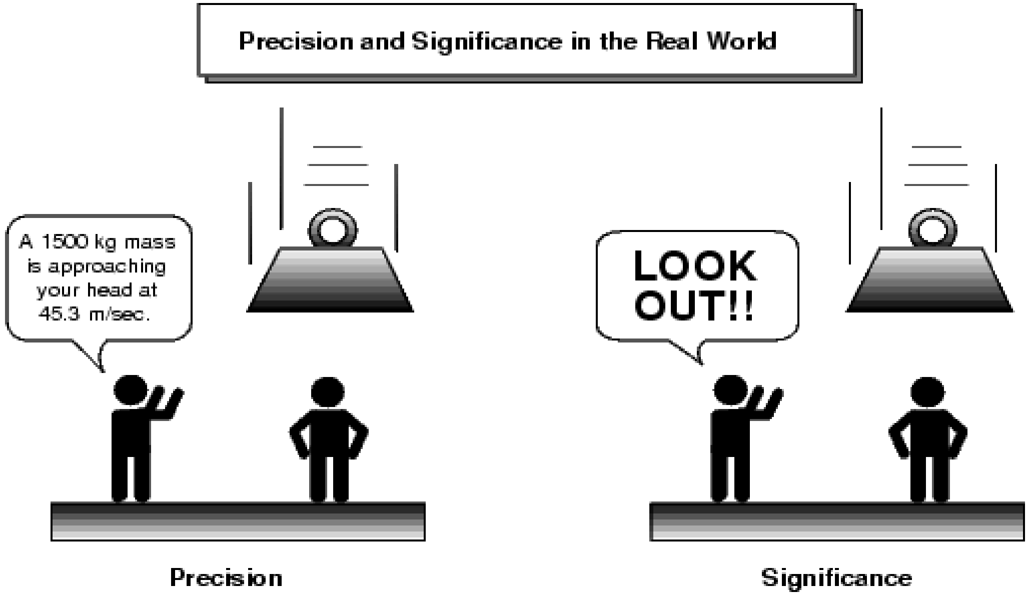
0.7 0 1.0 0.8 0 0.4 0.2

- “Who can drive very well?”
  $\mu(X)$ = MEMBERSHIP FUNCTION

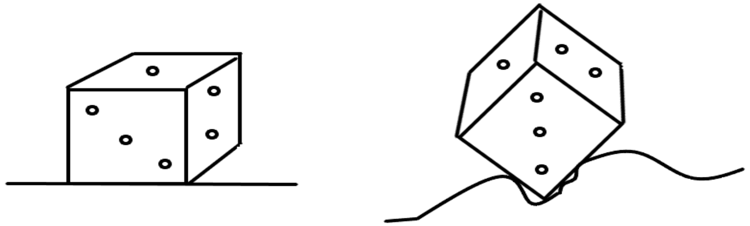
- Prof. Lotfi A. Zadeh (1921- 2017), UC Berkeley
  Proposed fuzzy set theory in 1965



- "As complexity rises precise statements lose meaning
  and meaningful statements lose precision"



Fuzzy Logic vs. Probability: how much vs. whether



- **ADVANTAGES:**

  - Mimics human reasoning and decision making process
  - Based on qualitative understanding of the phenomena under consideration
  - Easy to conceptualize
  - Tolerant to imprecise data

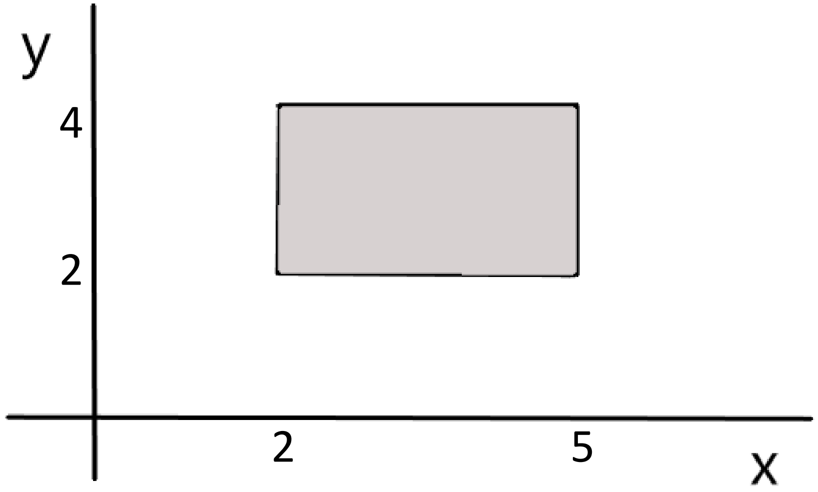
- **DISADVANTAGES:**

  - Usually lacks accuracy
  - Computationally expensive
  - Describing some phenomena even linguistically requires a lot of domain expertise
  - Very often depends on some optimizer (such as GA) to be more efficient
  - Computations increase exponentially as number of variables grows

Major Application Areas:

- ✓ Control systems and robotics
- ✓ Pattern classification and recognition
- ✓ Multi-criteria decision making and optimization
- ✓ Fault detection and structural health monitoring
- ✓ Image and speech processing
- ✓ Medical diagnosis
- ✓ Power and energy systems
- ✓ Finance and social sciences

A Brief Review of Classical/Crisp Sets



- A collection of objects

- A crisp set has sharp boundaries

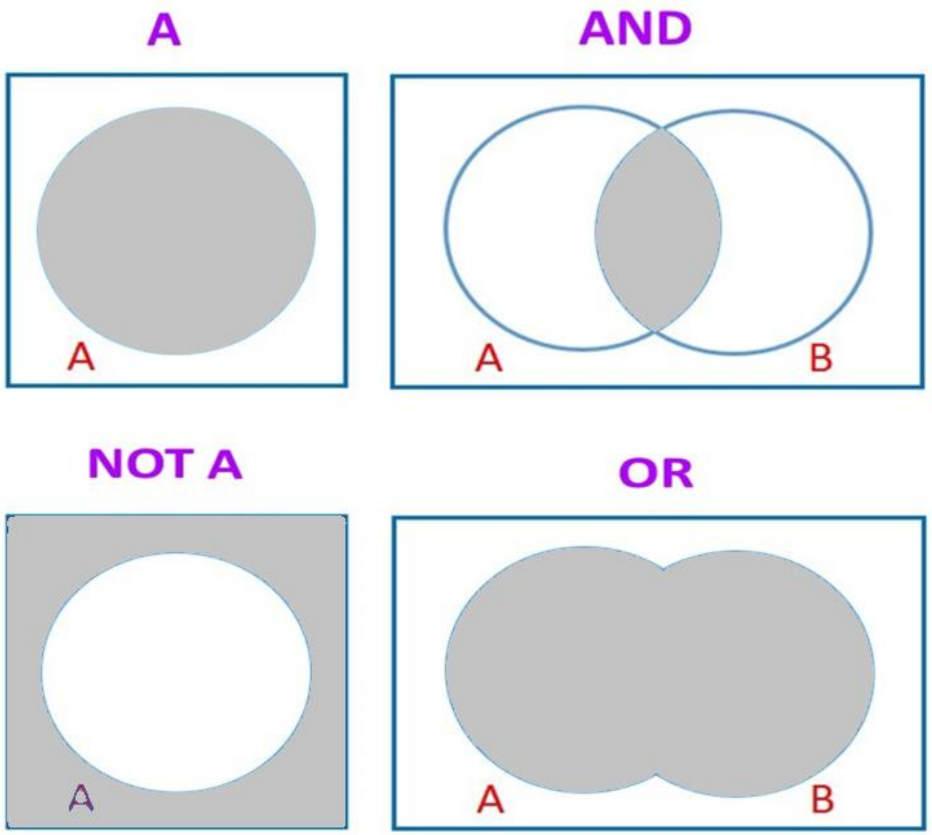
E.g. A = $\{(x,y)\ |2 \leq x \leq 5, 2 \leq y \leq 4\}$

- Cardinal No.: The number of elements of a set

- Null Set: A set with no member (Ø)

- Universal Set: A set with all possible members in a given context (X)

Three Fundamental Operations:



- $A \cdot B = \{x \mid x \in A \ \ and\ x \in B\}$
- $A + B = \{x \mid x \in A \ \ or\ x \in B\}$
- $\overline{A} = \{x \mid x \notin A\}$

Properties of Crisp Sets:

i) Commutativity: $A \cdot B = B \cdot A$; $A+B = B+A$

ii) Distributivity: $A+(B\cdot C) = (A+B) \cdot (A+C)$; $A \cdot (B+C) = (A\cdot B) + (A\cdot C)$

iii) Associativity: $A \cdot (B \cdot C) = (A \cdot B) \cdot C$; $A+(B+C) = (A+B)+C$

iv) Involution: $\overline{\overline{A}} = A$

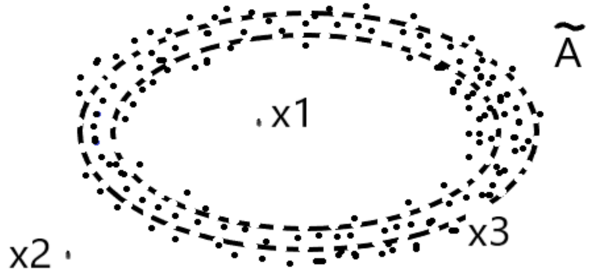
v) DeMorgan's laws: $\overline{A \cdot B} = \overline{A} + \overline{B}$; $\overline{A + B} = \overline{A} \cdot \overline{B}$

Law of Contradiction and Law of Excluded Middle:

vi) $A \cdot \overline{A} = \emptyset$

vii) $A + \overline{A} = X$

Fuzzy Sets



- For discrete domains a fuzzy set is described by assigning a membership value (μ between 0 and 1) to each element

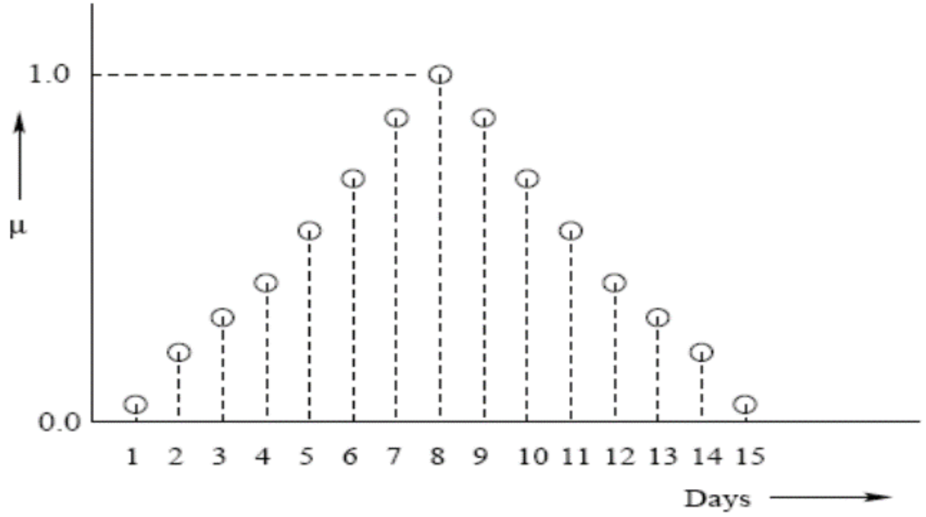
$$\widetilde{\mathrm{A}} = \left\{\frac{\mu(x_1)}{x_1} + \frac{\mu(x_2)}{x_2} + \cdots\right\}$$

E.g. X = {1,2,3,4,5,6} be the set of houses with number of rooms.

'comfortable house for a four member family' ,

$$\widetilde{\mathrm{A}} = \left\{\frac{0.1}{1} + \frac{0.5}{2} + \frac{0.8}{3} + \frac{1.0}{4} + \frac{0.9}{5} + \frac{0.4}{6}\right\}$$

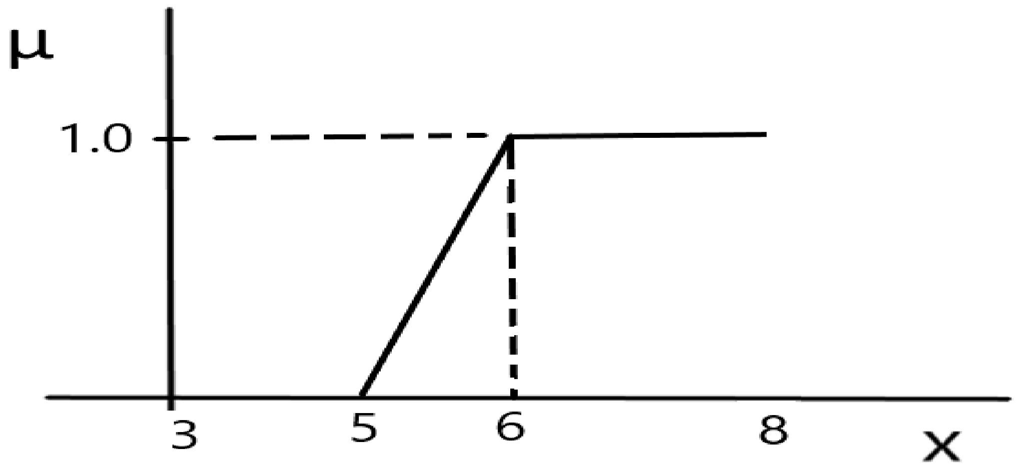
or 'an enjoyable vacation'



- For continuous domains, a fuzzy set is described by the membership function

E.g. X = [3, 8] is the height of people in foot

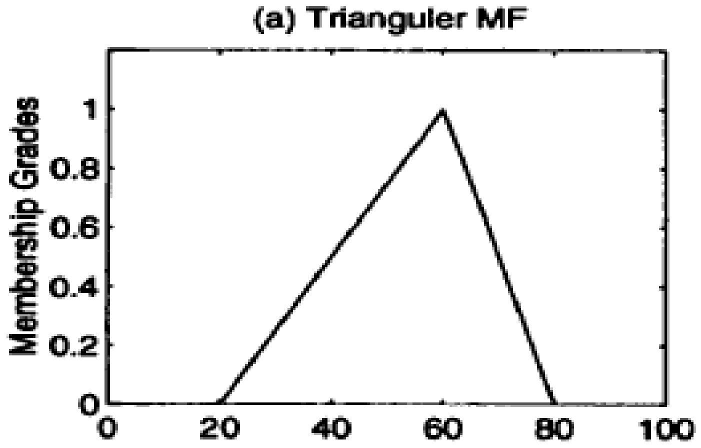
'a tall man', $\widetilde{\mathrm{A}} = \left\{\int \frac{\mu(x)}{x}\right\}$ where $\mu(x) = \begin{cases} 0.0\,, x \leq 5.0 \\ x-5\,, \quad 5.0 \leq x \leq 6.0 \\ 1.0\,, x \geq 6.0 \end{cases}$



