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1 Algorithms (57)

Searching, Sorting, Hashing, Asymptotic worst case time and Space complexity, Algorithm design techniques: Greedy, Dynamic programming, and Divide-and-conquer, Graph search, Minimum spanning trees, Shortest paths.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	3	2	3	2	0	2	2	3	3	0	2.2	3
2 Marks Count	3	4	4	2	4	2	3	2	3	2	3	4
Total Marks	9	10	11	6	8	6	8	7	9	6	8.2	11

1.1 Algorithm Design (2)

1.1.1 Algorithm Design: TIFR2011-B-29

https://gateoverflow.in/20576

You are given ten rings numbered from 1 to 10, and three pegs labeled A, B, and C. Initially all the rings are on peg A, arranged from top to bottom in ascending order of their numbers. The goal is to move all the rings to peg B in the minimum number of moves obeying the following constraints:

i. In one move, only one ring can be moved.
ii. A ring can only be moved from the top of its peg to the top of a new peg.
iii. At no point can a ring be placed on top of another ring with a lower number.

How many moves are required?

A. 501 B. 1023 C. 2011 D. 10079 E. None of the above.

tifr2011 algorithms algorithm-design

1.1.2 Algorithm Design: TIFR2019-A-5

https://gateoverflow.in/280505

Asha and Lata play a game in which Lata first thinks of a natural number between 1 and 1000. Asha must find out that number by asking Lata questions, but Lata can only reply by saying “Yes” or “no”. Assume that Lata always tells the truth. What is the least number of questions that Asha needs to ask within which she can always find out the number Lata has thought of?

A. 10 B. 32 C. 100 D. 999 E. None of the above

tifr2019 algorithm-design binary-search

Answers: Algorithm Design

1.1.1 Algorithm Design: TIFR2011-B-29

https://gateoverflow.in/20576

✔ I think its Tower of Hanoi problem.
Therefore, total number of function call $2^n - 1 = 1023$.

$\therefore$ Option B.

17 votes -- Umang Raman (12.2k points)

1.1.2 Algorithm Design: TIFR2019-A-5

https://gateoverflow.in/280505

By using **binary search** Asha needs only $\lceil \log_2 1000 \rceil = 10$ comparisons

6 votes -- Mk Utkarsh (25.7k points)

1.2 Asymptotic Notations (9)

1.2.1 Asymptotic Notations: TIFR2011-B-27

https://gateoverflow.in/20573

Let n be a large integer. Which of the following statements is **TRUE**?

A. $n^{1/\sqrt{\log_2 n}} < \sqrt{\log_2 n} < n^{1/100}$
B. $n^{1/100} < n^{1/\sqrt{\log_2 n}} < \sqrt{\log_2 n}$
C. $n^{1/\sqrt{\log_2 n}} < n^{1/100} < \sqrt{\log_2 n}$

D. $\sqrt{\log_2 n} < n^{1/\sqrt{\log_2 n}} < n^{1/100}$
E. $\sqrt{\log_2 n} < n^{1/100} < n^{1/\sqrt{\log_2 n}}$

tifr2011 asymptotic-notations

1.2.2 Asymptotic Notations: TIFR2012-B-6

https://gateoverflow.in/25106

Let n be a large integer. Which of the following statements is **TRUE**?

A. $2^{\sqrt{2\log n}} < \frac{n}{\log n} < n^{1/3}$
B. $\frac{n}{\log n} < n^{1/3} < 2^{\sqrt{2\log n}}$
C. $2^{\sqrt{2\log n}} < n^{1/3} < \frac{n}{\log n}$
D. $n^{1/3} < 2^{\sqrt{2\log n}} < \frac{n}{\log n}$
E. $\frac{n}{\log n} < 2^{\sqrt{2\log n}} < n^{1/3}$

tifr2012 algorithms asymptotic-notations

1.2.3 Asymptotic Notations: TIFR2014-B-8

https://gateoverflow.in/27192

Which of these functions grows fastest with n?

A. e^n/n.
B. $e^{n-0.9\log n}$.
C. 2^n.
D. $(\log n)^{n-1}$.
E. None of the above.

tifr2014 algorithms asymptotic-notations

1.2.4 Asymptotic Notations: TIFR2016-B-7

https://gateoverflow.in/30720

Let $n = m!$. Which of the following is **TRUE**?

A. $m = \Theta(\log n/\log\log n)$
B. $m = \Omega(\log n/\log\log n)$ but not $m = O(\log n/\log\log n)$
C. $m = \Theta(\log^2 n)$
D. $m = \Omega(\log^2 n)$ but not $m = \mathrm{O}((\log^2 n)$
E. $m = \Theta(\log^{1.5} n)$

tifr2016 asymptotic-notations

1.2.5 Asymptotic Notations: TIFR2017-A-4

https://gateoverflow.in/94943

Which of the following functions asymptotically grows the fastest as n goes to infinity?

A. $(\log\log n)!$
B. $(\log\log n)^{\log n}$
C. $(\log\log n)^{\log\log\log n}$
D. $(\log n)^{\log\log n}$
E. $2^{\sqrt{\log\log n}}$

tifr2017 algorithms asymptotic-notations

1.2.6 Asymptotic Notations: TIFR2018-A-3

https://gateoverflow.in/179272

Which of the following statements is TRUE for all sufficiently large integers n ?

A. $2^{2^{\sqrt{\log\log n}}} < 2^{\sqrt{\log n}} < n$
B. $2^{\sqrt{\log n}} < n < 2^{2^{\sqrt{\log\log n}}}$
C. $n < 2^{\sqrt{\log n}} < 2^{2^{\sqrt{\log\log n}}}$
D. $n < 2^{2^{\sqrt{\log\log n}}} < 2^{\sqrt{\log n}}$
E. $2^{\sqrt{\log n}} < 2^{2^{\sqrt{\log\log n}}} < n$

tifr2018 algorithms asymptotic-notations

1.2.7 Asymptotic Notations: TIFR2018-B-5

https://gateoverflow.in/179289

Which of the following functions, given by there recurrence, grows the fastest asymptotically?

A. $T(n) = 4T\left(\frac{n}{2}\right) + 10n$

B. $T(n) = 8T\left(\frac{n}{3}\right) + 24n^2$
C. $T(n) = 16T\left(\frac{n}{4}\right) + 10n^2$
D. $T(n) = 25T\left(\frac{n}{5}\right) + 20(n \log n)^{1.99}$
E. They all are asymptotically the same

tifr2018 asymptotic-notations recurrence

1.2.8 Asymptotic Notations: TIFR2019-B-5

https://gateoverflow.in/280490

Stirling's approximation for $n!$ states for some constants c_1, c_2

$$c_1 n^{n+\frac{1}{2}} e^{-n} \leq n! \leq c_2 n^{n+\frac{1}{2}} e^{-n}.$$

What are the tightest asymptotic bounds that can be placed on $n!$?

A. $n! = \Omega(n^n)$ and $n! = \mathcal{O}(n^{n+\frac{1}{2}})$
B. $n! = \Theta(n^{n+\frac{1}{2}})$
C. $n! = \Theta((\frac{n}{e})^n)$
D. $n! = \Theta((\frac{n}{e})^{n+\frac{1}{2}})$
E. $n! = \Theta(n^{n+\frac{1}{2}} 2^{-n})$

tifr2019 algorithms asymptotic-notations

1.2.9 Asymptotic Notations: TIFR2020-B-10

https://gateoverflow.in/333131

Among the following asymptotic expressions, which of these functions grows the slowest (as a function of n) asymptotically?

A. $2^{\log n}$
B. n^{10}
C. $(\sqrt{\log n})^{\log^2 n}$
D. $(\log n)^{\sqrt{\log n}}$
E. $2^{2^{\sqrt{\log \log n}}}$

tifr2020 algorithms asymptotic-notations time-complexity

Answers: Asymptotic Notations

1.2.1 Asymptotic Notations: TIFR2011-B-27

https://gateoverflow.in/20573

✔ Let $n = 2^x$. Then, $\log_2 n = x$.

$$\begin{aligned} f(n) &= n^{1/\sqrt{\log_2 n}} = (2^x)^{1/\sqrt{x}} = 2^{x/\sqrt{x}} = 2^{\sqrt{x}} \\ g(n) &= \sqrt{\log_2 n} = \sqrt{\log_2 (2^x)} = \sqrt{x} \\ h(n) &= n^{1/100} = (2^x)^{1/100} = 2^{x/100} \end{aligned}$$

Since exponentials grow faster than polynomials, $h(n) > g(n)$ for large n.
Since linear functions grow faster than square roots, $\frac{x}{100} > \sqrt{x}$ for large x. Thus, $h(n) > f(n)$ for large n.
Since exponentials grow faster than polynomials, $2^{\sqrt{x}} > \sqrt{x}$ for large $\sqrt{x}$. Thus, $f(n) > g(n)$ for large n.
Hence, the relation is,
$g(n) < f(n) < h(n)$
Thus, option (D) is correct.

👍 48 votes -- Pragy Agarwal (18.3k points)

1.2.2 Asymptotic Notations: TIFR2012-B-6

https://gateoverflow.in/25106

✔ Answer will be (C).

Take $n = 2^{1024}$

Now, $2^{\sqrt{(2 \log n)}} \approx 2^{45}$

$n^{\frac{1}{3}} \approx 2^{341}$

$n/\log n = 2^{1024}/1024 \approx 2^{1014}$

Just one value is not enough to confirm growth rate. So, take $n = 1024$.

Now, $2^{\sqrt{(2\log n)}} \approx 2^4$

$n^{\frac{1}{3}} \approx 2^3$

$n/\log n = 2^{10}/10 \approx 2^7$

So, as n increases, the gap between second and third function increases and also the second function overtakes the first. So, $f1 < f2 < f3$.

18 votes -- srestha (85.3k points)

1.2.3 Asymptotic Notations: TIFR2014-B-8 https://gateoverflow.in/27192

✔ Assuming that the base of the $\log$ in the question is e.

Let us try to rewrite each of these functions in the form $e^{\text{something}}$, to make the comparison easier.

$a.$	$e^n \ / \ n$	$= \dfrac{e^n}{e^{(\ln n)}}$	$= e^{(n - \ln n)}$
$b.$	$e^{n-0.9 \ln n}$		$= e^{(n - 0.9 \ln n)}$
$c.$	2^n	$= \left(e^{\ln 2}\right)^n$	$= e^{(n \ln 2)}$
$d.$	$(\ln n)^{n-1}$	$= \left(e^{\ln \ln n}\right)^{n-1}$	$= e^{(n \ln \ln n - \ln \ln n)}$

Now, if we just compare the exponents of all, we can clearly see that $(n \ln \ln n - \ln \ln n)$ grows faster than the rest. Note that in option (C), the multiplicative $\ln 2$ is a constant, and hence grows slower than the multiplicative $\ln \ln n$ from option (D).

This implies that $e^{(n \ln \ln n - \ln \ln n)}$ grows the fastest, and hence, $(\ln n)^{n-1}$ grows the fastest.

Thus, option (D) is the correct answer.

55 votes -- Pragy Agarwal (18.3k points)

1.2.4 Asymptotic Notations: TIFR2016-B-7 https://gateoverflow.in/30720

✔

m	$n = m!$	$\log n/\log\log n$
4	24	$4/2 = 2$
6	720	$9/3 = 3$
8	720×56	$15/3 = 5$
10	$720 \times 56 \times 90$	$21/4 = 5$

If we see, m is growing at same rate as $\log n/\log\log n$.

Correct Answer: A

22 votes -- Arjun Suresh (334k points)

1.2.5 Asymptotic Notations: TIFR2017-A-4 https://gateoverflow.in/94943

✔ Let $N = 2^{256}$

A. $(\log\log N)! = 8!$
B. $(\log\log N)^{\log N} = (8)^{256} = 2^{768}$
C. $(\log\log N)^{\log\log\log N} = 8^3 = 512$

D. $(\log N)^{\log \log N} = (256)^8 = 2^{64}$
E. $2^{\sqrt{\log \log N}} = 2^{2\sqrt{2}}$

Let $N = 2^{16}$

A. $(\log \log N)! = 4!$
B. $(\log \log N)^{\log N} = (4)^{16} = 2^{32}$
C. $(\log \log N)^{\log \log \log N} = 4^2 = 16$
D. $(\log N)^{\log \log N} = (16)^4 = 2^{16}$
E. $2^{\sqrt{\log \log N}} = 2^2 = 4$

Taking ratio for both N values,

A. $(\log \log N)! \rightarrow 8!/4!$
B. $(\log \log N)^{\log N} \rightarrow 2^{736}$
C. $(\log \log N)^{\log \log \log N} \rightarrow 32$
D. $(\log N)^{\log \log N} \rightarrow 2^{48}$
E. $2^{\sqrt{\log \log N}} \rightarrow \approx 2$

Option $B = (\log \log N)^{\log N}$ asymptotically grows the fastest, as for the same change in N, the value increased the most (growth) for option B and this growth is monotonic (continuous).

32 votes -- Kapil Phulwani (35.2k points)

1.2.6 Asymptotic Notations: TIFR2018-A-3

https://gateoverflow.in/179272

✔ Take $n = 2^{2^k}$

Now, option (A) gives $n = 2^{2^k}$

Option (B) gives $2^{\sqrt{\log 2^{2^k}}} = 2^{\sqrt{2^k}}$

Option (C) gives $2^{2^{\sqrt{\log(\log 2^{2^k})}}} = 2^{2^{\sqrt{k}}}$

Now, check power only for (B) and (C).

Take log of power of both functions, i.e.,

$(\log(2^{\sqrt{k}}) = O(\log(\sqrt{2^k})$

$\sqrt{k} = O(\frac{k}{2})$

Clearly, $(C) < (B) < (A)$.

Option (A) is answer.

16 votes -- Anu007 (14.4k points)

1.2.7 Asymptotic Notations: TIFR2018-B-5

https://gateoverflow.in/179289

✔ By applying master theorem,

(a) T(n) = $\Theta(n^2)$

(b) T(n) = $\Theta(n^2)$

(c) T(n) = $\Theta(n^2 * \log n)$

(d) T(n) = $\Theta(n^2)$

C is growing fastest.

15 votes -- Hemant Parihar (11.9k points)

1.2.8 Asymptotic Notations: TIFR2019-B-5

https://gateoverflow.in/280490

Given that -

$c_1 n^{n+\frac{1}{2}} e^{-n} \leq n! \leq c_2 n^{n+\frac{1}{2}} e^{-n}$.

$n! = \Theta\left(\frac{n^{n+1/2}}{e^n}\right)$

We can divide it by $e^{1/2}$ (multiplying or dividing by a +ve constant doesn't affect the asymptotic nature of a function)

$\implies n! = \Theta\left(\frac{n^{n+1/2}}{e^{n+1/2}}\right) = \Theta\left(\left(\frac{n}{e}\right)^{n+\frac{1}{2}}\right)$

Option (D) is the answer.

15 votes

-- Ashish verma (7.3k points)

1.2.9 Asymptotic Notations: TIFR2020-B-10

https://gateoverflow.in/333131

For simplicity, considering log with base 2.

$f_1 = 2^{\lg n} = n^{\lg 2}$

$f_2 = n^{10}$

So, f_1 has slower growth rate than f_2 i.e.
--- $f_1 << f_2$

$f_3 = (\sqrt{\lg n})^{\lg^2 n} = (\lg n)^{\frac{\lg^2 n}{2}} = (\lg n)^{\frac{(\lg n)^2}{2}}$

$f_4 = (\lg n)^{\sqrt{\lg n}} = (\lg n)^{(\lg n)^{\frac{1}{2}}}$
$\because (\lg n)^{\frac{1}{2}} < \frac{(\lg n)^2}{2}$. (For large values of $'n'$). So,
--- $f_4 << f_3$.
Between f_1 and f_4 :
f_4 can be rewritten as $2^{\lg ((\lg n)^{\sqrt{\lg n}})} = 2^{\sqrt{\lg n}\ \lg\lg n}$
So, comparison is between $2^{\lg n}$ and $2^{\sqrt{\lg n}\ \lg\lg n}$
$\because \lg n > \sqrt{\lg n}\ \lg\lg n$ ($\sqrt{\lg n}$ is common in both functions). So,
--- $f_4 << f_1$.
Till now, among f_1, f_2, f_3, f_4, slowest function is f_4. If we get $f_5 << f_4$ then f_5 will be slowest otherwise f_4.
Between f_4 and f_5 :
$f_5 = 2^{2^{\sqrt{lglgn}}}$
f_4 can also be rewritten as $2^{2^{\lg(\sqrt{\lg n}\ \lg\lg n)}}$
Now, $\lg(\sqrt{\lg n}\ \lg\lg n) = \lg\sqrt{\lg n} + \lg\lg\lg n = \frac{1}{2}\lg\lg n + \lg\lg\lg n > \sqrt{\lg\lg n}$.
-- $f_5 << f_4$.
Hence, function
f_5 grows slowest asymptotically.
Now, to get the correct order, we have to compare f_2 and f_3.
Between f_2 and f_3 :
set $n = 2^m$
$f_2 = (2^m)^{10}$
$f_3 = m^{\frac{m^2}{2}} = (m^m)^{m/2}$
Since, $m^m >> m! >> 2^m$. So,

-- $f_2 << f_3$.

Or, we can do like this :

$f_2 = n^{10} = 2^{\lg n^{10}} = 2^{10 \lg n}$

$f_3 = (\lg n)^{\frac{(\lg n)^2}{2}} = 2^{\lg((\lg n)^{\frac{(\lg n)^2}{2}})} = 2^{\frac{(\lg n)^2}{2} \lg \lg n}$

$\because 10 \lg n < \frac{(\lg n)^2}{2} \lg \lg n$. So, $f_2 << f_3$.

Therefore, increasing order of growth rate is :

$f_5 << f_4 << f_1 << f_2 << f_3$

5 votes -- ankitgupta.1729 (15.2k points)

1.3 Graph Algorithms (3)

1.3.1 Graph Algorithms: TIFR2013-B-15

https://gateoverflow.in/25798

Let G be an undirected graph with n vertices. For any subset S of vertices, the set of neighbours of S consists of the union of S and the set of vertices S' that are connected to some vertex in S by an edge of G. The graph G has the nice property that every subset of vertices S of size at most $n/2$ has at least $1.5|S|$-many neighbours. What is the length of a longest path in G?

A. $O(1)$
B. $O(\log \log n)$ but not $O(1)$
C. $O(\log n)$ but not $O(\log \log n)$
D. $O(\sqrt{n})$ but not $O(\log n)$
E. $O(n)$ but not $O(\sqrt{n})$

tifr2013 graph-algorithms

1.3.2 Graph Algorithms: TIFR2013-B-5

https://gateoverflow.in/25666

Given a weighted directed graph with n vertices where edge weights are integers (positive, zero, or negative), determining whether there are paths of arbitrarily large weight can be performed in time

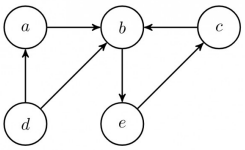
A. $O(n)$
B. $O(n.\log(n))$ but not $O(n)$
C. $O(n^{1.5})$ but not $O(n \log n)$
D. $O(n^3)$ but not $O(n^{1.5})$
E. $O(2^n)$ but not $O(n^3)$

tifr2013 algorithms graph-algorithms

1.3.3 Graph Algorithms: TIFR2014-B-3

https://gateoverflow.in/27137

Consider the following directed graph.



Suppose a depth-first traversal of this graph is performed, assuming that whenever there is a choice, the vertex earlier in the alphabetical order is to be chosen. Suppose the number of tree edges is T, the number of back edges is B and the number of cross edges is C. Then

A. $B = 1, C = 1$, and $T = 4$.
B. $B = 0, C = 2$, and $T = 4$.
C. $B = 2, C = 1$, and $T = 3$.
D. $B = 1, C = 2$, and $T = 3$.
E. $B = 2, C = 2$, and $T = 1$.

tifr2014 algorithms graph-algorithms

Answers: Graph Algorithms

1.3.1 Graph Algorithms: TIFR2013-B-15

https://gateoverflow.in/25798

Here we have to find the longest path in Graph.

Let's start with 1 node(chose any one) & find the number of hops in which whole Graph is reachable i.e. all the n vertices are reachable.

From First node (say A) -

(1) within 1 hop(1 edge traversal) we can reach atleast = 3/2 nodes

{as neighbourhood of A has atleast 3/2 nodes}

(2) within 2 hops we can reach atleast
$\left(\frac{3}{2}\right)^2$ nodes

(3) within 3 hops we can reach atleast
$\left(\frac{3}{2}\right)^3$ nodes

and so on.. like this when our nodes reach
$\left(\frac{n}{2}\right)$ within 1 more hop we can reach upto
$\left(\frac{n}{2}\right) * \frac{3}{2} = \frac{3n}{4}$ nodes..

uptill here we will take O(log n) hops {to reach
$\frac{3n}{4}$ nodes}.

But this property is valid upto a subset of size atmost n/2 .. so, here I stuck to proceed..

Hope it helps.. Also any suggestions to proceed further are welcome..

1 votes -- Himanshu Agarwal (12.4k points)

1.3.2 Graph Algorithms: TIFR2013-B-5

https://gateoverflow.in/25666

✔ I think arbitrary large weights means having positive weight cycle.

So, Bellman Ford algorithm can be used.

$O(VE)$

Changing sign of weights of edges.

Correct Answer: D

14 votes -- papesh pathare (573 points)

1.3.3 Graph Algorithms: TIFR2014-B-3

https://gateoverflow.in/27137

✔ Since they said that whenever there is a choice we will have to select the node which is alphabetically earlier, therefore we choose the starting node as A.

The tree then becomes $A - B - E - C$. Therefore number of tree edges is 3, that is, $(T = 3)$.

Now, there is one cycle $B - E - C$, so, we will get a back edge from C to B while performing DFS. Hence $B = 1$.

Now, D becomes disconnected node and it can only contribute in forming cross edge . There are 2 cross edges $D - A$, $D - B$. Therefore $C = 2$.

Answer is Option D.

Correct me if am going wrong.

35 votes -- Riya Roy(Arayana) (5.3k points)

1.4 Identify Function (4)

1.4.1 Identify Function: TIFR2010-B-24

https://gateoverflow.in/18742

Consider the following program operating on four variables u, v, x, y, and two constants X and Y.

```
x, y, u, v:= X, Y, Y, X;
While (x ≠ y)
do
    if (x > y) then x, v := x - y, v + u;
    else if (y > x) then y, u:= y - x, u + v;
od;
print ((x + y) / 2);  print ((u + v) / 2);
```

Given $X > 0 \wedge Y > 0$, pick the true statement out of the following:

A. The program prints $\gcd(X, Y)$ and the first prime larger than both X and Y.
B. The program prints $\gcd(X, Y)$ followed by $\text{lcm}(X, Y)$.
C. The program prints $\gcd(X, Y)$ followed by $\frac{1}{2} \times \text{lcm}(X, Y)$.
D. The program prints $\frac{1}{2} \times \gcd(X, Y)$ followed by $\frac{1}{2} \times \text{lcm}(X, Y)$.

E. The program does none of the above.

tifr2010 algorithms identify-function

1.4.2 Identify Function: TIFR2014-B-2

https://gateoverflow.in/27136

Consider the following code.

```
def brian(n):
    count = 0

    while ( n ! = 0 )
          n = n & ( n-1 )
         count = count + 1

  return count
```

Here n is meant to be an unsigned integer. The operator & considers its arguments in binary and computes their bit wise AND. For example, 22 & 15 gives 6, because the binary (say 8-bit) representation of 22 is 00010110 and the binary representation of 15 is 00001111, and the bit-wise AND of these binary strings is 00000110, which is the binary representation of 6. What does the function brian return?

A. The highest power of 2 dividing n, but zero if n is zero.
B. The number obtained by complementing the binary representation of n.
C. The number of ones in the binary representation of n.
D. The code might go into an infinite loop for some n.
E. The result depends on the number of bits used to store unsigned integers.

tifr2014 algorithms identify-function

1.4.3 Identify Function: TIFR2014-B-20

https://gateoverflow.in/27354

Consider the following game. There is a list of distinct numbers. At any round, a player arbitrarily chooses two numbers a, b from the list and generates a new number c by subtracting the smaller number from the larger one. The numbers a and b are put back in the list. If the number c is non-zero and is not yet in the list, c is added to the list. The player is allowed to play as many rounds as the player wants. The score of a player at the end is the size of the final list.

Suppose at the beginning of the game the list contains the following numbers: $48, 99, 120, 165$ and 273. What is the score of the best player for this game?

A. 40 B. 16 C. 33 D. 91 E. 123

tifr2014 algorithms identify-function

1.4.4 Identify Function: TIFR2017-A-12

https://gateoverflow.in/95299

Consider the following program modifying an $n \times n$ square matrix A:

```
for i=1 to n:
    for j=1 to n:
        temp=A[i][j]+10
        A[i][j]=A[j][i]
        A[j][i]=temp-10
    end for
end for
```

Which of the following statements about the contents of matrix A at the end of this program must be TRUE?

A. the new A is the transpose of the old A
B. all elements above the diagonal have their values increased by 10 and all the values below have their values decreased by 10
C. all elements above the diagonal have their values decreased by 10 and all the values below have their values increased by 10
D. the new matrix A is symmetric, that is, $A[i][j] = A[j][i]$ for all $1 \leq i, j \leq n$
E. A remains unchanged

tifr2017 algorithms identify-function

Answers: Identify Function

1.4.1 Identify Function: TIFR2010-B-24

https://gateoverflow.in/18742

Greatest Common Divisor of two number can be calculated using Euclid's Algorithm [1]

$$\begin{aligned} gcd(X,0) &= X \\ gcd(X,Y) &= gcd(Y, X\%Y) \end{aligned}$$

For $X, Y > 0$ this algorithm can be simplified as follows

$$gcd(X,Y) = \begin{cases} X & X = Y \\ gcd(X-Y,Y) & X > Y \\ gcd(X,Y-X) & X < Y \end{cases}$$

Out of observation we learn that the given program is performing the same operation on two variable x and y initialized to X and Y respectively. Loop terminated with $x = y$, therefore $(x+y)/2$ i.e., average of two will return x (or y), which is essentially $gcd(X,Y)$.

Next, let us propose a loop invariant[2] for this program[3]

$$2XY = (x \times v) + (y \times u)$$

The correctness of this program depends on this invariant, which must be true prior to the first iteration of the loop, during the execution of loop and with the termination of the loop it must express some useful property that program exhibits. Let's briefly show these three things.

1. **Initialization:** The fact that this loop invariant holds prior to the first iteration of the loop can be seen by plugging in initial values of x, y, u and v.
2. **Maintenance:** Let us assume this loop invariant holds before i^{th} iteration and we need to show that it holds after i^{th} iteration as well. During i^{th} iteration for $x > y$, $x = (x-y)$ and $u = (u+v)$. Substitue these new values in loop invariant we get

   $$\begin{aligned} 2XY &= (x-y) \times v + y(u+v) \\ &= (x \times v) + (y \times u) \end{aligned}$$

   And same will also be true for $x < y$.
3. **Termination**: Loop terminates with $x = y$ and as shown earlier that $x = y = gcd(X,Y)$. Therefore,

   $$\begin{aligned} 2XY &= (x \times v) + (x \times u) = x(u+v) \\ 2XY &= gcd(X,Y)(u+v) \end{aligned}$$

Rearranging above equation gives us[4]

$$\frac{XY}{gcd(X,Y)} = LCM(X,Y) = \frac{u+v}{2}$$

HTH

REFERENCES

1. GCD using Euclid's Algorithm
2. Introduction to Algorithms, CLRS, 3rd edition, 2.1, Pg.18
3. Edsger Dijkstra lecture at University of Newcastle 1992
4. Reduction by the greatest common divisor

P.S In competitive examinations like GATE/TIFR the better strategy for such unfamiliar program is to try executing the program on different values, observe the result and compare with given options. However, in my opinion, understanding the underlining crux of such problems (when we are outside examination hall) is more important than just getting answers.

References

12 votes -- Prateek Dwivedi (3.5k points)

1.4.2 Identify Function: TIFR2014-B-2

https://gateoverflow.in/27136

✔ Option **C**. It returns no of 1's in binary representation of n.
Here, $n\&(n-1)$ reset rightmost bit of n in each iteration.

For example,

Suppose $n = 15 = 00001111$(binary)

$n - 1 = 14(00001110)$

```
 00001111
^ 00001110
----------
  00001110
```

9 votes -- Avdhesh Singh Rana (2.3k points)

1.4.3 Identify Function: TIFR2014-B-20

https://gateoverflow.in/27354

✔ Option **D** is correct

Here the list is $(48, 99, 120, 165, 273)$.

$Gcd(48, 99) = 3$,means if we subtract $(99 - 48 = 51)$ then that number is also % 3,

So the numbers like $(3, 6, 9 - - - -99)$ are added. Total numbers $= 99/3 = 33$

//y $Gcd(48, 120) = 24$,so the numbers $\%24$ are added like $(24, 48, - - -120)$. Total numbers $= 120/24 = 5$

//y $Gcd(48, 165) = 3$,so the numbers $(3, 6, 9, - - 24 - -48 - -99 - -120 - - - 165)$ are added. Totally, $165/3 = 55$

At end, $Gcd(48, 273) = 3$,so the numbers $(3, 6, 9 - -24 - - - 48 - -99 - - - 120 - -165 - - - 273)$ are added(which covers all the above numbers)

So total numbers added to this list $= 273/3 = 91$

28 votes -- venky.victory35 (741 points)

1.4.4 Identify Function: TIFR2017-A-12

https://gateoverflow.in/95299

✔

```
1.for i=1 to n:
2.    for j=1 to n:
3.        temp=A[i][j]+10
4.        A[i][j]=A[j][i]
5.        A[j][i]=temp-10
6.    end for
7.end for
```

The $3, 4, 5$ lines swap $A[j][i]$ and $A[i][j]$.

The same variables are swapped twice. For eg when: $i = 5$, $j = 10$. $A[10][5]$ and $A[5][10]$ will be swapped. They will be swapped again when $i = 10$, $j = 5$.

Two times swap of same elements will lead to A remaining unchanged.

Hence, **E** is correct.

10 votes -- tarun_svbk (1.4k points)

1.5 Minimum Maximum (3)

1.5.1 Minimum Maximum: TIFR2014-B-10

https://gateoverflow.in/27198

Given a set of n distinct numbers, we would like to determine both the smallest and the largest number. Which of the following statements is TRUE?

A. These two elements can be determined using $O\left(\log^{100} n\right)$ comparisons.
B. $O\left(\log^{100} n\right)$ comparisons do not suffice, however these two elements can be determined using $n + O(\log n)$ comparisons.
C. $n + O(\log n)$ comparisons do not suffice, however these two elements can be determined using $3\lceil n/2 \rceil$ comparisons.
D. $3\lceil n/2 \rceil$ comparisons do not suffice, however these two elements can be determined using $2(n-1)$ comparisons.
E. None of the above.

tifr2014 algorithms minimum-maximum

1.5.2 Minimum Maximum: TIFR2014-B-6

https://gateoverflow.in/27183

Consider the problem of computing the minimum of a set of n distinct numbers. We choose a permutation uniformly at random (i.e., each of the n! permutations of $\langle 1, \ldots, n \rangle$ is chosen with probability $(1/n!)$ and we inspect the numbers in the order given by this permutation. We maintain a variable MIN that holds the minimum value seen so far. MIN is initialized to ∞ and if we see a value smaller than MIN during our inspection, then MIN is updated. For example, in the inspection given by the following sequence, MIN is updated four times.

5 9 4 2 6 8 0 3 1 7

What is the expected number of times MIN is updated?

A. $O(1)$ B. $H_n = \sum_{i=1}^{n} 1/i$ C. $\sqrt{n}$ D. $n/2$ E. n

tifr2014 algorithms minimum-maximum

1.5.3 Minimum Maximum: TIFR2014-B-9

https://gateoverflow.in/27194

Given a set of n distinct numbers, we would like to determine the smallest three numbers in this set using comparisons. Which of the following statements is TRUE?

A. These three elements can be determined using $O\left(\log^2 n\right)$ comparisons.
B. $O\left(\log^2 n\right)$ comparisons do not suffice, however these three elements can be determined using $n + O(1)$ comparisons.
C. $n + O(1)$ comparisons do not suffice, however these three elements can be determined using $n + O(\log n)$ comparisons.
D. $n + O(\log n)$ comparisons do not suffice, however these three elements can be determined using $O(n)$ comparisons.
E. None of the above.

tifr2014 algorithms minimum-maximum

Answers: Minimum Maximum

1.5.1 Minimum Maximum: TIFR2014-B-10

https://gateoverflow.in/27198

✔ I think answer will be **C.**

To be accurate, it will need $3n/2 - 2$ comparisons .

12 votes -- Pranay Datta (7.8k points)

1.5.2 Minimum Maximum: TIFR2014-B-6

https://gateoverflow.in/27183

✔ Let us consider 3 numbers $\{1, 2, 3\}$

We will consider the permutation along with min no of times MIN is updated.

Permutation	Minimum of times MIN is updated
$1,2,3$	1
$1,3,2$	1
$2,1,3$	2
$2,3,1$	2
$3,1,2$	2
$3,2,1$	3

Total number of times MIN updated is : 11.

Average no of times MIN updated is : $(11/6)$

Now going by the options i am getting B .

$H_3 = 1 + 1/2 + 1/3 = 11/6$.

H_3 is the answer and that is option **B** .

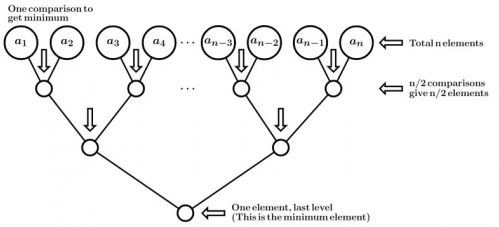
36 votes

-- Riya Roy(Arayana) (5.3k points)

1.5.3 Minimum Maximum: TIFR2014-B-9

https://gateoverflow.in/27194

✔ **Option (C)** is correct. Reason is as follows :



Here, at first level we are Given n elements, out of which we have to find smallest 3 numbers.

We compare $2-2$ elements as shown in figure & get $n/2$ elements at Second level.

Note: Minimum element is in these $n/2$ elements.

So, comparisons for this is $n/2$.

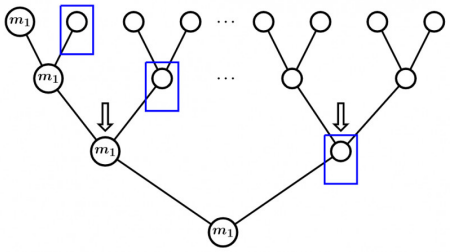
Similarly for next level we have $n/4$ Comparisons & $n/2$ elements and so on.

Total Comparisons till now is $n/2 + n/4 + n/8 + \ldots + 4 + 2 + 1 = (2^{\log n} - 1) = n - 1$ {Use G.P. sum}

Now we have to get smallest 3.

We have 1st smallest at last level already $\Rightarrow 0$ Comparison for this.

=> 2nd & 3rd smallest can be found in $O(\log n)$ time as shown below:



Minimum Element must have descended down from some path from top to Bottom.

SQUARES represent Candidates for 2^{nd} minimum.

Every element that is just below m_1 (first minimum) is a candidate for second minimum.

So, $O(\log n)$ Comparisons for finding second smallest.

Similarly for 3^{rd} minimum we get $O(\log n)$ time. As every element that is just below 1st & 2nd minimum is a candidate for 3^{rd} minimum.

40 votes -- Himanshu Agarwal (12.4k points)

1.6 Minimum Spanning Trees (2)

1.6.1 Minimum Spanning Trees: TIFR2018-B-13

https://gateoverflow.in/179297

Let $n \geq 3$, and let G be a simple, connected, undirected graph with the same number n of vertices and edges. Each edge of G has a distinct real weight associated with it. Let T be the minimum weight spanning tree of G. Which of the following statements is NOT ALWAYS TRUE ?

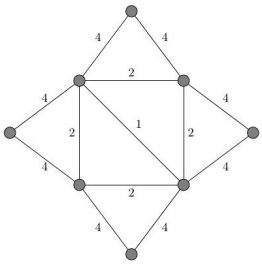
A. The minimum weight edge of G is in T.
B. The maximum weight edge of G is not in T.
C. G has a unique cycle C and the minimum weight edge of C is also in T.
D. G has a unique cycle C and the maximum weight edge of C is not in T.
E. T can be found in $O(n)$ time from the adjacency list representation of G.

tifr2018 graph-algorithms minimum-spanning-trees

1.6.2 Minimum Spanning Trees: TIFR2019-B-2

https://gateoverflow.in/280493

How many distinct minimum weight spanning trees does the following undirected, weighted graph have ?



A. 8 B. 16 C. 32 D. 64 E. None of the above

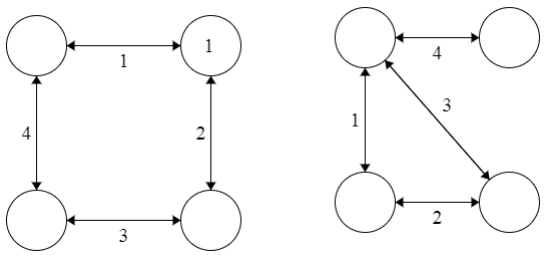
tifr2019 algorithms minimum-spanning-trees

Answers: Minimum Spanning Trees

1.6.1 Minimum Spanning Trees: TIFR2018-B-13

https://gateoverflow.in/179297

✔ Take 2 Random graph with same number of vertices and edges:



Now check option 1 by 1:

1. The minimum weight edge of G is in T. Always true
2. The maximum weight edge of G is not in T. True in 1st graph but false in 2nd graph Maximum weight edge will be in T if it is not part of the cycle.
3. G has a unique cycle C and the minimum weight edge of C is also in T. Always true
4. G has a unique cycle C and the maximum weight edge of C is not in T. Always true
5. T can be found in $O(n)$ time from the adjacency list representation of G. Always true. The given graph is actually a tree

with one extra edge. So, using BFS we can find the cycle in the graph and just eliminate the maximum edge weight in it. Time complexity will be $O(n)$.

Hence, B is answer

9 votes -- Anu007 (14.4k points)

1.6.2 Minimum Spanning Trees: TIFR2019-B-2 https://gateoverflow.in/280493

✔ There are 64 minimum spanning trees possible.

Two choices for edge of weight 4 for each of the outer triangle, ie, $\binom{2}{1} * \binom{2}{1} * \binom{2}{1} * \binom{2}{1} = 2^4$ and two choices for edge of weight 2 for each of the inner triangle ie, $\binom{2}{1} * \binom{2}{1} = 2^2$.

$2^4 * 2^2 = 64$

8 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

1.7 P Np Npc Nph (9)

1.7.1 P Np Npc Nph: TIFR2010-B-39 https://gateoverflow.in/18754

Suppose a language L is NP complete. Then which of the following is FALSE?

A. $L \in \mathrm{NP}$
B. Every problem in P is polynomial time reducible to L.
C. Every problem in NP is polynomial time reducible to L.
D. The Hamilton cycle problem is polynomial time reducible to L.
E. $\mathrm{P} \neq \mathrm{NP}$ and $L \in \mathrm{P}$.

tifr2010 algorithms p-np-npc-nph

1.7.2 P Np Npc Nph: TIFR2011-B-37 https://gateoverflow.in/20922

Given an integer $n \geq 3$, consider the problem of determining if there exist integers $a, b \geq 2$ such that $n = a^b$. Call this the forward problem. The reverse problem is: given a and b, compute a^b (mod b). Note that the input length for the forward problem is $\lfloor \log n \rfloor + 1$, while the input length for the reverse problem is $\lfloor \log a \rfloor + \lfloor \log b \rfloor + 2$. Which of the following statements is TRUE?

A. Both the forward and reverse problems can be solved in time polynomial in the lengths of their respective inputs.
B. The forward problem can be solved in polynomial time, however the reverse problem is NP-hard.
C. The reverse problem can be solved in polynomial time, however the forward problem is NP-hard.
D. Both the forward and reverse problem are NP-hard.
E. None of the above.

tifr2011 algorithms p-np-npc-nph

1.7.3 P Np Npc Nph: TIFR2012-B-20 https://gateoverflow.in/26480

This question concerns the classes P and NP. If you are familiar with them, you may skip the definitions and go directly to the question.
Let L be a set. We say that L is in P if there is some algorithm which given input x decides if x is in L or not in time bounded by a polynomial in the length of x. For example, the set of all connected graphs is in P, because there is an algorithm which, given a graph graph, can decide if it is connected or not in time roughly proportional to the number of edges of the graph.

The class NP is a superset of class P. It contains those sets that have membership witnesses that can be verified in polynomial time. For example, the set of composite numbers is in NP. To see this take the witness for a composite number to be one of its divisors. Then the verification process consists of performing just one division using two reasonable size numbers. Similarly, the set of those graphs that have a Hamilton cycle, i.e. a cycle containing all the vertices of the graph, is in in NP. To verify that the graph has a Hamilton cycle we just check if the witnessing sequence of vertices indeed a cycle of the graph that passes through all the vertices of the graph. This can be done in time that is polynomial in the size of the graph.

More precisely, if L is a set in P consisting of elements of the form (x, w), then the set

$M = \{x : \exists w,$ |

is in N P .
Let $G = (V, E)$ be a graph. G is said to have perfect matching if there is a subset M of the edges of G so that

i. No two edges in M intersect (have a vertex in common); and
ii. Every vertex of G has an edge in M.

Let MATCH be the set of all graphs that have a perfect matching. Let $\overline{\mathrm{MATCH}}$ be the set of graphs that do not have a perfect matching. Let $o(G)$ be the number of components of G that have an odd number of vertices.

Tutte's Theorem: $G \in \mathrm{MATCH}$ if and only if for all subsets S of V, the number of components in $G - S$ (the graph formed by deleting the vertices in S) with an odd number of vertices is at most $|S|$. That is,

$G \in \mathrm{MATCH} \leftrightarrow \forall S \subseteq Vo(G - S) \leq$ |

Which of the following is true?

A. $\mathrm{MATCH} \in NP$ and $\overline{\mathrm{MATCH}} \notin NP$
B. $\overline{\mathrm{MATCH}} \in NP$ and $\mathrm{MATCH} \notin NP$
C. $\mathrm{MATCH} \in NP$ and $\overline{\mathrm{MATCH}} \in NP$
D. $\mathrm{MATCH} \notin P$ and $\overline{\mathrm{MATCH}} \notin P$
E. none of the above

tifr2012 algorithms p-np-npc-nph

1.7.4 P Np Npc Nph: TIFR2013-B-7

https://gateoverflow.in/25668

Which of the following is not implied by $P = NP$?

A. 3SAT can be solved in polynomial time.
B. Halting problem can be solved in polynomial time.
C. Factoring can be solved in polynomial time.
D. Graph isomorphism can be solved in polynomial time.
E. Travelling salesman problem can be solved in polynomial time.

tifr2013 algorithms p-np-npc-nph

1.7.5 P Np Npc Nph: TIFR2015-B-13

https://gateoverflow.in/30076

Two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if there exist a bijection $\pi : V_1 \to V_2$ such that for all $u, v \in V_1, (u, v) \in E_1$ if and only $(\pi(u), \pi(v)) \in E_2$. Consider the following language.

L={(G, H)| G and H are undirected graphs such that a subgraph of G is isomorphic to H}

Then which of the following are true?

(i) $L \in NP$.

(ii) L is NP- hard.

(iii) L is undecidable.

A. Only (i) B. Only (ii) C. Only (iii) D. (i) and (ii) E. (ii) and (iii)

tifr2015 p-np-npc-nph non-gate

1.7.6 P Np Npc Nph: TIFR2017-B-15

https://gateoverflow.in/95828

A multivariate polynomial in n variables with integer coefficients has a binary root if it is possible to assign each variable either 0 or 1, so that the polynomial evaluates to 0. For example, the multivariate polynomial $-2x_1^3 - x_1x_2 + 2$ has the binary root $(x_1 = 1, x_2 = 0)$. Then determining whether a multivariate polynomial, given as the sum of monimials, has a binary root:

A. is trivial: every polynomial has a binary root
B. can be done in polynomial time
C. is NP-hard, but not in NP
D. is in NP, but not in P and not NP-hard
E. is both in NP and NP-hard

tifr2017 algorithms p-np-npc-nph

1.7.7 P Np Npc Nph: TIFR2017-B-2

https://gateoverflow.in/95673

Consider the following statements:

i. Checking if a given $undirected$ graph has a cycle is in P
ii. Checking if a given $undirected$ graph has a cycle is in NP
iii. Checking if a given $directed$ graph has a cycle is in P
iv. Checking if a given $directed$ graph has a cycle is in NP

Which of the above statements is/are TRUE? Choose from the following options.

A. Only i and ii B. Only ii and iv C. Only ii, iii, and iv D. Only i, ii and iv E. All of them

tifr2017 algorithms p-np-npc-nph

1.7.8 P Np Npc Nph: TIFR2018-B-15

https://gateoverflow.in/179299

G respresents an undirected graph and a cycle refers to a simple cycle (no repeated edges or vertices).
Define the following two languages.

$SCYCLE = \{(G,k) \mid G \text{ contains a cycle of length at most k}\}$
and
$LCYCLE = \{(G,k) \mid G \text{ contains a cycle of length at least k}\}$

Which of the following is NOT known to be TRUE (to the best of our current knowledge) ?

A. $SCYCLE \in P$
B. $LCYCLE \in NP$.
C. $LCYCLE \leq_p SCYCLE$ (i.e, there is a polynomial time many-to -one reduction from $LCYCLE$ to $SCYCLE$).
D. $LCYCLE$ is NP-complete.
E. $SCYCLE \leq_p LCYCLE$ (i.e, there is a polynomial time many-to-one reduction from $SCYCLE$ to $LCYCLE$).

tifr2018 algorithms p-np-npc-nph non-gate

1.7.9 P Np Npc Nph: TIFR2019-B-7

https://gateoverflow.in/280488

A formula is said to be a 3-CF-formula if it is a conjunction (i.e., an AND) of clauses, and each clause has at most 3 literals. Analogously, a formula is said to be a 3-DF-formula if it is a disjunction (i.e., an OR) of clauses of at most 3 literals each.

Define the languages 3-CF-SAT and 3-DF-SAT as follows:

$$3\text{-CF-SAT} = \{\Phi \mid \Phi \text{ is a satisfiable } 3\text{-CF formula}\}$$

$$3\text{-DF-SAT} = \{\Phi \mid \Phi \text{ is a satisfiable } 3\text{-DF formula}\}$$

Which of the following best represents our current knowledge of these languages ?

A. Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-CF-SAT is NP-complete
B. Both 3-CF-SAT and 3-DF-SAT are in NP-complete
C. Both 3-CF-SAT and 3-DF-SAT are in P
D. Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-DF-SAT is NP-complete
E. Neither 3-CF-SAT nor 3-DF-SAT are in P

tifr2019 algorithms p-np-npc-nph

Answers: P Np Npc Nph

1.7.1 P Np Npc Nph: TIFR2010-B-39

https://gateoverflow.in/18754

✔ **Option E leads to a contradiction, hence is false.**

We know that L is NPC , hence $\in \mathrm{NP}$. If $\mathrm{P} \neq \mathrm{NP}$, then L can't be in P

👍 8 votes

-- Pragy Agarwal (18.3k points)

1.7.2 P Np Npc Nph: TIFR2011-B-37

https://gateoverflow.in/20922

✔ The reverse problem can be solved in polynomial time as a^b requires at most $\log b$ recursive calls using the approach given below:

```
pow(int a, int b)
{
    if(b%2)
        return a* pow(a*a, b/2);
    else
        return pow(a*a, b/2);
}
```

Now, the forward problem is also solvable in polynomial time. We need to check for all the roots of n (from $\sqrt{n}$ till $n^{\frac{1}{\log n}}$) whether it is an integer . But each of these check can be done in $\log n$ time using a binary search on the set of integers from $2..n$ and so, the overall complexity will be $(\log n)^2$ which is polynomial in $\log n$ ($\log n$ is the size of input). So, (**a**) must be the answer.

9 votes

-- gatecse (63.8k points)

1.7.3 P Np Npc Nph: TIFR2012-B-20

https://gateoverflow.in/26480

✔ First of all let us see some things asked in question:

1. We say $\text{MATCH} \in P$ if given a graph G we can say in polynomial time using a **deterministic** Turing machine if G has a perfect matching.
2. We say $\text{MATCH} \in NP$ if given a graph G we can say in polynomial time using a **non-deterministic** Turing machine if G has a perfect matching.
    1. Equivalently given a subset M of edges of G we can say in polynomial time using a deterministic Turing machine, if it is a perfect matching for G.
    2. Similarly, we say $\overline{\text{MATCH}} \in NP$ if given a subset M of edges of G we can say in polynomial time using a deterministic Turing machine, if it is NOT a perfect matching for G.

So, we can see if point 2.1 given above is possible. To do this for the given subset of edges M we have to see

1. No two edges in M intersect (have a vertex in common);
   Can be done just by traversing the edges and seeing for common vertex. Naive implementation can be done in $O(V|M|)$ time by marking the two vertices of each edge and see if any vertex gets marked twice which is polynomial.
2. Every vertex of G has an edge in M.
   We can do the same procedure done for above and finally see if any vertex is left unmarked. So, polynomial time possible.

If the answer to steps 1 and 2 are "Yes", then M is a perfect matching for G and we proved that $\text{MATCH} \in NP$.

If the answer to any one of steps 1 or 2 above is "No", then M is not a perfect matching for G and we proved that $\overline{\text{MATCH}} \in NP$.

Thus, both the problems are in NP.

Option C is correct.

Further Read: http://mathworld.wolfram.com/PerfectMatching.html

References

2 votes

-- Arjun Suresh (334k points)

1.7.4 P Np Npc Nph: TIFR2013-B-7

https://gateoverflow.in/25668

✔ I believe Umang is right, option **B** is the correct answer.

Intractability : We are looking for EFFICIENT algorithms.

Intractable Problems are problems that are decidable, although the algorithm to decide that problem might be efficient (P) or inefficient (NP), but at least an algorithm exists to solve these problems.

Here we talk about efficient vs inefficient computations.

Thus the language of problems in P and NP classes is the language of Decidable Problems i.e. Recursive Language.

Undecidability: We are looking for algorithms.

Undecidable Problems are problems for which there is no algorithm to solve these problems.

Here we talk about what can or can not be computed.

The language of Undecidable Problems are "Recursively Enumerable but not recursive languages" & "Not Recursively Enumerable Languages".

Clearly we can talk about the intractability of any problem if we know at least one algorithm t o solve the problem, if there is no algorithm to solve a problem how can we talk about efficiency?

Halting Problem is undecidable.

I guess, all other problems mentioned here are decidable.

I don't know the most efficient algorithms to solve these problems but at least I can say that Brute force approach will work on all the other options except the Halting Problem.

What P = NP implies?

"Any problem that is solved by a non deterministic Turing machine in polynomial time also be solved by some deterministic Turing machine in polynomial time, even if the degree of the polynomial is higher."

and

There is neither a Non Deterministic Turing Machine nor Deterministic Turing Machine that can solve the Halting Problem.

So **any inference about P & NP is not going to affect the solvability of Halting Problem, since it is undecidable.**

11 votes -- Anurag Pandey (10.5k points)

1.7.5 P Np Npc Nph: TIFR2015-B-13 https://gateoverflow.in/30076

graph isomporphism is np complete and decidable so ans is d

1 votes -- Pooja Palod (24.1k points)

1.7.6 P Np Npc Nph: TIFR2017-B-15 https://gateoverflow.in/95828

This problem is similar to **Boolean Satisfiability Problem (SAT)** problem which is First NP-Complete problem.

A problem is NP-Complete it means it is in NP as well as NP-Hard, i.e. intersection of both NP and NP-Hard is NP-Complete.

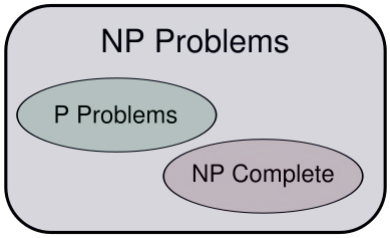
Hence, **option (E) is right**.

3 votes -- SHIVESH KUMAR ROY (785 points)

1.7.7 P Np Npc Nph: TIFR2017-B-2 https://gateoverflow.in/95673

✔ E. All of them. Because all of them can be solved by Depth first traversal.

Every P problem is a subset of NP.



11 votes -- Motamarri Anusha (8.6k points)

1.7.8 P Np Npc Nph: TIFR2018-B-15

https://gateoverflow.in/179299

L cycle problem can be reducible to finding Hamiltonian cycle which is np complete

Where as S Cycle is an class P problem

So 3 in directly ask us P=NP which is not known

1 votes -- Keval Malde (13.3k points)

1.7.9 P Np Npc Nph: TIFR2019-B-7

https://gateoverflow.in/280488

3 SAT where all the clauses are in conjunctive normal form is the well known NP-complete problem.

3 SAT with clauses in Disjunctive normal form lies in P class. Now one doubt you might get while solving this question is, if 3 SAT with CNF is NP-complete and 3 SAT with DNF belongs to P, then why not to convert CNF formula to equivalent DNF formula?? If we can do this conversion, then we can solve 3 SAT CNF too in polynomial time using the algorithm for checking 3 SAT DNF.

But please note that this conversion process is not polynomial. When you convert a CNF formula to equivalent DNF formula, it can have much much larger terms than that of original CNF formula, so this conversion process is not polynomial, it is exponential. Hence the idea of converting 3CNF to 3DNF won't work.

Option A is correct.

Nice read: https://math.stackexchange.com/questions/159591/solving-sat-by-converting-to-disjunctive-normal-form

References

3 votes -- Utkarsh Joshi (5.3k points)

1.8 Quicksort (1)

1.8.1 Quicksort: TIFR2018-B-7

https://gateoverflow.in/179291

Consider the recursive quicksort algorithm with "random pivoting". That is, in each recursive call, a pivot is chosen uniformly at random from the sub-array being sorted.When this randomized algorithm is applied to an array of size n all whose elements are distinct, what is the probability that the smallest and the largest elements in the array are compared during a run of the algorithm ?

A. $\left(\frac{1}{n}\right)$

B. $\left(\frac{2}{n}\right)$

C. $\Theta\left(\frac{1}{n \log n}\right)$

D. $O\left(\frac{1}{n^2}\right)$

E. $\Theta\left(\frac{1}{n \log^2 n}\right)$

tifr2018 algorithms sorting quicksort

Answers: Quicksort

1.8.1 Quicksort: TIFR2018-B-7

https://gateoverflow.in/179291

✔ Probability that the smallest and the largest elements in the array are compared during a run of the randomized quicksort algorithm.

After the first split, first subpart of array will contain the minimum element and second subpart contains the maximum element. (Why?)

Choose any element say x, greater than or equal to smallest element. After partition, smallest element will be present in the left of it. The largest element will be present on the right sub array.

After the first split, there will be no more comparison between the largest and smallest element.

To have comparison between largest and smallest element we must choose anyone of the as the pivot element in the first split

itself.

Among the N elements we have to choose either the smallest element or the largest element. Selection will be uniform.

Probability $= 2/n.$

28 votes -- Hemant Parihar (11.9k points)

1.9 Recurrence (3)

1.9.1 Recurrence: TIFR2014-B-11

https://gateoverflow.in/27308

Consider the following recurrence relation:

$$T(n) = \begin{cases} T\left(\frac{n}{k}\right) + T\left(\frac{3n}{4}\right) + n & \text{if } n \geq 2 \\ 1 & \text{if } n = 1 \end{cases}$$

Which of the following statements is FALSE?

A. $T(n)$ is $O(n^{3/2})$ when $k = 3$.
B. $T(n)$ is $O(n \log n)$ when $k = 3$.
C. $T(n)$ is $O(n \log n)$ when $k = 4$.
D. $T(n)$ is $O(n \log n)$ when $k = 5$.
E. $T(n)$ is $O(n)$ when $k = 5$.

tifr2014 algorithms recurrence

1.9.2 Recurrence: TIFR2015-B-1

https://gateoverflow.in/29657

Consider the following recurrence relation:

$$T(n) = \begin{cases} 2T(\lfloor \sqrt{n} \rfloor) + \log n & \text{if } n \geq 2 \\ 1 & \text{if } n = 1 \end{cases}$$

Which of the following statements is TRUE?

A. $T(n)$ is $O(\log n)$.
B. $T(n)$ is $O(\log n . \log \log n)$ but not $O(\log n)$.
C. $T(n)$ is $O(\log^{3/2} n)$ but not $O(\log n . \log \log n)$.
D. $T(n)$ is $O(\log^2 n)$ but not $O(\log^{3/2} n)$.
E. $T(n)$ is $O(\log^2 n . \log \log n)$ but not $O(\log^2 n)$.

tifr2015 algorithms recurrence time-complexity

1.9.3 Recurrence: TIFR2017-A-15

https://gateoverflow.in/95664

Let $T(a, b)$ be the function with two arguments (both nonnegative integral powers of 2) defined by the following recurrence:

- $T(a, b) = T\left(\frac{a}{2}, b\right) + T\left(a, \frac{b}{2}\right)$ if $a, b \geq 2$;
- $T(a, 1) = T\left(\frac{a}{2}, 1\right)$ if $a \geq 2$;
- $T(1, b) = T\left(1, \frac{b}{2}\right)$ if $b \geq 2$;
- $T(1, 1) = 1$.

What is $T(2^r, 2^s)$?

A. rs
B. $r + s$
C. $\binom{2^r + 2^s}{2^r}$
D. $\binom{r+s}{r}$
E. 2^{r-s} if $r \geq s$, otherwise 2^{s-r}

tifr2017 algorithms recurrence

Answers: Recurrence

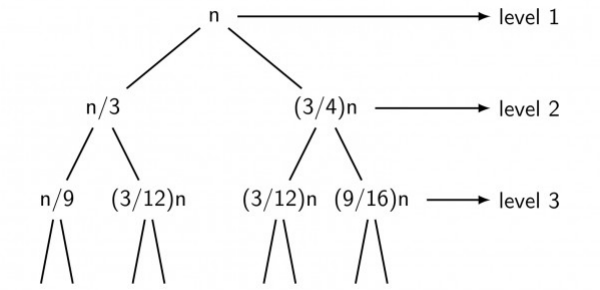
1.9.1 Recurrence: TIFR2014-B-11

https://gateoverflow.in/27308

✔ Considering when $k = 3$

$T(n) = T(\frac{n}{3}) + T(\frac{3}{4}) + n$

Below is the recursion tree



Since, the combine work according to recurrence is linear in term of the size of the sub-problem

so at Level 1, total combine work is n

Combined work at level 2$= \frac{n}{3} + \frac{3n}{4} = \frac{13n}{12}$

At level 3 combine work $= \frac{n}{9} + \frac{3n}{12} + \frac{3n}{12} + \frac{9n}{16} = (\frac{13}{12})^2 n$

So, at level k, when the recursion bottoms out at $T(1)$, the combine work will be $(\frac{13}{12})^{k-1}.n$

The left height of the tree is when the subproblem $\frac{n}{3}$ would bottom out and that is when $\frac{n}{3^k} = 1$

gives $k = \log_3 n$

The right height of the tree is $\log_{\frac{4}{3}} n$

for the purpose of upper bound consider right height and then sum up total combine work from root till the leaf level

$n + \frac{13}{12}n + (\frac{13}{12})^2.n + \ldots + (\frac{13}{12})^{\log_{\frac{4}{3}} n - 1} n$

$= n\left[1 + \frac{13}{12} + \ldots + (\frac{13}{12})^{\log_{\frac{4}{3}} n - 1}\right]$

$= n\left[1.\frac{(\frac{13}{12})^{\log_{\frac{4}{3}} n} - 1}{\frac{13}{12} - 1}\right]$

Using property

$a^{\log_b c} = c^{\log_b a}$

and ignoring denominator as constant term for upper bound

$n.\left[n^{\log_{\frac{4}{3}} \frac{13}{12}} - 1\right] = n^{1.27}$ (approximately)

Option (A) : $O(n^{1.5})$ is surely an upper bound when k=3. Correct option.

Option (B): $O(n \log n)$

this means $n.n^{0.27} \leq c.n.\log n$

$n^{0.27} \leq c.\log n$

This is clearly false as polynomials grow faster than poly-logarithms.

Option(B) can never be an upper bound, yes lower bound possible when $k = 3$.

When $k = 4$.

$T(n) = T(\frac{n}{4}) + T(\frac{3n}{4}) + n$

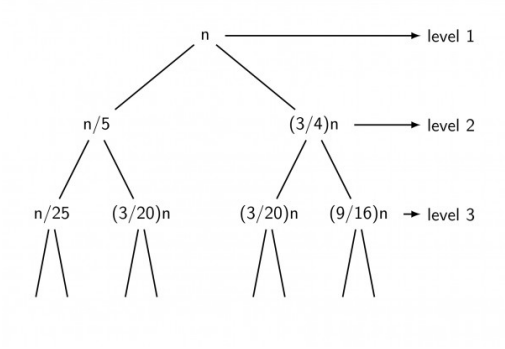
If we draw recursion tree for this, at every level cost comes to be n and there would be maximum $\log_{\frac{4}{3}} n$ levels

So, in this case $T(n) = O(n \log n)$. Option (C) correct.

When $k = 5$,

$T(n) = T(\frac{n}{5}) + T(\frac{3n}{4}) + n$

It's recursion tree would look like below:



Level 1 Total cost $= n$

Level 2 total cost $= \frac{19n}{20}$

Level 3 total cost $= (\frac{19}{20})^2 . n$

Level k cost $= (\frac{19}{20})^{k-1} . n$

Number of levels on right $= \log_{\frac{4}{3}} n$

Number of levels on left $= \log_5 n$

For the purpose of upper bound I'll consider right height.

Computing total cost from root till leaf

$n. \left[1 + \frac{19}{20} + \ldots + (\frac{19}{20})^{\log_{\frac{4}{3}} n - 1}\right]$

$= n. \left[\frac{1-(\frac{19}{20})^{\log_{\frac{4}{3}} n}}{1-\frac{19}{20}}\right]$

Ignoring denominator as we are only interested in some function of n.

$= n. \left[1 - (\frac{19}{20})^{\log_{\frac{4}{3}} n}\right]$

$= n \left[1 - n^{\log_{\frac{4}{3}} \frac{19}{20}}\right]$

$= n[1 - n^{-0.17}]$

$= n - n^{0.82}$

So, option(E) is correct.

Option(D) which says $O(n \log n)$ might not look convincing but

$n - n^{0.82} \leq c. n \log n$

$1 - \frac{1}{n^{0.17}} \leq c \log n$

for $c = 1, n_o = 2$ this satisfies big-oh.

Option (D) is also correct.

Answer: option(B)

47 votes

-- Ayush Upadhyaya (28.5k points)

1.9.2 Recurrence: TIFR2015-B-1

https://gateoverflow.in/29657

✔ Let $n = 2^k$

$T\left(2^k\right) = 2T\left(2^{k/2}\right) + k$

Let $T\left(2^k\right) = S(k) \implies T\left(2^{k/2}\right) = S(k/2)$

$S(k) = 2S(k/2) + k$

This gives $S(k) = \Theta(k \log k)$ (Master theorem case 2)

$= \Theta(\log n \log \log n)$

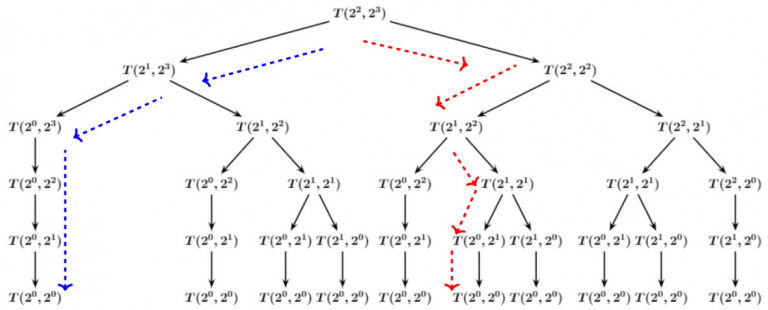
So ans is **b**

22 votes -- Pooja Palod (24.1k points)

1.9.3 Recurrence: TIFR2017-A-15

https://gateoverflow.in/95664

✔ Value of $T(2^r, 2^s)$ is nothing but the number of leaf nodes in its recursion tree. The reason for this is that there is only one base condition $T(1,1)$ which returns 1 which are all ultimately added. Here is an example recursion tree for $T(2^2, 2^3)$.



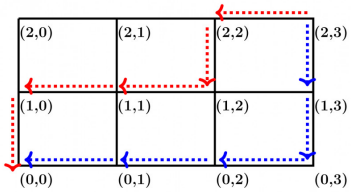
We will try to use some combinatorial technique to count the number of leaf nodes. One way of counting the number of leaf nodes in the recurrence tree is by counting all possible paths that one can take from the root to the leaf node. The reason for doing this would be clear in a moment. This idea would be used to establish a link with a very common combinatorial problem.

Claim: If we consider a grid of size $r \times s$ then the possible paths from (r, s) to $(0, 0)$ in that grid such that we can either move backwards or downward is nothing but all possible paths in recurrence tree from the root node to leaf node.

This happens to be true because arguments of recurrence relation are in powers of two, and at each level of recurrence tree, the power of one of the arguments get reduced by one.

$$\begin{aligned} T(2^r, 2^s) &= T(2^{r-1}, 2^s) + T(2^r, 2^{s-1}) \\ T(1, 2^s) &= T(1, 2^{s-1}) \\ T(2^r, 1) &= T(2^{r-1}, 1) \\ T(1,1) &= 1 \end{aligned}$$

Continuing from our previous example here is a grid of size 2×3 with two valid paths highlighted.



The number of valid paths possible in this grid will give us the count of the number of leaf nodes in the recurrence tree. Interestingly this problem, when interpreted exactly in opposite way, happens to be a very well known problem of combinatory.

In a grid of size $r \times s$, count the number of possible paths from $(0, 0)$ to (r, s) such that we can either move forward(F) or upward(U).

Video:

So there are $\binom{r+s}{r}$ many different valid paths on this grid to reach (r, s) from $(0, 0)$ or vice versa. And in turn, there are same numbers of leaf nodes in the recursion tree. Therefore, we can conclude with -

$$T(2^r, 2^s) = \binom{r+s}{r}$$

Video:

References

18 votes -- Prateek Dwivedi (3.5k points)

1.10 Searching (2)

1.10.1 Searching: TIFR2010-B-29

https://gateoverflow.in/18752

Suppose you are given an array A with $2n$ numbers.

The numbers in odd positions are sorted in ascending order, that is, $A[1] \leq A[3] \leq \ldots \leq A[2n-1]$.

The numbers in even positions are sorted in descending order, that is, $A[2] \geq A[4] \geq \ldots \geq A[2n]$.

What is the method you would recommend for determining if a given number is in the array?

A. Sort the array using quick-sort and then use binary search.
B. Merge the sorted lists and perform binary search.
C. Perform a single binary search on the entire array.
D. Perform separate binary searches on the odd positions and the even positions.
E. Search sequentially from the end of the array.

tifr2010 searching

1.10.2 Searching: TIFR2012-B-11

https://gateoverflow.in/25140

Consider the following three version of the binary search program. Assume that the elements of type T can be compared with each other; also assume that the array is sorted.

```
i, j, k : integer;
a : array [1....N] of T;
x : T;

Program 1 :   i := 1; j := N;
              repeat
                    k := (i + j) div 2;
                    if a[k] < x then i := k else j := k
              until (a[k] = x) or (i > j)
Program 2 :   i := 1; j := N;
              repeat
                    k := (i + j) div 2;
                   if x < a[k] then j := k - 1;
                   if a[k] < x then i := k + 1;
              until i > j
Program 3 :=  i := 1; j := N
              repeat
                    k := (i + j) div 2;
                   if x < a[k] then j := k else i := k + 1
              until i > j
```

A binary search program is called correct provided it terminates with $a[k] = x$ whenever such an element exists, or it terminates with $a[k] \neq x$ if there exists no array element with value x. Which of the following statements is correct?

A. Only Program 1 is correct
B. Only Program 2 is correct
C. Only Program 1 and 2 are correct.
D. Both Program 2 and 3 are correct
E. All the three programs are wrong

tifr2012 algorithms searching

Answers: Searching

1.10.1 Searching: TIFR2010-B-29

https://gateoverflow.in/18752

✔ **Option D is the correct answer.**

We can simply use clever indexing to binary search the element in the odd positions, and in the even positions separately.

This will take $O(\log n)$ time and $O(1)$ space in the worst case.

A: Sorting using Quicksort will take $O(n^2)$ time.
B: Merging will take $O(n)$ time and $O(n)$ space.
C: Binary search only works on a sorted array.

E: Sequential search will take $O(n)$ time.

37 votes -- Pragy Agarwal (18.3k points)

1.10.2 Searching: TIFR2012-B-11 https://gateoverflow.in/25140

✔ First program wont work if array has elements same..it may go into infinite loop .To make it work it properly we have to do following changes $j = k - 1$ and $i = k + 1$

For second program $a[k] == x$ condition is missing so it is wrong

Third program is also wrong as $j! = k - 1$ and condition $a[k] == x$ is missing

So, answer is **E.**

20 votes -- Pooja Palod (24.1k points)

1.11 Shortest Path (1)

1.11.1 Shortest Path: TIFR2018-B-9 https://gateoverflow.in/179293

Let $G = (V, E)$ be a DIRECTED graph, where each edge e has a positive weight $\omega(e)$, and all vertices can be reached from vertex s. For each vertex v, let $\phi(v)$ be the length of the shortest path from s to v. Let $G' = (V, E)$ be a new weighted graph with the same vertices and edges, but with the edge weight of every edge $e = (u \to v)$ changed to $\omega'(e) = \omega(e) + \phi(v) - \phi(u)$. Let P be a path from s to a vertex v, and let $\omega(P) = \sum_{e \in P} \omega_e$, and $\omega'(P) = \sum_{e \in P} \omega'_e$.

Which of the following options is NOT NECESSARILY TRUE ?

A. If P is a shortest path in G, then P is a shortest path in G'.
B. If P is a shortest path in F', then P is a shortest path in G.
C. If P is a shortest path in G, then $\omega'(P) = 2 \times \omega(P)$.
D. If P is NOT a shortest path in G, then $\omega'(P) < 2 \times \omega(P)$.
E. All of the above options are necessarily TRUE.

tifr2018 graph-algorithms shortest-path

Answers: Shortest Path

1.11.1 Shortest Path: TIFR2018-B-9 https://gateoverflow.in/179293

✔ Answer should be E, i.e., all statements A, B, C, D are correct.

Explanation:

$\omega(P) = \sum_e \omega_e$

$\omega'(P) = \sum_e \omega'_e$

$= \sum_e (\omega(e) + \phi(v) - \phi(u))$

$= \sum \omega(e) + \sum (\phi(v) - \phi(u))$

Given that P is a path from s to v. Let P be comprised of $S \to V_1 \to V_2 \to V_3 \cdots \to V_n \to V$

$\omega'(P) = \sum_e \omega(e) + \phi(v) - \cancel{\phi(v_n)}$

$+ \cancel{\phi(v_n)} - \cancel{\phi(v_{n-1})}$

$+$

$\vdots$

$+ \cancel{\phi(v_1)} - \phi(v_s)$

$\omega'(P) = \sum \omega(e) + \phi(v) - \phi(s)$

Since, s is the source (in the context of the shortest path) $\phi(s) = 0$.

$\therefore \omega'(P) = \sum \omega(e) + \phi(v) = \omega(P) + \phi(v)$

If P is the shortest path in G, $\omega'(P) = \sum \omega(e) + \phi(v)$

$= \phi(v) + \phi(v)$
$= 2.\phi(v)$
$= 2.\sum_e \omega_e$
$= 2.\omega(P)$

(C is correct)

$\omega'(P) = \omega(P) + \phi(v)$
$\leq \omega(P) + \omega(P)$
$= 2 * \omega(P)$
or $\omega'(P) \leq 2\omega(P)$

(D is correct)

If P is the shortest path in G, (Say from s to v)

- $\omega(P) = \phi(v)$
- $\omega'(P) = \omega(P) + \phi(v)$

Say P' is the shortest path in G' (not equal to P)
$\omega'(P') \leq \omega'(P)$
$\Rightarrow \omega(P') + \phi(v) \leq \omega(P) + \phi(v)$
$\Rightarrow \omega(P') \leq \omega(P) \Rightarrow$ Contradiction as we assumed P to be shortest path in G
$\therefore P$ is the shortest path in G' as well.

(A is correct)

Similarly say P is the shortest path in G' (s to v)
$\omega'(P) = \omega(P) + \phi(v)$
We claim P is the shortest path in G as well. By contradiction let us assume P' is the shortest path in G.
$\omega(P') \leq \omega(P)$
$\omega(P') + \phi(v) \leq \omega(P) + \phi(v)$
$\omega'(P') \leq \omega'(P) \Rightarrow$ Contradiction
Here, what we assumed was wrong and P is the shortest path in G

(B is correct)

$\therefore$ E is the answer

12 votes -- Yash Khanna (397 points)

1.12 Sorting (9)

1.12.1 Sorting: TIFR2010-B-23

https://gateoverflow.in/18623

Suppose you are given n numbers and you sort them in descending order as follows:

First find the maximum. Remove this element from the list and find the maximum of the remaining elements, remove this element, and so on, until all elements are exhausted. How many comparisons does this method require in the worst case?

A. Linear in n.
B. $O\left(n^2\right)$ but not better.
C. $O\left(n \log n\right)$
D. Same as heap sort.
E. $O\left(n^{1.5}\right)$ but not better.

tifr2010 algorithms time-complexity sorting

1.12.2 Sorting: TIFR2010-B-27

https://gateoverflow.in/19036

Consider the Insertion Sort procedure given below, which sorts an array L of size $n\left(\geq 2\right)$ in ascending order:

```
begin
    for xindex:= 2 to n do
        x := L [xindex];
        j:= xindex - 1;
        while j > 0 and L[j] > x do
            L[j + 1]:= L[j];
            j:= j - 1;
        end {while}
        L [j + 1]:=X;
```

```
    end{for}
end
```

It is known that insertion sort makes at most $n(n-1)/2$ comparisons. Which of the following is true?

A. There is no input on which insertion Sort makes $n(n-1)/2$ comparisons.
B. Insertion Sort makes $n(n-1)/2$ comparisons when the input is already sorted in ascending order.
C. Insertion Sort makes $n(n-1)/2$ comparisons only when the input is sorted in descending order.
D. There are more than one input orderings where insertion sort makes $n(n-1)/2$ comparisons.
E. Insertion Sort makes $n(n-1)/2$ comparisons whenever all the elements of L are not distinct.

tifr2010 algorithms sorting

1.12.3 Sorting: TIFR2011-B-21

https://gateoverflow.in/20324

Let $S = \{x_1, \ldots, x_n\}$ be a set of n numbers. Consider the problem of storing the elements of S in an array $A[1...n]$ such that the following min-heap property is maintained for all $2 \leq i \leq n : A[\lfloor i/2 \rfloor] \leq A[i]$. (Note that $\lfloor x \rfloor$ is the largest integer that is at most x). Which of the following statements is TRUE?

A. This problem can be solved in $O(\log n)$ time.
B. This problem can be solved in $O(n)$ time but not in $O(\log n)$ time.
C. This problem can be solved in $O(n \log n)$ time but not in $O(n)$ time.
D. This problem can be solved in $O\left(n^2\right)$ time but not in $O(n \log n)$ time.
E. None of the above.

tifr2011 algorithms sorting

1.12.4 Sorting: TIFR2011-B-31

https://gateoverflow.in/20617

Given a set of $n = 2^k$ distinct numbers, we would like to determine the smallest and the second smallest using comparisons. Which of the following statements is TRUE?

A. Both these elements can be determined using $2k$ comparisons.
B. Both these elements can be determined using $n-2$ comparisons.
C. Both these elements can be determined using $n+k-2$ comparisons.
D. $2n-3$ comparisons are necessary to determine these two elements.
E. nk comparisons are necessary to determine these two elements.

tifr2011 algorithms sorting

1.12.5 Sorting: TIFR2011-B-39

https://gateoverflow.in/20935

The first n cells of an array L contain positive integers sorted in decreasing order, and the remaining $m-n$ cells all contain 0. Then, given an integer x, in how many comparisons can one find the position of x in L?

A. At least n comparisons are necessary in the worst case.
B. At least $\log m$ comparisons are necessary in the worst case.
C. $O(\log(m-n))$ comparisons suffice.
D. $O(\log n)$ comparisons suffice.
E. $O(\log(m/n))$ comparisons suffice.

tifr2011 algorithms sorting

1.12.6 Sorting: TIFR2012-B-13

https://gateoverflow.in/25207

An array A contains n integers. We wish to sort A in ascending order. We are told that initially no element of A is more than a distance k away from its final position in the sorted list. Assume that n and k are large and k is much smaller than n. Which of the following is true for the worst case complexity of sorting A?

A. A can be sorted with constant $.kn$ comparison but not with fewer comparisons.
B. A cannot be sorted with less than constant $.n \log n$ comparisons.
C. A can be sorted with constant $.n$ comparisons.
D. A can be sorted with constant $.n \log k$ comparisons but not with fewer comparisons.
E. A can be sorted with constant $.k^2 n$ comparisons but not fewer.

tifr2012 algorithms sorting

1.12.7 Sorting: TIFR2012-B-14

https://gateoverflow.in/25209

Consider the quick sort algorithm on a set of n numbers, where in every recursive subroutine of the algorithm, the algorithm chooses the median of that set as the pivot. Then which of the following statements is TRUE?

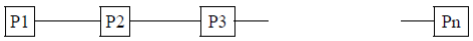
A. The running time of the algorithm is $\Theta(n)$.
B. The running time of the algorithm is $\Theta(n \log n)$.
C. The running time of the algorithm is $\Theta(n^{1.5})$.
D. The running time of the algorithm is $\Theta(n^2)$.
E. None of the above.

tifr2012 algorithms sorting

1.12.8 Sorting: TIFR2013-B-20

https://gateoverflow.in/25878

Suppose n processors are connected in a linear array as shown below. Each processor has a number. The processors need to exchange numbers so that the numbers eventually appear in ascending order (the processor $\mathrm{P1}$ should have the minimum value and the the processor Pn should have the maximum value).



The algorithm to be employed is the following. Odd numbered processors and even numbered processors are activated alternate steps; assume that in the first step all the even numbered processors are activated. When a processor is activated, the number it holds is compared with the number held by its right-hand neighbour (if one exists) and the smaller of the two numbers is retained by the activated processor and the bigger stored in its right hand neighbour.
How long does it take for the processors to sort the values?

