RBE/CS 549 Computer Vision

HW1 - AutoCalib

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***Abstract*—In this assignment we work on Calibrating Camera by estimating camera intrinsic and extrinsic and modelling the possible distortion caused in images by using a radial-tangential model based on Zhang's Camera Calibration paper. Based on model of Camera intrinsic and extrinsic and distortion, we estimate the relation between a 2D point in space and corresponding image point and work on optimizing the error between actual points and points after distortion correction.**

***Index Terms*—Camera Calibration, Camera Intrinsic, Camera Extrinsic, Distortion Correction, Projection**

I. Camera Calibration

Multiple techniques have been used for Camera Calibration over the years. Camera calibration techniques can be broadly classified into Photogrammetric and self calibration techniques. We are using Camera Calibration method presented by Zhengyou Zhang of Microsoft. This method lies between the two. The method uses 2D metric information rather than 3D or purely implicit one. Compared with classical techniques, the proposed technique is considerably more flexible. Compared with self-calibration, it gains considerable degree of robustness. The constraints on the camera's intrinsic parameters provided by observing a single plane.

A 2D point is denoted by $m = [u, v]^T$. A 3D point is denoted by $M = [X, Y, Z]^T$. We use x to denote the augmented vector by adding 1 as the last element: $m = [u, v, 1]^T$ and $M = [X, Y, Z, 1]^T$

A. Camera Intrinsic Estimation

The camera calibration matrix A is given as follows

$$A = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

We use another matrix $B = K^{-T}K^{-1}$.

In order to find K, we need to find B. In order find B, we use the Homography matrix H information and obtain another matrix V. Where $V_{i,j}$ is given as follows

B. Camera Extrinsic Estimation

From the Homography Matrix H and Camera intrinsic K matrix, we can find the transformation matrix that relates the point on the world and camera as follows.

$$\mathbf{B} = \mathbf{A}^{-T}\mathbf{A}^{-1} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} \\ \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} & \frac{(v_0\gamma - u_0\beta)^2}{\alpha^2\beta^2} + \frac{v_0^2}{\beta^2} + 1 \end{bmatrix}$$

Fig. 1. B Matrix

$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = \mathbf{0} \, .$$

Fig. 2. V Matrix

C. Distortion Model

We use a radial model. Here (u,v) are the ideal pixel coordinates without distortion and (x,y) are the ideal normalised image coordinates without distortion.

D. Error Estimation

Steps involved in Error Estimation:

1. Find the Image Corners of Checkerboard using cv2.findChessboardCorners and Actual 2D world Coordinates of the Checkerboard. The Checkerboard image used as 10*7

$$\mathbf{v}_{ij} = [h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3}]^T$$

Fig. 3. $V_{i,j}$ Matrix

$$\begin{aligned} \mathbf{r}_1 &= \lambda \mathbf{A}^{-1}\mathbf{h}_1 \\ \mathbf{r}_2 &= \lambda \mathbf{A}^{-1}\mathbf{h}_2 \\ \mathbf{r}_3 &= \mathbf{r}_1 \times \mathbf{r}_2 \\ \mathbf{t} &= \lambda \mathbf{A}^{-1}\mathbf{h}_3 \end{aligned}$$

Fig. 4. Transformation from Image to Camera - Camera Extrinsic

$$\breve{u} = u + (u - u_0)[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]$$
$$\breve{v} = v + (v - v_0)[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2] \ .$$

Fig. 5. Distortion Model

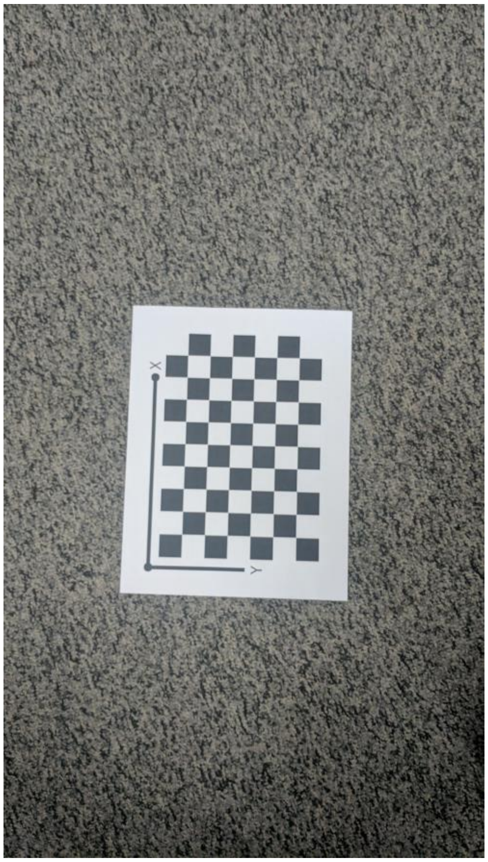

Fig. 6. Input Image- Google Pixel Phone

checkers and hence we find a total of 9*6 points from each of the Checkerboard.