Common Shapes of Membership Functions:

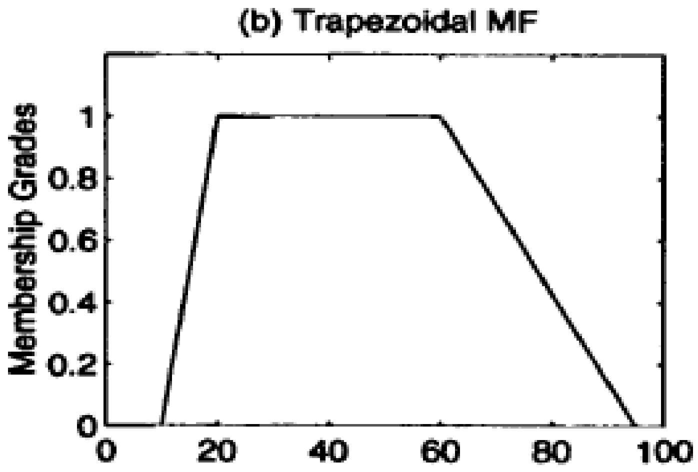
a)

$$\text{triangle}(x;a,b,c) = \begin{cases} 0, & x \le a. \\ \frac{x-a}{b-a}, & a \le x \le b. \\ \frac{c-x}{c-b}, & b \le x \le c. \\ 0, & c \le x. \end{cases}$$

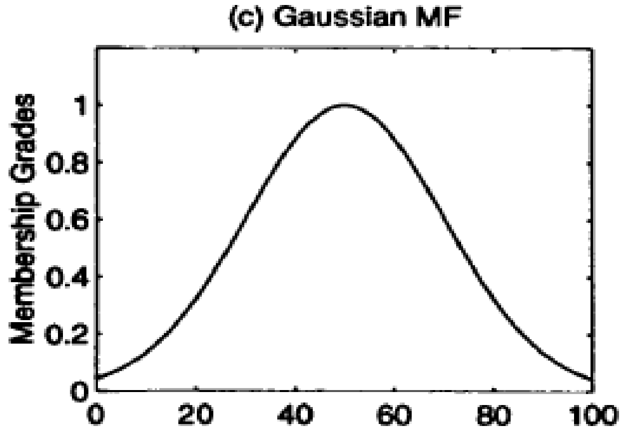


b)

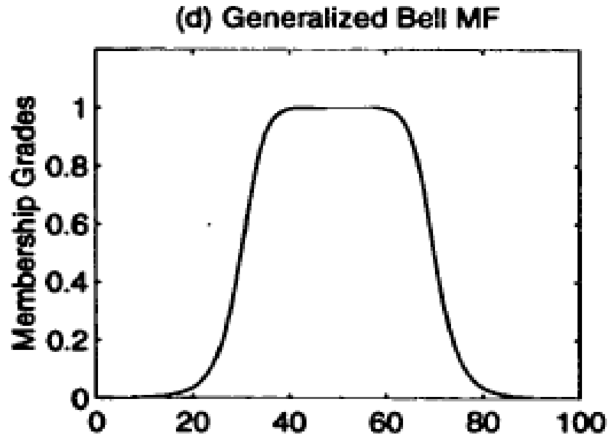
$$\text{trapezoid}(x;a,b,c,d) = \begin{cases} 0, & x \le a. \\ \frac{x-a}{b-a}, & a \le x \le b. \\ 1, & b \le x \le c. \\ \frac{d-x}{d-c}, & c \le x \le d. \\ 0, & d \le x. \end{cases}$$



c) $$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$

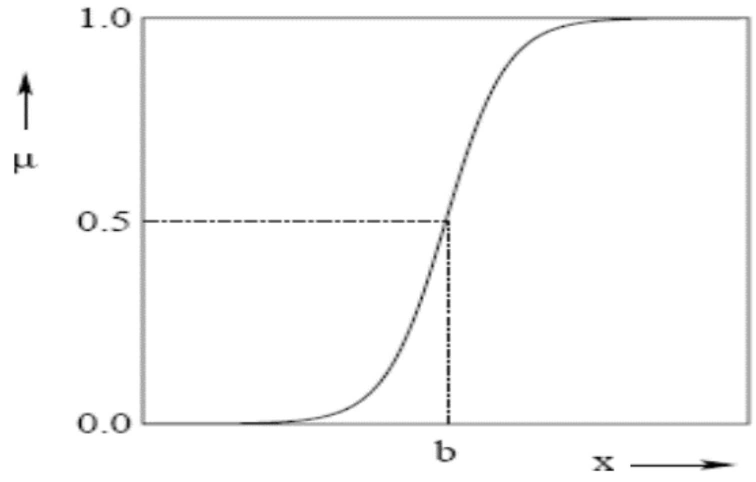


d) $$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



e) **sigmoidal**

$$\mu_{Sigmoid} = \frac{1}{1+e^{-a(x-b)}}$$



f) **singleton**

A Few More Terms:

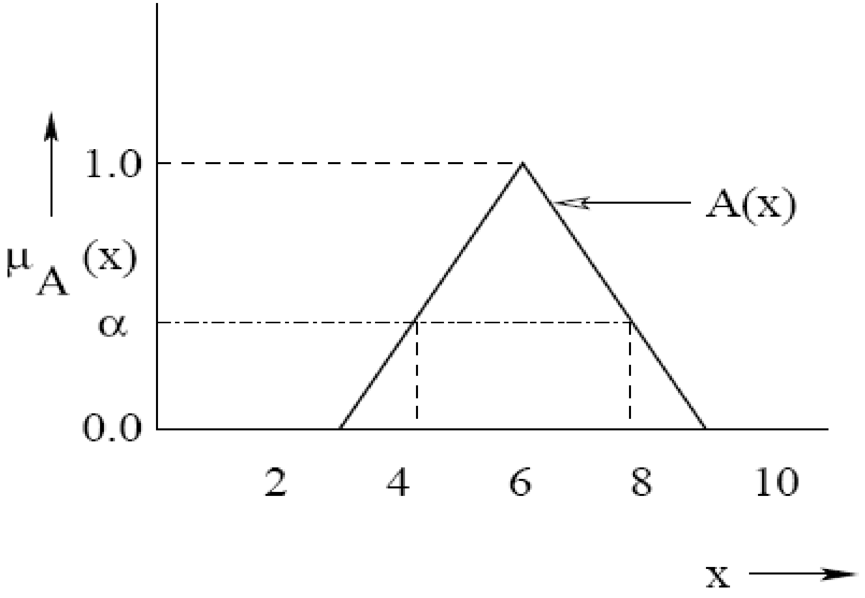
- **α- Cut of a fuzzy set** $\alpha_{\mu_A}(x)$

  A set consisting of elements *x* of the Universal set X, whose membership values are either greater than or equal to the value of α

$$\alpha_{\mu_A}(x) = \{ x \mid \mu_{\tilde{A}}(x) \geq \alpha \}$$

- **Strong α-Cut of a fuzzy set**

$$\alpha^{+}_{\mu_A}(x) = \{ x \mid \mu_{\tilde{A}}(x) > \alpha \}$$

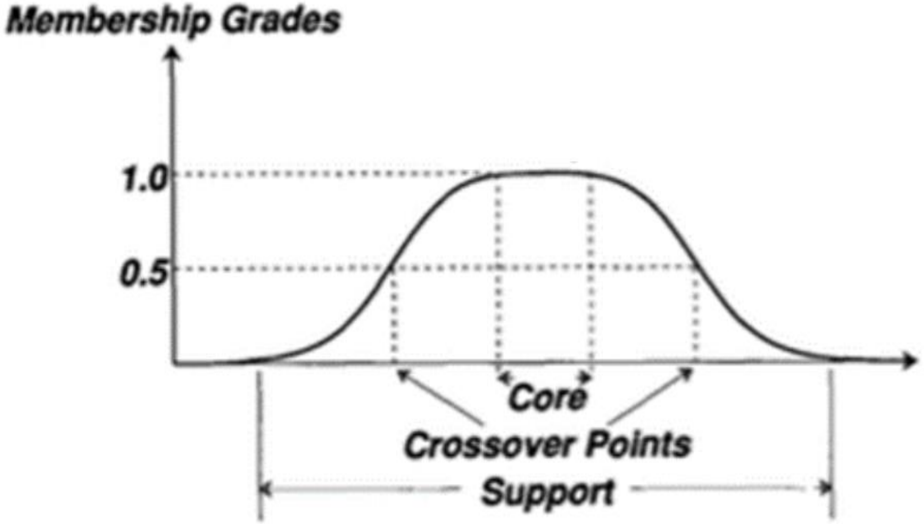


- **Support of a fuzzy set A(*x*)**

It is defined as the set of all $x \in X$, such that $\mu_A(x) > 0$

$\text{supp}(A) = \{ x \mid \mu_{\tilde{A}}(x) > 0 \}$

It is nothing but its Strong 0- cut



- **Core of a fuzzy set A(*x*)**

$\text{core}(\tilde{A}) = \{ x \mid \mu_{\tilde{A}}(x) = 1.0 \}$

It is nothing but its 1-cut

- **Crossover points**

$\text{crossover}(\tilde{A}) = \{ x \mid \mu_{\tilde{A}}(x) = 0.5 \}$

- **Height of a fuzzy set**

  It is the largest membership values of the elements contained in that set

- **Normal fuzzy set**

  For a normal fuzzy set, h(A) = 1.0

- **Sub-Normal fuzzy set**

  For a sub-normal fuzzy set, h(A) < 1.0

- **Concentration:** $\mu_{very\,\tilde{A}}(x) = (\mu_{\tilde{A}}(x))^{2}$

- **Dilation:** $\mu_{slightly\,\tilde{A}}(x) = (\mu_{\tilde{A}}(x))^{0.5}$

Numerical Example:

- Assume X={ a, b, c, d, e }

  A={ 1/a ,0.3/b , 0.2/c , 0.8/d, 0/e } ,

  B= { 0.6/a, 0.9/b , 0.1/c , 0.3/d, 0.2/e } .

- Find out whether A and B are normal sets or sub-normal sets

- Find out the height, support and core of A and B.

Answer:

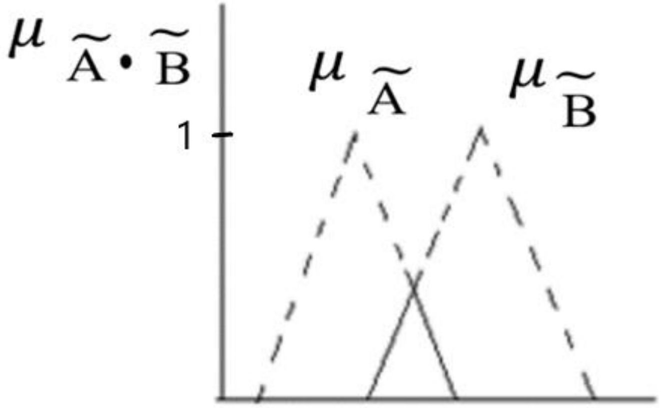
i) $h(A) = 1.0$; $h(B) = 0.9$

ii) $Supp(A) = \{a,b,c,d\}$; $Supp(B) = \{a,b,c,d,e\}$

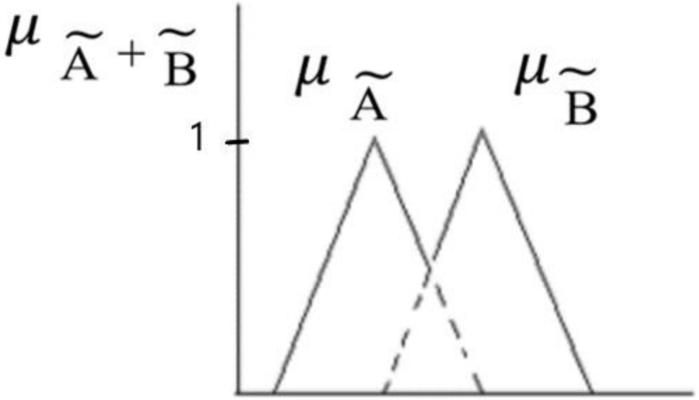
iii) $Core(A) = \{a\}$; $Core(B) = \emptyset$

Three Fundamental Operations on Fuzzy Sets

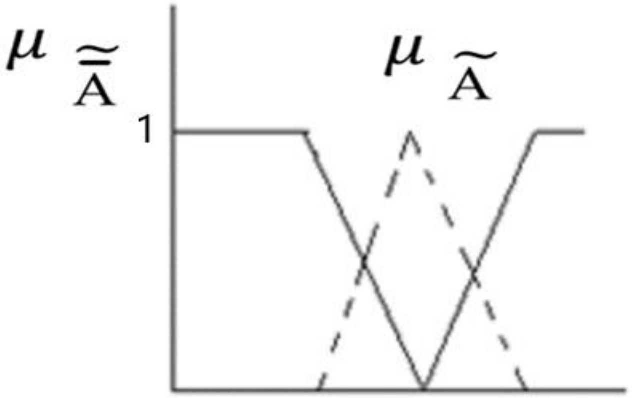
$$\mu_{\tilde{\mathrm{A}}\,.\,\tilde{B}}(x) = \min\left(\mu_{\tilde{\mathrm{A}}}(x), \mu_{\tilde{\mathrm{B}}}(x)\right)$$