A. $n \log n$ steps
B. n^2 steps
C. n steps
D. $n^{1.5}$ steps
E. The algorithm is not guaranteed to sort

tifr2013 algorithms sorting

1.12.9 Sorting: TIFR2017-B-7

https://gateoverflow.in/95699

An array of n distinct elements is said to be un-sorted if for every index i such that $2 \leq i \leq n-1$, either $A[i] > max\{A[i-1], A[i+1]\}$, or $A[i] < min\{A[i-1], A[i+1]\}$. What is the time-complexity of the fastest algorithm that takes as input a sorted array A with n distinct elements, and un-sorts A?

A. $O(n \log n)$ but not $O(n)$
B. $O(n)$ but not $O(\sqrt{n})$
C. $O(\sqrt{n})$ but not $O(\log n)$
D. $O(\log n)$ but not $O(1)$
E. $O(1)$

tifr2017 algorithms sorting

Answers: Sorting

1.12.1 Sorting: TIFR2010-B-23

https://gateoverflow.in/18623

✔ The given procedure resembles something like Bubble sort or Selection Sort.
Every time, for every input it will take $O(n^2)$ so, **B** is the answer.

If you are thinking about heap sort, where in the procedure, it is talked about building a heap? You can do extract_max only on a heap. Right ? The procedure is nearly same as bubble sort or selection sort.

Read qsn again: *"First find the maximum. Remove this element from the list and find the maximum of the remaining elements, remove this element, and so on, until all elements are exhausted."*

Just follow the steps. Find max, take it to right side of array. Shift.
So $n-1$ comparisons & $n-1$ shifts in the worst case.
Total $n-1+n-2+n-3+\ldots+1+n-1+n-2+n-3+..+1$ which is $O(n^2)$
Only number of comparisons is asked... which is also $\mathrm{O}(\mathrm{n}^2)$

18 votes -- Ahwan Mishra (10.2k points)

1.12.2 Sorting: TIFR2010-B-27

https://gateoverflow.in/19036

✔ Option D: In the worst case, the number of comparisons for given Insertion Sort are $\frac{n(n-1)}{2}$.

Consider the set of numbers $1, 2, 3, 4, 5$. Some examples for which the number of comparisons are maximum are

I. Descending order: $5, 4, 3, 2, 1$
II. $3, 5, 4, 2, 1$
III. $3, 5, 4, 1, 2$

There is more than 1 sequence for which the number of comparisons is maximum.

$\therefore$ Correct answer: D

Let us see why other options are false.

Option A: Since option D is true, option A is false.

Option B: Given Insertion Sort makes $(n-1)$ comparisons when the input is already sorted in ascending order.

Hence option B is false.

Option C: Insertion Sort makes $\frac{n(n-1)}{2}$ comparisons **only** when the input is sorted in descending order.

Since the word **only** is used, this option is false.

Option E: Insertion Sort makes $\frac{n(n-1)}{2}$ comparisons whenever all the elements of L are **not distinct**.

Not distinct means either all the elements are same or few elements are same and few are distinct.

When elements are same, number of comparisons are $n-1$.

When few elements are same and few are distinct, number of comparisons are always less than $\frac{n(n-1)}{2}$ because while comparing same elements we don't enter while loop.

Hence option E is false.

Note: There can be more than 1 sequence for maximum number of comparisons in insertion sort but maximum number of movements or swaps occur **only** when input is in Descending Order for this question.

7 votes -- Shiva Sagar Rao (4.1k points)

1.12.3 Sorting: TIFR2011-B-21

https://gateoverflow.in/20324

✔ Store the elements in an array and then call build_heap(A). the build_heap takes $O(n)$ time.

So, option 'b' is correct.

But, if we try building heap by inserting each element one by one, the total complexity will be then $O(n \log n)$. Because insertion takes $O(\log n)$ and inserting 'n' elements will take $O(n \log n)$.

26 votes -- Sujit Kumar Muduli (283 points)

1.12.4 Sorting: TIFR2011-B-31

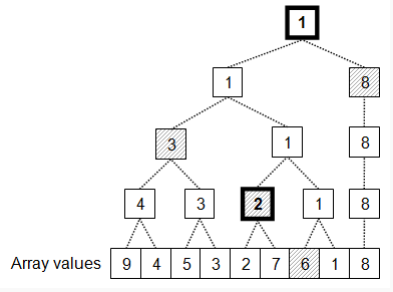
https://gateoverflow.in/20617

✔ Option **(c)** $n + k - 2$

Here is a nice explanation of the algorithm: http://www.codinghelmet.com/?path=exercises/two-smallest

> it is solution to the problem known for ages, and it has to do with tennis tournaments. The question was, knowing the outcome of the tennis tournament, how can we tell which player was the second best? The defeated finalist is a good candidate, but there are other players that were defeated directly by the tournament winner and any of them could also be a good candidate for the second best. So the solution to the problem is quite simple: Once the tournament finishes, pick up the
> $\log N$ competitors that were beaten by the tournament winner and hold a mini-tournament to find which one is the best among them. If we imagine that better players correspond with smaller numbers, the algorithm now goes like this. Hold the tournament to find the smallest number (requires
> $N - 1$ comparisons). During this step, for each number construct the list of numbers it was smaller than. Finally, pick the list of numbers associated with the smallest number and find their minimum in
> $\log N - 1$ steps. This algorithm requires
> $N + \log N - 2$ comparisons to complete, but unfortunately it requires additional space proportional to N (each element except the winner will ultimately be added to someone's list); it also requires more time per step because of the

relatively complex enlisting logic involved in each comparison. When this optimized algorithm is applied to example array, we get the following figure.



Tournament held among numbers promotes value
1 as the smallest number. That operation, performed on an array with nine numbers, requires exactly eight comparisons. While promoting the smallest number, this operation has also flagged four numbers that were removed from competition by direct comparison with the future winner:
$6, 2, 3$ and
8 in that order. Another sequence of three comparisons is required to promote number
2 as the second-smallest number in the array. This totals
11 comparisons, while naive algorithm requires
17 comparisons to come up with the same result. All in all, this algorithm that minimizes number of comparisons looks to be good only for real tournaments, while number cracking algorithms should keep with the simple logic explained above. Implementation of simple algorithm may look like this:

```
a - array containing n elements
min1 = a[0] - candidate for the smallest value
min2 = a[1] - candidate for the second smallest value
if min2 < min1
    min1 = a[1]
    min2 = a[0]
for i = 2 to n - 1
    if a[i] < min1
        min2 = min1
        min1 = a[i]
    else if a[i] < min2
        min2 = a[i]
```

by Zoran Horvat @zoranh75

References

47 votes -- Pragy Agarwal (18.3k points)

1.12.5 Sorting: TIFR2011-B-39

https://gateoverflow.in/20935

✔ **(d) $O(\log n)$ comparisons suffice.**

Since it is possible that $m \ggg n$, we need to restrict ourselves to the first $O(n)$ elements to perform the binary search.

We start with the first element (index $i = 1$), and check if it is equal to 0. If not, we double the value of i, and check again. We repeat this process until we hit a 0.

```
i = 1;
while(arr[i] != 0)
    i *= 2;
```

Once we hit a 0, the largest possible value (worst case) of i can be $2n - 2$. This will happen if $n = 2^k + 1$ for some k. Then, our 2nd last value of i will be 2^k, and then we get 2^{k+1}, which is equal to $2n - 2$.

Now that we've hit a 0, and the array contains positive numbers in decreasing order, if x is present in L, it must be in the first i elements.

We can binary search the first i elements in $O(\log i)$ comparisons.

Since the largest possible value of $i = 2n - 2$, our algorithm takes $O\Big(\log(2n - 2)\Big) = O(\log n)$ comparisons.

27 votes -- Pragy Agarwal (18.3k points)

1.12.6 Sorting: TIFR2012-B-13

https://gateoverflow.in/25207

✔ Let Array element be $\{4,3,2,1,7,5,6,9,10,8\}$ and K be 3 here no element is more than 3 distance away from its final position
So if we take
$arr(1\ to\ 6)$ and sort then surely first three element will be sorted in its final position $\{12345769108\}$ $O(6\log 6)$
then sort $arr(3\ to\ 9)$ then 3 to 6 will be sorted $\{12345679108\}$ $O(6\log 6)$
then at last $arr(6\ to\ 9)$ less than $O(6\log 6)$ $\{12345678910\}$

in general
Sort $arr(0\ to\ 2k)$
Now we know that $arr[0\ to\ k)$ are in their final sorted positions
and $arr(k\ to\ 2k)$ may be not sorted.

Sort $arr(k\ to\ 3k)$
Now we know that $arr[k\ to\ 2k)$ are in their final sorted positions
and $arr(2k\ to\ 3k)$ may be not sorted.
.
.
.
.

sort till $arr(ik..N)$
in final sorting there will be less than $2k$ element.

in each step it will take $O(2k\log 2k)$
and there will $\frac{n}{k}$ steps so $O(n\log k)$
option **D**.

18 votes -- Umang Raman (12.2k points)

1.12.7 Sorting: TIFR2012-B-14

https://gateoverflow.in/25209

✔ Algorithm is choosing $\text{median} = n/2$ smallest element as pivot.

Hence, the array is divided as:

$\left(\frac{n}{2}-1\right)$ elements	Median at $\frac{n}{2}$th location	$\left(n-\frac{n}{2}\right)$ elements

Therefore quick sort recurrence relation is given by:

$$T(n) = T\left(\frac{n}{2}-1\right) + T\left(n-\frac{n}{2}\right) + \Theta(n)$$

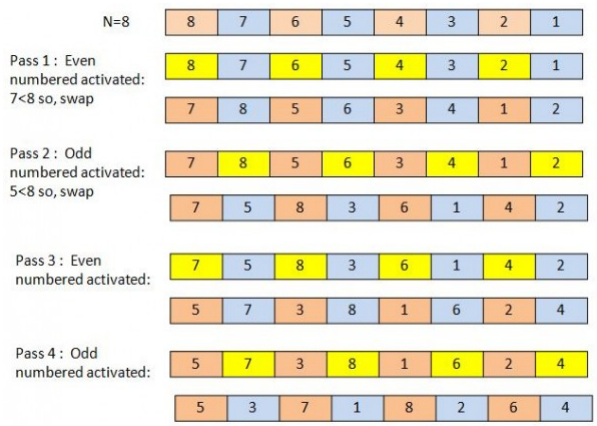
$$= \Theta(n\log n)$$

Hence, Option **B** is the correct answer.

24 votes -- Umang Raman (12.2k points)

1.12.8 Sorting: TIFR2013-B-20

https://gateoverflow.in/25878



Answer: **C.**

17 votes -- Vidhi Sethi (8.3k points)

1.12.9 Sorting: TIFR2017-B-7

https://gateoverflow.in/95699

A pairwise swap will make the sorted array unsorted. Hence, the option (**B**) is correct.

For eg - if an array is $1\ 2\ 3\ 4\ 5\ 6\ 7\ 8$

The array will become after a pair wise swap to $2\ 1\ 4\ 3\ 6\ 5\ 8\ 7$. For all i between 2 and $n-1$, $a[i]$ is either lower, or either greater than their adjacent elements.

Since, each element is being swapped exactly once. The operation has $O(n)$ time complexity.

19 votes -- tarun_svbk (1.4k points)

1.13 Spanning Tree (5)

1.13.1 Spanning Tree: TIFR2011-B-35

https://gateoverflow.in/20842

Let G be a connected simple graph (no self-loops or parallel edges) on $n \geq 3$ vertices, with distinct edge weights. Let $e_1, e_2, \ldots, e_m$ be an ordering of the edges in decreasing order of weight. Which of the following statements is FALSE?

A. The edge e_1 has to be present in every maximum weight spanning tree.
B. Both e_1 and e_2 have to be present in every maximum weight spanning tree.
C. The edge e_m has to be present in every minimum weight spanning tree.
D. The edge e_m is never present in any maximum weight spanning tree.
E. G has a unique maximum weight spanning tree.

tifr2011 algorithms graph-algorithms spanning-tree

1.13.2 Spanning Tree: TIFR2013-B-17

https://gateoverflow.in/25860

In a connected weighted graph with n vertices, all the edges have distinct positive integer weights. Then, the maximum number of minimum weight spanning trees in the graph is

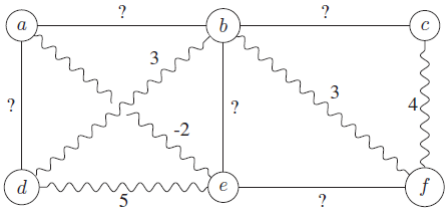
A. 1
B. n
C. equal to number of edges in the graph.
D. equal to maximum weight of an edge of the graph.
E. n^{n-2}

tifr2013 spanning-tree

1.13.3 Spanning Tree: TIFR2014-B-4

https://gateoverflow.in/27174

Consider the following undirected graph with some edge costs missing.



Suppose the wavy edges form a Minimum Cost Spanning Tree for G. Then, which of the following inequalities NEED NOT hold?

A. $\text{cost}(a, b) \geq 6$.
B. $\text{cost}(b, e) \geq 5$.
C. $\text{cost}(e, f) \geq 5$.
D. $\text{cost}(a, d) \geq 4$.
E. $\text{cost}(b, c) \geq 4$.

tifr2014 algorithms graph-algorithms spanning-tree

1.13.4 Spanning Tree: TIFR2014-B-5

https://gateoverflow.in/27180

Let $G = (V, E)$ be an undirected connected simple (i.e., no parallel edges or self-loops) graph with the weight function $w : E \to \mathbb{R}$ on its edge set. Let $w(e_1) < w(e_2) < \cdots < w(e_m)$, where $E = \{e_1, e_2, \ldots, e_m\}$. Suppose T is a minimum spanning tree of G. Which of the following statements is FALSE?

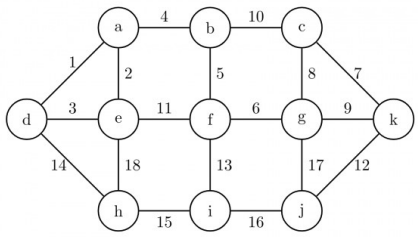
A. The tree T has to contain the edge e_1.
B. The tree T has to contain the edge e_2.
C. The minimum weight edge incident on each vertex has to be present in T.
D. T is the unique minimum spanning tree in G.
E. If we replace each edge weight $w_i = w(e_i)$ by its square w_i^2, then T must still be a minimum spanning tree of this new instance.

tifr2014 algorithms spanning-tree

1.13.5 Spanning Tree: TIFR2015-B-2

https://gateoverflow.in/29844

Consider the following undirected connected graph G with weights on its edges as given in the figure below. A minimum spanning tree is a spanning tree of least weight and a maximum spanning tree is one with largest weight. A second best minimum spanning tree whose weight is the smallest among all spanning trees that are not minimum spanning trees in G.



Which of the following statements is TRUE in the above graph? (Note that all the edge weights are distinct in the above graph)

A. There is more than one minimum spanning tree and similarly, there is more than one maximum spanning tree here.
B. There is a unique minimum spanning tree, however there is more than one maximum spanning tree here.
C. There is more than one minimum spanning tree, however there is a unique maximum spanning tree here.
D. There is more than one minimum spanning tree and similarly, there is more than one second-best minimum spanning tree here.
E. There is unique minimum spanning tree, however there is more than one second-best minimum spanning tree here.

tifr2015 spanning-tree algorithms graph-algorithms

Answers: Spanning Tree

1.13.1 Spanning Tree: TIFR2011-B-35

https://gateoverflow.in/20842

✔ **a) & c)** are trivially true. Edge with max value e_1 must be present in Maximum spanning tree & same with minimum.

e) This is true, because all edge weights are distinct. maximum spanning tree is unique.

b) e_1 & e_2 must be present in Maximum spanning tree. I'll prove it using Kruskal Algorithm.

We will first insert weight with biggest value, e_1. Then we insert e_2 (second highest) . 2 edges do not create **cycle**. Then we can go on from there inserting edges according to edge weights. As they have just asked for top 2 edges, using Kruskal Algo we can say that top 2 edges must be in Maximum spanning tree.

d) This is false. There are chances that this e_m weight edge is **cut edge(Bridge)** Then it must be inserted to from any spanning tree.

D is answer.

We can not say the same for Top 3 as they can create cycle & They we can not take a_3 to make spanning tree.

Kruskal Algo Reference: http://stackoverflow.com/questions/4992664/how-to-find-maximum-spanning-tree

References

16 votes -- Akash Kanase (36.1k points)

1.13.2 Spanning Tree: TIFR2013-B-17

https://gateoverflow.in/25860

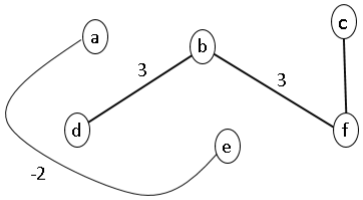
✔ There will be unique min weight spanning tree since all weights are distinct.
Option is **A**.

18 votes -- Umang Raman (12.2k points)

1.13.3 Spanning Tree: TIFR2014-B-4

https://gateoverflow.in/27174

✔



Now check this diagram, this is forest obtained from above given graph using Kruskal's algorithm for MST.

So, according to the question edge $d-e$ has weight 5 and it is included in the formation of MST. Now if edges $b-e$ and $e-f$ has weight greater than 5 than it is not a problem for our MST because still we will get the given tree as Kruskal's algorithm takes the smallest weighted edge without forming a cycle.

Cost of edge $b-c \geq 4$ may also lead us to the same tree as above though Kruskal's algorithm will have choice between $c-f$ and $b-c$.

Now if the edge weight of $a-d$ becomes 4, it is guaranteed that Kruskal's algorithm will not select edge $d-e$ because its edge cost is 5, and hence the tree structure will change. But there can be the case where edge weight is greater than 4 and we still get the same tree (happens when $a-d \geq 5$). Because in the question they asked to point out an unnecessary condition this case is not the answer as we need $a-d \geq 5$ which implies $a-d \geq 4$.

Now notice option A. Put $a-b=5$. The given MST would not change. So, this condition is not always necessary and hence is the answer.

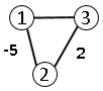
Therefore, option **A** is the answer.

21 votes -- Riya Roy(Arayana) (5.3k points)

1.13.4 Spanning Tree: TIFR2014-B-5

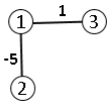
https://gateoverflow.in/27180

✔ Answer is **E**. The catch here is edge-weights belongs to real number. Therefore, edge weight can be negative. In that case the minimum spanning tree may be different.



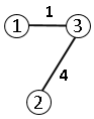
- $e1= -5$
- $e2 = 1$
- $e3 = 2$

T is:



New edge weights:

- $e1 = 25$
- $e2 = 1$
- $e3 = 4$



EDIT:

(Here every edge weight is distinct, therefore MST is unique.)

Option A is True. If we apply Kruskal's algorithm, it will choose e_1

Option B is True. If we apply Kruskal's algorithm then it will also choose e_2, and 2 edges can not form a cycle. (e_3 is not guaranteed in MST, as it may form a cycle)

Option C is also true. If we apply Prims algorithm also on any vertex (say u), it chooses minimum weight edge incident on vertex u.

Option D is true. Because every edge weight is distinct.

40 votes -- Riya Roy(Arayana) (5.3k points)

1.13.5 Spanning Tree: TIFR2015-B-2

https://gateoverflow.in/29844

✔ In the graph we have all edge weights are distinct so we will get unique minimum and maximum spanning tree.
Each Cycle must exclude maximum weight edge in minimum spanning tree.
Here, we have two cycle of 3 edges, $a-d-e$ and $c-g-k$.
For second best minimum spanning tree, exclude $a-e$ edge and include $d-e$ edge

Other way for second best minimum spanning tree: exclude $c-g$ edge and include $g-k$ edge.

So, **e** should be the answer.

26 votes -- papesh (18k points)

1.14 Time Complexity (4)

1.14.1 Time Complexity: TIFR2013-B-12

https://gateoverflow.in/25774

It takes $O(n)$ time to find the median in a list of n elements, which are not necessarily in sorted order while it takes only $O(1)$ time to find the median in a list of n sorted elements. How much time does it take to find the median of $2n$ elements which are given as two lists of n sorted elements each?

A. $O(1)$
B. $O(\log n)$ but not $O(1)$
C. $O(\sqrt{n})$ but not $O(\log n)$
D. $O(n)$ but not $O(\sqrt{n})$
E. $O(n \log n)$ but not $O(n)$

tifr2013 algorithms time-complexity

1.14.2 Time Complexity: TIFR2013-B-18

https://gateoverflow.in/25865

Let S be a set of numbers. For $x \in S$, the rank of x is the number of elements in S that are less than or equal to x. The procedure Select(S, r) takes a set S of numbers and a rank $r\,(1 \leq r \leq |S|)$ and returns the element in S of rank r. The procedure MultiSelect(S, R) takes a set of numbers S and a list of ranks $R = \{r_1 < r_2 < \ldots < r_k\}$, and returns the list $\{x_1 < x_2 < \ldots < x_k\}$ of elements of S, such that the rank of x_i is r_i. Suppose there is an implementation for Select(S, r) that uses at most "constant $\cdot |S|$" binary comparisons between elements of S. The minimum number of comparisons needed to implement MultiSelect(S, R) is

A. constant $\cdot\, |S| \log |S|$
B. constant $\cdot\, |S|$
C. constant $\cdot\, |S||R|$
D. constant $\cdot\, |R| \log |S|$
E. constant $\cdot\, |S|(1 + \log |R|)$

tifr2013 algorithms time-complexity

1.14.3 Time Complexity: TIFR2014-B-7

https://gateoverflow.in/27189

Which of the following statements is TRUE for all sufficiently large n?

A. $(\log n)^{\log \log n} < 2^{\sqrt{\log n}} < n^{1/4}$

B. $2^{\sqrt{\log n}} < n^{1/4} < (\log n)^{\log \log n}$

C. $n^{1/4} < (\log n)^{\log \log n} < 2^{\sqrt{\log n}}$

D. $(\log n)^{\log \log n} < n^{1/4} < 2^{\sqrt{\log n}}$

E. $2^{\sqrt{\log n}} < (\log n)^{\log \log n} < n^{1/4}$

tifr2014 algorithms time-complexity

1.14.4 Time Complexity: TIFR2015-B-3

https://gateoverflow.in/29846

Consider the following code fragment in the C programming language when run on a non-negative integer n.

```
int f (int n)
{
     if (n==0 || n==1)
         return 1;
     else
         return f (n - 1) + f(n - 2);
}
```

Assuming a typical implementation of the language, what is the running time of this algorithm and how does it compare to the optimal running time for this problem?

A. This algorithm runs in polynomial time in n but the optimal running time is exponential in n.
B. This algorithm runs in exponential time in n and the optimal running time is exponential in n.
C. This algorithm runs in exponential time in n but the optimal running time is polynomial in n.
D. This algorithm runs in polynomial time in n and the optimal running time is polynomial in n.
E. The algorithm does not terminate.

tifr2015 time-complexity

Answers: Time Complexity

1.14.1 Time Complexity: TIFR2013-B-12

https://gateoverflow.in/25774

✔

1. Calculate the medians $m1$ and $m2$ of the input arrays $ar1[\,]$ and $ar2[\,]$ respectively.
2. If $m1$ and $m2$ both are equal,
   return $m1$ (or $m2$)
3. If $m1$ is greater than $m2$, then median is present in one of the below two subarrays.
   (a) From first element of $ar1$ to $m1$ $(ar1[0 \text{ to } n/2])$
   (b) From $m2$ to last element of $ar2$ $(ar2[n/2 \text{ to } n-1])$
4. If $m2$ is greater than $m1$, then median is present in one of the below two subarrays.

(a) From $m1$ to last element of $ar1$ $(ar1[n/2 \text{ to } n-1])$
(b) From first element of $ar2$ to $m2$ $(ar2[0 \text{ to } n/2])$

5. Repeat the above process until size of both the subarrays becomes 2.
6. If size of the two arrays is 2 then,
   Median $= (\max(ar1[0], ar2[0]) + \min(ar1[1], ar2[1]))/2$
   Time complexity $O(\log n)$

http://www.geeksforgeeks.org/median-of-two-sorted-arrays/

Correct Answer: B

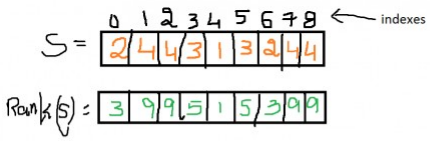
References

31 votes -- Umang Raman (12.2k points)

1.14.2 Time Complexity: TIFR2013-B-18

https://gateoverflow.in/25865

1) Find rank of elements of S is Constant*|S|



Given that, Rank of a member = no.of members less than or equal to it.

Rank(1) = 1 --> { at index 4 }
Rank(2) = 3 --> { at index 4, index 0, index 6 }
Rank(3) = 5 --> { at index 4, index 0, index 6, index 3, index 5 }
Rank(4) = 9 --> { at index 4, index 0, index 6, index 3, index 1, index 2, index 7,index 8, index 5}

How to calculate it in O(S) ?

step 1 :- calculate max. element ===> O(S)
step 2 :- Now, you know the maximum of your elements, take PASS,Current_Pass_zeros and Previous_count as variables.
int PASS = max. element, C_P_Z=0, P_C=0;

step 3 :- Make a duplicate array of members as A ===> O(S)

```
step 4 :-  for ( j=1; j<=PASS; j++){
                for( i=0;i<|S|; i++){
                    A.element--;
                    if(A.element == 0)
                        C_P_Z ++;
                }
                for( i=0;i<|S|; i++){
                    if(S.element == j)
                        Rank[i] = (C_P_Z + P_C);
                }
                P_C += C_P_Z;
                C_P_Z = 0;
            }
```

step 5 :- TC = O(S) + O(S) + PASS * (O(S) + O(S)) = const * |S|

2) Check for each member, take it's rank of element and search in input list of required ranks; if found print it !
we have an entry in list of Ranks R, As R is given as sorted we perform binary search over it .
So Comparison : Constant * |S| * Log |R|

Total Comparison : Constant * |S| + Constant * |S| * Log |R| = Constant * |S| (1 + log |R|) Ans (E)

For clarity image : https://drive.google.com/open?id=1hnIVb86YsTdDVPDbOXfOwlMCKL48z1Ks
Note :- assuming those are positive integers.

References

6 votes -- nitish (823 points)

1.14.3 Time Complexity: TIFR2014-B-7

https://gateoverflow.in/27189

✔ Let us take log for each function.

1. $\log (\log n)^{\log \log n} = (\log \log n)^2$
2. $\log 2^{\sqrt{\log n}} = \sqrt{\log n} = (\log n)^{0.5}$
3. $\log n^{1/4} = 1/4 \log n$

Here, If we consider $\log n$ as a term (which is common in all 3), first 1 is a log function, second one is sqrt function and third one is linear function of $\log n$. Order of growth of these functions are well known and log is the slowest growing followed by sqrt and then linear. So, option A is the correct answer here.

PS: After taking log is we arrive at functions distinguished by some constant terms only, then we can not conclude the order of grpwth of the original functions using the log function. Examples are $f(n) = 2^n, g(n) = 3^n$.

References

21 votes -- srestha (85.3k points)

1.14.4 Time Complexity: TIFR2015-B-3

https://gateoverflow.in/29846

✔ Answer: **C.**

It is fibanacci series generation. it takes exponential time if we won't use dynamic programming.

If we use dynamic programming then it takes $O(n)$

19 votes -- pramod (2.8k points)

Answer Keys

1.1.1	B	1.1.2	A	1.2.1	D	1.2.2	C	1.2.3	D
1.2.4	A	1.2.5	B	1.2.6	A	1.2.7	C	1.2.8	D
1.2.9	E	1.3.1	C	1.3.2	D	1.3.3	D	1.4.1	B
1.4.2	C	1.4.3	D	1.4.4	E	1.5.1	C	1.5.2	B
1.5.3	C	1.6.1	B	1.6.2	D	1.7.1	E	1.7.2	A
1.7.3	C	1.7.4	B	1.7.5	D	1.7.6	E	1.7.7	E
1.7.8	C	1.7.9	A	1.8.1	B	1.9.1	B	1.9.2	B
1.9.3	D	1.10.1	D	1.10.2	E	1.11.1	E	1.12.1	B
1.12.2	D	1.12.3	B	1.12.4	C	1.12.5	D	1.12.6	D
1.12.7	B	1.12.8	C	1.12.9	B	1.13.1	D	1.13.2	A
1.13.3	A	1.13.4	E	1.13.5	E	1.14.1	B	1.14.2	E
1.14.3	A	1.14.4	C						

2 Compiler Design (3)

Lexical analysis, Parsing, Syntax-directed translation, Runtime environments, Intermediate code generation.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	2	2	2	1	2	2	1	1	1	1.5	2
2 Marks Count	3	2	1	2	2	2	1	3	2	1	2	3
Total Marks	7	6	4	6	5	6	4	7	5	4	5.5	7

2.1 Parsing (3)

2.1.1 Parsing: TIFR2012-B-17

https://gateoverflow.in/25215

Which of the following correctly describes $LR(k)$ parsing?

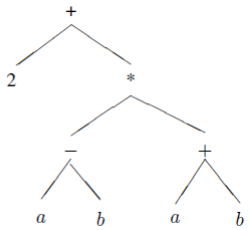
A. The input string is alternately scanned left to right and right to left with k reversals.
B. Input string is scanned once left to right with rightmost derivation and k symbol look-ahead.
C. $LR(k)$ grammers are expressively as powerful as context-free grammers.
D. Parser makes k left-to-right passes over input string.
E. Input string is scanned from left to right once with k symbol to the right as look-ahead to give left-most derivation.

tifr2012 compiler-design parsing

2.1.2 Parsing: TIFR2012-B-8

https://gateoverflow.in/25108

Consider the parse tree



Assume that $*$ has higher precedence than $+$, $-$ and operators associate right to left (i.e $(a+b+c=(a+(b+c))$). Consider

i. $2+a-b$
ii. $2+a-b*a+b$
iii. $(2+((a-b)*(a+b)))$
iv. $2+(a-b)*(a+b)$

The parse tree corresponds to

A. Expression (i)
B. Expression (ii)
C. Expression (iv) only
D. Expression (ii), (iii), and (iv)
E. Expression (iii) and (iv) only

tifr2012 compiler-design parsing

2.1.3 Parsing: TIFR2015-B-15

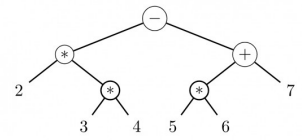
https://gateoverflow.in/30079

Consider the following grammar (the start symbol is E) for generating expressions.

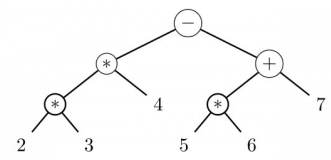
- $E \rightarrow T-E \mid T+E \mid T$
- $T \rightarrow T*F \mid F$
- $F \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

With respect to this grammar, which of the following trees is the valid evaluation tree for the expression $2*3*4-5*6+7$?

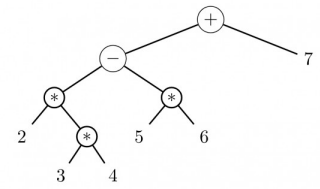
A.



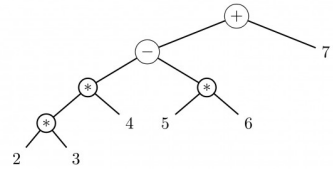
B.



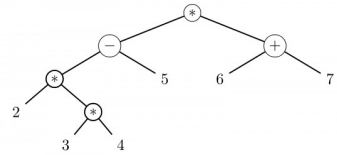
C.



D.



E.



tifr2015 parsing

Answers: Parsing

2.1.1 Parsing: TIFR2012-B-17

https://gateoverflow.in/25215

✔

a. Does not make any sense. false.
b. This is definition of LR(k) Parser. True
c. False. LR(k) is subset of CFL.
d. False.
e. LR(k) , bottom up parser . We have Right most derivation. This is False.

Answer :- **B**

17 votes -- Akash Kanase (36.1k points)

2.1.2 Parsing: TIFR2012-B-8

https://gateoverflow.in/25108

✔ **e** is correct

Because as the precedence is right to left, expression evaluated in (ii) is $2 + (a - ((b * a) + b))$, which is not a correct evaluation as the given parse tree.

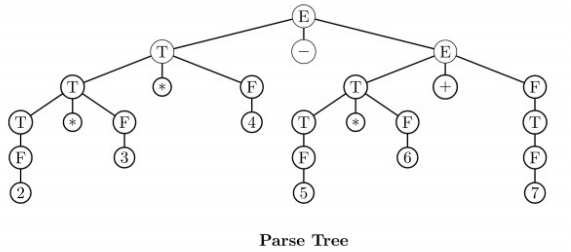
14 votes -- srestha (85.3k points)

2.1.3 Parsing: TIFR2015-B-15

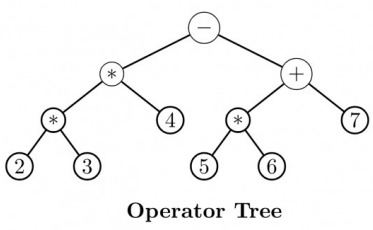
https://gateoverflow.in/30079

✔ Answer is option **B**.

The corresponding parse tree is drawn for the given expression according to the given grammar.



Parse Tree



Operator Tree

22 votes -- Riya Roy(Arayana) (5.3k points)

Answer Keys

2.1.1	B	2.1.2	E	2.1.3	B

3 Computer Networks (1)

Concept of layering.OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit-switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, **Email**.

.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	1	2	1	3	2	3	2	3	1	2	3
2 Marks Count	4	3	2	4	2	3	1	4	3	1	2.8	4
Total Marks	9	7	6	9	7	8	5	10	9	5	7.7	10

3.1 Network Security (1)

3.1.1 Network Security: TIFR2011-B-36

https://gateoverflow.in/20918

Consider malware programs. Which of the following is true?

A. A worm is a parasite.
B. A virus cannot affect a linux operating system.
C. A trojan can be in the payload of only a worm.
D. A worm and virus are self replicating programs.
E. There is no difference between a virus and a worm.

tifr2011 computer-networks network-security

Answers: Network Security

3.1.1 Network Security: TIFR2011-B-36

https://gateoverflow.in/20918

✔

- A worm is a parasite. **False**

ANS: worm is a standalone malware computer program that replicates itself in order to spread to other computers.

- A virus cannot affect a Linux operating system. **False**

ANS: virus can affect any operating system.

- A trojan can be in the payload of only a worm. **False**
- A worm and virus are self-replicating programs. **True**

ANS: worm and virus is self-replicating programs But trojan is not.

- There is no difference between a virus and a worm. **False**

ANS: since worm are standalone software and do not require a host program or human help to propagate.

So, D is choice.

16 votes -- Prashant Singh (47.2k points)

Answer Keys

3.1.1	D

4 Databases (2)

ER-model. Relational model:Relational algebra, Tuple calculus, SQL. Integrity constraints, Normal forms. File organization, Indexing (e.g., B and B+ trees). Transactions and concurrency control.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	2	1	2	2	2	2	2	3	2	1	2	3
2 Marks Count	3	3	3	3	2	3	3	1	2	1	2.5	3
Total Marks	8	7	8	8	6	8	8	5	6	6	7.1	8

4.1 Relational Algebra (2)

4.1.1 Relational Algebra: TIFR2010-B-33

https://gateoverflow.in/19246

In a relational database there are three relations:

- Customers = C (C Name)
- Shops = S (S Name)
- Buys = B (C Name, S Name)

Then the Relational Algebra expression (Π is the projection operator).

$$C - \Pi_{CName}((C \times S) - B)$$

returns the names of

A. Customers who buy from at least one shop.
B. Customers who buy from at least two shops.
C. Customers who buy from all shops.
D. Customers who do not buy buy anything at all.
E. None of the above.

tifr2010 databases relational-algebra

4.1.2 Relational Algebra: TIFR2013-B-19

https://gateoverflow.in/25872

In a relational database there are three relations:

- Customers = C(CName),
- Shops = S(SName),
- Buys = B(CName, SName).

Which of the following relational algebra expressions returns the names of shops that have no customers at all? [Here Π is the projection operator.]

A. $\Pi_{SName}B$
B. $S - B$
C. $S - \Pi_{SName}B$
D. $S - \Pi_{SName}((C \times S) - B)$
E. None of the above

tifr2013 databases relational-algebra

Answers: Relational Algebra

4.1.1 Relational Algebra: TIFR2010-B-33

https://gateoverflow.in/19246

✔ It is division in relational algebra

Division = $\pi_{AB}(R)/\pi_B(S)$ Results in 'A' values for which here should be 'B' in R for every 'B' of S.

$\pi_{AB}(R)/\pi_B(S) = \Pi_A(R) - \pi_A(\pi_A(R) \times S - R)$ Retrieve all A's who are related to every B

$C - \Pi_{CName}((C \times S) - B)$

$C \times S$ gives the complete relation of each customer to every shop

$(C \times S) - B)$:gives the relation of the customer which is not related to every shop.

$\Pi_{CName}((C \times S) - B)$: gives the customer name who is not related to every shop.

$C - \Pi_{CName}((C \times S) - B)$: gives the customer who is related to every shop.

Option C) Customers who buy from all shops.

31 votes -- Umang Raman (12.2k points)

4.1.2 Relational Algebra: TIFR2013-B-19

https://gateoverflow.in/25872

✔ Answer will be (**C**)

It subtract shop names to those shop which sells something.

So as a result we are getting shops which have no customer.

27 votes

-- srestha (85.3k points)

Answer Keys

4.1.1	C	4.1.2	C

5 Digital Logic (10)

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point)

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	2	3	2	4	2	3	2	3	3	2	2.6	4
2 Marks Count	2	2	1	2	2	0	4	2	0	0	1.6	4
Total Marks	6	7	4	8	6	3	10	7	3	3	6	10

5.0.1 TIFR2020-B-1

https://gateoverflow.in/333120

Consider the following Boolean valued function on n Boolean variables: $f(x_1, \ldots, x_n) = x_1 + \cdots + x_n \pmod 2$, where addition is over integers, mapping 'FALSE' to 0 and 'TRUE' to 1. Consider Boolean circuits (with no feedback) that use only logical AND and OR gates, and where each gate has two input bits, each of which is either an input bit of f or the output bit of some other gate of the circuit. The circuit has a distinguished gate whose value is the output of the circuit. The minimum size of such a circuit computing f (asymptotically in n) is :

A. $2^{o(\log n)}$
B. n^c, for some fixed constant c
C. $n^{\omega(1)}$, but $n^{O(\log n)}$
D. $2^{\Theta(n)}$
E. None of the others

tifr2020

5.1 Boolean Algebra (3)

5.1.1 Boolean Algebra: TIFR2010-B-21

https://gateoverflow.in/18621

For $x \in \{0,1\}$, let $\neg x$ denote the negation of x, that is

$$\neg x = \begin{cases} 1 & \text{iff } x = 0 \\ 0 & \text{iff } x = 1 \end{cases}$$

.

If $x \in \{0,1\}^n$, then $\neg x$ denotes the component wise negation of x; that is:

$$(\neg x)_i = (\neg x_i \mid i \in [1..n])$$

Consider a circuit C, computing a function $f : \{0,1\}^n \to \{0,1\}$ using **AND** $(\wedge)$, **OR** $(\vee)$,and **NOT** $(\neg)$ gates. Let D be the circuit obtained from C by replacing each **AND** gate by an **OR** gate and replacing each **OR** gate by an **AND**. Suppose D computes the function g. Which of the following is true for all inputs x?

A. $g(x) = \neg f(x)$
B. $g(x) = f(x) \wedge f(\neg x)$
C. $g(x) = f(x) \vee f(\neg x)$
D. $g(x) = \neg f(\neg x)$
E. None of the above.

tifr2010 digital-logic boolean-algebra

5.1.2 Boolean Algebra: TIFR2014-B-17

https://gateoverflow.in/27344

Let $f : \{0,1\}^n \to \{0,1\}$ be a boolean function computed by a logical circuit comprising just binary AND and binary OR gates (assume that the circuit does not have any feedback). Let PARITY : $\{0,1\}^n \to \{0,1\}$ be the boolean function that outputs 1 if the total number of input bits set to 1 is odd. Similarly, let MAJORITY be the boolean function that outputs 1 if the number of input bits that are set to 1 is at least as large as the number of input bits that are set to 0. Then, which of the following is NOT possible?

A. $f(0,0,\ldots,0) = f(1,1,\ldots,1) = 0$.
B. $f(0,0,\ldots,0) = f(1,1.\ldots,1) = 1$
C. f is the MAJORITY function.
D. f is the PARITY function.
E. f outputs 1 at exactly one assignment of the input bits.

tifr2014 boolean-algebra

5.1.3 Boolean Algebra: TIFR2016-B-1

https://gateoverflow.in/97626

A Boolean formula is said to be a $tautology$ if it evaluates to TRUE for all assignments to its variables. Which one of the following is NOT a tautology?

A. $((p \vee q) \wedge (r \vee s)) \Rightarrow ((p \wedge r) \vee q \vee s)$
B. $((p \vee q) \wedge (r \vee s)) \Rightarrow (q \vee s)$
C. $((p \vee q) \wedge (r \vee s)) \Rightarrow (r \vee q \vee s)$
D. $((p \vee q) \wedge (r \vee s)) \Rightarrow (p \vee q \vee s)$
E. $((p \vee q) \wedge (r \vee s)) \Rightarrow (p \vee q)$

tifr2016 boolean-algebra

Answers: Boolean Algebra

5.1.1 Boolean Algebra: TIFR2010-B-21

https://gateoverflow.in/18621

✔ Option (**D**) is answer.

The circuit D is the dual of the function $f(x)$ (i.e., replace AND by OR and vice versa)

We can find dual of any function $f(x)$ by doing $\neg f(\neg x)$.

19 votes -- Mari Ganesh Kumar (1.5k points)

5.1.2 Boolean Algebra: TIFR2014-B-17

https://gateoverflow.in/27344

Note: NOT is absent in function f.

for two boolean variables, $p_1 = 1, p_2 = 1$, neither $p_1 \wedge p_2$ nor $p_1 \vee p_2$ is 0. ie, $f(1,1)$ is never 0.

for $i = 1$ to n. $f(p_i)$ is a function of AND,OR operations on p_i. if all $p_i = 1$, then f can never be 0;

similarly, if all $p_i = 0$, f can never be 1;

Therefore A, B are not possible.

for $j < n$, if all $p_j = 0$ and $p_{n-j} = 1$,then $f(p_j, p_{n-j})$ =majority if each 0 is AND with each 1.The remaining $1's$ or $0's$ are OR with the result.

Hence, MAJORITY can be computed from f.

Option C is possible.

To check odd number of $1's$, for PARITY function, we have to get the result 0 for even number of $1's$ which is not possible with just AND and OR operations, how might we combine(since NOT is absent in f);

D is not possible.

For option E, we check by symmetry. When the inputs are complemented among $0's$ and $1's$, can f change to f'? f is not always fixed for a particular input,. example, $f(0,1) = 0 \vee 1 = 1 \quad 0 \wedge 1 = 0$,hence f can take multiple values for same input. Therefore E is also not right.

The only possible answer is C .'Hence A,B,D,E are not possible.

7 votes -- Vikranth Inti (579 points)

5.1.3 Boolean Algebra: TIFR2016-B-1

https://gateoverflow.in/97626

ANSWER: ***B)*** option as it does not evaluate to true

A) evaluate to TRUE

C) evaluate to TRUE
D) evaluate to TRUE
E) evaluate to TRUE

7 votes -- kunal chalotra (13.6k points)

5.2 Canonical Normal Form (1)

5.2.1 Canonical Normal Form: TIFR2015-B-9

https://gateoverflow.in/30030

A Boolean expression is an expression made out of propositional letters (such as p, q, r) and operators $\wedge$, $\vee$ and $\neg$; e.g. $p \wedge \neg(q \vee \neg r)$. An expression is said to be in sum of product form (also called disjunctive normal form) if all $\neg$ occur just before letters and no $\vee$ occurs in scope of $\wedge$; e.g. $(p \wedge \neg q) \vee (\neg p \wedge q)$. The expression is said to be in product of sum form (also called conjunctive normal form) if all negations occur just before letters and no $\wedge$ occurs in the scope of $\vee$; e.g. $(p \vee \neg q) \wedge (\neg p \vee q)$. Which of the following is not correct?

A. Every Boolean expression is equivalent to an expression in sum of product form.
B. Every Boolean expression is equivalent to an expression in product of sum form.
C. Every Boolean expression is equivalent to an expression without $\vee$ operator.
D. Every Boolean expression is equivalent to an expression without $\wedge$ operator.
E. Every Boolean expression is equivalent to an expression without $\neg$ operator.

tifr2015 canonical-normal-form

Answers: Canonical Normal Form

5.2.1 Canonical Normal Form: TIFR2015-B-9

https://gateoverflow.in/30030

✔ Answer will be (E)

a) True. Every expression can be written in SOP form

b) True. Every expression can be written in POS form

c)True. We can write OR in the form of AND. e.g.(A+B)=not(not(A) . not(B))

d)True.We can write AND in the form of OR e.g. (A .B)=not(not(A) + not(B))

(e)False. We cannot convert NOT gate to any other gate. We cannot also get NAND , NOR such universal gate without NOT gate. So, without NOT gate we cannot get every boolean expression

35 votes -- srestha (85.3k points)

5.3 Digital Circuits (1)

5.3.1 Digital Circuits: TIFR2015-A-4

https://gateoverflow.in/29162

The Boolean function obtained by adding an inverter to each and every input of an AND gate is:

A. OR B. XOR C. $NAND$ D. NOR E. None of the above

tifr2015 digital-logic digital-circuits

Answers: Digital Circuits

5.3.1 Digital Circuits: TIFR2015-A-4

https://gateoverflow.in/29162

✔ Invert-AND = NOR

For example, $A'B' = \overline{A + B}$

[Note : Invert-OR = NAND, $A' + B' = \overline{A.B}$]

16 votes -- Praveen Saini (42k points)

5.4 Gray Code (1)

5.4.1 Gray Code: TIFR2017-B-8

https://gateoverflow.in/95703

For any natural number n, an ordering of all binary strings of length n is a Gray code if it starts with 0^n, and any successive strings in the ordering differ in exactly one bit (the first and last string must also differ by one bit). Thus, for $n = 3$, the ordering (000, 100, 101, 111, 110, 010, 011, 001) is a Gray code. Which of the following must be TRUE for all Gray codes over strings of length n?

A. the number of possible Gray codes is even
B. the number of possible Gray codes is odd
C. In any Gray code, if two strings are separated by k other strings in the ordering, then they must differ in exactly $k + 1$ bits
D. In any Gray code, if two strings are separated by k other strings in the ordering, then they must differ in exactly k bits
E. none of the above

tifr2017 digital-logic binary-codes gray-code

Answers: Gray Code

5.4.1 Gray Code: TIFR2017-B-8

https://gateoverflow.in/95703

✔

1. In the question it is stated that -> Thus, for $n = 3$, the ordering $(000, 100, 101, 111, 110, 010, 011, 001)$ is a Gray code.
2. We have to find orderings such that they start with 0^n, must contain all bit strings of length n and successive strings must differ in one bit.

Options **c and d** are clearly wrong.
Now, consider $n = 1$. The only Gray code possible is $\{0, 1\}$. Hence no of Gray code $=$ odd for $n = 1$.
For $n = 2$ only two Gray code exists $\{00, 10, 11, 01\}$ and $\{00, 01, 11, 10\}$. Thus no of Gray code $=$ even for $n = 2$.
Thus it is not that gray codes could be only even or only odd.
Hence, option **e** is correct.

PS: Official answer given was A - probably a mistake

17 votes -- tarun_svbk (1.4k points)

5.5 Number Representation (2)

5.5.1 Number Representation: TIFR2011-A-16

https://gateoverflow.in/20253

A variable that takes thirteen possible values can be communicated using?

A. Thirteen bits.
B. Three bits.
C. $\log_2 13$ bits.
D. Four bits.
E. None of the above.

tifr2011 number-representation

5.5.2 Number Representation: TIFR2019-B-1

https://gateoverflow.in/280494

Which of the following decimal numbers can be exactly represented in binary notation with a finite number of bits ?

A. 0.1 B. 0.2 C. 0.4 D. 0.5 E. All the above

tifr2019 digital-logic number-representation

Answers: Number Representation

5.5.1 Number Representation: TIFR2011-A-16

https://gateoverflow.in/20253

✔ As there are only 13 possible values a variable can take, we need to use $\lceil 13 \rceil = 4 - bits.$

PS: As variable can take only 13 values, we don't need to worry what those values are.

Answer : Option D

15 votes -- Akash Kanase (36.1k points)

5.5.2 Number Representation: TIFR2019-B-1 https://gateoverflow.in/280494

Binary representation of 0.5, 0.1_2 is the only fraction that is terminating in the given options. So it can be represented exactly.

All other options have non-terminating fraction part and cannot be represented exactly with finite number of bits.

4 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

5.6 Number System (1)

5.6.1 Number System: TIFR2020-B-3 https://gateoverflow.in/333122

Consider the (decimal) number 182, whose binary representation is 10110110. How many positive integers are there in the following set?

$$\{n \in \mathbb{N} : n \leq 182 \text{ and n has exactly fourones in its binary representation}\}$$

A. 91 B. 70 C. 54 D. 35 E. 27

tifr2020 digital-logic number-system number-representation

Answers: Number System

5.6.1 Number System: TIFR2020-B-3 https://gateoverflow.in/333122

Total Bits = 8

case 1 : Starts with 0, then it must be less than 182

So, in the remaining 7 Bits, you need to choose 4 bits as 1 ==> $\binom{7}{4}$ = 35

case 2 : Starts with 1, then next bit should be 0 for total decimal value ≤ 182.

So, in the remaining 6 Bits, you need to choose 3 bits as 1 ==> $\binom{6}{3}$ = 20

but in this 20 patterns, 10111000 is grater than given number. SO only 19 patterns are matching with our condition.

Total = 35+19 = 54

4 votes -- Shaik Masthan (50.6k points)

Answer Keys

5.0.1	E	5.1.1	D	5.1.2	D	5.1.3	B	5.2.1	E
5.3.1	D	5.4.1	A	5.5.1	D	5.5.2	D	5.6.1	C

6 Discrete Mathematics: Combinatory (16)

Syllabus: Combinatorics: Counting, Recurrence relations, Generating functions.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	0	0	2	1	0	0	1	0	0	0.55	2
2 Marks Count	0	1	1	0	1	0	1	2	1	0	0.77	2
Total Marks	1	2	2	2	3	0	2	5	2	0	2.11	5

6.0.1 TIFR2011-A-2

https://gateoverflow.in/19829

In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

A. $\frac{(6+3)!}{2!}$ B. $\frac{6!}{2!}$ C. $\frac{3!3!}{2!}$ D. $\frac{4!3!}{2!}$ E. None of the above

tifr2011 combinatory

6.0.2 TIFR2015-A-7

https://gateoverflow.in/29568

A 1×1 chessboard has one square, a 2×2 chessboard has five squares. Continuing along this fashion, what is the number of squares on the regular 8×8 chessboard?

A. 64 B. 65 C. 204 D. 144 E. 256

tifr2015 combinatory

Answers:

6.0.1 TIFR2011-A-2

https://gateoverflow.in/19829

✔ Take AAU together and treat it like 1 entity. Now arrange $\boxed{\text{AAU}}\text{BCS}$ in $4!$ ways.

Then, the AAU can be arranged in $\frac{3!}{2!}$ ways because A has been repeated twice.

So, total arrangements $= \frac{4!3!}{2!}$

Option (**D**) is the correct answer.

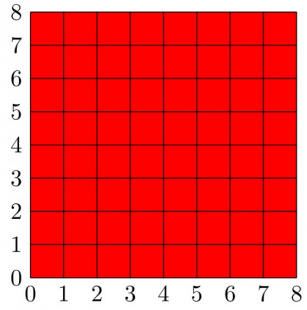
26 votes -- yes (1.1k points)

6.0.2 TIFR2015-A-7

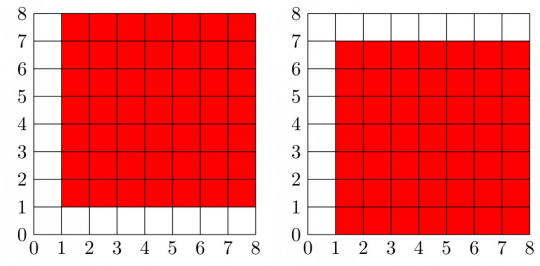
https://gateoverflow.in/29568

✔

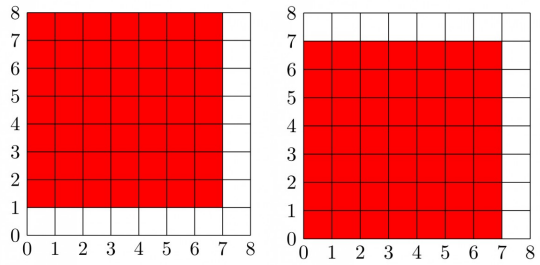
8×8 squares possible is 1



Now lets see how many 7×7 squares are possible

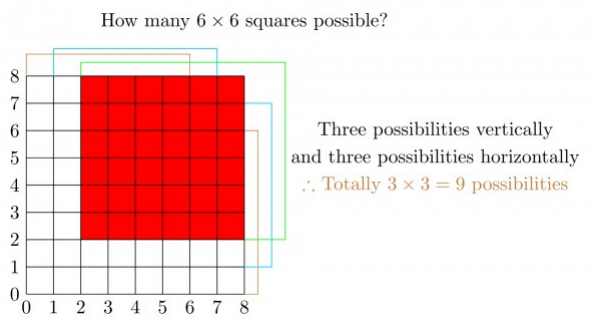


These two patterns can shift to right as well as follows:



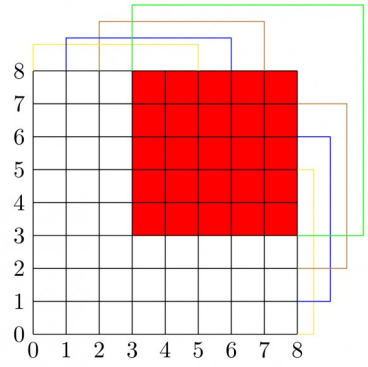
So, 7×7 squares possible is 4

Now lets see how many 6×6 squares are possible



So, 6×6 squares possible is 9

Now lets see how many 5×5 squares are possible:



4 vertical moves $\times 4$ horizontal moves $= 4^2$ possibilities.

Proceeding like this,

- 8×8 squares possible : $1 \times 1 = 1$
- 7×7 squares possible : $2 \times 2 = 4$
- 6×6 squares possible : $3 \times 3 = 9$
- 5×5 squares possible : $4 \times 4 = 16$
- 4×4 squares possible : $5 \times 5 = 25$
- 3×3 squares possible : $6 \times 6 = 36$
- 2×2 squares possible : $7 \times 7 = 49$
- 1×1 squares possible : $8 \times 8 = 64$

Total squares : 204

Now we can generalize like with $n \times n$ chess board total squares $= 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Correct Answer: C

28 votes -- srestha (85.3k points)

6.1 Balls In Bins (4)

6.1.1 Balls In Bins: TIFR2012-A-7 https://gateoverflow.in/21004

It is required to divide the $2n$ members of a club into n disjoint teams of 2 members each. The teams are not labelled. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$ B. $\frac{(2n)!}{n!}$ C. $\frac{(2n)!}{2^n.n!}$ D. $\frac{n!}{2}$ E. None of the above

tifr2012 combinatory balls-in-bins

6.1.2 Balls In Bins: TIFR2013-A-9 https://gateoverflow.in/25431

There are n kingdoms and $2n$ champions. Each kingdom gets 2 champions. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$ B. $\frac{(2n)!}{n!}$ C. $\frac{(2n)!}{2^n.n!}$ D. $\frac{n!}{2}$ E. None of the above

tifr2013 combinatory discrete-mathematics normal balls-in-bins

6.1.3 Balls In Bins: TIFR2015-A-8 https://gateoverflow.in/29571

There is a set of $2n$ people: n male and n female. A good party is one with equal number of males and females (including the one where none are invited). The total number of good parties is.

A. 2^n B. n^2 C. $\binom{n}{\lfloor n/2 \rfloor}^2$ D. $\binom{2n}{n}$ E. None of the above

tifr2015 combinatory discrete-mathematics normal balls-in-bins

6.1.4 Balls In Bins: TIFR2017-A-5 https://gateoverflow.in/94953

How many distinct ways are there to split 50 identical coins among three people so that each person gets at least 5 coins?

A. 3^{35} B. $3^{50} - 2^{50}$
C. $\binom{35}{2}$ D. $\binom{50}{15} \cdot 3^{35}$
E. $\binom{37}{2}$

tifr2017 combinatory discrete-mathematics normal balls-in-bins

Answers: Balls In Bins

6.1.1 Balls In Bins: TIFR2012-A-7 https://gateoverflow.in/21004

✔ $2n$ member to be n teams with 2 member each and teams are unordered so we can exchange n team member among them.

$$= \frac{(2n)!}{\underbrace{2!.2!.2!\ldots 2!}_{n \text{ times}} \times n!}$$
$$= \frac{(2n)!}{2^n \times n!}$$

Option C.

26 votes -- Umang Raman (12.2k points)

6.1.2 Balls In Bins: TIFR2013-A-9 https://gateoverflow.in/25431

✔ We have n Kingdoms as $k_1, k_2, \ldots, k_n$.

Firstly we can select 2 champions from $2n$ champions and assign to $k_1 = \binom{2n}{2}$ ways (Say w_1)

Then we can select next 2 champions and assign to $k_2 = \binom{2n-2}{2}$ ways (Say w_2)

and so on..

For last kingdom , we have 2 champions left$= \binom{2}{2}$ ways (Say w_n)

$$\begin{aligned}\text{Total ways for assigning } 2n \text{ champions to } n \text{ kingdoms} &= w_1 * w_2 * \ldots * w_n \\ &= \binom{2n}{2} * \binom{2n-2}{2} * \ldots * \binom{2}{2} \\ &= \frac{(2n)!}{2^n}\end{aligned}$$

So, Option **A** **(Ans) .**

👍 36 votes -- Himanshu Agarwal (12.4k points)

6.1.3 Balls In Bins: TIFR2015-A-8

https://gateoverflow.in/29571

✔ There are n men and n women

Now we can select 1 woman from n in ${}^nC_1$

Same way 1 man can be selected ${}^nC_1$ ways

So, for 1 woman and 1 man we can get ${}^nC_1 \times^n C_1$ ways $\rightarrow (1)$

Similarly, we can select 2 woman from n women in ${}^nC_2$

2 man can be selected in ${}^nC_2$ ways

So, for 2 woman and 2 man we can get ${}^nC_2 \times^n C_2$ ways $\rightarrow (2)$

$\vdots$

For n woman and n man we can get ${}^nC_n \times^n C_n$ ways $\rightarrow (n)$

Now, by adding these equations $(1), (2), \ldots, (n)$ we get ,

${}^nC_0 \times^n C_0 +^n C_1 \times^n C_1 +^n C_2 \times^n C_2 +^n C_3 \times^n C_3 + \ldots +^n C_n \times^n C_n = \left({}^{2n}C_n\right)$

Hence, Ans will be (**D**).

👍 26 votes -- srestha (85.3k points)

6.1.4 Balls In Bins: TIFR2017-A-5

https://gateoverflow.in/94953

✔ Distinct ways are there to split 50 identical coins among three people so that each person gets at least 5 coins

$x_1 + 5 + x_2 + 5 + x_3 + 5 = 50$

$x_1 + x_2 + x_3 = 35$

Solving Non-integral solution $n = 35, r = 3$

${}^{(n+r-1)}C_{r-1} = {}^{(35+3-1)}C_{3-1} = {}^{37}C_2$.

Hence E is Answer

👍 31 votes -- Prajwal Bhat (7.6k points)

6.2 Discrete Mathematics (4)

6.2.1 Discrete Mathematics: TIFR2012-A-10

https://gateoverflow.in/25014

In how many different ways can r elements be picked from a set of n elements if

i. Repetition is not allowed and the order of picking matters?
ii. Repetition is allowed and the order of picking does not matter?

A. $\frac{n!}{(n-r)!}$ and $\frac{(n+r-1)!}{r!(n-1)!}$, respectively.
B. $\frac{n!}{(n-r)!}$ and $\frac{n!}{r!(n-1)!}$, respectively.
C. $\frac{n!}{r!(n-r)!}$ and $\frac{(n-r+1)!}{r!(n-1)!}$, respectively.
D. $\frac{n!}{r!(n-r)!}$ and $\frac{n!}{(n-r)!}$, respectively.
E. $\frac{n!}{r!}$ and $\frac{r!}{n!}$, respectively.

tifr2012 combinatory discrete-mathematics normal

6.2.2 Discrete Mathematics: TIFR2016-A-15

https://gateoverflow.in/97624

In a tournament with 7 teams, each team plays one match with every other team. For each match, the team earns two points if it wins, one point if it ties, and no points if it loses. At the end of all matches, the teams are ordered in the descending order of their total points (the order among the teams with the same total are determined by a whimsical tournament referee). The first three teams in this ordering are then chosen to play in the next round. What is the minimum total number of points a team must earn in order to be guaranteed a place in the next round?

A. 13 B. 12 C. 11 D. 10 E. 9

tifr2016 combinatory discrete-mathematics normal

6.2.3 Discrete Mathematics: TIFR2017-A-6

https://gateoverflow.in/95033

How many distinct words can be formed by permuting the letters of the word $ABRACADABRA$?

A. $\frac{11!}{5!\,2!\,2!}$ B. $\frac{11!}{5!\,4!}$ C. $11!\,5!\,2!\,2!$ D. $11!\,5!\,4!$ E. $11!$

tifr2017 combinatory discrete-mathematics easy

6.2.4 Discrete Mathematics: TIFR2019-B-13

https://gateoverflow.in/280482

A row of 10 houses has to be painted using the colours red, blue, and green so that each house is a single colour, and any house that is immediately to the right of a red or a blue house must be green. How many ways are there to paint the houses?

A. 199 B. 683 C. 1365 D. $3^{10} - 2^{10}$ E. 3^{10}

tifr2019 engineering-mathematics discrete-mathematics combinatory

Answers: Discrete Mathematics

6.2.1 Discrete Mathematics: TIFR2012-A-10

https://gateoverflow.in/25014

✔

i. Repetition is not allowed and the order of picking matters $=$r arrangement with no repetition
$={}^{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!}$

ii. Repetition is allowed and the order of picking does not matter $=$ combination with unlimited repetition
$={}^{(n-1+r)}\mathrm{C}_{r} = \frac{n-1+r!}{(n-1)!r!}$

Hence, ans is Option (**A**).

19 votes -- Umang Raman (12.2k points)

6.2.2 Discrete Mathematics: TIFR2016-A-15

https://gateoverflow.in/97624

✔ Let the 7 Teams be A, B, C, D, E, F, G and so each team plays total 6 matches.

Suppose, Team A wins over E, F, G and draws with B, C, D hence total points scored by Team A $= 9$ points

Now, Team B wins over E, F, G and draws with A, C, D hence total points scored by Team B $= 9$ points

Similarly, happens for next two teams C and D.

Hence, Finalized scores are $\Rightarrow$

```
A = 9
B = 9
C = 9
D = 9
E = ? (Less than or equal to 4)
F = ? ("...")
G = ? ("...")
```

Given that the order among the teams with the same total are determined by a whimsical tournament referee.

So, He/She can order the top 3 teams like $ABC, ABD, BCD, ACD, \ldots$

But, Question says " team must earn in order to be guaranteed a place in the next round "

Hence, Not to depend on that whimsical referee, the minimum total number of points a team must earn in order to be guaranteed a place in the next round $= 9 + 1 = 10$ points

Correct Answer: D

30 votes -- Kapil Phulwani (35.2k points)

6.2.3 Discrete Mathematics: TIFR2017-A-6

https://gateoverflow.in/95033

✔ ABRACADABRA

$A \rightarrow 5B \rightarrow 2R \rightarrow 2$

Total Permutation of words $= 11!$

Now,we have to remove word from total permutation of words which have repetition of letter,

$$= \frac{11!}{5!2!2!}$$

Hence option (**A**) is correct.

13 votes -- kunal chalotra (13.6k points)

6.2.4 Discrete Mathematics: TIFR2019-B-13

https://gateoverflow.in/280482

✔ Let $T(n)$ denote the number of ways to color $'n'$ houses such that house immediately right to the red or the blue is green.

Consider the first house in the row. It can be either red, green or blue. (Dividing the problem into three cases).

Suppose the first house is painted green so for next house, we don't have any condition, It can be painted by using any of the colors,$(R, G$ or $B)$.

Hence If the first house is green we can paint the rest of the houses in $T(n-1)$ ways.

Suppose the first house is red, then for the next house we don't have any choice, it must be green.

So, if the first house is red, the number of ways of painting the rest of the houses is $T(n-2)$. (excluding the second house).

Same applies if the first house is blue, in that case also in $T(n-2)$ ways we can paint the houses.

Summing up all the mutually exclusive cases,

1. $T(n) = T(n-1) + T(n-2) + T(n-2)$.
2. $T(n) = T(n-1) + 2T(n-2)$.

Now consider the base case:

If we have 1 house to paint it can be either red, green or blue. So $T(1) = 3$.

For 2 houses, GR, GB, GG, BG (or) RG are the 5 possible ways. So, $T(2) = 5$.

$T(1)$	$T(2)$	$T(3)$	$T(4)$	$T(5)$	$T(6)$	$T(7)$	$T(8)$	$T(9)$	$T(10)$
3	5	11	21	43	85	171	341	683	1365

Option (C).

42 votes -- Utkarsh Joshi (5.3k points)

6.3 Generating Functions (1)

6.3.1 Generating Functions: TIFR2010-A-12

https://gateoverflow.in/18391

The coefficient of x^3 in the expansion of $(1+x)^3(2+x^2)^{10}$ is.

A. 2^{14}
B. 31
C. $\binom{3}{3} + \binom{10}{1}$
D. $\binom{3}{3} + 2\binom{10}{1}$
E. $\binom{3}{3}\binom{10}{1}2^9$

tifr2010 generating-functions

Answers: Generating Functions

6.3.1 Generating Functions: TIFR2010-A-12

https://gateoverflow.in/18391

✔ **Using the Binomial Theorem:**

$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$

$(1+x)^3(2+x^2)^{10} = \sum_{r=0}^{3} \binom{3}{r}(1)^{3-r}(x)^r \cdot \sum_{k=0}^{10} \binom{10}{k}(2)^{10-k}(x^2)^k$

$(1+x)^3(2+x^2)^{10} = \sum_{r=0}^{3} \binom{3}{r}(x)^r \cdot \sum_{k=0}^{10} \binom{10}{k}(2)^{10-k}(x^2)^k$

Case1 :If $r=3, k=0$ we get x^3

So, Coefficient of x^3 is $= \binom{3}{3} \cdot \binom{10}{0}(2^{10}) = 2^{10}$

Case2 :If $r=1, k=1$ we get x^3

So, Coefficient of x^3 is $= \binom{3}{1} \cdot \binom{10}{1}(2)^9 = 30 \times 2^9$

So, finally Coefficient of x^3 is $= 2^{10} + 30 \times 2^9$

$= 2^9(2+30)$

$= 2^9 \times 32$

$= 2^9 \times 2^5$

$= 2^{14}$

Correct Answer: **A**

26 votes -- Lakshman Patel (69.5k points)

6.4 Modular Arithmetic (1)

6.4.1 Modular Arithmetic: TIFR2018-B-1

https://gateoverflow.in/179285

What is the remainder when 4444^{4444} is divided by 9?

A. 1 B. 2 C. 5 D. 7 E. 8

tifr2018 modular-arithmetic combinatory

Answers: Modular Arithmetic

6.4.1 Modular Arithmetic: TIFR2018-B-1

https://gateoverflow.in/179285

✔ Remainder theorem can be used to find the remainder

Firstly we have to calculate the Euler Totient function of 9 i.e $\phi(9)$

Euler totient function of of n gives the number of positive integers upto n which are relatively prime to n

Euler number of 9=3^2 i.e. $\phi(9) = 9 * [1 - \frac{1}{3}] = 6$ which are 1,2,4,5,7,8

Given $\frac{4444^{4444}}{9}$

Using remainder theorem

$\frac{rem(4444/9)^{rem(4444/6)}}{9} = \frac{(7)^4}{9} = 7$(remainder)

Hence option d) is the correct answer

14 votes -- Ashwani Kumar (13.1k points)

6.5 Pigeonhole Principle (2)

6.5.1 Pigeonhole Principle: TIFR2014-A-5

https://gateoverflow.in/25990

The rules for the University of Bombay five-a-side cricket competition specify that the members of each team must have birthdays in the same month. What is the minimum number of mathematics students needed to be enrolled in the department to guarantee that they can raise a team of students?

A. 23 B. 91 C. 60 D. 49 E. None of the above

tifr2014 combinatory discrete-mathematics normal pigeonhole-principle

6.5.2 Pigeonhole Principle: TIFR2018-A-6

https://gateoverflow.in/179275

What is the minimum number of students needed in a class to guarantee that there are at least 6 students whose birthdays fall in the same month ?

A. 6 B. 23 C. 61 D. 72 E. 91

tifr2018 pigeonhole-principle combinatory

Answers: Pigeonhole Principle

6.5.1 Pigeonhole Principle: TIFR2014-A-5

https://gateoverflow.in/25990

✔ There are 12 months and we have to get 5 people having birthdays in the same month in order to form a team. Pigeon hole principle can be applied here :

$\lceil \frac{N}{12} \rceil = 5,$

$N = 49.$

Hence answer is **D**

26 votes -- Riya Roy(Arayana) (5.3k points)

6.5.2 Pigeonhole Principle: TIFR2018-A-6

https://gateoverflow.in/179275

✔ Using pigeonhole principle: With n pigeon and m holes atleast p pigeo will be on m holes are:

$\frac{n-1}{m} + 1 = p$

$n - 1 + 12 = 6 \times 12$

$n - 1 = 72 - 12 = 60$

$n = 60 + 1 = 61$

Correct Answer: C

14 votes -- Anu007 (14.4k points)

6.6 Recurrence (2)

6.6.1 Recurrence: TIFR2014-A-3

https://gateoverflow.in/25988

The Fibonacci sequence is defined as follows: $F_0 = 0, F_1 = 1,$ and for all integers $n \geq 2, F_n = F_{n-1} + F_{n-2}$. Then which of the following statements is FALSE?

A. $F_{n+2} = 1 + \sum_{i=0}^{n} F_i$ for any integer $n \geq 0$
B. $F_{n+2} \geq \emptyset^n$ for any integer $n \geq 0$, where $\emptyset = \left(\sqrt{5} + 1\right)/2$ is the positive root of $x^2 - x - 1 = 0$.
C. F_{3n} is even, for every integer $n \geq 0$.
D. F_{4n} is a multiple of 3, for every integer $n \geq 0$.
E. F_{5n} is a multiple of 4, for every integer $n \geq 0$.

tifr2014 recurrence easy

6.6.2 Recurrence: TIFR2017-A-7

https://gateoverflow.in/95037

Consider the sequence $S_0, S_1, S_2, \ldots$ defined as follows: $S_0 = 0,\ S_1 = 1$ and $S_n = 2S_{n-1} + S_{n-2}$ for $n \geq 2$. Which of the following statements is FALSE?

A. for every $n \geq 1$, S_{2n} is even
B. for every $n \geq 1$, S_{2n+1} is odd
C. for every $n \geq 1$, S_{3n} is multiple of 3
D. for every $n \geq 1$, S_{4n} is multiple of 6
E. none of the above

tifr2017 recurrence

Answers: Recurrence

6.6.1 Recurrence: TIFR2014-A-3

https://gateoverflow.in/25988

✔

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	1	1	2	3	5	8	13

Option (**e**) is FALSE. F_{5n} is a multiple of 4, for every integer $n \geq 0$.

14 votes

-- Umang Raman (12.2k points)

6.6.2 Recurrence: TIFR2017-A-7

https://gateoverflow.in/95037

✔ $S_n = 2S_{n-1} + S_{n-2}$

Characterstic polynomial for this recurrence is $x^2 = 2x + 1$

$x^2 - 2x - 1 = 0 \Rightarrow x_1 = (1 + \sqrt{2}), x_2 = (1 - \sqrt{2})$

The solution to the recurrence relation is of the form : $S_n = C_1 \times x_1^n + C_2 \times x_2^n$

Putting $S(0) = 0$, $C_1 + C_2 = 0$

Putting $S(1) = 1$, $C_1 \times (1 + \sqrt{2}) + C_2 \times (1 - \sqrt{2}) = 1$

Solving these two, we get $C_1 = \frac{1}{2\sqrt{2}}$ and $C_2 = -\frac{1}{2\sqrt{2}}$

$$S_n = \frac{1}{2\sqrt{2}}\left((1 + \sqrt{2})^n - (1 - \sqrt{2})^n\right)$$

$S_0 = 0, S_1 = 1, S_2 = 2, S_3 = 5, S_4 = 12, S_5 = 29, S_6 = 70$

Clearly S_3 and S_6 are not a multiple of 3

Hence (**C**) is correct answer.

17 votes

-- Manish Joshi (20.5k points)

Answer Keys

6.0.1	D	6.0.2	C	6.1.1	C	6.1.2	A	6.1.3	D
6.1.4	E	6.2.1	A	6.2.2	D	6.2.3	A	6.2.4	C
6.3.1	A	6.4.1	D	6.5.1	D	6.5.2	C	6.6.1	E
6.6.2	C								

7 Discrete Mathematics: Graph Theory (17)

Syllabus: Connectivity, Matching, Coloring.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	0	0	1	1	0	1	0	1	0	0.55	1
2 Marks Count	1	0	1	1	1	0	0	0	0	0	0.44	1
Total Marks	3	0	2	3	3	0	1	0	1	0	1.44	3

7.1 Counting (1)

7.1.1 Counting: TIFR2017-B-12

https://gateoverflow.in/95819

An undirected graph is complete if there is an edge between every pair of vertices. Given a complete undirected graph on n vertices, in how many ways can you choose a direction for the edges so that there are no directed cycles?

A. n
B. $\frac{n(n-1)}{2}$
C. $n!$
D. 2^n
E. 2^m, where $m = \frac{n(n-1)}{2}$

tifr2017 graph-theory counting

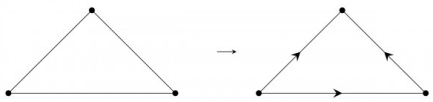
Answers: Counting

7.1.1 Counting: TIFR2017-B-12

https://gateoverflow.in/95819

✔ They are asking to convert Complete Undirected graph into Directed graph without cycle by choosing direction for the edges.

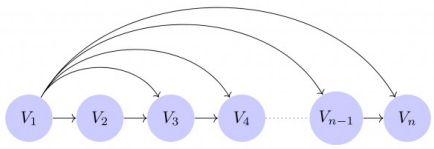
See this K_3 graph-



(Image ref)

By this time you must have got the problem statement meaning- our resultant graph should be acyclic.

Lets say we have a complete graph G which has n vertices, $v_1, v_2, \ldots v_n$. To convert it into the resultant graph we have to assign direction to each edge. Now see, our resultant graph is acyclic therefore it must have a topological order.

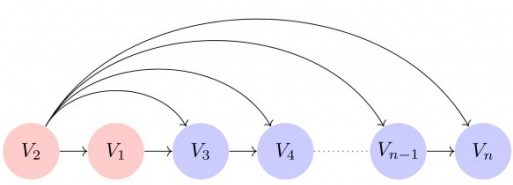


(I have not drawn all edges except V_1 edges.)

here every rearrangement of vertices in topological sort leads to one particular combination to choose the direction of edges.

Hence - $n!$ is answer.

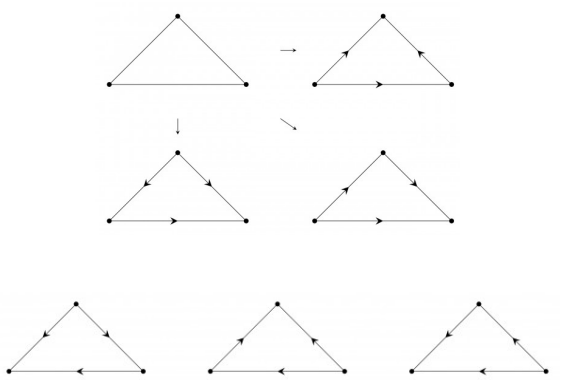
Just to illustrate more, see one of the permutation out of $n!$



These two permutation shows that undirected edge between V_1 and V_2, was first chosen as $V_1 \to V_2$ and then $V_2 \to V_1$

Don't think about the labeling of vertices, If I do unlabelling of all $n!$ permutations then all structures are same. But it doesn't matter If I am arriving at the same structure, What matters is, In how many ways I can reach to that.

See this-



All the above 3 structures are Isomorphic. But still, there are $3!$ ways to reach such structure.

Let there be 3 vertices,

In how many ways we can arrage them? It is $3! = 6$

i. a, b, c
ii. a, c, b
iii. b, a, c
iv. b, c, a
v. c, a, b
vi. c, b, a

Let's take each order, and check in how many ways we can make it as directed edges (remember there are 3 edges in K3) with it

ex :- b, a, c : we make $b \rightarrow a \rightarrow c$. Now the remaining edge must be $b \rightarrow c$; there is no other possibility for it, as if we make $c \rightarrow b$ then it is cyclic.

So, there is only one way to assign edges, to each order !

Now, you got doubt that " **There may be many assignment of direction of edges than permutations ?** "

Answer for this question is, Should be NO. (think about it, let there be an edge $u \rightarrow v$, then u should be before v)

In how many ways we can choose direction for the edges so that there are no directed cycles? $\rightarrow$ Number of permutations of vertices $= n!$

Correct Answer: C

References

68 votes

-- Sachin Mittal (15.9k points)

7.2 Degree Of Graph (3)

7.2.1 Degree Of Graph: TIFR2010-B-36

https://gateoverflow.in/19248

In a directed graph, every vertex has exactly seven edges coming in. What can one always say about the number of edges going out of its vertices?

A. Exactly seven edges leave every vertex.
B. Exactly seven edges leave some vertex.
C. Some vertex has at least seven edges leaving it.
D. The number of edges coming out of vertex is odd.
E. None of the above.

tifr2010 graph-theory degree-of-graph

7.2.2 Degree Of Graph: TIFR2012-B-2

https://gateoverflow.in/25047

In a graph, the degree of a vertex is the number of edges incident (connected) on it. Which of the following is true for

every graph G?

