CSCI 5654, Spring 2020: Assignment #1 (75 points).
Assigned date: Friday 1/17/2020,
Due date: Tuesday, 01/28/2020 (before class)
Instructions: Please upload your HW as a PDF file along with code that you wrote for running the LP pr

---

P1. (5 points) Convert the following problem to the standard form:

$$\begin{array}{rlcl} \textbf{minimize} & 3x_1 - 5x_2 & & \\ \textbf{s.t.} & 4x_1 + x_2 & \geq & -4 \\ & 2x_1 - x_2 & \geq & -8 \\ & x_1 + 2x_2 & \leq & 4 \\ & x_1 & \geq & 0 \end{array}$$

P2. (10 points) (a) Add slack variables to the standard form that you obtained in (P1) above and write down the **initial dictionary**. (b) Perform one step of simplex on the initial dictionary. (c) Write down clearly which variable is entering and which variable is leaving. (d) Show the resulting dictionary after pivoting.

P3.(20 points) An investment adviser wishes to recommend an ideal investment for her client. After lots of market research, she has compiled a table of investment as expressed in the spreadsheet that has been distributed with this assignment. The total investment is \$10,000.

The data in Fig. 1 shows the investment options and categorizes them based on risk $A - D$, market segment and whether the investment is in an eco-friendly business. The client has specified minimum and maximum investment limits for each category. This data is also given as a CSV file `investmentData.csv` for your convenience.

Setup and solve a linear program that maximizes the expected profit while respecting the maximum and minimum investment percentages for each category.

Is the problem feasible? Is it bounded? What is the optimal solution? **Note:** Upload the code you used to solve the LP either as a GLPK model, matlab files or Python code. Also turn in the outputs that the solver gave you as a `problem3.txt` file.

P4 (10 points) A set S is *convex* if any two points $\mathbf{x}_1, \mathbf{x}_2 \in S$, the point $\lambda \mathbf{x}_1 + (1 - \lambda)\mathbf{x}_2 \in S$ for all $\lambda \in [0, 1]$.

Consider the linear program:

$$\mathsf{max}\ \mathbf{c}^t\mathbf{x}\ \mathsf{s.t.}\ A\mathbf{x} \leq \mathbf{b}\,.$$

(A) Prove that the set of feasible solutions $F : \{\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}\}$ is a convex set. **Note:** The proof is just 5 lines at most.

(B) Let $\mathbf{x}_1 \neq \mathbf{x}_2$ be two distinct optimal solutions for the problem both achieving the optimal objective value $z^* = \mathbf{c}^t\mathbf{x_1} = \mathbf{c}^t\mathbf{x_2}$. Show that there are infinitely many optimal solutions for the linear program. **Note:** The proof requires 3-5 sentences at most.

P5 (15 points) This problem explores how to formulate constraints in linear programming. Let $\mathbf{x} : (x_1, \ldots, x_n)$ be n decision variables such that $x_1, \ldots, x_n \geq 0$. We wish to formulate the following constraints. Which of them can be expressed as constraints in a linear programming problem? If yes, explain how you would formulate them. If no, give a short proof. Note that linear programming problems allow constraints of the form $\sum a_j x_j \leq b$ for real numbers a_j, for $j = 1, \ldots, n$ and b.

(a) (2 points) x_5 must be between x_1 and x_3.

(b) (3 points) x_1 must be between 40% and 60% of the sum of all $x_1, \ldots, x_n$.

(c) (10 points) At least one of the variables $x_1, \ldots, x_n$ must be 0. (**Hint:** Is this a "convex" constraint?).

P6 (15 points, HARDER) Let x_1, x_2 be two real-valued variables which satisfy the constraints $0 \leq x_1 \leq 10$ and $0 \leq x_2 \leq 15$. Using indicator variables $w_1, w_2 \in \{0, 1\}$ write down a system of linear inequalities involving (x_1, x_2, w_1, w_2) that express the nonlinear constraint $x_1 x_2 = 0$. (**Hint:** Suppose we want to make sure that $w_1 = 1$ indicates that $x_1 = 0$ but $w_1 = 0$ places no constraints on x_1. How do we achieve this? Just write down the constraints involving x_1, x_2, w_1, w_2 with at most 4 lines of explanations).

ID	Expected Profit/ Unit	Price / Unit	Risk Category	Investment Market	Eco Friendly ?
1	1.451	2.563	A	Tech	Y
2	2.683	4.307	B	Finance	Y
3	5.898	6.422	C	Finance	Y
4	2.102	3.488	A	PetroChem	N
5	5.709	6.581	B	Finance	N
6	4.519	8.993	D	Finance	Y
7	7.176	11.481	C	Finance	Y
8	6.075	11.730	B	Tech	Y
9	5.718	9.270	B	PetroChem	Y
10	7.442	10.160	A	Automobile	Y
11	1.234	1.961	C	Tech	Y
12	4.680	9.300	D	Automobile	N
13	7.229	11.672	A	PetroChem	Y
14	9.589	10.877	B	Automobile	N
15	6.497	12.137	C	Finance	N

Risk Categories	Min	Max
A	1500	3500
B	4500	6500
C	1000	3000
D	500	2500

Investment Market	Min	Max
Tech	0	3000
Finance	0	4000
PetroChem	0	5000
Automobile	0	7000

EcoFriendly	Min	Max
Y	2000	10000

Figure 1: Investment Data.