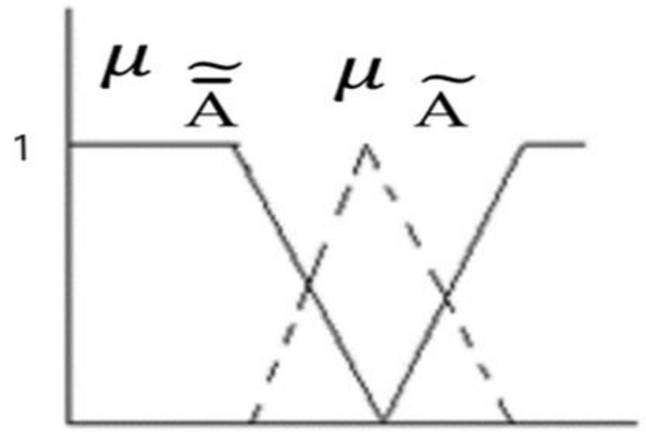
$$\mu_{\tilde{\mathrm{A}}+\tilde{B}}(x) = \max\left(\mu_{\tilde{\mathrm{A}}}(x), \mu_{\tilde{\mathrm{B}}}(x)\right)$$



$$\mu_{\bar{\tilde{\mathrm{A}}}}(x) = 1 - \mu_{\tilde{\mathrm{A}}}(x)$$



Properties of fuzzy sets

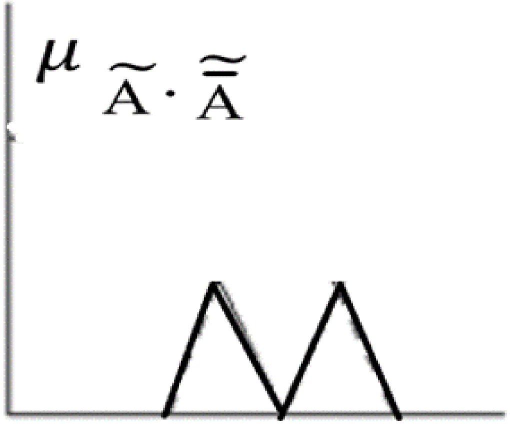


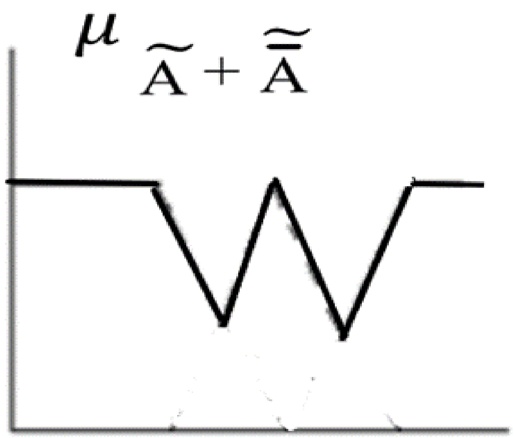
Fuzzy sets follow all the properties of crisp sets except for the following two:

1. Law of excluded middle

For a crisp set, $A \cup \overline{A} = X$

For a fuzzy set, $A \cup \overline{A} \neq X$





2. Law of contradiction

For a crisp set, $A \cap \overline{A} = O$

For a fuzzy set, $A \cap \overline{A} \neq O$

Numerical Example:

Given the two fuzzy sets

$$\widetilde{A} = \left\{ \frac{1}{1} + \frac{0.6}{2} + \frac{0.3}{3} + \frac{0.2}{4} \right\} \quad \text{and } \widetilde{B} = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$$

i) Verify DeMorgan's laws

ii) Compute $\widetilde{A} \cdot \overline{\widetilde{B}}$ and $\widetilde{B} + \overline{\widetilde{B}}$

Answer:

ii) $$\left\{\frac{1.0}{1}+\frac{0.5}{2}+\frac{0.3}{3}+\frac{0.2}{4}+\frac{0}{5}\right\}$$

$$\left\{\frac{1}{1}+\frac{0.5}{2}+\frac{0.7}{3}+\frac{0.8}{4}+\frac{0.6}{5}\right\}$$

Various T-norms (AND Operators):

T(0,0) = 0 ; T(a,1) = T(1,a) = a : boundary

T(a,b) $\leq$ T(c,d) if a $\leq$ c and b $\leq$ d : monotonicity

T(a,b) = T(b,a) : commutativity

T(a, T(b,c)) = T(T(a,b),c) : associativity

Minimum: T(a,b) = min(a,b)

Algebraic Product: T(a,b) = a.b

Bounded Product: T(a,b) = max(0, a+b-1)

Drastic Product: T(a,b) = $\begin{cases} a\ , if\ b = 1 \\ b\ , if\ a = 1 \\ 0\ , if\ a, b < 1 \end{cases}$

Various T-conorms or S-norms (OR Operators):

S(1,1) = 1 ; S(a,0) = S(0,a) = a : boundary

S(a,b) $\leq$ S(c,d) if a $\leq$ c and b $\leq$ d : monotonicity

S(a,b) = S(b,a) : commutativity

S(a, S(b,c)) = S(S(a,b),c) : associativity

Maximum: S(a,b) = max(a,b)

Algebraic Sum: S(a,b) = a+b-ab

Bounded Sum: S(a,b) = min(1, a+b)

Drastic Sum: S(a,b) = $\begin{cases} a\,, if\ b = 0 \\ b\,, if\ a = 0 \\ 1\,, if\ a, b > 0 \end{cases}$

Sugeno's Complement Operator:

N(0) = 1 ; N(1) = 0 : boundary

N(a) $\geq$ N(b) if a $\leq$ b : monotonicity

Sugeno: N(a) = $\frac{1-a}{1+s\cdot a}$ where s > -1

Numerical Example:

Verify DeMorgan's laws for the two fuzzy sets

$\widetilde{\mathrm{A}} = \left\{\frac{1}{1} + \frac{0.6}{2} + \frac{0.3}{3} + \frac{0.2}{4}\right\}$ and $\widetilde{\mathrm{B}} = \left\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}$ using Algebraic product T-norm and Algebraic Sum S-norm

Crisp Relation

- Sometimes elements of a set may be related to each other

  E.g. A = {Delhi, Rupee, Hindi}

- Two sets defined on two different universes but their elements are related to each other

  E.g. A = {China, India, Germany, Japan}

  B = {Berlin, Beijing, Tokyo, Delhi}

- A binary relation (relation between two sets) is a structure that represents the presence or absence of interaction between the elements of the two sets

Let A = {India, Japan, Canada, Egypt}

B = {Tokyo, Cairo, Delhi}

then the relation **‘B is capital of A’** is given by

$$R = \begin{array}{c c} & \begin{array}{ccc} T & C & D \end{array} \\ \begin{array}{c} I \\ J \\ C \\ E \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

- **Composition:** If R1 is the relation between X and Y and R2 is the relation between Y and Z then the relation R between X and Z is computed from the rule of composition as

$$R = R1 \circ R2 \quad \text{where } r(x,z) = \max\{ \min (r1(x,y), r2(y,z))\}$$

Let $A = \{India, Japan, Canada, Egypt\}$

$B = \{Tokyo, Cairo, Delhi\}$

$C = \{Hindi, Bengali\}$

$$R1 = \begin{array}{c} \\ I \\ J \\ C \\ E \end{array} \begin{array}{c} \begin{array}{ccc} T & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$R2 = \begin{array}{c} \\ T \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} H & B \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ I \\ J \\ C \\ E \end{array} \begin{array}{c} \begin{array}{cc} H & B \end{array} \\ \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Fuzzy Relation

- Crisp relation only reveals if the elements of a set or of different sets are related or not
- Fuzzy relation reveals if the elements are weakly or strongly related i.e. the strength of the relations
- The strength is represented by a membership value in the interval [0,1]

Let X = {Delhi, Bombay, Calcutta, Madras}

and Y = {Vizag, Pune, Kochi, Guwahati}

the relation 'far' may be given by

$$\tilde{R} = \begin{array}{c c} & \begin{array}{cccc} \text{V} & \text{P} & \text{K} & \text{G} \end{array} \\ \begin{array}{c} \text{D} \\ \text{B} \\ \text{C} \\ \text{M} \end{array} & \begin{bmatrix} 0.7 & 0.7 & 0.8 & 0.8 \\ 0.5 & 0.1 & 0.5 & 0.9 \\ 0.4 & 0.6 & 0.7 & 0.2 \\ 0.2 & 0.4 & 0.2 & 0.8 \end{bmatrix} \end{array}$$

Fuzzy Composition

- Fuzzy composition is defined in the same line as crisp composition
- Let $\widetilde{\mathrm{R}}$ be a fuzzy relation in the cross space $\mathrm{X} \times \mathrm{Y}$

  and $\tilde{\mathrm{S}}$ be a fuzzy relation in the cross space $\mathrm{Y} \times \mathrm{Z}$

  then the fuzzy relation $\widetilde{\mathrm{T}}$ in the cross space $\mathrm{X} \times \mathrm{Z}$ is given by

$$\widetilde{\mathrm{T}} = \widetilde{\mathrm{R}} \circ \tilde{\mathrm{S}}$$

where

$$\mu_{\tilde{T}}(x, z) = \max_{y} \; [\min\{\mu_{\tilde{R}}(x, y), \mu_{\tilde{S}}(y, z)\}]$$: max-min composition rule

$$\mu_{\tilde{T}}(x, z) = \max_{y} \; [\mu_{\tilde{R}}(x, y) \, . \, \mu_{\tilde{S}}(y, z)]$$: max-product composition rule

Numerical Example:

Let X={x1, x2}, Y={y1, y2} and Z={z1, z2, z3} be three different universes of discourse.

Fuzzy relation matrices on the cross spaces $X \times Y$ and $Y \times Z$ are given by

$$\tilde{R} = \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \text{ and } \tilde{S} = \begin{bmatrix} 0.9 & 0.6 & 0.2 \\ 0.1 & 0.7 & 0.5 \end{bmatrix}$$

Obtain the relation matrix relating X and Z by

i) max-min rule of composition

ii) max-product rule of composition

Answer:

$$\widetilde{\mathrm{T}} = \widetilde{\mathrm{R}} \circ \tilde{\mathrm{S}}$$

$$= \begin{bmatrix} 0.7 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.4 \end{bmatrix} \qquad \text{(i)}$$

$$= \begin{bmatrix} 0.63 & 0.42 & 0.25 \\ 0.72 & 0.48 & 0.20 \end{bmatrix} \qquad \text{(ii)}$$

Fuzzy Extension Principle

- A function (y=f(x)) maps one universe to another (in crisp sense)

- Same concept can be extended to fuzzy sets

- Let $\widetilde{\mathrm{R}}$ be the fuzzy relation between two universes X and Y.

  If $\widetilde{\mathrm{A}}_1$ is a fuzzy set from X then its mapping onto universe Y is given by

$$\widetilde{\mathrm{B}}_1 = \widetilde{\mathrm{A}}_1 \circ \widetilde{\mathrm{R}}$$

Numerical Example:

Let X = {1, 2, 3, 4, 5} and Y = {10, 20, 30, 40, 50} be two universes.

The relation ‘about ten times larger‘ is given by

$$\mathrm{R} = \begin{bmatrix} 1.00 & 0.50 & 0.33 & 0.25 & 0.20 \\ 0.50 & 1.00 & 0.70 & 0.50 & 0.40 \\ 0.33 & 0.70 & 1.00 & 0.75 & 0.60 \\ 0.25 & 0.50 & 0.75 & 1.00 & 0.80 \\ 0.20 & 0.40 & 0.60 & 0.80 & 1.00 \end{bmatrix}$$

Obtain the mapping of the fuzzy set

$\tilde{A}_1 = '\,close\ to\ three'\ = \left\{\frac{0.2}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.6}{4} + \frac{0.2}{5}\right\}$ onto the universe Y.

Answer: $\tilde{B}_1 = 'close\ to\ thirty\,'$ = $\left\{\frac{0.5}{10} + \frac{0.7}{20} + \frac{1}{30} + \frac{0.75}{40} + \frac{0.6}{50}\right\}$

Numerical Example:

Let $A = \left\{\frac{0.2}{1} + \frac{0.3}{-1} + \frac{0.1}{2} + \frac{0.5}{3} + \frac{0.4}{-3}\right\}$ and $f(x) = 1+x^2$

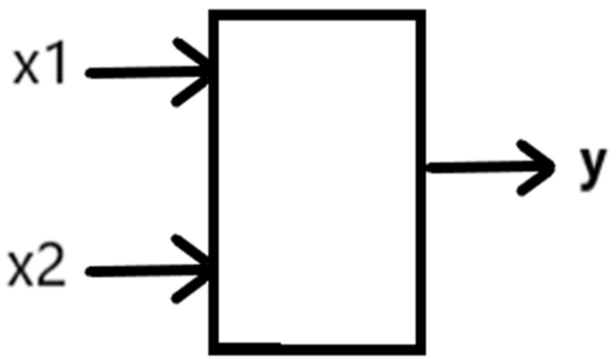
Find B = f(A)

Solution:

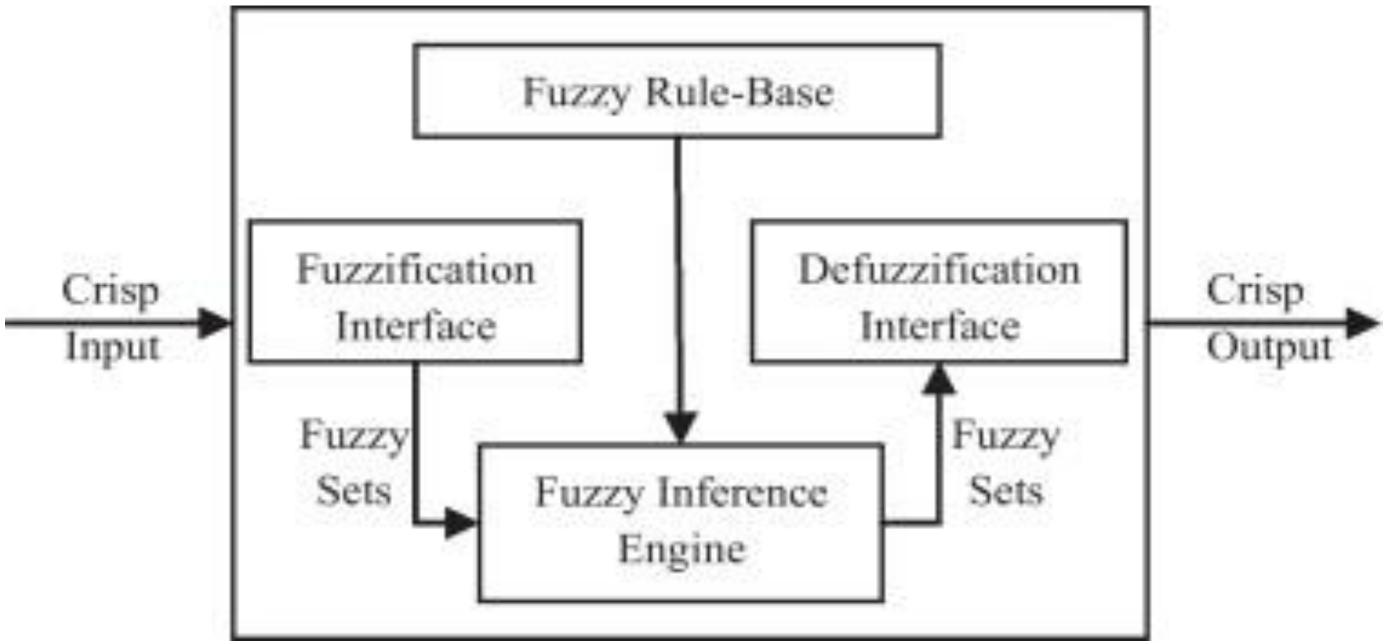
$$R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \left\{\frac{0.3}{2} + \frac{0.1}{5} + \frac{0.5}{10}\right\}$$

Fuzzy Reasoning Process



- Fuzzification
- Fuzzy Rule Base
- Fuzzy Inference
- Defuzzification



Fuzzification (Crisp to fuzzy conversion)