A. There are even number of vertices of even degree.
B. There are odd number of vertices of even degree.
C. There are even number of vertices of odd degree.
D. There are odd number of vertices of odd degree.
E. All the vertices are of even degree.

tifr2012 graph-theory degree-of-graph

7.2.3 Degree Of Graph: TIFR2018-B-8

https://gateoverflow.in/179292

In an undirected graph G with n vertices, vertex 1 has degree 1, while each vertex $2, \ldots, n-1$ has degree 10 and the degree of vertex n is unknown, Which of the following statement must be TRUE on the graph G?

A. There is a path from vertex 1 to vertex n.
B. There is a path from vertex 1 to each vertex $2, \ldots, n-1$.
C. Vertex n has degree 1.
D. The diameter of the graph is at most $\frac{n}{10}$
E. All of the above choices must be TRUE

tifr2018 graph-theory degree-of-graph

Answers: Degree Of Graph

7.2.1 Degree Of Graph: TIFR2010-B-36

https://gateoverflow.in/19248

✔ Since 7 edges come to every vertex, total no. of edges leaving n vertices must be $7n$. So, option (A) is a possibility but it needn't be always true. We can have 8 edges leave one vertex and 6 edges leave another (and similarly any other combination of outgoing edges ensuring total no. of outgoing edges remain constant). But option (C) must always be true as if none of the n vertices have at least 7 edges leaving, sum of outgoing edges can never be $7n$.

👍 37 votes -- Arjun Suresh (334k points)

7.2.2 Degree Of Graph: TIFR2012-B-2

https://gateoverflow.in/25047

✔ As we know that sum of degree of vertex $= 2 \times edges$.
let there is u vertex with odd degrees and v vertex with even degrees.

Then $\sum(u) + \sum(v) = 2e$.
now $2e = \text{even}$.
$\sum(v) =$ sum of even number will be even.
$\sum(u) =$ if you consider odd number of vertices of odd degree then sum will be odd and this will violate $2e$
so there will be always the even number of vertices with odd degree

Hence, Ans is (**c**)There are the even number of vertices of odd degree.

👍 17 votes -- Umang Raman (12.2k points)

7.2.3 Degree Of Graph: TIFR2018-B-8

https://gateoverflow.in/179292

✔ We have to recall couple of theorems before solving this question.

1. Number of odd degree vertices in any connected component of a graph is even.
2. The degree sum of all the vertices in any connected component of a graph is even.

Now in this question graph G has $n-2$ vertices with degree 10, one vertex with degree 1.
Because of theorem (1), vertex n and vertex 1 should be in the same connected component.

Proof: Lets suppose vertex 1 and vertex n are not in the same connected component. Also assume that vertex 1 is in component C_1. Now C_1 has only one vertex with odd degree which is vertex 1, because vertex 2 to vertex $n-1$ has even degree (10) and this violates theorem (1). So, C_1 must have vertex 1 as well as vertex n and vertex n should have odd degree.

Now, since vertex 1 and vertex n are in the same connected component and the given graph is undirected so, there must be a path from vertex 1 to vertex n. Hence option (a) is true.

Option (b) is false. Proof by counter example:
Lets take a graph G where $n = 13$ with two connected components C_1 and C_2. C_1 has two vertices vertex 1 and vertex n and only one edge $(\text{vertex } 1, \text{vertex } n)$. Hence, degree of vertex 1 as well as vertex n is 1. C_2 has rest 11 vertices (vertex 2 to vertex $n-1$) and C_2 is a complete component (there is exactly one edge between every pair of vertices of C_2), hence degree of vertex $2, \ldots,$ vertex $n-1$ is 10. In this graph there is no path from vertex 1 to vertex $2, \ldots,$ vertex $n-1$..

Option (c) is False. Proof by counter example:

Lets take graph G where $n = 12$ and each pair of vertices from vertex 2 to vertex n are connected with each other by an edge and additionally there is an ede between vertex n and vertex 1. Hence, vertex 1 has degree 1 and vertex $2, \ldots,$ vertex $n-1$ has degree 10 and vertex n has degree 11. In this graph vertex n has degree 11, and hence option C is wrong.

Option (d) is false.
Proof: As we have seen an example as part of proving that option (b) is wrong where, graph G can be disconnected and a disconnected graph has infinite diameter. Hence, option (d) is wrong.

Option (e) is false since options b, c, d are false.

40 votes -- Sourav Basu (2.7k points)

7.3 Graph Coloring (5)

7.3.1 Graph Coloring: TIFR2013-B-1 https://gateoverflow.in/25508

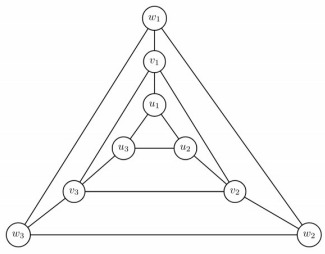
Let $G = (V, E)$ be a simple undirected graph on n vertices. A colouring of G is an assignment of colours to each vertex such that endpoints of every edge are given different colours. Let $\chi(G)$ denote the chromatic number of G, i.e. the minimum numbers of colours needed for a valid colouring of G. A set $B \subseteq V$ is an independent set if no pair of vertices in B is connected by an edge. Let $a(G)$ be the number of vertices in a largest possible independent set in G. In the absence of any further information about G we can conclude.

A. $\chi(G) \geq a(G)$
B. $\chi(G) \leq a(G)$
C. $a(G) \geq \frac{n}{\chi(G)}$
D. $a(G) \leq \frac{n}{\chi(G)}$
E. None of the above

tifr2013 graph-theory graph-coloring

7.3.2 Graph Coloring: TIFR2017-B-1 https://gateoverflow.in/95669

A vertex colouring with three colours of a graph $G = (V, E)$ is a mapping $c : V \to \{R, G, B\}$ so that adjacent vertices receive distinct colours. Consider the following undirected graph.



How many vertex colouring with three colours does this graph have?

A. 3^9 B. 6^3 C. 3×2^8 D. 27 E. 24

tifr2017 graph-theory graph-coloring

7.3.3 Graph Coloring: TIFR2017-B-10 https://gateoverflow.in/95817

A vertex colouring of a graph $G = (V, E)$ with k coulours is a mapping $c : V \to \{1, \ldots, k\}$ such that $c(u) \neq c(v)$ for every $(u, v) \in E$. Consider the following statements:

i. If every vertex in G has degree at most d then G admits a vertex coulouring using $d + 1$ colours.

ii. Every cycle admits a vertex colouring using 2 colours
iii. Every tree admits a vertex colouring using 2 colours

Which of the above statements is/are TRUE? Choose from the following options:

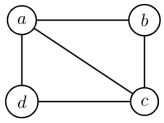
A. only i B. only i and ii C. only i and iii D. only ii and iii E. i, ii, and iii

tifr2017 graph-theory graph-coloring

7.3.4 Graph Coloring: TIFR2018-A-9

https://gateoverflow.in/179388

How many ways are there to assign colours from range $\{1, 2, \ldots, r\}$ to vertices of the following graph so that adjacent vertices receive distinct colours?



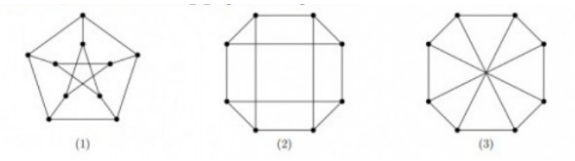
A. r^4
B. $r^4 - 4r^3$
C. $r^4 - 5r^3 + 8r^2 - 4r$
D. $r^4 - 4r^3 + 9r^2 - 3r$
E. $r^4 - 5r^3 + 10r^2 - 15r$

tifr2018 graph-theory graph-coloring

7.3.5 Graph Coloring: TIFR2020-B-11

https://gateoverflow.in/333132

Which of the following graphs are bipartite?



A. Only (1)
B. Only (2)
C. Only (2) and (3)
D. None of $(1), (2), (3)$
E. All of $(1), (2), (3)$

tifr2020 engineering-mathematics graph-theory graph-coloring

Answers: Graph Coloring

7.3.1 Graph Coloring: TIFR2013-B-1

https://gateoverflow.in/25508

✔ **Independence number :** Size of largest maximum independent set: $a(G)$ (it covers all adjacent vertices)
Chromatic Number : Minimum No. of color required to properly color the graph: $\chi(G)$

The vertices of G can be partitioned into $\chi(G)$ monochromatic classes. Each class is an independent set, and hence cannot have size larger than $\alpha(G)$.

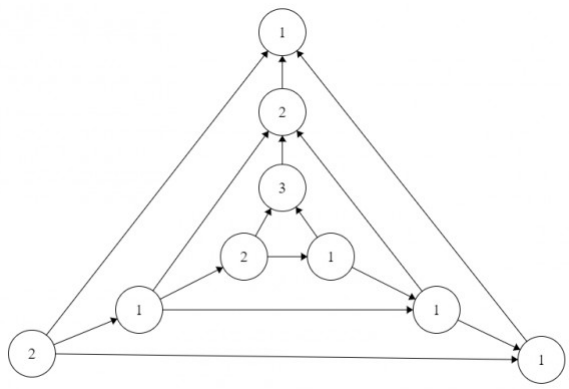
$a(G)\chi(G) \geq n$. (It is a theorem),
Option C.

25 votes

-- Umang Raman (12.2k points)

7.3.2 Graph Coloring: TIFR2017-B-1

https://gateoverflow.in/95669



Start with the Inner one which can be filled in $\Rightarrow 3 \times 2 \times 1 = 6$ ways.

Then, middle one can be filled in $\Rightarrow 2 \times 1 \times 1 = 2$ ways.

Then, similarly outermost can be filled in $\Rightarrow 2 \times 1 \times 1 = 2$ ways.

Hence, Total number of ways to fill this figure $\Rightarrow 6 \times 2 \times 2 = 24$ ways.

Correct Answer: E

43 votes -- Kapil Phulwani (35.2k points)

7.3.3 Graph Coloring: TIFR2017-B-10

https://gateoverflow.in/95817

i. is true, since in worst case the graph can be complete. So, $d+1$ colours are necessary for graph containing vertices with degree atmost $'d'$. Example : Consider a complete graph of 4 vertices ... K_4

ii. is false since cyles with odd no of vertices require 3 colours.

iii. is true, since each level of the tree must be coloured in an alternate fashion. We can do this with two colours. Its a theorem that a tree is 2 colourable ...

Therefore, option **C** is correct.

23 votes -- tarun_svbk (1.4k points)

7.3.4 Graph Coloring: TIFR2018-A-9

https://gateoverflow.in/179388

The graph can be colored using minimum 3 colors and maximum 4 colors.

No. of ways using 3 colors $= r(r-1)(r-2)$.

No. of ways using 4 colors $= r(r-1)(r-2)(r-3)$.

Total no. of ways $= r(r-1)(r-2) + r(r-1)(r-2)(r-3)$

$= r^4 - 5r^3 + 8r^2 - 4r$

OPTION (C)

28 votes -- ZAHID WAKEEL (1.6k points)

7.3.5 Graph Coloring: TIFR2020-B-11

https://gateoverflow.in/333132

A graph $G = (V(G), E(G))$ is said to be Bipartite if and only if there exists a partition $V(G) = A \cup B$ and $A \cap B = \emptyset$. Hence all edges share a vertex from both set A and B, and there are no edges formed between two vertices in the set A, and there are not edges formed between the two vertices in B.

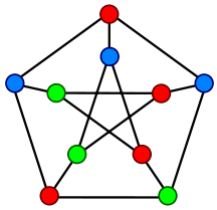
(OR)

A bipartite graph, also called a bigraph, is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent.

- G is bipartite if and only if all cycles in G are of even length.

- Every tree is bipartite.
- A graph is bipartite if and only if it is 2-colorable , $(\text{i.e. its chromatic number is less than or equal to } 2).$

The first graph is known as the Peterson graph.



It is not a bipartite graph.

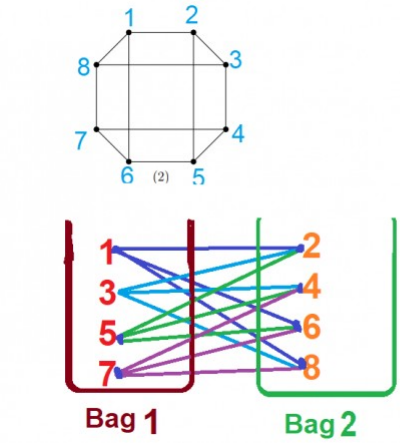
The Petersen graph has chromatic number 3, meaning that its vertices can be colored with three colors — but not with two — such that no edge connects vertices of the same color. It has a list colouring with 3 colours, by Brooks' theorem for list colourings.

The Petersen graph has chromatic index 4; coloring the edges requires four colors. As a connected bridgeless cubic graph with chromatic index four, the Petersen graph is a snark.

Reference:

- https://en.wikipedia.org/wiki/Petersen_graph
- https://www.win.tue.nl/~aeb/graphs/Petersen.html
- https://www.win.tue.nl/~aeb/graphs/Petersen.html
- https://graph.subwiki.org/wiki/Petersen_graph

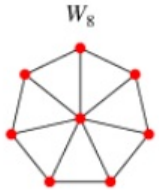
The second graph is a bipartite graph because it is 2-colorable.



The third graph is the wheel graph.

In a wheel graph, the hub has degree $n-1$, and other nodes have degree 3. Wheel graphs are 3-connected. $W_4 = K_4$, where K_4 is the complete graph of order four. The chromatic number of W_n is

$$\chi(W_n) = \begin{cases} 3 & \text{for n odd} \\ 4 & \text{for n even} \end{cases}$$



It is not a bipartite graph.

Reference:

- http://mathworld.wolfram.com/BipartiteGraph.html
- http://discrete.openmathbooks.org/dmoi2/sec_matchings.html

So, the correct answer is (B).

References

1 votes -- Lakshman Patel (69.5k points)

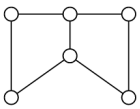
7.4 Graph Connectivity (4)

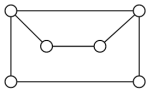
7.4.1 Graph Connectivity: TIFR2015-B-5

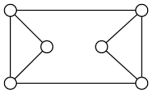
https://gateoverflow.in/29858

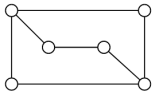
Suppose $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$

is the adjacency matrix of an undirected graph with six vertices: that is, the rows and columns are indexed by vertices of the graph, and an entry is 1 if the corresponding vertices are connected by an edge and is 0 otherwise; the same order of vertices is used for the rows and columns. Which of the graphs below has the above adjacency matrix?

i. 

ii. 

iii. 

iv. 

A. Only (i)
B. Only (ii)
C. Only (iii)
D. Only (iv)
E. (i) and (ii)

tifr2015 graph-connectivity graph-theory

7.4.2 Graph Connectivity: TIFR2019-B-12

https://gateoverflow.in/280483

Let $G = (V, E)$ be a directed graph with $n(\geq 2)$ vertices, including a special vertex r. Each edge $e \in E$ has a strictly positive edge weight $w(e)$. An arborescence in G rooted at r is a subgraph H of G in which every vertex $u \in V \backslash \{r\}$ has a directed path to the special vertex r. The weight of an arborescence H is the sum of the weights of the edges in H.

Let H^* be a minimum arborescence rooted at r, and w^* the weight of H^*. Which of the following is NOT always true?

A. $w^* \geq \sum_{u \in V \backslash \{r\}} \min_{(u,v) \in E} w((u,v))$
B. $w^* \geq \sum_{u \in V \backslash \{r\}} \min_{(v,u) \in E} w((v,u))$
C. H^* has exactly $n-1$ edges
D. H^* is acyclic
E. w^* is less than the weight of the minimum weight directed Hamiltonian cycle in G, when G has a directed Hamiltonian cycle

tifr2019 graph-connectivity graph-theory difficult

7.4.3 Graph Connectivity: TIFR2019-B-15

https://gateoverflow.in/280480

Consider directed graphs on n labelled vertices $\{1, 2, \ldots, n\}$, where each vertex has exactly one edge coming in and exactly one edge going out. We allow self-loops. How many graphs have exactly two cycles ?

A. $\sum_{k=1}^{n-1} k!(n-k)!$

B. $\frac{n!}{2}\left[\sum_{k=1}^{n-1} \frac{1}{k(n-k)}\right]$

C. $n!\left[\sum_{k=1}^{n-1} \frac{1}{k}\right]$

D. $\frac{n!(n-1)}{2}$

E. None of the above

tifr2019 graph-connectivity graph-theory

7.4.4 Graph Connectivity: TIFR2019-B-3

https://gateoverflow.in/280492

A graph is d – regular if every vertex has degree d. For a d – regular graph on n vertices, which of the following must be TRUE?

A. d divides n

B. Both d and n are even

C. Both d and n are odd

D. At least one of d and n is odd

E. At least one of d and n is even

tifr2019 graph-connectivity graph-theory

Answers: Graph Connectivity

7.4.1 Graph Connectivity: TIFR2015-B-5

https://gateoverflow.in/29858

✔ Yes, Option (**E**) must be the right answer.

Number of edges in the graph:

Since the graphs are undirected, it can be observed that there will be two 1's in the adjacency matrix corresponding to each edge in the graph.

For example, suppose two there is an edge between nodes A & B, then there will be 1 in position $[A, B]$ & there will be a 1 in position $[B, A]$ of the adjacency matrix.

That's why the given adjacency matrix is symmetric.

So the number of edges in the graph must be equal to half the number of 1's in the adjacency matrix.

Hence number of edges will be 7 in the graph.

All the other graphs except (iii), have 7 edges.So it is clear that the adjacency matrix does not represents graph (iii).

Isomorphism:

From the definition of Isomorphic graphs, it can be inferred that,

Isomorphic graphs must have same (adjacency matrix) representation.

Thus after eliminating graph (iii) we have to check for isomorphism among graphs (i), (ii) & (iv).

It can clearly be observed that graphs (ii) & (iv) are not isomorphic to each other.

It can also be observed that graph (i) & (ii) are isomorphic(Rotate graph (i) by 90 degree left/right.

Graph (ii) is looking like a closed envelope in the figure, try to view it like an open envelope, like a trapezium over a rectangle.)

So now it can be inferred that either the adjacency matrix is representing both graphs (i) & (ii) or it is only representing (iv).

Cycles of length 6 :

Now from the adjacency matrix it can be observed that there should be a cycle of length 6 in the graph, since $[1, 2], [2, 3], [3, 4], [4, 5], [5, 6], [6, 1]$ are all 1's in the matrix.(as 1 at any position $[x, y]$ represents an edge between x & y in the graph).

& both graphs (i) & (ii) have cycles of length 6, but graph (iv) does not has any cycle of length 6, it has cycles of length 4 & 5 only.

Thus graph (iv) can not have the above adjacency matrix.

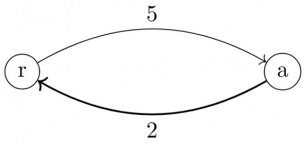
Hence the adjacency matrix represents graphs (i) & (ii).

28 votes -- Anurag Pandey (10.5k points)

7.4.2 Graph Connectivity: TIFR2019-B-12

https://gateoverflow.in/280483

For these questions it is better to get some counter examples by trying small graphs. Consider the following graph:



Now, as per the question, an arborescence is a subgraph with all incoming edges to the special vertex r. So, for the above graph it is just the edge $a - r$ which is of weight 2.

Now, option A says that if we sum up all the outgoing edge weights (minimum weight in case multiple outgoing edges exist) of all vertices excluding r, the sum will be greater than or equal to the weight of the minimum arborescence. This is true because all the edges in the minimum arborescence will be included here as the outgoing edges of the other vertices.

Now, **option B is false**. Because instead of outgoing edges we are adding the weights of incoming edges. And these edges are not part of the arborescence. For example, in the above graph, only one vertex $a \in V \backslash \{r\}$.

$\sum_{u \in V \backslash \{r\}} \min_{(v,u) \in E} w((v,u)) = w(r,a) = 5 > w^*$.

Options C and D are straightforward TRUE as for minimum arborescence we only need to consider the incoming edges to r from $n-1$ other vertices thus making it acyclic too.

For option E, suppose a graph has a directed Hamiltonian cycle. In an arborescence every vertex is connected and we need "one more" edge to make it a cycle. But this may or may not be a Hamiltonian cycle as a Hamiltonian cycle requires that **in the path from the start and end of any vertex, no vertex is repeated**. But if we take any directed Hamiltonian cycle, every vertex has a directed path to every other vertex (special vertex r can be any of the given vertices) and so it is an arborescence. What will happen if we remove an edge (u,v) from such a minimum weighted directed Hamiltonian cycle?. Then we no longer have a path to vertex v but all vertices still have a path to vertex u and so u can be the special vertex making it still an arborescence. Since the weight of this edge (u,v) is guaranteed to be positive, this means the weight of this arborescence must be STRICTLY less than the weight of the minimum Hamiltonian cycle. Thus option E is TRUE.

So, Correct answer: B.

9 votes

-- Arjun Suresh (334k points)

7.4.3 Graph Connectivity: TIFR2019-B-15

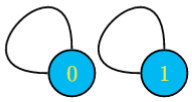
https://gateoverflow.in/280480

✔ We need to find from $\{1, 2, \ldots, n\}$ vertices how many graphs exist which are having exactly two cycles when self loops are allowed.

When $n = 1$ there is no such graph with two cycles.

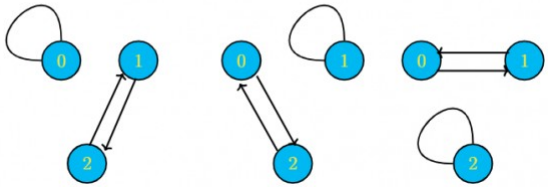
0 graph.

When $n = 2$



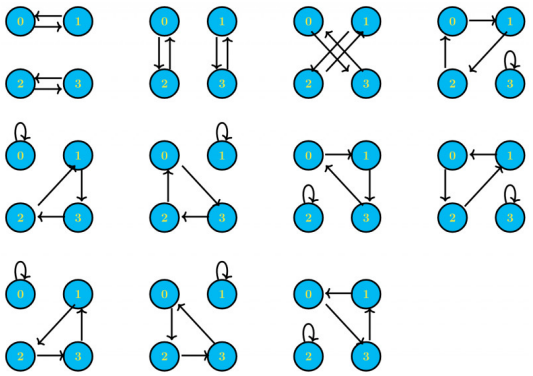
1 graph exists.

When $n = 3$



3 graphs exist.

When $n = 4$



11 graphs exist.

For $n = 4$

(A) $\sum_{k=1}^{n-1} k!(n-k)! = 16$

(B) $\frac{n!}{2}[\sum_{k=1}^{n-1} \frac{1}{k(n-k)}] = 11$

(C) $n![\sum_{k=1}^{n-1} \frac{1}{k}] = 44$

(D) $\frac{n!(n-1)}{2} = 36$

Option B is answer.

14 votes -- Shubhgupta (6.5k points)

7.4.4 Graph Connectivity: TIFR2019-B-3

https://gateoverflow.in/280492

✔ We know that the sum of degrees is twice the number of edges.

Now, sum of degrees is nd, as there are n vertices of degree d each.

As nd is even, either one of them should be definitely even.

14 votes -- Gokulnath (993 points)

7.5 Line Graph (1)

7.5.1 Line Graph: TIFR2017-B-13

https://gateoverflow.in/95821

For an undirected graph $G = (V, E)$, the line graph $G' = (V', E')$ is obtained by replacing each edge in E by a vertex, and adding an edge between two vertices in V' if the corresponding edges in G are incident on the same vertex. Which of the following is TRUE of line graphs?

A. the line graph for a complete graph is complete
B. the line graph for a connected graph is connected
C. the line graph for a bipartite graph is bipartite
D. the maximum degree of any vertex in the line graph is at most the maximum degree in the original graph
E. each vertex in the line graph has degree one or two

tifr2017 graph-theory line-graph

Answers: Line Graph

7.5.1 Line Graph: TIFR2017-B-13

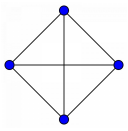
https://gateoverflow.in/95821

✔ Option **B** is the right answer

We can solve this question by eliminating options

Option (A) False

Let us take a complete graph of 4 vertices:



In the corresponding line graph, number of edges is

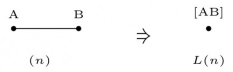
$\sum_{i=0}^{n} {}^{d_i}C_2, \quad d_i$ is degree of each vertex

$= \frac{3\times2}{2} + \frac{3\times2}{2} + \frac{3\times2}{2} + \frac{3\times2}{2} = 3 \times 4 = 12$

No. of vertices in line graph $=$ No. of edges in original graph $= 6$.
So, no. of edges to make complete graph with 6 vertices $= \frac{6\times5}{2} = 3 \times 5 = 15$
But we got only 12 edges for the line graph.
Contradiction.

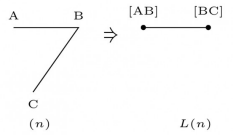
Option (B) True

Smallest line graph for the original graph with just one edge

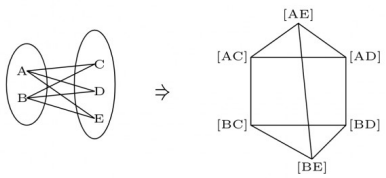


which is also connected graph

If a graph is connected with more then one edge, it's line graph will never be disconnected



Option (C) False

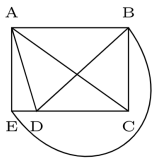


This cannot be 2-colorable and hence is not bipartite.

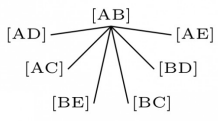
Option (D) False

Because in the line graph degree of a vertex depends on the number of neighbors its corresponding edge has in the original graph.

e.g., $[A, B]$ as a point in the line graph, then the degree of this vertex depends on the degree of A and degree of B in the original graph.



I'm drawing a degree for a point $[AB]$ in the line graph.



So, this is wrong.

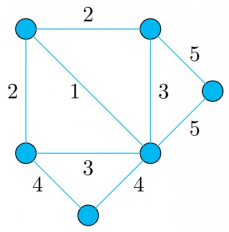
Option (E) is false (wrong as proved in the above option (D))

14 votes -- SAKET NANDAN (4.2k points)

7.6 Minimum Spanning Trees (1)

7.6.1 Minimum Spanning Trees: TIFR2018-B-3 https://gateoverflow.in/179287

How many distinct minimum weight spanning trees does the following undirected, weighted graph have ?



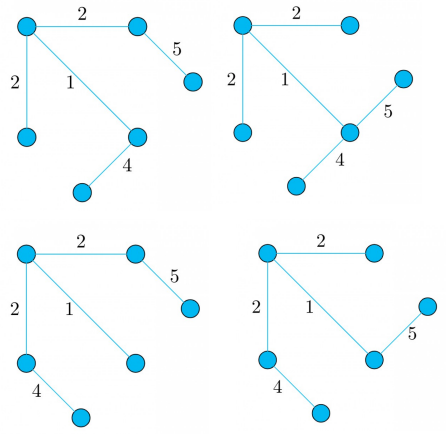
A. 1 B. 2 C. 4 D. 6 E. 8

tifr2018 graph-theory minimum-spanning-trees

Answers: Minimum Spanning Trees

7.6.1 Minimum Spanning Trees: TIFR2018-B-3 https://gateoverflow.in/179287

✔ Minimum Weight Spanning Trees of the given graph are



4 Minimum Weight Spanning Trees each having cost of 14

Hence, option (C) is the correct answer

8 votes -- Ashwani Kumar (13.1k points)

7.7 Spanning Tree (1)

7.7.1 Spanning Tree: TIFR2015-B-11 https://gateoverflow.in/30043

Let K_n be the complete graph on n vertices labeled $\{1, 2, \ldots, n\}$ with $m = \frac{n(n-1)}{2}$ edges. What is the number of spanning trees of K_n?

A. $\frac{m}{n-1}$ B. m^{n-1} C. n^{n-2} D. n^{n-1} E. None of the above

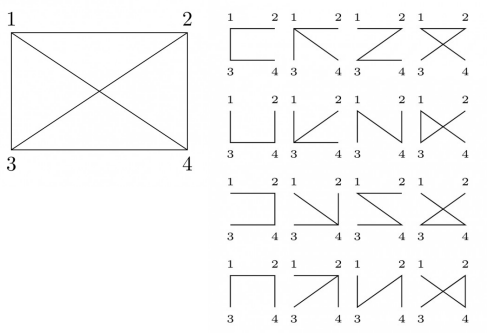
tifr2015 graph-theory spanning-tree

Answers: Spanning Tree

7.7.1 Spanning Tree: TIFR2015-B-11 https://gateoverflow.in/30043

✔ **Answer will be (C)**

e.g. for K_4 no. of spanning tree will be 16.



Number of Spanning trees of a Complete Graph K_n is given by Cayley's theorem = n^{n-2}.

12 votes -- srestha (85.3k points)

7.8 Trees (1)

7.8.1 Trees: TIFR2011-B-33

https://gateoverflow.in/20624

Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

A. G is connected and has $n-1$ edges.
B. G is acyclic and connected.
C. G is acyclic and has $n-1$ edges.
D. G is acyclic, connected and has $n-1$ edges.
E. G has $n-1$ edges.

tifr2011 graph-theory trees

Answers: Trees

7.8.1 Trees: TIFR2011-B-33

https://gateoverflow.in/20624

✔ **Option a $\iff$ Option b $\iff$ Option c $\iff$ Option d.**

- You need at least $n-1$ edges to have a connected graph. This leaves no edges to make any cycles. Thus, Option a $\implies$ G is acyclic.
- A connected graph with $n-1$ edges is acyclic, as shown above. Now, if we add any more edges to this graph, we will be connecting two vertices that are already connected. Thus, adding any more than edges to a connected graph will cause cycles. So, if a graph is acyclic and connected, it has exactly $(n-1)$ edges.
- You can't fit $(n-1)$ edges between $(n-1)$ vertices without causing cycles. Thus, if graph with $(n-1)$ edges is acyclic, it must connect n vertices. Hence, an acyclic graph with $(n-1)$ edges is connected.

Thus, all options, a to d are equivalent.

Option b $\iff$ G is a tree.

- Any acyclic connected graph is a tree by definition.
- A graph G is a tree if it is both acyclic and connected by definition.

Thus, all options a to d are both necessary and sufficient for an undirected graph G to be a tree.

Option e $\not\implies$ G is a tree.

- Since G is not constrained to be acyclic, we can create a cyclic graph with $(n-1)$ edges. This graph will be cyclic, and it won't be connected. And thus, it won't be a tree.

Hence, option E is the correct answer.

30 votes -- Pragy Agarwal (18.3k points)

Answer Keys

7.1.1	C	7.2.1	C	7.2.2	C	7.2.3	A	7.3.1	C
7.3.2	E	7.3.3	C	7.3.4	C	7.3.5	B	7.4.1	E

7.4.2	B	7.4.3	B	7.4.4	E	7.5.1	B	7.6.1	C
7.7.1	C	7.8.1	E						

8 Discrete Mathematics: Mathematical Logic (15)

Syllabus: Propositional and first order logic.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	1	0	0	0	2	1	1	1	0	0.77	2
2 Marks Count	0	0	1	1	1	1	0	0	1	0	0.55	1
Total Marks	1	1	2	2	2	4	1	1	3	1	1.88	4

8.1 First Order Logic (7)

8.1.1 First Order Logic: TIFR2010-A-8

https://gateoverflow.in/18239

Which of the following is NOT necessarily true? { Notation: The symbol "$\neg$"notes negation; $P(x,y)$ means that for given x and y, the property $P(x,y)$ is true }.

A. $(\forall x \forall y P(x,y)) \Rightarrow (\forall y \forall x P(x,y))$
B. $(\forall x \exists y \neg P(x,y)) \Rightarrow \neg(\exists x \forall y P(x,y))$
C. $(\exists x \exists y P(x,y)) \Rightarrow (\exists y \exists x P(x,y))$
D. $(\exists x \forall y P(x,y)) \Rightarrow (\forall y \exists x P(x,y))$
E. $(\forall x \exists y P(x,y)) \Rightarrow (\exists y \forall x P(x,y))$

tifr2010 mathematical-logic first-order-logic

8.1.2 First Order Logic: TIFR2012-A-2

https://gateoverflow.in/20939

If $Mr.\,M$ is guilty, then no witness is lying unless he is afraid. There is a witness who is afraid. Which of the following statements is true?

(Hint: Formulate the problem using the following predicates

- $G - Mr.\,M$ is guilty
- $W(x) - x$ is a witness
- $L(x) - x$ is lying
- $A(x) - x$ is afraid)

A. $Mr.\,M$ is guilty.
B. $Mr.\,M$ is not guilty.
C. From these facts one cannot conclude that $Mr.\,M$ is guilty.
D. There is a witness who is lying.
E. No witness is lying.

tifr2012 mathematical-logic first-order-logic

8.1.3 First Order Logic: TIFR2012-B-3

https://gateoverflow.in/25048

For a person p, let $w(p)$, $A(p,y)$, $L(p)$ and $J(p)$ denote that p is a woman, p admires y, p is a lawyer and p is a judge respectively. Which of the following is the correct translation in first order logic of the sentence: "All woman who are lawyers admire some judge"?

A. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda w(y) \Lambda A(x,y)))]$
B. $\forall x : [(w(x) \Rightarrow L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$
C. $\forall x \forall y : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
D. $\exists y \forall x : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
E. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$

tifr2012 mathematical-logic first-order-logic

8.1.4 First Order Logic: TIFR2016-B-4

https://gateoverflow.in/97634

In the following, A stands for a set of apples, and $S(x,y)$ stands for "x is sweeter than y. Let

$$\Psi \equiv \exists x : x \in A$$

$$\Phi \equiv \forall x \in A : \exists y \in A : S(x,y).$$

Which of the following statements implies that there are infinitely many apples (i.e.,, A is an inifinite set)?

A. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)]$
B. $\Psi \wedge \Phi \wedge [\forall x \in A : S(x,x)]$
C. $\Psi \wedge \Phi \wedge [\forall x,y \in A : S(x,x) \wedge S(x,y) \rightarrow S(y,y)]$
D. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)] \wedge [\forall x,y,z \in A : S(x,y) \wedge S(y,z) \rightarrow s(y,x)]$
E. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)] \wedge [\forall x,y,z \in A : S(x,y) \wedge S(y,z) \rightarrow s(x,z)]$

tifr2016 mathematical-logic first-order-logic

8.1.5 First Order Logic: TIFR2017-B-11

https://gateoverflow.in/95818

Given that

- $B(x)$ means "x is a bat",
- $F(x)$ means "x is a fly", and
- $E(x,y)$ means "x eats y",

what is the best English translation of

$$\forall x(F(x) \rightarrow \forall y(E(y,x) \rightarrow B(y)))?$$

A. all flies eat bats
B. every fly is eaten by some bat
C. bats eat only flies
D. every bat eats flies
E. only bats eat flies

tifr2017 first-order-logic

8.1.6 First Order Logic: TIFR2017-B-6

https://gateoverflow.in/95689

Consider the First Order Logic (FOL) with equality and suitable function and relation symbols. Which of the following is FALSE?

A. Partial orders cannot be axiomatized in FOL
B. FOL has a complete proof system
C. Natural numbers cannot be axiomatized in FOL
D. Real numbers cannot be axiomatized in FOL
E. Relational numbers cannot be axiomatized in FOL

tifr2017 first-order-logic normal

8.1.7 First Order Logic: TIFR2019-B-4

https://gateoverflow.in/280491

Let φ be a propositional formula on a set of variables A and φ be a propositional formula on a set of variables B, such that $\varphi \Rightarrow \psi$. A Craig interpolant of φ and ψ is a propositional formula μ on variables $A \cap B$ such that $\varphi \Rightarrow \mu$ and $\mu \Rightarrow \psi$. Given propositional formula $\varphi = q \vee (r \wedge s)$ on the set of variables $A = \{q,r,s\}$ and propositional formula $\psi = \neg q \rightarrow (s \vee t)$ on the set of variables $B = \{q,s,t\}$, which of the following is a Craig interpolant for φ and ψ?

A. q B. φ itself C. $q \vee s$ D. $q \vee r$ E. $\neg q \wedge s$

tifr2019 engineering-mathematics discrete-mathematics mathematical-logic first-order-logic

Answers: First Order Logic

8.1.1 First Order Logic: TIFR2010-A-8

https://gateoverflow.in/18239

✔

a. is TRUE as both LHS and RHS are equivalent- English would be for every x, and for every $y, P(x,y)$ is TRUE. Changing y and x wouldn't change the meaning.

b. is TRUE as both LHS and RHS are equivalent- RHS is obtained by double negation of LHS.

c. Similar to (a), both are equivalent.

d. LHS: For some x, for all $y, P(x,y)$ is TRUE.
RHS: For all y and for some $x, P(x,y)$ is TRUE.

Both are not equivalent. LHS is stronger and implies RHS. For example, on the natural number set, we have $x = 1$ such that for every $y, P(x \leq y)$ is TRUE. Clearly, this implies for all y there exists some x (here x could be different for different y but on LHS, it must be the same).

e. LHS: For all x and for some $y, P(x, y)$ is TRUE.
RHS: For some y and for all $x, P(x, y)$ is TRUE.

As explained in d, these are not equivalent and here RHS is stronger than LHS, making the implication false. For example, consider the "<=" relation on the integer set. LHS is true here as for every integer we have another integer which is greater. But RHS is false as there is no single integer (infinity is not an integer) which is greater than all other integers.

Hence, (**e**). is not necessarily TRUE.

21 votes -- Arjun Suresh (334k points)

8.1.2 First Order Logic: TIFR2012-A-2

https://gateoverflow.in/20939

✔ If Mr. M is guilty, then if we pick a witness, we know that the witness won't lie unless he is afraid. If the witness is afraid, it may lie or it may lot lie (nothing is guaranteed).

However, unless we know what the victim said in the court (whether he said that Mr. M was guilty or not guilty), we can't say anything about Mr. M.

All we know is that we've a witness who is afraid, so he may or may not lie in the court. We haven't been told anything about what actually happened in the court proceeding.

So, we can't logically conclude anything about Mr. M being guilty or not guilty.

Thus, options (a) and (b) are False.

Furthermore, that witness who was afraid, he may or may not lie. Since he is afraid, we know that he "can" lie, but we're not guaranteed that he will lie.

Thus, options (d) and (e) are False too.

This leaves option c, and as we have seen earlier, we cannot conclude anything about Mr. M being guilty or not guilty.

Hence, option (c) is the correct answer.

Although not necessary, the logic equivalent of the given statement will be:

$$\Big[G \Rightarrow \neg\exists x : (W(x) \wedge L(x) \wedge \neg A(x))\Big] \equiv$$

$$\Big[G \Rightarrow \forall x : (W(x) \Rightarrow (\neg A(x) \Rightarrow \neg L(x)))\Big]$$

32 votes -- Pragy Agarwal (18.3k points)

8.1.3 First Order Logic: TIFR2012-B-3

https://gateoverflow.in/25048

✔ Just translating to English:

a. Every women who is a lawyer admires some women judge.
b. If a person being women implies she is a lawyer then she admires some judge. OR If a person is not women or is a lawyer he/she admires some judge.
c. Every women who is a lawyer admires every judge.
d. There is some judge who is admired by every women lawyer.
e. Every women lawyer admire some judge.

So, option (**e**) is the answer.

37 votes -- Arjun Suresh (334k points)

8.1.4 First Order Logic: TIFR2016-B-4

https://gateoverflow.in/97634

✔ **(E) is the answer**

Let A be $\{1, 2\}$ (say apple 1, apple 2)

There is at least one element in A : satisfied
For every element x in A there is a y in A such that $S(x, y)$: Since nothing is told about the symmetry of the relation, we

can have $S(1,2)$ and $S(2,1)$, so, satisfied

So, as of now, $A = \{1,2\}$ and $S = \{(1,2),(2,1)\}$

Now we can go through the options :

A. $S(x,x)$ is not possible ... I do not have that in S ... so satisfied ... and still finite A
B. $S(x,x)$ should be there ... well, I will make $S = \{(1,2)(2,1),(1,1),(2,2)\}$ ... satisfied and still finite A
C. take the same case as above ... satisfied and still finite A
D. Now I can not have $(1,1)$ and $(2,2)$
but even with $S = \{(1,2).(2,1)\}$ this condition is satisfied ... still finite A
E. I can not do this with $(1,2)$ and $(2,1)$ because the transitivity makes it $(1,1)$ which should not be there and whatever elements I add the transitivity will lead me to (x,x) because any element added to A should occur at least once on the left side of an ordered pair ... so only solution is infinite A.

15 votes -- Anand Vijayan (729 points)

8.1.5 First Order Logic: TIFR2017-B-11

https://gateoverflow.in/95818

✔ If x is a fly, then for all y which eats x, y is a bat. This means only bats eat flies.

Option (E).

39 votes -- Arjun Suresh (334k points)

8.1.6 First Order Logic: TIFR2017-B-6

https://gateoverflow.in/95689

✔ For the relation operator $\leq$,

- **Transitive**: $\forall x \forall y \forall z; x \leq y \wedge y \leq z \implies x \leq z$
- **Reflexive**: $\forall x; x \leq x$
- **Antisymmetric**: $\forall x \forall y; x \leq y \wedge y \leq x \implies x = y$

Now, the axioms of Partial Order is just $\text{Transitive} \wedge \text{Reflexive} \wedge \text{Antisymmetric}$

So, A is FALSE.

For Option B, quoting from Wikipedia:

> In general, logical consequence in first-order logic is only semidecidable: if a sentence A logically implies a sentence B then this can be discovered (for example, by searching for a proof until one is found, using some effective, sound, complete proof system). However, if A does not logically imply B, this does not mean that A logically implies the negation of B. There is no effective procedure that, given formulas A and B, always correctly decides whether A logically implies B.

So, B is TRUE.

Axioms of natural numbers are called Peano Axioms. There are 9 of them and only one of them (axiom of induction) cannot be represented in FOL (requires second order).

So, C is TRUE.

Options D and E are more theoretical and related to fields. If interested you can read here.

Correct option: A

Reference: https://en.wikipedia.org/wiki/First-order_logic

References

5 votes -- Arjun Suresh (334k points)

8.1.7 First Order Logic: TIFR2019-B-4

https://gateoverflow.in/280491

Answer is C.
Set of variables for A=$\{q, r, s\}$
Set of variables for B=$\{q, s, t\}$
$\therefore$Variables in $A \cap B$=$\{q, s\}$
$\varphi = q \vee (r \wedge s)$
$\Psi = \neg q(s \vee t)$
$\Psi = \neg\neg q \vee (s \vee t)$
$\Psi = q \vee s \vee t$

According to problem we have two conditions:

i. $\varphi \implies \mu$

So, when φ is true μ must be true.
Therefore, $\mu = q \vee s$ (Since r is not in the domain of μ)

ii. $\mu \implies \Psi$

Taking μ as $q \vee s$ also satisfy the above constraint as

$q \vee s \implies q \vee s \vee t$

7 votes

-- Abhishek Shaw (1.1k points)

8.2 Logical Reasoning (6)

8.2.1 Logical Reasoning: TIFR2010-A-4

https://gateoverflow.in/18212

- If the bank receipt is forged, then Mr. M is liable.
- If Mr. M is liable, he will go bankrupt.
- If the bank will loan him money, he will not go bankrupt.
- The bank will loan him money.

Which of the following can be concluded from the above statements?

A. Mr. M is liable
B. The receipt is not forged
C. Mr. M will go bankrupt
D. The bank will go bankrupt
E. None of the above

tifr2010 logical-reasoning mathematical-logic

8.2.2 Logical Reasoning: TIFR2011-A-1

https://gateoverflow.in/237

- If either wages or prices are raised, there will be inflation.
- If there is inflation, then either the government must regulate it or the people will suffer.
- If the people suffer, the government will be unpopular.
- Government will not be unpopular.

Which of the following can be validly concluded from the above statements.

A. People will not suffer
B. If the inflation is not regulated, then wages are not raised
C. Prices are not raised
D. If the inflation is not regulated, then the prices are not raised
E. Wages are not raised

tifr2011 mathematical-logic normal logical-reasoning

8.2.3 Logical Reasoning: TIFR2011-A-12

https://gateoverflow.in/20221

The action for this problem takes place in an island of Knights and Knaves, where Knights always make true statements and Knaves always make false statements and everybody is either a Knight or a Knave. Two friends A and B lives in a house. The census taker (an outsider) knocks on the door and it is opened by A. The census taker says "I need information about you and your friend. Which if either is a Knight and which if either is a Knave?". "We are both Knaves" says A angrily

and slams the door. What, if any thing can the census taker conclude?

A. A is a Knight and B is a Knave.
B. A is a Knave and B is a Knight.
C. Both are Knaves.
D. Both are Knights.
E. No conclusion can be drawn.

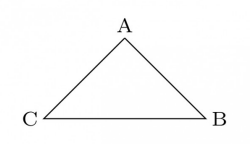
tifr2011 mathematical-logic logical-reasoning

8.2.4 Logical Reasoning: TIFR2012-A-3

https://gateoverflow.in/20981

Long ago,in a planet far far away, there lived three races of intelligent inhabitants: the blues (who always tell the truth), the whites (who always lie), and the pinks (who, when asked a series of questions, start with a lie and then tell the truth and lie alternately). To three creatures, chosen from the planet and seated facing each other at A, B, and C (see figure), the following three questions are put:

i. What race is your left-hand neighbour?
ii. What race is your right-hand neighbour?
iii. What race are you?



Here are their answers:

A. (i) White (ii) Pink (iii) Blue
B. (i) Pink (ii) Pink (iii) Blue
C. (i) White (ii) Blue (iii) Blue

What is the actual race of each of the three creatures?

A. A is Pink, B is White, C is Blue.
B. A is Blue, B is Pink, C is White.
C. A is Pink, B is Blue, C is Pink.
D. A is White, B is Pink, C is Blue.
E. Cannot be determined from the above data.

tifr2012 mathematical-logic logical-reasoning

8.2.5 Logical Reasoning: TIFR2013-A-3

https://gateoverflow.in/25384

Three candidates, Amar, Birendra and Chanchal stand for the local election. Opinion polls are conducted and show that fraction a of the voters prefer Amar to Birendra, fraction b prefer Birendra to Chanchal and fraction c prefer Chanchal to Amar. Which of the following is impossible?

A. $(a, b, c) = (0.51, 0.51, 0.51)$;
B. $(a, b, c) = (0.61, 0.71, 0.67)$;
C. $(a, b, c) = (0.68, 0.68, 0.68)$;
D. $(a, b, c) = (0.49, 0.49, 0.49)$;
E. None of the above.

tifr2013 set-theory&algebra logical-reasoning

8.2.6 Logical Reasoning: TIFR2014-A-8

https://gateoverflow.in/25994

All that glitters is gold. No gold is silver.

Claims:

1. No silver glitters.
2. Some gold glitters.

Then, which of the following is TRUE?

A. Only claim 1 follows.
B. Only claim 2 follows.
C. Either claim 1 or claim 2 follows but not both.
D. Neither claim 1 nor claim 2 follows.
E. Both claim 1 and claim 2 follow.

tifr2014 mathematical-logic logical-reasoning

Answers: Logical Reasoning

8.2.1 Logical Reasoning: TIFR2010-A-4

https://gateoverflow.in/18212

✔ Let us denote sentences with variables:

- F : Bank receipt is forged
- L : Mr. M is liable
- B : He will go bankrupt
- M : Bank loan him money

1. $F \rightarrow L \quad \rightarrow (1)$
2. $L \rightarrow B \quad \rightarrow (2)$
3. $M \rightarrow B' \quad \rightarrow (3)$
4. $M \quad \rightarrow (4)$

From 3 and 4 modus ponens we get

- $B' \quad \rightarrow (5)$

From 2 and 5 modus tollens we get

- $L' \quad \rightarrow (6)$

From 1 and 6 modus tollens we get

- F'

Ans is (**b**). Bank receipt is not forged.

👍 24 votes

-- Pooja Palod (24.1k points)

8.2.2 Logical Reasoning: TIFR2011-A-1

https://gateoverflow.in/237

✔ It is told in the question "If the people suffer, the government will be unpopular". And "government will not be unpopular" means, people will not suffer.
It is like $A \rightarrow B$ is true and ~B is given. So, ~A must be true.

So, (A) is valid (always true).

Lets take the English meaning

Government will not be unpopular

$\Longrightarrow$ People will not suffer
$\Longrightarrow$ Either no inflation or government regulates it
$\Longrightarrow$ If no regulation then no inflation
$\Longrightarrow$ if no regulation then no wage or price rise

So, (B) and (D) are valid (always true) and (C) and (E) are not valid.

👍 18 votes

-- Arjun Suresh (334k points)

8.2.3 Logical Reasoning: TIFR2011-A-12

https://gateoverflow.in/20221

✔ Option (**B**) should be the correct answer, that is A is a Knave & B is a Knight.

A must be either a Knight or a Knave.

Suppose A is a Knight, it means that the statement "We are both Knaves." must be true.

This is contradicting our assumption.

So the assumption that "A is a Knight" is not logically satisfiable simultaneously with the statement he made, which implies that A must be a Knave.

Now since A is a Knave, the statement made by him : "We are both Knaves." must be false.

The statement "We are both Knaves." will be false in any one of the following 3 conditions :

1. A is a Knight, B is a Knave.
2. A is a Knave, B is a Knight.
3. A is a Knight, B is a Knight.

Bus since we have already deduced that A is a Knave so in order to make the statement "We are both Knaves." false, we are only left with condition 2.

So B must be a Knight.

👍 29 votes -- Anurag Pandey (10.5k points)

8.2.4 Logical Reasoning: TIFR2012-A-3

https://gateoverflow.in/20981

✔ If A is Blue (honest), then

- Whatever A says about B and C must be True.
- A says that B is White(liar) and C is Pink(alternating). So, if A is Blue, B must be White and C must be Pink.
- B says that C is Pink. But B is a liar, and B agrees with A on the race of C (they must not agree). Thus, we reached a contradiction.

So, A can't be Blue.

If B is Blue (honest), then

- Whatever B says about A and C must be True.
- B says that A is Pink(alternating) and C is Pink(alternating). So, if B is Blue, A must be Pink and C must be Pink.
- Since A is pink, it must lie about B, say the truth about C and then lie about itself. Which it does.
- Since C is pink, it must lie about A, say the truth about B, and then lie about itself. Which it does.

So we see that Blue B, Pink A and Pink C is a possible solution!

Thus, option (C) is correct.

However, there is another option (E), which says Cannot be determined from the above data.

So, what if there are multiple solutions that satisfy these constraints? If that is the case, option e will be correct. Sadly, there is no way of proving that no other solutions work except checking each one of them (using branch and bound to somewhat improve). Sadly, that will be lengthy.

Here is a Python3 program that finds all solutions to this problem: http://ideone.com/7EFXCn

References

👍 27 votes -- Pragy Agarwal (18.3k points)

8.2.5 Logical Reasoning: TIFR2013-A-3

https://gateoverflow.in/25384

✔ 6 preference order for voter are possible:

$ABC, ACB, BCA, BAC, CAB, CBA$

Also given that

$a = ABC + ACB + CAB (A \text{ prefer over } B) - - - (1)$

$b = BCA + BAC + ABC (B \text{ prefer over } C) - - - (2)$

$c = CAB + CBA + BCA (C \text{ prefer over } A) - - - (3)$

Adding $1, 2$ and 3 we get

$a + b + c = 2(ABC + BCA + CAB) + ACB + BAC + CBA$

Now we know that $ABC + ACB + BAC + BCA + CAB + CBA = 1$ therefore

$[ABC + ACB + BAC + BCA + CAB + CBA] < [2(ABC + BCA + CAB) + ACB + BAC + CBA] < 2(ABC +$

Hence we can say that value of $a + b + c$ must be between 1 and 2

option (c) value greater than 2 hence correct answer is (**c**)

👍 29 votes -- Saurav Shrivastava (1.3k points)

8.2.6 Logical Reasoning: TIFR2014-A-8

https://gateoverflow.in/25994

✔ **The correct answer is option (a) Only claim 1 follows.**

$\text{Glitters}(x) \implies \text{Gold}(x) \implies \neg\, \text{Silver}(x)$. Hence, Claim 1 follows. If something Glitters, it cannot be Silver.

For claim 2:

The set of things that Glitter could be empty.

We can still assert that All that Glitters is Gold, because nothing Glitters in the first place.

So, in the case when nothing Glitters, there is no Gold that Glitters. Glitters is still a subset of Gold, but there is no element in the subset Glitters.

64 votes

-- Pragy Agarwal (18.3k points)

8.3 Propositional Logic (2)

8.3.1 Propositional Logic: TIFR2015-A-5

https://gateoverflow.in/29454

What is logically equivalent to "If Kareena and Parineeti go to the shopping mall then it is raining":

A. If Kareena and Parineeti do not go to the shopping mall then it is not raining.
B. If Kareena and Parineeti do not go to the shopping mall then it is raining.
C. If it is raining then Kareena and Parineeti go to the shopping mall.
D. If it is not raining then Kareena and Parineeti do not go to the shopping mall.
E. None of the above.

tifr2015 mathematical-logic propositional-logic

8.3.2 Propositional Logic: TIFR2018-B-4

https://gateoverflow.in/179288

The notation "$\Rightarrow$" denotes "implies" and "$\wedge$" denotes "and" in the following formulae.

- Let X denote the formula: $(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
- Let Y denote the formula: $(a \Rightarrow b) \wedge b$

Which of the following is TRUE?

A. X is satisfiable and Y is not satisfiable.
B. X is satisfiable and Y is tautology.
C. X is not tautology and Y is not satisfiable.
D. X is not tautology and Y is satisfiable.
E. X is a tautology and Y is satisfiable,

tifr2018 mathematical-logic propositional-logic

Answers: Propositional Logic

8.3.1 Propositional Logic: TIFR2015-A-5

https://gateoverflow.in/29454

✔ Answer will be (D)

"If Kareena and Parineeti go to the shopping mall then it is raining"

Let "Kareena and Parineeti go to the shopping mall" be represented by p and "it is raining" by q

Now, the statement says that $p \rightarrow q$

a. a. If Kareena and Parineeti do not go to the shopping mall then it is not raining.
   i.e., $\neg p \rightarrow \neg q$
   Not matching with the given implication.
b. b.If Kareena and Parineeti do not go to the shopping mall then it is raining.
   i.e., $\neg p \rightarrow q$
   Not matching with the given implication.
c. If it is raining then Kareena and Parineeti go to the shopping mall.
   i.e., $q \rightarrow p$
   Not matching with the given implication.
d. If it is not raining then Kareena and Parineeti do not go to the shopping mall.
   i.e., $\neg q \rightarrow \neg p \equiv q \vee \neg p \equiv p \rightarrow q$
   Matces with the given implication.

So, correct option is (D).

👍 28 votes

-- srestha (85.3k points)

8.3.2 Propositional Logic: TIFR2018-B-4

https://gateoverflow.in/179288

✔ Since we only have to deal with 2 variables (a and b), solving this with truth table is feasible.

Truth table for X will be:

a	b	$b \Rightarrow a$	$a \Rightarrow b$	$(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	1	1	1

So, clearly, X is satisfiable but not a tautology.

Truth table for Y will be:

a	b	$a \Rightarrow b$	$(a \Rightarrow b) \wedge b$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	1	1

So, Y is also satisfiable but not a tautology.

This gives the **OPTION (D)** as the correct answer.

👍 6 votes

-- Rishabh Gupta (12.5k points)

Answer Keys

8.1.1	E	8.1.2	C	8.1.3	E	8.1.4	E	8.1.5	E
8.1.6	A	8.1.7	C	8.2.1	B	8.2.2	Q-Q	8.2.3	B
8.2.4	C	8.2.5	C	8.2.6	E	8.3.1	D	8.3.2	D

9 Discrete Mathematics: Set Theory & Algebra (22)

Syllabus: Sets, Relations, Functions, Partial orders, Lattices, **Monoids,** Groups.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	0	1	2	1	1	0	2	0	0	0	0.77	2
2 Marks Count	2	1	0	0	1	1	0	1	2	0	0.88	2
Total Marks	4	3	2	1	3	2	2	2	4	1	2.55	4

9.1 Convex Sets Functions (1)

9.1.1 Convex Sets Functions: TIFR2019-A-6

https://gateoverflow.in/280504

A function $f : \mathbb{R} \to \mathbb{R}$ is said to be convex if for all $x, y \in \mathbb{R}$ and λ such that $0 \leq \lambda \leq 1$,

$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$.

Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function , and define the following functions:

$$p(x) = f(-x), \quad q(x) = -f(-x), \text{ and } r(x) = f(1-x).$$

Which of the functions p, q and r must be convex?

A. Only p B. Only q C. Only r D. Only p and r E. Only q and r

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra functions convex-sets-functions non-gate

Answers: Convex Sets Functions

9.1.1 Convex Sets Functions: TIFR2019-A-6

https://gateoverflow.in/280504

A twice differentiable function of one variable is convex on an interval if and only if its second derivative is non-negative.

For eg :- $f(x) = x^2 \implies f''(x) = 2 > 0$ so f is a convex function.

Now , we know $f(x) = x^2$ is a convex function.

$p(x) = f(-x) = x^2 \implies p''(x) = 2 > 0$

$q(x) = -f(-x) = -x^2 \implies q''(x) = -2 < 0$

$r(x) = -f(1-x) = (1-x)^2 \implies r''(x) = 2(1-x) = 2 > 0$

Hence $p(x)$ and $r(x)$ are convex functions.

$\therefore$ Option $D.$ is correct choice.

References :-

https://en.wikipedia.org/wiki/Convex_function

References

3 votes -- Satbir Singh (21.2k points)

9.2 Functions (6)

9.2.1 Functions: TIFR2012-B-1

https://gateoverflow.in/25046

For $x, y \in \{0,1\}^n$, let $x \oplus y$ be the element of $\{0,1\}^n$ obtained by the component-wise exclusive-or of x and y. A Boolean function $F : \{0,1\}^n \to \{0,1\}$ is said to be linear if $F(x \oplus y) = F(x) \oplus F(y)$, for all x and y. The number of linear functions from $\{0,1\}^n$ to $\{0,1\}$ is.

A. 2^{2n} B. 2^{n+1} C. $2^{n-1}+1$ D. $n!$ E. 2^n

tifr2012 set-theory&algebra functions

9.2.2 Functions: TIFR2013-B-16

https://gateoverflow.in/25859

Consider a function $T_{k,n} : \{0,1\}^n \to \{0,1\}$ which returns 1 if at least k of its n inputs are 1. Formally, $T_{k,n}(x) = 1$ if $\sum_1^n x_i \geq k$. Let $y \in \{0,1\}^n$ be such that y has exactly k ones. Then, the function $T_{k,n-1}(y_1, y_2, \ldots . y_{i-1}, y_{i+1}, \ldots, y_n)$ (where y_i is omitted) is equivalent to

A. $T_{k-1}, n(y)$ B. $T_{k,n}(y)$ C. y_i D. $\neg y_i$ E. None of the above

tifr2013 set-theory&algebra functions

9.2.3 Functions: TIFR2014-B-18

https://gateoverflow.in/27351

Let k be an integer at least 4 and let $[k] = \{1, 2, \ldots, k\}$. Let $f : [k]^4 \to \{0,1\}$ be defined as follows: $f(y_1, y_2, y_3, y_4) = 1$ if an only if the $y_i's$ are all distinct. For each choice $z = (z_1, z_2, z_3) \in [k]^3$, let $g_z : [k] \to \{0,1\}$ be defined by $g_z(Y) = f(Y, z_1, z_2, z_3)$. Let N be the number of distinct functions g_z that are obtained as z varies in $\{1, 2, \ldots, k\}^3$, that is, $N = \left| \left\{ g_z : z \in \{1, 2, \ldots, k\}^3 \right\} \right|$. What is N?

A. $k^3 + 1$ B. $2^{\binom{k}{3}}$ C. $\binom{k}{3}$ D. $\binom{k}{3} + 1$ E. $4\binom{k}{3}$

tifr2014 set-theory&algebra functions

9.2.4 Functions: TIFR2017-A-11

https://gateoverflow.in/95289

Let $f \circ g$ denote function composition such that $(f \circ g)(x) = f(g(x))$. Let $f : A \to B$ such that for all $g : B \to A$ and $h : B \to A$ we have $f \circ g = f \circ h \Rightarrow g = h$. Which of the following must be true?

A. f is onto (surjective)
B. f is one-to-one (injective)
C. f is both one-to-one and onto (bijective)
D. the range of f is finite
E. the domain of f is finite

tifr2017 set-theory&algebra functions

9.2.5 Functions: TIFR2018-B-10

https://gateoverflow.in/179294

For two n bit strings $x, y \in \{0,1\}^n$, define $z = x \oplus y$ to be the bitwise XOR of the two strings (that is, if x_i, y_i, z_i denote the i^{th} bits of x, y, z respectively, then $z_i = x_i + y_i \bmod 2$). A function $h : \{0,1\}^n \to \{0,1\}^n$ is called linear if $h(x \oplus y) = h(x) \oplus h(y)$, for every $x, y \in \{0,1\}^n$. The number of such linear functions for $n \geq 2$ is:

A. 2^n B. 2^{n^2} C. $2^{\frac{n}{2}}$ D. 2^{4n} E. 2^{n^2+n}

tifr2018 functions

9.2.6 Functions: TIFR2019-A-12

https://gateoverflow.in/280498

Let f be a function with both input and output in the set $\{0, 1, 2, \ldots, 9\}$, and let the function g be defined as $g(x) = f(9 - x)$. The function f is non-decreasing, so that $f(x) \geq f(y)$ for $x \geq y$. Consider the following statements:

i. There exists $x \in \{0, \ldots, 9\}$ so that $x = f(x)$
ii. There exists $x \in \{0, \ldots, 9\}$ so that $x = g(x)$
iii. There exists $x \in \{0, \ldots, 9\}$ so that $x = (f(x) + g(x)) \bmod 10$

Which of the above statements must be TRUE for ALL such functions f and g?

A. Only (i)
B. Only (i) and (ii)
C. Only (iii)
D. None of them
E. All of them

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra functions

Answers: Functions

9.2.1 Functions: TIFR2012-B-1

https://gateoverflow.in/25046

✔ Take an example

Suppose the function f is from $\{0,1\}^3$ to $\{0,1\}$

Now, here we have to find the number of functions possible from $\{0,1\}^3$ to $\{0,1\}$ such that

$f(x \oplus y) = f(x) \oplus f(y)$

Now, observe that if we maintain the linearity of XOR then the values $f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$ (Reason given below)

So, if we fix the values of $f(100), f(010)$ and $f(001)$ then we will get the whole function.

Now, each of $f(100), f(010)$ and $f(001)$ has 2 options, either 0 or 1.

So, the number of linear functions possible is $2^3 = 8$.

Now, we can see how the values $f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$.

Let $f(100) = 0, f(010) = 0, f(001) = 0$

Now,

- $f(000) = f(100 \oplus 100) = f(100) \oplus f(100) = 0 \oplus 0 = 0$
- $f(101) = f(100 \oplus 001) = f(100) \oplus f(001) = 0 \oplus 0 = 0$
- $f(110) = f(100 \oplus 010) = f(100) \oplus f(010) = 0 \oplus 0 = 0$
- $f(011) = f(010 \oplus 001) = f(010) \oplus f(001) = 0 \oplus 0 = 0$
- $f(111) = f(100 \oplus 011) = f(100) \oplus f(011) = f(100) \oplus f(010) \oplus f(001) = 0$

So, we have seen how the values $f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$

Now, suppose the function is from $\{0,1\}^n$ to $\{0,1\}$

In this case if we fix the values of $f(100\ldots0), f(010\ldots0), f(001\ldots0), \ldots, f(000\ldots1)$ then we will get the whole function since rest of the values are dependent on the above n values.

Now, each of the n values $f(100\ldots0), f(010\ldots0), f(001\ldots0), \ldots, f(000\ldots1)$ has 2 options - either 0 or 1.

So, the number of linear functions possible is 2^n.

Correct Answer: E

13 votes -- Kushagra Chatterjee (6.8k points)

Here, x, y are n length binary strings.

If $n = 2$, we have $x, y \in \{00, 01, 10, 11\}$

For a generic function mapping to $\{0,1\}$ we have 2 options for each of the four 2 bit strings thus giving 2^4 functions which is 2^{2^n}. But here, our function must be linear. Lets see all the possibilites:

$00 \oplus 00 = 00$
$00 \oplus 01 = 01$
$00 \oplus 10 = 10$
$00 \oplus 11 = 11$
$01 \oplus 01 = 00$
$01 \oplus 10 = 11$
$01 \oplus 11 = 10$
$10 \oplus 10 = 00$
$10 \oplus 11 = 01$
$11 \oplus 11 = 00$

Since, $\oplus$ is commutative not considering the remaining 6 combinations.

If F maps every thing to 0, it does preserve linearity. This way we get 1 linear function.

Now, $F(00)$ must map to 0, because it equals $F(01) \oplus F(01)$ which is 0 irrespective of the value of $F(01)$. Now, lets assume $F(01) = 1$. Since $F(01) \oplus F(10) = F(11)$, either $F(10)$ or $F(11)$ but not both must be 1. Both of these are

valid. Now, if we take $F(01) = 0$, both $F(10)$ and $F(11)$ must be 1 (both can also be 0 but this we considered as our first case). Thus, we get 4 linear functions in total

1. $\{F(00) = 0, F(01) = 0, F(10) = 0, F(11) = 0\}$
2. $\{F(00) = 0, F(01) = 1, F(10) = 1, F(11) = 0\}$
3. $\{F(00) = 0, F(01) = 1, F(10) = 0, F(11) = 1\}$
4. $\{F(00) = 0, F(01) = 0, F(10) = 1, F(11) = 1\}$

So, 2^n should be the answer. General case discussion can be seen here:
https://math.stackexchange.com/questions/496185/number-of-linear-binary-function-in-a-vector-space

https://math.stackexchange.com/questions/2244907/the-number-of-linear-functions-from-left-0-1-right-n-to-left-0-1-ri?noredirect=1&lq=1

References

6 votes -- Arjun Suresh (334k points)

9.2.2 Functions: TIFR2013-B-16

https://gateoverflow.in/25859

✔ $T_{k,n} = 1$ iff there are at least k $1's$ in n bit binary string.

Similarly, $T_{k,n-1} = 1$ iff k $1's$ are there out of $(n-1)$ bits

It is given that $\langle y_1, y_2, y_3, y_4, y_5 \ldots y_n \rangle$ has exactly k $1's$ which means out of n bits, k bits are $1's$.

Now, suddenly one-bit y_i is removed (that one bit can be 1 or 0, we don't know). Removing one bit will make the number of bits as $(n-1)$.

So, $T_{k,n-1}$ really depends on whether i^{th} bit which we removed is a 0 or 1. If 0, then we have k $1's$ in our remaining $(n-1)$ bits and function will return 1.
If 1, then we have only $(k-1)$ $1's$ out of $(n-1)$ bits and so the function will return 0.

So, $T_{k,n-1} = \neg y_i$ which depends on the value of y_i .

Option D is correct.

15 votes -- Vicky rix (7.1k points)

9.2.3 Functions: TIFR2014-B-18

https://gateoverflow.in/27351

✔ The function $g_z(Y)$ is defined as $[k] \rightarrow \{0,1\}$ where $[k]$ is the set of positive integers till k. That is, given a triplet (z_1, z_2, z_3), Y can take any value from 1 to k. If Y happen to be any of z_1, z_2, z_3, $g_z(Y) = 0$ due to the definition of f and g_z. Now even for different z, g_z may be the same. Otherwise, the answer would have been how many ways we can form a triplet z - which gives k^3 and for each z we get a function g_z.

For all unique combinations of z_1, z_2, z_3 are unique, we are guaranteed that we get a distinct function g_z. This is clear from the definition of g_z. For example, suppose $k = 4$. The triplets are

$(1,2,3)$
$(1,2,4)$
$(1,3,4)$
$(2,3,4)$
For the triplet $(1,2,3)$, Y can be made in 4 ways as $(1,2,3,1), (1,2,3,2), (1,2,3,3)$ and $(1,2,3,4)$. Now, as per definition of g_z, we get $g_{(1,2,3)} = \{\{1 \rightarrow 0\}, \{2 \rightarrow 0\}, \{3 \rightarrow 0\}, \{4 \rightarrow 1\}\}$.

Similarly, for the next three triplets, g_z are different as in second only 3 maps to 1, in third one only 2 maps to 1 and in fourth one only 1 maps to 1.

So, in general, for any given k, we have ${}^kC_3$ ways of forming distinct triplets and each of them guarantees a unique function g_z where exactly $k-3$ elements map to 1 and 3 elements map to 0. Now, if any of the elements in the triplet are same, then the function becomes $\{\{1 \rightarrow 0\}, \{2 \rightarrow 0\}, \ldots, \{k \rightarrow 0\}\}$, (all k elements mapping to 0) and this remains the same for any triplet. So, total number of possible functions are

$$^{k}C_{3}+1$$

Correct Answer: D

10 votes -- Arjun Suresh (334k points)

9.2.4 Functions: TIFR2017-A-11

https://gateoverflow.in/95289

✔ $f: A \rightarrow B$ is injective if and only if, given any functions $g, h: B \rightarrow A$ whenever $f \circ g = f \circ h$, $f \circ g = f \circ h, then\ g = h$.

Refer to properties of Injective functions: https://en.wikipedia.org/wiki/Injective_function

Let us prove $(\forall g, h: f(g(x)) = f(h(x)) \rightarrow g(x) = h(x)) \rightarrow f\ is\ one-to-one$ is true.

This is equivalent to, $f\ is\ not\ one-to-one \rightarrow (\exists g, h: f(g(x)) = f(h(x)) \wedge g(x) \neq h(x))$

Let us assume LHS is true, i.e. $f\ is\ not\ one-to-one$.

Then there exists some $c, d \in A$ such that,

$$f(c) = f(d) = a, \text{ where } a \text{ is an arbitrary element which belongs to B}$$

Let g and h be some functions out of all possible functions from B to A such that $g \neq h$,

$$\text{i.e. } g(x) = c\ and\ h(x) = d\ \exists c, d \in A\ and\ \exists x \in B$$

$\therefore f(g(x)) = f(c) = a\ and\ f(h(x)) = f(d) = a$ and $g(x) \neq h(x)$, i.e. RHS is also true.

Thus, whenever $\forall g, h\ f \circ g = f \circ h \rightarrow g = h$ is true, $f\ is\ one-to-one$.

Domain of f need not be finite. Let $f: A \rightarrow B$ be identity function and A and B be infinite sets. Assume that $f \circ g = f \circ h$ is true,

then $f(g(x)) = f(h(x)) \rightarrow g(x) = h(x))$ will be true $\forall g, h$ since f is an identity function. So, even if domain of f is not finite, the condition holds true.

Correct Answer: B

References

10 votes -- V S Silpa (2.8k points)

9.2.5 Functions: TIFR2018-B-10

https://gateoverflow.in/179294

✔ Here, x, y are n length binary strings.

If $n = 2$, we have $x, y \in \{00, 01, 10, 11\}$

For a generic function mapping to $\{0,1\}^2$ we have $2^2 = 4$ options for each of the four 2 bit strings thus giving $2^4 = 16$ functions (in general 2^{2^2}). But here, our function must be linear.

Lemma: Once we assign values to $h(x)$ for all x having exactly one 1, the other values are fixed.

For example, for $n = 2$ we can only fix $h(01)$ and $h(10)$ and the remaining two values $h(00)$ and $h(11)$ are fixed as $h(00)$ must be $h(01) \oplus h(01) = 00$ and $h(11) = h(01) \oplus h(10)$.

In this way for any n we have only n combinations possible from the domain set for a linear function instead of 2^n for a normal function. Here, each of these n possible combinations can be mapped to any of the 2^n bit strings thus giving rise to

$(2^n)^n$ possible LINEAR functions which equals 2^{n^2}.

Correct Option: B.

4 votes -- Arjun Suresh (334k points)

9.2.6 Functions: TIFR2019-A-12 https://gateoverflow.in/280498

Option A.

We have 10 elements for the domain set to map to. If we want to ensure $x \neq f(x)$ lets do as follows:

$f(0) = 1, f(1) = 2, f(2) = 3, \ldots, f(8) = 9$. Now the only option is for $f(9) = 9$ or else we will break the non-decreasing condition.

Like this for any mapping we will get at least one x such that $x = f(x)$.

Now consider $f(0) = 0, f(1) = 0, f(2) = 0, \ldots, f(8) = 0, f(9) = 1$. This implies,
$g(9) = g(8) = g(7) = \ldots = g(1) = 0, g(0) = 1$. That is we have a non-decreasing f and for no x, $x = g(x)$. So, (ii) is FALSE.

Let $f(0) = f(1) = f(2) = \ldots = f(8) = 0, f(9) = 1$. Now, for no x, $x = f(x) + g(x) \mod 10$. So, (iii) is FALSE.

10 votes -- Arjun Suresh (334k points)

9.3 Lattice (1)

9.3.1 Lattice: TIFR2012-B-4 https://gateoverflow.in/25090

Let $\wedge$, $\vee$ denote the meet and join operations of lattice. A lattice is called distributive if for all x, y, z,
$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
It is called complete if meet and join exist for every subset. It is called modular if for all x, y, z
$z \leq x \Rightarrow x \wedge (y \vee z) = (x \wedge y) \vee z$
The positive integers under divisibility ordering i.e. $p \leq q$ if p divides q forms a.

A. Complete lattice.
B. Modular, but not distributive lattice.
C. Distributive lattice.
D. Lattice but not a complete lattice.
E. Under the give ordering positive integers do not form a lattice.

tifr2012 set-theory&algebra lattice

Answers: Lattice

9.3.1 Lattice: TIFR2012-B-4 https://gateoverflow.in/25090

Consider the lattice with gcd as meet and lcm as join.

It is distributive

- $gcd(a, lcm(b, c)) = lcm(gcd(a, b), gcd(a, c))$
- $lcm(a, gcd(b, c)) = gcd(lcm(a, b), lcm(a, c))$

It is complete semi-meet lattice. As there is no upper bound it is not complete semi-join lattice.
ANS: C

6 votes -- pramod (2.8k points)

9.4 Number Theory (1)

9.4.1 Number Theory: TIFR2014-A-14 https://gateoverflow.in/26392

Let m and n be any two positive integers. Then, which of the following is FALSE?

A. $m+1$ divides $m^{2n}-1$.
B. For any prime p, $m^p \equiv m(\mod p)$.
C. If one of m, n is prime, then there are integers x, y such that $mx+ny=1$.
D. If $m<n$, then $m!$ divides $n(n-1)(n-2)\ldots(n-m+1)$.
E. If 2^n-1 is prime, then n is prime.

tifr2014 number-theory set-theory&algebra

Answers: Number Theory

9.4.1 Number Theory: TIFR2014-A-14

https://gateoverflow.in/26392

✔ (a) m divides m^{2n}. Now we can observe that now if we divide m^{2n} by $m+1$ then remainder will always be 1. Now to make remainder 0 we have to subtract 1 from m^{2n} so in that way we can say that $m+1$ divides $m^{2n}-1$.

(b) It is popular theorem by Mathematician Fermat

https://en.wikipedia.org/wiki/Fermat%27s_little_theorem

(c) To make $mx+ny=1$, n and m both must have to be relatively prime. If only one is prime and then other, when it is a multiple of that prime, sum cannot be 1.

like $5x+ny=1$ if n is $10, 15, 20\ldots$ result cannot be 1. Basically the theorem is

For any positive integers a and b, there exist integers x and y such that
mx+ny= gcd(m, n).

Now when m and n are relatively prime then their $\gcd$ will be 1 so eventually equation would be like $mx+ny=1$

(d) We know that ${}^nC_m$ means choose m out of n and result of this will always be integer. If we see carefully what's being asked is ${}^nC_m$, which is always an integer so yes $m!$ will have to divide $n \times (n-1)(n-2)\ldots(n-m+1)$.

(e) https://en.wikipedia.org/wiki/Mersenne_prime

So, option C must be false.

References

3 votes -- Rupendra Choudhary (11.4k points)

9.5 Partial Order (4)

9.5.1 Partial Order: TIFR2012-B-5

https://gateoverflow.in/25092

Let R be a binary relation over a set S. The binary relation R is called an equivalence relation if it is reflexive transitive and symmetric. The relation is called partial order if it is reflexive, transitive and anti symmetric. (Notation: Let aRb denote that order pair $(a,b) \in R$.) The relation R is called a well-order if R is a partial order and there does not exist an infinite descending chain (with respect to R) within S. An infinite sequence $x_1, x_2\ldots$ of elements of S is called an infinite descending chain if for all i we have $x_{i+1}Rx_i$ and $x_i \neq x_{i+1}$.

Take $S = \aleph \times \aleph$ and let the binary relation $\sqsubseteq$ over S be such that $(i_1, j_1) \sqsubseteq (i_2, j_2)$ if and only if either $(i_1 < i_2)$ or $((i_1 = i_2) \wedge (j_1 \leq j_2))$. Which statement is true of $\sqsubseteq$?

A. $\sqsubseteq$ is an equivalence relation but not a well order.
B. $\sqsubseteq$ is a partial order but not a well order.
C. $\sqsubseteq$ is a partial order and a well order.
D. $\sqsubseteq$ is an equivalence relation and a well order.
E. $\sqsubseteq$ is neither a partial order nor an equivalence relation.

tifr2012 set-theory&algebra partial-order

9.5.2 Partial Order: TIFR2013-B-4

https://gateoverflow.in/25664

A set S together with partial order $\ll$ is called a well order if it has no infinite descending chains, i.e. there is no infinite sequence $x_1, x_2, \ldots$ of elements from S such that $x_{i+1} \ll x_i$ and $x_{i+1} \neq x_i$ for all i.

Consider the set of all words (finite sequence of letters $a-z$), denoted by W, in dictionary order.

A. Between "aa" and "az" there are only 24 words.
B. Between "aa" and "az" there are only 2^{24} words.
C. W is not a partial order.
D. W is a partial order but not a well order.
E. W is a well order.

tifr2013 set-theory&algebra partial-order

9.5.3 Partial Order: TIFR2014-B-15

https://gateoverflow.in/27322

Consider the set N^* of finite sequences of natural numbers with $x \leq_p y$ denoting that sequence x is a prefix of sequence y. Then, which of the following is true?