2. After we have the data for image corners and world corners, we estimate the homography matrix between the image corners and world corners using cv2.findHomography.

3. From the obtained H, we calculate V matrix based on the formula below.

4. Further we apply single value decomposition on V to find the b using te formula given below.

E. Error Optimization

We use a maximum likelihood estimation for minimizing the difference between observed point $x_{i,j}$ and point after distortion correction $\hat{x}_{i,j}$, given camera intrinsic K, Camera Extrinsic R_i, t_i and distortion value k.

$$\sum_{i=1}^{N} \sum_{j=1}^{M} ||x_{i,j} - \hat{x}_{i,j}(K, R_i, t_i, X_j, k)||$$

We have used scipy optimize least squares function. We solve the non-linear minimization problem using Levenberg-Marquardt algorithm.

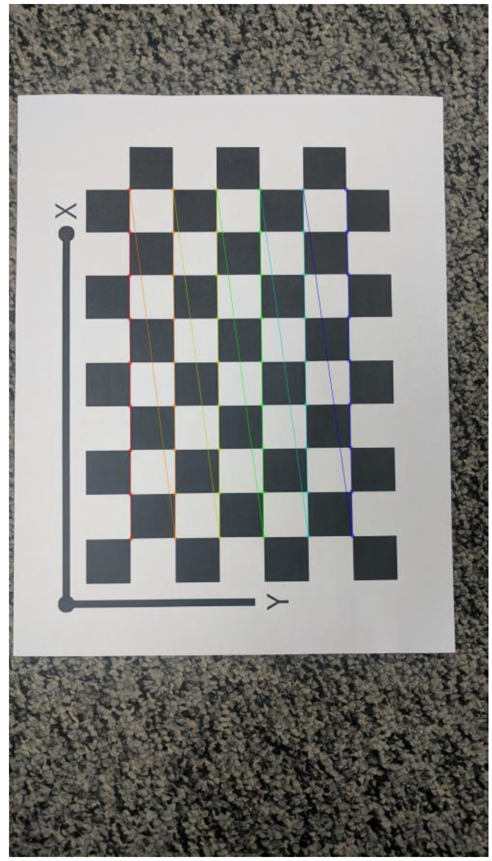

Fig. 7. Corners from cv2.findChessboardCorners

$$\operatorname{argmin}_{f_x, f_y, c_x, c_y, k_1, k_2} \sum_{i=1}^{N} \sum_{j=1}^{M} ||x_{i,j} - \hat{x}_{i,j}(K, R_i, t_i, X_j, k)||$$

F. Conclusion

Initial Value of K Matrix

$$K: \begin{bmatrix} 2.07564188e+03 & -3.76040852e+00 & 7.56285509e+02 \\ 0.00000000e+00 & 2.06494924e+03 & 1.36120530e+03 \\ 0.00000000e+00 & 0.00000000e+00 & 1.00000000e+00 \end{bmatrix}$$

Initial RMS Projection Error : 0.7275062450739974

Updated K matrix

$$\begin{bmatrix} 2.07564040e+03 & -3.76096818e+00 & 7.56290689e+02 \\ 0.00000000e+00 & 2.06494930e+03 & 1.36121079e+03 \\ 0.00000000e+00 & 0.00000000e+00 & 1.00000000e+00 \end{bmatrix}$$

Updated Distortion matrix

$$\begin{bmatrix} 0.00812859 \\ -0.05789797 \end{bmatrix}$$

Final RMS Projection Error : 0.7088373871012587

References

[1] Zhang, Zhengyou. "A flexible new technique for camera calibration." IEEE Transactions on pattern analysis and machine intelligence 22.11 (2000): 1330-1334.

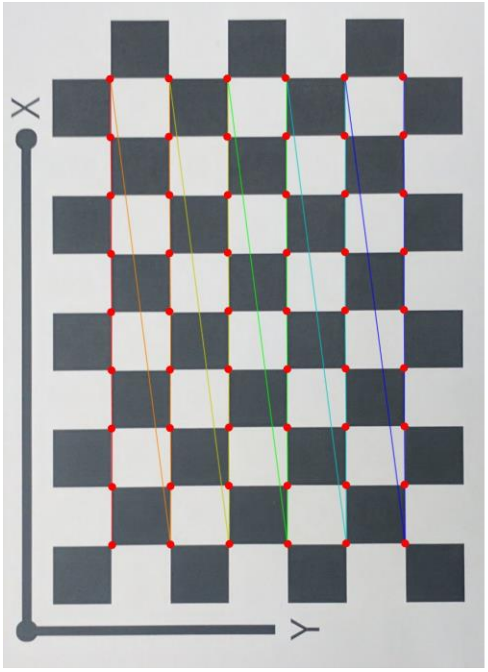

Fig. 8. Reprojected Corners

[2] https://www.ipb.uni-bonn.de/html/teaching/photo12-2021/2021-pho1-22-Zhang-calibration.pptx.pdf
[3] https://learnopencv.com/camera-calibration-using-opencv/
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