- Deciding on the spaces for each input and output variable
  (may be normalized universes coupled with input and output scaling factors)

- Deciding on the number of fuzzy sets for all the input and output variables

N P

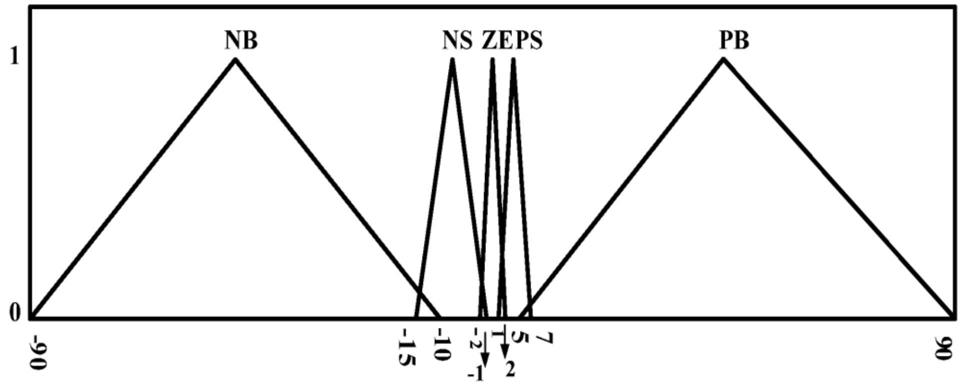
N Z P

NB NS Z PS PB

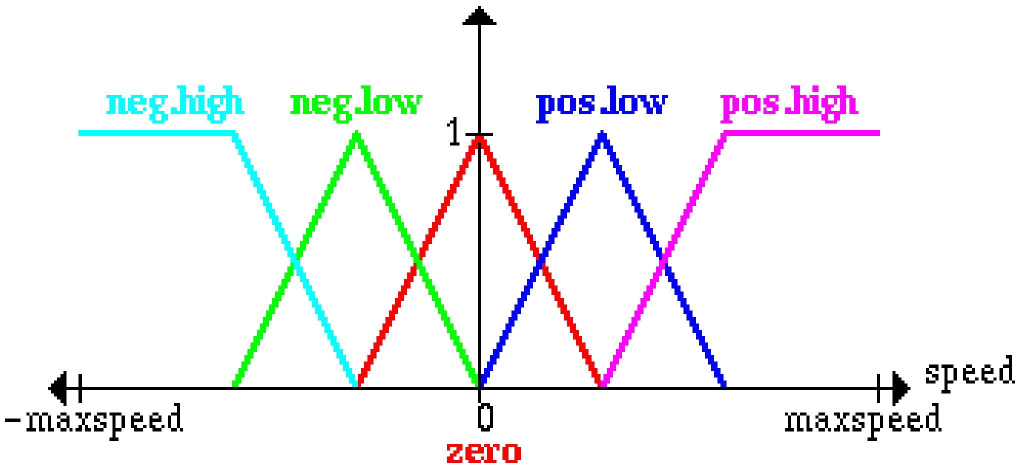
NB NM NS Z PS PM PB

- Deciding on the shapes of the fuzzy sets

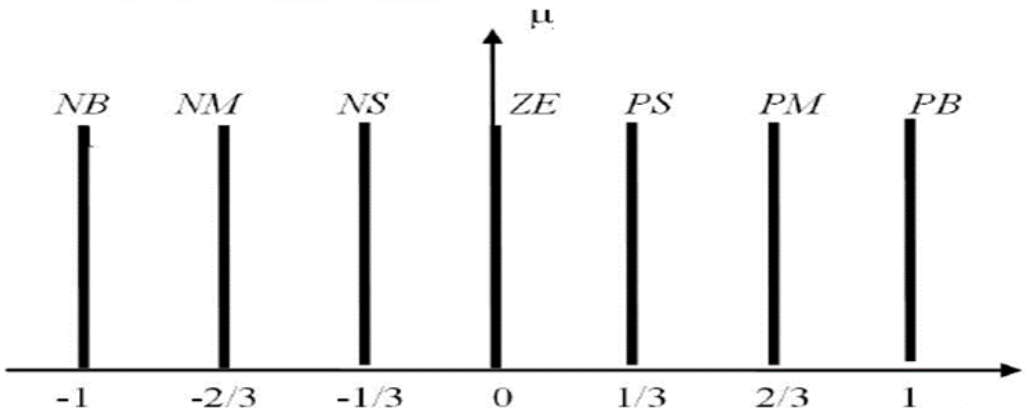
(or the nature of the membership functions)



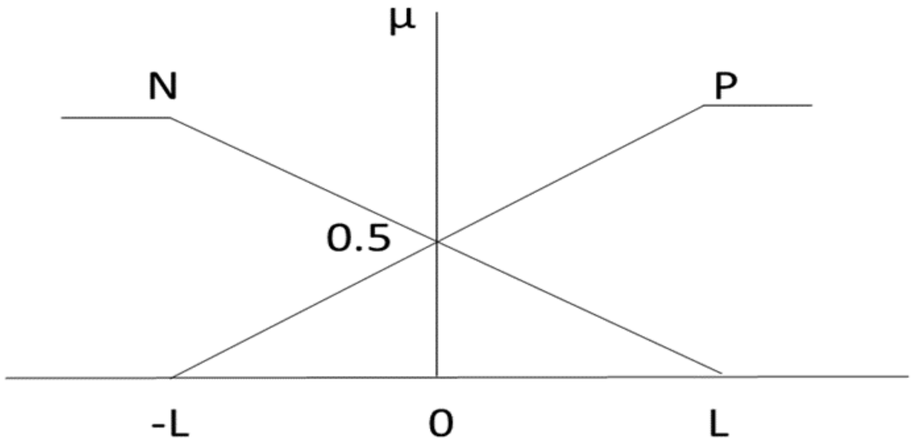
Asymmetric

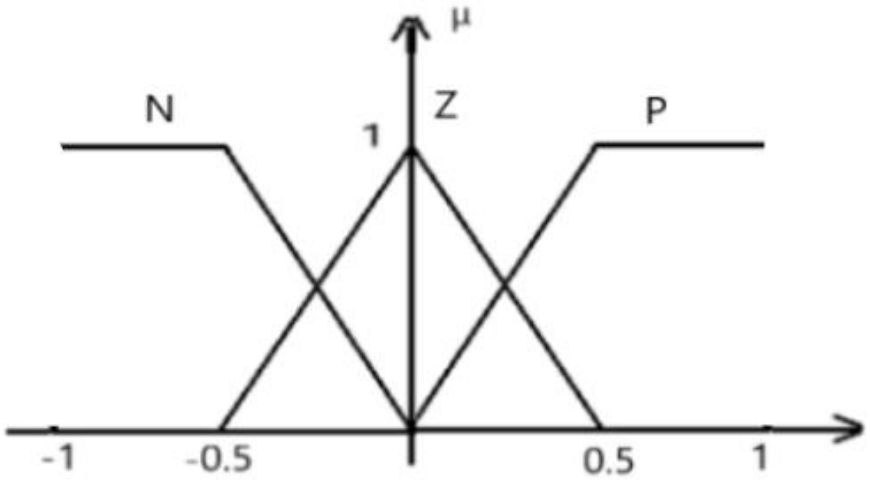


Trapezoidal at the ends



Singleton





Fuzzy Rule Base (look up table for reasoning)

- Rule base for Mamdani inference (1976):

  If Temp. is High And Humidity is High Then Speed is Very High

  If Temp. is High And Humidity is Low Then Speed is Medium

  …

	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	ZE
NM	NB	NB	NM	NS	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PM
ZE	NM	NS	NS	ZE	PS	PS	PM
PS	NM	NS	ZE	PS	PS	PM	PB
PM	NS	ZE	PS	PS	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

- Linear Fuzzy Rules

If X_1 is A_i And X_2 is B_j Then Y is C_{i+j}

$-J \leq i, j \leq +J$

(2J+1 numbers of fuzzy sets for each i/p variable and

4J+1 numbers of fuzzy sets for o/p variable)

- Rule base for Sugeno (or TS) Inference (1985)

If Temp. is High And Humidity is High Then Speed is f1(T, H)

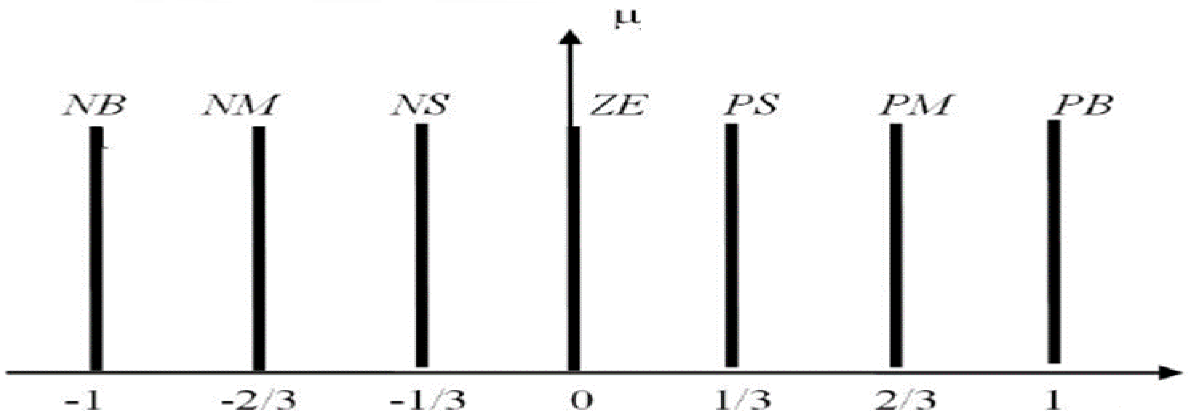
If Temp. is High And Humidity is Low Then Speed is f2(T, H)

…

1st order: $y = f1(T, H) = a1*T+b1*H+c1$

2nd order: $y = f1(T, H) = a1*T^2+a2*T+b1*H^2+b2*H+c1$

- Zero-th order Sugeno model and Mamdani with singleton output fuzzification are same



Fuzzy Inference

- Mamdani Model

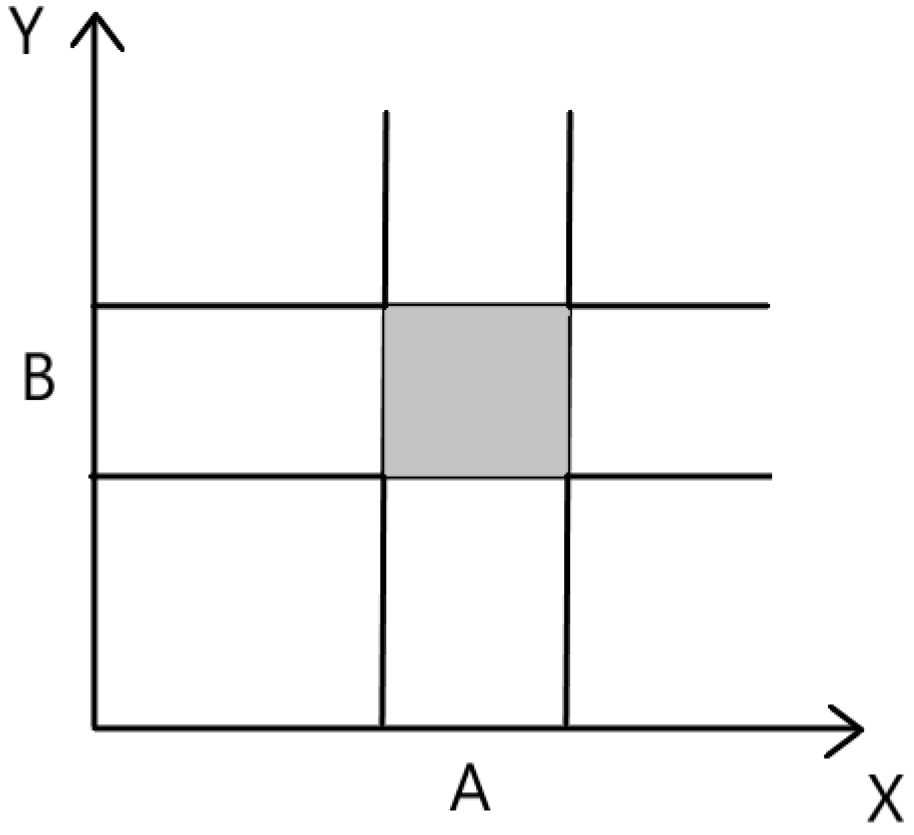
$$\underbrace{\text{IF } x \text{ is } A}_{\text{antecedent}} \quad \underbrace{\text{THEN } y \text{ is } B}_{\text{consequent}}$$

$A \rightarrow B = A \cdot B$: A is coupled with B

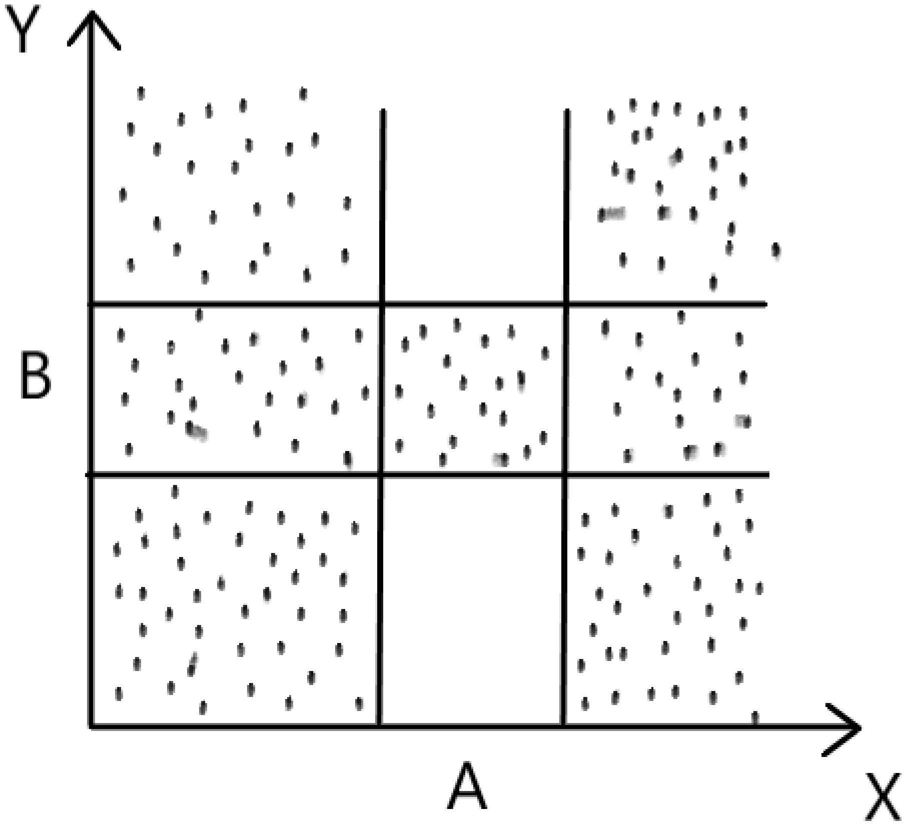
Another interpretation:

$A \rightarrow B = \overline{A} + B$: A entails B

(i.e. If x is A Then y is not $\overline{B}$)



$$A \rightarrow B = A \cdot B$$



$$A \rightarrow B = \overline{A} + B$$

$$(\overline{A \cdot \overline{B}} = \overline{A} + B)$$

$A \rightarrow B = A \cdot B$

- Mamdani minimum inference
- Larsen product inference
- Bounded product inference
- Drastic product inference

✓ Logical OR of the outputs of the individual rules

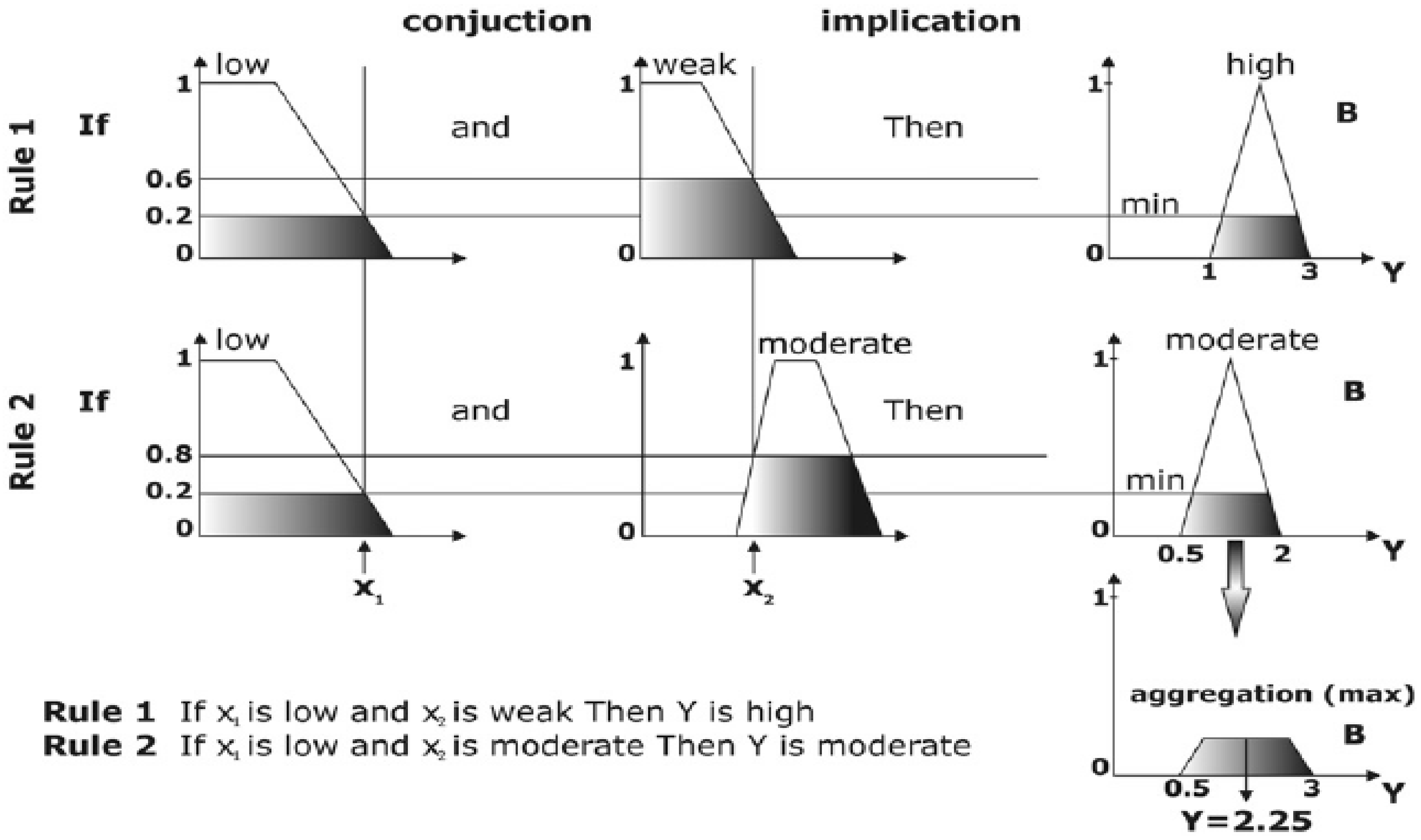
$A \rightarrow B = \overline{A} + B$

- Zadeh implication: $\overline{A} + (A \cdot B)$; max and min operators

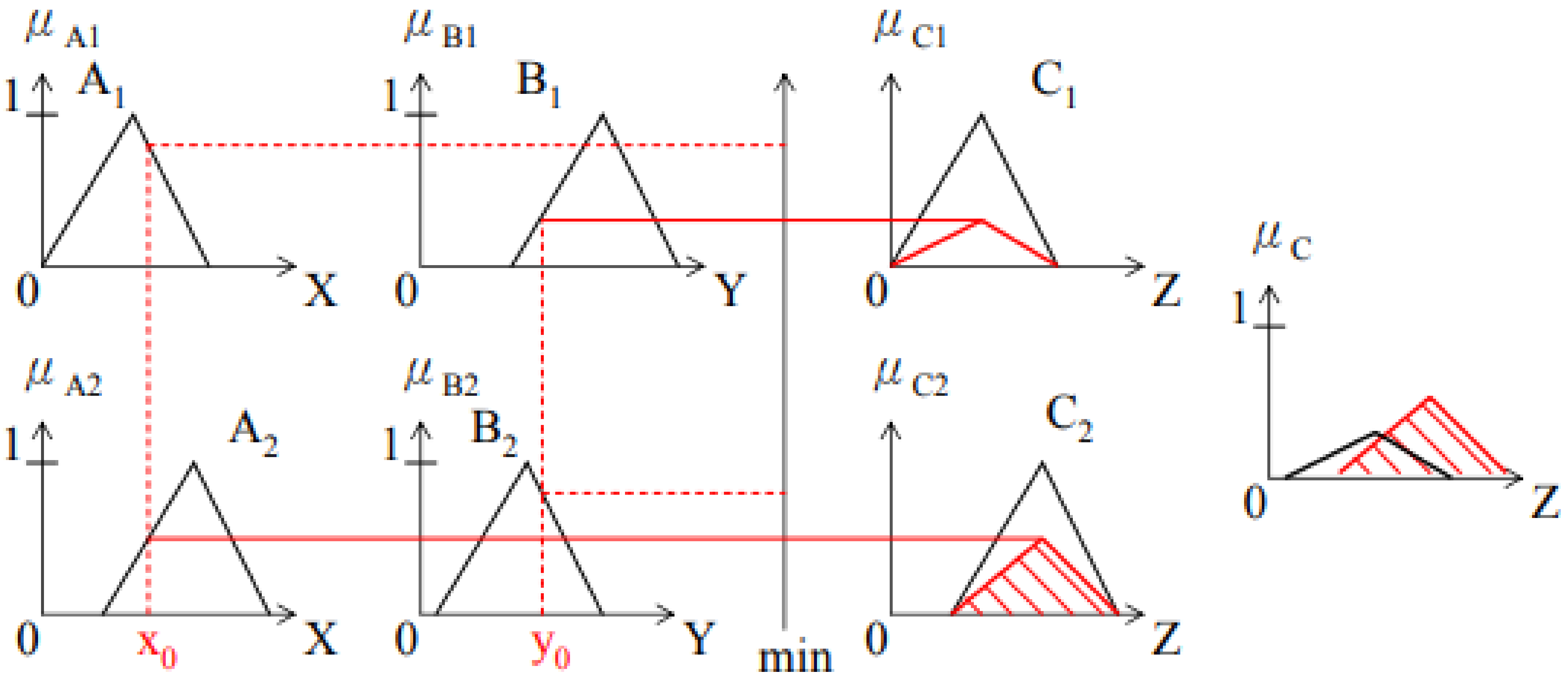
- Lukasiewicz implication: Bounded sum operator for OR

✓ Logical OR of the outputs of the individual rules

- **Mamdani Inference**

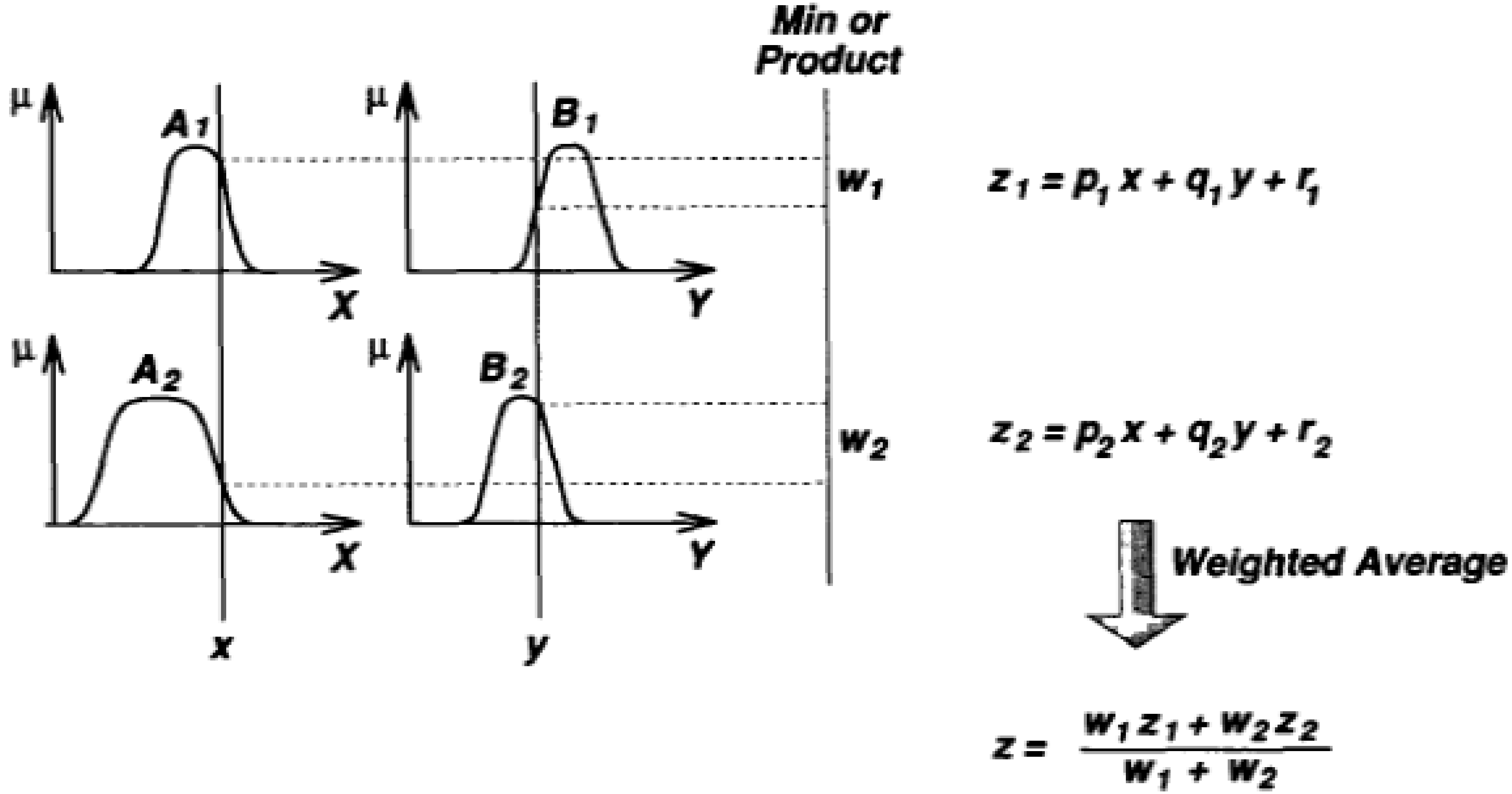


- **Larsen’s Product Inference**



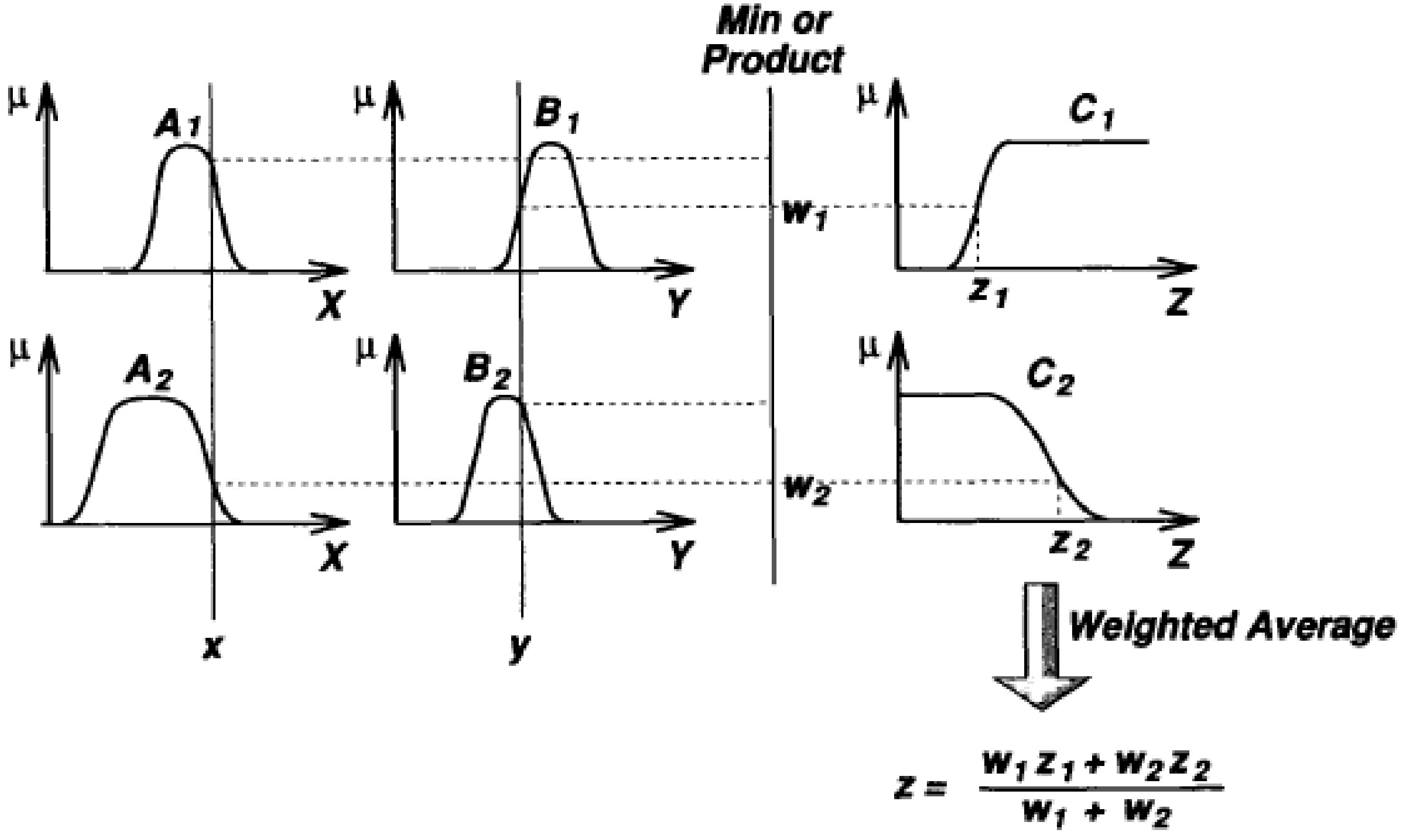
- **Sugeno Model (or Sugeno or TS Inference)**