A. N^* is uncountable.
B. $\leq_p$ is a total order.
C. Every non-empty subset of N^* has a least upper bound.
D. Every non-empty subset of N^* has a greatest lower bound.
E. Every non-empty finite subset of N^* has a least upper bound.

tifr2014 set-theory&algebra partial-order

9.5.4 Partial Order: TIFR2014-B-16

https://gateoverflow.in/27341

Consider the ordering relation $x \mid y \subseteq N \times N$ over natural numbers N such that $x \mid y$ if there exists $z \in N$ such that $x \bullet z = y$. A set is called lattice if every finite subset has a least upper bound and greatest lower bound. It is called a complete lattice if every subset has a least upper bound and greatest lower bound. Then,

A. $\mid$ is an equivalence relation.
B. Every subset of N has an upper bound under $\mid$.
C. $\mid$ is a total order.
D. $(N, \mid)$ is a complete lattice.
E. $(N, \mid)$ is a lattice but not a complete lattice.

tifr2014 set-theory&algebra partial-order

Answers: Partial Order

9.5.1 Partial Order: TIFR2012-B-5

https://gateoverflow.in/25092

Answer: C

$S = (i_1 j_1) \sqsubseteq (i_2 j_2)$ iff $(i_1 < i_2)$ or $((i_1 = i_2) \wedge (j_1 \leq j_2))$

1. $(m,n)R(m,n)$?

yes, here $m \not< n$, so we go at second criteria.

Now, $m = n$ & $n = n$. So, this is reflexive.

2. Antisymmetric

$(1,2)R(2,3) \implies (2,3)R(1,2)$?
No, as $2 \not< 1$.

If you see the definition, it is clear that other than diagonal element no other element is related to itself. So, antisymmetric.

3. Transitivity
$(1,2)R(2,3)$ & $(2,3)R(2,4) \implies (1,2)R(2,4)$?
Yes. It can be seen easily from the definition of S. Same for any other pairs. Not going to prove this formally.

4. It is not symmetric $(1,2)R(2,3)$ but $(2,3)R(1,2)$

5. This is well ordered. We do not have infinite descending chain. As we have least element $(0,0)$ our chain stops there.

Ref :-

https://en.wikipedia.org/wiki/Infinite_descending_chain
https://en.wikipedia.org/wiki/Well-order#Examples_and_counterexamples

References

8 votes -- Akash Kanase (36.1k points)

9.5.2 Partial Order: TIFR2013-B-4

https://gateoverflow.in/25664

Answer = D.

The set A of all words (finite sequence of letters $a - z$), denoted by W, in dictionary order.

W is a Poset as it is Reflexive, Antisymmetric and transitive. W is even a Total Ordered structure as Every Two elements of Set A are comparable.

Let's catch the bigger fish here i.e. Well-order. :

> A set S together with partial order $\lll$ is called a well order if it has no infinite descending chains, i.e. there is no infinite sequence x1,x2,..... of elements from S such that xi+1 $\lll$ xi and xi+1≠xi for all i.

We know a different definition of Well-ordered set which is "A well-ordered set is a structure of the form $(S, \leq)$ such that $\leq$ is a partial order on S and Every nonempty subset of S has a $\leq -smallest\ element$...We will call this definition as **Definition 1 of Well Order.** If $(S, \leq)$ is a well-ordered set, we may express this by saying that the relation $\leq$ well-orders S. (NOTE that We need not say that S first should be a Total ordered set to be a Well Ordered set Because the first and second condition of the definition together itself imply that $(S, \leq)$) is a Total ordered structure)

Now, there is an Equivalent definition to the above definition of Well-Ordered set which is "A total ordered structure is well-ordered if and only if it does not contain infinite descending chains; that is, a linearly ordered set $(S, \leq)$ is a well-ordered set if and only if there does not exist a sequence $a_0, a_1, a_2, \ldots$ of elements of S such that $a_0 > a_1 > a_2 > \ldots .$ ..We will call it as **Definition 2 of well order.**

We can prove that Definition 1 and Definition 2 are Equivalent (Proof is given below, after the answer to the asked question).

Since now that we know both the definitions are equivalent. We can use the Definition 1 to check for Well-ordering.

Now consider the following Subset of the given set A :

$S = \{a^n b \mid n \geq 0\}$ .. this Subset has No least element. Hence, The given Structure $(A, dictionary\ order)$ is NOT Well-Ordered Structure.

Proving that Definition 1 and Definition 2 are Equivalent :

> Prove that a structure is well-ordered if and only if it does not contain infinite descending chains; that is, prove that a linearly ordered set (S, ≤) is a well-ordered set if and only if there does not exist a sequence a0, a1, a2, . . . of elements of S such that a0 > a1 > a2 > . . . .

(Only if Part) If $a_0 > a_1 > a_2, \ldots$ is an infinite descending sequence in S, then the set $\{a0, a1, a2, \ldots\}$ does not have a minimum element, so S is not well-ordered.

(If part) Suppose that S is not well-ordered, and fix a nonempty $A \subseteq S$ that does not have a minimum element. Fix $a_0 \in A$. Since a_0 is not a minimum element of A, there exists $a_1 \in A$. such that $a_0 > a_1$. Since a_1 is not a minimum element of A, there exists $a_2 \in A$ such that $a_1 > a_2$. Continuing this construction inductively, we find an infinite descending chain in A.

8 votes -- Deepak Poonia (23.8k points)

9.5.3 Partial Order: TIFR2014-B-15

https://gateoverflow.in/27322

Consider two sequences $\langle 43, 9\rangle$ and $\langle 52, 2\rangle$. These two are not comparable as there is no common prefix and also there can be no sequence which can be higher than these in the prefix relation - so no upper bound for these. So, options B, C and E are ruled out. Option D is correct as for any two sequences their common prefix acts as the greatest lower bound, and in case there is nothing common, empty sequence will act as the greatest lower bound.

So, option D is true.

14 votes -- Arjun Suresh (334k points)

9.5.4 Partial Order: TIFR2014-B-16

https://gateoverflow.in/27341

A. Taking an example, $\{4, 2\}$ is not symmetric as $4|2$ is not equal to $2|4$. So, it cannot be an equivalence.

B. It is not necessary that every subset will have an upper bound.

C. For being total every element in the subset should be comparable have it is not total as well. We have counter example:

$\{1, 2, 3, 9, 15\}$. Here, 2&3 are incomparable. 9&15 are also not comparable in this division relation.

D. Not every subset is a complete lattice. Counter example: $\{1, 2, 3, 24, 30\}$ Here, 2&3 have no LUB.

So, E is the answer.

4 votes -- bharti (2.3k points)

9.6 Polynomials (1)

9.6.1 Polynomials: TIFR2012-A-12

https://gateoverflow.in/25035

For the polynomial $p(x) = 8x^{10} - 7x^3 + x - 1$ consider the following statements (which may be true or false)

i. It has a root between $[0, 1]$.
ii. It has a root between $[0, -1]$.
iii. It has no roots outside $(-1, 1)$.

Which of the above statements are true?

A. Only (i).
B. Only (i) and (ii).
C. Only (i) and (iii).
D. Only (ii) and (iii).
E. All of (i), (ii) and (iii).

tifr2012 set-theory&algebra polynomials

Answers: Polynomials

9.6.1 Polynomials: TIFR2012-A-12

https://gateoverflow.in/25035

✔ At $f(0)$ it is negative, at $f(1)$ it is positive, and at $f(-1)$ it is positive, which means there will be roots between $(0, 1)$ and $(-1, 0)$. Any values below -1 and above 1 will always yield positive values for $f(x)$, which means no roots available.

Correct Answer: E

13 votes -- Shaun Patel (6.1k points)

9.7 Sets (8)

9.7.1 Sets: TIFR2010-A-15

https://gateoverflow.in/18394

Let A, B be sets. Let $\bar{A}$ denote the compliment of set A (with respect to some fixed universe), and $(A - B)$ denote the set of elements in A which are not in B. Set $(A - (A - B))$ is equal to:

A. B B. $A \cap \bar{B}$ C. $A - B$ D. $A \cap B$ E. $\bar{B}$

tifr2010 set-theory&algebra sets

9.7.2 Sets: TIFR2010-A-18

https://gateoverflow.in/18496

Let X be a set of size n. How many pairs of sets (A, B) are there that satisfy the condition $A \subseteq B \subseteq X$?

A. 2^{n+1} B. 2^{2n} C. 3^n D. $2^n + 1$ E. 3^{n+1}

tifr2010 sets

9.7.3 Sets: TIFR2011-A-10

https://gateoverflow.in/20039

Let m, n denote two integers from the set $\{1, 2, \ldots, 10\}$. The number of ordered pairs (m, n) such that $2^m + 2^n$ is divisible by 5 is.

A. 10 B. 14 C. 24 D. 8 E. None of the above

tifr2011 set-theory&algebra sets

9.7.4 Sets: TIFR2011-B-23

https://gateoverflow.in/20400

Suppose $(S_1, S_2, \ldots, S_m)$ is a finite collection of non-empty subsets of a universe U. Note that the sets in this collection need not be distinct. Consider the following basic step to be performed on this sequence. While there exist

sets S_i and S_j in the sequence, neither of which is a subset of the other, delete them from the sequence, and

i. If $S_i \cap S_j \neq \emptyset$, then add the sets $S_i \cup S_j$ and $S_i \cap S_j$ to the sequence;
ii. If $S_i \cap S_j = \emptyset$, then add only the set $S_i \cup S_j$ to the sequence.

In each step we delete two sets from the sequence and add at most two sets to the sequence. Also, note that empty sets are never added to the sequence. Which of the following statements is TRUE?

A. The size of the smallest set in the sequence decreases in every step
B. The size of the largest set in the sequence increases in every step
C. The process always terminates
D. The process terminates if U is finite but might not if U is infinite
E. There is a finite collection of subsets of a finite universe U and a choice of S_i and S_j in each step such that the process does not terminate

tifr2011 set-theory&algebra sets

9.7.5 Sets: TIFR2012-A-8

https://gateoverflow.in/21007

How many pairs of sets (A, B) are there that satisfy the condition $A, B \subseteq \{1, 2, \ldots, 5\}, A \cap B = \{\}$?

A. 125 B. 127 C. 130 D. 243 E. 257

tifr2012 set-theory&algebra sets

9.7.6 Sets: TIFR2016-A-8

https://gateoverflow.in/97234

Let A and B be finite sets such that $A \subseteq B$. Then, what is the value of the expression:

$$\sum_{C: A \subseteq C \subseteq B} (-1)^{|C \setminus A|},$$

Where $C \setminus A = \{x \in C : x \notin A\}$?

A. Always 0
B. Always 1
C. 0 if $A = B$ and 1 otherwise
D. 1 if $A = B$ and 0 otherwise
E. Depends on the size of the universe

tifr2016 set-theory&algebra sets

9.7.7 Sets: TIFR2017-A-10

https://gateoverflow.in/95272

For a set A define $P(A)$ to be the set of all subsets of A. For example, if $A = \{1, 2\}$ then $P(A) = \{\emptyset, \{1, 2\}, \{1\}, \{2\}\}$. Let $A \to P(A)$ be a function and A is not empty. Which of the following must be TRUE?

A. f cannot be one-to-one (injective)
B. f cannot be onto (surjective)
C. f is both one-to-one and onto (bijective)
D. there is no such f possible
E. if such a function f exists, then A is infinite

tifr2017 set-theory&algebra sets functions easy

9.7.8 Sets: TIFR2019-A-1

https://gateoverflow.in/280509

Let X be a set with n elements. How many subsets of X have odd cardinality?

A. n
B. 2^n
C. $2^{n/2}$
D. 2^{n-1}
E. Can not be determined without knowing whether n is odd or even

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra sets

Answers: Sets

9.7.1 Sets: TIFR2010-A-15

https://gateoverflow.in/18394

✔ $(A-(A-B)) = A \cap (A \cap B')'$ Since $A - B = A \cap B'$

$= A \cap (A'UB)$ Since $(A \cap B)' = A'UB'$

$= A \cap B$ Option D

12 votes -- Umang Raman (12.2k points)

9.7.2 Sets: TIFR2010-A-18

https://gateoverflow.in/18496

✔ Option C i.e. 3^n must be the right answer.

It is given that there are n elements in the set X.

Consider an element p of set X.

What are the choices it will have?

1. Either it can be present in set A & set B both.
2. Or it can absent from set A & present in set B.
3. Or it can be absent from both set A & set B.

But since it is given that A must be a subset of B, it is not possible that it can be present in A & absent from B.

So, it each of the n elements of set X have 3 choices available.

So, total choices available for formation of sets A & $B = 3^n$, which will give 3^n such different (A, B) pairs.

55 votes -- Anurag Pandey (10.5k points)

9.7.3 Sets: TIFR2011-A-10

https://gateoverflow.in/20039

✔ Ending in 2 : $\{2^1, 2^5, 2^9\}$

Ending in 4 : $\{2^2, 2^6, 2^{10}\}$

Ending in 6 : $\{2^4, 2^8\}$

Ending in 8 : $\{2^3, 2^7\}$

To make $2^m + 2^n$ divisible by 5, it must end in either a 0 or a 5.

Since, $m, n > 1$, all numbers $2^m, 2^n$ are even. Since sum of even numbers is even, $2^m + 2^n$ cannot end in a 5.

Thus, $2^m + 2^n$ must end in a 0.

Possible ways to achieve a number ending with 0 are:

$$\begin{array}{llll} 2^m + 2^n: & m \in \{1,5,9\}, & n \in \{3,7\} & \implies 3 \times 2 = 6 \text{ pairs} \\ 2^m + 2^n: & m \in \{3,7\}, & n \in \{1,5,9\} & \implies 2 \times 3 = 6 \text{ pairs} \\ & & & \\ 2^m + 2^n: & m \in \{2,6,10\}, & n \in \{4,8\} & \implies 3 \times 2 = 6 \text{ pairs} \\ 2^m + 2^n: & m \in \{4,8\}, & n \in \{2,6,10\} & \implies 2 \times 3 = 6 \text{ pairs} \end{array}$$

$$\text{Total} = 6 + 6 + 6 + 6 = 24 \text{ ordered pairs}$$

Thus, option C is correct.

54 votes -- Pragy Agarwal (18.3k points)

9.7.4 Sets: TIFR2011-B-23 https://gateoverflow.in/20400

✔ There are m sets in the initial sequence. If we pick two sets S_i and S_j NEITHER of which are subsets of each other we add

1. $S_i \cup S_j$
2. $S_i \cap S_j$ if it is not NULL.

So, the two added sets are not eligible to be picked as a pair as INTERSECTION is always a subset of UNION. So, in each step we are removing a pair of eligible sets. But the addition of set(s) can cause more eligible pairs to be created.

With m sets we can only have ${}^mC_2$ pairs possible. So, if we repeat the procedure for ${}^mC_2$ times, and number of times $S_i \cap S_j = \phi$ is k,

- Number of sets reduces by k
- Number of ineligible pairs will be $\geq {}^mC_2 - k$
- Number of possible eligible pairs $\leq {}^{m-k}C_2 - ({}^mC_2 - k)$
  $\implies \frac{(m-k)(m-k-1)}{2} - \frac{m(m-1)}{2} + k$
  $= \frac{m^2-mk-m-mk+k^2+k-m^2+m}{2} + k$
  $= \frac{k^2-2mk+k}{2} + k$
  $= k\left[\frac{k-2m+1}{2} + 1\right]$
  Maximum value of k is m. If we put $k = m$ we get
  $m\left[\frac{m-2m+1}{2} + 1\right] = \frac{-m^2}{2} + \frac{3m}{2} \leq 0$ for $m \geq 3$
- i.e., for all $m \geq 3$ after some finite moves we are left with 0 eligible pairs to choose.
- For $m = 1$ since there are no eligible pairs, process cannot start. For $m = 2$ there can be maximum one eligible pair and after one step this must reduce to no eligible pairs.
- So, the process always terminates.
- PS: Even if each of the sets is infinite it won't affect the termination of the process.

Option C.

👍 2 votes -- Arjun Suresh (334k points)

9.7.5 Sets: TIFR2012-A-8 https://gateoverflow.in/21007

✔ Correct Option: C 243.

First take A as $\emptyset$ and B as power set of $\{1,2,3,4,5\}$ which is 2^5. Then take A as set of one element ex: when A= $\{1\}$ then set B could be any of the 2^4 elements of power set. This will give us 16×5 . In similar fashion when A consists of 2 elements we get total pairs $5C2 \times 2^3$; when A is of three elements we get 40; for 4 elements we get 10 pairs and when A is of 5 elements we get one pair which is A=$\{1,2,3,4,5\}$ and $B = \{\}$. So, in total
$= 32 + 80 + 80 + 40 + 10 + 1 = 243$

Alternative Solution:

for each element in $[n]$, you have 3 choices:

1. Include it in A but not in B
2. Include it in B, but not in A
3. Include it in neither

so this gives 3^n pairs.

for set $\{1,2,,\ldots,5\}$, $n = 5$, $3^5 = 243$

👍 51 votes -- Shaun Patel (6.1k points)

9.7.6 Sets: TIFR2016-A-8 https://gateoverflow.in/97234

✔ Let set B be of cardinality n.
Total subsets(A) possible are : $nC_0 + nC_1 + nC_2 + \ldots + nC_n$. i.e nC_r number of subsets exist with r cardinality.

Note that for each r, summation has 2^{n-r} terms to sum.
Case 1: $r = 0$. which is $\emptyset$.

Total terms $= 2^n$.
Total terms when $|C \backslash A|$ even $= nC_0 + nC_2 + nC_4 + \ldots + nC(n-1)$ { if n is odd, nC_n otherwise }
Similarly, for odd $= 2^n - |C \backslash A|$
Even contributes to 1 whereas odd contributes to -1.

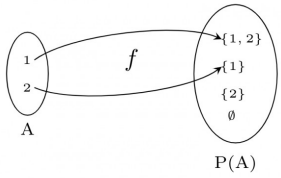
Therefore, Summation $= 0$ as 2^n terms are present with half as 1 & half as -1.
Case 2: $r = 1$, total terms $= 2^{n-1}$

Total terms when $|C \backslash A|$ even $= nC_r + nC_{r+2} + \ldots + nC_n$ { if n is odd $n-1$ otherwise }
Total terms when $|C \backslash A|$ odd $= nC_{r+1} + nC_{r+3} + \ldots$
Again we are end up with total even terms with half contributing to 1 & half -1.
same situation will arise for every $r \neq n$ (as for such r, 2^r is always even) i.e summation $= 0$ for all $r, r \neq n$.
Case n: $r = n$, total terms $= 2^{n-n} = 1$. This is the case when $B = A$.
$\therefore |C \backslash A| = 0$ as both are equal.
Summation $= 1$.
Therefore, answer is : 1 if $A = B, 0$ otherwise.
Correct Answer: D

8 votes -- Aghori (4.4k points)

9.7.7 Sets: TIFR2017-A-10 https://gateoverflow.in/95272

✔ Even if it can be **one-to-one** in the following way,



But, It cannot be onto,because, the number of elements in domain (A) $<$ the number of elements in co-domain ($P(A)$) . For a function to be **onto**, the domain should be able to cover all elements of co-domain with each element of the domain having exactly one image in co-domain.
So, option(B)

23 votes -- Motamarri Anusha (8.6k points)

9.7.8 Sets: TIFR2019-A-1 https://gateoverflow.in/280509

✔ X is a set of n elements then there are total 2^n subsets out of which 2^{n-1} have odd cardinality and 2^{n-1} have even cardinality.

D is correct option.

12 votes -- Mk Utkarsh (25.7k points)

Answer Keys

9.1.1	D	9.2.1	E	9.2.2	D	9.2.3	D	9.2.4	B
9.2.5	B	9.2.6	B	9.3.1	C	9.4.1	C	9.5.1	C
9.5.2	E	9.5.3	D	9.5.4	E	9.6.1	E	9.7.1	D
9.7.2	C	9.7.3	C	9.7.4	C	9.7.5	D	9.7.6	D
9.7.7	B	9.7.8	D						

10 Engineering Mathematics: Calculus (21)

Syllabus: Limits, Continuity, and Differentiability, Maxima and minima, Mean value theorem, Integration.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	1	1	1	1	0	1	1	1	0	0.88	1
2 Marks Count	0	0	0	0	0	1	0	0	0	0	0.1	1
Total Marks	1	1	1	1	1	2	1	1	1	1	1.1	2

10.0.1 TIFR2020-A-13

https://gateoverflow.in/333116

What is the area of the largest rectangle that can be inscribed in a circle of radius R?

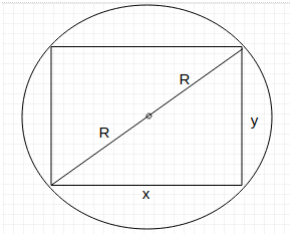
A. $R^2/2$ B. $\pi \times R^2/2$ C. R^2 D. $2R^2$ E. None of the above

tifr2020

Answers:

10.0.1 TIFR2020-A-13

https://gateoverflow.in/333116



$(R+R)^2 = x^2 + y^2 \Rightarrow y^2 = 4R^2 - x^2$

Area of Rectange, $A = xy \Rightarrow A^2 = x^2y^2 = x^2(4R^2 - x^2) = 4R^2x^2 - x^4$

For area of largest rectange, $\frac{d(A^2)}{dx} = 0$

So, $8R^2x - 4x^3 = 0 \Rightarrow x = 0, \pm\sqrt{2}R$. Only $x = +\sqrt{2}R$ is possible.

For $x = \sqrt{2}R$, $\frac{d^2(A^2)}{dx^2} < 0$

So, at $x = \sqrt{2}R$, the area of rectangle which is inscribed in a circle of radius R is maximum.

Now, $y^2 = 4R^2 - x^2 \Rightarrow y^2 = 4R^2 - 2R^2$. So, possible value of y is also $\sqrt{2}R$.

Hence, the area of the largest rectangle that can be inscribed in a circle of radius R is : $\sqrt{2}R \times \sqrt{2}R = 2R^2$

Edit:

Alternatively, we can take the help of Lagrangian Multiplier,

$\mathcal{L} = xy - \lambda(x^2 + y^2 - 4R^2)$

$\frac{\partial \mathcal{L}}{\partial x} = 0 \Rightarrow x = \frac{y}{2\lambda}$ and $\frac{\partial \mathcal{L}}{\partial y} = 0 \Rightarrow y = \frac{x}{2\lambda}$

So, $x = \pm y$ for $\lambda \neq 0$

$x^2 + y^2 = 4R^2 \Rightarrow x^2 + x^2 = 4R^2 \Rightarrow x^2 = 2R^2$

Hence, Area = $xy = x^2 = 2R^2$

1 votes -- ankitgupta.1729 (15.2k points)

10.1 Convergence (1)

10.1.1 Convergence: TIFR2014-A-15

https://gateoverflow.in/26402

Consider the following statements:

1. $b_1 = \sqrt{2}$, series with each $b_i = \sqrt{b_{i-1} + \sqrt{2}}, i \geq 2$, converges.
2. $\sum_{i=1}^{\infty} \frac{\cos(i)}{i^2}$ converges.
3. $\sum_{i=0}^{\infty} b_i$ converges if $\lim_{i \to \infty} \frac{|b_{i+1}|}{|b_i|} < 1$

Which of the following is TRUE?

A. Statements (1) and (2) but not (3).
B. Statements (2) and (3) but not (1).
C. Statements (1) and (3) but not (2).
D. All the three statements.
E. None of the three statements.

tifr2014 convergence non-gate

Answers: Convergence

10.1.1 Convergence: TIFR2014-A-15

https://gateoverflow.in/26402

1. for a large value of i, b_i and $b_i + 1$ becomes equal... so $bi^2 = bi + \sqrt{2}$. this is quadratic. solving this results to a fix number.
2. $cos(i) <= i$, So $cos(i) <= i^2$ that means series is decreasing that will return again a fix value.
3. it is the condition of convergence. if $b(i+1) < b(i)$ then series will always convergent.

1 votes -- Digvijay (44.9k points)

10.2 Differentiation (1)

10.2.1 Differentiation: TIFR2018-A-5

https://gateoverflow.in/179274

Which of the following is the derivative of $f(x) = x^x$ when $x > 0$?

A. x^x
B. $x^x \ln\ x$
C. $x^x + x^x \ln\ x$
D. $(x^x)(x^x \ln\ x)$
E. None of the above; function is not differentiable for $x > 0$

tifr2018 calculus differentiation

Answers: Differentiation

10.2.1 Differentiation: TIFR2018-A-5

https://gateoverflow.in/179274

✔ Let $y = x^x$

Taking natural log on both the sides, we get

$\ln\ y = x \ln\ x$

Differentiate both the sides wrt to x we get

$\frac{1}{y}\frac{dy}{dx} = (1) \ln\ x + x\frac{1}{x}$ (Product Rule)

$\frac{dy}{dx} = y \ln\ x + y$

Substitute $y = x^x$

$\frac{dy}{dx} = x^x \ln\ x + x^x$

Hence, option C is correct answer.

8 votes -- Ashwani Kumar (13.1k points)

10.3 Integration (2)

10.3.1 Integration: TIFR2011-A-11

https://gateoverflow.in/20219

$$\int_0^1 \log_e(x)dx =$$

A. 1 B. -1 C. ∞ D. $-\infty$ E. None of the above

tifr2011 calculus integration

10.3.2 Integration: TIFR2019-A-13

https://gateoverflow.in/280497

Consider the integral

$$\int_0^1 \frac{x^{300}}{1+x^2+x^3}dx$$

What is the value of this integral correct up to two decimal places?

A. 0.00 B. 0.02 C. 0.10 D. 0.33 E. 1.00

tifr2019 engineering-mathematics calculus integration

Answers: Integration

10.3.1 Integration: TIFR2011-A-11

https://gateoverflow.in/20219

✔ Use Integration by Parts

$\int \ln(x)dx$

set

$u = \ln(x), \quad dv = dx$

then we find

$du = \left(\frac{1}{x}\right)dx, \quad v = x$

substitute

$\int \ln(x)dx = \int u\,dv$

and use integration by parts

$= uv - \int v\,du$

substitute $u = \ln(x), v = x$, and $du = \left(\frac{1}{x}\right)dx$

$= \ln(x)\,x - \int x\left(\frac{1}{x}\right)dx$

$= \ln(x)x - \int dx$

$= \ln(x)x - x + C$

$= x\ln(x) - x + C.$

Now Put Limits

$[\ln(1) - 1 + C] - [0 - 0 + C] = -1$

Note-

$\lim[x \ln x] = 0.$

$x = 0$

Correct Answer: B

9 votes

-- sonu (1.8k points)

10.3.2 Integration: TIFR2019-A-13

https://gateoverflow.in/280497

✔ $\int_0^1 \frac{x^{300}}{1+x^2+x^3}dx \leq \int_0^1 x^{300}dx (\because 1+x^2+x^3 \geq 1)$

$\leq \left[\frac{x^{301}}{301}\right]_0^1 \leq \frac{1}{301} \leq 0.0033$

Only option matching is Option A.

17 votes

-- Arjun Suresh (334k points)

10.4 Limits (7)

10.4.1 Limits: TIFR2010-A-7

https://gateoverflow.in/18234

The limit of $\frac{10^n}{n!}$ as $n \to \infty$ is.

A. 0 B. 1 C. e D. 10 E. ∞

tifr2010 calculus limits

10.4.2 Limits: TIFR2011-A-14

https://gateoverflow.in/20224

The limit

$$\lim_{x \to 0} \frac{d}{dx} \frac{\sin^2 x}{x}$$

is

A. 0 B. 2 C. 1 D. $\frac{1}{2}$ E. None of the above

tifr2011 calculus limits

10.4.3 Limits: TIFR2011-A-17

https://gateoverflow.in/20254

What is

$$\lim_{x \to 0} \frac{2^x - 1}{x}$$

A. 0 B. $\log_2(e)$ C. $\log_e(2)$ D. 1 E. None of the above

tifr2011 limits

10.4.4 Limits: TIFR2012-A-14

https://gateoverflow.in/25037

The limit $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n \right)$ equals.

A. ∞ B. 1 C. $1/2$ D. 0 E. None of the above

tifr2012 calculus limits

10.4.5 Limits: TIFR2014-A-16

https://gateoverflow.in/27107

Let $x_0 = 1$ and

$x_{n+1} = \frac{3+2x_n}{3+x_n}, n \geq 0$.

$x_\infty = \lim_{n \to \infty} x_n$ is

A. $\left(\sqrt{5} - 1\right)/2$
B. $\left(\sqrt{5} + 1\right)/2$
C. $\left(\sqrt{13} - 1\right)/2$
D. $\left(-\sqrt{13} - 1\right)/2$
E. None of the above

tifr2014 limits

10.4.6 Limits: TIFR2014-A-18

https://gateoverflow.in/27128

We are given a collection of real numbers where a real number $a_i \neq 0$ occurs n_i times. Let the collection be enumerated as $\{x_1, x_2, \ldots x_n\}$ so that $x_1 = x_2 = \ldots = x_{n_1} = a_1$ and so on, and $n = \sum_i n_i$ is finite. What is

$$\lim_{k \to \infty} \left(\sum_{i=1}^{n} \frac{1}{|x_i|^k} \right)^{-1/k} ?$$

A. $\max_i \left(n_i |a_i|\right)$
B. $\min_i |a_i|$
C. $\min_i \left(n_i |a_i|\right)$
D. $\max_i |a_i|$
E. None of the above

tifr2014 limits

10.4.7 Limits: TIFR2019-A-15

https://gateoverflow.in/280495

Consider the matrix

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

What is $\lim_{n\to\infty} A^n$?

A. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

D. $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

E. The limit exists, but it is none of the above

tifr2019 engineering-mathematics calculus limits

Answers: Limits

10.4.1 Limits: TIFR2010-A-7

https://gateoverflow.in/18234

✔ $\lim_{n\to\infty} \frac{10^n}{n!}$

$$= \frac{\overbrace{10 \times 10 \times 10 \times \ldots \times 10 \times 10}^{n \text{ times}}}{1 \times 2 \times 3 \times \cdots \times (n-1) \times n}$$

$$= \overbrace{\frac{10}{1} \cdot \frac{10}{2} \cdots \frac{10}{10}}^{\approx 2755} \cdot \underbrace{\frac{10}{11} \cdots \frac{10}{100} \cdot \frac{10}{101}}_{\ll 1} \cdots \overbrace{\frac{10}{10000} \cdot \frac{10}{10001}}^{\ll 1} \cdots \text{ goes forever}$$

Now we can see that after the $\frac{10}{10}$ term, all subsequent terms are < 1, and keep decreasing. As we increase the value of n the product will get close to 0.

So as $n \to \infty$ $\frac{10^n}{n!} \to 0$.

Hence, the answer is option A.

26 votes -- minal (13.1k points)

10.4.2 Limits: TIFR2011-A-14

https://gateoverflow.in/20224

✔ Answer is 1.

$$\lim_{x\to 0} \frac{d}{dx} \frac{sin^2 x}{x}$$

Find $\frac{d}{dx}\left(\frac{\sin^2 x}{x}\right)$

$= \frac{(\sin^2 x)^1 x - (x)^1 \sin^2 x}{x^2}$

$= \frac{[2 \sin x . \cos x]\, x - sin^2 x}{x^2}$

$= \frac{x \sin 2x - \sin^2 x}{x^2}$

$\lim_{x \to 0}\left(\frac{x \sin 2x - \sin^2 x}{x^2}\right)$

$= \lim_{x \to 0}\left(\frac{x \sin 2x}{x^2} - \frac{\sin^2 x}{x^2}\right)$

$= \lim_{x \to 0} \frac{x \sin 2x}{x^2} - \lim_{x \to 0}\left(\frac{\sin x}{x}\right)^2$

$= \lim_{x \to 0} \frac{\cos 2x . 2}{1} - 1$

$= 2[\cos 2(0)] - 1$

$= 2(1) - 1 \Rightarrow 1.$

17 votes

-- Prabhanjan_1 (8.5k points)

10.4.3 Limits: TIFR2011-A-17

https://gateoverflow.in/20254

✔ Since we have a $\frac{0}{0}$ form, we can apply the L'Hôpital's rule.

$$\begin{aligned} L &= \lim_{x \to 0} \frac{2^x - 1}{x} \\ &= \lim_{x \to 0} \frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx} x} \\ &= \lim_{x \to 0} \frac{2^x \ln 2}{1} \\ &= \ln 2 \\ L &= \log_e (2) \end{aligned}$$

Hence, option c is correct.

18 votes

-- Pragy Agarwal (18.3k points)

10.4.4 Limits: TIFR2012-A-14

https://gateoverflow.in/25037

✔ $L = \lim_{n \to \infty} \sqrt{n^2 + n} - n$

$= \lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right) \times \left(\frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n}\right)$

$= \lim_{n \to \infty} \frac{n^2 + n - n^2}{\sqrt{n^2 + n} + n}$

$= \lim_{n \to \infty} \frac{n}{n\left(\sqrt{1 + \frac{1}{n}} + 1\right)}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1}$$
$$= \frac{1}{\sqrt{1+\frac{1}{\infty}}+1}$$
$$L = \frac{1}{2}$$

Hence, option C is the correct answer.

17 votes -- Jagdish Singh (413 points)

10.4.5 Limits: TIFR2014-A-16

https://gateoverflow.in/27107

✔ Answer : C

$$x_{n+1} = 1 + \frac{x(n)}{3 + x(n)}$$

As n tends to infinity, $x_{n+1} = x_n = x$

$$x = 1 + \frac{x}{3+x}$$

$$\implies x^2 + 3x = 3 + 2x$$

$$\implies x^2 + x - 3 = 0$$

The roots are: $\frac{-1+\sqrt{13}}{2}, \frac{-1-\sqrt{13}}{2}$

Since, x_n is positive,

$$x = \frac{-1+\sqrt{13}}{2}$$

7 votes -- Vikranth Inti (579 points)

10.4.6 Limits: TIFR2014-A-18

https://gateoverflow.in/27128

✔ $$\lim_{k\to\infty} \left(\sum_{i=1}^{n} \frac{1}{|x_i|^k} \right)^{-1/k}$$

$= \lim_{k\to\infty} \left(\sum_{i} n_i \frac{1}{|a_i|^k} \right)^{-1/k}$ (The i here is different from the previous expression and denote the i unique real values repeated n_i times each.

$$= \lim_{k\to\infty} \left(\sum_{i} n_i |a_i|^{-k} \right)^{-1/k}$$

Since, k is very large $(\to \infty)$, the summation above will be dominated by the largest value for $n_i|a_i|^{-k}$ which will come for $\min|a_i|$ due to $-k$ in exponent. We can also say that the summation series converges to $n_i(\min|a_i|)^{-k}$. So, our limit becomes

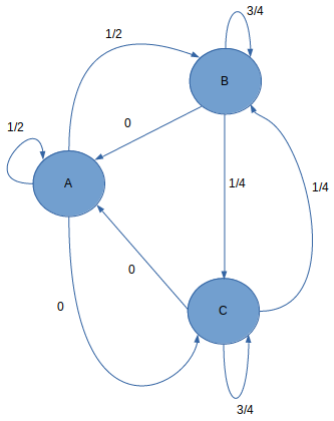
$$= \lim_{k\to\infty} \left(n_i(\min|a_i|)^{-k} \right)^{-1/k} = n_i^{-1/k}((\min|a_i|)^{-k})^{-1/k}$$
$$= n_i^0 \min|a_i| = \min|a_i|$$

Correct Option: B.

3 votes -- Arjun Suresh (334k points)

10.4.7 Limits: TIFR2019-A-15 https://gateoverflow.in/280495

We observe that given matrix A is a right stochastic matrix. That is, for every row the row sum is 1. What does that mean? It means the given matrix corresponds to some Markov system. which is: A 3 state system as follows : (with each edge being the probability of the system going from a state to other.)



Now we find the steady-state probability vector as follows :

$$[x_1, x_2, x_3] \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix} = [x_1, x_2, x_3]$$

As well as, we have $x_1 + x_2 + x_3 = 1$
Solving the above equation, we get $x_1 = 0, x_2 = \frac{1}{2}, x_3 = \frac{1}{2}$

We know that, in a Markov chain, A^∞ is nothing but all rows steady-state vector.

Therefore, $A^\infty = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Ref: If you want to know more about Markov chains and stochastic matrices:
https://en.wikipedia.org/wiki/Stochastic_matrix#Definition_and_properties

References

4 votes -- severustux (111 points)

10.5 Maxima Minima (9)

10.5.1 Maxima Minima: TIFR2010-A-3 https://gateoverflow.in/18209

The function $f(x) = 2.5 \log_e \left(2 + \exp\left(x^2 - 4x + 5\right)\right)$ attains a minimum at $x =$?

A. 0 B. 1 C. 2 D. 3 E. 4

tifr2010 calculus maxima-minima

10.5.2 Maxima Minima: TIFR2011-A-4

https://gateoverflow.in/20002

Consider the problem of maximizing $x^2 - 2x + 5$ such that $0 < x < 2$. The value of x at which the maximum is achieved is:

A. 0.5 B. 1 C. 1.5 D. 1.75 E. None of the above

tifr2011 calculus maxima-minima

10.5.3 Maxima Minima: TIFR2012-A-13

https://gateoverflow.in/25036

The maximum value of the function

$f(x,y,z) = (x-1/3)^2 + (y-1/3)^2 + (z-1/3)^2$

subject to the constraints

$x + y + z = 1, \quad x \geq 0, y \geq 0, z \geq 0$

is

A. $1/3$ B. $2/3$ C. 1 D. $4/3$ E. $4/9$

tifr2012 calculus maxima-minima

10.5.4 Maxima Minima: TIFR2012-A-15

https://gateoverflow.in/25040

Consider the differential equation $dx/dt = (1-x)(2-x)(3-x)$. Which of its equilibria is unstable?

A. $x = 0$ B. $x = 1$ C. $x = 2$ D. $x = 3$ E. None of the above

tifr2012 calculus maxima-minima

10.5.5 Maxima Minima: TIFR2013-A-16

https://gateoverflow.in/25496

The minimum of the function $f(x) = x \log_e(x)$ over the interval $[\frac{1}{2}, \infty)$ is

A. 0 B. $-e$ C. $\frac{-\log_e(2)}{2}$ D. $\frac{-1}{e}$ E. None of the above

tifr2013 calculus maxima-minima

10.5.6 Maxima Minima: TIFR2014-A-9

https://gateoverflow.in/25996

Solve min $x^2 + y^2$ subject to

$$\begin{aligned} x + y &\geq 10, \\ 2x + 3y &\geq 20, \\ x &\geq 4, \\ y &\geq 4. \end{aligned}$$

A. 32 B. 50 C. 52 D. 100 E. None of the above

tifr2014 calculus maxima-minima

10.5.7 Maxima Minima: TIFR2015-A-10

https://gateoverflow.in/29579

Let $f(x), x \in [0,1]$, be any positive real valued continuous function. Then

$$\lim_{n\to\infty} (n+1) \int_0^1 x^n f(x) \mathrm{d}x$$

equals.

A. $\max_{x\in[0,1]} f(x)$ B. $\min_{x\in[0,1]} f(x)$
C. $f(0)$ D. $f(1)$
E. ∞

tifr2015 maxima-minima calculus non-gate integration

10.5.8 Maxima Minima: TIFR2015-A-11

https://gateoverflow.in/29581

Suppose that $f(x)$ is a continuous function such that $0.4 \leq f(x) \leq 0.6$ for $0 \leq x \leq 1$. Which of the following is always true?

A. $f(0.5) = 0.5$.
B. There exists x between 0 and 1 such that $f(x) = 0.8x$.
C. There exists x between 0 and 0.5 such that $f(x) = x$.
D. $f(0.5) > 0.5$.
E. None of the above statements are always true.

tifr2015 maxima-minima calculus

10.5.9 Maxima Minima: TIFR2020-A-8

https://gateoverflow.in/333108

Consider a function $f : [0, 1] \rightarrow [0, 1]$ which is twice differentiable in $(0, 1)$. Suppose it has exactly one global maximum and exactly one global minimum inside $(0, 1)$. What can you say about the behaviour of the first derivative f' and and second derivative f'' on $(0, 1)$ (give the most precise answer)?

A. f' is zero at exactly two points, f'' need not be zero anywhere
B. f' is zero at exactly two points, f'' is zero at exactly one point
C. f' is zero at at least two points, f'' is zero at exactly one point
D. f' is zero at at least two points, f'' is zero at at least one point
E. f' is zero at at least two points, f'' is zero at at least two points

tifr2020 engineering-mathematics calculus maxima-minima

Answers: Maxima Minima

10.5.1 Maxima Minima: TIFR2010-A-3

https://gateoverflow.in/18209

✔ Digvijay is right, $f(x)$ will be minimum at $x = 2$,
Here is another approach.
Since log and exponent are monotonically increasing functions, the problem of minimizing $f(x)$ can be reduced to just minimizing the quadratic expression

$x^2 - 4x + 5$,
this quadratic expression can be written as $(x^2 - 4x + 4) + 1$ which is equal to $(x - 2)^2 + 1$.
now since $(x - 2)^2$ can not be less than 0, so $(x - 2)^2 + 1$ can not be less than 1.
Also $(x - 2)^2 + 1$ will be at its minimum value $(= 1)$, when $x = 2$.

so value of $f(x)$ will be minimum at $x = 2$.

26 votes -- Anurag Pandey (10.5k points)

10.5.2 Maxima Minima: TIFR2011-A-4

https://gateoverflow.in/20002

✔

$$P(x) = x^2 - 2x + 5$$

Since a polynomial is defined and continuous everywhere, we only need to check the critical point and the boundaries.

$\frac{d}{dx}P(x) = 2x - 2$

Critical point: $2x - 2 = 0 \implies x = 1$ gives $P(x) = 4$, which is the minimum.

Boundaries: $\lim_{x \to 0^+} P(x) = \lim_{x \to 2^-} P(x) = 5$

Since $P(x)$ increases as x goes farther away from the 1. But $P(x)$ **being defined on an open interval, it never attains a maximum!**

Hence, **e. None of the above** is the correct answer.

14 votes -- Pragy Agarwal (18.3k points)

10.5.3 Maxima Minima: TIFR2012-A-13

https://gateoverflow.in/25036

✔ Expanding given equation

$x^2+y^2+z^2-\frac{2}{3}(x+y+z)+\frac{1}{3}$

$=x^2+y^2+z^2-\frac{2}{3}+\frac{1}{3}$

$=(x+y+z)^2-\frac{1}{3}-2(xy+yz+xz)$

$=1-\frac{1}{3}-2(xy+yz+xz)$

$=\frac{2}{3}-2(xy+yz+xz)$

Now to maximize it, we need to minimize $(xy+yz+xz)$. As all x, y and z are non-negative $xy+yz+xz \geq 0$. So, the maximum value is $\frac{2}{3}$.

Correct Option: B.

7 votes

-- Dhruv Patel (1.6k points)

10.5.4 Maxima Minima: TIFR2012-A-15

https://gateoverflow.in/25040

✔ Here, dx/dt is neither constant nor function of variable t. So, on solving for x, we get exponential decay function something similar to radioactive decay $dN/dt=-KN; K>0$.

For any equilibrium point, we have $dx/dt=0 \implies x=1,2,3.$

Now if we consider sign scheme of dx/dt for different ranges of x, we have :

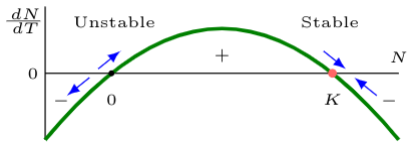
- $dx/dt>0$ for $x<1$;
- $dx/dt<0$ for $1<x<2$;
- $dx/dt>0$ for $2<x<3$;
- $dx/dt<0$ for $x>3$;

For $x \to 2, dx/dt$ changes from negative to positive. So, $x=2$ is a point of unstable equilibrium.

Since, here dx/dt is a function of x not t, so condition for checking stable / unstable points (aka local minima / local maxima) is opposite to what we apply in case of $dy/dx=f(x)$. Here we have $dy/dx=f(y)$ which is actually exponential function in y and x.

We can understand equilibrium in terms of radioactive decay.

Let $dN/dt=-KN; K>0$ its significance is that an element is loosing energy so it is getting stability because we know more energy an element gets,more de-stability it gains and vice versa.



Correct Answer: C

12 votes

-- Shashank Kumar (2.3k points)

10.5.5 Maxima Minima: TIFR2013-A-16

https://gateoverflow.in/25496

Minimum value of function occurs at end points or critical points

$f'(x)=1+\log x$

Equate it to 40

$x=\frac{1}{e}$

$f''(x) = \frac{1}{x}$

Put $x = \frac{1}{e}$ $f''(x) = e$ so minima at $\frac{1}{e}$.

But $\frac{1}{e} = 0.36$

But $x \in \left[\frac{1}{2}, \infty\right]$

So min occurs at $\frac{1}{2}$

So min value=$\frac{1}{2}\log \frac{1}{2} = \frac{1}{2} * \log_e -2$

So, answer is C

12 votes -- Pooja Palod (24.1k points)

10.5.6 Maxima Minima: TIFR2014-A-9

https://gateoverflow.in/25996

✔ Answer: Option (B) 50

$x \geq 4$ and $y \geq 4$, So we can take both $x = 5$ & $y = 5$

$x + y \geq 10$, Satisfied , $5 + 5 = 10$

$2x + 3y \geq 20$. Satisfied.

This is in fact the minimum value.

Other options:

$4, 4 \implies x + y \geq 10$ constraint fail

$4, 5 \implies x + y \geq 10$ fail

$6, 4 \implies$ Sum $= 52$ which is more than 50 and hence cannot be answer.

$7, 3 \implies 49 + 9 = 58$ which is again greater than 50 and hence cannot be the answer.

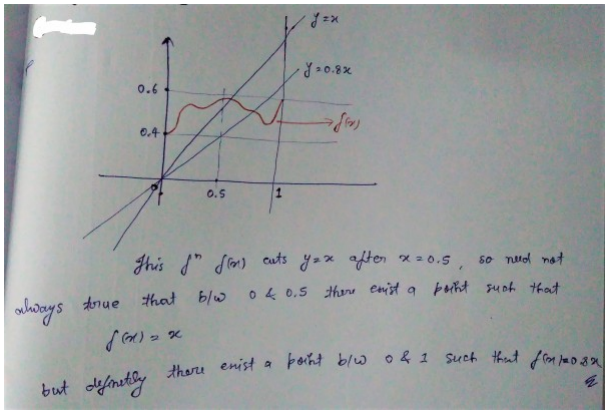
7 votes -- Akash Kanase (36.1k points)

10.5.8 Maxima Minima: TIFR2015-A-11

https://gateoverflow.in/29581

Another approach...

It may be possible that... ->



14 votes -- Gyanendra Singh (2.3k points)

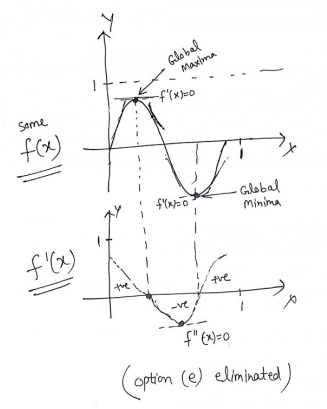
10.5.9 Maxima Minima: TIFR2020-A-8 https://gateoverflow.in/333108

Function f is differentiable in domain $[0,1]$, it means f is continuous in the given domain and no corner edge is present.

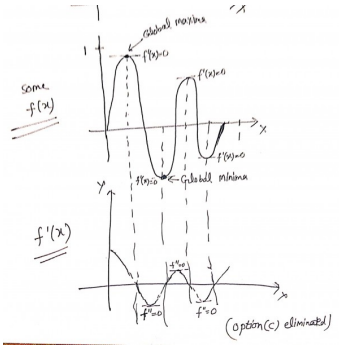
Since, f has exactly one global maxima and one global minima but it can have many local maxima/minima, So, f' must be zero at atleast 2 points which is shown below.

Now, possible options are : b, c, d.

Now, consider some function $f(x)$ as given below which eliminates option (e) :



Now, consider some function $f(x)$ as given below which eliminates option (c) :



1 votes -- ankitgupta.1729 (15.2k points)

Answer Keys

10.0.1	D	10.1.1	D	10.2.1	C	10.3.1	B	10.3.2	A
10.4.1	A	10.4.2	C	10.4.3	C	10.4.4	C	10.4.5	C
10.4.6	B	10.4.7	D	10.5.1	C	10.5.2	E	10.5.3	B
10.5.4	C	10.5.5	C	10.5.6	B	10.5.7	D	10.5.8	B
10.5.9	D								

11 Engineering Mathematics: Linear Algebra (14)

Syllabus: Matrices, determinants, System of linear equations, Eigenvalues and eigenvectors, LU decomposition.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	0	1	0	1	1	1	1	1	2	0	0.88	2
2 Marks Count	1	1	1	1	1	2	1	0	0	0	0.88	2
Total Marks	2	3	2	3	3	5	3	1	2	1	2.66	5

11.0.1 TIFR2020-A-12

https://gateoverflow.in/333114

The hour needle of a clock is malfunctioning and travels in the anti-clockwise direction, i.e., opposite to the usual direction, at the same speed it would have if it was working correctly. The minute needle is working correctly. Suppose the two needles show the correct time at 12 noon, thus both needles are together at the 12 mark. After how much time do the two needles meet again?

A. $\frac{10}{11}$ hour B. $\frac{11}{12}$ hour C. $\frac{12}{13}$ hour D. $\frac{19}{22}$ hour E. One hour

tifr2020

Answers:

11.0.1 TIFR2020-A-12

https://gateoverflow.in/333114

Since, minute needle takes 60 minutes to cover 360°, So, in 1 minute, it will cover 6°. Similarly, hour needle takes $12 \times 60 = 720$ minutes to cover 360°, So, in 1 minute, it will cover $(\frac{1}{2})^\circ$.

When both needles are at 12 and starts traveling in their direction. Suppose, after t minutes, minute needle covers x° with the speed of $6^\circ/min$ and in the same time, hour needle covers $(360-x)^\circ$ with the speed of $(\frac{1}{2})^\circ/min$.

When they meet, then $t = \frac{x^\circ}{6^\circ/min} = \frac{(360-x)^\circ}{(\frac{1}{2})^\circ/min}$

$\Rightarrow x = (\frac{12*360}{13})^\circ$

Since, $t = \frac{x^\circ}{6^\circ/min}$

So, $t = \frac{(\frac{12*360}{13})^\circ}{6^\circ}\ min$

$t = \frac{12}{13} \times 60\ min$

$t = \frac{12}{13} Hour$

2 votes -- ankitgupta.1729 (15.2k points)

11.1 Eigen Value (1)

11.1.1 Eigen Value: TIFR2019-A-3

https://gateoverflow.in/280507

A is $n \times n$ square matrix for which the entries in every row sum to 1. Consider the following statements:

i. The column vector $[1, 1, \ldots, 1]^T$ is an eigen vector of A.
ii. $\det(A - I) = 0$.
iii. $\det(A) = 0$.

Which of the above statements must be **TRUE**?

A. Only (i) B. Only (ii) C. Only (i) and (ii) D. Only (i) and (iii) E. $(i), (ii)$ and (iii)

tifr2019 engineering-mathematics linear-algebra eigen-value

Answers: Eigen Value

11.1.1 Eigen Value: TIFR2019-A-3

https://gateoverflow.in/280507

✔ (iii) is clealy incorrect. Example : $\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

Determinant of this matrix is -1

Let the matrix $A = \begin{bmatrix}x_{11} & x_{12} & x_{13} & \dots\\x_{21} & x_{22} & x_{23} & \dots\\x_{31} & x_{32} & x_{33} & \dots\\\dots & \dots & \dots & \dots\end{bmatrix}$ of order $n * n$

$\sum_{j=1}^{n} x_{ij} = 1 \; \forall \; i = 1, \dots, n$

We know, $Av = \lambda v$, where λ are the eigenvalues and v is the eigenvector

As, $Av = v$

$$\begin{bmatrix}x_{11} & x_{12} & x_{13} & \dots\\x_{21} & x_{22} & x_{23} & \dots\\x_{31} & x_{32} & x_{33} & \dots\\\dots & \dots & \dots & \dots\end{bmatrix}\begin{bmatrix}1\\1\\1\\\dots\end{bmatrix} = \begin{bmatrix}x_{11}+x_{12}+x_{13}+\dots\\x_{21}+x_{22}+x_{23}+\dots\\x_{31}+x_{32}+x_{33}+\dots\\x_{41}+x_{42}+x_{43}+\dots\end{bmatrix} = \begin{bmatrix}1\\1\\1\\\dots\end{bmatrix} = v$$

v is an eigen vector and eigen value is 1

So, (i) is *correct*

$$A - \lambda I = \begin{bmatrix}x_{11}-1 & x_{12} & x_{13} & \dots\\x_{21} & x_{22}-1 & x_{23} & \dots\\x_{31} & x_{32} & x_{33}-1 & \dots\\\dots & \dots & \dots & \dots\end{bmatrix}$$

Now, $\sum_{j=1}^{n} x_{ij} = 0 \; \forall \; i = 1, \dots, n$

Applying Matrix transformation,

$C_1 \leftarrow \sum_{j=1}^{n} C_j$

The matrix now becomes = $\begin{bmatrix}0 & x_{12} & x_{13} & \dots\\0 & x_{22}-1 & x_{23} & \dots\\0 & x_{32} & x_{33}-1 & \dots\\\dots & \dots & \dots & \dots\end{bmatrix}$

As C_1 is completely 0 , so $|A - \lambda I| = 0$

(ii) is *correct*

So, option (C).

9 votes

-- Shobhit Joshi (3.9k points)

11.2 Matrices (8)

11.2.1 Matrices: TIFR2010-A-16

https://gateoverflow.in/18492

Let the characteristic equation of matrix M be $\lambda^2 - \lambda - 1 = 0$. Then.

A. M^{-1} does not exist.
B. M^{-1} exists but cannot be determined from the data.
C. $M^{-1} = M + I$

D. $M^{-1} = M - I$
E. M^{-1} exists and can be determined from the data but the choices (c) and (d) are incorrect.

tifr2010 linear-algebra matrices

11.2.2 Matrices: TIFR2010-A-5 https://gateoverflow.in/18216

A is symmetric positive definite matrix (i.e., $x^T Ax > 0$ for all non zero x). Which of the following statements is false?

A. At least one element is positive.
B. All eigen values are positive real.
C. Sum of the diagonal elements is positive.
D. *det* (A) is positive.
E. None of the above.

tifr2010 linear-algebra matrices

11.2.3 Matrices: TIFR2012-B-12 https://gateoverflow.in/25141

Let A be a matrix such that $A^k = 0$. What is the inverse of $I - A$?

A. 0
B. I
C. A
D. $1 + A + A^2 + \ldots + A^{k-1}$
E. Inverse is not guaranteed to exist.

tifr2012 linear-algebra matrices

11.2.4 Matrices: TIFR2013-B-3 https://gateoverflow.in/25659

How many 4×4 matrices with entries from $0, 1$ have odd determinant?

Hint: Use modulo 2 arithmetic.

A. 20160 B. 32767 C. 49152 D. 57343 E. 65520

tifr2013 linear-algebra matrices

11.2.5 Matrices: TIFR2015-A-14 https://gateoverflow.in/29588

Consider the following 3×3 matrices.

$$M_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

How may $0 - 1$ column vectors of the form

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

are there such that $M_1 X = M_2 X$ (modulo 2)? (modulo 2 means all operations are done modulo 2, i.e, $3 = 1$ (modulo 2), $4 = 0$ (modulo 2)).

A. None B. Two C. Three D. Four E. Eight

tifr2015 matrices

11.2.6 Matrices: TIFR2018-A-12 https://gateoverflow.in/179281

An $n \times n$ matrix M with real entries is said to be positive definite if for every non-zero n-dimensional vector x with real entries, we have $x^T Mx > 0$. Let A and B be symmetric, positive definite matrices of size $n \times n$ with real entries.

Consider the following matrices, where I denotes the $n \times n$ identity matrix:

1. $A + B$

2. ABA
3. $A^2 + I$

Which of the above matrices must be positive definite?

A. Only (2)
B. Only (3)
C. Only (1) and (3)
D. None of the above matrices are positive definite
E. All of the above matrices are positive definite

tifr2018 matrices linear-algebra

11.2.7 Matrices: TIFR2018-A-14

https://gateoverflow.in/179377

Let A be an $n \times n$ invertible matrix with real entries whose row sums are all equal to c. Consider the following statements:

1. Every row in the matrix $2A$ sums to $2c$.
2. Every row in the matrix A^2 sums to c^2.
3. Every row in the matrix A^{-1} sums to c^{-1}.

Which of the following is **TRUE**?

A. none of the statements $(1), (2), (3)$ is correct
B. statement (1) is correct but not necessarily statements (2) or (3)
C. statement (2) is correct but not necessarily statements (1) or (3)
D. statement (1) and (2) are correct but not necessarily statement (3)
E. all the three statements $(1), (2)$, and (3) are correct

tifr2018 matrices linear-algebra

11.2.8 Matrices: TIFR2020-A-5

https://gateoverflow.in/333104

Let A be am $n \times n$ invertible matrix with real entries whose column sums are all equal to 1. Consider the following statements:

1. Every column in the matrix A^2 sums to 2
2. Every column in the matrix A^3 sums to 3
3. Every column in the matrix A^{-1} sums to 1

Which of the following is TRUE?

A. none of the statements $(1), (2), (3)$ is correct
B. statement (1) is correct but not statements (2) or (3)
C. statement (2) is correct but not statements (1) or (3)
D. statement (3) is correct but not statements (1) or (2)
E. all the 3 statements $(1), (2)$, and (3) are correct

tifr2020 engineering-mathematics linear-algebra matrices

Answers: Matrices

11.2.1 Matrices: TIFR2010-A-16

https://gateoverflow.in/18492

✔ We can solve using Cayley- Hamilton Theorem

$\lambda^2 - \lambda - 1 = 0$
$\implies M^2 - M - I = 0$
$\implies I = M^2 - M$

Now premultiplying by M^{-1}

$M^{-1}I = M^{-1}M^2 - M^{-1}M$
$M^{-1} = M - I$

Correct Answer: D

18 votes -- Umang Raman (12.2k points)

11.2.2 Matrices: TIFR2010-A-5

https://gateoverflow.in/18216

✔ $x^TAx > 0$ means that matrix is positive definite. And properties of positive definite matrix (relevant to question) is :

- All its eigen values are positive.

Let's consider a $3 * 3$ matrix for simplicity.

1. Trace(A) = sum of eigen values = positive
2. Det(A) = product of eigen values = positive

Take any matrix and try to anlayse the validity of these points.

So, (E) is the answer here.

3 votes -- Manish Joshi (20.5k points)

11.2.3 Matrices: TIFR2012-B-12

https://gateoverflow.in/25141

✔ Given $A^k = 0$

Subtract from I on both sides,

$I - A^k = I \qquad \rightarrow (1)$
Now $I - A^k$ can be written as,

$I - A^k = (I - A)(I + A + A^2 + A^3 + \ldots + A^{k-1})$ (simplifying RHS we get LHS)

Putting this in (1),

$I = (I - A)(I + A + A^2 + A^3 + \ldots A^{k-1})$

Now question is $(I - A)^{-1}$ so we multiply with $(I - A)^{-1}$ on both sides,

$(I - A)^{-1} = (I + A + A^2 + A^3 + \ldots + A^{k-1})$

Hence, (D) is the Answer.

34 votes -- Leen Sharma (28.7k points)

11.2.4 Matrices: TIFR2013-B-3

https://gateoverflow.in/25659

✔ OPTION A is correct

Whenever the 1^{st} row is 0 then its determent is 0, and similarly if any 2 or more rows are linearly dependent then its $|det| = 0$

In order to find the odd determinant the 1^{st} row must be non zero
$\Rightarrow$ totally$(2^4 - 1)$ possibilities $\left|\begin{matrix}0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1\end{matrix}\right|$ like totally $= 16 - 1$

$2nd$ row must be non zero and not linearly depending on 1^{st} row $\Rightarrow$ totally $(2^4 - 2)$ possibilities
For $3rd$ row it must be non-zero as well as not linearly depending on first 2 rows (not start with 0) $\Rightarrow$ totally $(2^4 - 4)$

For 4th row $\Rightarrow (2^4 - 8)$

Total possibilities $= (2^4 - 2^0) * (2^4 - 2^1) * (2^4 - 2^2) * (2^4 - 2^3) = 15 * 14 * 12 * 8 = 20160.$

18 votes -- venky.victory35 (741 points)

11.2.5 Matrices: TIFR2015-A-14

https://gateoverflow.in/29588

✔ $M_1X = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 + x_3 \\ x_1 + x_3 \\ x_1 + x_2 \end{pmatrix}$

$M_2X = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_3 \\ 0 \\ x_1 + x_3 \end{pmatrix}$

Given $M_1X = M_2X \mod 2$

So, by comparing both matrices, we can say

$(x_2 + x_3) \mod 2 = (x_1 + x_3) \mod 2$

$(x_1 + x_3) \mod 2 = 0$ // so either $x_1 = x_3 = 0$ OR $x_1 = x_3 = 1$

$(x_1 + x_2) \mod 2 = (x_1 + x_3) \mod 2$

So, we can see when $x_1 = x_3 = 0$ then $x_2 = 0$ and when $x_1 = x_3 = 1$ then $x_2 = 1$

Eventually there can be two $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $0-1$ column vectors for X.

Correct Answer: B

11 votes

-- Rupendra Choudhary (11.4k points)

11.2.6 Matrices: TIFR2018-A-12

https://gateoverflow.in/179281

Note

- The sum of two positive definite matrices is positive definite. $(x^T(A+B)x = x^TAx + x^TBx > 0)$.

$i)$ $A + B$ is positive definite from above property.

$ii)$ $x^T(ABA)x = (x^TA)B(Ax) = (Ax)^TB(Ax) > 0$

$iii)$ Since A is symmetric A^2 is also symmetric. And $A^2 + I$ is also positive definite since I is positive definite.

Thus, all of the above matrices are positive definite.

6 votes

-- krish__ (4.6k points)

11.2.7 Matrices: TIFR2018-A-14

https://gateoverflow.in/179377

Take a matrix:

$\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

Sum of each row$= a + b$ [matrix is invertible hence determinant is not 0 i.e. a and b are different]

Now lets prove:

$2 \times \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 2b & 2a \end{bmatrix}$: Sum of each row $= 2(a+b)$

$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \times \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+ba \\ ab+ba & a^2+b^2 \end{bmatrix}$: Sum of each rows $= (a^2 + b^2 + 2ab) = (a+b)^2$

$\frac{1}{a^2-b^2}\begin{bmatrix} a & -b \\ -b & a \end{bmatrix} = \frac{(a-b)}{a^2-b^2} = \frac{(a-b)}{(a+b)(a-b)} = \frac{1}{a+b}$

Hence, all are true

Correct Answer: E

14 votes -- Anu007 (14.4k points)

11.2.8 Matrices: TIFR2020-A-5

https://gateoverflow.in/333104

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- Every column in the matrix A^2 sums to 2. $\implies$ False

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- Every column in the matrix A^3 sums to 3. $\implies$ False

$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Every column in the matrix A^{-1} sums to 1. $\implies$ True

Lets take another example for statement 3 :

$A = \begin{bmatrix} 3 & -6 \\ -2 & 7 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

In other words: swap the positions of a and d, put **negatives** in front of b and c, and **divide** everything by the determinant $(ad - bc)$.

$A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & 6 \\ 2 & 3 \end{bmatrix}$

- Every column in the matrix A^{-1} sums to 1. $\implies$ False

So, the correct answer is (D).

References

3 votes -- Lakshman Patel (69.5k points)

11.3 Rank Of Matrix (1)

11.3.1 Rank Of Matrix: TIFR2020-A-2

https://gateoverflow.in/333102

Let M be a real $n \times n$ matrix such that for *every* non-zero vector $x \in \mathbb{R}^n$, we have $x^T Mx > 0$. Then

A. Such an M cannot exist
B. Such Ms exist and their rank is always n
C. Such Ms exist, but their eigenvalues are always real
D. No eigenvalue of any such M can be real
E. None of the above

tifr2020 engineering-mathematics linear-algebra rank-of-matrix eigen-value

Answers: Rank Of Matrix

11.3.1 Rank Of Matrix: TIFR2020-A-2

https://gateoverflow.in/333102

-- For $a)$ and $d)$, counter-examples are identity matrices like $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for $n = 2$.

Because, considering a non-zero vector x as $= \begin{bmatrix} a \\ b \end{bmatrix}$ where, $a > 0, b > 0$

Now, $x^T Mx = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = a^2 + b^2 > 0.$ So, such Ms exist and eigen values can be real.

-- For (c), counter-example is : $M = \begin{bmatrix} 2k & 2k \\ -2k & 2k \end{bmatrix}$ where $k \geq 1$

Here, also on considering a non-zero vector x as $= \begin{bmatrix} a \\ b \end{bmatrix}$ where, $a > 0, b > 0$

Now, $x^T Mx = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 2k & 2k \\ -2k & 2k \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 2k(a^2 + b^2) > 0.$ So, such Ms exist and eigen values can be complex here.

-- Now, for (b), Suppose, $x \in \mathbb{R}^n$ is a non-zero eigen vector associated with matrix M.

Then, $Mx = \lambda x$

$\Rightarrow x^T Mx = x^T \lambda x$

$\Rightarrow x^T Mx = \lambda x^T x$

Since, $x^T x > 0$ and If $\lambda > 0$ then $x^T Mx > 0$

So, For positive eigenvalues λ and a non-zero eigen-vectors x, $x^T Mx > 0$ is always true.

Since, determinant is the product of eigen values. So, if all eigen values are positive then determinant will be positive and matrix M will be non-singular and has full rank n. So, (b) is true.

1 votes -- ankitgupta.1729 (15.2k points)

11.4 Vector Space (3)

11.4.1 Vector Space: TIFR2010-A-11

https://gateoverflow.in/18503

The length of a vector $X = (x_1, \ldots, x_n)$ is defined as

$$\|X\| = \sqrt{\sum_{i=1}^{n} x_i^2}$$

Given two vectors $X = (x_1, \ldots, x_n)$ and $Y = (y_1, \ldots, y_n)$, which of the following measures of discrepancy between X and Y is insensitive to the length of the vectors?

A. $\|X - Y\|$
B. $\|X - Y\| \,/\, \|x\| \, \|y\|$
C. $\|X\| - \|Y\|$
D. $\left\| \frac{X}{\|X\|} - \frac{Y}{\|Y\|} \right\|$
E. None of the above

tifr2010 linear-algebra vector-space

11.4.2 Vector Space: TIFR2017-A-2

https://gateoverflow.in/94938

For vectors x, y in $\mathbb{R}^n$, define the inner product $\langle x, y \rangle = \Sigma_{i=1}^{n} x_i y_i$, and the length of x to be $\|x\| = \sqrt{\langle x, x \rangle}$. Let a, b be two vectors in $\mathbb{R}^n$ so that $\|b\| = 1$. Consider the following statements:

i. $\langle a, b \rangle \leq \|b\|$
ii. $\langle a, b \rangle \leq \|a\|$
iii. $\langle a, b \rangle = \|a\| \|b\|$
iv. $\langle a, b \rangle \geq \|b\|$
v. $\langle a, b \rangle \geq \|a\|$

Which of the above statements must be TRUE of a, b? Choose from the following options.

A. ii only B. i and ii C. iii only D. iv only E. iv and v

tifr2017 linear-algebra vector-space

11.4.3 Vector Space: TIFR2020-A-3

https://gateoverflow.in/333103

Let $d \geq 4$ and fix $w \in \mathbb{R}$. Let

$$S = \{a = (a_0, a_1, \ldots, a_d) \in \mathbb{R}^{d+1} \mid f_a(w) = 0 \text{ and } f_a'(w) = 0\},$$

where the polynomial function $f_a(x)$ is defined as $f_a(x) := \sum_{i=0}^{d} a_i x^i$ and $f_a'(w)$ denotes the derivative of $f_a(x)$ with respect to x, evaluated at w. Then,

A. S is finite or infinite depending on the value of α
B. S is a 2-dimensional vector subspace of $\mathbb{R}^{d+1}$
C. S is a d-dimensional vector subspace of $\mathbb{R}^{d+1}$
D. S is a $(d-1)$-dimensional vector subspace of $\mathbb{R}^{d+1}$
E. None of the other options

tifr2020 engineering-mathematics linear-algebra vector-space

Answers: Vector Space

11.4.1 Vector Space: TIFR2010-A-11

https://gateoverflow.in/18503

✔ **Option D is the correct answer.**

The following statement

> discrepancy between x and y is insensitive to the length of the vectors

means that The discrepancy, as measured by the formula, between two vectors x and y is same as the discrepancy between the vectors $c_1 x$ and $c_2 y$, for any **constant scalars** c_1, c_2. That is,

$D(x, y) = D(c_1 x, c_2 y), \quad \forall c_1, c_2 \in \mathbb{R}$

Now, lets think about which formula achieves that.

Let us also define two pairs of vectors as follows:

$x_1 = (0.1067, 0.9619, 0.0046, 0.7749, 0.8173)$

$y_1 = (0.8687, 0.0844, 0.3998, 0.2599, 0.8001)$

$x_2 = 0.4314 \times x_1 = (0.0460, 0.4150, 0.0020, 0.3343, 0.3526)$

$y_2 = 0.9106 \times y_1 = (0.7911, 0.0769, 0.3641, 0.2367, 0.7286)$

A) $\|x - y\|$

Since the definition of $\|x\|$ is sensitive to scaling, option A won't be insensitive to scaling either.

For example,

$D(x_1, y_1) = \|x_1 - y_1\| \approx 1.3313$

$D(x_2, y_2) = \|x_2 - y_2\| \approx 0.9754$

$D(x_1, y_1) \neq D(x_2, y_2)$

B) $\frac{\|x-y\|}{\|x\|\|y\|}$

Once we've subtracted the vectors, scaling them according to their original lengths won't help at all.

For example,

$D(x_1, y_1) = \frac{\|x_1 - y_1\|}{\|x_1\|\|y_1\|} \approx 0.7024$

$D(x_2, y_2) = \frac{\|x_2 - y_2\|}{\|x_2\|\|y_2\|} \approx 1.3099$

$D(x_1, y_1) \neq D(x_2, y_2)$

C) $\|x\| - \|y\|$

We aren't doing any scaling in this definition of discrepancy. So, this definition is certainly sensitive to scaling, and thus, not the correct answer.

For example,

$D(x_1, y_1) = \|x_1\| - \|y_1\| \approx 0.2086$

$D(x_2, y_2) = \|x_2\| - \|y_2\| \approx -0.5217$

$D(x_1, y_1) \neq D(x_2, y_2)$

D) $\left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\|$

In this, we first scale each vector x and y down to their unit vectors, and then calculate the discrepancy.

Since $x_2 = c_1 x_1$, x_2 will have the same unit vector as x_1.
Similarly, y_2 will have the same unit vector as y_1.

Thus, no matter how we scale x_2 and y_2, as long as they are derived from x_1 and y_1, their discrepancy will be the same.

Therefore, our formula will be insensitive to scaling, which is exactly what we want!

For example,

$D(x_1, y_1) = \left\| \frac{x_1}{\|x_1\|} - \frac{y_1}{\|y_1\|} \right\| \approx 0.9551$

$D(x_2, y_2) = \left\| \frac{x_2}{\|x_2\|} - \frac{y_2}{\|y_2\|} \right\| \approx 0.9551$

$D(x_1, y_1) = D(x_2, y_2)$

Thus, **option D** is the correct answer.

You can use this matlab code to test the options with randomly generated vectors.

```
%% Get two random vectors x1 and y1, each of length 5
x1 = rand(5,1);
y1 = rand(5,1);

%% Create two more vectors x2 and y2, which are multiples of x1 and y1
x2 = rand()*x1;
y2 = rand()*y1;

%% Define the modd function
modd = @(z) sqrt(sum(z.^2));

%% Define the answers function that computes the values
%  obtained from options A, B, C and D
answers = @(x,y) [
                  modd(x-y);
                  modd(x-y)/(modd(x)*modd(y));
                  modd(x) - modd(y);
                  modd(x/modd(x)  - y/modd(y))
              ];

%% Define function to perform floating point comparision
%  Copied from stackoverflow.com/a/2203483/2570622
isequalRel = @(x,y,tol) ( abs(x-y) <= ( tol*max(abs(x),abs(y)) + eps) );

%% Calculate the answers for (x1, y1) and (x2, y2) and see which option
%  remains unaffected.
isequalRel(answers(x1,y1), answers(x2,y2), 1e-6)
```

13 votes

-- Pragy Agarwal (18.3k points)

11.4.2 Vector Space: TIFR2017-A-2

https://gateoverflow.in/94938

Let $n = 1$ then,

if $a = \{2\}, b = \{1\}$

$< a, b > = 2$

$|| b || = 1$

Hence, i is incorrect.

Let a = {-1} , b ={1}

$< a, b >= -1$

$|| a || || b || = 1$ x 1 = 1

Hence, iii and iv is incorrect, if iv is incorrect d and e both can't be right.

So, by elimination (a) is correct.

5 votes

-- tarun_svbk (1.4k points)

Answer Keys

11.0.1	C	11.1.1	C	11.2.1	D	11.2.2	E	11.2.3	D
11.2.4	A	11.2.5	B	11.2.6	E	11.2.7	E	11.2.8	D
11.3.1	B	11.4.1	D	11.4.2	A	11.4.3	D		

12 Engineering Mathematics: Probability (40)

Syllabus: Random variables, Uniform, Normal, Exponential, Poisson and Binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	1	0	2	1	1	0	1	1	0	0.88	2
2 Marks Count	2	2	1	1	1	0	3	1	0	0	1.22	3
Total Marks	5	5	2	4	3	1	6	3	1	1	3.33	6

12.0.1 TIFR2020-A-11

https://gateoverflow.in/333113

Suppose we toss $m = 3$ labelled balls into $n = 3$ numbered bins. Let A be the event that the first bin is empty while B be the event that the second bin is empty. $P(A)$ and $P(B)$ denote their respective probabilities. Which of the following is true?

A. $P(A) > P(B)$

B. $P(A) = \dfrac{1}{27}$

C. $P(A) > P(A \mid B)$

D. $P(A) < P(A \mid B)$

E. None of the above

tifr2020

Answers:

12.0.1 TIFR2020-A-11

https://gateoverflow.in/333113

Suppose, we have 3 labelled balls as $\{B_1, B_2, B_3\}$ and 3 numbered bins as $\{U_1, U_2, U_3\}$. Now, number of ways to throw these balls into bins is same as total number of functions from set $\{B_1, B_2, B_3\}$ to set $\{U_1, U_2, U_3\}$. So, sample size = 3^3

Now, A be the event that first bin is empty and we have to throw balls into bins. So, number of favorable outcomes is total number of functions from set $\{B_1, B_2, B_3\}$ to set $\{U_2, U_3\}$ which is 2^3. So, $\mathbb{P}(A) = \frac{2^3}{3^3}$.

Now, B be the event that second bin is empty and we have to throw balls into bins. So, in this case, number of favorable outcomes is total number of functions from set $\{B_1, B_2, B_3\}$ to set $\{U_1, U_3\}$ which is 2^3. So, $\mathbb{P}(B) = \frac{2^3}{3^3}$.

Now, $\mathbb{P}(A|B)$ means given second bin is empty, what is the probability that first bin is empty. So,
$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\frac{1}{3^3}}{\frac{2^3}{3^3}} = \frac{1}{2^3}$

So, $\mathbb{P}(A) \neq \frac{1}{27}$, $\mathbb{P}(A) = \mathbb{P}(B)$ and $\mathbb{P}(A) > \mathbb{P}(A|B)$

4 votes

-- ankitgupta.1729 (15.2k points)

12.1 Binomial Distribution (3)

12.1.1 Binomial Distribution: TIFR2010-A-6

https://gateoverflow.in/18222

Given 10 tosses of a coin with probability of head = . 4 = (1 - the probability of tail), the probability of at least one head is?

A. $(.4)^{10}$ B. $1 - (.4)^{10}$ C. $1 - (.6)^{10}$ D. $(.6)^{10}$ E. $10(.4)(.6)^9$

tifr2010 probability binomial-distribution

12.1.2 Binomial Distribution: TIFR2010-B-38

https://gateoverflow.in/19050

Suppose three coins are lying on a table, two of them with heads facing up and one with tails facing up. One coin is chosen at random and flipped. What is the probability that after the flip the majority of the coins(i.e., at least two of them) will have heads facing up?

A. $\left(\frac{1}{3}\right)$ B. $\left(\frac{1}{8}\right)$ C. $\left(\frac{1}{4}\right)$ D. $\left(\frac{1}{4} + \frac{1}{8}\right)$ E. $\left(\frac{2}{3}\right)$

tifr2010 probability binomial-distribution

12.1.3 Binomial Distribution: TIFR2011-A-3

https://gateoverflow.in/20000

The probability of three consecutive heads in four tosses of a fair coin is

A. $\left(\frac{1}{4}\right)$ B. $\left(\frac{1}{8}\right)$ C. $\left(\frac{1}{16}\right)$ D. $\left(\frac{3}{16}\right)$ E. None of the above

tifr2011 probability binomial-distribution

Answers: Binomial Distribution

12.1.1 Binomial Distribution: TIFR2010-A-6

https://gateoverflow.in/18222

✔ 10 tosses of coin are there.

Probability of head = 0.4

Probability of tail = 0.6

Probability of at least one head $= 1 - P_{\text{no head occur}} = 1 - (0.6)^{10}$.

Correct Answer: C

10 votes -- Digvijay (44.9k points)

12.1.2 Binomial Distribution: TIFR2010-B-38

https://gateoverflow.in/19050

✔ **(e) is correct**

Table has 3 coins with H, H, T facing up.

Now, probability of choosing any coin is $\frac{1}{3}$, as we can chose any of the three coins.

Case A: 1^{st} coin : either H or T can come.
so, HHT, THT possible.only HHT is favorable.
which gives $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right) = \frac{1}{6}$.

Case B: $2nd$ coin : either H or T can come.
so, HHT, HTT possible.only HHT is favourable.
which gives $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right) = \frac{1}{6}$.

Case C: $3rd$ coin : Table already contains two $H's$ so, whatever comes is favourable.
which gives $\left(\frac{1}{3}\right) \times 1 = \frac{1}{3}$.

Summing up the total gives $\frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$.

25 votes -- Himanshu Agarwal (12.4k points)

12.1.3 Binomial Distribution: TIFR2011-A-3

https://gateoverflow.in/20000

✔ Let the 4 tosses be named P, Q, R and S

To have 3 consecutive heads:

- Q, R must be both heads.
- At least one of $\{P, S\}$ must be a head.

Thus, the probability of getting 3 consecutive heads is given by:

$$P = P_q \times P_r \times \underbrace{(P_p + P_s - P_p P_s)}_{\text{atleast one}}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \right)$$

$P = \frac{3}{16}$.

Hence, option D is the correct answer.