  - Output is expressed as a crisp function of inputs



- **Tsukamoto Fuzzy Model**

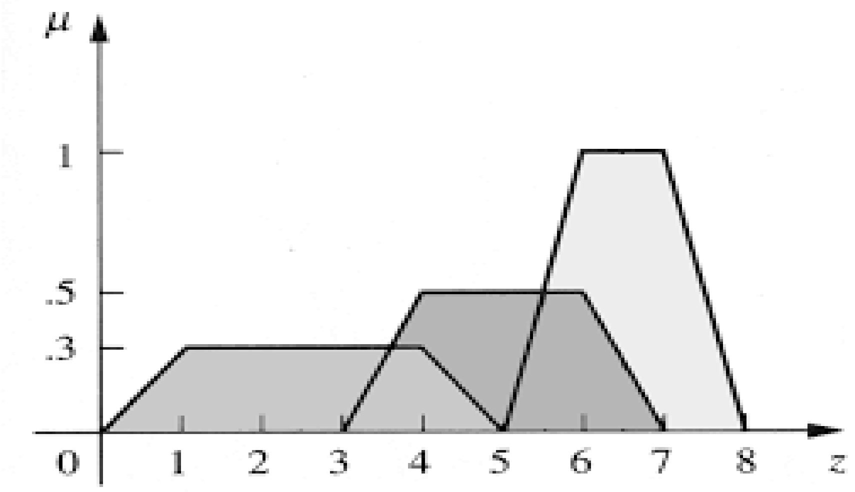
  - output sets have monotonic membership functions



Defuzzification (fuzzy to crisp conversion)

- Mamdani Model:
  - ✓ COG (Centre of Gravity) or COA

$$z^* = \frac{\int_Z \; z\,\mu(z)dz}{\int_Z \; \mu(z)dz}$$

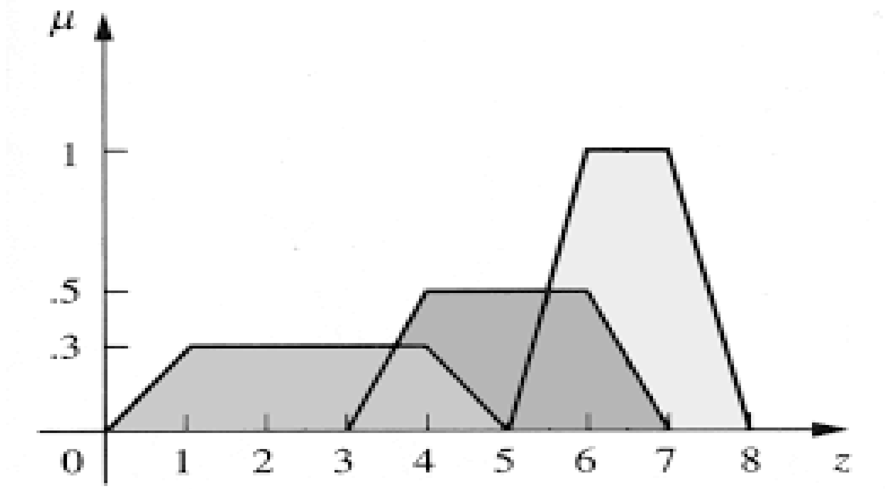


  - ✓ COS (Centre of Sums)

$$z^* = \frac{\sum_{Rules} \; z_i \, A_i}{\sum_{Rules} \; A_i}$$

computationally less expensive than COA

✓ MOM (Mean of Maximum)

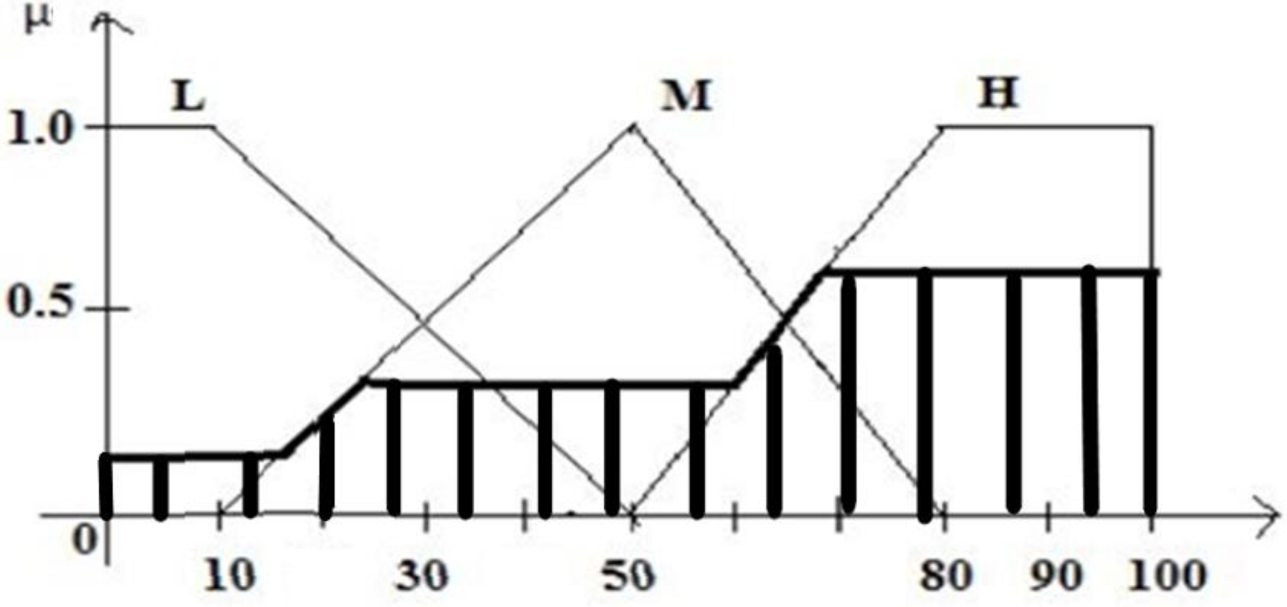


Low accuracy but computationally less expensive

- Sugeno Model:

$$z^* = \frac{w_1 f_1 + w_2 f_2 + \cdots}{w_1 + w_2 + \cdots}$$

Defuzzification through Sampling:



$$z^* = \frac{w_1 z_1 + w_2 z_2 + \cdots}{w_1 + w_2 + \cdots}$$

- **Mamdani Inference vs. Sugeno Inference**

  - Mamdani is completely intuitive

  - Sugeno requires some supporting mathematical formulation or input-output data

  - Defuzzification is much simpler in Sugeno (and Tsukamoto)

A Numerical Example on Mamdani Inference

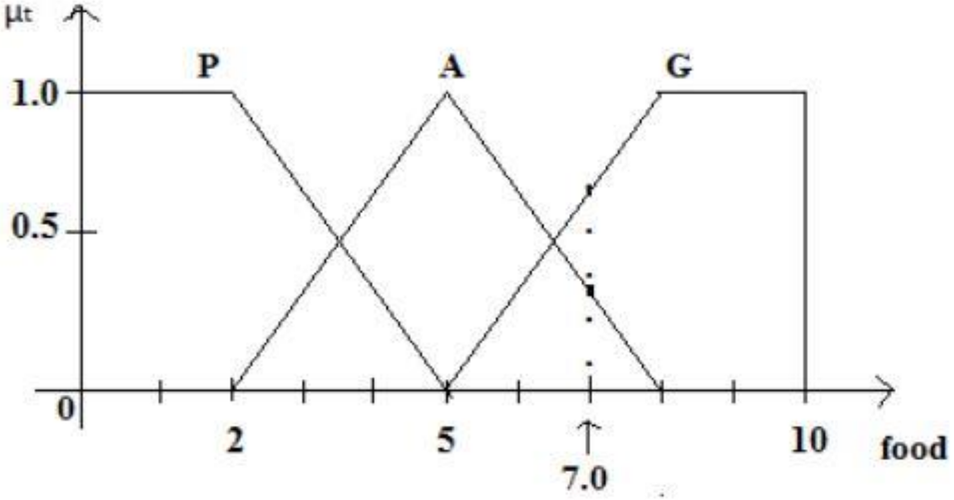
Consider the restaurant tipping problem where the amount of tip depends on the 'quality of food' and 'quality of service' which are graded as Good, Average and Poor on a scale of 0 to 10.

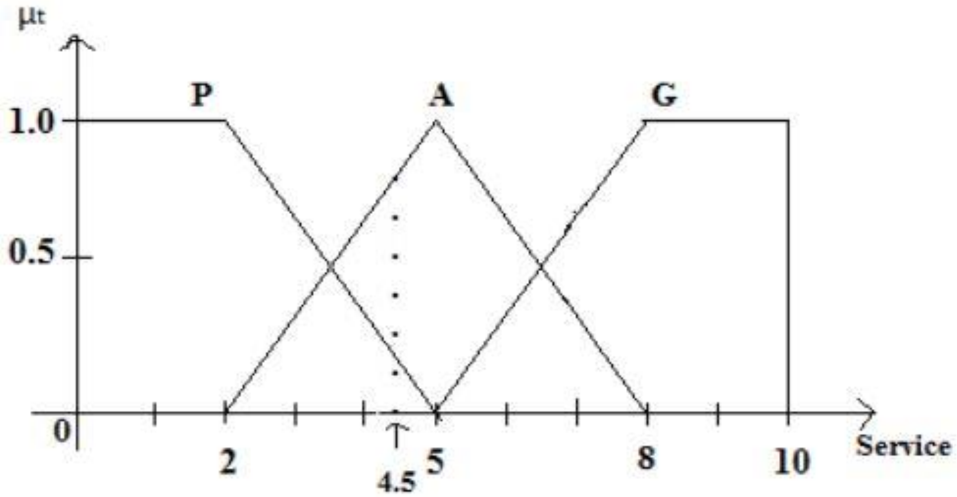
Amount of tip is graded as Low, Medium and High on a scale of 0 to 100 rupees.

Determine the amount of tip when quality of food and quality of service get scores of 7.0 and 4.5 respectively.

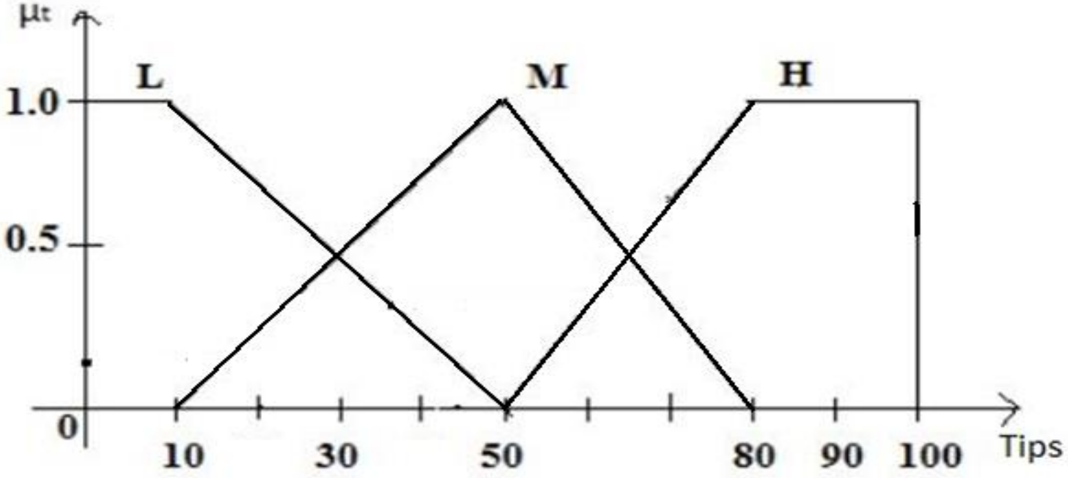
i) Use COS defuzzification method

ii) Use COA defuzzification method





S / F	G	A	P
G	H	H	M
A	H	M	L
P	M	L	L



F \ S	G	A	P
G	H	H^{R1}	M^{R2}
A	H	M^{R3}	L^{R4}
P	M	L	L

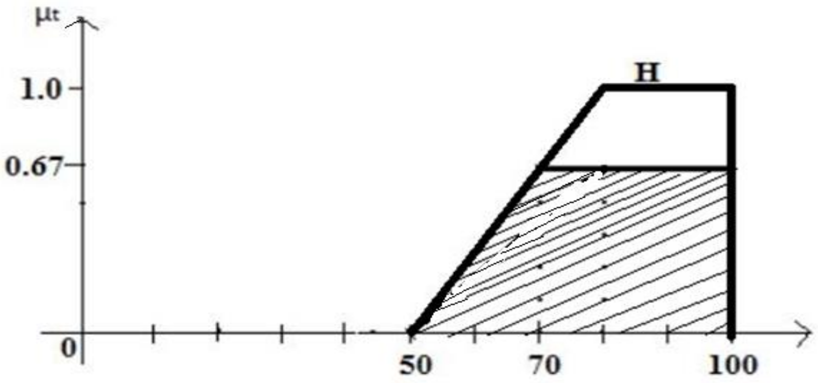
$$R1 \rightarrow \quad \mu f = \underline{0.67}, \quad \mu s = 0.83$$