Another way of looking at it is:

$$P = P_{HHHT} + P_{THHH} + P_{HHHH} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$$

32 votes -- Pragy Agarwal (18.3k points)

12.2 Conditional Probability (3)

12.2.1 Conditional Probability: TIFR2010-A-19, TIFR2014-A-6

https://gateoverflow.in/18499

Karan tells truth with probability $\frac{1}{3}$ and lies with probability $\frac{2}{3}$. Independently, Arjun tells truth with probability $\frac{3}{4}$ and lies with probability $\frac{1}{4}$. Both watch a cricket match. Arjun tells you that India won, Karan tells you that India lost. What probability will you assign to India's win?

A. $\left(\frac{1}{2}\right)$ B. $\left(\frac{2}{3}\right)$ C. $\left(\frac{3}{4}\right)$ D. $\left(\frac{5}{6}\right)$ E. $\left(\frac{6}{7}\right)$

tifr2010 probability conditional-probability tifr2014

12.2.2 Conditional Probability: TIFR2012-A-1

https://gateoverflow.in/20938

Amar and Akbar both tell the truth with probability $\frac{3}{4}$ and lie with probability $\frac{1}{4}$. Amar watches a test match and talks to Akbar about the outcome. Akbar, in turn, tells Anthony, "Amar told me that India won". What probability should Anthony assign to India's win?

A. $\left(\frac{9}{16}\right)$ B. $\left(\frac{6}{16}\right)$ C. $\left(\frac{7}{16}\right)$ D. $\left(\frac{10}{16}\right)$ E. None of the above

tifr2012 probability conditional-probability

12.2.3 Conditional Probability: TIFR2013-A-6

https://gateoverflow.in/25390

You are lost in the National park of Kabrastan. The park population consists of tourists and Kabrastanis. Tourists comprise two-thirds of the population the park and give a correct answer to requests for directions with probability $\frac{3}{4}$. The air of Kabrastan has an amnesaic quality, however, and so the answers to repeated questions to tourists are independent, even if the question and the person are the same. If you ask a Kabrastani for directions, the answer is always wrong.

Suppose you ask a randomly chosen passer-by whether the exit from the park is East or West. The answer is East. You then ask the same person again, and the reply is again East. What is the probability of East being correct?

A. $\left(\frac{1}{4}\right)$ B. $\left(\frac{1}{3}\right)$ C. $\left(\frac{1}{2}\right)$ D. $\left(\frac{2}{3}\right)$ E. $\left(\frac{3}{4}\right)$

tifr2013 probability conditional-probability

Answers: Conditional Probability

12.2.1 Conditional Probability: TIFR2010-A-19, TIFR2014-A-6

https://gateoverflow.in/18499

✔ If really India wins, then Karan lies $\left(P = \frac{2}{3}\right)$ and Arjun tells truth $\left(P = \frac{3}{4}\right)$

Now probability of Karan lying and Arjun telling truth $= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

Now probability of Arjun lying and Karan telling truth $= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

So, by Bayes theorem,

Probability of India winning $= \dfrac{\dfrac{1}{2}}{\dfrac{1}{2}+\dfrac{1}{12}} = \dfrac{6}{7}$

So, answer is (e)

PS: Assuming superover in case of tie.

21 votes -- srestha (85.3k points)

12.2.2 Conditional Probability: TIFR2012-A-1

https://gateoverflow.in/20938

✔ Option D should be the correct answer.

Consider the following events,

W : India wins,

$W\neg$: India does not wins (India Lost/ Match Draw/ Match Tie/ Match Suspended etc.)

X : Akbar tells Anthony, "Amar told me that India won"

$X\neg$: Akbar tells Anthony, "Amar told me that India did not won"

Given X, we have to find W, that is we have to calculate $P\left(\frac{W}{X}\right)$.

$P\left(\frac{W}{X}\right)$ can be calculated using Bayes's theorem as follow:

$$P\left(\frac{\text{India Wins}}{\text{Akbar tells Anthony “Amar told me that India won”}}\right)$$

$$= \frac{P\left(\frac{\text{Akbar tells Anthony “Amar told me that India won”}}{\text{India Wins}}\right)}{P\left(\frac{\text{Akbar tells Anthony “Amar told me that India won”}}{\text{India Won}}\right) \cup P\left(\frac{\text{Akbar tells Anthony “Amar told me that India won”}}{\text{India didn't won}}\right)}$$

rewriting same equation using the events defined:

$$P\left(\frac{W}{X}\right) = \frac{P\left(\frac{X}{W}\right)}{P\left(\frac{X}{W}\right)+P\left(\frac{X}{W\neg}\right)}$$

Calculation of $P\left(\frac{X}{W}\right)$ and $P\left(\frac{X}{W\neg}\right)$:

$P\left(\frac{X}{W}\right) = P\left(\frac{Case\ 1}{W}\right) \cup P\left(\frac{Case\ 4}{W}\right)$

$P\left(\frac{Case\ 1}{W}\right) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$P\left(\frac{Case\ 4}{W}\right) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$So\ P\left(\frac{X}{W}\right) = \frac{9}{16} + \frac{1}{16} = \frac{10}{16}$

$P\left(\frac{X}{W\neg}\right) = P\left(\frac{Case\ 6}{W\neg}\right) \cup P\left(\frac{Case\ 7}{W\neg}\right)$

$P\left(\frac{Case\ 6}{W\neg}\right) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

$P\left(\frac{Case\ 7}{W\neg}\right) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

$So\ P\left(\frac{X}{W\neg}\right) = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$

$Hence,\ P\left(\frac{W}{X}\right) = \frac{\frac{10}{16}}{\frac{10}{16}+\frac{6}{16}} = \frac{10}{16}.$

14 votes -- Anurag Pandey (10.5k points)

12.2.3 Conditional Probability: TIFR2013-A-6

https://gateoverflow.in/25390

✔ We have to find the probability of East being correct given that we twice get the answer as East from some person.

Let's denote:

Probability of East being correct $= P(E)$

& Probability of West being correct $= P(W)$

Since, only East and West are possible answers we can assume $P(E) = P(W) = \frac{1}{2}$.

Now, the passer-by is giving me 2 times the answer as East. Lets denote the probability for this as $P(E'')$

As we need to find the probability of East being correct given that the passer-by gave $2-$ times East as answer, we can denote this as $P(E \mid E'')$

Now, as per Baye's Theorem we can break $P(E \mid E'')$ as

$$P(E \mid E'') = \frac{P(E \cap E'')}{P(E'')} = \frac{P(E)P(E'' \mid E)}{P(E'')}$$

Now, $P(E'') = P(E)P(E'' \mid E) + P(W)P(E'' \mid W)$

as we can get $2-$ times East in two ways

1. It is an East and someone answers 2 times East
2. It is a West and someone answers 2 times East

$$\therefore P(E \mid E'') = \frac{\frac{1}{2} \times P(E'' \mid E)}{\frac{1}{2} \times P(E'' \mid E) + \frac{1}{2} \times P(E'' \mid W)} = \frac{P(E'' \mid E)}{P(E'' \mid E) + P(E'' \mid W)}$$

Now, $P(E'' \mid E) =$ Probability of getting $2-$ times East given that East is correct.

This we can get in 2 ways -

1. When we're asking a tourist & she tells $2-$ times East as answer and East is correct. Probability of this $= \frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}$
2. When we're asking a Kabrastani & she tells $2-$ times East as answer and East is correct. Probability of this $= \frac{1}{3} \times 0 \times 0$ as Kabrastani always lies.

$$\therefore P(E'' \mid E) = \text{ Case 1} + \text{Case 2} = \left(\frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times 0 \times 0\right) = \frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}$$

$P(E'' \mid W) =$Probability of getting 2-times East given that West is correct.

This we can get in 2 ways -

1. When we're asking a tourist & she tells $2-$ times East as answer and East is wrong. Probability of this $= \frac{2}{3} \times \frac{1}{4} \times \frac{1}{4}$
2. When we're asking a Kabrastani & she tells $2-$ times East as answer and East is wrong. Probability of this $= \frac{1}{3} \times 1 \times 1$

$$P(E''|W) = \text{ Case 1} + \text{Case 2} = \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{3} \times 1 \times 1\right)$$

$$\begin{aligned}
\therefore P(E \mid E'') &= \frac{P(E'' \mid E)}{P(E'' \mid E) + P(E'' \mid W)} \\
&= \frac{\left(\frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}\right)}{\left(\frac{2}{3} \times \frac{3}{4} \times \frac{3}{4}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{3} \times 1 \times 1\right)} \\
&= \frac{\frac{3}{8}}{\left(\frac{3}{8}\right) + \left(\frac{1}{24}\right) + \left(\frac{1}{3}\right)} \\
&= \frac{\frac{3}{8}}{\frac{(9+1+8)}{24}}
\end{aligned}$$

$$= \frac{\left(\frac{3}{8}\right)}{\left(\frac{18}{24}\right)}$$
$$= \left(\frac{3}{8}\right) \times \left(\frac{24}{18}\right)$$
$$= \frac{1}{2}$$

Correct Answer: C

8 votes -- Subarna Das (11.2k points)

12.3 Expectation (5)

12.3.1 Expectation: TIFR2011-A-6

https://gateoverflow.in/20011

Assume that you are flipping a fair coin, i.e. probability of heads or tails is equal. Then the expected number of coin flips required to obtain two consecutive heads for the first time is.

a. 4 b. 3 c. 6 d. 10 e. 5

tifr2011 probability expectation

12.3.2 Expectation: TIFR2012-B-7

https://gateoverflow.in/25107

A bag contains 16 balls of the following colors: 8 red, 4 blue, 2 green, 1 black, and 1 white. Anisha picks a ball randomly from the bag, and messages Babu its color using a string of zeros and ones. She replaces the ball in the bag, and repeats this experiment, many times. What is the minimum expected length of the message she has to convey to Babu per experiment?

A. $\frac{3}{2}$ B. $\log 5$ C. $\frac{15}{8}$ D. $\frac{31}{16}$ E. 2

tifr2012 probability expectation

12.3.3 Expectation: TIFR2014-A-17

https://gateoverflow.in/27111

A fair dice (with faces numbered $1, \ldots, 6$) is independently rolled repeatedly. Let X denote the number of rolls till an even number is seen and let Y denote the number of rolls till 3 is seen. Evaluate $E(Y|X=2)$.

A. $6\frac{5}{6}$ B. 6 C. $5\frac{1}{2}$ D. $6\frac{1}{3}$ E. $5\frac{2}{3}$

tifr2014 expectation

12.3.4 Expectation: TIFR2015-A-6

https://gateoverflow.in/29567

Ram has a fair coin, i.e., a toss of the coin results in either head or tail and each event happens with probability exactly half $(1/2)$. He repeatedly tosses the coin until he gets heads in two consecutive tosses. The expected number of coin tosses that Ram does is.

A. 2 B. 4 C. 6 D. 8 E. None of the above

tifr2015 expectation

12.3.5 Expectation: TIFR2020-A-1

https://gateoverflow.in/333101

Two balls are drawn uniformly at random without replacement from a set of five balls numbered $1, 2, 3, 4, 5$. What is the expected value of the larger number on the balls drawn?

A. 2.5 B. 3 C. 3.5 D. 4 E. None of the above

tifr2020 engineering-mathematics probability expectation

Answers: Expectation

12.3.1 Expectation: TIFR2011-A-6

https://gateoverflow.in/20011

✔ **Answer is (C)**

Let the expected number of coin flips be X. The case analysis goes as follows:

a. If the first flip is a tails, then we have wasted one flip. The probability of this event is $\frac{1}{2}$ and the total number of flips required is $X+1$.
b. If the first flip is a heads and second flip is a tails, then we have wasted two flips. The probability of this event is $\frac{1}{4}$ and the total number of flips required is $X+2$. as the same scenario as beginning is there even after 2 tosses.
c. If the first flip is a heads and second flip is also heads, then we are done. The probability of this event is $\frac{1}{4}$ and the total number of flips required is 2.

Adding, the equation that we get is -
$X = \frac{1}{2}(X+1) + \frac{1}{4}(X+2) + \frac{1}{4}2$

Solving, we get $X = 6$.

Thus, the expected number of coin flips for getting two consecutive heads is 6.

48 votes -- Avdhesh Singh Rana (2.3k points)

12.3.2 Expectation: TIFR2012-B-7

https://gateoverflow.in/25107

✔ **Answer is (C)**

Using static Huffman compression you can encode the more common colours in fewer bits than the rare colours, that being the case on can expect that common colours will usually be chosen.

eg:

red	1
blue	01
green	001
white	0001
black	0000

On average from 16 draws there will be

8 reds	=	8 bits
4 blues	=	8 bits
2 greens	=	6 bits
1 white	=	4 bits
1 black	=	4 bits

for a total of $\frac{30}{16} = \frac{15}{8}$ bits on average.

28 votes -- sudipta roy (381 points)

12.3.3 Expectation: TIFR2014-A-17

https://gateoverflow.in/27111

✔ **Answer is (E)**

X: The value of X denotes the number of rolls till an even number is seen.

Y: The value of Y denotes the number of rolls till a 3 is seen.

For example:

$X = 2$ implies an even number first time occurred on second roll, or outcome of the first roll is odd & outcome of the second roll is even.

$Y = 4$ implies 3 appeared for first time in the 4^{th} die roll.

Ranges of Random Variables X & Y

$X : \{1, 2, 3, \ldots, \infty\}$

$Y : \{1, 2, 3, \ldots, \infty\}$

$E[Y \mid X = 2]$: Expected number of rolls till a 3 is seen given that an even number appeared for the first time in the second roll.

It is sure that 3 cannot appear on 2^{nd} toss, i.e. $P[Y = 2 \mid X = 2] = 0$ and henceforth $E[Y = 2 \mid X = 2] = 0.$

Now, there are two cases possible:

Case 1: 3 appears on the first toss given that outcome of first toss is odd.

i.e., $E[Y = 1 \mid X = 2]$

Here we need not to concern about outcomes of rolls other than the first roll.

Probability of getting 3 in first toss given that o/c of the first toss is odd $= P(Y = 1 \mid X = 2) = \frac{1}{3} = 0.33$

So, Expectation $E[Y = 1 \mid X = 2] = y \times P(Y = 1 \mid X = 2) = 1 \times 0.33 = 0.33$

Case 2: 3 appears on any toss after the second toss given that outcome of first toss is odd, & that of second toss is even

$P[Y = y \mid X = 2]$ = given that 1^{st} roll is an odd number and $2nd$ roll is an even number, Probability that out of y rolls,

None of the first $(y-1)$ roll's outcome is 3 &

Outcome of the y^{th} roll is 3.

So $P[Y = y \mid X = 2]$

$= \left(\frac{2}{3}\right)$ (for first o/c odd but not 3)

$\times \left(\frac{5}{6}\right)^{(y-3)}$ (for not getting a 3 $from$ 3^{rd} to $0(y-1)^{th} rolls$)

$\times \left(\frac{1}{6}\right)$ (for y^{th}o/c to be 3).

$P[Y = y \mid X = 2] = \left(\frac{2}{3}\right) \times \left(\frac{5}{6}\right)^{(y-3)} \times \left(\frac{1}{6}\right)$

So $E[Y = y \mid X = 2] =$ Summation from $y = 3$ to infinity$(y \times P(Y = y \mid X = 2)) = 5.33$ (where $y \geq 3$)

This summation will give sum of all the expectations from $Y = 3$ to infinity.

Now:

Net Expectation is given as:

$E[Y = y \mid X = 2] = E[Y = 1 \mid X = 2] + E[Y = 2 \mid X = 2] + E[Y = y' \mid X = 2]$ where $y' \geq 3.$

Putting all the values,

$E[Y = y \mid X = 2] = 0.33 + 0 + 5.33$

So, $E[Y = y \mid X = 2] = 5.66 = \frac{17}{3}.$

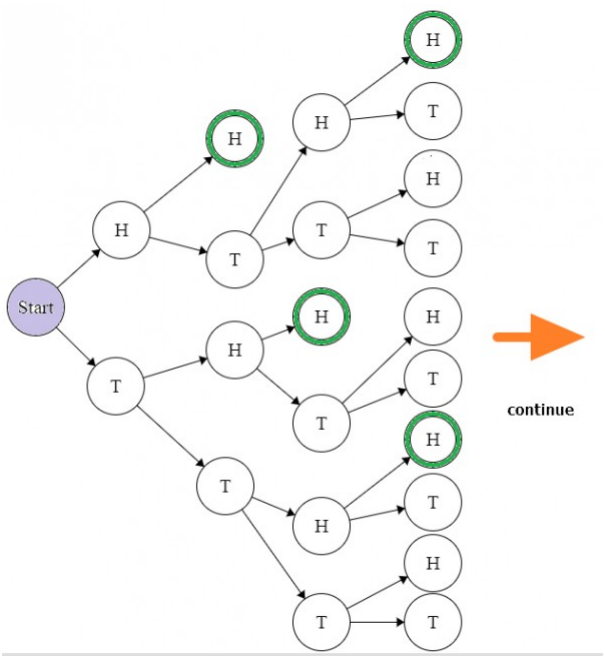
22 votes -- Anurag Pandey (10.5k points)

12.3.4 Expectation: TIFR2015-A-6

https://gateoverflow.in/29567

✔ **Answer is (C)**

By drawing the tree diagram we can find the following series :



$$E = \sum k.P(k)$$
$$= 2.(1.x^2) + 3.(1.x^3) + 4.(2.x^4) + 5.(3.x^5) + 6.(5.x^6) + 7.(8.x^7) + \ldots\infty$$

Above series is a nice combination of AP , generating function and Fibonacci numbers !!!!

- AP terms can be handled by integration or differentiation
- Fibonacci generating function is $= \dfrac{1}{1-x-x^2}$
- probability on each branch is $x = \frac{1}{2}$

$$\Rightarrow \frac{E}{x} = 2.(1.x^1) + 3.(1.x^2) + 4.(2.x^3) + 5.(3.x^4) + 6.(5.x^5) + 7.(8.x^6) + \ldots\infty$$
$$\Rightarrow \int \frac{E}{x}.dx = 1.x^2 + 1.x^3 + 2.x^4 + 3.x^5 + 5.x^6 + \ldots..\infty$$
$$\Rightarrow \int \frac{E}{x}.dx = x^2.\left(1.x^0 + 1.x^1 + 2.x^2 + 3.x^3 + 5.x^4 + \ldots\infty\right)$$
$$\Rightarrow \int \frac{E}{x}.dx = \frac{x^2}{1-x-x^2}$$
$$\Rightarrow \frac{E}{x} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2}{1-x-x^2}\right]$$
$$\Rightarrow \frac{E}{x} = \frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}$$
$$\Rightarrow E = x.\left\{\frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}\right\}$$
$$\Rightarrow E = \frac{1}{2}.\left\{\frac{2.\frac{1}{2}(1-\frac{1}{2}-\frac{1}{4}) + (1+2.\frac{1}{2}).\frac{1}{4}}{(1-\frac{1}{2}-\frac{1}{4})^2}\right\}$$
$$\Rightarrow E = 6$$

Similar Kind of Question as a Reference

- https://gateoverflow.in/3778/gate2005-it-32

References

18 votes -- Debashish Deka (40.8k points)

12.3.5 Expectation: TIFR2020-A-1 https://gateoverflow.in/333101

5 balls we have.

2 balls can be drawn in 10 ways

Ball1, Ball2

Ball1, Ball3

Ball1, Ball4

Ball1, Ball5

Ball2, Ball3

Ball2, Ball4

Ball2, Ball5

Ball3, Ball4

Ball3, Ball5

Ball4, Ball5

Let x be the larger number on the balls drawn

x	P(x)
1	0
2	1
3	2
4	3
5	4

We have to calculate $\sum_1^5 xP(x)$

$1(0) + 2(\frac{1}{10}) + 3(\frac{2}{10}) + 4(\frac{3}{10}) + 5(\frac{4}{10}) = 4$

3 votes -- Shashank Rustagi (5.4k points)

12.4 Independent Events (1)

12.4.1 Independent Events: TIFR2020-A-7 https://gateoverflow.in/333107

A lottery chooses four random winners. What is the probability that at least three of them are born on the same day of the week? Assume that the pool of candidates is so large that each winner is equally likely to be born on any of the seven days of the week independent of the other winners.

A. $\frac{17}{2401}$ B. $\frac{48}{2401}$ C. $\frac{105}{2401}$ D. $\frac{175}{2401}$ E. $\frac{294}{2401}$

tifr2020 engineering-mathematics probability independent-events

Answers: Independent Events

12.4.1 Independent Events: TIFR2020-A-7 https://gateoverflow.in/333107

$P(exactly3) = \binom{4}{3} * 1 * \frac{1}{7} * \frac{1}{7} * \frac{6}{7} = \frac{168}{2401} P(exactly4) = \binom{4}{4} * 1 * \frac{1}{7} * \frac{1}{7} * \frac{1}{7} = \frac{7}{2401} P(atleast3) = P(exactly$

P(exactly 3) means Probability of exactly 3 people out of 4 having their birthdays on the same day of the week. And so on.

3 votes -- undecided (35 points)

12.5 Probability (22)

12.5.1 Probability: TIFR2010-A-10 https://gateoverflow.in/26481

A drawer contains 2 Blue, 4 Red and 2 Yellow balls. No two balls have the same radius. If two balls are randomly selected from the drawer, what is the probability that they will be of the same colour?

A. $\left(\frac{2}{7}\right)$ B. $\left(\frac{2}{5}\right)$

C. $\left(\frac{3}{7}\right)$ D. $\left(\frac{1}{2}\right)$

E. $\left(\frac{3}{5}\right)$

tifr2010 probability

12.5.2 Probability: TIFR2010-A-13

https://gateoverflow.in/18392

A cube whose faces are colored is split into 1000 small cubes of equal size. The cubes thus obtained are mixed thoroughly. The probability that a cube drawn at random will have exactly two colored faces is:

A. 0.096 B. 0.12 C. 0.104 D. 0.24 E. None of the above

tifr2010 probability

12.5.3 Probability: TIFR2011-A-19

https://gateoverflow.in/26479

Three dice are rolled independently. What is the probability that the highest and the lowest value differ by 4?

A. $\left(\frac{1}{3}\right)$ B. $\left(\frac{1}{6}\right)$ C. $\left(\frac{1}{9}\right)$ D. $\left(\frac{5}{18}\right)$ E. $\left(\frac{2}{9}\right)$

tifr2011 probability

12.5.4 Probability: TIFR2011-A-9

https://gateoverflow.in/20020

You have to play three games with opponents A and B in a specified sequence. You win the series if you win two consecutive games. A is a stronger player than B. Which sequence maximizes your chance of winning the series?

A. AAB B. ABA C. BAB D. BAA E. All are the same.

tifr2011 probability

12.5.5 Probability: TIFR2012-A-17

https://gateoverflow.in/25042

A spider is at the bottom of a cliff, and is n inches from the top. Every step it takes brings it one inch closer to the top with probability $1/3$, and one inch away from the top with probability $2/3$, unless it is at the bottom in which case, it always gets one inch closer. What is the expected number of steps for the spider to reach the top as a function of n?

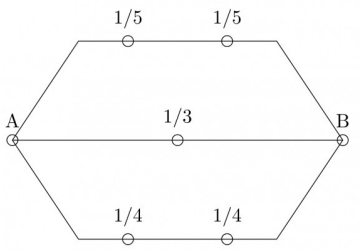
A. It will never reach the top.
B. Linear in n.
C. Polynomial in n.
D. Exponential in n.
E. Double exponential in n.

tifr2012 probability

12.5.6 Probability: TIFR2012-A-19

https://gateoverflow.in/25044

An electric circuit between two terminals A and B is shown in the figure below, where the numbers indicate the probabilities of failure for the various links, which are all independent.



What is the probability that A and B are connected?

A. $\left(\frac{6}{25}\right)$ B. $\left(\frac{379}{400}\right)$ C. $\left(\frac{1}{1200}\right)$ D. $\left(\frac{1199}{1200}\right)$ E. $\left(\frac{59}{60}\right)$

tifr2012 probability

12.5.7 Probability: TIFR2012-A-20

https://gateoverflow.in/25045

There are 1000 balls in a bag, of which 900 are black and 100 are white. I randomly draw 100 balls from the bag. What is the probability that the 101st ball will be black?

A. $9/10$
B. More than $9/10$ but less than 1.
C. Less than $9/10$ but more than 0.
D. 0

E. 1

tifr2012 probability

12.5.8 Probability: TIFR2012-A-9

https://gateoverflow.in/21008

The probability of throwing six perfect dices and getting six different faces is

A. $1-\frac{6!}{6^6}$ B. $\frac{6!}{6^6}$ C. 6^{-6} D. $1-6^{-6}$ E. None of the above

tifr2012 probability

12.5.9 Probability: TIFR2013-A-13

https://gateoverflow.in/25435

Doctors A and B perform surgery on patients in stages III and IV of a disease. Doctor A has performed a 100 surgeries (on 80 stage III and 20 stage IV patients) and 80 out of her 100 patients have survived (78 stage III and 2 stage IV survivors). Doctor B has also performed 100 surgeries (on 50 stage III and 50 stage IV patients). Her success rate is $\frac{600}{100}$ (49 stage III survivors and 11 stage IV survivors).A patient has been advised that she is equally likely to be suffering from stage III or stage IV of this disease. Which doctor would you recommend to this patient and why?

A. Doctor A since she has a higher success rate
B. Doctor A since she specializes in stage III patients and the success of surgery in stage IV patients is anyway too low
C. Doctor B since she has performed more stage IV surgeries
D. Doctor B since she appears to be more successful
E. There is not enough data since the choice depends on the stage of the disease the patient is suffering from.

tifr2013 probability

12.5.10 Probability: TIFR2013-A-14

https://gateoverflow.in/25437

An unbiased die is thrown n times. The probability that the product of numbers would be even is

A. $\frac{1}{(2n)}$ B. $\frac{1}{[(6n)!]}$ C. $1-6^{-n}$ D. 6^{-n} E. None of the above

tifr2013 probability

12.5.11 Probability: TIFR2013-A-17

https://gateoverflow.in/25497

A stick of unit length is broken into two at a point chosen at random. Then, the larger part of the stick is further divided into two parts in the ratio $4:3$. What is the probability that the three sticks that are left CANNOT form a triangle?

A. $1/4$ B. $1/3$ C. $5/6$ D. $1/2$ E. $\log_e(2)/2$

tifr2013 probability

12.5.12 Probability: TIFR2013-A-4

https://gateoverflow.in/25386

A biased coin is tossed repeatedly. Assume that the outcomes of different tosses are independent and probability of heads is $\frac{2}{3}$ in each toss. What is the probability of obtaining an even number of heads in 5 tosses, zero being treated as an even number?

A. $\left(\frac{121}{243}\right)$ B. $\left(\frac{122}{243}\right)$ C. $\left(\frac{124}{243}\right)$ D. $\left(\frac{125}{243}\right)$ E. $\left(\frac{128}{243}\right)$

tifr2013 probability

12.5.13 Probability: TIFR2013-B-10

https://gateoverflow.in/25771

Let m,n be positive integers with m a power of 2. Let $s=100n^2\log m$. Suppose $S_1,S_2,\ldots,S_m$ are subsets of $1,2,\ldots,s$ such that $|S_i|=10n\log m$ and $|S_i\cap S_j|\leq\log m$ for all $1\leq i<j\leq m$. Such a collection of sets $S_1,\ldots,S_m$ is an example of a so-called Nisan-Wigderson design. We now consider the set membership problem, where we have to store an arbitrary subset $T\subseteq\{1,2,\ldots,m\}, |T|=n$ as an array A of s bits so that given any integer $x,1\leq x\leq m$, we can discover whether $x\in T$ by reading only one bit of A. Consider the following strategy to solve this problem. Array A is initialized to all zeroes. Given the set T to be stored, we put a one in all the locations of A indexed by the union $\cup_{t\in T}S_t$. Now, given the integer x, we read a random location in A from S_x and declare that $x\in T$ if the bit in that location is one. This strategy gives the correct answer with probability

A. 1 if $x \in T$ and at most 0.1 if $x \notin T$.
B. At least 0.9 if $x \in T$ and at most 0.1 if $x \notin T$.
C. At least 0.9 if $x \in T$ and at least 0.9 if $x \notin T$.
D. 1 if $x \in T$ and at least 0.9 if $x \notin T$.
E. At least 0.9 if $x \in T$ and 1 if $x \notin T$.

tifr2013 probability

12.5.14 Probability: TIFR2015-A-1

https://gateoverflow.in/29156

Consider a 6-sided die with all sides not necessarily equally likely such that probability of an even number is $P(\{2,4,6\}) = \frac{1}{2}$, probability of a multiple of 3 is $P(\{3,6\}) = 1/3$ and probability of 1 is $P(\{1\}) = \frac{1}{6}$. Given the above conditions, choose the strongest (most stringent) condition of the following that must always hold about $P(\{5\})$, the probability of 5.

A. $P(\{5\}) = \frac{1}{6}$
B. $P(\{5\}) \geq \frac{1}{6}$
C. $P(\{5\}) \leq \frac{1}{6}$
D. $P(\{5\}) \leq \frac{1}{3}$
E. None of the above

tifr2015 probability

12.5.15 Probability: TIFR2016-A-12

https://gateoverflow.in/73498

There are two rocks A and B, located close to each other, in a lily pond. There is a frog that jumps randomly between the two rocks at time $t = 0, 1, 2, \ldots$. The location of the frog is determined as follows. Initially, at time $t = 0$, the frog is at A. From then on, the frog's location is determined as follows. If the frog is at A at time t, then at time $t+1$, with probability $2/3$ it jumps to B and with probability $1/3$, it jumps on the spot and stays at A. If the frog is at B at time t, then at time $t+1$, with probability $1/2$ it jumps to A and with probability $1/2$ it jumps on the spot and stays at B. What is the probability that the frog is at B at time 3 (just after its third jump)?

A. $\frac{1}{2}$ B. $\frac{31}{54}$ C. $\frac{14}{27}$ D. $\frac{61}{108}$ E. $\frac{2}{3}$

tifr2016 probability

12.5.16 Probability: TIFR2017-A-9

https://gateoverflow.in/95042

Consider the *majority* function on three bits, $\text{maj} : \{0,1\}^3 \to \{0,1\}$ where $\text{maj}(x_1, x_2, x_3) = 1$ if and only if $x_1 + x_2 + x_3 \geq 2$. Let $p(\alpha)$ be the probability that the output is 1 when each input is set to 1 independently with probability α. What is $p'(\alpha) = \frac{d}{d\alpha}p(\alpha)$?

A. 3α
B. α^2
C. $6\alpha(1-\alpha)$
D. $3\alpha^2(1-\alpha)$
E. $6\alpha(1-\alpha) + \alpha^2$

tifr2017 probability

12.5.17 Probability: TIFR2018-A-10

https://gateoverflow.in/179279

Let C be a biased coin such that the probability of a head turning up is p. Let p_n denote the probability that an odd number of heads occurs after n tosses for $n \in \{0, 1, 2, \ldots\}$, Then which of the following is TRUE ?

A. $p_n = \frac{1}{2}$ for all $n \in \{0, 1, 2, \ldots\}$.
B. $p_n = (1-p)(1-p_{n-1}) + p.p_{n-1}$ for $n \geq 1$ and $p_0 = 0$.
C. $p_n = \sum_{i=1}^{n} p(1-2p)^{i-1}$ for $n \geq 1$.
D. If $p = \frac{1}{2}$, then $p_n = \frac{1}{2}$ for all $n \in \{0, 1, 2, \ldots\}$.
E. $p_n = 1$ if n is odd and 0 otherwise.

tifr2018 probability

12.5.18 Probability: TIFR2018-A-13

https://gateoverflow.in/179371

A hacker knows that the password to the TIFR server is 10-letter string consisting of lower-case letters from the

English alphabet. He guesses a set of 5 distinct 10-letter strings (with lower-case letters) uniformly at random. What is the probability that one of the guesses of the hacker is correct password?

A. $\frac{5}{(26)^{10}}$
B. $1-\left(1-\frac{1}{(26)^{10}}\right)^{5}$
C. $1-\left\{\left(\frac{(26)^{10}-1}{(26)^{10}}\right)\left(\frac{(26)^{10}-2}{(26)^{10}}\right)\left(\frac{(26)^{10}-3}{(26)^{10}}\right)\left(\frac{(26)^{10}-4}{(26)^{10}}\right)\left(\frac{(26)^{10}-5}{(26)^{10}}\right)\right\}$
D. $\frac{1}{(26)^{10}}$
E. None of the above

tifr2018 probability

12.5.19 Probability: TIFR2018-A-15 https://gateoverflow.in/179366

Suppose a box contains 20 balls: each ball has a distinct number in $\{1,\ldots,20\}$ written on it. We pick 10 balls (without replacement) uniformly at random and throw them out of the box. Then we check if the ball with number ‘‘1 ” on it is present in the box. If it is present, then we throw it out of the box; else we pick a ball from the box uniformly at random and throw it out of the box.

What is the probability that the ball with number ‘‘2 ” on it is present in the box?

A. $9/20$ B. $9/19$ C. $1/2$ D. $10/19$ E. None of the above

tifr2018 probability

12.5.20 Probability: TIFR2019-A-14 https://gateoverflow.in/280496

A drawer contains 9 pens, of which 3 are red, 3 are blue, and 3 are green. The nine pens are drawn from the drawer one at at time (without replacement) such that each pen is drawn with equal probability from the remaining pens in the drawer. What is the probability that two red pens are drawn in succession ?

A. $7/12$ B. $1/6$ C. $1/12$ D. $1/81$ E. None of the above

tifr2019 engineering-mathematics probability

12.5.21 Probability: TIFR2019-A-4 https://gateoverflow.in/280506

What is the probability that a point $P=(\alpha,\beta)$ picked uniformly at random from the disk $x^2+y^2\leq 1$ satisfies $\alpha+\beta\leq 1$?

A. $\frac{1}{\pi}$
B. $\frac{3}{4}+\frac{1}{4}\cdot\frac{1}{\pi}$
C. $\frac{3}{4}+\frac{1}{4}\cdot\frac{2}{\pi}$
D. 1
E. $\frac{2}{\pi}$

tifr2019 engineering-mathematics discrete-mathematics probability

12.5.22 Probability: TIFR2020-A-10 https://gateoverflow.in/333109

In a certain year, there were exactly four Fridays and exactly four Mondays in January. On what day of the week did the 20^{th} of the January fall that year (recall that January has 31 days)?

A. Sunday B. Monday C. Wednesday D. Friday E. None of the others

tifr2020 engineering-mathematics probability

Answers: Probability

12.5.1 Probability: TIFR2010-A-10 https://gateoverflow.in/26481

✔ If any 2 balls selected from 8 balls then we can choose ${}^{8}C_{2}$ ways=28 ways

If selected 2 balls are same color then ${}^{2}C_{2}+{}^{4}C_{2}+{}^{2}C_{2}$ ways=$1+6+1$ ways=8 ways

So, required probability=$\frac{8}{28}=\frac{2}{7}$

Correct Answer: A

13 votes -- srestha (85.3k points)

12.5.2 Probability: TIFR2010-A-13

https://gateoverflow.in/18392

✔ 0.096 should be the correct answer, i.e. option (a)

Suppose that the side of larger cube is $10\ m$ then volume of the larger cube will be $10 \times 10 \times 10 = 1000\ m^3$.

After dividing the cube into 1000 equal sized small cubes, volume of each smaller cube will be

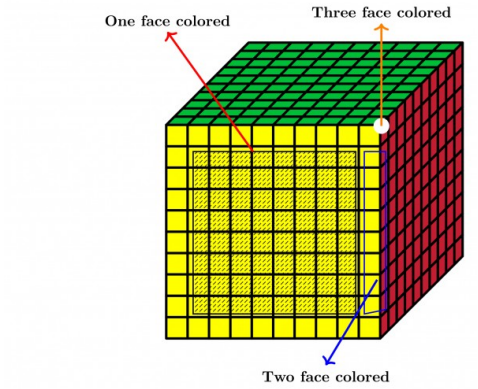
$\left(\frac{10 \times 10 \times 10}{1000}\right) m^3 = 1\, m^3$.

So the sides of the each of the smaller cube will be $1\ m$, which is 10 times less than the side length of larger cube.

So, each EDGE of the larger larger cube will contain 10 smaller cube edges.

Each FACE of the larger cube will contain $10 \times 10 = 100$ smaller cube faces.

Each CORNER of the larger cube will contain 1 smaller cube corner.



Position of each of the smaller cube can be as follows:

(A) It can be in the corners of the larger cube, In this case it would have three of its faces colored.

There are total

$8(\text{number of corners}) \times 1(\text{number of smaller cubes per corner}) = 8 \text{ such cubes.}$

(B) It can be in the edges of the larger cube, In this case it would have two of its faces colored.

There are total

$12(\text{number of edges}) \times 8(\text{number of smaller cubes per edge excluding corner cubes of the edge}) = 96.$

(C) It can be on the face of the larger cube but not in the edges of a face, in this case it would have one face colored.

There are total

$6(\text{number of faces}) \times 64(\text{number of smaller cubes per face excluding the edge \& corner cubes}) = 384.$

(D) It can be inside the core of the larger cube, in this case it will be uncolored.

There will be $512 = (1000 - (384 + 96 + 8))$ cubes.

Now since there are 96 cubes out of 1000 which have 2 colored faces, so required probability $= \frac{96}{1000} = 0.096$

Now, since total number of edges in the larger cube $= 12$,

So, total number of smaller cubes with two colored faces $= 12 \times 8 = 96.$

23 votes -- Anurag Pandey (10.5k points)

12.5.3 Probability: TIFR2011-A-19

https://gateoverflow.in/26479

✔ Case 1: largest is 5, smallest 1 and middle is 2 or 3 or 4 : $3 \times 3!$

Case 2: largest is 5, smallest 1 and middle is 1 or 5 : $\frac{3! \times 2}{2!}$

Case 3: largest is 6, smallest 2 and middle is 3 or 4 or 5 : $3 \times 3!$

Case 4: largest is 6, smallest 2 and middle is 6 or 2 : $\frac{3! \times 2}{2!}$

So, probability the highest and the lowest value differ by 4,

$$= \frac{\left(3 \times 3! + \frac{3! \times 2}{2!} + 3 \times 3! + \frac{3! \times 2}{2!}\right)}{6^3} = \frac{2}{9}.$$

Correct Option: E

22 votes -- Shreya Roy (3.8k points)

12.5.4 Probability: TIFR2011-A-9 https://gateoverflow.in/20020

✔ Let the three games in a series be called G_1, G_2 and G_3 respectively, and the probability of winning the game x be denoted as $P(x)$

You can win the series if and only if: You win G_2 and you win atleast one of $\{G_1, G_3\}$.

$$\boxed{P\left(\begin{matrix}\text{winning}\\ \text{the series}\\ G_1G_2G_3\end{matrix}\right) = P(G_2) \times \Big(\big(P(G_1) + P(G_3) - P(G_1) \times P(G_3)\big)\Big)}$$

Let the probability of winning against player A be a and the probability of winning against player B be b.

Then, $a < b$. (A is a stronger player than B, so probability of winning against A is smaller compared to B)

Let $P(xyz)$ be the probability of winning the series in which the games played are against x, y and z in order.

a. $P(AAB) = a(a + b - ab) = a^2 + ab - a^2b$
b. $P(ABA) = b(a + a - aa) = 2ab - a^2b$
c. $P(BAB) = a(b + b - b^2) = 2ab - ab^2$
d. $P(BAA) = a(b + a - ba) = a^2 + ab - a^2b$

We can see that **not all probabilities are equal, so option E is not correct**.

We can also see that options **A and D result in the same value, so they are not correct either** .

Comparing option B and option C.

Since $a < b$ and $a, b \geq 0$, we have that $2ab - a^2b > 2ab - ab^2$

Hence, option B is the correct answer.

40 votes -- Pragy Agarwal (18.3k points)

12.5.5 Probability: TIFR2012-A-17 https://gateoverflow.in/25042

It will be linear to n

It will go (n-1) inches in (n-1)*3 steps

last 1 inch goes in 1 step

so total (n-1)*3+1 steps

1 votes -- srestha (85.3k points)

12.5.6 Probability: TIFR2012-A-19 https://gateoverflow.in/25044

✔ Lets define the following events:

P : Uppermost link is working.

Q: Middle link is working.

R: Lowermost link is working.

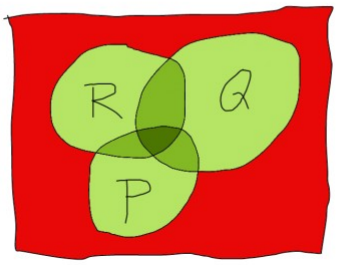
W : Terminals A & B are connected.

From the given information, we can calculate the probabilities of events P, Q and R as follow:

(as I already consumed letter (capital) P denoting an event so here by I will use letter (small) p to denote probabilities, e.g. $p(X)$ denotes probability of happening of event X).

$$p(P) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25},$$
$$\Rightarrow p\left(\widehat{P}\right) = 1 - \frac{16}{25} = \frac{9}{25},$$
$$p(Q) = \frac{2}{3},$$
$$\Rightarrow p\left(\widehat{Q}\right) = \frac{1}{3},$$
$$p(R) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16},$$
$$\Rightarrow p\left(\widehat{R}\right) = 1 - \frac{9}{16} = \frac{7}{16}.$$

The situation here can be represented using the following Venn Diagram:



Here Red region denotes the event $\widehat{W}$ in which $A\&B$ are disconnected.
Different gradients of Green colour represent the W in which terminals $A\&B$ are connected.
We have to find $p(W)$
Events W and $\widehat{W}$ are mutually exclusive & totally exhaustive,
since either terminals A and B will be connected or they will be disconnected.
So we can write $p(W)$ in terms of $p\left(\widehat{W}\right)$ as follow:

$$p(W) = 1 - p\left(\widehat{W}\right).$$

Also it can be seen that
$p\left(\widehat{W}\right) = p\left(\widehat{P}\right) \cap \left(\widehat{Q}\right) \cap \left(\widehat{R}\right)$ where $\widehat{P}$, $\widehat{Q}$ and $\widehat{R}$ are independent events.
That is terminals A & B will be disconnected only when all of the links will fail simultaneously.
Using independence we can write,

$$p\left(\widehat{W}\right) = p(W) = p\left(\widehat{P}\right) \times \left(\widehat{Q}\right) \times \left(\widehat{R}\right),$$
$$\Rightarrow p\left(\widehat{W}\right) = \frac{9}{25} \times \frac{1}{3} \times \frac{7}{16} = \frac{21}{400}$$
$$\Rightarrow p(W) = 1 - \frac{21}{400} = \frac{379}{400}.$$

Correct Answer: B

24 votes

-- Anurag Pandey (10.5k points)

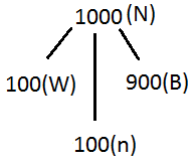
12.5.7 Probability: TIFR2012-A-20

https://gateoverflow.in/25045

✔ Here we are having a **total** of 1000 Balls**,** out of which we firstly **draw** 100 **balls** , and **then** 101^{st} **ball..**

Firstly we have to **find expected number of white and black balls in** drawn 100 **balls** , as both can occur in 100 balls..

We have a situation like this:



Expected number of **white balls** $=n \times \frac{W}{N} = 100 \times \left(\frac{100}{1000}\right) = 10$

Expected number of **black balls** $=n \times \frac{B}{N} = 100 \times \left(\frac{900}{1000}\right) = 90$

So, **we have drawn** 100 **balls(**90 **black,** 10 **white)**

Left balls = (810 Black , 90 White) = 900 **total**

Now,

probability for 101^{st} **ball to be black** = $\frac{810}{900} = \frac{9}{10}$.

So, option (A) is Correct

32 votes -- Himanshu Agarwal (12.4k points)

12.5.8 Probability: TIFR2012-A-9

https://gateoverflow.in/21008

✔ B) for all the 6 different faces $1, 2, 3, 4, 5, 6$

the probability is $\frac{1}{6^6}$ and then for $6!$ different permutations : $\frac{6!}{6^6}$.

9 votes -- Shaun Patel (6.1k points)

12.5.9 Probability: TIFR2013-A-13

https://gateoverflow.in/25435

✔ **Answer will be** d)

As, $\%$ of Doctor A successful for stage III = $\frac{78}{80} \times 100 = 97.5$

For stage IV= $\frac{2}{20} \times 100 = 10$

As, $\%$ of Doctor B successful for stage III = $\frac{49}{50} \times 100 = 98$

For stage IV= $\frac{11}{50} \times 100 = 22$

In both cases Doc B is more successful.

13 votes -- srestha (85.3k points)

12.5.10 Probability: TIFR2013-A-14

https://gateoverflow.in/25437

✔ Even number =$2, 4, 6$
odd number = $1, 3, 5$
Product will come even when even one time even number comes
odd product will be if every time odd comes

so, $P(Even) = 1 - P(odd)$

$= 1 - {}^nC_n \times \left(\frac{3}{6}\right)^0 \times \left(\frac{3}{6}\right)^n$

$= 1 - \left(\frac{1}{2}\right)^n$

Correct Answer: E

20 votes -- Umang Raman (12.2k points)

12.5.11 Probability: TIFR2013-A-17 https://gateoverflow.in/25497

✔ Let the stick be broken to x and $l-x$ with x being the larger part.

To not get a triangle, sum of two sides must be smaller than the third one.

$\implies (l-x)+3x/7 < 4x/7 \implies l < 8x/7 \implies x > 7l/8$.

It is given that $x > l/2$ (x being the larger part) $\implies (7l/8, l)$ is the favorable case from $(l/2, l)$ which gives probability $= (1/8)/(1/2) = 1/4$.

8 votes -- Arjun Suresh (334k points)

12.5.12 Probability: TIFR2013-A-4 https://gateoverflow.in/25386

✔ probability of obtaining an even number of heads in 5 tosses, zero being treated as an even number
number of event = 0 head or 2 head or 4 head

Probability of head $=\frac{2}{3}$

Probability of tail $=\frac{1}{3}$

Probability $= {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 + {}^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + {}^5C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1$

$=\frac{121}{243}.$

Option A

14 votes -- Umang Raman (12.2k points)

12.5.13 Probability: TIFR2013-B-10 https://gateoverflow.in/25771

Option D is correct

Let $m = 4(2^2), n = 2$ then $s = 100 \times 4 \times 2 = 800\ numbers$

Let S_1, S_2, S_3, S_4 be subsets to S, each having $(10 \times 2 \times 2 = 40)\ elements.$

Special condition is $|S_i \cap S_j| \leq 2\ (i.e\ \log m)$

Let

$S_1 = \{1, 2, \ldots, 37, 38, 39, 40\} S_2 = \{121, 122, \ldots, 160\} S_3 = \{37, 38, 41, 42, \ldots, 78\} \quad here\ |S_1 \cap S_2| = 2 S_4 = \{39, 40$

and A is an array of 800 locations, initially 0.

and $T \subseteq (S_1, S_2, S_3, S_4)\ and\ |T| = 2$ (i.e n)

Case 1: Let T $= (S_3, S_4)$

then A is like this

1	2	3	…	36	37	38	39	40	41	42	…	78	…	118	…	800
0	0	0	…	0	1	1	1	1	1	1	…	1	…	1	…	0

$Case$ a : If input is 3 (i.e. S_3) belongs to T
it went to one of the 40 locations $\{37 \ldots 78\}$
cmp bit.

$$P\ (S_3 \text{ present}) = \frac{40}{40} = 1$$

$$\text{If input is 1 (i.e. } S_1\text{) belongs to } T$$
$$\text{it went to one of the 40 locations } \{1 \ldots 40\}$$
$$\text{cmp bit}$$

$Case$ b :

$$P\left(S_1 \text{ present}\right) = \frac{4}{40} = \frac{1}{10} \text{ (wrong says)}$$

$$P\left(S_1 \text{ not present}\right) = \frac{36}{40} = \frac{9}{10} = 0.9 \text{ (correct says)}$$

4 votes -- venky.victory35 (741 points)

12.5.14 Probability: TIFR2015-A-1

https://gateoverflow.in/29156

✔ $P\{3,5\} = 1 - P\{2,4,6\} - P\{1\} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

Can $P3 = 0$? then $P\{6\} = \frac{1}{3}$ and $P\{2,4\} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

And

$P\{5\} = \frac{1}{3}$. Possible.

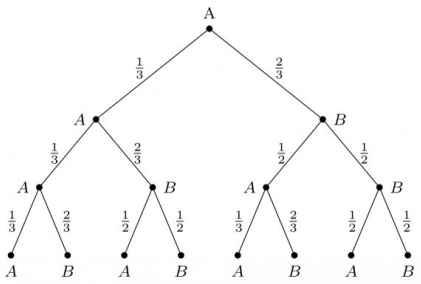
So, option D.

10 votes -- Arjun Suresh (334k points)

12.5.15 Probability: TIFR2016-A-12

https://gateoverflow.in/73498

✔ In such cases where the events are mutually exclusive and collectively exhaustive, it is preferable to use tree diagram to find the final outcome. The tree diagram for the given problem is as shown:



The frog is at B at $t = 3$

$\implies P(A.A.B) + P(A.B.B) + P(B.A.B) + P(B.B.B)$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{2}{27} + \frac{1}{9} + \frac{2}{9} + \frac{1}{6} = \frac{31}{54}$

Hence we have $P(\text{frog is at } B \text{ at } t = 3) = 31/54$

Correct Answer: B

17 votes -- HABIB MOHAMMAD KHAN (67.6k points)

12.5.16 Probability: TIFR2017-A-9

https://gateoverflow.in/95042

✔

$$\Rightarrow [\text{maj } \{x_i\} = 1] \Leftrightarrow \left(\left[\sum_{i=1}^{3} x_i \right] \geq 2 \right)$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i \right] \geq 2 \right) = Prob\left(\left[\sum_{i=1}^{3} x_i \right] = 2 \right) + Prob\left(\left[\sum_{i=1}^{3} x_i \right] = 3 \right)$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i \right] \geq 2 \right) = \binom{3}{2} * \{\alpha.\alpha.(1-\alpha)\} + \alpha^3$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i \right] \geq 2 \right) = 3.\alpha^2(1-\alpha) + \alpha^3$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i \right] \geq 2 \right) = 3.\alpha^2 - 2.\alpha^3$$

$$\Rightarrow P^{'}(\alpha) = \frac{\mathrm{d}}{\mathrm{d}\alpha}\left[3.\alpha^2 - 2.\alpha^3\right]$$

$$\Rightarrow P^{'}(\alpha) = 6.\alpha\,(1-\alpha)$$

Correct Answer: C

14 votes

-- Debashish Deka (40.8k points)

12.5.17 Probability: TIFR2018-A-10

https://gateoverflow.in/179279

✔ $p_0 = 0$

$p_1 = p$

$p_2 = {}^2C_1 p(1-p)$

$p_3 = {}^3C_1 p(1-p)^2 + {}^3C_3 p^3$

$\vdots$

$p_n = {}^nC_1 p(1-p)^{n-1} + {}^nC_3 p^3 (1-p)^{n-3} + \ldots$

$= \frac{1}{2}[((1-p)+p)^n - ((1-p)-p)^n]$

(Since $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + \ldots + {}^nC_n b^n$) and $(a-b)^n = {}^nC_0 a^n - {}^nC_1 a^{n-1} b + \ldots + (-1)^n {}^nC_n b^n$)

$= \frac{1}{2}[1 - (1-2p)^n]$

From option C,

$\sum_{i=1}^{n} p(1-2p)^{i-1} = p \sum_{i=1}^{n} (1-2p)^{i-1}$

$= p\frac{1-(1-2p)^n}{2p}$ (Sum to n terms of GP with $a = 1, r = (1-2p)$

$= \frac{1}{2}[1 - (1-2p)^n]$

So, correct option: C.

6 votes

-- Arjun Suresh (334k points)

12.5.18 Probability: TIFR2018-A-13

https://gateoverflow.in/179371

✔ Total number of 5 string sets is $\binom{26^{10}}{5}$

Out of which, total $\binom{26^{10}-1}{4}$ sets will contain password.

So required probability that his guessed set contains the password will be $\frac{\binom{26^{10}-1}{4}}{\binom{26^{10}}{5}}$.

And if you simplify $\frac{\binom{n-1}{r-1}}{\binom{n}{r}}$, it will be $\frac{r}{n} \rightarrow \frac{5}{26^{10}}$.

$\rightarrow$ So, Option A

17 votes -- Shivansh Gupta (2k points)

12.5.19 Probability: TIFR2018-A-15

https://gateoverflow.in/179366

Required probability has 2 parts.

a) When both 1 and 2 remain after 10 are chosen. Later 1 will be chosen with probability 1 and hence, 2 will remain.

b) When 1 is thrown along with some other 9, and 2 remains. Later we have to multiply with probability that 2 will not be thrown.

P(2 remains after all the events) = P(1 and 2 are not thrown out) + P(1 is thrown out , 2 remains) * P(from remaining 10 2 is not chosen)

$\rightarrow \frac{{}^{18}_{10}\mathrm{C}+{}^{18}_{9}\mathrm{C}*\frac{9}{10}}{{}^{20}_{10}\mathrm{C}}$

$\rightarrow \frac{9}{19}$

Hence, B.

13 votes -- Shivansh Gupta (2k points)

12.5.20 Probability: TIFR2019-A-14

https://gateoverflow.in/280496

✔ Here, we are drawing all 9 pens. So,

Number of different ways $=$ Number of permutations of 9 objects where 3 are of type A, next 3 of type B and final 3 of type C

$$= \frac{9!}{3!3!3!}$$

Now, when we have two red pens in succession, this is equivalent to 2 objects being close to each other in permutation. So, ignoring these two we can arrange the rest 7 in $\frac{7!}{3!3!}$ ways. Now, this group of 2 can be placed in 8 positions around the 7 objects -- wait -- here is the problem. A RED pen is already there and if we put RED,RED on its left or right side we get the same arrangement. So, effectively we have only 7 unique positions for the 2 RED pens. This gives our required number of permutations as $\frac{7!.7}{3!3!}$.

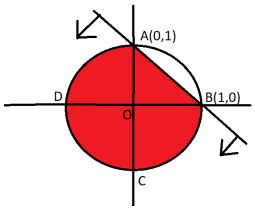
Thus our required probability will be

$$\frac{7!.7}{3!3!} \div \frac{9!}{3!3!3!} = \frac{3!.7}{8.9} = \frac{7}{12}$$

22 votes -- Arjun Suresh (334k points)

12.5.21 Probability: TIFR2019-A-4

https://gateoverflow.in/280506



The $Area$ inside $ADBC$ satisfies the $x^2 + y^2 \leq 1$

The line AB has $eq^n\ x + y = 1$, the arrow represents the points which statisfy $x + y \leq 1$

The $coloured\ region$ shows the area from where any point can be picked from the entire disk $ADBC$

$Area\ of\ the\ disk = \pi * 1^2 = \pi$

$Area\ of\ the\ coloured\ region = Area(ADCBO) + Area(AOB)$

$Area(ADCBO) = \frac{3}{4} * Area\ of\ disk = \frac{3}{4} * \pi$

$Area(AOB) = \frac{1}{2} * AO * OB = \frac{1}{2} * 1 * 1 = \frac{1}{2}$

$Area\ of\ the\ coloured\ region = \frac{3}{4} * \pi + \frac{1}{2}$

$Required\ Proabability = \frac{coloured\ region}{total\ area\ of\ disk} = \frac{\frac{3}{4} * \pi + \frac{1}{2}}{\pi} = \frac{3}{4} + \frac{1}{2} . \frac{1}{\pi}$

So, (c) should be correct. Correct me If I'm wrong

22 votes -- Shobhit Joshi (3.9k points)

12.5.22 Probability: TIFR2020-A-10 https://gateoverflow.in/333109

January has 31 days, no. of complete weeks in Jan $= \lfloor 31/7 \rfloor = 4$

Then remaining days $31 - 7(4) = 3$. Since mentioned there are exactly 4 Mondays and 4 Fridays then these Mondays and Fridays are already covered in the 4 complete weeks.Hence for these 3 days we need 3 consecutive days other than Monday and Friday.

The only 3 **consecutive** days other than Monday and Friday is :

$29^{th}\ Jan \rightarrow Tuesday$

$30^{th}\ Jan \rightarrow Wednesday$

$31^{st}\ Jan \rightarrow Thursday$

then the 4 Mondays are :

$28^{th}\ Jan \rightarrow Monday$

$21^{st}\ Jan \rightarrow Monday$

$14^{th}\ Jan \rightarrow Monday$

$7^{th}\ Jan \rightarrow Monday$

Thus

$20^{th}\ Jan \rightarrow$ Sunday

1 votes -- Sourajit25 (2k points)

12.6 Random Variable (2)

12.6.1 Random Variable: TIFR2011-A-7 https://gateoverflow.in/20012

Let X and Y be two independent and identically distributed random variables. Then $P(X > Y)$ is.

A. $\frac{1}{2}$
B. 1
C. 0
D. $\frac{1}{3}$

E. Information is insufficient.

tifr2011 probability random-variable

12.6.2 Random Variable: TIFR2014-A-19

https://gateoverflow.in/27130

Consider the following random function of x
$F(x) = 1 + Ux + Vx^2 \bmod 5$,
where U and V are independent random variables uniformly distributed over $\{0, 1, 2, 3, 4\}$. Which of the following is FALSE?

A. $F(1)$ is uniformly distributed over $\{0, 1, 2, 3, 4\}$.
B. $F(1), F(2)$ are independent random variables and both are uniformly distributed over $\{0, 1, 2, 3, 4\}$.
C. $F(1), F(2), F(3)$ are independent and identically distributed random variables.
D. All of the above.
E. None of the above.

tifr2014 probability random-variable

Answers: Random Variable

12.6.1 Random Variable: TIFR2011-A-7

https://gateoverflow.in/20012

✔ Let the probability $P(X = Y) > 0$. This can happen if X and Y are discrete random variables. Also, if X and Y are continuous random variables, it could be that some values have a non-zero probability of getting selected.

Then $P(X > Y) = P(Y > X) = \dfrac{1 - P(X = Y)}{2}$

Since nothing is said about the value of $P(X = Y)$, the correct answer will be option e. Information is insufficient.

10 votes -- Pragy Agarwal (18.3k points)

12.6.2 Random Variable: TIFR2014-A-19

https://gateoverflow.in/27130

Here as U and V are uniformly distributed, they will have equal probabilities for each point in {0,1,2,3,4} which by the rule of probability comes $1/5$. Now for being uniformly distributed F(1), F(2), F(3) must have same values at points {0,1,2,3,4}

So,

$F(1) = (1 + U + V) mod 5 = (1 + 1/5 + 1/5) mod 5$ for all {0,1,2,3,4}

$F(2) = (1 + 2U + 4V) mod 5 = (1 + 2/5 + 4/5) mod 5$ for all {0,1,2,3,4}

$F(3) = (1 + 3U + 9V) mod 5 = (1 + 3/5 + 9/5) mod 5$ for all {0,1,2,3,4}

so all three are uniformly distributed as they all same value for all points.

but not identical as they have values different from each other.

hence Option C is false.

2 votes -- Shivansh Gupta (2k points)

12.7 Uniform Distribution (3)

12.7.1 Uniform Distribution: TIFR2013-A-18

https://gateoverflow.in/25498

Consider three independent uniformly distributed (taking values between 0 and 1) random variables. What is the probability that the middle of the three values (between the lowest and the highest value) lies between a and b where $0 \leq a < b \leq 1$?

A. $3(1 - b)a(b - a)$
B. $3((b - a) - (b^2 - a^2)/2)$
C. $6(1 - b)a(b - a)$
D. $(1 - b)a(b - a)$
E. $6((b^2 - a^2)/2 - (b^3 - a^3)/3)$.

tifr2013 probability random-variable uniform-distribution

12.7.2 Uniform Distribution: TIFR2015-A-12

https://gateoverflow.in/29583

Consider two independent and identically distributed random variables X and Y uniformly distributed in $[0,1]$. For $\alpha \in [0,1]$, the probability that $\alpha \max(X,Y) < XY$ is

A. $1/(2\alpha)$ B. $\exp(1-\alpha)$ C. $1-\alpha$ D. $(1-\alpha)^2$ E. $1-\alpha^2$

tifr2015 probability random-variable uniform-distribution

12.7.3 Uniform Distribution: TIFR2020-A-4

https://gateoverflow.in/333105

Fix $n \geq 4$. Suppose there is a particle that moves randomly on the number line, but never leaves the set $\{1,2,\ldots,n\}$. Let the initial probability distribution of the particle be denoted by $\overrightarrow{\pi}$. In the first step, if the particle is at position i, it moves to one of the positions in $\{1,2,\ldots,i\}$ with uniform distribution; in the second step, if the particle is in location j, then it moves to one of the locations in $\{j,j+1,\ldots,n\}$ with uniform distribution.Suppose after two steps, the final ditribution of the particle is uniform. What is the initial distribution $\overrightarrow{\pi}$?

A. $\overrightarrow{\pi}$ is not unique
B. $\overrightarrow{\pi}$ is uniform
C. $\overrightarrow{\pi(i)}$ is non-zero for all even i and zero otherwise
D. $\overrightarrow{\pi}(1) = 1$ and $\overrightarrow{\pi}(i) = 0$ for $i \neq 1$
E. $\overrightarrow{\pi}(n) = 1$ and $\overrightarrow{\pi}(i) = 0$ for $i \neq n$

tifr2020 engineering-mathematics probability uniform-distribution

Answers: Uniform Distribution

12.7.1 Uniform Distribution: TIFR2013-A-18

https://gateoverflow.in/25498

✔ Let X, Y and Z be the three random variables. For the middle value to lie between a and b we have the following cases

1. All X, Y and Z are between a and b
2. 2 of X, Y and Z are between a and b and one is less than a
3. 2 of X, Y and Z are between a and b and one is greater than a
4. Exactly one of X, Y and Z is between a and b and one other is less than a and the remaining is greater than b

- Probability of case 1 : $(b-a)^3$ (Uniform distribution)
- Probability of case 2 : ${}^3C_2 \times (b-a)^2 \times a$
- Probability of case 3 : ${}^3C_2 \times (b-a)^2 \times (1-b)$
- Probability of case 4 : $3! \times a \times (b-a) \times (1-b)$

Since, all these cases are **mutually exclusive** (no overlapping) and **exhaustive** (no other favorable case) we can get the required probability by adding the probabilities for the four cases as

$$(b-a)\left[(b-a)^2 + 3(b-a)a + 3(b-a)(1-b) + 6a(1-b)\right]$$
$$= (b-a)\left[(b-a)^2 + 3(b-a)a + 3(1-b)(b+a)\right]$$
$$= (b-a)\left[(b-a)^2 + 3(ab - a^2 + b + a - b^2 - ab)\right]$$
$$= (b-a)\left[(b-a)^2 + 3(-a^2 - b^2 + b + a)\right]$$
$$= (b-a)\left[a^2 + b^2 - 2ab - 3a^2 - 3b^2 + 3b + 3a\right]$$
$$= (b-a)\left[-2a^2 - 2b^2 - 2ab + 3b + 3a\right]$$
$$= -2a^2b - 2b^3 - 2ab^2 + 3b^2 + 3ab + 2a^3 + 2ab^2 + 2a^2b - 3ab - 3a^2$$
$$= 2(a^3 - b^3) + 3(b^2 - a^2)$$
$$= 3(b^2 - a^2) - 2(b^3 - a^3)$$
$$= 6((b^2 - a^2)/2 - (b^3 - a^3)/3)$$

Correct Option: E

3 votes -- Arjun Suresh (334k points)

12.7.2 Uniform Distribution: TIFR2015-A-12

https://gateoverflow.in/29583

$P(\alpha \max(X,Y) < XY)$

$= P(\alpha X < XY \text{ AND } \alpha Y < XY)$

$= P(\alpha < Y \text{ AND } \alpha < X)$

$= P(\alpha < Y) \times P(\alpha < X)$ (Since, X and Y are independent)

$= (P(Y) - P(\alpha)) \times (P(X) - P(\alpha))$

$= (1-\alpha) \times (1-\alpha)$ (Uniform distribution in the interval 0 to 1)

$= (1-\alpha)^2$

Option D.

5 votes

-- Arjun Suresh (334k points)

Answer Keys

12.0.1	C	12.1.1	C	12.1.2	E	12.1.3	D	12.2.1	E
12.2.2	D	12.2.3	C	12.3.1	C	12.3.2	C	12.3.3	E
12.3.4	C	12.3.5	D	12.4.1	D	12.5.1	A	12.5.2	A
12.5.3	E	12.5.4	B	12.5.5	D	12.5.6	B	12.5.7	A
12.5.8	B	12.5.9	D	12.5.10	E	12.5.11	A	12.5.12	B
12.5.13	D	12.5.14	D	12.5.15	B	12.5.16	C	12.5.17	C
12.5.18	A	12.5.19	B	12.5.20	A	12.5.21	C	12.5.22	A
12.6.1	E	12.6.2	C	12.7.1	E	12.7.2	D	12.7.3	D

13 General Aptitude: Analytical Aptitude (10)

Logic: deduction and induction, Analogy, Numerical relations and reasoning

Mark Distribution in Previous GATE

Year	2021-1	2021-2	Minimum	Average	Maximum
1 Mark Count	0	0	0	0	0
2 Marks Count	1	1	1	1	1
Total Marks	2	2	2	2	2

13.1 Logical Reasoning (10)

13.1.1 Logical Reasoning: TIFR2010-A-1

https://gateoverflow.in/18202

A box contains 731 black balls and 2000 white balls. The following process is to be repeated as long as possible. Arbitrarily select two balls from the box. If they are of the same color, throw them out and put a black ball into the box (enough extra black balls are available to do this). If they are of different color, place the white ball back into the box and throw the black ball away. Which of the following is correct?

A. The process can be applied indefinitely without any prior bound
B. The process will stop with a single white ball in the box
C. The process will stop with a single black ball in the box
D. The process will stop with the box empty
E. None of the above

tifr2010 analytical-aptitude logical-reasoning

13.1.2 Logical Reasoning: TIFR2013-A-10

https://gateoverflow.in/25432

Three men and three rakhsasas arrive together at a ferry crossing to find a boat with an oar, but no boatman. The boat can carry one or at the most two persons, for example, one man and one rakhsasas, and each man or rakhsasas can row. But if at any time, on any bank, (including those who maybe are in the boat as it touches the bank) rakhsasas outnumber men, the former will eat up the latter. If all have to go to the other side without any mishap, what is the minimum number of times that the boat must cross the river?

A. 7 B. 9 C. 11 D. 13 E. 15

tifr2013 analytical-aptitude logical-reasoning

13.1.3 Logical Reasoning: TIFR2013-A-11

https://gateoverflow.in/25433

Let there be a pack of 100 cards numbered 1 to 100. The i^{th} card states: "There are at most $i-1$ true cards in this pack". Then how many cards of the pack contain TRUE statements?

A. 0 B. 1 C. 100 D. 50 E. None of the above

tifr2013 logical-reasoning

13.1.4 Logical Reasoning: TIFR2013-A-2

https://gateoverflow.in/25383

Consider the following two types of elections to determine which of two parties A and B forms the next government in the 2014 Indian elections. Assume for simplicity an Indian population of size $545545 (= 545 * 1001)$. There are only two parties A and B and every citizen votes.

TYPE C: The country is divided into 545 constituencies and each constituency has 1001 voters. Elections are held for each constituency and a party is said to win a constituency if it receive a majority of the vote in that constituency. The party that wins the most constituencies forms the next government.

TYPE P: There are no constituencies in this model. Elections are held throughout the country and the party that wins the most votes (among 545545 voters forms the government.

Which of the following is true?

A. If the party forms the govt. by election TYPE C winning at least two-third of the constituencies, then it will also forms the govt. by election TYPE P.
B. If a party forms govt. by election TYPE C, then it will also form the govt. by election TYPE P.
C. If a party forms govt. by election TYPE P, then it will also form the govt. by election TYPE C.
D. All of the above

E. None of the above

tifr2013 logical-reasoning

13.1.5 Logical Reasoning: TIFR2016-A-1

https://gateoverflow.in/96822

Suppose the following statements about three persons in a room are true.

Chandni, Sooraj and Tara are in a room. Nobody else is in the room. Chandni is looking at Sooraj. Sooraj is looking at Tara. Chandni is married. Tara is not married. A married person in the room is looking at an unmarried person.

Which of the following is necessarily true?

A. Sooraj is married
B. Sooraj is unmarried
C. The situation described is impossible
D. There is insufficient information to conclude if Sooraj is married or unmarried
E. None of the above

tifr2016 logical-reasoning

13.1.6 Logical Reasoning: TIFR2017-A-14

https://gateoverflow.in/95657

Consider the following game with two players, Aditi and Bharat. There are n tokens in a bag. The two players know n, and take turns removing tokens from the bag. In each turn, a player can either remove one token or two tokens. The player that removes the last token from the bag loses. Assume that Aditi always goes first. Further, we say that a player has a winning strategy if she or he can win the game, no matter what other player does. Which of the following statements is TRUE?

A. For $n = 3$, Bharath has a winning strategy. For $n = 4$, Aditi has a winning strategy.
B. For $n = 7$, Bharath has a winning strategy. For $n = 8$, Aditi has a winning strategy.
C. For both $n = 3$ and $n = 4$, Aditi has a winning strategy.
D. For both $n = 7$ and $n = 8$, Aditi has a winning strategy.
E. Bharat never has a winning strategy.

Edit : Option (D) is : For both $n = 7$ and $n = 8$, Bharat has a winning strategy. (Source)

tifr2017 analytical-aptitude logical-reasoning

13.1.7 Logical Reasoning: TIFR2018-A-11

https://gateoverflow.in/179280

We are given a (possibly empty) set of objects. Each object in the set is colored either black or white, is shaped either circular or rectangular, and has a profile that is either fat or thin, Those properties obey the following principles:

1. Each white object is also circular.
2. Not all thin objects are black.
3. Each rectangular object is also either thin or white or both thin and white.

Consider the following statements:

i. If there is a thin object in the set, then there is also a white object.
ii. If there is a rectangular object in the set, then there are at least two objects.
iii. Every fat object in the set is circular.

Which of the above statements must be TRUE for the set?

A. (i) only
B. (i) and (ii) only
C. (i) and (iii) only
D. None of the statements must be TRUE
E. All of the statements must be TRUE

tifr2018 analytical-aptitude logical-reasoning

13.1.8 Logical Reasoning: TIFR2018-A-8

https://gateoverflow.in/179277

A crime has been committed with four people at the scene of the crime. You are responsible for finding out who did it. You have recorded the following statements from the four witnesses, and you know one of them has committed the crime.

1. Anuj says that Binky did it.

2. Binky says that Anuj did it.
3. Chacko says that Binky is telling the truth.
4. Desmond says that Chacko is not lying.

You know that exactly three of the statements recorded are FALSE. Who committed the crime?

A. Anuj
B. Binky
C. Chacko
D. Desmond
E. Either Anuj or Binky; the information is insufficient to pinpoint the criminal

tifr2018 logical-reasoning

13.1.9 Logical Reasoning: TIFR2019-A-10

https://gateoverflow.in/280500

Avni and Badal alternately choose numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ without replacement (starting with Avni). The first person to choose numbers of which any 3 sum to 15 wins the game (for example, Avni wins if she chooses the numbers $8,3,5,2$ since $8+5+2=15$). A player is said to have a winning strategy if the player can always win the game, no matter what the other player does. Which of the following statements is TRUE?

As a hint, there are exactly 8 ways in which 3 numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ can sum up to 15, shown as the three rows, the three columns, and the two diagonals in the following square:

$$\begin{matrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{matrix}$$

A. Avni has a winning strategy
B. Badal has a winning strategy
C. Both of them have a winning strategy
D. Neither of them has a winning strategy
E. The Player that picks 9 has a winning strategy

tifr2019 general-aptitude analytical-aptitude logical-reasoning

13.1.10 Logical Reasoning: TIFR2019-A-11

https://gateoverflow.in/280499

Suppose there are n guests at a party (and no hosts). As the night progresses, the guests meet each other and shake hands. The same pair of guests might shake hands multiple times. for some parties stretch late into the night , and it is hard to keep track.Still, they don't shake hands with themselves. Let Odd be the set of guests who have shaken an odd number of hands, and let even be the set of guests who have shaken an even number of hands. Which of the following stays invariant throughout the night?

A. $|\text{Odd}| \mod 2$
B. $|\text{Even}|$
C. $|\text{Even}| - |\text{Odd}|$
D. $2 \mid |\text{Even}| - |\text{Odd}|$
E. $2 \mid |\text{Odd}| - |\text{Even}|$

tifr2019 general-aptitude analytical-aptitude logical-reasoning

Answers: Logical Reasoning

13.1.1 Logical Reasoning: TIFR2010-A-1

https://gateoverflow.in/18202

✔ The total number of balls is 2371 and in each round 1 ball is removed. So after 2370 steps, we will have 1 ball in the box and we must stop.

Here, white balls are even and every time either it gets removed in a pair or remains unchanged.

So, the last ball remaining can never be a white ball.

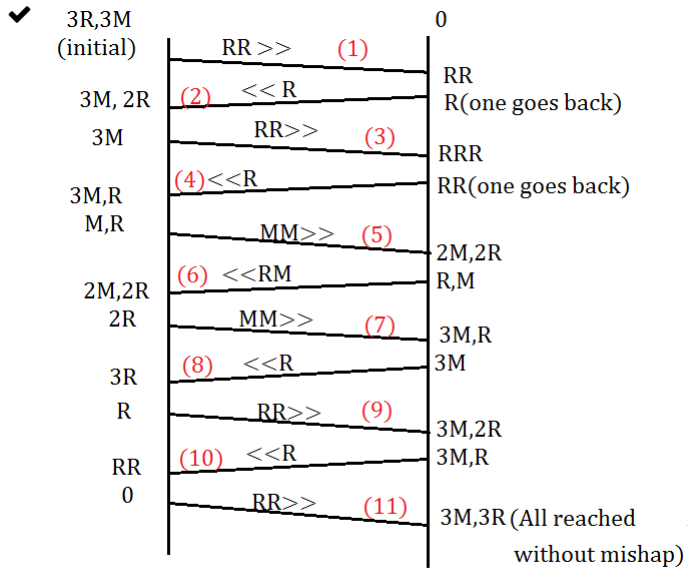
Since at each step the number of black balls changes from odd to even or even to odd, so at the end only 1 black ball will be in the box.

Hence, option C is the correct answer.

7 votes -- Umang Raman (12.2k points)

13.1.2 Logical Reasoning: TIFR2013-A-10

https://gateoverflow.in/25432



So, boat crosses river 11 times.. **Option (C)**

9 votes

-- Himanshu Agarwal (12.4k points)

13.1.3 Logical Reasoning: TIFR2013-A-11

https://gateoverflow.in/25433

Option D should be the correct one.

that is 50 cards of the pack contain true statements.

Why?

Because if the statement written on card number x is true then all the statements written in card numbers $x+1$ to 100 must be true.

For Example if Card number 3 is true, then according to the statement written in this card, "There are at most $3-1(=2)$ true cards in the pack".

This implies number of true cards must be less than 3.

Now the statement on card number 4 will imply that the number of true cards must be less than 4.

similarly the statement on card number 100 will imply that the number of true cards must be less than 100.

So if statement written on card number 3 is true then all the statements written on the card numbers 4 to 100 will vacuously be true.

But now the the number of true cards will be 98 (from card number 3 to 100) hence the statement on the card number 3 must be false.

Clearly this is inconsistent so card number 3 can not be a true card.

Conclusion:

If card number x is a true card then:

1. There are at least $(100-x)+1$ true cards. and

2.Total Number of true cards must be less then or equal to $x-1$

(where x belongs to Integers between 1 and 100).

For any value of $x \leq 50$ both of the above statements can not be true simultaneously, so none of the cards from 1 to 50 is a true card.

For any value of $x \geq 51$ both of the above statements can be true at the same time, so all of the cards from 51 to 100 must be true cards & the total number of true cards will be 50.

It can be observed that comparing one of the boundary cases of each of the above two statements will give us of the boundary cases of our answer.

that is, on solving:

$(100 - x) + 1 = x - 1$

we get $x = 51$ which indeed is our smallest true card.

13 votes -- Anurag Pandey (10.5k points)

13.1.4 Logical Reasoning: TIFR2013-A-2 https://gateoverflow.in/25383

✔ Minimum condition for Type C winning: if any party wins 273 constituencies out of 545 and with vote 501 out of 1001 for each.
Minimum condition for Type P winning: if any party wins 272773 votes out of 545545

Option A:

- Type C : Let party A wins $2/3$ of constituencies i.e., 364 wins by 1 vote and 181 loss by all vote $364 \times 501 + 181 \times 0 = 182364$
- Type P : If A wins it should have more than half of vote i.e., 272773

So, it is FALSE since A got only 182364 votes

Option B : Similar to option A

Option C:

- Type P: Let A win by 272773 votes
- Type C: If A wins 272 constituencies with 1001 votes and loss 272 with 1001 and 1 constituency with 1000 votes then, total received votes $= 272 \times 1001 + 272 \times 0 + 1 \times 1 = 272773$. But A lose the election since A won only 272 constituencies out of 545

So option E must be correct.

5 votes -- Umang Raman (12.2k points)

13.1.5 Logical Reasoning: TIFR2016-A-1 https://gateoverflow.in/96822

✔ ***"A married person in the room is looking at an unmarried person"*** *---this is satisfied when Sooraj is made* ***married***

$\underset{married}{\text{Chandni}} \longrightarrow \underset{married}{\text{Sooraj}} \longrightarrow \underset{unmarried}{\text{Tara}}$

"A married person in the room is looking at an unmarried person" *---this is satisfied when Sooraj is made* ***unmarried***

$\underset{married}{\text{Chandni}} \longrightarrow \underset{unmarried}{\text{Sooraj}} \longrightarrow \underset{unmarried}{\text{Tara}}$

As both are possible we cannot finalize it to A or B --- we need extra information **so answer should be D**

8 votes -- Pavan Kumar Munnam (7.4k points)

13.1.6 Logical Reasoning: TIFR2017-A-14 https://gateoverflow.in/95657

✔ Point to be noted-

1. Always Aditi starts the game.
2. A player loses the game if he/she has to remove the last item.
3. A player always removes 1 or 2 items with a winning strategy.

Method 1: shortcut- time complexity- O(1)

n = total no of tokens.

If **n mod 3 - 1 = 0, then Bharat has a winning strategy (1 is the last token being taken by the loser)**

if **n mod 3 - 1 = 1 or 2, then Aditi has a winning strategy.**

The trick here is with 4 tokens whoever plays next is not having a winning strategy. So, the strategy must be to ensure that the opponent always takes his turn with $3n + 1$ tokens. This is possible because if the opponent takes 1, I take 2 and vice verse.

Let's check each option-
[Here my strategy of solving this would be just contradicting the each option by considering all the possibilities].

A. **For n =3.** // Here I am trying to make Bharat win, Let's see if he can win or not.
Aditi - removes 2 item , n = 3-2 = 1

Now only 1 item remains, so Bharat is forced to remove the last item and loses the game. **Hence for n=3, Aditi has a winning strategy.**
The first part of A - False.

For n =4. // Here I am trying to make Aditi win, Let's see if she can win or not.

Case -1:
Aditi - removes 1 item , n = 4-1 = 3
Bharat- removes 2,item , n = 3 -2 =1

case 2:
Aditi - removes 2 item , n = 4-2 = 1
Bharat- removes 1,item , n = 2 -1 =1

Now only 1 item remains in both the case, so Aditi is forced to remove the last item and loses the game. **Hence for n=4, Bharat has a winning strategy.**
The 2nd part of A - True.

Hence, A- False.

B. For n =7. // Here I am trying to make Aditi win, Let's see if she can win or not.

Case 1:
Aditi - removes 1 item , n = 7- 1 = 6
Bharat- removes 2 item , n = 6 -2 = 4.

Case 2:
Aditi - removes 2 item , n = 7- 2 = 5
Bharat- removes 1 item , n = 5 -1 = 4.

Now remaining tokens in both the case, n = 4.

case 1:
Aditi - removes 1 item ,n=4-1 = 3
Bharat- removes 2,item ,n=3-2=**1**,

case 2:
Aditi - removes 2 item , n = 4-2= 2.
Bharat- removes 1,item , n = 2-1 = **1.**

Now in both the case, only 1 item remains, so Aditi is forced to remove the last item and loses the game. **Hence for n=7, Bharat has a winning strategy. Aditi could not win the game even taking all possibility of Aditi's turn.**

For n = 8. // Here I am trying to make Aditi win taking all possibility of Bharat's turn, Let's see if she can win or not.

Aditi - removes 1 item , remaining item n = 8 - 1 = 7

Case 1:
Bharat- removes 1 item , n = 7 - 1 = 6.
Aditi - removes 2 item , n = 6 - 2 = 4 ,

case 2:
Bharat- removes 2 item ,n = 7 - 2 = 5.
Aditi - removes 1 item , n = 5-1 = 4.

N**ow In both cases, remaining items = 4.**

case 1
Bharat- removes 1,item , n = 4-1 = **3**
Aditi - removes 2 item , n = 3-2 = 1 ,

case 2
Bharat- removes 2,item , n = 4-2 = 2 **.**
Aditi - removes 1 item , n = 2-1 = 1.

Now in both the case, only 1 item is remaining so Bharat is forced to remove the last item and loses the game. **Hence for n=8, Aditi has a winning strategy.**

B- True.

C. For both n=3 and n=4, Aditi has a winning strategy.

See explanation for A. where Bharat has a winning strategy for both n=3 and n=4.

Hence, False.

D. For both n=7 and n=8, Aditi has a winning strategy.

See explanation for B. where Bharat has a winning strategy for both n=7.

Hence, False.

E. Bharat never has a winning strategy.

See explanation for B where Aditi has a winning strategy for both n=8.

Hence, False.

Ans - B.

15 votes -- Dhananjay Kumar Sharma (18.8k points)

13.1.7 Logical Reasoning: TIFR2018-A-11

https://gateoverflow.in/179280

✔

1. Each white object is also circular.
2. Not all thin objects are black.
3. Each rectangular object is also either thin or white or both thin and white.

Consider the following statements:

i. If there is a thin object in the set, then there is also a white object.
"Not all thin objects are black." means there is a thin object which is not black and white is the only other color possible. So, this is TRUE.

ii. If there is a rectangular object in the set, then there are at least two objects.
"Each rectangular object is also either thin or white or both thin and white". So each rectangular object is either thin or white or both. But "Each white object is also circular" means, a rectangular object cannot be white and hence it must be thin. Now, "Not all thin objects are black" means there is a white object and this is circular as per (1). So, if there is a rectangular object there are at least two objects and this statement is TRUE.

iii. Every fat object in the set is circular.
Can a fat object be rectangular? No, because as seen in (ii) a rectangular object must be thin (as it cannot be white). This means every fat object must be circular as there are no other possibilities. This statement is also TRUE.

Correct Answer: E.

0 votes -- Arjun Suresh (334k points)

13.1.8 Logical Reasoning: TIFR2018-A-8

https://gateoverflow.in/179277

✔ Out of 4 statements, exactly 3 are false and 1 is true.

- If statement 4 is true then it implies statement 3 is also true (but only 1 statement can be true).
- If statement 3 is true then it implies statement 2 is also true (again only 1 statement can be true).
- If statement 2 is true then all other statements are false including statement 3 which states " **Chacko says that Binky is telling the truth**". This means Binky **is not** telling the truth, so statement 2 can not be true.

Thus statements $2, 3$ and 4 cannot be possibly true implying statement 1 is true.

Statement 1 states "**Anuj says that Binky did it**", so **Binky** is the culprit.

Option (B).

6 votes -- ZAHID WAKEEL (1.6k points)

13.1.9 Logical Reasoning: TIFR2019-A-10

https://gateoverflow.in/280500

1. $9+5+1$
2. $9+4+2$
3. $8+6+1$
4. $8+5+2$
5. $8+4+3$
6. $7+6+2$
7. $7+5+3$
8. $6+5+4$

Consider each digits from 1 to 9

1. For 1, $\{5,9\}$ and $\{8,6\}$ are the winning combinations. So, if a player picks 1 the other player should avoid picking any number from $\{5,6,8,9\}$ and if the first player then picks 5 the second player must pick 9 and if the first player picks 8 the second player must pick 6 (same for reverse order too) thus blocking all the winning combinations.
2. Like above for all digits from 2 to 9 the winning combinations can be blocked by the other player because if one element is picked there are no common elements in the winning combinations.

So, option D is correct. Neither has a winning strategy.

More read: https://puzzling.stackexchange.com/questions/48132/finding-digits-that-sum-to-15

References

4 votes -- Arjun Suresh (334k points)

13.1.10 Logical Reasoning: TIFR2019-A-11

https://gateoverflow.in/280499

According to Handshaking Lemma, number of person with odd number of handshake is always even.

hence |odd| mod 2 is always 0.

answer: A

please point out if any mistake is there.

7 votes -- Srishty Suman (2.4k points)

Answer Keys

13.1.1	C	13.1.2	C	13.1.3	D	13.1.4	E	13.1.5	D
13.1.6	B	13.1.7	E	13.1.8	B	13.1.9	D	13.1.10	A

14 General Aptitude: Quantitative Aptitude (55)

Syllabus: Numerical computation, Numerical estimation, Numerical reasoning and data interpretation

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	1	2	1	2	3	2	3	1	2	1	1.8	3
2 Marks Count	3	2	3	3	4	4	4	3	3	2	3.2	4
Total Marks	7	6	7	8	11	10	11	7	8	6	8.3	11

14.0.1 TIFR2020-A-14

https://gateoverflow.in/333117

A ball is thrown directly upwards from the ground at a speed of $10\ ms^{-1}$, on a planet where the gravitational acceleration is $10\ ms^{-2}$. Consider the following statements:

1. The ball reaches the ground exactly 2 seconds after it is thrown up
2. The ball travels a total distance of 10 metres before it reaches the ground
3. The velocity of the ball when it hits the ground is $10\ ms^{-1}$

What can you say now?

A. Only Statement 1 is correct
B. Only Statement 2 is correct
C. Only Statement 3 is correct
D. None of the Statements $1, 2$ or 3 is correct
E. All of the Statements $1, 2$ and 3 are correct

tifr2020

Answers:

14.0.1 TIFR2020-A-14

https://gateoverflow.in/333117

According to the equations of motion for a particle in a straight line under uniform acceleration :

$1)\ v = u + at$

$2)\ s = ut + \frac{1}{2}at^2$

$3)\ v^2 = u^2 + 2as$

$1)$: When ball is thrown directly upwards from the ground

In this case, $u = 10\ m/sec$, $a = g = -10\ m/s^2$ (because I assumed displacement s as positive in upward direction) and $v = 0$ at maximum height.

So, from (1), $0 = 10 - 10t \Rightarrow t = 1\ sec$

from (2), $s = 10 * 1 - \frac{1}{2} * 10 * 1^2 \Rightarrow s = 5\ m$

$2)$: When ball goes downwards after reaching the maximum height

In this case, $u = 0\ m/sec$, $a = g = +10\ m/s^2$ (because I assumed displacement s as negative in downward direction)

So, from (3), $v^2 = 0 + 2 * 10 * 5 \Rightarrow v = 10\ m/sec$

from (1), $10 = 0 + 10 * t \Rightarrow t = 1\ sec$

So, from (1) and (2)

$1)$ Total time in journey : $1 + 1 = 2\ sec$

$2)$ Total displacement in journey : $5 + 5 = 10\ m$

$3)$ Velocity of the ball when it hits the ground : $10\ m/sec$

2 votes -- ankitgupta.1729 (15.2k points)

14.1 Cartesian Coordinates (3)

14.1.1 Cartesian Coordinates: TIFR2013-B-9

https://gateoverflow.in/25675

Suppose n straight lines are drawn on a plane. When these lines are removed, the plane falls apart into several connected components called regions. A region R is said to be convex if it has the following property: whenever two points are in R, then the entire line segment joining them is in R. Suppose no two of the n lines are parallel. Which of the following is true?

A. $O(n)$ regions are produced, and each region is convex.
B. $O(n^2)$ regions are produced but they need not all be convex.
C. $O(n^2)$ regions are produced, and each region is convex.
D. $O(n \log n)$ regions are produced, but they need not all be convex.
E. All regions are convex but there may be exponentially many of them.

tifr2013 quantitative-aptitude geometry cartesian-coordinates

14.1.2 Cartesian Coordinates: TIFR2014-A-13

https://gateoverflow.in/26390

Let L be a line on the two dimensional plane. L's intercepts with the X and Y axes are respectively a and b. After rotating the co-ordinate system (and leaving L untouched), the new intercepts are a' and b' respectively. Which of the following is TRUE?

A. $\frac{1}{a}+\frac{1}{b}=\frac{1}{a}+\frac{1}{b}$.
B. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{a'^2}+\frac{1}{b'^2}$.
C. $\frac{b}{a^2}+\frac{a}{b^2}=\frac{b'}{a'^2}+\frac{a}{b'^2}$.
D. $\frac{b}{a}+\frac{a}{b}=\frac{b'}{a'}+\frac{a'}{b'}$.
E. None of the above.

tifr2014 geometry cartesian-coordinates

14.1.3 Cartesian Coordinates: TIFR2015-A-13

https://gateoverflow.in/29586

Imagine the first quadrant of the real plane as consisting of unit squares. A typical square has 4 corners: $(i,j),(i+1,j),(i+1,j+1)$,and $(i,j+1)$, where (i,j) is a pair of non-negative integers. Suppose a line segment l connecting $(0,0)$ to $(90,1100)$ is drawn. We say that l passes through a unit square if it passes through a point in the interior of the square. How many unit squares does l pass through?

A. $98,990$ B. $9,900$ C. $1,190$ D. $1,180$ E. $1,010$

tifr2015 quantitative-aptitude cartesian-coordinates

Answers: Cartesian Coordinates

14.1.1 Cartesian Coordinates: TIFR2013-B-9

https://gateoverflow.in/25675

Option C must be correct.

Regions divided by n lines = 1+1+2+3+4+.....+n-1 = 1 + n(n-1)/2 = O(n^2)

option B can be easily discarded as convex segments are obvious.

Hence C is correct.

2 votes -- Nitesh Tripathi (123 points)

14.1.2 Cartesian Coordinates: TIFR2014-A-13

https://gateoverflow.in/26390

✔ If $X-$intercept is a and $Y-$intercept is b then equation of line on $X-Y$ plane is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \rightarrow (1)$$

We can prove it by considering two points on the line, $(a,0)$ as (x_1,y_1) and $(0,b)$ as (x_2,y_2)

So, equation of line will be : $y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$

$\Rightarrow (y-0) = \left(\frac{b-0}{0-a}\right)(x-a)$

$\Rightarrow -ay = bx - ab$

On dividing by ab and rearranging,

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$

Now, if a point (x, y) lies on the $X-Y$ plane and if we rotate the co-ordinate system by θ degree then this point becomes (x', y') on new co-ordinate system and relationship is given by :

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

So,

- $x' = x\cos\theta + y\sin\theta$ and
- $y' = -x\sin\theta + y\cos\theta$

Now, on the new co-ordinate system, equation on line will be :

$\frac{x'}{a'} + \frac{y'}{b'} = 1$

$\Rightarrow \frac{x\cos\theta + y\sin\theta}{a'} + \frac{-x\sin\theta + y\cos\theta}{b'} = 1$

$\Rightarrow x\left(\frac{\cos\theta}{a'} - \frac{\sin\theta}{b'}\right) + y\left(\frac{\sin\theta}{a'} + \frac{\cos\theta}{b'}\right) = 1$

On comparing with equation (1),

$\left(\frac{\cos\theta}{a'} + \frac{-\sin\theta}{b'}\right) = \frac{1}{a}$ and

$\left(\frac{\sin\theta}{a'} + \frac{\cos\theta}{b'}\right) = \frac{1}{b}$

Now, on squaring both sides and adding above 2 equations :-

$\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = \left(\frac{\cos\theta}{a'} + \frac{-\sin\theta}{b'}\right)^2 + \left(\frac{\sin\theta}{a'} + \frac{\cos\theta}{b'}\right)^2$

$\Rightarrow \left(\frac{1}{a^2} + \frac{1}{b^2}\right) = \frac{\cos^2\theta}{a'^2} + \frac{\sin^2\theta}{b'^2} - \frac{2\sin\theta\cos\theta}{a'b'} + \frac{\sin^2\theta}{a'^2} + \frac{\cos^2\theta}{b'^2} + \frac{2\sin\theta\cos\theta}{a'b'}$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{a'^2} + \frac{1}{b'^2}$

So, **Answer is (b)**

5 votes -- ankitgupta.1729 (15.2k points)

14.1.3 Cartesian Coordinates: TIFR2015-A-13

https://gateoverflow.in/29586

✔ **Answer will be (d) 1,180**

If a line segment passes through unit square from $(0,0)$ to (i,j) the line intersects $(i + j - gcd(i,j))$

no. of squares $= (90 + 1100 - 10) = 1180.$

11 votes -- srestha (85.3k points)

14.2 Circles (1)

14.2.1 Circles: TIFR2011-A-18

https://gateoverflow.in/20255

The equation of the tangent to the unit circle at point ($\cos\alpha, \sin\alpha$) is

A. $x\cos\alpha - y\sin\alpha = 1$
B. $x\sin\alpha - y\cos\alpha = 1$
C. $x\cos\alpha + y\sin\alpha = 1$
D. $x\sin\alpha - y\cos\alpha = 1$
E. None of the above

tifr2011 quantitative-aptitude geometry circles

Answers: Circles

14.2.1 Circles: TIFR2011-A-18

https://gateoverflow.in/20255

✔ Assuming that the unit circle is centered at the origin, the equation of the unit circle is: $x^2 + y^2 = 1$

The slope of the tangent to the unit circle at point (x, y) can be derived by implicit differentiation as follows:

$$\begin{aligned} x^2 + y^2 &= 1 \\ \frac{d}{dx}\left(x^2 + y^2\right) &= \frac{d}{dx}1 \\ 2x + 2y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

Thus, the slope of the tangent at the point $(\cos\alpha, \sin\alpha)$ is $-\dfrac{\cos\alpha}{\sin\alpha}$

The equation of the tangent line then will be:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \sin\alpha &= -\frac{\cos\alpha}{\sin\alpha}(x - \cos\alpha) \\ y\sin\alpha - \sin^2\alpha &= -x\cos\alpha + \cos^2\alpha \\ y\sin\alpha + x\cos\alpha &= \sin^2\alpha + \cos^2\alpha = 1 \end{aligned}$$

Hence, option c is correct.

17 votes

-- Pragy Agarwal (18.3k points)

14.3 Clock Time (3)

14.3.1 Clock Time: TIFR2010-A-2

https://gateoverflow.in/18206

The hour hand and the minute hands of a clock meet at noon and again at mid-night. In between they meet N times, where N is.:

A. 6 B. 11 C. 12 D. 13 E. None of the above

tifr2010 quantitative-aptitude clock-time

14.3.2 Clock Time: TIFR2013-A-20

https://gateoverflow.in/25502

Consider a well functioning clock where the hour, minute and the seconds needles are exactly at zero. How much time later will the minutes needle be exactly one minute ahead ($1/60$ th of the circumference) of the hours needle and the seconds needle again exactly at zero?

Hint: When the desired event happens both the hour needle and the minute needle have moved an integer multiple of $1/60$ th of the circumference.

A. 144 minutes B. 66 minutes C. 96 minutes D. 72 minutes E. 132 minutes

tifr2013 quantitative-aptitude clock-time

14.3.3 Clock Time: TIFR2014-A-10

https://gateoverflow.in/25998

A person went out between 4pm and 5pm to chat with her friend and returned between 5pm and 6pm. On her return, she found that the hour-hand and the minute-hand of her (well-functioning) clock had just exchanged their positions with respect to their earlier positions at the time of her leaving. The person must have gone out to chat at

A. Twenty five minutes past 4pm.

B. Twenty six and $\frac{122}{143}$ minutes past 4pm.

C. Twenty seven and $\frac{1}{3}$ minutes past 4pm.

D. Twenty eight minutes past 4pm.

E. None of the above.

tifr2014 quantitative-aptitude clock-time

Answers: Clock Time

14.3.1 Clock Time: TIFR2010-A-2

https://gateoverflow.in/18206

✔ Let δ be the difference in minutes between hour and minute hand at $1:05$. So, the meeting times are

$1:05+\delta$

$2:10+2\delta$

$3:15+3\delta$

$4:20+4\delta$

$5:25+5\delta$

$6:30+6\delta$

$7:35+7\delta$

$8:40+8\delta$

$9:45+9\delta$

$10:50+10\delta$

$11:55+11\delta$

We have $11\delta = 5$

$\implies \delta = \frac{5}{11}$

minutes as the meeting time is $0:00$. So, we have $N = 10$.

Alternatively,

Speed of minute hand = 360 degrees per 60 minutes = 6 degrees per minute.

Speed of hour hand = 360 degree per $12 * 60$ minutes = 0.5 degree per minute.

For first meeting, distance traveled by minute hand = $360 +$ distance traveled by hour hand

Let, x be the minutes after which the hands intersect.

So, $6x = 360 + 0.5x$

$\implies 5.5x = 360$

$\implies x = \frac{720}{11}$

In 12 hours we have 12×60 minutes.

So, no. of intersections $= \frac{12 \times 60}{720/11} = 11$.

But the last intersection is at midnight and must be excluded as per given question. So, $N = 10$.

Correct Answer: E

11 votes

-- Arjun Suresh (334k points)

14.3.2 Clock Time: TIFR2013-A-20

https://gateoverflow.in/25502

✔ The minute needle should be exactly one minute ahead of hour needle.

Difference between min. needle and hr. needle is equal to one minute.

In 1 minute, distance covered by minute needle $= \frac{360^\circ}{60} = 6^\circ$

Suppose, after x minutes, hour needle and minute needle are separated by 6° .

In x minutes, distance covered by minute needle $= \frac{360^\circ}{60} \times x = 6x^\circ$

In x minutes, distance covered by hour needle $= \frac{360^\circ}{12 * 60} \times x = \frac{x}{2}^\circ$

$\therefore$ Difference between minute needle and hour needle in x minutes $= \left(6x^\circ - \frac{x}{2}^\circ\right)$

$\therefore (6x^\circ - \frac{x}{2}^\circ) = 6^\circ$

$\implies 12x - x = 12$

$\implies 11x = 12$

$\implies x = \frac{12}{11}$

It is given in question that second hand is 0 or minute hand has traversed an integral multiple of minutes. So, smallest possible value of $x = 12 \times 11 = 132$ minutes.

Option e.

10 votes -- Sukanya Das (10k points)

14.3.3 Clock Time: TIFR2014-A-10

https://gateoverflow.in/25998

✔ Let the time of departure be $4:x$ and time of arrival be $5:y$.

Angle made by hour hand during departure $= \frac{4}{12} \times 360 + x\frac{360}{12 \times 60} = 120 + \frac{x}{2}$.

This angle is equal to the angle made by the minute hand on arrival which is $6y$. So,

$$240 + x = 12y \rightarrow (1)$$

Similarly, the angle made by the hour hand on arrival is equal to the angle made by the minute hand on departure, which gives

$$\frac{5}{12} \times 360 + \frac{y}{2} = 6x \implies 300 + y = 12x \rightarrow (2).$$

Eliminating y from (1) and (2),

$$240 + x = 12(12x - 300) \implies 143x = 3840 \implies x = 26\frac{122}{143}$$

Correct Answer: B

13 votes -- Arjun Suresh (334k points)

14.4 Complex Number (3)

14.4.1 Complex Number: TIFR2011-A-13

https://gateoverflow.in/20223

If $z = \frac{\sqrt{3} - i}{2}$ and $\left(z^{95} + i^{67}\right)^{97} = z^n$, then the smallest value of n is

A. 1 B. 10 C. 11 D. 12 E. None of the above

tifr2011 quantitative-aptitude complex-number

14.4.2 Complex Number: TIFR2011-A-5

https://gateoverflow.in/20009

Three distinct points x, y, z lie on a unit circle of the complex plane and satisfy $x + y + z = 0$. Then x, y, z form the vertices of .

A. An isosceles but not equilateral triangle.
B. An equilateral triangle.
C. A triangle of any shape.
D. A triangle whose shape can't be determined.
E. None of the above.

tifr2011 quantitative-aptitude geometry complex-number non-gate

14.4.3 Complex Number: TIFR2013-A-7

https://gateoverflow.in/25429

For any complex number z, $arg\ z$ defines its phase, chosen to be in the interval $0 \leq argz < 360^\circ$. If z_1, z_2 and z_3 are three complex numbers with the same modulus but different phases ($argz_3 < argz_2 < argz_1 < 180^\circ$), then the quantity

$$\frac{arg(z_1/z_2)}{arg[(z_1-z_3)/(z_2-z_3)]}$$

is a constant, and has the value

A. 2 B. $\frac{1}{3}$ C. 1 D. 3 E. $\frac{1}{2}$

tifr2013 quantitative-aptitude complex-number non-gate

Answers: Complex Number

14.4.1 Complex Number: TIFR2011-A-13

https://gateoverflow.in/20223

✔

$$z = \frac{1}{2}\left(\sqrt{3} - i\right)$$

$$z^2 = \frac{1}{4}\left(3 - 1 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(1 - \sqrt{3}\,i\right)$$

$$z^4 = \frac{1}{4}\left(1 - 3 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 - \sqrt{3}\,i\right)$$

$$z^8 = \frac{1}{4}\left(1 - 3 + 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 + \sqrt{3}\,i\right)$$

$$z^{16} = \frac{1}{4}\left(1 - 3 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 - \sqrt{3}\,i\right) = z^4$$

$$z^{32} = z^{16} \times z^{16} = z^4 \times z^4 = z^8$$

$$z^{64} = z^{32} \times z^{32} = z^8 \times z^8 = z^{16} = z^4$$

$$z^{95} = z^{64} \times z^{16} \times z^{15}$$

$$= z^4 \times z^4 \times z^{15}$$

$$= z^{16} \times z^7$$

$$= z^4 \times z^7$$

$$= z^8 \times z^2 \times z$$

$$= \frac{1}{2}\left(-1 + \sqrt{3}\,i\right) \times \frac{1}{2}\left(1 - \sqrt{3}\,i\right) \times \frac{1}{2}\left(\sqrt{3} - i\right)$$

$$= \frac{1}{2}\left(\sqrt{3} + i\right)$$

$$i^{67} = i^{64} \times i^3$$

$$= 1 \times (-i)$$

$$= -i$$

$$z^{95} + i^{67} = \frac{1}{2}\left(\sqrt{3} + i\right) - i$$

$$= \frac{1}{2}\left(\sqrt{3} - i\right)$$

$= z$

$\left(z^{95}+i^{67}\right)^{97} = z^{97} = z^{95} \times z^2$

$= \frac{1}{2}\left(\sqrt{3}+i\right) \times \frac{1}{2}\left(1-\sqrt{3}\,i\right)$

$= \frac{1}{2}\left(\sqrt{3}-i\right)$

$= z$

Hence, option a is the correct answer.

15 votes -- Pragy Agarwal (18.3k points)

14.4.2 Complex Number: TIFR2011-A-5

https://gateoverflow.in/20009

$|x| = |y| = |z| = 1$ because points are on the unit circle.

Suppose, points x, y and z makes angles θ_1, θ_2 and θ_3 respectively from +ve x-axis in anticlockwise direction.

$\therefore x = e^{i\theta_1}, y = e^{i\theta_2}, z = e^{i\theta_3}$

$x + y + z = 0$

$\frac{x+y+z}{z} = \frac{0}{z}, z \neq 0 \Rightarrow \frac{x+y+z}{z} = 0$

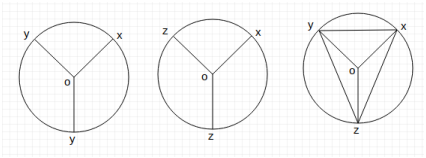
$1 + \frac{x}{z} + \frac{y}{z} = 0 \Rightarrow \frac{x}{z} + \frac{y}{z} = -1 \Rightarrow \left|\frac{x+y}{z}\right| = |-1| \Rightarrow \frac{|x+y|}{|z|} = 1 \Rightarrow |x+y| = 1$

$|e^{i\theta_1} + e^{i\theta_2}| = 1 \Rightarrow (\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_1 + \sin\theta_2)^2 = 1$

$\Rightarrow \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \frac{-1}{2} \Rightarrow \cos(\theta_1 - \theta_2) = \frac{-1}{2}$

For $\theta \in [0, 360^\circ), |\theta_1 - \theta_2| = 120^\circ \; or \; |\theta_1 - \theta_2| = 240^\circ$

So, there are 2 possibilities if we measure angle anticlockwise.



In the left figure, these 2 possibilities are shown for point y. The angle between lines ox and oy is 120° and 240°.

We do the same thing for point z, so, divide by y both sides in $x + y + z = 0$ and do the same thing as above and we get the middle figure. Now, out of 2 possibilities for each y and z, if we fix y then the other will be z, and suppose, we get the figure which is on the right side.

In the figure which is on the right side, since $ox = oy$, so angle between lines oy & yx and lines ox & yx will be same and it will be of 30° because angle between ox and oy is 120°.

If we do the same for triangles yoz and zox, we get all 3 angles $\measuredangle zyx, \measuredangle yxz, \measuredangle xzy = 60^\circ$.

Hence, Option (B)

1 votes -- ankitgupta.1729 (15.2k points)

14.4.3 Complex Number: TIFR2013-A-7

https://gateoverflow.in/25429

✔ Suppose, phases of z_1, z_2 and z_3 are θ_1, θ_2 and θ_3 respectively

and given: $|z_1| = |z_2| = |z_3| = r$

$\therefore z_1 = re^{i\theta_1}, z_2 = re^{i\theta_2}$ and $z_2 = re^{i\theta_3}$

$\arg\left(\frac{z_1}{z_2}\right) = \arg\left(\frac{re^{i\theta_1}}{re^{i\theta_2}}\right) = \arg\left(e^{i(\theta_1-\theta_2)}\right) = \theta_1 - \theta_2$

$\arg\left(\frac{z_1-z_3}{z_2-z_3}\right) = \arg\left(\frac{\frac{z_1}{z_3}-1}{\frac{z_2}{z_3}-1}\right) = \arg\left(\frac{e^{i(\theta_1-\theta_3)}-1}{e^{i(\theta_2-\theta_3)}-1}\right) = \arg\left(\frac{(\cos(\theta_1-\theta_3)-1)+i\sin(\theta_1-\theta_3)}{(\cos(\theta_2-\theta_3)-1)+i\sin(\theta_2-\theta_3)}\right)$

$= \arg\left(\frac{-2\sin^2\left(\frac{\theta_1-\theta_3}{2}\right)+i\sin(\theta_1-\theta_3)}{-2\sin^2\left(\frac{\theta_2-\theta_3}{2}\right)+i\sin(\theta_2-\theta_3)}\right)$

$= \arg(-2\sin^2(\frac{\theta_1-\theta_3}{2}) + i\sin(\theta_1-\theta_3)) - \arg(-2\sin^2(\frac{\theta_2-\theta_3}{2}) + i\sin(\theta_2-\theta_3))$

$\because -2\sin^2(\frac{\theta_{\{1,2\}}-\theta_3}{2}) < 0$ and $\sin(\theta_{\{1,2\}}-\theta_3) \geq 0$

$\therefore = \pi + \tan^{-1}\left(\frac{\sin(\theta_1-\theta_3)}{-2\sin^2(\frac{\theta_1-\theta_3}{2})}\right) - \pi - \tan^{-1}\left(\frac{\sin(\theta_2-\theta_3)}{-2\sin^2(\frac{\theta_2-\theta_3}{2})}\right)$

$= \tan^{-1}\left(\cot\frac{\theta_3-\theta_1}{2}\right) - \tan^{-1}\left(\cot\frac{\theta_3-\theta_2}{2}\right)$

$= \frac{\pi}{2} - \cot^{-1}\left(\cot\frac{\theta_3-\theta_1}{2}\right) - \frac{\pi}{2} + \cot^{-1}\left(\cot\frac{\theta_3-\theta_2}{2}\right)$

$= -(\frac{\theta_3-\theta_1}{2}) + \frac{\theta_3-\theta_2}{2} = \frac{\theta_1-\theta_2}{2}$

Therefore, $\arg(\frac{z_1}{z_2}) = (\theta_1-\theta_2)$ and $\arg(\frac{z_1-z_3}{z_2-z_3}) = \frac{(\theta_1-\theta_2)}{2}$

Answer: A

1 votes -- ankitgupta.1729 (15.2k points)

14.5 Convex Sets Functions (1)

14.5.1 Convex Sets Functions: TIFR2014-A-12

https://gateoverflow.in/26386

Let $f(x) = 2^x$. Consider the following inequality for real numbers a, b and $0 < \lambda < 1$:
$f(\lambda a + b) \leq \lambda f(a) + (1-\lambda) f(\frac{b}{1-\lambda})$.

Consider the following 3 conditions:

1. $\lambda = 0.5$
2. $0 < a \leq 2, b > 0$
3. $a/\lambda > 2, 0 < b \leq 1 - \lambda$

Which of the following statements is TRUE?

A. The above inequality holds under conditions (1) and (2) but not under condition (3).
B. The above inequality holds under conditions (2) and (3) but not under condition (1).
C. The above inequality holds under conditions (1) and (3) but not under condition (2).
D. The above inequality holds under all the three conditions.
E. The above inequality holds under none of the three conditions.

tifr2014 quantitative-aptitude convex-sets-functions non-gate

14.6 Cost Market Price (1)

14.6.1 Cost Market Price: TIFR2012-A-6

https://gateoverflow.in/21002

A certain pair of used shoes can be repaired for $Rs.1250$ and will last for 1 year. A pair of the same kind of shoes can be purchased new for $Rs.2800$ and will last for 2 years. The average cost per year of the new shoes is what percent greater than the cost of repairing the used shoes?

A. 5 B. 12 C. 15 D. 3 E. 24

tifr2012 cost-market-price

Answers: Cost Market Price

14.6.1 Cost Market Price: TIFR2012-A-6

https://gateoverflow.in/21002

✔ Repaired shoes lasts 1 year worth $Rs.1250$

New shoes lasts 2 year worth $Rs.2800$

So, for one year new shoes costs $Rs.\,1400$

% greater of new shoes cost than the cost of repaired shoes $(1400 - 1250/1250)$ = 12% (option b)

5 votes -- khush tak (5.9k points)

14.7 Factors (3)

14.7.1 Factors: TIFR2010-A-20

https://gateoverflow.in/18500

How many integers from 1 to 1000 are divisible by 30 but not by 16?

A. 29 B. 31 C. 32 D. 33 E. 25

tifr2010 quantitative-aptitude factors

14.7.2 Factors: TIFR2011-A-15

https://gateoverflow.in/20226

The exponent of 3 in the product $100!$ is

A. 27 B. 33 C. 44 D. 48 E. None of the above

tifr2011 quantitative-aptitude factors tricky

14.7.3 Factors: TIFR2013-A-12

https://gateoverflow.in/25434

Among numbers 1 to 1000 how many are divisible by 3 or 7?

A. 333 B. 142 C. 475 D. 428 E. None of the above

tifr2013 quantitative-aptitude factors normal

Answers: Factors

14.7.1 Factors: TIFR2010-A-20

https://gateoverflow.in/18500

✔ Option A. i.e., 29 should be the answer.

Number of integers between 1 to 1000 that are divisible by $30 = \lfloor \frac{1000}{30} \rfloor = 33.$

Now, since LCM of $30, 16 = 240$, only the numbers that are divisible by 240 between 1 to 1000 will divisible by both 30 & 16.
So, number of integers between 1 to 1000 that are divisible by 240 (i.e., divisible by both 30 and $16 = \lfloor \frac{1000}{240} \rfloor = 4.$

So, total number of integers that are divisible by 30 but not divisible by $16 = 33 - 4 = 29.$

15 votes -- Anurag Pandey (10.5k points)

14.7.2 Factors: TIFR2011-A-15

https://gateoverflow.in/20226

✔ Exponent of p in $n!$, where p is a prime number, and n is an integer greater than p is:

$$E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \cdots + \left\lfloor \frac{n}{p^S} \right\rfloor$$

where S is the largest positive integer such that $p^s \leq n \leq p^{s+1}$

So,

$$E_3(100!) = \left\lfloor \frac{100}{3} \right\rfloor + \left\lfloor \frac{100}{3^2} \right\rfloor + \left\lfloor \frac{100}{3^3} \right\rfloor + \left\lfloor \frac{100}{3^4} \right\rfloor + \left\lfloor \frac{100}{3^5} \right\rfloor$$

$$= 33 + 11 + 3 + 1 + 0$$
$$= 48$$

Correct Answer: D

16 votes -- sonu (1.8k points)

14.7.3 Factors: TIFR2013-A-12

https://gateoverflow.in/25434

✔ Divisible by $3 = \dfrac{1000}{3} = 333$

Divisible by $7 = \dfrac{1000}{7} = 142$

Divisible by both $= \dfrac{1000}{\text{LCM OF 3 \& 7}} = \dfrac{1000}{21} = 47$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 333 + 142 - 47 = 428.$$

Correct Answer: D

9 votes

-- Umang Raman (12.2k points)

14.8 Fractions (2)

14.8.1 Fractions: TIFR2014-A-11

https://gateoverflow.in/26329

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

A. $51 : 49$ B. $1 : 1$ C. $49 : 51$ D. $51 : 98$ E. $98 : 51$

tifr2014 quantitative-aptitude fractions tricky

14.8.2 Fractions: TIFR2017-A-1

https://gateoverflow.in/94931

A suitcase weighs one kilogram plus half of its weight. How much does the suitcase weigh?

A. $1.3333...$ kilograms
B. 1.5 kilograms
C. $1.666...$ kilograms
D. 2 kilograms
E. cannot be determined from the given data

tifr2017 quantitative-aptitude fractions normal

Answers: Fractions

14.8.1 Fractions: TIFR2014-A-11

https://gateoverflow.in/26329

✔ (A) should be the correct choice.

In the community we know that each set of parents will have exactly 1 boy.

The number of girls might differ.

To find the ratio of boys to girls in the community, we are going to find the the expected number of girls that each parent set can have.

Henceforth in this question we are going to use the word "family" to denote a "parent set".

Calculation of expected number of girls in any family

Let X be a random variable that denotes the number of girls that any family. Each family will have exactly 1 boy.

$P(X = 0)$ will denote : Probability that a family has 0 girls and 1 boy.

$P(X = 0) = (0.49)^0 (0.51)$

$P(X = 3)$ will denote : Probability that a family has 3 girls and 1 boy.

$P(X = 3) = (0.49)^3 (0.51)$

and so on.

In general we can say that

$P(X = i)$ will denote the probability that a family has i girls and 1 boy, and

$P(X = i) = (0.49)^i (0.51)$.

Now the expected number of girls in any family will be denoted by $E[X]$.

Here,

$E[X] = \sum_{i=0}^{\infty} i \cdot P(X = i)$.

but $P(X = i) = (0.49)^i (0.51)$. So, we get,

$E[X] = \sum_{i=0}^{\infty} i \cdot (0.49)^i (0.51)$.

This implies $E[X] = (0.51) \sum_{i=0}^{\infty} i \cdot (0.49)^i$.

The formula for summation of series of type $\sum_{k=0}^{\infty} k \cdot x^k$ can be found by differentiating $\sum_{k=0}^{\infty} x^k$ with respect to x.(See the reference below).

This gives

$\sum_{k=0}^{\infty} k \cdot x^k = \frac{x}{(x-1)^2}$

So, $\sum_{i=0}^{\infty} i \cdot (0.49)^i = \frac{0.49}{(0.49-1)^2}$

Hence,

$E[X] = (0.51) \cdot \frac{0.49}{(0.51)^2} = \frac{0.49}{0.51}$

Now the ratio of boys to girls can be given by number of boys in each family/expected number of girls in each family.

i.e., $Ratio\,(\text{B to G}) = \frac{1}{E[X]}$

So, $Ratio\,(\text{B to G}) = \frac{1}{\frac{0.49}{0.51}} = \frac{51}{49}$

Reference for series summation:

http://math.stackexchange.com/questions/629589/converge-of-the-sum-sum-k-1n-k-xk

References

19 votes -- Anurag Pandey (10.5k points)

14.8.2 Fractions: TIFR2017-A-1 https://gateoverflow.in/94931

✔ **D. 2 Kg**

Let x be the weight of suitcase.

A suitcase weighs one kilogram plus half of its weight. So,

$(1 + x/2 = x) \implies x = 2.$

13 votes -- Prajwal Bhat (7.6k points)

14.9 Geometry (9)

14.9.1 Geometry: TIFR2010-A-17 https://gateoverflow.in/18493

Suppose there is a sphere with diameter **at least** 6 inches. Through this sphere we drill a hole along a diameter. The part of the sphere lost in the process of drilling the hole looks like two caps joined to a cylinder, where the cylindrical part has length 6 inches. It turns out that the volume of the remaining portion of the sphere does not depend on the diameter of the sphere. Using this fact, determine the volume of the remaining part.

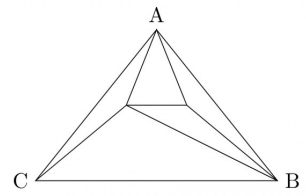
A. 24π cu. inches
B. 36π cu. inches
C. 27π cu. inches
D. 32π cu. inches
E. 35π cu. inches

tifr2010 quantitative-aptitude geometry

14.9.2 Geometry: TIFR2012-A-4 https://gateoverflow.in/20984

Let ABC be a triangle with n distinct points inside. A triangulation of ABC with respect to the n points is obtained by connecting as many points as possible, such that no more line segments can be added without intersecting other line segments. In other words ABC has been partitioned into triangles with end points at the n points or at the vertices A,B,C. For

example, the following figure gives one possible triangulation of ABC with two points inside it.



Although there are many different ways to triangulate ABC with the n points inside, the number of triangles depends only on n. In the above figure it is five. How many triangles are there in a triangulation of ABC with n points inside it?

A. $3n-1$ B. n^2+1 C. $n+3$ D. $2n+1$ E. $4n-3$

tifr2012 quantitative-aptitude geometry

14.9.3 Geometry: TIFR2012-A-5

https://gateoverflow.in/21001

What is the maximum number of points of intersection between the diagonals of a convex octagon (8-vertex planar polygon)? Note that a polygon is said to be convex if the line segment joining any two points in its interior lies wholly in the interior of the polygon. Only points of intersection between diagonals that lie in the interior of the octagon are to be considered for this problem.

A. 55 B. 60 C. 65 D. 70 E. 75

tifr2012 quantitative-aptitude geometry

14.9.4 Geometry: TIFR2013-A-5

https://gateoverflow.in/25387

The late painter Maqbool Fida Husain once coloured the surface of a huge hollow steel sphere, of radius 1 metre, using just two colours, Red and Blue. As was his style however, both the red and blue areas were a bunch of highly irregular disconnected regions. The late sculptor Ramkinkar Baij then tried to fit in a cube inside the sphere, the eight vertices of the cube touching only red coloured parts of the surface of the sphere. Assume $\pi = 3.14$ for solving this problem. Which of the following is true?

A. Baij is bound to succeed if the area of the red part is $10 sq. metres$;
B. Baij is bound to fail if the area of the red part is $10 sq. metres$;
C. Baij is bound to fail if the area of the red part is $11 sq. metres$;
D. Baij is bound to succeed if the area of the red part is $11 sq. metres$;
E. None of the above.

tifr2013 geometry quantitative-aptitude

14.9.5 Geometry: TIFR2015-A-2

https://gateoverflow.in/29158

Consider a circle with a circumference of one unit length. Let $d < \frac{1}{6}$. Suppose that we independently throw two arcs, each of length d, randomly on this circumference so that each arc is uniformly distributed along the circle circumference. The arc attaches itself exactly to the circumference so that arc of length d exactly covers length d of the circumference. What can be said about the probability that the two arcs do not intersect each other?

A. It equals $(1-d)$
B. It equals $(1-3d)$
C. It equals $(1-2d)$
D. It equals 1
E. It equals $(1-d)\ (1-d)$

tifr2015 geometry

14.9.6 Geometry: TIFR2015-A-9

https://gateoverflow.in/29575

Consider a square of side length 2. We throw five points into the square. Consider the following statements:

i. There will always be three points that lie on a straight line.
ii. There will always be a line connecting a pair of points such that two points lie on one side of the line and one point on the other.
iii. There will always be a pair of points which are at distance at most $\sqrt{2}$ from each other.

Which of the above is true:

A. (i) only B. (ii) only C. (iii) only D. (ii) and (iii) E. None of the above

tifr2015 geometry quantitative-aptitude easy

14.9.7 Geometry: TIFR2017-A-13

https://gateoverflow.in/95307

A set of points $S \subseteq \mathbb{R}^2$ is convex if for any points $x,\ y \in S$, every point on the straight line joining x and y is also in S. For two sets of points $S, T \subset \mathbb{R}^2$, define the sum $S+T$ as the set of points obtained by adding a point in S to a point T. That is, $S+T := \{(x_1,x_2) \in \mathbb{R}^2 : x_1 = y_1 + z_1,\ x_2 = y_2 + z_2,\quad (y_1,y_2) \in S, (z_1,z_2) \in T\}$. Similarly, $S-T := \{(x_1,x_2) \in \mathbb{R}^2 : x_1 = y_1 - z_1,\ x_2 = y_2 - z_2,\quad (y_1,y_2) \in S, (z_1,z_2) \in T\}$ is the set of points obtained by subtracting a point in T from a point in S. Which of the following statements is TRUE for all convex sets $S,\ T$?

A. $S+T$ is convex but not $S-T$
B. $S-T$ is convex but not $S+T$
C. exactle one of $S+T$ and $S-T$ is convex, but it depends on S and T which one
D. neither $S+T$ nor $S-T$ is convex
E. both $S+T$ and $S-T$ are convex

tifr2017 quantitative-aptitude geometry

14.9.8 Geometry: TIFR2017-A-8

https://gateoverflow.in/95039

In a tutorial on geometrical constructions, the teacher asks a student to construct a right-angled triangle ABC where the hypotenuse BC is 8 inches and the length of the perpendicular dropped from A onto the hypotenuse is h inches, and offers various choices for teh value of h. For which value of h can such a triangle NOT exist?

A. 3.90 inches
B. $2\sqrt{2}$ inches
C. $2\sqrt{3}$ inches
D. 4.1 inches
E. none of the above

tifr2017 quantitative-aptitude geometry

14.9.9 Geometry: TIFR2018-A-1

https://gateoverflow.in/179270

Consider a point A inside a circle C that is at distance 9 from the centre of a circle. Suppose you told that there is a chord of length 24 passing through A with A as its midpoint. How many distinct chords of C have integer length and pass through A?

A. 2 B. 6 C. 7 D. 12 E. 14

tifr2018 quantitative-aptitude geometry

Answers: Geometry

14.9.1 Geometry: TIFR2010-A-17

https://gateoverflow.in/18493

✔ This is the Napkin ring problem.

The volume of the remaining part is

$$V = \frac{\pi h^3}{6}$$

For $h = 6, V = 36\pi$
Hence, the answer is option (2).

References

5 votes -- admin (2.3k points)

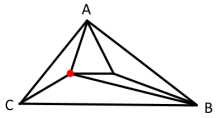
14.9.2 Geometry: TIFR2012-A-4

https://gateoverflow.in/20984

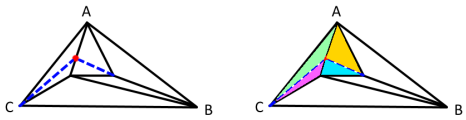
✔ Any polygon can be split into triangles, so any n–triangulate for any n will always be composed of triangles.

Given an $(n-1)$ triangulate, we can add the point in the following three ways:

I. **The point lies on a point that is already there.** In this case, the point has already been connected to all possible vertices that it can be connected to (since we started with a $(n-1)$ triangulate). Example:

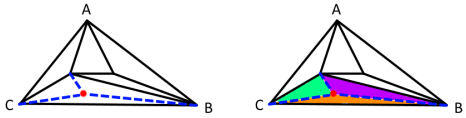


II. **The point lies on a line that is already there, but not on a point.** In this case, since the line is the common edge of at most 2 triangles, the point can only be connected to 2 vertices (the opposite ends of the triangles). For example:



III. This creates 4 new triangles, but destroys the original 2 triangles. Thus, the number of triangles increase by 2. This is an optimal case.

IV. **The point lies inside of a triangle.** The new point can then be connected to exactly 3 vertices of the bounding triangle. Example:



V. This creates 3 new triangles, but destroys the original triangle. So, the number of triangles increase by 2. So, this is also an optimal case.

We can see that the n^{th} triangulate has exactly 2 more triangles than the $(n-1)^{th}$ triangulate.
This gives us the following recurrence:

$$T(n) = T(n-1) + 2, T(2) = 5$$

Which solves to:

$$\boxed{T(n) = 2n + 1}$$

Hence, option d is the correct answer.

5 votes -- Pragy Agarwal (18.3k points)

14.9.3 Geometry: TIFR2012-A-5

https://gateoverflow.in/21001

${}^{n}c_4$ is the direct formulae to calculate no intersection made by polygon. Therefore Answer is D.

For more on this refer: http://www-math.mit.edu/~poonen/papers/ngon.pdf

References

2 votes -- Saurav Shrivastava (1.3k points)

14.9.4 Geometry: TIFR2013-A-5

https://gateoverflow.in/25387

✔ A similar problem which is used to solve the given question is given in the following link :

Question No. 9 here.

The solution is explained for the given problem by TA as :

Black colored surface : $x\%$

White colored surface : $(100-x)\% = y\% = y/100$

So, $x + y = 100$

Now, probability that a particular vertex has black color is = $\frac{x}{100}$

and probability that a particular vertex has white color is = $\frac{y}{100}$

Now, define a random variable X_i as :

$X_i = \begin{cases} 1 & if\ i^{th}\ vertex\ is\ colored\ white \\ 0 & otherwise \end{cases}$

Now, $P(X_i = 1) = \frac{y}{100}$ where $1 \leqslant i \leqslant 8$

Now, consider a new random variable $X = \sum_{i=1}^{8} X_i$.

So, Expected no. of vertices of color white = $E(X) = E(\sum_{i-1}^{8} X_i) = \frac{y}{100} * 8$ (Using linearity of expectation)

Since, all its '8' vertices are white which touches the sphere, so expected no. of vertices of color white should be strictly greater than 7

So, $\frac{8y}{100} > 7$

$\Rightarrow y > 700/8$

$\Rightarrow 100 - x > 700/8$

$\Rightarrow x > 100/8$

$\Rightarrow x < 12.5\%$

similar link.

Now, the same concept can be applicable for the given question.

Since, here the eight vertices of the cube touching only red colored parts of the surface of the sphere.

So, here also $y > \frac{700}{8}\%$

$\Rightarrow y > \frac{7}{8}$

So, It is saying that we need 7/8 of the total surface area of sphere to be red, so that all the 8 vertices of the cube touch the sphere.

Since, radius of the sphere is 1 metre.

So, $\frac{7}{8}$ of the total surface area of sphere = $\frac{7}{8} * 4\pi(1)^2 = 10.99\ metre^2$

So, area of the red part should be greater than $10.99\ metre^2$

So, I think answer should be (D).

same question is asked here

References

3 votes -- ankitgupta.1729 (15.2k points)

14.9.5 Geometry: TIFR2015-A-2

https://gateoverflow.in/29158

✔ $(1 - 2d)$ will be the correct answer.

Two points on the circumference of any circle divides the circle in two arcs, the length of smaller arc must be less than or equal to half of the circumference, & length of the larger arc must be greater than or equal to half the circumference.

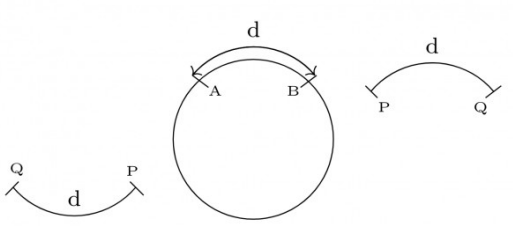
but since here given length of the arc under consideration is strictly less than $\frac{1}{2}$, so henceforth in this answer, whenever I will use the term "arc", I'll be referring to the smaller of those two arcs.

Process of Arc drawing: I am going to follow a specific procedure for drawing any arc of length d, which is as follows:

1. Pick any point on the circumference of the circle, this will be the starting point.
2. Move d units CLOCKWISE on the circumference of the circle & mark that point as the finishing point.

Suppose we choose our first arc AB of length d, randomly anywhere on the circumference on the circle.

Here A is the starting point & B is the end point.



After drawing the arc AB, we have to draw another arc PQ on the circle of length d, where P will be the starting point & Q will be the end point.

Now if we have to make sure that arc PQ does not intersects with arc AB, we have to keep following things in mind hile choosing our starting point P:

1. P can not lie within arc AB, otherwise AB and PQ will intersect each other.
2. P can not lie anywhere within the anticlockwise distance d from the point A other wise, end part of arc PQ will intersect with starting part of arc AB.

So, we can conclude that "If P lies anywhere on the circumference of the circle within a distance d from A then the arc PQ & AB will intersect."

So $Probability\,(\text{Arc PQ does not intersects with Arc AB}) = \frac{\text{P lies at least d distance away from A}}{\text{P lies anywhere in the circumference}}$

$\Rightarrow Probability\,(\text{Arc PQ does not intersects with Arc AB}) = \frac{1-2d}{1} = (1-2d)\,.$

8 votes -- Anurag Pandey (10.5k points)

14.9.6 Geometry: TIFR2015-A-9

https://gateoverflow.in/29575

✔

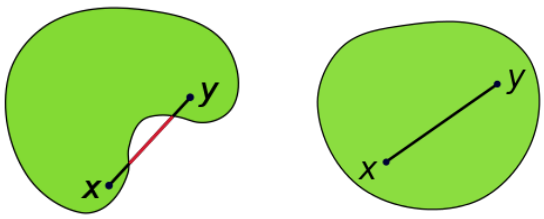
1. Not necessarily true. There can be a line connecting a pair of points such that two other points lie on one side of the line and the remaining point on the other.
2. This is also not necessarily true as even all the 5 points can be collinear (lie on same line).
3. This is TRUE.
   We are given a square of side 2 and so it takes 2×2 square units of area. This area can be divided into 4 equal squares of side 1 taking an area of 1 square units each. The maximum separation of two points in each square is the diagonal of the small square which is $\sqrt{2}$. This means we cannot put more than 2 points in each of these squares and since we have 5 points and only 4 such squares, at least one square must have more than 1 points (Pigeonhole Principle) which means there are at most $\sqrt{2}$ units apart.

2 votes -- Arjun Suresh (334k points)

14.9.7 Geometry: TIFR2017-A-13

https://gateoverflow.in/95307

Answer should be (E) Both will fill the space accordingly :
For ex :



Now since the points are set therefore the closure property would hold on union and intersection because the space will move accordingly.
Note : Take a bunch of points as set and try to perform the operation b/w those sets all points will come in linear straight line therefore it will be convex too.

3 votes -- Akshay Saxena (8.3k points)

14.9.8 Geometry: TIFR2017-A-8

https://gateoverflow.in/95039

✔ Area of the triangle is = 0.5*BC*h = 0.5*AB*AC

also, $AC^2+AB^2 = BC^2$

$AC^2+AB^2 = 64$

Since, AM>= GM

$(AC^2+AB^2)/2$ >= AC*AB

Hence, AC*AB <= 32

Therefore, area <= 16

Hence, 4*h <= 16

=> h <=4

Therefore, option (d) is correct.

6 votes -- tarun_svbk (1.4k points)

14.9.9 Geometry: TIFR2018-A-1

https://gateoverflow.in/179270

✔ Since A is the midpoint of the chord, the diameter bisects it. The diameter of the circle is 30 (Using Pythagoras Theorem). It is the shortest chord that passes through A and the longest chord is the diameter. All the integers between 24 and the diameter i.e. 30 account for 2 distinct chords. This is a consequence of Intermediate Value Theorem i.e., the length of the chord is a decreasing function of the smaller of the angles it makes with the diameter. Therefore we have the number of distinct chords as : $1 + 1 + 2 \times (30 - 24 - 1) = 12.$

Correct Option: D.

11 votes -- krish__ (4.6k points)

14.10 Logarithms (1)

14.10.1 Logarithms: TIFR2010-A-9

https://gateoverflow.in/18385

A table contains 287 entries. When any one of the entries is requested, it is encoded into a binary string and transmitted. The number of bits required is.

A. 8
B. 9
C. 10
D. Cannot be determined from the given information.
E. None of the above.

tifr2010 quantitative-aptitude theory-of-computation logarithms

Answers: Logarithms

14.10.1 Logarithms: TIFR2010-A-9

https://gateoverflow.in/18385

✔ D should be the correct answer.

Each Entry represents a row of table,in general.

In order to encode all the information of a row/entry, we must know what is the size of that information.But it is not mentioned in the question.

We can ADDRESS 287 entries by ceiling of $\log 2(287) = 9$ bits.

but we CAN NOT ENCODE an entry, without knowing the size of that entry

So I don't think it is possible to tell how many bits are required to encode & transmit an entry unless any information is given about the data contained in entries.

8 votes -- Anurag Pandey (10.5k points)

14.11 Modular Arithmetic (3)

14.11.1 Modular Arithmetic: TIFR2019-A-2

https://gateoverflow.in/280508

How many proper divisors (that is, divisors other than 1 or 7200) does 7200 have ?

A. 18 B. 20 C. 52 D. 54 E. 60

tifr2019 modular-arithmetic quantitative-aptitude

14.11.2 Modular Arithmetic: TIFR2019-A-7

https://gateoverflow.in/280503

What are the last two digits of $1! + 2! + \cdots + 100!$?

A. 00 B. 13 C. 30 D. 33 E. 73

tifr2019 modular-arithmetic quantitative-aptitude

14.11.3 Modular Arithmetic: TIFR2019-B-14

https://gateoverflow.in/280481

Let m and n be two positive integers. Which of the following is NOT always true?

A. If m and n are co-prime, there exist integers a and b such that $am + bn = 1$
B. $m^{n-1} \equiv 1 (\bmod n)$
C. The rational number $\frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \frac{n-2}{m-2} \cdots \frac{n-(m-2)}{m-(m-2)} \cdot \frac{n-(m-1)}{m-(m-1)}$ is an integer
D. $m + 1$ is a factor of $m^{n(n+1)} - 1$
E. If $2^n - 1$ is prime, then n is prime

tifr2019 general-aptitude quantitative-aptitude modular-arithmetic

Answers: Modular Arithmetic

14.11.1 Modular Arithmetic: TIFR2019-A-2

https://gateoverflow.in/280508

✔ 7200 can be written as $2^5 * 3^2 * 5^2$

Therefore, there are $(5 + 1) * (2 + 1) * (2 + 1) = 54$ divisors including 1, and 7200.

Hence, the total divisors (excluding 1 and 7200) $= 54 - 2 = 52$

Here's some justification to support the above method :

Let $n \in \mathbb{N}$. Then by fundamental theorem of arithmetic we can write $n \in \mathbb{N}, n \neq 1$ by $n = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$ where $p_1, p_2, \ldots p_k$ are prime and $a_k \in \mathbb{N}$, $k = 1, 2, \ldots, k$. Hence, number of divisors of $n =$ $(a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$.

More on this here.

Answer (**C**)

References

9 votes -- (points)

14.11.2 Modular Arithmetic: TIFR2019-A-7

https://gateoverflow.in/280503

✔ $1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120.$

After 4, factorials of all numbers end in 0.
So, **unit digit = Unit digit of (1+2+6+4) = Unit digit of 13 = 3** .

$6! = 720, 7! = \_\_40, 8! = \_\_20, 9! = \_\_80, 10! = \_\_\_800.$

After 9, factorials of all numbers end in 00.
Tens digit = Unit digit of (1+2+2+2+4+2+8) = Unit digit of 21 = 1 .

Last 2 digits = 13.
Source: https://www.quora.com/What-are-the-last-2-digits-of-1-2-3-100

References

11 votes -- Gokulnath (993 points)

14.11.3 Modular Arithmetic: TIFR2019-B-14 https://gateoverflow.in/280481

$m^{n-1} = 1(mod\ n)$ is not always true

Consider m and n are not co-primes. Then this result will not hold.

For example, $m = 9, n = 3$

$9^{3-1} \mod 3 \neq 1$

5 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

14.12 Number Representation (1)

14.12.1 Number Representation: TIFR2012-A-11 https://gateoverflow.in/25015

Let N be the sum of all numbers from 1 to 1023 except the five primes numbers: $2, 3, 11, 17, 31$. Suppose all numbers are represented using two bytes (sixteen bits). What is the value of the least significant byte (the least significant eight bits) of N?

A. 00000000 B. 10101110 C. 01000000 D. 10000000 E. 11000000

tifr2012 quantitative-aptitude number-representation

Answers: Number Representation

14.12.1 Number Representation: TIFR2012-A-11 https://gateoverflow.in/25015

✔ This is another way of saying , what will be the remainder when N is divided by $2^8 = 256$?

Here $N = 1023 \times \dfrac{1024}{2} - (2 + 3 + 11 + 17 + 31)$

$= 1023 \times 512 - 64$

$= 1022 \times 512 + (512 - 64)$

$= 1022 \times 512 + 448$

Now $448\%256 = 192 = 11000000$

So option e) is correct.

13 votes -- sudipta roy (381 points)

14.13 Number Series (5)

14.13.1 Number Series: TIFR2011-A-8 https://gateoverflow.in/20014

The sum of the first n terms of the series $1, 11, 111, 1111, \ldots,$ is.

A. $\frac{1}{81}(10^{n+1} - 9n - 10)$
B. $\frac{1}{81}(10^n - 9n)$
C. $\frac{1}{9}(10^{n+1} - 1)$
D. $\frac{1}{9}(10^{n+1} - n10^n)$
E. None of the above

tifr2011 quantitative-aptitude number-series

14.13.2 Number Series: TIFR2013-A-15 https://gateoverflow.in/25438

Let $\text{sgn}(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

What is the value of the following summation?

$$\sum_{i=0}^{50} \text{sgn}((2i-1)(2i-3)\ldots(2i-99))$$

A. 0 B. -1 C. $+1$ D. 25 E. 50

tifr2013 quantitative-aptitude number-series

14.13.3 Number Series: TIFR2013-A-8

https://gateoverflow.in/25430

Find the sum of the infinite series

$$\frac{1}{1\times 3\times 5}+\frac{1}{3\times 5\times 7}+\frac{1}{5\times 7\times 9}+\frac{1}{7\times 9\times 11}+\ldots\ldots$$

A. ∞ B. $\left(\frac{1}{2}\right)$ C. $\left(\frac{1}{6}\right)$ D. $\left(\frac{1}{12}\right)$ E. $\left(\frac{1}{14}\right)$

tifr2013 quantitative-aptitude number-series

14.13.4 Number Series: TIFR2014-A-7

https://gateoverflow.in/25992

Consider a sequence of non-negative numbers $x_n : n = 1, 2, \ldots$. Which of the following statements cannot be true?

A. $\sum_{n=1}^{\infty} x_n = \infty$ and $\sum_{n=1}^{\infty} x_n^2 = \infty$.
B. $\sum_{n=1}^{\infty} x_n = \infty$ and $\sum_{n=1}^{\infty} x_n^2 < \infty$.
C. $\sum_{n=1}^{\infty} x_n < \infty$ and $\sum_{n=1}^{\infty} x_n^2 < \infty$.
D. $\sum_{n=1}^{\infty} x_n \leq 5$ and $\sum_{n=1}^{\infty} x_n^2 \geq 25$.
E. $\sum_{n=1}^{\infty} x_n < \infty$ and $\sum_{n=1}^{\infty} x_n^2 = \infty$.

tifr2014 quantitative-aptitude number-series

14.13.5 Number Series: TIFR2015-A-3

https://gateoverflow.in/29159

Let $|z| < 1$. Define $M_n(z) = \sum_{i=1}^{10} z^{10^n(i-1)}$? what is

$$\prod_{i=0}^{\infty} M_i(z) = M_0(z) \times M_1(z) \times M_2(z) \times \ldots ?$$

A. Can't be determined B. $1/(1-z)$ C. $1/(1+z)$ D. $1-z^9$ E. None of the above

tifr2015 quantitative-aptitude numerical-computation number-series

Answers: Number Series

14.13.1 Number Series: TIFR2011-A-8

https://gateoverflow.in/20014

✔ $S = (1 + 11 + 111 + \cdots n \text{ terms})$

$= \frac{1}{9} \times (9 + 99 + 999 + \cdots n \text{ terms})$

$= \frac{1}{9}\Big(\big((10-1) + (100-1) + (1000-1) + \cdots n \text{ terms}\big)\Big)$

$= \frac{1}{9}\Big(\big((10 + 100 + \cdots 10^n) - (1 + 1 + \cdots n \text{ terms})\big)\Big)$

$= \frac{1}{9}\left(\frac{10^{n+1}-10}{10-1} - n\right)$

$= \frac{1}{9}\left(\frac{10^{n+1}-10-9n}{9}\right)$

$= \frac{10^{n+1}-9n-10}{81}$

So, the correct answer is option A.

13 votes -- Pooja Palod (24.1k points)

14.13.2 Number Series: TIFR2013-A-15

https://gateoverflow.in/25438

✔ $\sum_{i=0}^{50} sgn((2i-1)(2i-3)\ldots(2i-99))$

There are 50(even number) terms present in the product.

Now we need to open summation for $i = 0$ to 50

$i = 0, sgn((-1)(-3)\dots(-99)) = +1$

Because we know that there are 50 terms present in product which is an even number so it will be $+1$.

$i = 1, sgn((1)(-1)\dots(-97)) = -1$

Because in product there are 49 negative terms and 1 term is positive, answer will be negative.

From $i = 0$ and $i = 1$, we understood that summation is giving $+1$ for even values of i and -1 for odd values of i.

From 0 to 50 there are 26 even and 25 odd numbers are present.

So, answer will be $+1$.

1 votes -- Shubhgupta (6.5k points)

14.13.3 Number Series: TIFR2013-A-8

https://gateoverflow.in/25430

✔ We note that:

$$\begin{aligned}\frac{1}{1\times 3\times 5} &= \frac{1}{4}\cdot\left(\frac{5-1}{1\times 3\times 5}\right)\\ &= \frac{1}{4}\cdot\left(\frac{1}{1\times 3}-\frac{1}{3\times 5}\right)\end{aligned}$$

Now, we can rewrite the original series as a Telescoping series and simplify as follows:

$$\left(\frac{1}{1\times 3\times 5}+\frac{1}{3\times 5\times 7}+\frac{1}{5\times 7\times 9}+\cdots\right)$$

$$= \frac{1}{4}\cdot\left(\frac{1}{1\times 3}-\frac{1}{3\times 5}+\frac{1}{3\times 5}-\frac{1}{5\times 7}+\frac{1}{5\times 7}-\frac{1}{7\times 9}+\cdots\right)$$

$$= \frac{1}{4}\cdot\left(\frac{1}{1\times 3}-\cancel{\frac{1}{3\times 5}}+\cancel{\frac{1}{3\times 5}}-\cancel{\frac{1}{5\times 7}}+\cancel{\frac{1}{5\times 7}}-\cancel{\frac{1}{7\times 9}}+\cdots\right)$$

$$= \frac{1}{4}\cdot\left(\frac{1}{1\times 3}\right)$$

$$= \frac{1}{12}$$

Hence (D) is the Answer.

17 votes -- Leen Sharma (28.7k points)

14.13.4 Number Series: TIFR2014-A-7

https://gateoverflow.in/25992

I think (b) could be the answer, because if summation of Xn=infinite,then summation of Xn^2 cannot be less than Xn value

1 votes -- srestha (85.3k points)

14.13.5 Number Series: TIFR2015-A-3

https://gateoverflow.in/29159

✔ $M_n(z) = \sum_{i=1}^{10} z^{10^n(i-1)}$

$= z^{0*10^n} + z^{1*10^n} + z^{2*10^n} + \ldots + z^{9*10^n}$

$$= 1 + z^{1*10^n} + z^{2*10^n} + \ldots + z^{9*10^n}$$

$$= \frac{1 - (z^{10^n})^{10}}{1 - z^{10^n}}$$

$$= \frac{1 - z^{10^{(n+1)}}}{1 - z^{10^n}}$$

$$M_n(z) = \frac{1 - z^{10^{(n+1)}}}{1 - z^{10^n}}$$

$Now,$

$$\prod_{i=0}^{\infty} M_i(z) = M_0(z) \times M_1(z) \times M_2(z) \times \ldots$$

$$= \left(\frac{1 - z^{10^1}}{1 - z^{10^0}}\right) \times \left(\frac{1 - z^{10^2}}{1 - z^{10^1}}\right) \times \left(\frac{1 - z^{10^3}}{1 - z^{10^2}}\right) \times \ldots \times \left(\frac{1 - z^{10^k}}{1 - z^{10^{(k-1)}}}\right) \times \left(\frac{1 - z^{10^{(k+1)}}}{1 - z^{10^k}}\right) \times \ldots$$

$$= \frac{1}{1 - z}$$

for ending terms, as $|z| < 1, z^{\infty}$ tends to 0, $1 - z^{\infty}$ tends to 1.

Correct Answer: B

7 votes

-- Praveen Saini (42k points)

14.14 Number System (2)

14.14.1 Number System: TIFR2020-A-15

https://gateoverflow.in/333118

The sequence $s_0, s_1, \ldots, s_9$ is defined as follows:

- $s_0 = s_1 + 1$
- $2s_i = s_{i-1} + s_{i+1} + 2$ for $1 \leq i \leq 8$
- $2s_9 = s_8 + 2$

What is s_0?

A. 81 B. 95 C. 100 D. 121 E. 190

tifr2020 general-aptitude quantitative-aptitude number-system

14.14.2 Number System: TIFR2020-A-6

https://gateoverflow.in/333106

What is the maximum number of regions that the plane $\mathbb{R}^2$ can be partitioned into using 10 lines?

A. 25 B. 50 C. 55 D. 56 E. 1024

Hint: Let $A(n)$ be the maximum number of partitions that can be made by n lines. Observe that $A(0) = 1, A(2) = 2, A(2) = 4$ etc. Come up with a recurrence equation for $A(n)$.

tifr2020 general-aptitude quantitative-aptitude number-system

Answers: Number System

14.14.1 Number System: TIFR2020-A-15

https://gateoverflow.in/333118

- $s_0 = s_1 + 1 \rightarrow (1)$
- $2s_i = s_{i-1} + s_{i+1} + 2$ for $1 \leq i \leq 8 \rightarrow (2)$
- $2s_9 = s_8 + 2 \rightarrow (3)$

Now,

- $2s_8 = s_7 + s_9 + 2$
- $2s_7 = s_6 + s_8 + 2$

- $2s_6 = s_5 + s_7 + 2$
- $2s_5 = s_4 + s_6 + 2$
- $2s_4 = s_3 + s_5 + 2$
- $2s_3 = s_2 + s_4 + 2$
- $2s_2 = s_1 + s_3 + 2$
- $2s_1 = s_0 + s_2 + 2$

Adding all the terms, and we get

$2(s_8 + s_7 + s_6 + s_5 + s_4 + s_3 + s_2 + s_1) = s_7 + s_9 + 2 + s_6 + s_8 + 2 + s_5 + s_7 + 2 + s_4 + s_6 + 2 + s_3 + s_5 + 2 + s_2 +$

$\implies s_8 + s_1 = s_9 + s_0 + 16$

Given that: $2s_9 = s_8 + 2 \implies s_8 = 2s_9 - 2$

$\implies 2s_9 - 2 + s_0 - 1 = s_9 + s_0 + 16$

$\implies s_9 = 19 \rightarrow (4)$

Now, again

$2s_1 = s_0 + s_2 + 2$

From the equation (1), we get

$s_1 = s_0 - 1$

$2(s_0 - 1) = s_0 + s_2 + 2$

$\implies 2s_0 - 2 = s_0 + s_2 + 2$

$\implies s_0 = s_2 + 4$

$\implies s_2 = s_0 - 4(1 + 3)$

$2s_2 = s_1 + s_3 + 2$

$2(s_0 - 4) = s_0 - 1 + s_3 + 2$

$\implies 2s_0 - 8 = s_0 - 1 + s_3 + 2$

$\implies s_0 = s_3 + 9$

$\implies s_3 = s_0 - 9(4 + 5)$

$2s_3 = s_2 + s_4 + 2$

$2(s_0 - 9) = s_0 - 4 + s_4 + 2$

$\implies 2s_0 - 18 = s_0 - 4 + s_4 + 2$

$\implies s_0 = s_4 + 16$

$\implies s_4 = s_0 - 16(9 + 7)$

Similarly,

$\implies s_5 = s_0 - 25(16 + 9)$

$\implies s_6 = s_0 - 36(25 + 11)$

$\implies s_7 = s_0 - 49(36 + 13)$

$\implies s_8 = s_0 - 64(49 + 15)$

$\implies 2s_9 - 2 = s_0 - 64$

Put the value of s_9 from the equation (4) and we get

$s_0 = 2(19) - 2 + 64 = 36 + 64 = 100.$

So, the correct answer is (C).

0 votes -- Lakshman Patel (69.5k points)

14.14.2 Number System: TIFR2020-A-6

https://gateoverflow.in/333106

The recurrence is given by $A(n) = A(n-1) + n$. Each new n^{th} line drawn is creating n new partitions. While creating partitions,draw the new line in such a way that it cuts the all the previously drawn $n-1$ lines, then the n^{th} line will create n new partitions and previous $A(n-1)$ partitions will remain the same.

Then $A(3) = A(2) + 3 = 4 + 3 = 7$

$A(4) = A(3) + 4 = 7 + 4 = 11$

$\vdots \qquad \vdots \qquad \vdots$

$A(10) = A(9) + 10 = 46 + 10 = 56$

P.S. Try dividing $\mathbb{R}^2$ by $n = 3, n = 4, n = 5$ lines using the method above, you'll get the idea.

2 votes -- Sourajit25 (2k points)

14.15 Number Theory (1)

14.15.1 Number Theory: TIFR2014-A-20 https://gateoverflow.in/27132

Consider the equation $x^2 + y^2 - 3z^2 - 3t^2 = 0$. The total number of integral solutions of this equation in the range of the first 10000 numbers, i.e., $1 \leq x, y, z, t \leq 10000$, is

A. 200 B. 55 C. 100 D. 1 E. None of the above

tifr2014 number-theory quantitative-aptitude

Answers: Number Theory

14.15.1 Number Theory: TIFR2014-A-20 https://gateoverflow.in/27132

✔ We can use this property of squares to reach the solution

The Square of a natural number other than one is either a multiple of 3 or exceeds a multiple of 3 by 1.
In other words, a perfect square leaves remainder 0 or 1 on division by 3.

Given equation is

$x^2 + y^2 = 3(z^2 + t^2)$ This implies $x^2 + y^2$ would be a multiple of 3.

There are 4 possibilities for x^2 and y^2

1. $x^2 = 3a$ and $y^2 = 3b + 1$ so $x^2 + y^2 = 3(a + b) + 1$ won't be multiple of 3 so reject such type of x and y
2. $x^2 = 3a + 1$ and $y^2 = 3b + 1$ so $x^2 + y^2 = 3(a + b) + 2$ won't be multiple of 3 so reject such type of x and y
3. $x^2 = 3a + 1$ and $y^2 = 3b$ // same as possibility 1) so reject
4. $x^2 = 3a$ and $y^2 = 3b$ so $x^2 + y^2 = 3(a + b)$ // multiple of 3 so x and y would be of such type

Thus, here we can simplify the equation to $3(a + b) = 3(z^2 + t^2)$

$\implies a + b = z^2 + t^2$ // if we observe square of numbers then we can see that $a + b$ would also be a multiple of 3 and more clearly it would be like $3 \times p$ where p is not multiple of 3.

As the previous 4 possibilities there are again 4 possibilities for $z^2 + t^2$. Now as $a + b$ is a multiple of 3, $z^2 + t^2$ must also be a multiple of 3. So, $z^2 = 3c$ and $t^2 = 3d$.

Let $a + b = 3p$ and $3p = 3(c + d)$

so eventually $p = c + d$. According to my last claim $c + d$ must also be multiple of 3 but p won't be a multiple of three so this p can never be equal to $c + d$.

Hence No Solution. Option E

2 votes -- Rupendra Choudhary (11.4k points)

14.16 Numerical Computation (5)

14.16.1 Numerical Computation: TIFR2010-A-14 https://gateoverflow.in/18393

A marine biologist wanted to estimate the number of fish in a large lake. He threw a net and found 30 fish in the net. He marked all these fish and released them into the lake. The next morning he again threw the net and this time caught 40 fish, of which two were found to be marked. The (approximate) number of fish in the lake is:

A. 600 B. 1200 C. 68 D. 800 E. 120

tifr2010 quantitative-aptitude numerical-computation

14.16.2 Numerical Computation: TIFR2014-A-1 https://gateoverflow.in/25979

Consider the reactions

$$X + 2Y \rightarrow 3Z$$

$$2X + Z \rightarrow Y.$$

Let n_X, n_Y, n_Z denote the numbers of molecules of chemicals X, Y, Z in the reaction chamber. Then which of the following is conserved by both reactions?

A. $n_X + n_Y + n_Z$.
B. $n_X + 7n_Y + 5n_Z$.
C. $2n_X + 9n_Y - 3n_Z$.
D. $3n_X - 3n_Y + 13n_Z$.
E. None of the above.

tifr2014 quantitative-aptitude numerical-computation

14.16.3 Numerical Computation: TIFR2014-A-4

https://gateoverflow.in/25989

Consider numbers greater than one that satisfy the following properties:

i. They have no repeated prime factors;
ii. For all primes $p \geq 2$, p divides the number if and only if $p - 1$ divides the number.

The number of such numbers is

A. 0 B. 5 C. 100 D. Infinite E. None of the above

tifr2014 quantitative-aptitude difficult numerical-computation

14.16.4 Numerical Computation: TIFR2015-B-12

https://gateoverflow.in/30046

Let t_n be the sum of the first n natural numbers, for $n > 0$. A number is called triangular if it is equal to t_n for some n. Which of the following statements are true:

i. There exists three successive triangular numbers whose product is a perfect square.
ii. If the triangular number t_n is a perfect square, then so is $t_{4n(n+1)}$.
iii. The sum of the reciprocals of the first n triangular numbers is less than 2, i.e.

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \ldots + \frac{1}{t_n} < 2$$

A. (i) only
B. (ii) only
C. (iii) only
D. All of the above
E. None of the above

tifr2015 quantitative-aptitude normal numerical-computation

14.16.5 Numerical Computation: TIFR2019-A-9

https://gateoverflow.in/280501

Let A and B be two containers. Container A contains 50 litres of liquid X and container B contains 100 litres of liquid Y. Liquids X and Y are soluble in each other.

We now take 30 ml of liquid X from container A and put it into container B. The mixture in container B is then thoroughly mixed and 20 ml of the resulting mixture is put back into container A.At the end of this process let V_{AY} be the volume of liquid Y and V_{BX} be the volume of liquid X in container B. Which of the following must be TRUE ?

A. $V_{AY} < V_{BX}$
B. $V_{AY} > V_{BX}$
C. $V_{AY} = V_{BX}$
D. $V_{AY} + V_{BX} = 30$
E. $V_{AY} + V_{BX} = 20$

tifr2019 general-aptitude quantitative-aptitude numerical-computation

Answers: Numerical Computation

14.16.1 Numerical Computation: TIFR2010-A-14

https://gateoverflow.in/18393

✔ Answer should be 600, Option A.

The problem given is equivalent to the problem in which an urn contains some number of white balls in it. We take 30 balls out of it, mark them and put them back into the urn. Now, we randomly take 40 balls out of the urn, 2 of them are found to be marked. What is the approximate number of balls that were present in the urn initially?

Solution: Suppose the urn contained X balls initially. Then
if we take n ball out of urn, probably $n \times (30/X)$ balls will be marked out of n balls.
Here, $n = 40$.
So, Probably $40 * (30/X)$ out of 40 balls will be marked.

But it is given that there are 2 marked balls,
So, $2 = 40 \times (30/X)$, which gives

$X = (40 \times 30)/2 = 600.$

13 votes -- Anurag Pandey (10.5k points)

14.16.2 Numerical Computation: TIFR2014-A-1 https://gateoverflow.in/25979

✔ Basically here we need to find the number of molecules are same before and after the reaction ie. the conservation of mass.

To check that just take options and eliminate one by one. Following is the breakdown of options

In first reaction put the values of option (a) $n + 2n = 3n$ is true for first equation but if we put the values for second equation, we get $2n + n = n$ which is not possible. So, option (a) is conserving mass for equation one but not for equation two.

Similarly, eliminating all the options we can conclude option (b) is correct. As for equation one, $n + 2 * 7n = 3 * 5n$, for equation two, $2n + 5n = 7n$. So, mass is conserved for both reactions.

Correct option: B

3 votes -- curious_karan (75 points)

14.16.3 Numerical Computation: TIFR2014-A-4 https://gateoverflow.in/25989

✔ The prime factors of 30 are $2, 3, 5$, so it satisfies the 1st constraint.