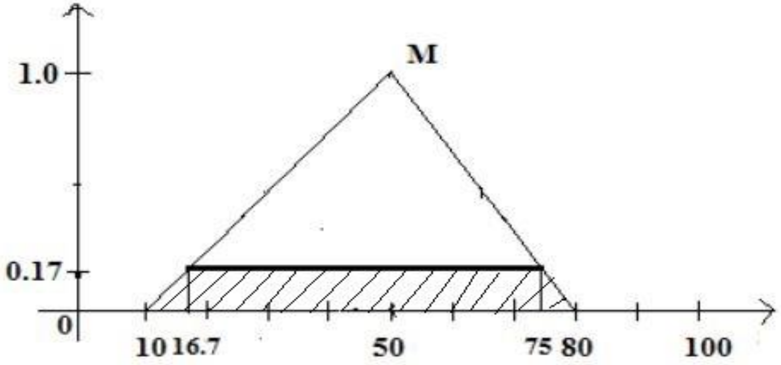
$$R2 \rightarrow \quad \mu f = 0.67, \quad \mu s = \underline{0.17}$$

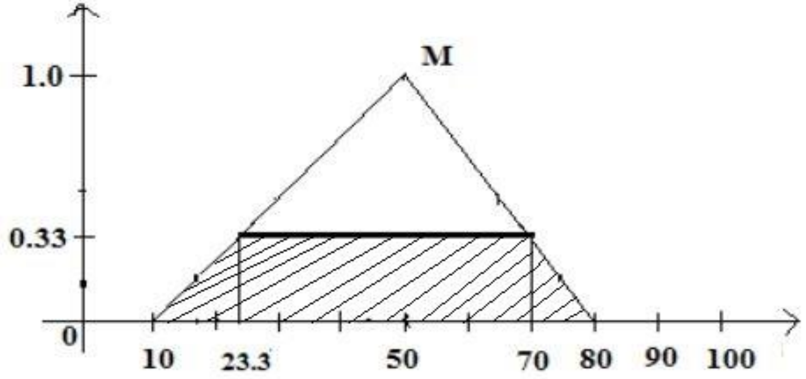
$$R3 \rightarrow \quad \mu f = \underline{0.33}, \quad \mu s = 0.83$$

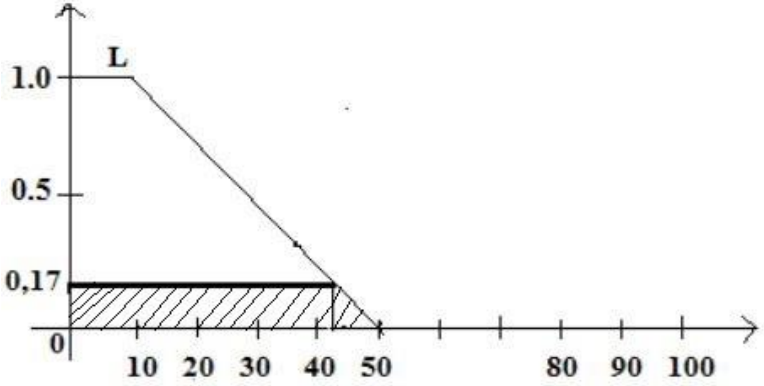
$$R4 \rightarrow \quad \mu f = 0.33, \quad \mu s = \underline{0.17}$$

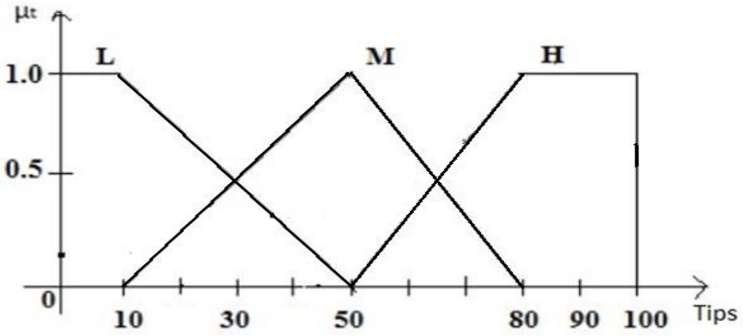
F \ S	G	A	P
G	H	H[R1]	M[R2]
A	H	M[R3]	L[R4]
P	M	L	L



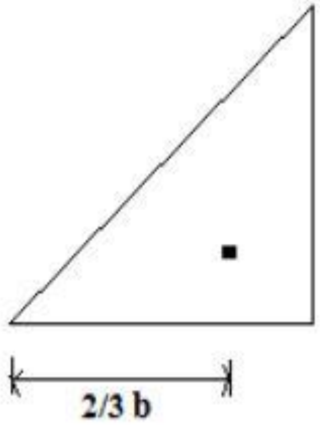




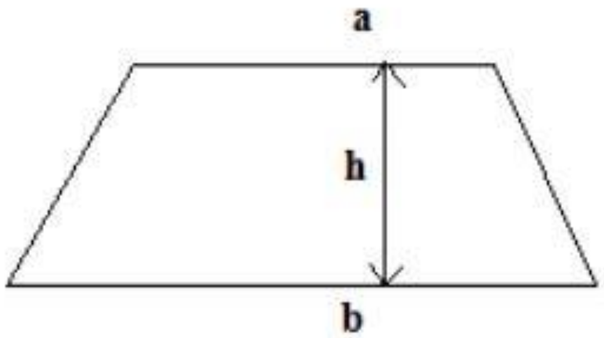




- Centroid of a right angle triangle = $\frac{2}{3}\ b$



- Area of a trapezoid = $\frac{a+b}{2} * h$



$$R1 \longrightarrow A_{11} = \frac{1}{2} * 20 * 0.67 = 6.7$$

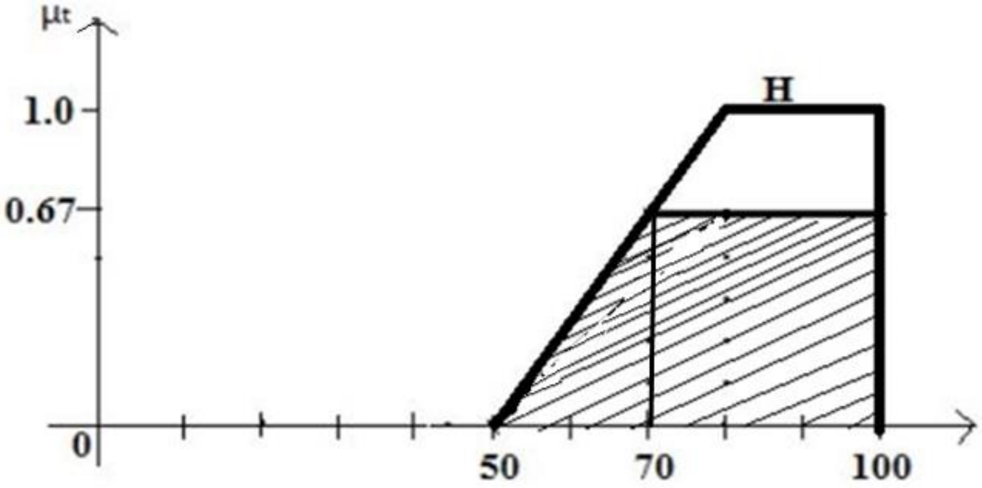
$$X_{11} = (\frac{2}{3} * 20) + 50 = 63.3$$

$$A_{12} = 30 * 0.67 = 20$$

$$X_{12} = 85$$

$$\therefore A_1 = 26.7$$

$$X_1 = \frac{A11\ x\ 11 + A\ 12\ x\ 12}{A\ 11 + A12} = 79.5$$



$$R\,2 \rightarrow A_{21} = \frac{1}{2} * 6.7 * 0.17 = 0.57$$

$$X_{21} = 10 + \frac{2}{3} * 6.7 = 14.4$$

$$A_{22} = 0.17 * (75 - 16.7) = 9.9$$

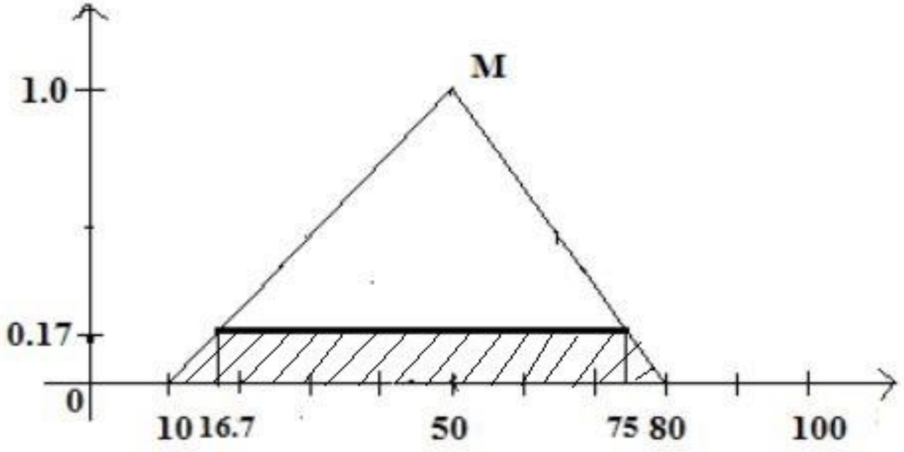
$$X_{22} = 45.8$$

$$A_{23} = \frac{1}{2} * 5 * 0.17 = 0.42$$

$$X_{23} = 75 + \frac{1}{3} * 5 = 76.7$$

$$\therefore A_2 = 0.57 + 9.9 + 0.42 = 10.9$$

$$X_2 = \frac{A21\ x21 + A\ 22\ x\ 22 + A23\ x23}{A\ 21 + A22 + A23} = 45.3$$



$R_3 \quad \rightarrow \quad A_{31} = \frac{1}{2} * 13.3 * 0.33 = 2.2$

$X_{31} = 10 + (\frac{2}{3} * 13.3) = 18.9$

$A_{32} = 0.33 * (70 - 23.3) = 15.4$

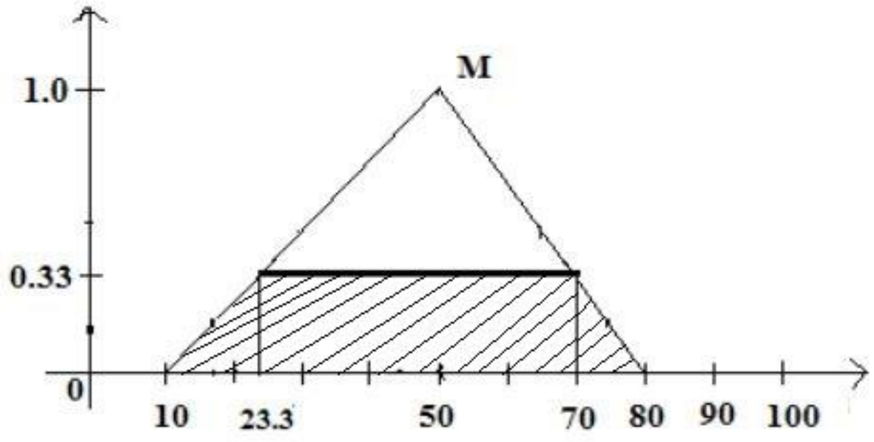
$X_{32} = 46.65$

$A_{33} = \frac{1}{2} * 10 * 0.33 = 1.67$

$X_{33} = 70 + \frac{1}{3} * 10 = 73.33$

$\therefore A_3 = 2.2 + 15.4 + 1.65 = 19.25$

$X_{33} = 45.76$



$R_4 \quad \rightarrow \quad A_{41} = 43.3 * 0.17 = 7.4$

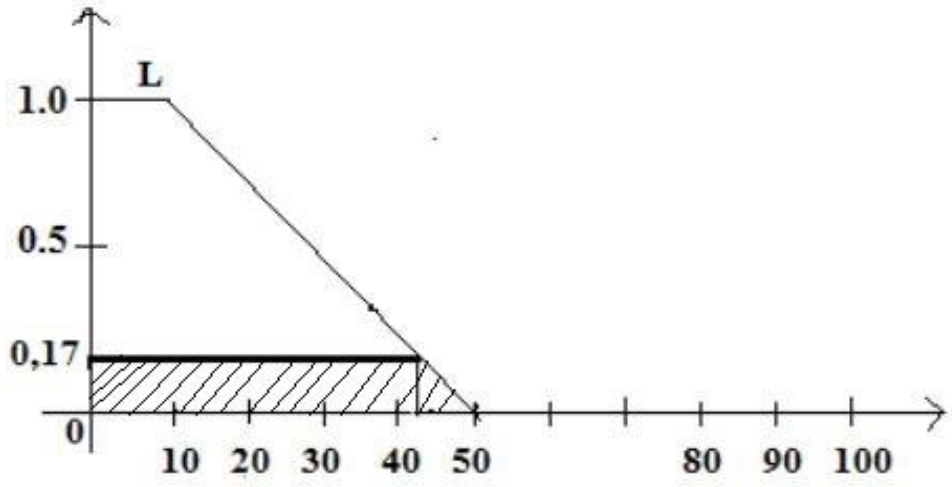
$$X_{41} = \frac{1}{2} * 43.3 = 21.65$$

$$A_{42} = \frac{1}{2} * 0.67 * 0.17 = 0.57$$

$$X_{42} = 43.3 + \frac{1}{3} * 6.7 = 45.5$$

$$\therefore \quad A_4 = 7.97$$

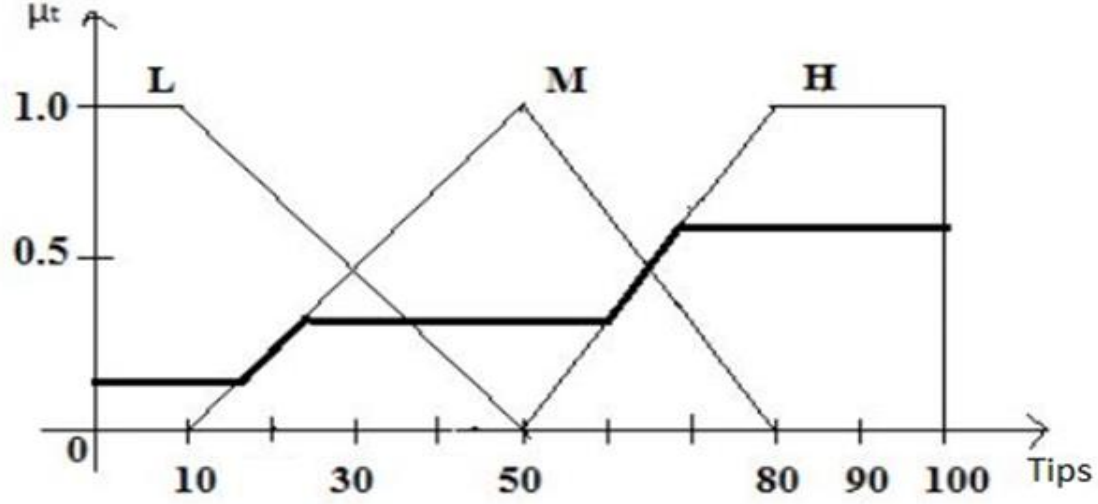
$$X_4 = 23.35$$



$$\therefore \text{ Tip } = \frac{A1x1+A2x2+A3x3+A4x4}{A1+A2+A3+A4} = \frac{3683.4}{64.8}$$