$(2-1)$ divides 30 ✓
However, $(3-1)$ divides 30 ✓ and thus it doesn't satisfy the 2nd constraint.
$(5-1)$ divides 30 ×
One can prove that for n to satisfy these properties, $n = p(p-1)$ for some prime p and that $(p-1)$ satisfies the properties too.

Some examples of the numbers that satisfy both constraints are:

$$\begin{aligned} & 2 \\ 2 \times 3 = & \ 6 \\ 6 \times 7 = & \ 42 \\ 42 \times 43 = & 1806 \end{aligned}$$

Now 1807 is not a prime and hence breaks the sequence. So, number of such numbers is 4.

Correct Answer: E.

6 votes -- Pragy Agarwal (18.3k points)

14.16.4 Numerical Computation: TIFR2015-B-12 https://gateoverflow.in/30046

✔ Triangular number, $t_n = \dfrac{n(n+1)}{2}$

Product of three consecutive Triangular numbers , $t_m \times t_{m+1} \times t_{m+2}$

$$= \frac{m(m+1)}{2} \times \frac{(m+1)(m+2)}{2} \times \frac{(m+2)(m+3)}{2}$$

$$= \left(\frac{m(m+1)}{2}\right)^2 \times \left(\frac{m(m+3)}{2}\right)$$

At $m = 3$, $t_3 \times t_4 \times t_5$ is a perfect square.

(i) is True.

$$t_{4n(n+1)} = t_{4n^2+4n} = \frac{(4n^2+4n)(4n^2+4n+1)}{2}$$

$$= 4 \times (2n+1)^2 \times \frac{n(n+1)}{2} = 2^2 \times (2n+1)^2 \times t_n$$

If t_n is a perfect square, then $t_{4n(n+1)}$ is also a perfect square

(ii) is True

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n}$$

$$= \frac{2}{1.(1+1)} + \frac{2}{2.(2+1)} + \frac{2}{3.(3+1)} + \frac{2}{4.(4+1)} + \ldots + \frac{2}{n.(n+1)}$$

$$= 2 \times \left(\frac{1}{1.(1+1)} + \frac{1}{2.(2+1)} + \frac{1}{3.(3+1)} + \frac{1}{4.(4+1)} + \ldots + \frac{1}{n.(n+1)} \right)$$

$$= 2 \times \left(\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots + \frac{1}{n.(n+1)} \right)$$

$$= 2 \times \left(\left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \ldots + \left(\frac{1}{n} - \frac{1}{(n+1)} \right) \right)$$

$$= 2 \times \left(1 - \frac{1}{(n+1)} \right)$$

$$= 2 \times \frac{n}{n+1}$$

for any $n > 0$, $\frac{n}{n+1}$ will be < 1, so $2 \times \frac{n}{n+1}$ will be < 2.

So, $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n} < 2$

(iii) is also True.

Correct Answer: D

20 votes

-- Praveen Saini (42k points)

14.16.5 Numerical Computation: TIFR2019-A-9

https://gateoverflow.in/280501

✔ Volume of liquid X in container $A = 50\,l$, Volume of liquid Y in container $B = 100\,l$

$\rightarrow$ After mixing $30\,ml$ of liquid from A to B
In Container A :

- Volume of liquid $X = 50l - 30\,ml = 49970\ ml$

In Container B :

- Volume of liquid $X = 30\,ml$, Volume of liquid $Y = 100\,l$
- % of liquid $X = \frac{30}{100030}$, % of liquid $Y = \frac{100000}{100030}$

$\rightarrow$ After mixing $20\,ml$ of liquid from container B to A:
In Container A :

- Volume of $Y\,(V_{AY}) = 20 * \frac{100000}{100030} = \frac{2000000}{100030}$

In Container B :

- Volume of $X\left(V_{BX}\right) = 30 - \left(\frac{30}{100030} * 20\right) = \frac{3000300}{100030}$

So, $V_{AY} < V_{BX}$, option (A) should be correct.

4 votes -- Shobhit Joshi (3.9k points)

14.17 Polynomials (1)

14.17.1 Polynomials: TIFR2013-B-2 https://gateoverflow.in/25657

Consider polynomials in a single variable x of degree d. Suppose $d < n/2$. For such a polynomial $p(x)$, let C_p denote the n-tuple $(P(i))_{1 \leq i \leq n}$. For any two such distinct polynomials p, q, the number of coordinates where the tuples C_p, C_q differ is.

A. At most d
B. At most $n - d$
C. Between d and $n - d$
D. At least $n - d$
E. None of the above.

tifr2013 polynomials non-gate

Answers: Polynomials

14.17.1 Polynomials: TIFR2013-B-2 https://gateoverflow.in/25657

let p(x)=x and q(x)=2x-1 these are two polynomials of degree 1 suppose n=4 so n/2=2 so d<n/2 both these polynomials are satisfying these relation of degree if we put different values p(1) will be same for both the polynomials but p(2), p(3),p(4) will be different for both the polynomials hence 3 different values 1 same value hence i can say that n-d that 4-1=3 differ values. Hence option d is the answer.

0 votes -- Yash wadhwani (831 points)

14.18 Quantitative Aptitude (2)

14.18.1 Quantitative Aptitude: TIFR2011-A-20 https://gateoverflow.in/20260

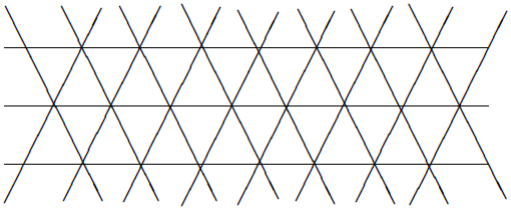
Let $n > 1$ be an odd integer. The number of zeros at the end of the number $99^n + 1$ is

A. 1 B. 2 C. 3 D. 4 E. None of the above

tifr2011 quantitative-aptitude combinatory

14.18.2 Quantitative Aptitude: TIFR2013-A-1 https://gateoverflow.in/25382

An infinite two-dimensional pattern is indicated below.



The smallest closed figure made by the lines is called a unit triangle. Within every unit triangle, there is a mouse.

At every vertex there is a laddoo. What is the average number of laddoos per mouse?

A. 3 B. 2 C. 1 D. $\left(\frac{1}{2}\right)$ E. $\left(\frac{1}{3}\right)$

tifr2013 quantitative-aptitude combinatory

Answers: Quantitative Aptitude

14.18.1 Quantitative Aptitude: TIFR2011-A-20 https://gateoverflow.in/20260

✔ For odd n,

$$99^n = (100-1)^n$$

$$= 100^n - \binom{n}{1}100^{n-1} + \quad \cdots \quad - \binom{n}{n-2}100^2 + \binom{n}{n-1}100^1 - 1$$

$$99^n + 1 = 100^n - \binom{n}{1}100^{n-1} + \quad \cdots \quad - \binom{n}{n-2}100^2 + \binom{n}{n-1}100^1 - 1 + 1$$

$$= 100\left(100^{n-1} - \binom{n}{1}100^{n-2} + \cdots - \binom{n}{n-2}100 + n\right)$$

Since n is odd, it cannot end in a 0

Thus, $99^n + 1 = 100\,(\ldots$ doesn't end with $0)$

which means $99^n + 1$ ends with exactly 2 zeros

Hence, option b) is correct.

Alternative way:

$$\begin{aligned}
99 \times 01 &= 99\\
99 \times 099 &= 9801\\
99 \times (\ldots)801 &= (\ldots)299\\
99 \times (\ldots)299 &= (\ldots)601\\
99 \times (\ldots)601 &= (\ldots)499\\
99 \times (\ldots)499 &= (\ldots)401\\
99 \times (\ldots)401 &= (\ldots)699\\
99 \times (\ldots)699 &= (\ldots)201\\
99 \times (\ldots)201 &= (\ldots)899\\
99 \times (\ldots)899 &= (\ldots)001
\end{aligned}$$

Thus, 99^n always ends in a 99 when n is odd, but never in a 999.

Hence, $99^n + 1$ will always end with exactly 2 zeros.

Note: We couldn't just say that $99^3 + 1$ ends with exactly 2 zeros, so b must be correct. This is because we also have an option e which says None of the above. Had it not been there, we could have marked b without having to prove that the pattern will continue.

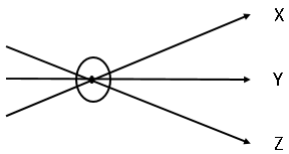
Correct Answer: B

13 votes -- Pragy Agarwal (18.3k points)

14.18.2 Quantitative Aptitude: TIFR2013-A-1

https://gateoverflow.in/25382

✔ Let the number of lines per direction be 1 as shown bellow:

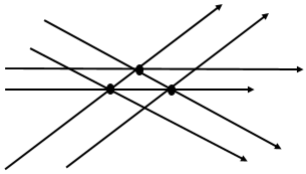


Here x, y, z depict the directions of the line.

$\eta_{laddoo} = 1$

$\eta_{mouse} = 0$

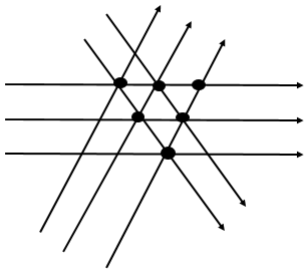
Add one more parallel line to each dimension x, y, z as shown bellow:



Encircled points represent laddoo $\Rightarrow \eta_{laddoo} = 3$

and triangle enclosed by them represent mouse $\implies \eta_{mouse} = 1$

Similarly for 3 lines in each direction



$\eta_{laddoo} = 6 \quad (1+2+3)$

$\eta_{mouse} = 4 \quad (2^2)$

As we continue we get a series which depends upon the no. of lines per direction $\left(\text{let say } l\right)$

So, $\eta_{laddoo} = \frac{l(l+1)}{2}$

$\eta_{mouse} = (l-1)^2$

$\lim\limits_{l \to \infty} \frac{\eta_{laddoo}}{\eta_{mouse}} = 1/2.$

So, $D : 1/2$ is the correct answer.

4 votes

-- Ritesh Kumar (177 points)

14.19 Ratio Proportion (2)

14.19.1 Ratio Proportion: TIFR2012-A-18

https://gateoverflow.in/25043

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

A. $51 : 49$ B. $1 : 1$ C. $49 : 51$ D. $51 : 98$ E. $98 : 51$

tifr2012 quantitative-aptitude ratio-proportion

14.19.2 Ratio Proportion: TIFR2014-A-2

https://gateoverflow.in/25987

A body at a temperature of 30 Celsius is immersed into a heat bath at 0 Celsius at time $t = 0$. The body starts cooling at a rate proportional to the temperature difference. Assuming that the heat bath does not change in temperature throughout the process, calculate the ratio of the time taken for the body to reach 1 Celsius divided by the time taken for the body to reach 5 Celsius.

A. $\log 5$ B. $\frac{\log 29}{\log 25}$ C. e^5 D. $1 + \log_6 5$ E. None of the above

tifr2014 quantitative-aptitude ratio-proportion

Answers: Ratio Proportion

14.19.1 Ratio Proportion: TIFR2012-A-18

https://gateoverflow.in/25043

✔ Let, X be the expected number of children a parent has.

So, expected number of boys $= 1$ and expected number of girls $= X - 1$.

The probability of having a baby boy $= 0.51$.

And the probability of having a baby girl $= 0.49$.

So,

$X = 1 \times (0.51) + 2 \times (0.49) \times (0.51) + 3 \times (0.49)^2 \times (0.51) + 4 \times (0.49)^3 \times (0.51)$

$0.49X = 1 \times (0.49) \times (0.51) + 2 \times (0.49)^2 \times (0.51) + 3 \times (0.49)^3 \times (0.51)$

$X - 0.49X = (0.51)[1 + (0.49) \times (0.51) + (0.49)^2 \times (0.51) + (0.49)^3 \times (0.51) + \ldots]$

$0.51X = (0.51)[1/0.51]$

$\implies X = 100/51.$

So, No of girl children $= X - 1.$

$= [100/51] - 1 = 49/51.$

No. of boy children $= 1.$

Hence, Ratio of Boys and Girls $= 51 : 49.$

[Ans - A]

10 votes -- Dhananjay Kumar Sharma (18.8k points)

14.19.2 Ratio Proportion: TIFR2014-A-2

https://gateoverflow.in/25987

✔ Let's say the Temperature of body at time t be T_t, which means $T_0 = 30$.

Now let the time at which temperature is 5 be t_1, which means $T_{t_1} = 5$

and the time at which temperature is 1 be t_2 , which means $T_{t_2} = 1$.

Question Asks us to find $\frac{t_2}{t_1}$.

Now, $\text{Temperature decrease(D) at time}t \propto \text{Body Temperature} - \text{Bath Temperature}$

$D \propto T_t$ {As Bath Temp. is 0 & Body Temp. is Tt}

$D = k \times T_t$ {where k is proportionality constant}

now, it gives $T_{t+1} = T_t - D = T_t - k \times T_t = (1-k)T_t$

Now,$T_0 = 30$

$T_1 = 30(1-k)$

$T_2 = 30(1-k)^2$

$T_{t_1} = 30(1-k)^{t_1}$ & $T_{t_2} = 30(1-k)^{t_2}$

$\Rightarrow 30(1-k)^{t_1} = 5$

$\Rightarrow t_1 \times \log(1-k) = \log\left(\frac{5}{30}\right) = \log\left(\frac{1}{6}\right)$

Similarly $\Rightarrow t_2 \times \log(1-k) = \log\left(\frac{1}{30}\right)$

$\frac{t_2}{t_1} = \frac{\log\left(\frac{1}{30}\right)}{\log\left(\frac{1}{6}\right)}$

$= \frac{\log(30-1)}{\log(6-1)} = \frac{\log(30)}{\log(6)} = \log_6 30$

$= \log_6(6 \times 5) = \log_6 6 + \log_6 5 = 1 + \log_6 5$

which is OPTION (D)

7 votes -- Himanshu Agarwal (12.4k points)

14.20 Sequence Series (1)

14.20.1 Sequence Series: TIFR2013-A-19

https://gateoverflow.in/25500

Consider a sequence of numbers $(\epsilon_n : n = 1, 2, \dots)$, such that $\epsilon_1 = 10$ and

$$\epsilon_{n+1} = \frac{20\epsilon_n}{20 + \epsilon_n}$$

for $n \geq 1$. Which of the following statements is true?

Hint: Consider the sequence of reciprocals.

A. The sequence $(\epsilon_n : n = 1, 2, \dots)$ converges to zero.
B. $\epsilon_n \geq 1$ for all n
C. The sequence $(\epsilon_n : n = 1, 2, \dots)$ is decreasing and converges to 1.
D. The sequence $(\epsilon_n : n = 1, 2, \dots)$ is decreasing and then increasing. Finally it converges to 1.
E. None of the above.

tifr2013 quantitative-aptitude sequence-series

Answers: Sequence Series

14.20.1 Sequence Series: TIFR2013-A-19

https://gateoverflow.in/25500

✔ ϵ_1 is positive.

In the formula for ϵ_{n+1}, we only add, multiply and divide positive numbers. Thus, all ϵ_n are positive.

Also, $\epsilon_{n+1} < \epsilon_n$

Proof:

$$\begin{aligned}
\epsilon_{n+1} - \epsilon_n &= \frac{20 \cdot \epsilon_n}{20 + \epsilon_n} - \epsilon_n \\
&= \frac{20 \cdot \epsilon_n - 20 \cdot \epsilon_n - (\epsilon_n)^2}{20 + \epsilon_n} \\
&= \frac{-(\epsilon_n)^2}{20 + \epsilon_n} \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\epsilon_{n+1} - \epsilon_n &< 0 \\
\epsilon_{n+1} &< \epsilon_n
\end{aligned}$$

Thus, the sequence is decreasing.

Since the sequence is decreasing and is bounded below by 0, we know that the sequence converges (Monotone Convergence Theorem).

The only fixed point of the sequence can be found as follows:

$$\epsilon_f = \frac{20 \cdot \epsilon_f}{20 + \epsilon_f}$$

$$20 \cdot \epsilon_f + (\epsilon_f)^2 = 20 \cdot \epsilon_f$$

$$(\epsilon_f)^2 = 0$$

$$\epsilon_f = 0$$

Hence, the sequence converges to 0.

Option a is correct.

2 votes

-- Pragy Agarwal (18.3k points)

14.21 Speed Time Distance (2)

14.21.1 Speed Time Distance: TIFR2012-A-16

https://gateoverflow.in/25041

Walking at $4/5$ is normal speed a man is 10 minute too late. Find his usual time in minutes.

A. 81
B. 64
C. 52
D. 40
E. It is not possible to determine the usual time from given data.

tifr2012 quantitative-aptitude speed-time-distance

14.21.2 Speed Time Distance: TIFR2017-A-3

https://gateoverflow.in/94941

On planet TIFR, the acceleration of an object due to gravity is half that on planet earth. An object on planet earth dropped from a height h takes time t to reach the ground. On planet TIFR, how much time would an object dropped from height h take to reach the ground?

A. $\left(\frac{t}{\sqrt{2}}\right)$ B. $\sqrt{2}t$ C. $2t$ D. $\left(\frac{h}{t}\right)$ E. $\left(\frac{h}{2t}\right)$

tifr2017 quantitative-aptitude speed-time-distance

Answers: Speed Time Distance

14.21.1 Speed Time Distance: TIFR2012-A-16

https://gateoverflow.in/25041

✔ Let the man travel distance d with speed s in time t.

$d = st \quad \rightarrow (1)$

According to the question we get second equation,

$d = \frac{4}{5} \times s(t + 10) \quad \rightarrow (2)$

From (1) and (2)

$\frac{5}{4}t - t = 10$

So, $t = 40$

Ans is d.

5 votes -- Pooja Palod (24.1k points)

14.21.2 Speed Time Distance: TIFR2017-A-3

https://gateoverflow.in/94941

✔ Let, The acceleration due to gravity on earth = g.

and the acceleration due to gravity on $TIFR = G = \frac{g}{2}$.

Time taken to reach the ground on earth $= t = \sqrt{\frac{2h}{g}}$.

Similarly, on TIFR planet, time taken = T = $\sqrt{\frac{2h}{G}}$.= $\sqrt{\frac{4h}{g}}$.

$\Rightarrow T = \sqrt{2}t.$

Ans - Option (b)

8 votes -- Dhananjay Kumar Sharma (18.8k points)

14.22 Statistics (1)

14.22.1 Statistics: TIFR2015-A-15

https://gateoverflow.in/29611

Let A and B be non-empty disjoint sets of real numbers. Suppose that the average of the numbers in the first set is μ_A and the average of the numbers in the second set is μ_B; let the corresponding variances be v_A and v_B respectively. If the average of the elements in $A \cup B$ is $\mu = p.\mu_A + (1-p).\mu_B$, what is the variance of the elements in $A \cup B$?

A. $p.v_A + (1-p).v_B$
B. $(1-p).v_A + p.v_B$
C. $p.[v_A + (\mu_A - \mu)^2] + (1-p).[v_B + (\mu_B - \mu)^2]$
D. $(1-p).[v_A + (\mu_A - \mu)^2] + p.[v_B + (\mu_B - \mu)^2]$
E. $p.v_A + (1-p).v_B + (\mu_A - \mu_B)^2$

tifr2015 statistics

Answers: Statistics

14.22.1 Statistics: TIFR2015-A-15

https://gateoverflow.in/29611

✔ We have $v = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i^2 - 2x_i\mu + \mu^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - 2\mu\frac{1}{n}\sum_{i=1}^{n}x_i + \mu^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - 2\mu^2 + \mu^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \mu^2$$

Thus,

- $v_A = \frac{1}{n_A}\left(\sum_{i=1}^{n_A}x_i^2\right) - \mu_A^2 \quad \rightarrow (1)$
- $v_B = \frac{1}{n_B}\left(\sum_{i=1}^{n_B}x_i^2\right) - \mu_B^2 \quad \rightarrow (2)$

$$v = \frac{1}{n}\left(\sum_{i=1}^{n}x_i^2\right) - \mu^2$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n_A}x_i^2 + \sum_{i=1}^{n_B}x_i^2\right) - \mu^2$$

$$= \frac{1}{n}\left(n_A(v_A + \mu_A^2) + n_B(v_B + \mu_B^2)\right) - \mu^2 \text{ (From (1) and (2))}$$

$$= \frac{n_A}{n}\left(v_A + \mu_A^2 - \mu^2\right) + \frac{n_B}{n}\left(v_B + \mu_B^2 - \mu^2\right) \text{ (Since, } n_A + n_B = n)$$

$$= p[v_A + \mu_A^2 - \mu^2] + (1-p)[v_B + \mu_B^2 - \mu^2] \text{ (Since, } \tfrac{n_A}{n} = p)$$

$$= p[v_A + (\mu_A - \mu)^2 + 2\mu\mu_A - 2\mu^2] + (1-p)[v_B + (\mu_B - \mu)^2 + 2\mu\mu_B - 2\mu^2]$$

$$= p[v_A + (\mu_A - \mu)^2] + (1-p)[v_B + (\mu_B - \mu)^2] + 2p\mu\mu_A + 2(1-p)\mu\mu_B - 2\mu^2$$

$$= p[v_A + (\mu_A - \mu)^2] + (1-p)[v_B + (\mu_B - \mu)^2] + 2\mu\left[p\mu_A + (1-p)\mu_B\right] - 2\mu^2$$

$$= p[v_A + (\mu_A - \mu)^2] + (1-p)[v_B + (\mu_B - \mu)^2] + 2\mu^2 - 2\mu^2$$

$$= p[v_A + (\mu_A - \mu)^2] + (1-p)[v_B + (\mu_B - \mu)^2]$$

So, Correct option C.

0 votes -- Arjun Suresh (334k points)

14.23 Three Dimensional Geometry (1)

14.23.1 Three Dimensional Geometry: TIFR2018-A-2 https://gateoverflow.in/179271

Consider the following subset of $\mathbb{R}^3$ (the first two are cylinder, the third is a plane):

- $C_1 = \{(x, y, z) : y^2 + z^2 \leq 1\}$;
- $C_2 = \{(x, y, z) : x^2 + z^2 \leq 1\}$;
- $H = \{(x, y, z) : z = 0.2\}$;

Let $A = C_1 \cap C_2 \cap H$. Which of the following best describe the shape of set A?

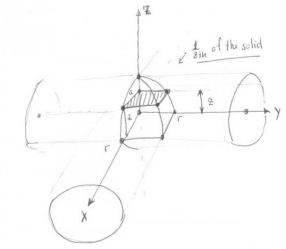
A. Circle
B. Ellipse
C. Triangle
D. Square
E. An octagonal convex figure with curved sides

tifr2018 quantitative-aptitude geometry three-dimensional-geometry non-gate

Answers: Three Dimensional Geometry

14.23.1 Three Dimensional Geometry: TIFR2018-A-2 https://gateoverflow.in/179271

The intersection of two perpendicular cylinders forms a solid called Steinmetz Solid. And the intersection with the plane $z = 2$ can be visualized as below.



The enclosed intersection is a ***square*** of sides a, where $a^2 = r^2 + z^2$ (Apply Pythagoras Theorem). And here $a = \sqrt{1 - z^2}$.

[[Image source: math.stackexchange.com]]

References

2 votes -- krish__ (4.6k points)

Answer Keys

14.0.1	E	14.1.1	C	14.1.2	B	14.1.3	D	14.2.1	C
14.3.1	E	14.3.2	E	14.3.3	B	14.4.1	A	14.4.2	B
14.4.3	A	14.5.1	D	14.6.1	B	14.7.1	A	14.7.2	D
14.7.3	D	14.8.1	A	14.8.2	D	14.9.1	B	14.9.2	D
14.9.3	D	14.9.4	D	14.9.5	C	14.9.6	C	14.9.7	E
14.9.8	D	14.9.9	D	14.10.1	D	14.11.1	C	14.11.2	B
14.11.3	B	14.12.1	E	14.13.1	A	14.13.2	C	14.13.3	D
14.13.4	E	14.13.5	B	14.14.1	C	14.14.2	D	14.15.1	E

14.16.1	A	14.16.2	B	14.16.3	E	14.16.4	D	14.16.5	A
14.17.1	D	14.18.1	B	14.18.2	D	14.19.1	A	14.19.2	D
14.20.1	A	14.21.1	D	14.21.2	B	14.22.1	C	14.23.1	D

15 General Aptitude: Verbal Aptitude (2)

Syllabus: English grammar, Sentence completion. Verbal analogies, Word groups. Instructions, Critical reasoning and Verbal deduction

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Min.	Avg.	Max.
1 Mark Count	2	2	4	3	2	3	2	4	3	2	2.7	4
2 Marks Count	1	1	1	2	0	1	1	2	2	0	1.2	2
Total Marks	4	4	6	7	2	5	4	8	7	2	5.2	8

15.1 Quantitative Aptitude (2)

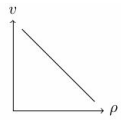
15.1.1 Quantitative Aptitude: TIFR2019-A-8

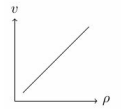
https://gateoverflow.in/280502

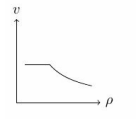
Consider the following toy model of traffic on a straight , single lane, highway. We think of cars as points, which move at the maximum speed v that satisfies the following constraints:

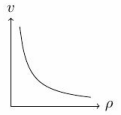
1. The speed is no more than the speed limit v_{max} mandated for the highway.
2. The speed is such that when traveling at this speed, it takes at least time t_0 (where t_0 is a fixed time representing the reaction time of drivers) to reach the car ahead, in case the car ahead stops suddenly.

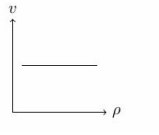
Let as assume that in the steady state, all cars on the highway move at the same speed v satisfying both the above constraints, and the distance between any two successive cars is the same. Let ρ denote the "density" , that is, the number of card per unit length of the highway. Which of the following graphs most accurately captures the relationship between the speed v and the density ρ in this model ?

A. 

B. 

C. 

D. 

E. 

tifr2019 general-aptitude quantitative-aptitude

15.1.2 Quantitative Aptitude: TIFR2020-A-9

https://gateoverflow.in/333110

A contiguous part, i.e., a set of adjacent sheets, is missing from Tharoor's GRE preparation book. The number on the first missing page is 183, and it is known that the number on the last missing page has the same three digits, but in a different order. Note that every sheet has two pages, one at the front and one at the back. How many pages are missing from Tharoor's book?

A. 45 B. 135 C. 136 D. 198 E. 450

tifr2020 general-aptitude quantitative-aptitude

Answers: Quantitative Aptitude

15.1.1 Quantitative Aptitude: TIFR2019-A-8

https://gateoverflow.in/280502

✔ As cars are moving at constant speed there is no chance of collision until all cars occupy $v * t_0$ space. So upto certain limit which is $v * t_0$ there is no effect of density of speed. After that, the system seems to follow mass flow rate equation which implies $v \propto \frac{1}{\rho}$. So, C is correct.

References

1 votes -- 2019_Aspirant (1.1k points)

15.1.2 Quantitative Aptitude: TIFR2020-A-9

https://gateoverflow.in/333110

The last missing page number is 318.Hence number of pages missing is given by $318 - 183 + 1 = 136$.

Few more points:

- Since each sheet has page numbers on front and back.Hence if we start numbering the pages from 1(front) and 2(back) of the 1^{st} sheet, then odd page numbers will be on front of every sheet and even page number in the back.As given last page of the missing sheet,hence it should be even page number(8 as last digit)
- The $+1$ for counting the number of pages(as shown above) is for counting page no. 183 as well.

2 votes -- Sourajit25 (2k points)

Answer Keys

15.1.1	C	15.1.2	C

16 Non GATE: Object Oriented Programming (1)

16.1 Object Oriented Programming (1)

16.1.1 Object Oriented Programming: TIFR2011-B-40

https://gateoverflow.in/20937

Consider the class of object oriented languages. Which of the following is true?

A. Pascal is an object oriented language.
B. Object oriented languages require heap management.
C. Object oriented languages cannot be implemented in language C.
D. Object oriented languages are more powerful than declarative programming languages.
E. Parallelism cannot be realized in object oriented languages.

tifr2011 programming object-oriented-programming non-gate

Answers: Object Oriented Programming

16.1.1 Object Oriented Programming: TIFR2011-B-40

https://gateoverflow.in/20937

ans is B .in programming language like java when object is created it cannot reside on stack so it uses heap.....
A is wrong because pascal is procedure oriented but object pascal is object oriented.
C is also wrong coz we can implement some oops concept through c.
d and e also wrong.....

3 votes -- khamer (333 points)

Answer Keys

16.1.1	B

17 Operating System (11)

System calls, Processes, Threads, Inter-process communication, Concurrency and synchronization. Deadlock. CPU scheduling. Memory management and Virtual memory. File systems. Disks is also under this

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	4	2	2	2	3	2	2	1	1	1	2.1	4
2 Marks Count	1	3	4	4	3	2	2	4	3	1	2.8	4
Total Marks	6	8	10	10	9	6	6	9	7	6	7.8	10

17.1 Page Replacement (1)

17.1.1 Page Replacement: TIFR2013-B-14

https://gateoverflow.in/25794

Assume a demand paged memory system where ONLY THREE pages can reside in the memory at a time. The following sequence gives the order in which the program references the pages.

$1, 3, 1, 3, 4, 2, 2, 4$

Assume that least frequently used page is replaced when necessary. If there is more than one least frequently used pages then the least recently used page among them is replaced. During the program's execution, how many times will the pages $1, 2, 3$ and 4 be brought to the memory?

A. $2, 2, 2, 2$ times, respectively
B. $1, 1, 1, 2$ times, respectively
C. $1, 1, 1, 1$ times, respectively
D. $2, 1, 2, 2$ times, respectively
E. None of the above

tifr2013 operating-system page-replacement

Answers: Page Replacement

17.1.1 Page Replacement: TIFR2013-B-14

https://gateoverflow.in/25794

✔ Page reference order: $1, 3, 1, 3, 4, 2, 2, 4$

First 2 pages causes page fault. I.e. 1 and 3

Next 2 pages no fault.

Next page ie 4 fault occurs.

Now for page no. 2 we have fault. We will replace less frequently used I.e. 4

Next page is again 2 so no. page fault.

Now page no. 12 and 3 all are used 2 times so, we will replace page 1 to accomodate page 4 (least recently used is 1)

So, page 123 are brought once in memory and page 4 is brought two times so, answer is (**B**).

30 votes -- Pooja Palod (24.1k points)

17.2 Process Synchronization (8)

17.2.1 Process Synchronization: TIFR2010-B-28

https://gateoverflow.in/18751

Consider the **concurrent** program:

```
x: 1;
    cobegin
        x:= x + 3 || x := x + x + 2
    coend
```

Reading and writing of variables is atomic, but the evaluation of an expression is not atomic.

The set of possible values of variable x at the end of the execution of the program is:

A. $\{4\}$ B. $\{8\}$ C. $\{4, 7, 10\}$ D. $\{4, 7, 8, 10\}$ E. $\{4, 7, 8\}$

tifr2010 process-synchronization

17.2.2 Process Synchronization: TIFR2010-B-32

https://gateoverflow.in/19244

Consider the following solution (expressed in Dijkstra's guarded command notation) to the mutual exclusion problem.

```
process P1 is
begin
    loop
        Non_critical_section;
        while not (Turn=1) do skip od;
        Critical_section_1;
        Turn:=2;
    end loop
end
```

‖

```
process P2 is
begin
    loop
        Non_critical_section;
        while not (Turn=2) do skip od;
        Critical_section_2;
        Turn:=1;
    end loop
end
```

Initially, Turn $= 1$, Assume that the two process run forever and that no process stays in its critical and non-critical section infinitely. A mutual exclusion program is correct if it satisfies the following requirements.

1. Only one process can be in a critical region at a time.
2. Program is a dead-lock free, i.e., if both processes are trying to enter the critical region then at least one of them does enter the critical region.
3. Program is starvation-free; i.e, a process trying to enter the critical region eventually manages to do so.

The above mutual exclusion solution.

A. Does not satisfy the requirement (1).
B. Satisfy the requirement (1) but does not satisfy the requirement (2).
C. Satisfies the requirements (1) and (2), but does not satisfies the requirement (3).
D. Satisfies the requirement (1) and (3), but does not satisfies the requirement (2).
E. Satisfies all the requirement (1), (2), and (3).

tifr2010 operating-system process-synchronization

17.2.3 Process Synchronization: TIFR2011-B-22

https://gateoverflow.in/20330

Consider the program

```
P:: x:=1; y:=1; z:=1; u:=0
```

And the program

```
Q:: x, y, z, u := 1, 1, 1, 1; u:= 0
```

Which of the following is true?

A. P and Q are equivalent for sequential processors.
B. P and Q are equivalent for all multi-processor models.
C. P and Q are equivalent for all multi-core machines.
D. P and Q are equivalent for all networks of computers.
E. None of the above

tifr2011 operating-system process-synchronization

17.2.4 Process Synchronization: TIFR2011-B-26

https://gateoverflow.in/20572

Consider the following two scenarios in the dining philosophers problem:

i. First a philosopher has to enter a room with the table that restricts the number of philosophers to four.
ii. There is no restriction on the number of philosophers entering the room.

Which of the following is true?

A. Deadlock is possible in (i) and (ii).
B. Deadlock is possible in (i).
C. Starvation is possible in (i).
D. Deadlock is not possible in (ii).
E. Starvation is not possible in (ii)

tifr2011 operating-system process-synchronization

17.2.5 Process Synchronization: TIFR2011-B-28

https://gateoverflow.in/20575

Consider a basic block:

```
x:= a[i]; a[j]:= y; z:= a[j]
```

optimized by removing common sub expression a[i] as follows:

```
x:= a[i]; z:= x; a[j]:= y.
```

Which of the following is true?

A. Both are equivalent.
B. The values computed by both are exactly the same.
C. Both give exactly the same values only if i is not equal to j.
D. They will be equivalent in concurrent programming languages with shared memory.
E. None of the above.

tifr2011 process-synchronization operating-system normal

17.2.6 Process Synchronization: TIFR2011-B-34

https://gateoverflow.in/20838

Consider the class of synchronization primitives. Which of the following is false?

A. Test and set primitives are as powerful as semaphores.
B. There are various synchronizations that can be implemented using an array of semaphores but not by binary semaphores.
C. Split binary semaphores and binary semaphores are equivalent.
D. All statements a - c are false.
E. Petri nets with and without inhibitor arcs have the same power.

tifr2011 operating-system process-synchronization

17.2.7 Process Synchronization: TIFR2012-B-9

https://gateoverflow.in/25109

Consider the concurrent program

```
x := 1;
cobegin
    x := x + x + 1 || x := x + 2
coend;
```

Reading and writing of a variable is atomic, but evaluation of an expression is not atomic. The set of possible values of variable x at the end of execution of the program is

A. $\{3\}$ B. $\{7\}$ C. $\{3,5,7\}$ D. $\{3,7\}$ E. $\{3,5\}$

tifr2012 process-synchronization operating-system

17.2.8 Process Synchronization: TIFR2015-B-14

https://gateoverflow.in/30077

Consider the following concurrent program (where statements separated by | | with-in cobegin-coend are executed concurrently).

```
x:=1
cobegin
   x:= x + 1 ||     x:= x + 1  ||     x:= x + 1
coend
```

Reading and writing of variables is atomic but evaluation of expressions is not atomic. The set of possible values of x at the end of execution of the program is

A. $\{4\}$
B. $\{2,3,4\}$
C. $\{2,4\}$
D. $\{2,3\}$

E. $\{2\}$

tifr2015 process-synchronization operating-system normal

Answers: Process Synchronization

17.2.1 Process Synchronization: TIFR2010-B-28

https://gateoverflow.in/18751

✔

```
cobegin
    x:= x + 3 || x := x + x + 2
coend
```

This implies that the two statements x:=x+3 and x :=x+x+2 can execute sequentially as well as parallelly..

Now,

Sequential part :

$x := x + 3 \,||\, x := x + x + 2$

$x = 1$ (initial value)

First run $x = x + 3$, value of x becomes 4

Now $x = 4$, run $x = x + x + 2$ value of x will be $4 + 4 + 2 = 10$

Finally x becomes 10.

Parallel part:

Initialized value of $x = 1$

Both will take x as 1 initially and then run independently, so

$x = x + 3 = 1 + 3 = 4$

$x = x + x + 2 = 1 + 1 + 2 = 4$

But final write will be done by the process $x = x + x + 2$

It will give value 4.

$x = x + x + 2$ completed its execution before $x = x + 3$.

Then, x= x+3 will read value of x as 4 and then $x = x + 3$ i.e. $x = 4 + 3 = 7$

So, here we get $\{4, 7\}$

Final answer is combination of both sequential and parallel part:

which is $\{4, 7, 10\}$

Option (C) .

15 votes -- Soumya Jain (12.5k points)

17.2.2 Process Synchronization: TIFR2010-B-32

https://gateoverflow.in/19244

✔ Process P1 code can be realized as

```
begin
    loop
    Non_critical_section;
    while (Turn!=1) do{
        skip od;}
        Critical_section_1;
        Turn=2;
    end loop
end
```

Process P2 code is

```
begin
    loop
    Non_critical_section;
    while (turn!=2) do{
        skip od;}
```

```
        Critical_section_2;
        Turn=1;
    end loop
end
```

Given assumption

> Assume that the two process run forever and that no process stays in its critical and non-critical section infinitely.

This means that a process will execute its non-critical section for a finite amount of time and it will execute the critical section for a finite amount of time. But will a process get to execute its critical section in the finite amount of time? let's find out.!!

Initially Turn=1.

Only one process will be in critical region at any point of time: How? : The process which gets to execute CS must have completed its while loop. For process P1, this will be true when turn=1 and for process P2 it will be true when turn =2. However, the turn variable can be only 1 or 2 at a time but not both, means one of the processes among P1 and P2 must be waiting in while loop, while another one will get to execute CS. So **Mutual Exclusion is ensured.**

Yes, **the program is deadlock free**, for the program to be in a deadlock, both the processes should have been executing in their while loop continuously which is possible only when turn=1 and turn =2 both at the same time which is impossible!!.

Now we need to check whether

> Program is **starvation-free**; i.e, a process trying to enter the critical region **eventually** manages to do so.

Focus on the word eventually in the definition of "**starvation free**". A process **eventually** manages to enter CS. After how long does it get to execute CS, it is not told but it is told it will do so. **So, starvation-freedom does not ensure bounded waiting but bounded waiting does ensure starvation freedom because a process will then "eventually" gets to execute CS (it is mentioned in question that no process stays in CS forever).**

Suppose that turn=2. Process P1 will keep waiting in while loop. Process P2 will get to execute CS and it is given in assumption that CS will be executed for a finite amount of time. Means after a "**finite amount of time**" process P2 will come out of CS and set turn=1. This will give chance to process P1. Hence the entry of P1 into CS, it bounded by 1 entry of process P2 into CS.

The same reasoning goes when turn =1.

So, **bounded waiting is being ensured**. $\rightarrow$ "**starvation free**".

Is **Progress being satisfied**?

Assume turn=2. Process P2 is not ready to enter CS. Process P1 wants to enter CS, but it will keep looping in it's while loop.

So, a process which is requesting to enter CS while no process is executing in CS is not being allowed to enter CS.

Progress is not Satisfied. Why? Process P2 is running outside it's CS is blocking process P1 to enter CS and P1 can only enter CS, once P2 has executed it's CS and set turn=1. and this is what the term progress means. **No process which wants to enter CS, should be blocked by the process which are not executing in CS**.

Final Verdict: (E)

21 votes -- Ayush Upadhyaya (28.5k points)

17.2.3 Process Synchronization: TIFR2011-B-22

https://gateoverflow.in/20330

✔ Both the programs are equivalent in the sense that the output will be the same at the end of execution. Q just writes 1 to u but this will be overwritten by the following write of 0. So, in any computer both P and Q should produce the same result at the end of execution.

Correct Answer: *D*

14 votes -- Arjun Suresh (334k points)

17.2.4 Process Synchronization: TIFR2011-B-26

https://gateoverflow.in/20572

✔

i. is a solution to Dining Philosophers problem mentioned in Galvin. Of course this assumes that the number of

forks is 5 and the number of philosophers is 4. This guarantees no deadlock, but starvation is possible as here a philosopher who finishes his meal can again get access to the forks whereas some philosopher may not get fork ever leading to his starvation. So, option (**C**) is TRUE.

ii. is the Dining Philosophers problem with no restriction on the number of philosophers which can cause deadlock and deadlock implies starvation. So, options (D) and (E) are FALSE.

19 votes -- Arjun Suresh (334k points)

17.2.5 Process Synchronization: TIFR2011-B-28 https://gateoverflow.in/20575

✔ **(E)**
As in 1^{st} $z = a[j] = y$

whereas in 2^{nd} $z = x = a[i]$

So, z is getting different values if $i \neq j$

(Also there's typo in question, expression $a[j]$ has been removed)

7 votes -- Vertika Srivastava (645 points)

17.2.6 Process Synchronization: TIFR2011-B-34 https://gateoverflow.in/20838

(a) True. A test and set primitive can be used in place of a semaphore

(b) True (Example: multiple process synchronization to a critical section access)

(c) False. A split binary semaphore is essentially an array of binary semaphores
(http://bluehawk.monmouth.edu/rclayton/web-pages/u03-598/us.html)
(d) False as (a) and (b) are true
(e) False as seen from the definition of inhibitor arc here
(https://en.wikipedia.org/wiki/Petri_net

References

6 votes -- Arjun Suresh (334k points)

17.2.7 Process Synchronization: TIFR2012-B-9 https://gateoverflow.in/25109

✔ $1. x = 1$, Run $x = x + 2$ then $x = x + x + 1$ finally x will be 7.
$2. x = 1,$ run $x = x + 2$ and $x = x + x + 1$, parallely ..
$x = 5, 3$
Final answer would be $\{3, 5, 7\}$

17 votes -- Digvijay (44.9k points)

17.2.8 Process Synchronization: TIFR2015-B-14 https://gateoverflow.in/30077

✔ Reading and writing is atomic but evaluation not atomic

So,

1. read 1, evaluate 1 time, write 2
2. read 1, evaluate 2 time, write 3
3. read 1, evaluate 3 time, write 4

Answer will be **(B)** $2, 3, 4$

13 votes -- srestha (85.3k points)

17.3 Round Robin (1)

17.3.1 Round Robin: TIFR2020-B-8 https://gateoverflow.in/333128

Jobs keep arriving at a processor. A job can have an associated time length as well as a priority tag. New jobs may arrive while some earlier jobs are running. Some jobs may keep running indefinitely. A $\textit{starvation free} job-scheduling policy guarantees that no job waits indefinitely for service. Which of the following job-scheduling policies is starvation free?

A. Round – robin
B. Shortest job first
C. Priority queuing
D. Latest job first
E. None of the others

tifr2020 operating-system process-scheduling round-robin

Answers: Round Robin

17.3.1 Round Robin: TIFR2020-B-8 https://gateoverflow.in/333128

Option A) is correct

- Shortest Job First: Starvation for Longer Jobs
- Priority Queuing: Starvation for Lower Priority Jobs
- Latest Job First: Starvation for Older Jobs
- Round Robin: No starvation

1 votes -- Ashwani Kumar (13.1k points)

17.4 Semaphores (1)

17.4.1 Semaphores: TIFR2012-B-10 https://gateoverflow.in/25110

Consider the blocked-set semaphore where the signaling process awakens any one of the suspended process; i.e.,

Wait (S): If $S > 0$ then $S \leftarrow S - 1$, else suspend the execution of this process.

Signal (S): If there are processes that have been suspended on semaphore S, then wake any one of them, else $S \leftarrow S + 1$

Consider the following solution of mutual exclusion problem using blocked-set semaphores.

```
s := 1;
cobegin
P(1)    ||     P(2)      ||     .....    ||      P(N)
coend
```

Where the task body P(i) is

```
begin
while true do
begin
 < non critical section >
 Wait (S)
<critical section>
 Signal (S)
end
end
```

Here N is the number of concurrent processors. Which of the following is true?

A. The program fails to achieve mutual exclusion of critical regions.
B. The program achieves mutual exclusion, but starvation freedom is ensured only for $N \leq 2$
C. The program does not ensure mutual exclusion if $N \geq 3$
D. The program achieves mutual exclusion, but allows starvation for any $N \geq 2$
E. The program achieves mutual exclusion and starvation freedom for any $N \geq 1$

tifr2012 operating-system process-synchronization semaphores

Answers: Semaphores

17.4.1 Semaphores: TIFR2012-B-10

https://gateoverflow.in/25110

✔ **(B) is correct option!**

Wait (S) : If $S > 0$ then $S \leftarrow S - 1$, else suspend the execution of this process.

Signal (S) : If there are processes that have been suspended on semaphore S, then wake any one of them, else $S \leftarrow S + 1$

Suppose there are only 2 processes in the system i.e., $N = 2$.

```
s :=1
cobegin
    P(1) | P(2);
coend;
```

There are **two concurrent processes** lets say P1 and P2, which are trying to to access critical section, by executing $P(1)$ and $P(2)$ respectively.

Suppose $P(1)$ gets executed first, and it makes $s = 0$. when $P(2)$ executes $Wait(S)$ then **P2 will be blocked** when $P1$ executes $Signal(S)$, there is one suspended process, hence $P2$ will be waken up and be placed into ready queue, now it can execute its critical section. **The value of S is still 0, hence ME is satisfied** .

There is only 1 process in the suspended state, hence there is no chance that it will starve because as soon as the other process performs signal operation this process will be waken up.

If $N > 2$, then there is a chance **that multiple processes are blocked**, and a random process will be unblocked when a process perform $signal(S)$ operation, there is a chance that one blocked process is not getting its chance hence **it might starve.** So, (E) is not correct option**.**

PS: A blocked queue semaphore awakens processes in the order they are suspended- hence no starvation even for $N > 2$ if used in this question.

21 votes

-- Manu Thakur (34k points)

Answer Keys

17.1.1	B	17.2.1	C	17.2.2	E	17.2.3	D	17.2.4	C
17.2.5	E	17.2.6	Q-Q	17.2.7	C	17.2.8	B	17.3.1	A
17.4.1	B								

18 Others: Others (62)

18.0.1 TIFR2021-A: 1

https://gateoverflow.in/358967

A box contains 5 red marbles, 8 green marbles, 11 blue marbles, and 15 yellow marbles. We draw marbles uniformly at random without replacement from the box. What is the minimum number of marbles to be drawn to ensure that out of the marbles drawn, at least 7 are of the same colour?

A. 7 B. 8 C. 23 D. 24 E. 39

tifr2021

18.0.2 TIFR2016-B-10

https://gateoverflow.in/98003

A *vertex cover* in an undirected graph G is a subset $C \subseteq V(G)$ such that every edge of G has an endpoint in C. An independent set in G is a subset $I \subseteq V(G)$ such that no edge has both its endpoints in I. Which of the following is TRUE of every graph G and every vertex cover C of G?

A. There exists an independent set of size $|C|$
B. $V(G) - C$ is an independent set
C. $|C| \geq |E(G)|/2$
D. $|C| \geq |V(G)|/2$
E. C intersects every independent set

tifr2016

18.0.3 TIFR2021-A: 2

https://gateoverflow.in/358966

What is the area of a rectangle with the largest perimeter that can be inscribed in the unit circle (i.e., all the vertices of the rectangle are on the circle with radius 1)?

A. 1 B. 2 C. 3 D. 4 E. 5

tifr2021

18.0.4 TIFR2021-B: 15

https://gateoverflow.in/358938

Let $A[i] : i = 0, 1, 2, \ldots, n-1$ be an array of n distinct integers. We wish to sort A in ascending order. We are given that each element in the array is at a position that is at most k away from its position in the sorted array, that is, we are given that $A[i]$ will move to a position in $\{i-k, i-k+1, \ldots, i, \ldots, i+k-1, i+k\}$ after the array is sorted in ascending order. Suppose insertion sort is used to sort this array: that is, in the i th iteration, $A[i]$ is compared with the elements in positions $A[i-1], A[i-2], \ldots$ until one that is smaller is found and $A[i]$ is inserted after that element. Note that elements can be moved back when later insertions are made before them. Let $t(n)$ be the worst-case number of comparisons made by insertion sort for such inputs. Then,

A. $t(n) = \Theta\left(n^2\right)$
B. $t(n) = \Theta\left(n \log_2 n\right)$
C. $t(n) = \Theta\left(nk \log k\right)$
D. $t(n) = \Theta\left(n \log_2 k\right)$
E. $t(n) = \Theta\left(nk\right)$

tifr2021

18.0.5 TIFR2020-B-15

https://gateoverflow.in/333135

Suppose $X_{1a}, X_{1b}, X_{2a}, X_{2b}, \ldots, X_{5a}, X_{5b}$ are ten Boolean variables each of which can take the value TRUE or FLASE. Recall the Boolean XOR $X \oplus Y := (X \wedge \neg Y) \vee (\neg X \wedge Y)$. Define the Boolean logic formulas

$$F := (X_{1a} \vee X_{1b}) \wedge (X_{2a} \vee X_{2b}) \wedge (X_{3a} \vee X_{3b}) \wedge (X_{4a} \vee X_{4b}) \wedge (X_{5a} \vee X_{5b}),$$

$$G_i := (X_{i,a} \oplus X_{i+1,a}) \vee (X_{i,b} \oplus X_{i+1,b}), \quad 1 \leq i \leq 4$$

$$G_5 := (X_{5a} \oplus X_{1a}) \vee (X_{5b} \oplus X_{1b}),$$

$$H := F \wedge G_1 \wedge G_2 \wedge G_3 \wedge G_4 \wedge G_5.$$

A truth assignment to the ten Boolean variables $X_{ia}, X_{ib},\ 1 \leq i \leq 5$ is said to be a satisfying assignment if H takes the value TRUE for example,

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \ldots, X_{5a}, X_{5b}) = (F, T, T, F, F, T, T, T, T, F)$$

is a satisfying assignment,

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \ldots, X_{5a}, X_{5b}) = (F, T, T, T, F, T, T, T, T, F)$$

is another satisfying assignment, while

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \ldots, X_{5a}, X_{5b}) = (F, T, T, F, F, T, T, F, T, F)$$

is not a satisfying assignment.

How many satisfying assignments does H have?

A. 20 B. 30 C. 32 D. 160 E. 1024

tifr2020

18.0.6 TIFR2021-B: 2

https://gateoverflow.in/358951

Let L be a singly-linked list X and Y be additional pointer variables such that X points to the first element of L and Y points to the last element of L. Which of the following operations cannot be done in time that is bound above by a constant?

A. Delete the first element of L.
B. Delete the last element of L.
C. Add an element after the last element of L.
D. Add an element before the first element of L.
E. Interchange the first two elements of L.

tifr2021

18.0.7 TIFR2016-A-9

https://gateoverflow.in/97247

Suppose a rectangular farm has area 100 square meters. The lengths of its sides are not known. It is known, however, that all the edges are at least 2 meters in length. Which of the following statements about the rectangle's perimeter p (in meters) is FALSE?

A. p can take all values between 45 and 50
B. p can be 52 for some configuration
C. p can take all values between 55 and 60
D. p can be 70 for some configuration
E. p can be 39 for some configuration

tifr2016

18.0.8 TIFR2016-A-10

https://gateoverflow.in/97370

Consider the sequence $\langle s_n : n \geq 0 \rangle$ defined as follows: $s_0 = 0, s_1 = 1, s_2 = 1$, and $s_n = s_{n-1} + s_{n-2} + s_{n-3}$, for $n \geq 3$. Which of the following statements is FALSE?

A. s_{4k} is even, for any $k \geq 0$
B. s_{4k+1} is odd, for any $k \geq 0$
C. s_{4k+2} is odd, for any $k \geq 0$
D. s_n is a multople of 3, for only finitely many values of n
E. s_{4k+3} is even, for any $k \geq 0$

tifr2016

18.0.9 TIFR2016-B-5

https://gateoverflow.in/97643

Consider the recursive function mc91.

```
int mc91(int n)
{
```

```
    print n
    if (n > 100) {
            return n-10;
    }
    else {
        return mc91(mc91(n+11));
    }
}
```

Let

$\mathsf{Out} = \{n : \text{ there is an } x \in \{0, 1, \ldots, 100\} \text{ such that } n \text{ is one of the integers printed by } \mathsf{mc91}(x)\}$

Then which of the following is Out?

A. $\{n : -\infty < n \leq 100\}$
B. $\{n : 0 \leq n \leq 101\}$
C. $\{n : 0 \leq n \leq 110\}$
D. $\{n : 0 \leq n \leq 111\}$
E. $\{n : 0 \leq n < +\infty\}$

tifr2016

18.0.10 TIFR2016-B-6

https://gateoverflow.in/97646

A subset X of $\mathbb{R}^n$ is convex if for all $x, y \in X$ and all $\lambda \in (0, 1)$, we have $\lambda x + (1 - \lambda)y \in X$. If X is a convex set, which of the following statements is necessarily TRUE?

A. For every $x \in X$ there exist $y, z \in X - \{x\}$ and $\lambda \in (0, 1)$ so that $x = \lambda y + (1 - \lambda)z$
B. If $x, y \in X$ and $\lambda \geq 0$, then $\lambda x + (1 - \lambda)y \in X$
C. If $x_1, \ldots, x_n \in X (n \geq 1)$, then $(x_1 + \cdots + x_n)/n \in X$
D. If $x \in X$, then $\lambda x \in X$ for all scalars λ
E. If $x, y \in X$, then $x - y \in X$

tifr2016

18.0.11 TIFR2020-B-13

https://gateoverflow.in/333134

Let G be an undirected graph. An Eulerian cycle of G is a cycle that traverses each edge of G exactly once. A Hamiltonian cycle of G is a cycle that traverses each vertex of G exactly once. Which of the following must be true?

A. Checking if G has a Eulerian cycle can be done in polynomial time
B. Deciding if G has a Hamiltonian cycle is not NP-complete
C. If G has an Eulerian cycle, then it has a Hamiltonian cycle
D. A complete graph always has both an Eulerian cycle and a Hamiltonian cycle
E. All of the other statements are true

tifr2020

18.0.12 TIFR2016-B-11

https://gateoverflow.in/98006

Let $n \geq 4$ be an integer. Regard the set $\mathbb{R}^n$ as a vector space over $\mathbb{R}$. Consider the following undirected graph H.

$$V(H) = \{S \subseteq \mathbb{R}^n : S \text{ is a basis for } \mathbb{R}^n\};$$

$$E(H) = \{\{S, T\} : | S \setminus T | = 1 \text{ and } | T \setminus S | = 1\},$$

where $S \setminus T = \{x \in S : x \notin T\}$. Which of the following statements is FALSE?

A. H has an inifinite number of vertices
B. The diameter of H is infinite
C. H is conneceted
D. H contains an infinite clique
E. H contains an infinite independent set

tifr2016

18.0.13 TIFR2021-A: 4

https://gateoverflow.in/358964

What is the probability that at least two out of four people have their birthdays in the same month, assuming their birthdays are uniformly distributed over the twelve months?

A. $\frac{25}{48}$
B. $\frac{5}{8}$

C. $\frac{5}{12}$

D. $\frac{41}{96}$

E. $\frac{55}{96}$

tifr2021

18.0.14 TIFR2016-B-12

https://gateoverflow.in/98010

A computer program computes a function $f\{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$. Suppose $f(a,b)$ ahs length $|b|^2$, where $|a|$ and $|b|$ are the lengths of a and b. Suppose, using this program, the following computation is performed.

```
x="01"
for i=1, ... , n do
    x=f("01", x)
```

Suppose at the end, the length of the string x is t. Which of the following is TRUE (assume $n \geq 10)?

A. $t \leq 2n$
B. $n < t \leq n^2$
C. $n^2 < t \leq n^{\log_2 n}$
D. $n^{\log_2 n} < t \leq 2^{(2n)}$
E. $2^{(2n)} < t$

tifr2016

18.0.15 TIFR2016-B-13

https://gateoverflow.in/98012

An undorected graph $G = (V, E)$ is said to be k-colourable if there exists a mapping $c : V \rightarrow \{1, 2, \ldots k\}$ such that for every edge $\{u, v\} \in E$ we have $c(u) \neq c(v)$. Which of the following statements is FALSE?

A. G is $|V|$ -colourable
B. G is 2-colourable if there are no odd cycles in G
C. G is $(\Delta + 1)$-colourable where Δ is the maximum degree in G
D. There is a polynomial time algorithm to check if G is 2-colourable
E. If G has no triangle then it is 3-colourable

tifr2016

18.0.16 TIFR2016-B-14

https://gateoverflow.in/98014

Consider a family $\mathcal{F}$ of subsets of $\{1, 2, \ldots, n\}$ such that for any two distinct sets A and B in $\mathcal{F}$ we have: $A \subset B$ or $B \subset A$ or $A \cap B = \emptyset$. Which of the following statements is TRUE? (Hint: what does the Venn diagram of this family look like?)

A. $|\mathcal{F}| \leq 2n$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2n$
B. $|\mathcal{F}| \leq n^2$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = n^2$
C. $|\mathcal{F}| \leq 2n^2$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2n^2$
D. $|\mathcal{F}| \leq 2^{n-1}$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2^{n-1}$
E. None of the above

tifr2016

18.0.17 TIFR2016-B-15

https://gateoverflow.in/98018

Let G be an undirected graph. For a pair (x, y) of distinct vertices of G, let $\mathsf{mincut}(x, y)$ be the least number of edges that should be delted from G so that the resulting graph has no $x - y$ path. Let a, b, c be three vertices in G such that $\mathsf{mincut}(a, b) \leq \mathsf{mincut}(b, c) \leq maths f mincut(c, a)$. Consider the following possibilities:

i. $\mathsf{mincut}(a, b) < \mathsf{mincut}(b, c) < \mathsf{mincut}(c, a)$
ii. $\mathsf{mincut}(a, b) = \mathsf{mincut}(b, c) < \mathsf{mincut}(c, a)$
iii. $\mathsf{mincut}(a, b) < \mathsf{mincut}(b, c) = \mathsf{mincut}(c, a)$
iv. $\mathsf{mincut}(a, b) = \mathsf{mincut}(b, c) = \mathsf{mincut}(c, a)$

Which of the following is TRUE?

A. All of i, ii iii, iv are possible

B. i, ii, iii are possible but not iv
C. i and iv are possible but neither ii nor iii
D. ii and iv are possible but neither i not iii
E. iii and iv are possible but neither i nor ii

tifr2016

18.0.18 TIFR2020-B-4

https://gateoverflow.in/333123

A clamp gate is an analog gate parametrized by two real numbers a and b, and denoted as $\text{clamp}_{a,b}$. It takes as input two non-negative real numbers x and y. Its output is defined as

$$\text{clamp}_{a,b}(x,y) = \begin{cases} ax+by & \text{when } ax+by \geq 0, \text{ and} \\ 0 & \text{when } ax+by < 0. \end{cases}$$

Consider circuits composed only of clamp gates, possibly parametrized by different pairs (a,b) of real numbers. How many clamp gates are needed to construct a circuit that on input non-negative reals x and y outputs the maximum of x and y?

A. 1
B. 2
C. 3
D. 4
E. No circuit composed only of clamp gates can compute the max function

tifr2020

18.0.19 TIFR2020-B-5

https://gateoverflow.in/333125

Let u be a point on the unit circle in the first quadrant (i.e., both coordinates of u are positive). Let θ be the angle subtended by u and the x axis at the origin. Let ℓ_u denote the infinite line passing through the origin and u. Consider the following operation O_u on points in the plane.

Operation O_u

INPUT: a point v on the plane

1. Reflect v in the x axis, obtaining $\tilde{v}$.
2. Reflect $\tilde{v}$ in ℓ_u, obtaining $\hat{v}$.
3. Output $\hat{v}$.

If $\hat{v}$ is the output of applying O_u on v, we write $O_u(v) = \hat{v}$. Further, we denote by O_u^k the iterates of O_u, i.e., $O_u^1(v) := O_u(v)$ and $O_u^k(v) := O_u(O_u^{k-1}(v))$ for all integers $k > 1$.

Consider a point v in the first quadrant such that v and the x-axis subtend an angle ϕ at the origin. Define $w = O_u^8(v)$. Assuming $\theta = 5^\circ$ and $\phi = 10^\circ$, what is the angle subtended by w and the x-axis at the origin?

A. 50° B. 85° C. 90° D. 145° E. 165°

tifr2020

18.0.20 TIFR2020-B-6

https://gateoverflow.in/333126

Consider the context-free grammar below (ϵ denotes the empty string, alphabet is $\{a,b\}$):

$$S \to \epsilon \mid aSb \mid bSa \mid SS.$$

What language does it generate?

A. $(ab)^* + (ba)^*$
B. $(abba)* + (baab)^*$
C. $(aabb)^* + (bbaa)^*$
D. Strings of the form a^nb^n or b^na^n, n any positive integer
E. Strings with equal numbers of a and b

tifr2020

18.0.21 TIFR2020-B-7

https://gateoverflow.in/333127

Consider the following algorithm (Note: For positive integers, $p, q, p/q$ denotes the floor of the rational number $\frac{p}{q}$, assume that given $p, q, p/q$ can be computed in one step):

Input: Two positive integers $a, b, a \geq b$.

Output: A positive integers g.

```
while(b>0) {
                x = a - (a/b)*b;
                a = b;
                b = x;
          }

g = a;
```

Suppose K is an upper bound on a. How many iterations does the above algorithm take in the worst case?

A. $\Theta(\log K)$ B. $\Theta(K)$ C. $\Theta(K \log K)$ D. $\Theta(K^2)$ E. $\Theta(2^K)$

tifr2020

18.0.22 TIFR2020-B-9

https://gateoverflow.in/333129

A particular Panini-Backus-Naur Form definition for a $< \text{word} >$ is given by the following rules:

- $< \text{word} > ::= < \text{letter} > | < \text{letter} > < \text{pairlet} > | < \text{letter} > < \text{pairdig} >$
- $< \text{pairlet} > ::= < \text{letter} > < \text{letter} > | < \text{pairlet} > < \text{letter} > < \text{letter} >$
- $< \text{pairdig} > ::= < \text{digit} > < \text{digit} > | < \text{pairdig} > < \text{digit} > < \text{digit} >$
- $< \text{letter} > ::= a \mid b \mid c \mid \cdots \mid y \mid z$
- $< \text{digit} > ::= 0 \mid 1 \mid 2 \mid \cdots \mid 9$

Which of the following lexical entities can be derived from $< \text{word} >$?

I. word
II. words
III. $c22$

A. None of I, II or III B. I and II only C. I and III only D. II and III only E. I, II and III

tifr2020

18.0.23 TIFR2020-B-12

https://gateoverflow.in/333133

Given the pseudocode below for the function $\text{remains}()$, which of the following statements is true about the output, if we pass it a positive integer $n > 2$?

```
int remains(int n)
{
     int x = n;
     for (i=(n-1);i>1;i--)  {
        x = x % i ;
   }
     return x;
}
```

A. Output is always 0
B. Output is always 1
C. Output is 0 only if n is NOT a prime number
D. Output is 1 only if n is a prime number
E. None of the above

tifr2020

18.0.24 TIFR2021-A: 3

https://gateoverflow.in/358965

Let M be an $n \times m$ real matrix. Consider the following:

- Let k_1 be the smallest number such that M can be factorized as $A \cdot B$, where A is an $n \times k_1$ and B is a $k_1 \times m$ matrix.
- Let k_2 be the smallest number such that $M = \sum_{i=1}^{k_2} u_i v_i$, where each u_i is an $n \times 1$ matrix and each v_i is an $1 \times m$ matrix.
- Let k_3 be the column-rank of M.

Which of the following statements is TRUE?

A. $k_1 < k_2 < k_3$
B. $k_1 < k_3 < k_2$
C. $k_2 = k_3 < k_1$
D. $k_1 = k_2 = k_3$
E. No general relationship exists among k_1, k_2 and k_3

tifr2021

18.0.25 TIFR2016-A-3

https://gateoverflow.in/96828

Consider the following set of $3n$ linear equations in $3n$ variables:

$$\begin{array}{lll} x_1 - x_2 = 0 & x_4 - x_5 = 0 & \ldots \quad x_{3n-2} - x_{3n-1} = 0 \\ x_2 - x_3 = 0 & x_5 - x_6 = 0 & \quad\quad x_{3n-1} - x_{3n} = 0 \\ x_1 - x_3 = 0 & x_4 - x_6 = 0 & \quad\quad x_{3n-2} = x_{3n} = 0 \end{array}$$

Let $S \subseteq \mathbb{R}^{3n}$ be the set of solutions to this set of equations. Then,

A. S is empty
B. S is a subspace of $\mathbb{R}^{3n}$ of dimension 1
C. S is a subspace of $\mathbb{R}^{3n}$ of dimension n
D. S is a subspace of $\mathbb{R}^{3n}$ of dimension $n-1$
E. S has exactly n elements

tifr2016

18.0.26 TIFR2021-A: 5

https://gateoverflow.in/358963

Let n, m and k be three positive integers such that $n \geq m \geq k$. Let S be a subset of $\{1, 2, \ldots, n\}$ of size k. Consider sampling a function f uniformly at random from the set of all functions mapping $\{1, \ldots, n\}$ to $\{1, \ldots, \mathrm{m}\}$. What is the probability that f is not injective on the set S, i.e., there exist $i, j \in S$ such that $f(i) = f(j)$?

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $1 - \frac{k!}{k^k}$
B. $1 - \frac{m!}{m^k}$
C. $1 - \frac{k!\binom{m}{k}}{m^k}$
D. $1 - \frac{k!\binom{n}{k}}{n^k}$
E. $1 - \frac{k!\binom{n}{k}}{m^k}$

tifr2021

18.0.27 TIFR2021-A: 13

https://gateoverflow.in/358955

What are the last two digits of 7^{2021}?

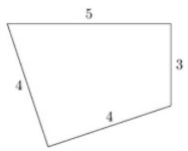
A. 67 B. 07 C. 27 D. 01 E. 77

tifr2021

18.0.28 TIFR2021-A: 15

https://gateoverflow.in/358953

Let P be a convex polygon with sides $5, 4, 4, 3$. For example, the following:



Consider the shape in the plane that consists of all points within distance 1 from some point in P. If ℓ is the perimeter of the shape, which of the following is always correct?

A. ℓ cannot be determined from the given
B. $20 \leq \ell < 21$

information.

C. $21 \leq \ell < 22$
D. $22 \leq \ell < 23$
E. $23 \leq \ell < 24$

tifr2021

18.0.29 TIFR2021-B: 5

https://gateoverflow.in/358948

For a language L over the alphabet $\{a, b\}$, let $\overline{L}$ denote the complement of L and let L^* denote the Kleene-closure of L. Consider the following sentences.

i. $\overline{L}$ and L^* are both context-free.
ii. $\overline{L}$ is not context-free but L^* is context-free.
iii. $\overline{L}$ is context-free but L^* is regular.

Which of the above sentence(s) is/are true if $L = \{a^n b^n \mid n \geq 0\}$?

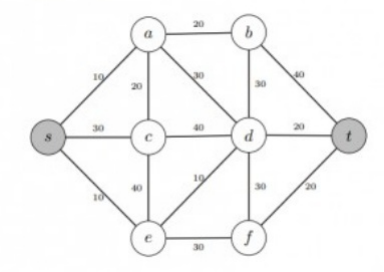
A. Both (i) and (iii) B. Only (i) C. Only (iii) D. Only (ii) E. None of the above

tifr2021

18.0.30 TIFR2020-B-14

https://gateoverflow.in/333136

The figure below describes the network of streets in a city where Motabhai sells pakoras from his cart. The number next to an edge is the time (in minutes) taken to traverse the corresponding street.



At present, the cart is required to start at point s and, after visiting each street at least once, reach point t. For example, Motabhai can visit the streets in the following order

$$s - a - c - s - e - c - d - a - b - d - f - e - d - b - t - f - d - t$$

in order to go from s to t. Note that the streets $\{b, d\}$ and $\{d, f\}$ are both visited twice in this strategy. The total time taken for this trip is 440 minutes [which is, 380 (the sum of traversal times of all streets in the network) $+60$ (the sum of the traversal times of streets $\{b, d\}$ and $\{d, f\}$)].

Motabhai now wants the cart to return to s at the end of the trip. So the previous strategy is not valid, and he must find a new strategy. How many minutes will Motabhai now take if he uses an optimal strategy?

Hint: s, t, b and f are the only odd degree nodes in the figure above.

A. 430 B. 440 C. 460 D. 470 E. 480

tifr2020

18.0.31 TIFR2021-A: 14

https://gateoverflow.in/358954

Five married couples attended a party. In the party, each person shook hands with those they did not know. Everyone knows his or her spouse. At the end of the party, Shyamal, one of the attendees, listed the number of hands that other attendees including his spouse shook. He got every number from 0 to 8 once in the list. How many persons shook hands with Shyamal at the party?

A. 2
B. 4
C. 6
D. 8
E. Insufficient information

tifr2021

18.0.32 TIFR2021-A: 6

https://gateoverflow.in/358962

A matching in a graph is a set of edges such that no two edges in the set share a common vertex. Let G be a graph on n vertices in which there is a subset M of m edges which is a matching. Consider a random process where each vertex in the graph is independently selected with probability $0 < p < 1$ and let B be the set of vertices so obtained. What is the probability that there exists at least one edge from the matching M with both end points in the set B?

A. p^2
B. $1 - \left(1 - p^2\right)^m$
C. p^{2m}
D. $\left(1 - p^2\right)^m$
E. $1 - (1 - p\,(1 - p))^m$

tifr2021

18.0.33 TIFR2021-B: 6

https://gateoverflow.in/358947

Consider the following pseudocode:

```
procedure HowManyDash(n)
 if n=0 then
  print '-'
 else if n=1 then
  print '-'
 else
  HowManyDash(n-1)
  HowManyDash(n-2)
 end if
end procedure
```

How many '-' does HowManyDash (10) print?

A. 9
B. 10
C. 55
D. 89
E. 1024

tifr2021

18.0.34 TIFR2021-B: 7

https://gateoverflow.in/358946

Which of the following regular expressions defines a language that is different from the other choices?

A. $b^*(a+b)^*a(a+b)^*ab^*(a+b)^*$
B. $a^*(a+b)^*ab^*(a+b)^*a(a+b)^*$
C. $(a+b)^*ab^*(a+b)^*a(a+b)^*b^*$
D. $(a+b)^*a(a+b)^*b^*a(a+b)^*a^*$
E. $(a+b)^*b^*a(a+b)^*b^*(a+b)^*$

tifr2021

18.0.35 TIFR2021-B: 8

https://gateoverflow.in/358945

Let A and B be two matrices of size $n \times n$ and with real-valued entries. Consider the following statements.

1. If $AB = B$, then A must be the identity matrix.
2. If A is an idempotent (i.e. $A^2 = A$) nonsingular matrix, then A must be the identity matrix.
3. If $A^{-1} = A$, then A must be the identity matrix.

Which of the above statements MUST be true of A?

A. 1, 2 and 3
B. Only 2 and 3
C. Only 1 and 2
D. Only 1 and 3
E. Only 2

tifr2021

18.0.36 TIFR2021-B: 9

https://gateoverflow.in/358944

Let L be a context-free language generated by the context-free grammar $G = (V, \Sigma, R, S)$ where V is the finite set of variables, Σ the finite set of terminals (disjoints from V), R the finite set of rules and $S \in V$ the start variable. Consider the context-free grammar G' obtained by adding $S \to SS$ to the set of rules in G. What must be true for the language L' generated by G'?

A. $L' = LL$
B. $L' = L$
C. $L' = L^*$
D. $L' = \{xx \mid x \in L\}$
E. None of the above

tifr2021

18.0.37 TIFR2021-A: 12

https://gateoverflow.in/358956

How many numbers in the range $0, 1, \ldots, 1365$ have exactly four 1's in their binary representation? (Hint: 1365_{10} is 10101010101_2, that is,

$$1365 = 2^{10} + 2^8 + 2^6 + 2^4 + 2^2 + 2^0.)$$

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $\binom{6}{4}$
B. $\binom{10}{4}$
C. $\binom{10}{4} + \binom{8}{3} + \binom{6}{2} + \binom{5}{1}$
D. $\binom{11}{4} + \binom{9}{3} + \binom{7}{2} + \binom{5}{1}$
E. 1024

tifr2021

18.0.38 TIFR2021-B: 4

https://gateoverflow.in/358949

Consider the following two languages.

$$\begin{aligned} \text{PRIME} &= \{1^n \mid n \text{ is a prime number}\}, \\ \text{FACTOR} &= \{1^n 0 1^a 0 1^b \mid n \text{ has a factor in the range } [a, b]\}. \end{aligned}$$

What can you say about the languages PRIME and FACTOR?

A. PRIME is in P, but FACTOR is not in P.
B. Neither PRIME nor FACTOR are in P.
C. Both PRIME and FACTOR are in P.
D. PRIME is not in P, but FACTOR is in P.
E. None of the above since we can answer this question only if we resolve the status of the NP vs. P question.

tifr2021

18.0.39 TIFR2021-B: 10

https://gateoverflow.in/358943

Let G be a connected bipartite simple graph (i.e., no parallel edges) with distinct edge weights. Which of the following statements on MST (minimum spanning tree) need NOT be true?

A. G hasa unique MST.
B. Every MST in G contains the lightest edge.
C. Every MST in G contains the second lightest edge.
D. Every MST in G contains the third lightest edge.
E. No MST in G contains the heaviest edge.

tifr2021

18.0.40 TIFR2021-B: 11

https://gateoverflow.in/358942

Suppose we toss a fair coin (i.e., both beads and tails have equal probability of appearing) repeatedly until the first time by which at least two heads and at least two tails have appeared in the sequence of tosses made. What is the expected number of coin tosses that we would have to make?

A. 8 B. 4 C. 5.5 D. 7.5 E. 4.5

tifr2021

18.0.41 TIFR2021-B: 12

https://gateoverflow.in/358941

Let G be an undirected graph. For any two vertices u, v in G, let $\text{cut}(u, v)$ be the minimum number of edges that should be deleted from G so that there is no path between u and v in the resulting graph. Let a, b, c, d be 4 vertices in G. Which of the following statements is impossible?

A. $\text{cut}(a,b)=3$, $\text{cut}(a,c)=2$ and $\text{cut}(a,d)=1$
B. $\text{cut}(a,b)=3$, $\text{cut}(b,c)=1$ and $\text{cut}(b,d)=1$
C. $\text{cut}(a,b)=3$, $\text{cut}(a,c)=2$ and $\text{cut}(b,c)=2$
D. $\text{cut}(a,c)=2$, $\text{cut}(b,c)=2$ and $\text{cut}(c,d)=2$
E. $\text{cut}(b,d)=2$, $\text{cut}(b,c)=2$ and $\text{cut}(c,d)=1$

tifr2021

18.0.42 TIFR2021-B: 13

https://gateoverflow.in/358940

Let A be a 3×6 matrix with real-valued entries. Matrix A has rank 3. We construct a graph with 6 vertices where each vertex represents distinct column in A, and there is an edge between two vertices if the two columns represented by the vertices are linearly independent. Which of the following statements MUST be true of the graph constructed?

A. Each vertex has degree at most 2.
B. The graph is connected.
C. There is a clique of size 3.
D. The graph has a cycle of length 4.
E. The graph is 3-colourable.

tifr2021

18.0.43 TIFR2021-B: 14

https://gateoverflow.in/358939

Consider the following greedy algorithm for colouring an n-vertex undirected graph G with colours $c_1, c_2, \cdots$: consider the vertices of G in any sequence and assign the chosen vertex the first colour that has not already been assigned to any of its neighbours. Let $m(n,r)$ be the minimum number of edges in a graph that causes this greedy algorithm to use r colours. Which of the following is correct?

A. $m\left(n,r\right)=\Theta\left(r\right)$
B. $m\left(n,r\right)=\Theta\left(r\left\lceil \log_2\ r\right\rceil\right)$
C. $m\left(n,r\right)=\binom{r}{2}$
D. $m\left(n,r\right)=nr$
E. $m\left(n,r\right)=n\binom{r}{2}$

tifr2021

18.0.44 TIFR2021-A: 11

https://gateoverflow.in/358957

Find the following sum.

$$\frac{1}{2^2-1}+\frac{1}{4^2-1}+\frac{1}{6^2-1}+\cdots+\frac{1}{40^2-1}$$

A. $\frac{20}{41}$
B. $\frac{10}{41}$
C. $\frac{10}{21}$
D. $\frac{20}{21}$
E. 1

tifr2021

18.0.45 TIFR2021-A: 10

https://gateoverflow.in/358958

Lavanya and Ketak each flip a fair coin (i.e., both heads and tails have equal probability of appearing) n times. What is the probability that Lavanya sees more heads than ketak?

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $\frac{1}{2}$
B. $\frac{1}{2}\left(1-\frac{\sum_{i=0}^{n}\binom{n}{i}^2}{2^{2n}}\right)$

C. $\frac{1}{2}\left(1-\sum_{i=0}^{n}\frac{\binom{n}{i}}{2^{2n}}\right)$

D. $\frac{1}{2}\left(1-\frac{1}{2^{2n}}\right)$

E. $\sum_{i=0}^{n}\frac{\binom{n}{i}}{2^{n}}$

tifr2021

18.0.46 TIFR2021-A: 9

https://gateoverflow.in/358959

Fix $n \geq 6$. Consider the set $\mathcal{C}$ of binary strings $x_1, x_2 \ldots x_n$ of length n such that the bits satisfy the following set of equalities, all modulo 2: $x_i + x_{i+1} + x_{i+2} = 0$ for all $1 \leq i \leq n-2, x_{n-1} + x_n + x_1 = 0,$ and $x_n + x_1 + x_2 = 0$. What is the size of the set $\mathcal{C}$?

A. 1 for all $n \geq 6$
B. 4 for all $n \geq 6$
C. 0 for all $n \geq 6$
D. If $n \geq 6$ is divisible by 3 $|\mathcal{C}| = 1$. If $n \geq 6$ is not divisible by 3 then $|\mathcal{C}| = 4$.
E. If $n \geq 6$ is divisible by 3 $|\mathcal{C}| = 4$. If $n \geq 6$ is not divisible by 3 then $|\mathcal{C}| = 1$.

tifr2021

18.0.47 TIFR2021-A: 8

https://gateoverflow.in/358960

Consider the sequence

$$y_n = \frac{1}{\int_1^n \frac{1}{(1+x/n)^3}dx}$$

for $\mathrm{n} = 2, 3, 4, \ldots$ Which of the following is TRUE?