$$= 56.8 \approx 57/-$$

- **COA Defuzzification:**



Rectangle $_1$ → $A_1 = 0.17 \times 23.3 = 3.9$

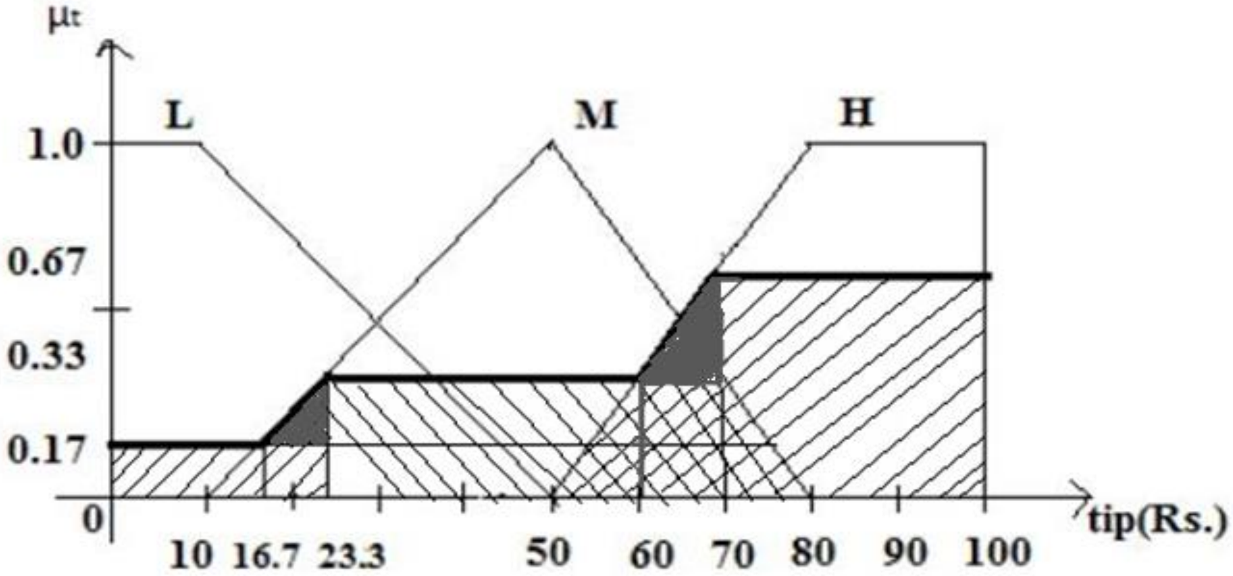
$X_1 = \frac{23.3}{2} = 11.65$

Rectangle $_2$ → $A_2 = 0.33 \times (70-23.3) = 15.4$

$X_2 = 46.65$

Rectangle $_3$ → $A_3 = 30 \times 0.67 = 20$

$X_3 = 85$



Triangle $_1$ $\rightarrow$ $A_4 = \frac{1}{2}(23.3 - 16.7) * 0.16 = 0.53$

$$X_4 = 16.7 + \frac{2}{3}(6.7) = 21.2$$

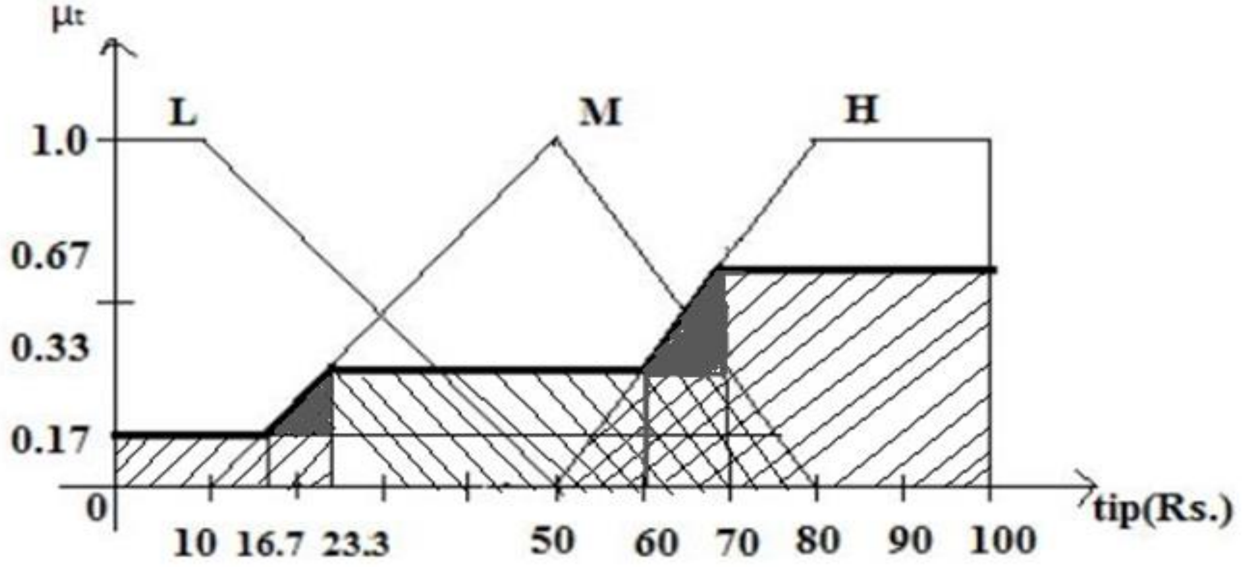
Triangle $_2$ $\rightarrow$ $A_5 = \frac{1}{2} * 10 * 0.34 = 1.7$

$$X_5 = 60 + \frac{2}{3} * 10 = 66.7$$

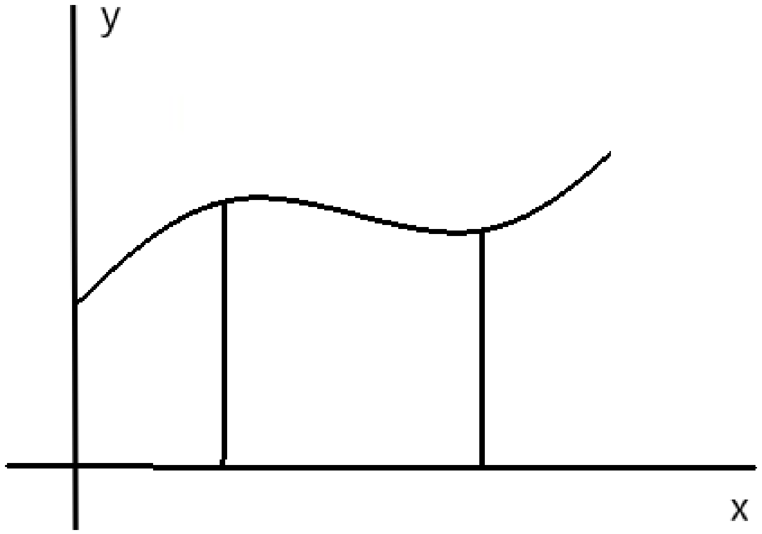
$$\therefore \; x = \frac{A1x1+A2x2+A3x3+A4x4+A5x5}{A1+A2+A3+A4+A5}$$

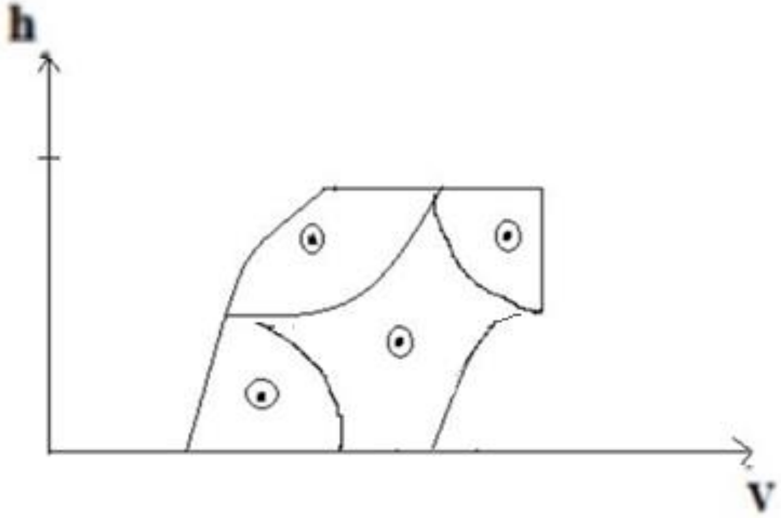
$$= \frac{2588.47}{41.53} = 62.3 \approx 62$$

$\therefore$ Tip = Rs. 62/-



Interpolation among Linear Dynamic Models using T-S Fuzzy System:





$$y = f(x)$$

$$\Delta y = \left.\frac{\partial f}{\partial x}\right|_{x_0} \Delta x$$

$$\dot{X} = f(X, u)$$

$$\Delta \dot{X} = \left.\frac{\partial f}{\partial X}\right|_{(x_0,u_0)} \Delta X + \left.\frac{\partial f}{\partial u}\right|_{(x_0,u_0)} \Delta u$$

- **Numerical Example**

A 2nd order nonlinear dynamic system is described by the following four locally linear models in four regions of the state space –

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \qquad b_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

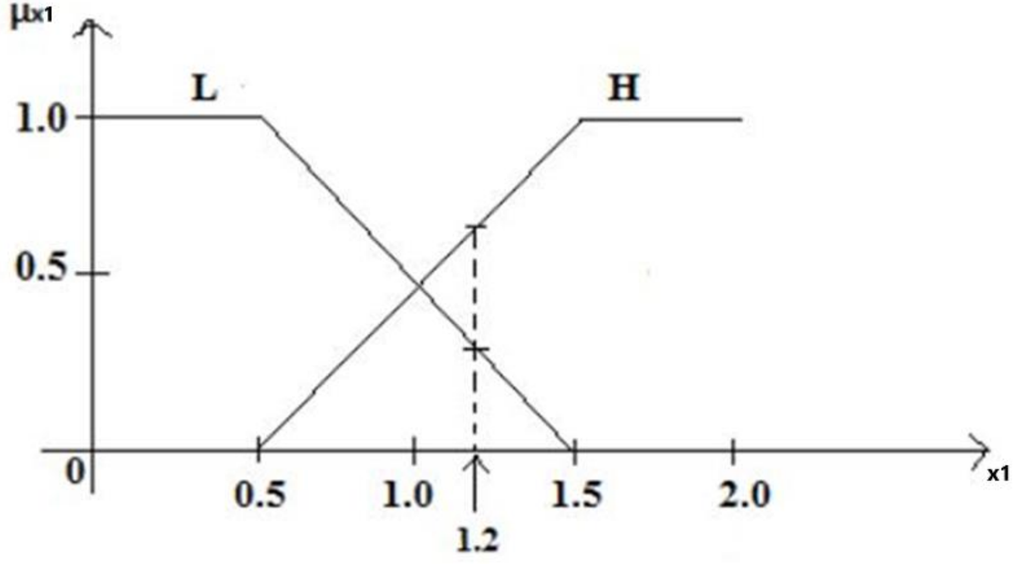
$$A_2 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad b_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

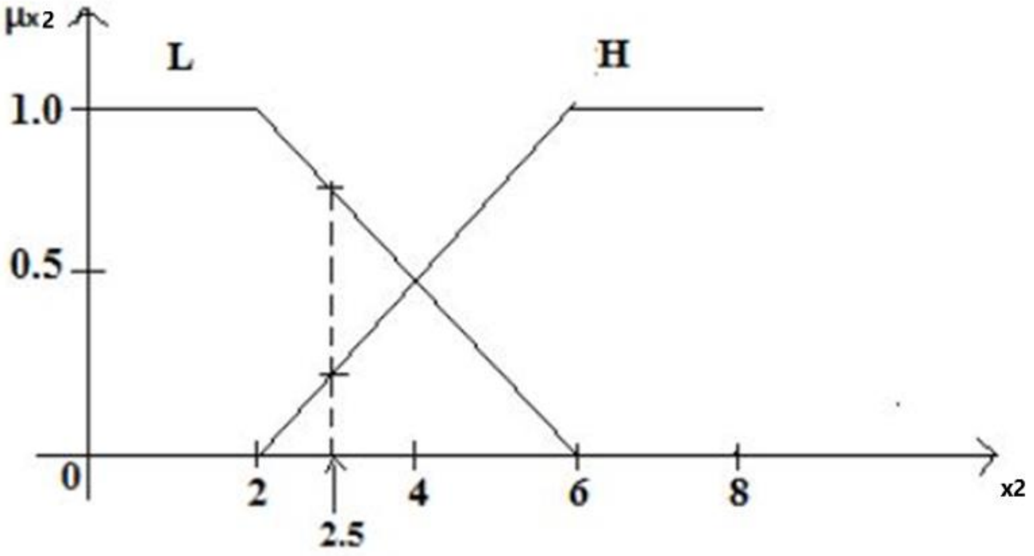
$$A_3 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \qquad b_3 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix}, \qquad b_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Considering the rule base and fuzzification for the state variables x1 and x2 as given below, obtain a linear model when x1(t) = 1.2 and x2(t) = 2.5

(L ≡ Low, H ≡ High)





<u>Rule Base</u>:

x2 x1	L	H
L	A1, b1	A2, b2
H	A3, b3	A4, b4

$$\dot{x} = A_1\, x \;+\; B_1 u$$

$$\dot{x} = A_2\, x \;+\; B_2 u$$

…

Clearly , for the given crisp inputs , all four rules are fired

Weightage of Rule 1, $w^1 = \mu x_1 . \mu x_2$ = 0.3 x 0.875 = 0.2625

Weightage of Rule 2, w^2=0.3 x 0.125= 0.0375

Weightage of Rule 3, w^3=0.7 x 0.875= 0.6125

Weightage of Rule 4, w^4= 0.7 x 0.125= 0.0875

$$w^1+w^2+w^3+w^4 = 0.2625 +0.0375+0.6125+0.0875 = 1.0$$

Hence the interpolated model is given by,

$$\dot{x} = \left\{0.2625\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} + 0.0375\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} + 0.6125\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} + 0.0875\begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix}\right\}x$$

$$+ \left\{0.2625\begin{bmatrix} 0 \\ -1 \end{bmatrix} + 0.0375\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 0.6125\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0.0875\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}u$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1.43 & -1.56 \end{bmatrix} x + \begin{bmatrix} -0.05 \\ 0.40 \end{bmatrix} u$$