A. The sequence $\{y_n\}$ does not have a limit as $n \to \infty$.
B. $y_n \leq 1$ for all $\mathrm{n} = 2, 3, 4, \ldots$
C. $\lim_{n\to\infty} y_n$ exists and is equal to $6/\pi^2$.
D. $\lim_{n\to\infty} y_n$ exists and is equal to 0.
E. The sequence $\{y_n\}$ first increases and then decreases as n takes values $2, 3, 4, \ldots$

tifr2021

18.0.48 TIFR2021-A: 7

https://gateoverflow.in/358961

Let d be the positive square integers (that is, it is a square of some integer) that are factors of $20^5 \times 21^5$. Which of the following is true about d?

A. $50 \leq d < 100$
B. $100 \leq d < 150$
C. $150 \leq d < 200$
D. $200 \leq d < 300$
E. $300 \leq d$

tifr2021

18.0.49 TIFR2021-B: 3

https://gateoverflow.in/358950

What is the prefix expression corresponding to the expression:
$((9+8)*7+(6*(5+4))*3)+2$?
You may assume that $*$ has precedence over $+$?

A. $*++987**6++5432$
B. $*++++\ 987**6+\ 5432$
C. $+*++\ 987**6+\ 5432$
D. $++*+\ 987**6+\ 5432$
E. $+*+*\ 987++6*\ 5432$

tifr2021

Answers:

18.0.1 TIFR2021-A: 1

https://gateoverflow.in/358967

✔ Consider the worst case scenario. We have 6 each of different colours(but note that there are only 5 red ones).

Therefore pick $5(red) + 6(green) + 6(blue) + 6(yellow) = 23$.

Now adding one more marble ensures that we have at least 7 of the same colour (By pigeonhole principle).

Threrefore, $23 + 1 = 24$

Option (D)

3 votes -- vishnu_m7 (745 points)

18.0.2 TIFR2016-B-10

https://gateoverflow.in/98003

option A: False
There exists an independent set of size $|C|$

This may not be true always as the independent set can be empty .ie, all the vertices may be included in the vertex cover(even both the end points of every edge as it is not a minimum vertex cover).

option B True

For any graph V(G)−C is an independent set

Reason: there will be no edge between the vertices of I set as per the definition of vertex cover. There is no edge with both sides belonging to independent set(if so then the subset is not vertex cover)

option C & D can be proved wrong using a complete graph with a vertex in the centre.

option E : False

Consider a graph with an isolated vertices. It falls in independent set. So intersection is empty.

1 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

18.0.3 TIFR2021-A: 2

https://gateoverflow.in/358966

The answer will be option B) 2.

Solution :

For max area, the rectangle will be a square. So, if x is the side of the square, we have

$$x^2 + x^2 = (2(1))^2$$

Here, the diagonal of the square will be equal to the diameter of the circle.

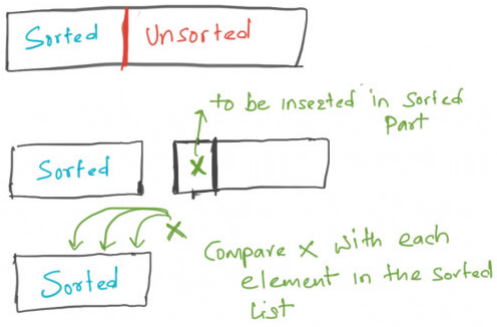
Solving this, we get $x = \sqrt{2}$ or the area of the square as 2 sq.units.

0 votes -- shashanks1999 (11 points)

18.0.4 TIFR2021-B: 15

https://gateoverflow.in/358938

Let me recall that in insertion sort we have two parts in the array. First one is where element are sorted and second one is in which elements are not sorted. In each step but we do we pick an element (starting element) from the unsorted array and put it into the right place in the sorted array part by comparisons. To insert an element into its exact position elements from the sorted part might need to be shift.



Coming to the question we are promised that each element is at most k distance from its actual position. Now when an element (Say x) will be picked from the unsorted part. To insert x into its correct position we only need to compare it with O(k) many elements and also need to move/shift at most O(k) elements (as given in the question statement that element is k distance away from its actual position). Now to insert n elements intio their right position it will take O(nk) time.

1 votes -- Shivdutt (31 points)

18.0.5 TIFR2020-B-15

https://gateoverflow.in/333135

Here, we have,

$G_1 = (X_{1a} \oplus X_{2a}) \vee (X_{1b} \oplus X_{2b})$

$G_2 = (X_{2a} \oplus X_{3a}) \vee (X_{2b} \oplus X_{3b})$

$G_3 = (X_{3a} \oplus X_{4a}) \vee (X_{3b} \oplus X_{4b})$

$G_4 = (X_{4a} \oplus X_{5a}) \vee (X_{4b} \oplus X_{5b})$

$G_5 = (X_{5a} \oplus X_{1a}) \vee (X_{5b} \oplus X_{1b})$

$F = (X_{1a} \vee X_{1b}) \wedge (X_{2a} \vee X_{2b}) \wedge (X_{3a} \vee X_{3b}) \wedge (X_{4a} \vee X_{4b}) \wedge (X_{5a} \vee X_{5b})$

$H = F \wedge G_1 \wedge G_2 \wedge G_3 \wedge G_4 \wedge G_5$

Now, we have to assign boolean variables X_{1a} to X_{5a} and X_{1b} to X_{5b} as 1 or 0 in such a way that H becomes 1

To make H as 1, F and all G_1 to G_5 should be 1. Now, either we start with G_1 or G_2 or any $G_i, 1 \leq i \leq 5$, answer will remain same.

So, first to make both G_1 and F as 1 , possibilities are :

$(i)\ X_{1a} = 1$ and $X_{1b} = 1$

$(ii)\ X_{1a} = 1$ and $X_{1b} = 0$

$(iii)\ X_{1a} = 0$ and $X_{1b} = 1$

So, case $(i)\ X_{1a} = 1$ and $X_{1b} = 1$

Now, to make $F = 1$ and $G_1 = 1 \Rightarrow\ (1 \oplus X_{2a}) \vee (1 \oplus X_{2b}) = 1$ means both $(1 \oplus X_{2a})$ and $(1 \oplus X_{2b})$ should not be 0. As we know, that $x \oplus x = 0$.. So, $X_{2a} \neq 1$ and $X_{2b} \neq 1$.

So, when $X_{1a} = 1$ and $X_{1b} = 1$,

$X_{2a} = 1, X_{2b} = 0$

$X_{2a} = 0, X_{2b} = 1$.

So, $(1,1)$ maps to either $(1,0)$ or $(0,1)$ i.e.

$(1,1) \mapsto (1,0)$ (or) $(1,1) \mapsto (0,1)$

Similarly,

To make $F = 1$ and $G_2 = 1 \Rightarrow\ (1 \oplus X_{3a}) \vee (0 \oplus X_{3b}) = 1$ (or) $(0 \oplus X_{3a}) \vee (1 \oplus X_{3b}) = 1$ means both $(1 \oplus X_{3a})$ and $(0 \oplus X_{3b})$ should not be 0 and . As we know, that $x \oplus x = 0$.. So, $X_{3a} \neq 1$ and $X_{3b} \neq 0$ (or) $X_{3a} \neq 0$ and $X_{3b} \neq 1$.

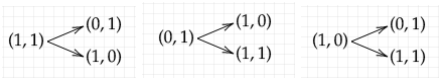
So, when $X_{2a} = 1, X_{2b} = 0$ then $X_{3a} = 0$, $X_{3b} = 1$ (or) $X_{3a} = 1$, $X_{3b} = 1$ and

when $X_{2a} = 0, X_{2b} = 1$ then $X_{3a} = 1$, $X_{3b} = 0$ (or) $X_{3a} = 1$, $X_{3b} = 1$

So, $(1,0) \mapsto (0,1)$ (or) $(1,0) \mapsto (1,1)$ and

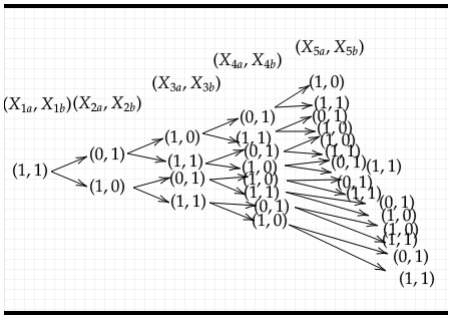
$(0,1) \mapsto (1,0)$ (or) $(0,1) \mapsto (1,1)$.

Like this we can find the values of other boolean variables very easily if we know the following mappings :



Using these mappings, we can get the possible values of other boolean variables.

So, here, (X_{ia}, Y_{ib}) represents the value of boolean variables X_{ia} and X_{ib} at level i where $1 \leq i \leq 5$

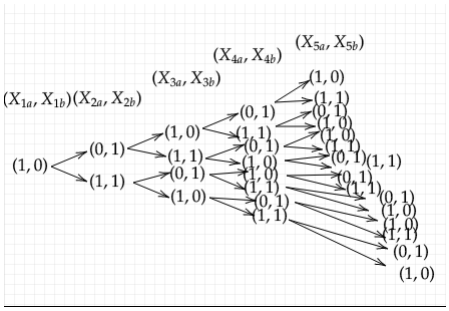


Now, G_5 will be 0 when $(X_{1a}, X_{1b}) \oplus (X_{5a}, X_{5b}) = 0$ which says those leaf nodes which have value same as $(1,1)$, G_5 will be zero. So, we have to exclude $(1,1)$ from leaf nodes. Out of 16 leaf nodes, 6 has values as $(1,1)$. So, remaining values will be $16 - 6 = 10$.

So, when $X_{1a} = 1$ and $X_{1b} = 1$, $H = 1$ for 10 assignments.

Now for case $(ii) X_{1a} = 1$ and $X_{1b} = 0$

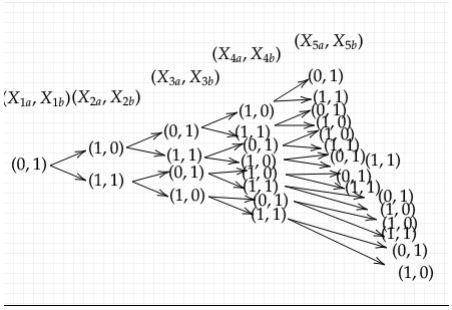
Tree structure will be like :



Here, total 6 $(1,0)$(starting node) in leaf nodes. So, total $16 - 6 = 10$ assignments for which $H = 1$

Now, for case $(iii) X_{1a} = 0$ and $X_{1b} = 1$

Tree structure will be like :



Here, also 6 $(0,1)$(starting node) in leaf nodes. So, total $16 - 6 = 10$ assignments for which $H = 1$

So, Total assignments in all 3 cases = $10 + 10 + 10 = 30$.

Now, since, structure is symmetric, so, you can start with either (X_{2a}, X_{2b}) or (X_{3a}, X_{3b}) or take any case, answer will remain the same.

2 votes -- ankitgupta.1729 (15.2k points)

18.0.6 TIFR2021-B: 2

https://gateoverflow.in/358951

(A)
$Y \rightarrow next = Y \rightarrow next \rightarrow next$
$X = Y \rightarrow next$

(C)

$temp = (\text{int *})(\text{malloc}(\text{sizeof}(\text{int})))$
$temp \rightarrow data = val$
$temp \rightarrow next = X$
$Y \rightarrow next = temp$
$Y = temp$

(D)
$temp = (\text{int *})(\text{malloc}(\text{sizeof}(\text{int})))$
$temp \rightarrow data = val$
$temp \rightarrow next = X$
$Y \rightarrow next = temp$
$X = temp$

(E)
$temp = X \rightarrow next \rightarrow data$
$X \rightarrow next \rightarrow data = X \rightarrow data$
$X \rightarrow data = temp$

Now, let's look at the Part (B)
Let's copy the data of X in Y and then delete X.

$Y \rightarrow data = X \rightarrow data$
$Y \rightarrow next = X \rightarrow next$
$X = Y$

Now we need to point Y to the previous node of the newly set X. How do we do that in constant time?
For that, we need to iterate starting from X to the last node which will take $O(n)$ time where n is the number of nodes in the list.

So, the answer should be (B)

0 votes -- JATIN MITTAL (2.2k points)

18.0.7 TIFR2016-A-9

https://gateoverflow.in/97247

perimeter cant be 39.
min possible value of length and breadth with l>2 and b>2 which satisfy area l*b = 100 is length=10 and breadth =10
with this min value perimeter = 2(l+b) = 2(10+10) = 40
min possible value of perimeter = 40
so perimeter cant be 39

2 votes -- Motamarri Anusha (8.6k points)

18.0.8 TIFR2016-A-10

https://gateoverflow.in/97370

First note that:

- Even + Odd + Odd = Even.
- Odd + Odd + Even = Even.
- Odd + Even + Even = Odd.

Now, the given series in Even Odd terms is:

E,O,O,E,E,O,O,E,...

Therefore all options except D is true. D is false since there are infinite multiples of 3 in the series.

1 votes -- Aghori (4.4k points)

18.0.9 TIFR2016-B-5

https://gateoverflow.in/97643

✔ For using the values of
$x \in \{0, 1, 2, 3, 4, \ldots\ldots\ldots\ldots, 100\}$, the n ranges
$\{n : 0 \leq n \leq 111\}$.

- Taking
  $x = 0$

```c
#include <stdio.h>
int mc91(int );
int main(void)
{
 int i = mc91(0);
 printf("%d",i);
 return 0;
}

int mc91(int n)
{
    printf("%d\n",n);
    if (n > 100)
    {
        return (n-10);
    }
    else
    {
        return (mc91(mc91(n+11)));
    }
}
```

Output :

0 // Minimum Value printed for the given range of x
11
22
33
44
55
66
77
88
99
110
100
111 // Maximum Value printed for the given range of x
101
91
102
92
103
93
104
94
105
95
106
96
107
97
108
98
109
99
110
100
111
101 // Maximum value returned for the given range of x

.....

.....

.....

- Taking
  $x = 100$

```c
#include <stdio.h>
int mc91(int );

int main(void)
{
 int i = mc91(100);
 printf("%d",i);
 return 0;
}

int mc91(int n)
{
    printf("%d\n",n);
    if (n > 100)
    {
        return (n-10);
    }
    else
    {
        return (mc91(mc91(n+11)));
    }
}
```

Output :

100
111 // Maximum Value printed for the given range of x
101 // Maximum value returned for the given range of x
91

3 votes -- Kapil Phulwani (35.2k points)

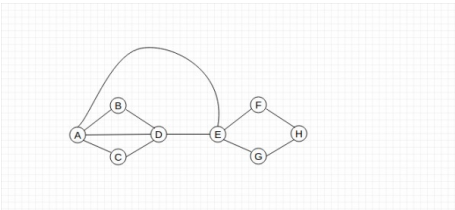
18.0.11 TIFR2020-B-13 https://gateoverflow.in/333134

(a) **True.** To check whether a given graph has Eulerian cycle or not, can be done in **linear time** in terms of input of the graph. So, we can check it in polynomial time. We can store the given undirected graph G in adjacency list and first using DFS, check whether the graph is connected or not. If the graph is connected then check whether each vertices has **even degree** or not in the adjacency list representation of the graph and if the graph is not connected then do the same thing for each component of the graph. It takes $\mathcal{O}(|V| + |E|)$ time.

(b) **False.** To check whether the graph G has Hamiltonian cycle or not, is an NPC problem.

(c) **False.**

Counter-Example :



Eulerian Cycle : **ABDACDEFHGEA**

But it does not have any Hamiltonian cycle.

(d) **False.**

Complete graph K_4 does not have a Eulerian cycle because each vertex has degree 3 which is not even.

But every complete graph always have a Hamiltonian cycle because there exist a path from one vertex to every other vertices.

References

0 votes -- ankitgupta.1729 (15.2k points)

18.0.12 TIFR2016-B-11

https://gateoverflow.in/98006

Option (B)

I would like to strongly recommend to watch this video of legendary 3b1b for understanding what are basis vectors.

Basis vectors are linearly independent vectors that can span full space.

Lets first understand what this graph is about.

Vertices of this graph are set of basis vectors. (Note: In nth dimensional vector space there are n linearly independent basis vectors).

So each vertex will be of the form S={v1,v2,v3,..vn} where S is set of basis vectors.

and here Vi = [x1,x2,x3,x4 , xn] can be any n dimensional vector.

Edge between any two vectors is present if only if there is exactly one basis vector different among two vertices.

Now we can see for the given vertex there can be infinite adjacent vertices cause we have infinite options for replacing one of the basis vectors. Hence (a) is true.

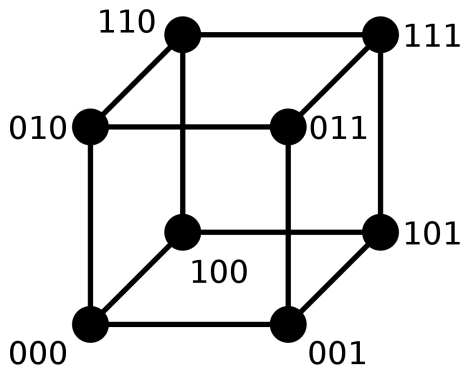
Graph will be connected cause suppose even if we have two vertices Vi not equals Vj such that each basis vector among two of them is different , even then we can reach to other vertex by replacing **one of the basic vector at time** (See the condition of Adjacency) and eventually we will reach our desired vertex in at max **n** steps (changing each basis vertex at time).

Hence the diameter of the graph will be finite and **n.**

Graph has infinite vertices and there will infinite cliques of size 3 , for example Vi , Vj , Vk and each of them has exactly one basis vector different.

Similarly, we can show it has infinite independent set by considering all the vertices with two or more different basis vectors cause such vertices are not connected to each other.

We can relate this graph with the graph of hamming distances even there, an edge is present between two vertices when only one bit is swapped (here in this case : **one basis vector is changed**).



References

1 votes

-- Tushar Kadam (387 points)

18.0.13 TIFR2021-A: 4

https://gateoverflow.in/358964

Answer (D)

$P(\text{Atleast two have same birthday month}) = 1 - P(\text{No one have birthday in the same month})$
$= 1 - \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{10}{12} \cdot \frac{9}{12}$

$= \frac{41}{96}$

0 votes -- JATIN MITTAL (2.2k points)

18.0.14 TIFR2016-B-12

https://gateoverflow.in/98010

Initially x is of 2 bits.

In next iteration of loop x has size 4

lly, after n iterations(at the end of loop) the size of x is $2^{(2*2*2*2*.....n\ times)}$

ie, $t = 2^{2^n}$

option A, B, C is wrong as they are small compared to t.

option E is wrong as it exceeds the value of t.

So, answer is option D

0 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

18.0.15 TIFR2016-B-13

https://gateoverflow.in/98012

Option E

https://gateoverflow.in/30708/graph-coloring

References

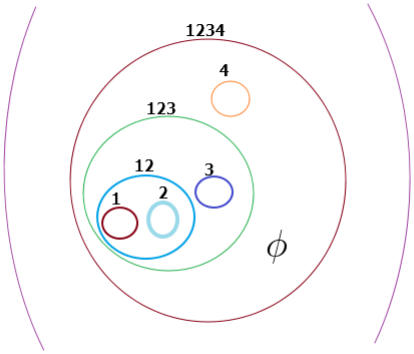
1 votes -- KISHALAY DAS (4k points)

18.0.16 TIFR2016-B-14

https://gateoverflow.in/98014

This is what I thought ..please correct if wrong ! Assuming $\{1, 2, \ldots, 4\}$

F =



With following recurrence relation:

$f(n) = 2 + f(n-1)\ \ n \geq 2$ and $\quad f(1) = 2$

5 votes -- Debashish Deka (40.8k points)

18.0.17 TIFR2016-B-15

https://gateoverflow.in/98018

The answer is d. Consider a path a->c->b , then mincut(a,b) >= min(mincut(b,c),mincut(a,c)) . So if considering this condition to be always true , option i and iii violate the conditions.

0 votes -- Vikram Bhat (737 points)

18.0.18 TIFR2020-B-4

https://gateoverflow.in/333123

$max(x,y) = \frac{1}{2}(x+y+|x-y|)$

Here, $x \geq 0$,$y \geq 0$

Now,

Case 1 : When $x \geq y$

$max(x,y) = \frac{1}{2}(x+y+x-y) = x = clamp_{1,0}(x,y)$

Case 2 : When $x < y$

$max(x,y) = \frac{1}{2}(x+y+y-x) = y = clamp_{0,1}(x,y)$

So, we need only 2 clamp gates as $clamp_{1,0}(x,y)$ and $clamp_{0,1}(x,y)$ to find $max(x,y)$.

1 votes -- ankitgupta.1729 (15.2k points)

18.0.19 TIFR2020-B-5

https://gateoverflow.in/333125

The reflection matrix, to reflect a point (x,y) about the line $y = mx$ where $m = tan\theta$ is given by,

$$\begin{bmatrix} cos2\theta & sin2\theta \\ sin2\theta & -cos2\theta \end{bmatrix}$$

So, if we have a point (x,y) and after reflection, new point is (x',y') then we can write it as :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos2\theta & sin2\theta \\ sin2\theta & -cos2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Now, here in the given question, given point is $(vcos\phi, vsin\phi)$

After reflecting about $x-$axis, It becomes, $(vcos\phi, -vsin\phi)$

Now, according to given information for $\theta = 5^\circ$ and $\phi = 10^\circ$,

$$O_u(v) = v \begin{bmatrix} cos10 & sin10 \\ sin10 & -cos10 \end{bmatrix} \begin{bmatrix} cos10 \\ -sin10 \end{bmatrix} = v \begin{bmatrix} cos20 \\ sin20 \end{bmatrix}$$

$$O_u^2(v) = v \begin{bmatrix} cos10 & sin10 \\ sin10 & -cos10 \end{bmatrix} \begin{bmatrix} cos20 \\ -sin20 \end{bmatrix} = v \begin{bmatrix} cos30 \\ sin30 \end{bmatrix}$$

$$O_u^3(v) = v \begin{bmatrix} cos10 & sin10 \\ sin10 & -cos10 \end{bmatrix} \begin{bmatrix} cos30 \\ -sin30 \end{bmatrix} = v \begin{bmatrix} cos40 \\ sin40 \end{bmatrix}$$

Similarly,

$$O_u^8(v) = v \begin{bmatrix} cos10 & sin10 \\ sin10 & -cos10 \end{bmatrix} \begin{bmatrix} cos80 \\ -sin80 \end{bmatrix} = v \begin{bmatrix} cos90 \\ sin90 \end{bmatrix}$$

It means new point is $w = (vcos90^\circ, vsin90^\circ)$.

So, angle subtended by w and $x-$axis at origin is 90°.

0 votes -- ankitgupta.1729 (15.2k points)

18.0.20 TIFR2020-B-6

https://gateoverflow.in/333126

$S \rightarrow \epsilon \mid aSb \mid bSa \mid SS$

- $S \rightarrow a\underline{S}b$
- $S \rightarrow a\overline{S}\underline{S}b$
- $S \rightarrow aa\overline{S}\underline{S}bSb$
- $S \rightarrow aab\overline{S}\underline{S}abSb$
- $S \rightarrow aabab\underline{S}b$
- $S \rightarrow aababb$

Stings with an equal number of $a's$ and $b's$.

References:

- https://www.cs.wcupa.edu/rkline/fcs/cfls.html#equal-as-bs
- https://stackoverflow.com/questions/55374248/cfg-of-language-which-contains-equal-of-as-and-bs
- https://www.cs.umd.edu/~gasarch/COURSES/250/S15/equal.pdf
- https://courses.engr.illinois.edu/cs373/sp2009/lectures/disc_07.pdf
- https://cs.nyu.edu/courses/fall03/V22.0453-001/ans3.pdf
- https://www.cs.usfca.edu/~galles/cs411/lecture/lecture8.printable.pdf
- https://www.cs.odu.edu/~toida/nerzic/390teched/cfl/cfg.html

So, the correct answer is (E).

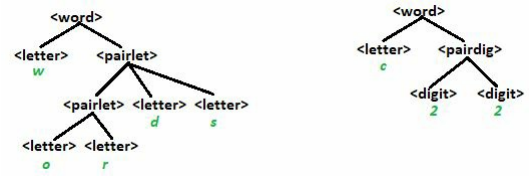
References

0 votes -- Lakshman Patel (69.5k points)

18.0.22 TIFR2020-B-9

https://gateoverflow.in/333129

II) and III) can be generated



1 votes -- Ashwani Kumar (13.1k points)

18.0.23 TIFR2020-B-12

https://gateoverflow.in/333133

BASIC point :-

1%1 = 0

1% x = 1, for x > 1

x+1%x = 1, for all x.

```
for(i=(n-1);i>1;i--)
    x=x%i;
```

when i=(n-1) ===> **x will updated to 1** due to x%i returns 1

after that it never changed.

if i=1, then x will updated to 0 but when i=1, condition evaluates to false, So x can't updated one more time.

2 votes -- Shaik Masthan (50.6k points)

18.0.24 TIFR2021-A: 3

https://gateoverflow.in/358965

first method and second method are basically same things. They are just alternative ways to look at matrix multiplication.

Matrix multiplication can be treated as taking linear combinations of first matrix with scalars as columns of second matrix. or another way to look at it is taking linear combinations of second matrix with scalars as rows of first matrix, Consider either way of matrix multiplication in this example.

Rank represents number of independent columns/rows present in matrix.

for a 3*3 matrix M if rank is 3 the same matrix can be given as M*I.

for rank<3, suppose 2, columns can be treated as c1,c2= x*c1, c3.(or r1, x*r1, r3).

now putting c1 and c3 in one matrix and multiplying them with appropriate matrix we can get original matrix as c2(x*c1) is scalar multiple of c1. And c1,c3 can be derived in this multiplication from themselves easily. similar is case if we consider the row way. just the way of multiplication will be changed(i.e. first matrix will be holding multiples and second will be holding independent rows of original matrix).

hence rank can be used to decompose matrix in two matrices.

From all of this we can conclude that k1=k2=k3.

0 votes

-- Amartyap (33 points)

18.0.25 TIFR2016-A-3

https://gateoverflow.in/96828

$x_1 = x_2 = x_3$

$x_4 = x_5 = x_6$

. . . . . .

. . . . . .

$x_{3n-2} = x_{3n-1} = x_{3n}$

So, solution space is $\left\{ \begin{pmatrix} x_1 \\ x_1 \\ x_1 \\ x_4 \\ x_4 \\ x_4 \\ \cdots \\ x_{3n-2} \\ x_{3n-2} \\ x_{3n-2} \end{pmatrix} \right\}$

$$= \left\{ x_1 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ \cdots \\ 0 \\ 0 \\ 0 \end{pmatrix} + \ldots\ldots + x_{3n} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\} \rightarrow (1)$$

So, solution space contains $3n/3 = n$ basis vectors which are:

$$\left\{\begin{pmatrix}1\\1\\1\\0\\0\\0\\\dots\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\\1\\1\\\dots\\0\\0\\0\end{pmatrix}, \dots\dots, \begin{pmatrix}0\\0\\0\\0\\0\\0\\\dots\\1\\1\\1\end{pmatrix}\right\}$$

These vectors are orthogonal and so they form a linearly independent set and

from (1), it is clear that they span the whole solution space.

Hence, subspace S has dimension n.

0 votes -- ankitgupta.1729 (15.2k points)

18.0.26 TIFR2021-A: 5

https://gateoverflow.in/358963

Option (C)

$P(\text{f is not injective}) = 1 - P(\text{f is injective})$

$= 1 - \frac{\text{injective functions}}{\text{total fuctions}}$

$= 1 - \frac{(\text{choosing k values from m}) * (\text{permuting each value from set S to these k values})}{m^k}$

$= 1 - \frac{\binom{m}{k} \times k!}{m^k}$

0 votes -- JATIN MITTAL (2.2k points)

18.0.27 TIFR2021-A: 13

https://gateoverflow.in/358955

Option (B)

Last two digits are looping as:

$07, 49, 43, 01 | 07, 49, 43, 01 | \dots..$

So, 7^{2020} will give last two digits as 01 and 7^{2021} will give 07 as the last two digits.

2 votes -- JATIN MITTAL (2.2k points)

18.0.29 TIFR2021-B: 5

https://gateoverflow.in/358948

Option (B)

$\bar{L} = \{a^n b^m | n \neq m; n, m \geq 0\}$ This is CFL

$L^* = (a^n b^n)^*$ will always be CFL as CFL are closed under kleene-closure but it is not regular as it requires b to match with the a before starting with another runs of a.

0 votes -- JATIN MITTAL (2.2k points)

18.0.30 TIFR2020-B-14 https://gateoverflow.in/333136

The way to approach this question is to know that proof of "Euler Circuit" problem. Know the proof to get the idea why the following that I am writing, makes sense.
Here we have 4 vertices of odd degree, so, Euler circuit doesn't exist But if you still want to go from S to S by visiting every edge at least once and with minimum cost (i.e. visiting every edge at least once and as minimum repetition as possible) then for every odd degree vertex you have to re-visit one edge incident on that vertex. So, to get minimum cost,
For S, you can revisit(i.e. visit two times) s-a (cost 10)
For t, f, you can revisit(i.e. visit two times) t-f (cost 20)
For b, you can revisit(i.e. visit two times) b-a (cost 20)

So, 380 + 20 + 20 + 10 = 430

2 votes -- Deepak Poonia (23.8k points)

18.0.31 TIFR2021-A: 14 https://gateoverflow.in/358954

There are 10 attendees in total,Shyamal counts all the hands shaken by everyone other than him.

no. of hands that other attendees including his spouse shook (expcept him)

$= \sum_{i=0}^{8}(i) = \frac{8\times 9}{2} = 36.$

Hand shaking lemma state that,The sum of total hand shakes(sum of degrees) should be even, total no. of handsh

Therefore, No of persons who shook hands with Shyamal should be even. (Let it be n)

There are 5 different possible values of n {0,2,4,6,8}, and 5 different degree sequences as follows:

$n = 8,$ degree sequence: $8,8,7,6,5,4,3,2,1,0$ $0,7,6,5,4,3,2,1,0,0$ → Not possible, we can't dele

$n = 6,$ degree sequence: $8,7,6,6,5,4,3,2,1,0$ $0,6,5,5,4,3,2,1,0,0$ $0,0,4,4,3,2,1,0,0,0$ 0,0,(

$n = 4,$ degree sequence: $8,7,6,5,4,4,3,2,1,0$ $0,6,5,4,3,3,2,1,0,0$ $0,0,4,3,2,2,1,0,0,0$ 0,0,(

$n = 2,$ degree sequence: $8,7,6,5,4,3,2,2,1,0$ $0,6,5,4,3,2,1,1,0,0$ $0,0,4,3,2,1,0,0,0,0$ → N

$n = 0,$ degree sequence: $8,7,6,5,4,3,2,1,0,0$ → Not possible, we can't even delete 8

Only possible value for n is 4.

Hence B is Answer.

0 votes -- Nikhil Dhama (2.7k points)

18.0.32 TIFR2021-A: 6 https://gateoverflow.in/358962

Probability(selecting any vertex) = p ,
Probability(getting any particular edge) = p^2,
Probability(not getting any particular edge) = $(1-p^2)$,
as there are total m edges in the Matching set M,
Probability(not getting any of these m edges) = $(1-p^2)^m$,
Probability(getting at-least 1 edge from M) = 1 – Probability(not getting any edge from M) = $1-(1-p^2)^m$
Option B.

0 votes -- Nikhil Dhama (2.7k points)

18.0.33 TIFR2021-B: 6 https://gateoverflow.in/358947

Answer (D)
Array index starts from 0 and $A[i]$ stores the value of the function for index i –

1	1	2	3	5	8	13	21	34	55	89

$A[0] = A[1] = 1$

for rest of the table use, $A[i] = A[i-1] + A[i-2]$

Answer $= 89$

0 votes -- JATIN MITTAL (2.2k points)

18.0.34 TIFR2021-B: 7

https://gateoverflow.in/358946

✔ Language of options $(a), (b), (c)$ and (d) is the set of strings over $\{a, b\}$ containing at least two $a's$.

Language of option (e) is the set of strings over $\{a, b\}$ containing at least one a.

1 votes -- JATIN MITTAL (2.2k points)

18.0.35 TIFR2021-B: 8

https://gateoverflow.in/358945

(1). **If**
$AB = B$**, then A must be the identity matrix.** $\rightarrow$ **False**

If B is invertible, then B^{-1} exists,

and $ABB^{-1} = BB^{-1} \implies AI = I \implies A = I$

But B may not be invertible, in such case A may or may not be identity matrix.

For example, If $A = B$, then we have $A^2 = A$, here A can be a non-singular matrix.

For $A = B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, we have $AB = B$, and A is not identity.

Hence this is False.

(2). **If**
A **is an idempotent nonsingular matrix, then**
A **must be the identity matrix.** $\rightarrow$ **True.**

A is nonsingular, means A^{-1} exists,

$A^2 = A \implies AAA^{-1} = AA^{-1} \implies AI = I \implies A = I$

Hence true.

(3). **If**
$A^{-1} = A$**, then**
A **must be the identity matrix.** $\rightarrow$ **False.**

A is an Involutory matrix, It not necessary that $A = I$

For example: $P = P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (Interchange row/column 2 and 3)

Hence False.

Option (E) is correct.

0 votes -- Nikhil Dhama (2.7k points)

18.0.36 TIFR2021-B: 9

https://gateoverflow.in/358944

E

For starters, the language $L^{'}$ isn't really the complement of L, it is defined by $G^{'}$ defined by adding a new-rule $S \rightarrow SS$.

So, for this problem let's consider $L = \{a, b, c\}$ (RG is a CFG) so we've a simple derivation $S \rightarrow a/b/c$, for $L^{`}$ use the added rule which generates $L^{`} = \{a, b, c, aa, ab \ldots, cc, aaa, aab \ldots . ccc \ldots\}$.

A. Concatenation of every string of L with every string of L itself will produce $\{aa, ab, \ldots, cc\}$ which is only a subset of the language $L^{`}$.
B. B.If this was true, we can't generate aa for instance, the $L^{`}$ doesn't only comprise the original elements but newly-generated too.C.

C. The original language needn't necessarily contain ϵ, which in-turn can't be used generated a ϵ int the new-language. The above Language provides the counter-example for this statement. So, not true...D.
D. All strings generated by the above grammar do belong in $L^{'}$ but, there are other elements such as $\{ab, ac, bc\}$ which aren't generated so it isn't true.E.
E. None are true so ?

0 votes -- Cringe is my middle name... (973 points)

18.0.37 TIFR2021-A: 12 https://gateoverflow.in/358956

$1365 = (10101010101)_2$

$(\text{total number of bits} = 11)$

Now if I keep $MSB = 0$, then from the remaining 10 bits, I can select any 4 bits ans set them as 1. So total cases become $\binom{10}{4}$.

Now keep the $MSB = 1$, now we cannot replace 0 on the second last bit from LHS with 1 as it will increase the number. This will apply to every 0 as we move forward. So, now we have 10 on the LHS and the third bit from LHS, we set to 0, now we have 8 bits remaning from which we have to select 3 bits (Why 3? as we are keeping MSB as 1 and in total we only need 4 bits to be set to 1). Total cases will become $\binom{8}{3}$.

Similarly, we will move forward and will get $\binom{6}{2}$ and $\binom{4}{1}$.

So, the answer should be $\binom{10}{4} + \binom{8}{3} + \binom{6}{2} + \binom{4}{1}$

I think there is a typo in the (C) option. They have the last term as $\binom{5}{1}$ instead of $\binom{4}{1}$.

0 votes -- JATIN MITTAL (2.2k points)

18.0.38 TIFR2021-B: 4 https://gateoverflow.in/358949

✔ **PRIME is in P :**

References: AKS primality test

Primality testing with Gaussian periods

FACTOR is also in P :

Algorithm:

if $(b - a + 1) \geq n$: return "Yes"else if $(a \bmod n == 0)$ or $(a \bmod n > b \bmod n)$: return "Yes"else : r

Time complexity is $O(1)$: polynomial time.

(C) is correct.

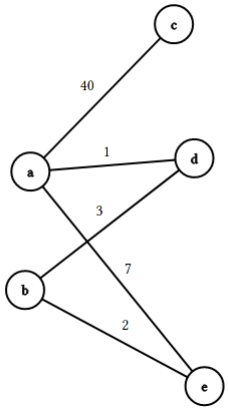
References

1 votes -- Nikhil Dhama (2.7k points)

18.0.39 TIFR2021-B: 10 https://gateoverflow.in/358943

Option E is not necessarily True.

Example: edge $a - c$ is heaviest, but will be present in every MST of this bipartite graph.

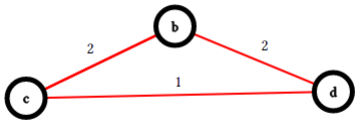


It's a simple connected graph and edge weights are given distinct, therefore we'll have a unique MST for every such graph.

0 votes -- Nikhil Dhama (2.7k points)

18.0.41 TIFR2021-B: 12 https://gateoverflow.in/358941

Option E is not possible.
following configuration is not possible,



Here, egde weight denotes the number of path,
If there are 2 paths from between b & c, and 2 paths between b & d, then there will be at-least 2 paths between c

0 votes -- Nikhil Dhama (2.7k points)

18.0.42 TIFR2021-B: 13 https://gateoverflow.in/358940

Option C) There is a clique of size 3.

rank of matrix A is 3, Therefore 3 of the columns must be linearly independent of each other, Hence a clique of size 3.

0 votes -- Nikhil Dhama (2.7k points)

18.0.43 TIFR2021-B: 14 https://gateoverflow.in/358939

option (C). $m(n,r) = \binom{r}{2}$

An assumption that there's some order of these r colors,..

to newly selected vertex we'll assign first color that has not already been assigned to any of its neighbors,

we'll get a new color only if new node is connected to all the other vertices which have a distinct color assigned to them,

recursively to get r^{th} color , new vertex should be connected to at-least all vertices having $(r-1)$ distinct colors,

minimum value of $m(n,r)$ is given by,

$T(r) = T(r-1) + (r-1)T(1) = 0$

$T(r) = T(r-i) + (r-i) + (r-(i-1)) + \ldots + (r-1) \quad r-i = 1 \Rightarrow i = r-1$

substituting the value of i, we get

$T(r) = T(r-(r-1)) + (r-(r-1)) + (r-(r-2)) + \ldots + (r-1) \quad \Rightarrow T(1) + 1 + 2 + \ldots + (r-1) \quad \Rightarrow 0$ -

0 votes -- Nikhil Dhama (2.7k points)

18.0.44 TIFR2021-A: 11

https://gateoverflow.in/358957

Option (A)

$= \frac{1}{2^2-1} + \frac{1}{4^2-1} + \ldots + \frac{1}{40^2-1}$

$= \frac{1}{(2+1)(2-1)} + \frac{1}{(4+1)(4-1)} + \ldots + \frac{1}{(40+1)(40-1)}$

$= \left[\frac{1}{(2+1)(2-1)} + \frac{1}{(4+1)(4-1)} + \ldots + \frac{1}{(40+1)(40-1)}\right]\frac{2}{2}$

$= \left[\frac{2}{(2+1)(2-1)} + \frac{2}{(4+1)(4-1)} + \ldots + \frac{2}{(40+1)(40-1)}\right]\frac{1}{2}$

$= \left[\frac{(2+1)-(2-1)}{(2+1)(2-1)} + \frac{(4+1)-(4-1)}{(4+1)(4-1)} + \ldots + \frac{(40+1)-(40-1)}{(40+1)(40-1)}\right]\frac{1}{2}$

$= \left[\frac{1}{(2-1)} - \frac{1}{(2+1)} + \frac{1}{(4-1)} - \frac{1}{(4+1)} + \ldots + \frac{1}{(40-1)} - \frac{1}{(40+1)}\right]\frac{1}{2}$

$= \left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \ldots + \frac{1}{39} - \frac{1}{41}\right]\frac{1}{2}$

$= \left[1 - \frac{1}{41}\right]\frac{1}{2}$

$= \frac{20}{41}$

2 votes -- JATIN MITTAL (2.2k points)

18.0.45 TIFR2021-A: 10

https://gateoverflow.in/358958

Option (B) is the correct answer.

$P(H(L) > H(K)) + P(H(K) > P(L)) + P(H(K) == H(K)) = 1$

Now since, $P(H(L) > H(K))$ and $P(H(K) > P(L))$ are symmetrical, they will be equal (lets say = Y).

Assume, $P(H(K) == H(K)) = X$

So, $X + 2 * Y = 1$

$\Rightarrow Y = (1 - X)/2$

And, $X = P(H(L) == 0 \text{ AND } H(K) == 0) + P(H(L) == 1 \text{ AND } H(K) == 1) + ... + P(H(L) == n \text{ AND } H(K) == n)$

$X = \binom{n}{0}^2/2^{2n} + \binom{n}{1}^2/2^{2n} + \ldots + \binom{n}{n}^2/2^{2n}$

$X = \sum_{i=0}^{n} \binom{n}{i}^2/2^{2n}$

$Y = \frac{1}{2}(1 - \sum_{i=0}^{n} \binom{n}{i}^2/2^{2n})$

1 votes -- JATIN MITTAL (2.2k points)

18.0.46 TIFR2021-A: 9

https://gateoverflow.in/358959

For $n \geq 6$, If n is divisible by 3 then $\mathcal{C}$ contains :
$(000)^{\frac{n}{3}}, (011)^{\frac{n}{3}}, (101)^{\frac{n}{3}}$ and $(110)^{\frac{n}{3}}$
example: for n=6, $\mathcal{C} = \{000000, 011011, 101101, 110110\}$
$|\mathcal{C}| = 4$

If n is not divisible by 3 then $\mathcal{C}$ only contains $(0)^n$

example: for n=7, $\mathcal{C} = \{0000000\}$

$|\mathcal{C}| = 1$

E is correct.

Note: Observe the cyclic property defined for $\mathcal{C}$, It should be cyclic in group of 3.

0 votes -- Nikhil Dhama (2.7k points)

18.0.48 TIFR2021-A: 7

https://gateoverflow.in/358961

Answer (E)

$= 20^5 \times 21^5$

$= (2^2 \times 5^1)^5 \times (7 \times 3)^5$

$= 2^{10} \times 5^5 \times 7^5 \times 3^5$

Now to form the numbers which are perfect squares, for each of $2, 3, 5, 7$ we should pick ONLY even powers.

So the numbers possible to be present in the perfect square are $\{2^2, 2^4, 2^6, 2^8, 2^{10}, 3^2, 3^4, 5^2, 5^4, 7^2, 7^4\}$

Now, we can take any subset of this and multiply the numbers present in that subset to get the perfect square number. For example, if we pick the subset $\{2^4, 7^2\}$, then the number will be $2^4 \times 7^2 = 784 = (28)^2$

Total subsets possible = 2^{11}

But, we don't have to consider the empty set (as 0 is only factor of itself and question also asks for positive numbers only).

So the answer should be $2^{11} - 1 = 2047$.

0 votes -- JATIN MITTAL (2.2k points)

18.0.49 TIFR2021-B: 3

https://gateoverflow.in/358950

Option (D)

Steps to convert Infix to Prefix:

1. Reverse the Infix expression, replace '(' with ')' and vice-versa.
2. Convert into Postfix expression.
3. Reverse the postfix expression.

Reversed expression: $2 + (3 * ((4 + 5) * 6)) + 7 * (8 + 9))$

Check here https://gateoverflow.in/2633/gate1995-2-21 to see how to convert infix to postfix expression.

Postfix expression using operator stack: $2345 + 6 * *789 + * + +$

Reverse the above postfix expression to get the prefix expression: $+ + * + 987 * *6 + 5432$

References

0 votes -- JATIN MITTAL (2.2k points)

18.1 3 Sat (1)

18.1.1 3 Sat: TIFR2016-B-3

https://gateoverflow.in/97629

Assume $P \neq NP$. Which of the following is not TRUE?

A. 2-SAT in NP
B. 2-SAT in coNP
C. 3-SAT is polynmial-time reducible to 2-SAT
D. 4-SAT is polynmial-time reducible to 3-SAT
E. 2-SAT in P

tifr2016 p-np-npc-nph 3-sat 2-sat

Answers: 3 Sat

18.1.1 3 Sat: TIFR2016-B-3

https://gateoverflow.in/97629

✔ 3-SAT is NPC problem.

2-SAT is P problem.

NPC cannot be reduced to P (When P≠NP).

So C is false.

0 votes

-- Akash Sheoran (1.5k points)

18.2 Binomial Theorem (1)

18.2.1 Binomial Theorem: TIFR2016-A-13

https://gateoverflow.in/97622

Let $n \geq 2$ be any integer. Which of the following statements is not necessarily true?

A. $\binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$, where $1 \leq i \leq n-1$
B. $n!$ divides the product of any n consecutive integers
C. $\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$
D. n divides $\binom{n}{i}$, for all $i \in \{1, 2, \ldots, n-1\}$
E. If n is an odd prime, then n divides $2^{n-1} - 1$

tifr2016 binomial-theorem

Answers: Binomial Theorem

18.2.1 Binomial Theorem: TIFR2016-A-13

https://gateoverflow.in/97622

A,B,C are true due to usual reasons.

Now,

Let a = 2 which is not divisible by n, then using Fermat's little theorem

$2^{n-1} \equiv 1 (mod n)$

$\Rightarrow 2^{n-1} - 1 \equiv 0 (mod n)$

Therefore **E** is true.

D. But it also seems to be correct within given range.

For n= 1 : n divides $\frac{n(n-1)}{2}$ and so for n-1.

For i = k, k between 1 & n-1. n divides $\frac{n(n-1)(n-2)...(n-k+1)}{k!}$, Since there is n at numerator always. Therefore True.

Please tell me where I'm wrong.

1 votes -- Aghori (4.4k points)

18.3 Closure Property (1)

18.3.1 Closure Property: TIFR2016-B-2

https://gateoverflow.in/97627

Which language class has the following properties?

It is closed under union and intersection but not complement.

A. Regular language
B. Context-free language
C. Recursive language
D. Recursively enumerable language
E. Languages that are not recursively enumerable

tifr2016 theory-of-computation closure-property

Answers: Closure Property

18.3.1 Closure Property: TIFR2016-B-2

https://gateoverflow.in/97627

✔ A) regular language is closed under union ,intersection , complement

B) context free language is closed under union but not closed under intersection and complement

C)recursive language is closed under union ,intersection, complment

D)recursive enemurable language is closed under union intersection but it is not closed under complement

E)language NOT RE we can't say anything about it.

SO ***D) option answer***

2 votes -- kunal chalotra (13.6k points)

18.4 Complex Number (1)

18.4.1 Complex Number: TIFR2016-A-6

https://gateoverflow.in/96901

Which of the following statements about the eigen values of I_n, the $n \times n$ identity matrix (over complex numbers), is true?

A. The eigen values are $1, \omega, \omega^2, \ldots, \omega^{n-1}$, where ω is a primitive n-th root of unity
B. The only eigen value is -1
C. Both 0 and 1 are eigen values, but there are no other eigen values
D. The eigen values are $1, 1/2, 1/3, \ldots, 1/n$
E. The only eigen value is 1

tifr2016 matrices complex-number

Answers: Complex Number

18.4.1 Complex Number: TIFR2016-A-6

https://gateoverflow.in/96901

✔ Be it real matrix or complex matrix , the identity matrix remains the same ..So we will have in In , 1's only in the principal diagonal elements and the rest of the elements of the matrix will be 0..So the characteristic equation will be :

$[A - \lambda I] = 0$

$\Longrightarrow (1 - \lambda) . (1 - \lambda) .....(1 - \lambda)$ [n times] $= 0$

$\Longrightarrow (1 - \lambda)^n = 0$

$\Longrightarrow \lambda = 1$ as the only solution the reason being complex roots of unity only holds if it were $\lambda^n - 1 = 0$ which is not the case here..

Hence E) is the correct answer..

3 votes -- HABIB MOHAMMAD KHAN (67.6k points)

18.5 Divergence (1)

18.5.1 Divergence: TIFR2016-A-5

https://gateoverflow.in/96898

For a positive integer $N \geq 2$, let

$$A_N := \Sigma_{n=2}^{N} \frac{1}{n};$$

$$B_N := \int_{x=1}^{N} \frac{1}{x} dx$$

Which of the following statements is true?

A. As $N \to \infty$, A_N increases to infinity but B_N coverages to a finite number
B. $A_N < B_N$ and the difference decreases as $N \to \infty$
C. $A_N < B_N < A_N + 1$
D. $B_N < A_N < B_N + 1$
E. As $N \to \infty$, B_N increases to infinity but A_N coverages to a finite number

tifr2016 convergence divergence integration

Answers: Divergence

18.5.1 Divergence: TIFR2016-A-5

https://gateoverflow.in/96898

$$\text{We know , } (\ln n) < H_n < (\ln n + 1)$$
$$\Rightarrow B_n < H_n < (B_n + 1)$$
$$\text{Given, } A_n = H_n - 1$$
$$\Rightarrow B_n < (A_n + 1) < (B_n + 1)$$

Option C

For more : concrete mathematics by knuth.chapter 6

6 votes

-- Debashish Deka (40.8k points)

18.6 Dynamic Programming (1)

18.6.1 Dynamic Programming: TIFR2016-A-7

https://gateoverflow.in/97222

Let S be the 4×4 square grid $\{(x, y) : x, y \in \{0, 1, 2, 3\}\}$. A *monotone path* in this grid starts at $(0, 0)$ and at each step either moves one unit up or one unit right. For example, fromthe point (x, y) one can in one step either move to $(x + 1, y) \in S$ or $(x, y + 1) \in S$, but never leave S. Let the number of distinct monotone paths to reach point $(2, 2)$ starting from $(0, 0)$ bt z. How many distinct monotone paths are there to reach point $(3, 3)$ starting from $(0, 0)$?

A. $2z + 6$ B. $3z + 6$ C. $2z + 8$ D. $3z + 8$ E. $3z + 4$

tifr2016 combinatory dynamic-programming

Answers: Dynamic Programming

18.6.1 Dynamic Programming: TIFR2016-A-7

https://gateoverflow.in/97222

✔ Ans : C] 2z+8

Clearly a path of the desired type must consist of 3 moves to right and 3 moves to up.

Therefore, each such path can be represented by a bit string of 3 0's and 3 1's, with the 0's representing moves to the right and the 1's representing moves up.

The number of bit strings of length 3+3 containing exactly 3 0's and 3 1's is $\left(\frac{6!}{3!*3!}\right) = 20$.

The number of bit strings of length 2+2 containing exactly 2 0's and 2 1's is z = $\left(\frac{4!}{2!*2!}\right) = 6$.

Now, by substituting z=6 in options, only C gives 20.

6 votes -- Kantikumar (3.4k points)

18.7 Euler Graph (1)

18.7.1 Euler Graph: TIFR2016-B-9

https://gateoverflow.in/98001

Which of the following graphs DOES NOT have an Eulerian circuit? (Recall that an Eulerian circuit in an undirected graph is a walk in the graph that starts at a vertex ans returns to the vertex after tracelling on each edge exactly once.)

A. $K_{9,9}$
B. $K_{8,8}$
C. $K_{12,12}$
D. K_9
E. The graph G on vertex set $\{1, 2, \ldots, 9\}$ with edge set

$$E(G) = \{\{i, j\} : 1 \leq i < j \leq 5 \text{ or } 5 \leq i < j \leq 9\}.$$

tifr2016 discrete-mathematics graph-theory euler-graph normal

Answers: Euler Graph

18.7.1 Euler Graph: TIFR2016-B-9

https://gateoverflow.in/98001

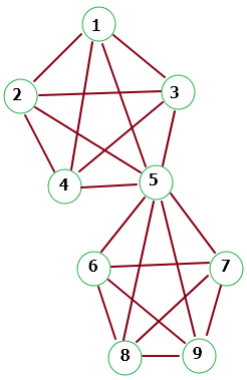
✔ **A connected undirected multi graph has Euler's circuit iff all vertices are of Even degree**

1.$\mathbf{K_{9,9}}$ is Bipartite graph with each side having 9 vertices and each of the vertex on one side are connected to all vertices of other side,hence degree of all vertices are 9 which is odd degree**, Therefore No Eulers Circuit**

2 &3 .$\mathbf{K_{8,8}}$ **and** $\mathbf{K_{12,12}}$ is Bipartite graph with each side having 8 vertices and each of them are connected to all vertices of other side,hence degree of all vertices are 8 and 12 respectively which is Even**, Therefore Eulers Circuit exists**

4. **K9** Complete graph with 9 vertices with each vertex connected to all other therefore degree is 8, **Therefore Eulers Circuit exists**

5.



In this graph also all vertices have even degree. So, Euler circuit exists.

Hence Option A doesn't have Euler's circuit

7 votes -- Prajwal Bhat (7.6k points)

18.8 Generalaptitude (1)

18.8.1 Generalaptitude: TIFR2016-A-14

https://gateoverflow.in/97623

A *diagonal* in a polygon is a straight line segment that connects two non-adjacent vertices, and is contained in the interior of the polygon (except for its points). Two such diagonals are said to cross if they have a point in common in the interior of the polygon. In one such polygon with n vertices, a certain number (say k) of non-crossing diagonals were

drawn to cut up the inside of the polygon into regions, each of which was a quadrilateral. how many diagonals were drawn, that is, what is k?

A. cannot be determined from the information given B. $\frac{n}{2}-2$ C. $\frac{n}{4}-1$ D. $n-4$ E. $n^2-9.5n+22$

tifr2016 graph-theory generalaptitude

Answers: Generalaptitude

18.8.1 Generalaptitude: TIFR2016-A-14

https://gateoverflow.in/97623

We have drawn k diagonals on a n-sided polygon.

Now, total edges = n(sides)+k(diagonals) = n+k

For each diagonal we draw, one extra region will be formed. Initially, there are 2 regions (entire region inside polygon and the external region)

total regions = k+2

Now, using the law, sum of degrees of all regions = 2*no of edges.

Here, r-1 regions will have degree of 4 (all the regions formed inside polygon) and the external region has degree (n+k) as it is bounded by (n) edges (sides of the polygon).

So, 4*(r-1)+(n)*1=2*e
4*(k+2-1)+(n)*1=2*(n+k)
4k+4+n=2n+2k
k=(n-4)/2
Option B is correct

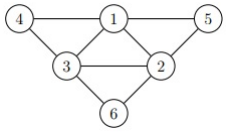
3 votes -- Chandrashis Mazumdar (83 points)

18.9 Graph (1)

18.9.1 Graph: TIFR2016-A-2

https://gateoverflow.in/96825

Consider the graph shown below:



The following experiment is performed using this graph. First, an edge $e=\{i,j\}$ of the graph is chosen uniformly at random from the set of 9 possibilities. Next, a common neighbour k of i and j is chosen, again uniformly from the set of possibilities. (Note that the set of possibilities is always non-empty.) Thus, $\{i,j,k\}$ is a triangle in the graph. What is the probability that the triangle finally picked is $\{1,2,3\}$?

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$ E. $\frac{5}{6}$

tifr2016 graph probability

Answers: Graph

18.9.1 Graph: TIFR2016-A-2

https://gateoverflow.in/96825

✔ First we select edge 1-3 with probability $\frac{1}{9}$ now common neighbor of (1,3) are (4,2)

Probability of choosing 2 as neighbor is so that {1,2,3} can from triangle is $\frac{1}{2}$ so total probabiltiy is $\frac{1}{9}*\frac{1}{2}$

similar for edge 1-2 and edge 2-3 as both have only 2 neighbour common {3,5} and {1,6} respectively

total probability is $\frac{1}{9}*\frac{1}{2}*3=\frac{1}{6}$

5 votes -- Keval Malde (13.3k points)

18.10 P Np Npc Nph (1)

18.10.1 P Np Npc Nph: TIFR2016-B-8

https://gateoverflow.in/97996

Consider the following language

$$\text{PRIMES} = \left\{ \underbrace{111\ldots11}_{p \text{ times}} : p \text{ is prime} \right\}$$

Then, which of the following is TRUE?

A. PRIMES is regular
B. PRIMES is undecidable
C. PRIMES is decidable in polynomial time
D. PRIMES is context free but not regular
E. PRIMES is NP-complete and P $\neq$ NP

tifr2016 decidability p-np-npc-nph

Answers: P Np Npc Nph

18.10.1 P Np Npc Nph: TIFR2016-B-8

https://gateoverflow.in/97996

The language is $\{1^p \mid p\ is\ prime\}$, which is wel-known to be CSL.

Option C

0 votes

-- JashanArora (12.3k points)

18.11 Probability (1)

18.11.1 Probability: TIFR2016-A-11

https://gateoverflow.in/97618

In one of the islands that his travels took him to, Gulliver noticed that the probability that a (uniformly) randomly chosen inhabitant has height at least 2 meters is 0.2. Also, 0.2 is the probability that a a (uniformly) randomly chosen inhabitant has height at most 1.5 meters. What can we conclude about the average height h in meters of the inhabitants of the island?

i. $1.5 \leq h \leq 2$
ii. $h \geq 1.3$
iii. $h \leq 2.2$

Which of the above statements is necessarily true?

A. ii only B. iii only C. i, ii and iii D. ii and iii only E. None of the above

tifr2016 probability

Answers: Probability

18.11.1 Probability: TIFR2016-A-11

https://gateoverflow.in/97618

✔ we can approach the question in this way:

say there are 100 inhabitants

1.) 20 inhabitants have height atleast 2 m.

2.) 20 inhabitants have height atmost 1.5m

3.) 60 people have height between 1.5m and 2m

Consider the lower limit of height in case 1. It is 2m.(upper limit is infinity)

In case 2 the lower limit can be 0.(upper limit is 1.5m)

In case 3 the lower limit is 1.5m(upper limit 2m)

On taking the average of minimum values:(as the options are comparing values of heights <= or >=)

2 x 20+0 x 20+1.5 x 60 = 130

So lower limit of height is : h>=1.3

So answer is option A

1 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

18.12 Uniform Hashing (1)

18.12.1 Uniform Hashing: TIFR2016-A-4

https://gateoverflow.in/96893

There are n balls $b_1, \ldots, b_n$ and n boxes. Each ball is placed in box chosen independently and uniformly at random. We say that (b_i, b_j) is a colliding pair if $i < j$, and b_i and b_j are placed in the same box. WHat is the expected number of colliding pairs?

A. $\frac{n-1}{2}$ B. 0 C. 1 D. $\frac{n}{4}$ E. $\binom{n}{2}$

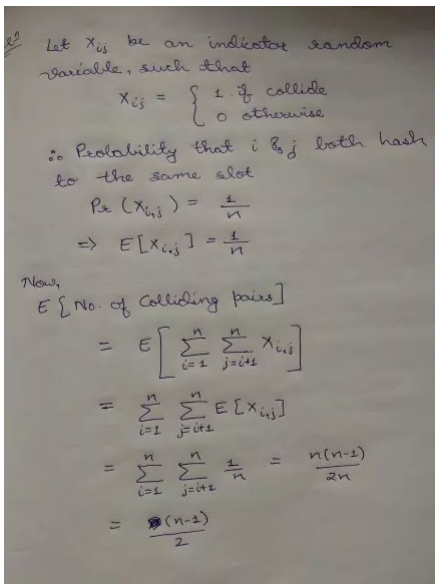
tifr2016 probability uniform-hashing

Answers: Uniform Hashing

18.12.1 Uniform Hashing: TIFR2016-A-4

https://gateoverflow.in/96893

I think A.



7 votes -- Aghori (4.4k points)

18.13 Work Time (1)

18.13.1 Work Time: TIFR2018-A-4

https://gateoverflow.in/179273

The distance from your home to your office is 4 kilometers and your normal walking speed is 4 Km/hr. On the first day, you walk at your normal walking speed and take time T_1 to reach office.

On the second day, you walk at a speed of 3 Km/hr from 2 Kilometers, and at a speed of 5 Km/hr for the remaining 2 Kilometers and you take time T_2 to reach office.

On the third day, you walk at a speed of 3 Km/hr for 30 minutes, and at 5 Km/hr for the remaining time and take time T_3 to reach office.

What can you say about the ordering of T_1, T_2 and T_3

A. $T_1 > T_2$ and $T_1 < T_3$
B. $T_1 = T_2 = T_3$
C. $T_1 < T_2$ and $T_1 > T_3$
D. $T_1 = T_2$ and $T_1 < T_3$
E. $T_1 < T_2$ and $T_1 = T_3$

tifr2018 quantitative-aptitude work-time

Answers: Work Time

18.13.1 Work Time: TIFR2018-A-4

https://gateoverflow.in/179273

✔ option E

$T_1 = \frac{4km}{4km/hr} = 1hr$

$T_2 = \frac{2km}{3km/hr} + \frac{2km}{5km/hr} = 1.067hr$

Distance travelled in 30 minutes$= 3 * \frac{30}{60} = \frac{3}{2}km$

$T_3 = \frac{\frac{3}{2}km}{3km/hr} + \frac{4-\frac{3}{2}km}{5km/hr} = 1hr$

$T_1 = T_3$ and $T_1 < T_2$

1 votes

-- Manoja Rajalakshmi Aravindakshan (7.7k points)

Answer Keys

18.0.1	D	18.0.2	B	18.0.3	B	18.0.4	E	18.0.5	B
18.0.6	B	18.0.7	E	18.0.8	D	18.0.9	D	18.0.10	C
18.0.11	A	18.0.12	B	18.0.13	D	18.0.14	D	18.0.15	E
18.0.16	A	18.0.17	D	18.0.18	B	18.0.19	C	18.0.20	E
18.0.21	A	18.0.22	D	18.0.23	B	18.0.24	D	18.0.25	C
18.0.26	C	18.0.27	B	18.0.28	Q-Q	18.0.29	B	18.0.30	A
18.0.31	B	18.0.32	B	18.0.33	D	18.0.34	E	18.0.35	E
18.0.36	E	18.0.37	Q-Q	18.0.38	C	18.0.39	E	18.0.40	Q-Q
18.0.41	E	18.0.42	C	18.0.43	C	18.0.44	A	18.0.45	B
18.0.46	E	18.0.47	Q-Q	18.0.48	E	18.0.49	D	18.1.1	C
18.2.1	D	18.3.1	D	18.4.1	E	18.5.1	C	18.6.1	C
18.7.1	A	18.8.1	B	18.9.1	A	18.10.1	C	18.11.1	A
18.12.1	A	18.13.1	E						

19 Programming and DS: DS (10)

Arrays, Stacks, Queues, Linked lists, Trees, Binary search trees, Binary heaps, Graphs.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	4	2	2	0	2	3	1	1	1	0	1.7	4
2 Marks Count	1	0	3	2	0	0	1	3	3	0	1.2	3
Total Marks	6	2	4	4	2	3	3	7	7	2	4.2	7

19.1 Arrays (1)

19.1.1 Arrays: TIFR2011-B-30

https://gateoverflow.in/20611

Consider an array $A[1...n]$. It consists of a permutation of numbers $1....n$. Now compute another array $B[1...n]$ as follows: $B[A[i]] := i$ for all i. Which of the following is true?

A. B will be a sorted array.
B. B is a permutation of array A.
C. Doing the same transformation twice will not give the same array.
D. B is not a permutation of array A.
E. None of the above.

tifr2011 data-structures arrays

Answers: Arrays

19.1.1 Arrays: TIFR2011-B-30

https://gateoverflow.in/20611

✔ **Option (b) B is a permutation of array A.**

In fact, B gives the reverse index of all the elements of array A. Since the array A contains numbers $[1..n]$ mapped to the locations $[1..n]$ and A is a permutation of the numbers $[1..n]$, the array B will also be a permutation of the numbers $[1..n]$.

For example:

index	1	2	3	4	5	6	7	8
A	5	1	3	7	6	2	8	4
B	2	6	3	8	1	5	4	7

To see that option c is incorrect, let array C be the array attained from doing the same transformation twice, that is, $C[B[i]] = i, \forall i \in [1..n]$. We get,

index	1	2	3	4	5	6	7	8
A	5	1	3	7	6	2	8	4
B	2	6	3	8	1	5	4	7
C	5	1	3	7	6	2	8	4

We can see that $C = A$, which makes option c incorrect.

👍 36 votes

-- Pragy Agarwal (18.3k points)

19.2 Binary Search Tree (1)

19.2.1 Binary Search Tree: TIFR2010-B-26

https://gateoverflow.in/18749

Suppose there is a balanced binary search tree with n nodes, where at each node, in addition to the key, we store the number of elements in the sub tree rooted at that node.

Now, given two elements a and b, such that $a < b$, we want to find the number of elements x in the tree that lie between a and b, that is, $a \leq x \leq b$. This can be done with (choose the best solution).

A. $O(\log n)$ comparisons and $O(\log n)$ additions.
B. $O(\log n)$ comparisons but no further additions.
C. $O(\sqrt{n})$ comparisons but $O(\log n)$ additions.
D. $O(\log n)$ comparisons but a constant number of additions.
E. $O(n)$ comparisons and $O(n)$ additions, using depth-first- search.

tifr2010 binary-search-tree

Answers: Binary Search Tree

19.2.1 Binary Search Tree: TIFR2010-B-26

https://gateoverflow.in/18749

$O(\log n)$ comparisons and $O(\log n)$ additions. The algorithm is :

1. Find a and b : This will take $O(\log n)$ comparisons as tree is balanced BST.

2. Follow path from a to b, and along the path, keep adding the required number of nodes to result by looking at number stored at each node. Path length is $O(\log n)$, hence number of additions will also be $O(\log n)$.

15 votes -- Happy Mittal (8.2k points)

19.3 Binary Tree (5)

19.3.1 Binary Tree: TIFR2012-B-16

https://gateoverflow.in/25214

Consider a complete binary tree of height n, where each edge is one Ohm resistor. Suppose all the leaves of the tree are tied together. Approximately how much is the effective resistance from the root to this bunch of leaves for very large n?

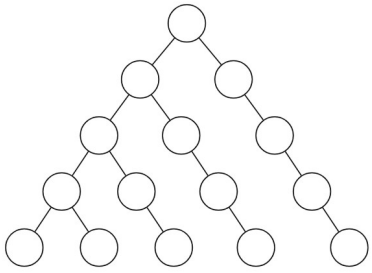
A. Exponential in n.
B. Cubic in n.
C. Linear in n.
D. Logarithmic in n.
E. Of the order square root of n.

tifr2012 binary-tree

19.3.2 Binary Tree: TIFR2013-B-13

https://gateoverflow.in/25775

Given a binary tree of the following form and having n nodes, the height of the tree is



A. $\Theta(\log n)$
B. $\Theta(n)$
C. $\Theta(\sqrt{n})$
D. $\Theta(n/\log n)$
E. None of the above.

tifr2013 binary-tree data-structures

19.3.3 Binary Tree: TIFR2014-B-1

https://gateoverflow.in/27133

Let T be a rooted binary tree whose vertices are labelled with symbols $a, b, c, d, e, f, g, h, i, j, k$. Suppose the in-order (visit left subtree, visit root, visit right subtree) and post-order (visit left subtree, visit right subtree, visit root) traversals of T produce the following sequences.

in-order:$a, b, c, d, e, f, g, h, i, j, k$

post-order:$a, c, b, e, f, h, j, k, i, g, d$

How many leaves does the tree have?

A. THREE.
B. FOUR.
C. FIVE.
D. SIX.
E. Cannot be determined uniquely from

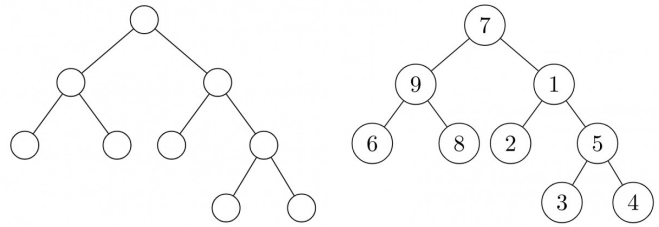
the given information.

tifr2014 binary-tree data-structures easy

19.3.4 Binary Tree: TIFR2015-B-4

https://gateoverflow.in/29849

First, consider the tree on the left.



On the right, the nine nodes of the tree have been assigned numbers from the set $\{1, 2, \ldots, 9\}$ so that for every node, the numbers in its left subtree and right subtree lie in disjoint intervals (that is, all numbers in one subtree are less than all numbers in the other subtree). How many such assignments are possible? Hint: Fix a value for the root and ask what values can then appear in its left and right subtrees.

A. $2^9 = 512$ B. $2^4.3^2.5.9 = 6480$ C. $2^3.3.5.9 = 1080$ D. $2^4 = 16$ E. $2^3.3^3 = 216$

tifr2015 binary-tree combinatory

19.3.5 Binary Tree: TIFR2018-B-6

https://gateoverflow.in/179290

Consider the following implementation of a binary tree data strucrure. The operator $+$ denotes list-concatenation. That is, $[a, b, c] + [d, e] = [a, b, c, d, e]$.

```
struct TreeNode:
 int value
 TreeNode leftChild
 TreeNode rightChild

function preOrder(T):
 if T == null:
  return []
 else:
  return [T.value] + preOrder(T.leftChild) + preOrder(T.rightChild)

function inOrder(T):
 if T == null:
  return []
 else:
  return inOrder(T.leftChild) + [T.value] + inOrder(T.rightChild)

function postOrder(T):
 if T == null:
  return []
 else:
  return postOrder(T.leftChild) + postOrder(T.rightChild) + [T.value]
```

For some T the functions inOrder(T) and preOrder(T) return the following:

inOrder(T) : $[12, 10, 6, 9, 7, 2, 15, 5, 1, 13, 4, 3, 8, 14, 11]$

preOrder(T) : $[5, 2, 10, 12, 9, 6, 7, 15, 13, 1, 3, 4, 14, 8, 11]$

What does postOrder(T) return ?

A. $[12, 6, 10, 7, 15, 2, 9, 1, 4, 13, 8, 11, 14, 3, 5]$
B. $[11, 8, 14, 4, 3, 1, 13, 15, 7, 6, 9, 12, 10, 2, 5]$
C. $[11, 14, 8, 3, 4, 13, 1, 5, 15, 2, 7, 9, 6, 10, 12]$
D. $[12, 6, 7, 9, 10, 15, 2, 1, 4, 8, 11, 14, 3, 13, 5]$
E. Cannot be uniquely determined from given information.

tifr2018 data-structures binary-tree

Answers: Binary Tree

19.3.1 Binary Tree: TIFR2012-B-16

https://gateoverflow.in/25214

✔ Sum of resistors when in series $r_{serial} = r_1 + r_2$

Sum of resistors when in parallel, $\frac{1}{r_{parallel}} = \frac{1}{r1} + \frac{1}{r2}$

$\implies r_{parallel} = \frac{r_1 \cdot r_2}{r_1 + r_2}$

Every node siblings are in parallel and the sum of each level are in series and all node of the last level are tied so all are in series.

So, total sum of resistor = root + total of level 2 + total of level $3 + \ldots +$ total of level $n-1+$ total of level n (in series)

Level	Number of nodes
1	$1(2^0)$
2	$2(2^1)$
3	$4(2^2)$
4	$8(2^3)$
.	
.	
$n-1$	2^{n-2}
n	2^{n-1}

So, total sum of resistor = root + total of level 2 + total of level $3 + \ldots. +$ total of level $n-1+$ total of level n (series)

$= 1 + \frac{r}{2} + \frac{r}{4} + \frac{r}{8} + \frac{r}{16} + \cdots + \frac{r}{2^{n-2}} + \underbrace{(r + r + r + r + \cdots + r)}_{2^{n-1}\ times}$ (here $r = 1$)

$= 1 + \left\{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-2}}\right\}(\text{decreasing GP}) + \underbrace{(1 + 1 + 1 + \ldots + 1)}_{2^{n-1}\ times}$

$\approx (2^n)$ approx

Option (A) is the correct choice.

5 votes

-- Umang Raman (12.2k points)

19.3.2 Binary Tree: TIFR2013-B-13

https://gateoverflow.in/25775

✔ The correct answer is option c, $\Theta(\sqrt{n})$.

$$\begin{aligned} n &= 1+2+3+\cdots+(h+1) \\ &= \frac{(h+1)(h+2)}{2} \\ 2n &= h^2+3h+2 \\ 0 &= h^2+3h+(2-2n) \end{aligned}$$

$$\implies h = \frac{-3+\sqrt{3^2-4\cdot(2-2n)}}{2} \qquad \because h \geq 0$$

$$= \frac{-3+\sqrt{8n+1}}{2}$$

$$\implies h = \Theta(\sqrt{n})$$

70 votes -- Pragy Agarwal (18.3k points)

19.3.3 Binary Tree: TIFR2014-B-1

https://gateoverflow.in/27133

✔ We can construct binary tree by postorder and inorder traversal

There we get 5 leaves of the tree are a, c, e, h, j

So, answer (**C**) 5

13 votes -- srestha (85.3k points)

19.3.4 Binary Tree: TIFR2015-B-4

https://gateoverflow.in/29849

✔ Option is **B.**

for every node -all numbers in one subtree are less than all numbers in the other subtree .

Firstly **chose** a value for **root** -9 elements = 9 **ways**

Now, we hv 8 **elements left** - we hv to **chose** 3 **for left subtree &** 5 **for right subtree.**

Note: Here we can either chose 3 nodes from beginning or end out of 8 elements we have ! $= 2$ **ways**

Now,we hv 3 elements for left subtree & 5 for right(**Consider subtrees of subtree**).

Left Subtree :

whatever way we place , always one side is smaller than other { 6 is smaller than 8 in above example given in question} so, total ways $= 3!$ {three places put one by one} $= 6$ **ways**

Right Subtree :

Right subtree has two more sub-trees ,so that elements on one side should be smaller than other **

Steps :

1. Select one element for root $= 5$ **ways**
2. 4 elements left ,Select one element for left $= 2$ **ways** {Either we can chose from left or right}
3. 3 elements left, for right subtree $= 3!$ ways $= 6$ **ways**

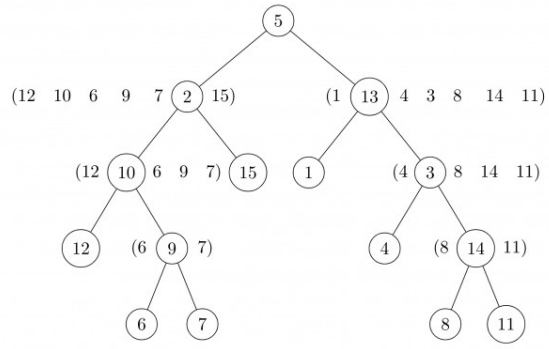
Total ways $= 9 * 2 * 3! * 5 * 2 * 3! = 2^4 * 3^2 * 5 * 9 = 6480 =$ **B (Ans)**

58 votes -- Himanshu Agarwal (12.4k points)

19.3.5 Binary Tree: TIFR2018-B-6

https://gateoverflow.in/179290

✔ **OPTION (D)**



Post Order: 12, 6, 7, 9, 10, 15, 2, 1, 4, 8, 11, 14, 3, 13, 5

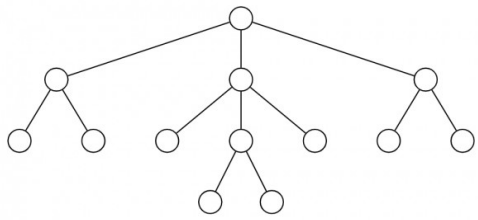
3 votes -- ZAHID WAKEEL (1.6k points)

19.4 Heap (1)

19.4.1 Heap: TIFR2014-B-19

https://gateoverflow.in/27352

Consider the following tree with 13 nodes.



Suppose the nodes of the tree are randomly assigned distinct labels from $\{1, 2, \ldots, 13\}$, each permutation being equally likely. What is the probability that the labels form a min-heap (i.e., every node receives the minimum label in its subtree)?

A. $\left(\frac{1}{6!}\right)\left(\frac{1}{3!}\right)^2$
B. $\left(\frac{1}{3!}\right)^2\left(\frac{1}{2!}\right)^3$
C. $\left(\frac{1}{13}\right)\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^3$
D. $\frac{2}{13}$
E. $\frac{1}{2^{13}}$

tifr2014 heap

Answers: Heap

19.4.1 Heap: TIFR2014-B-19

https://gateoverflow.in/27352

✔ **(C)** is Correct -

Probability $= \frac{Number\ of\ Favorable\ Outcomes(minheaps)}{Total\ Trees(13!)}$

Now, total Min-Heaps :

Firstly we select **minimum element** i.e. 1 **for root = 1 way**

We have 3 subtrees of $3, 6, 3$ sizes respectively

First Subtree with 3 elements:

Steps:

1. Choose 3 elements $= {}^{12}C_3$ **ways**
2. Give minimum to root = **1 way**
3. 2 elements , 2 children(left & right) = **2 ways** { Any of the 2 elements can be given to left or right child}

Second Subtree with 3 elements :

Steps:

1. Choose 3 elements from 9 left elements = ${}^{9}C_3$ **ways**
2. Similar to above assign these = **2 ways**

Subtree with 6 elements: { We already have 6 elements left now}

1 Root , 1left_most child , 1 right_most child , 3 in middle **

Steps:

1. Choose root = **1 way** {minimum}
2. Now 5 elements left, choose leftmost child = **5 ways** (choose anyone)
3. Now 4 elements left, choose right_most child = **4 ways** (choose anyone)
4. Now 3 elements left for middle :

    a) assign root = **1 way**

    b) assign left and right anyone = **2 ways**

For, Total Heaps multiply all ways **above.**

Total probability $= \frac{Total\ Heaps}{Total\ Trees(13!)}$

$= \frac{12! * 2 * 9! * 2 * 5 * 4 * 2}{13! * 3! * 9! * 3! * 6!}$

$= \left(\frac{1}{13}\right)\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^3$

👍 58 votes

-- Himanshu Agarwal (12.4k points)

19.5 Stack (1)

19.5.1 Stack: TIFR2017-B-3

https://gateoverflow.in/95679

We have an implementation that supports the following operations on a stack (in the instructions below, s is the name of the stack).

- isempty(s) : returns True if s is empty, and False otherwise.
- top(s) : returns the top element of the stack, but does not pop the stack; returns null if the stack is empty.
- push(s, x) : places x on top of the stack.
- pop(s) : pops the stack; does nothing if s is empty.

Consider the following code:

```
pop_ray_pop(x):
    s=empty
    for i=1 to length(x):
        if (x[i] == '('):
            push(s, x[i])
        else:
            while (top(s)=='('):
                pop(s)
            end while
            push(s, ')')
        end if
    end for
    while not isempty(s):
        print top(s)
        pop(s)
    end while
```

What is the output of this program when

```
pop_ray_pop("((((()(()))((((")
```

is executed?

A. ((((B.))) ((((C.))) D. ((()())) E. ()()

tifr2017 data-structures stack

Answers: Stack

19.5.1 Stack: TIFR2017-B-3

https://gateoverflow.in/95679

✔ (**D**) is option.

First push ((((on stack. Now when) comes pop all ((((and push) on stack. Now push (((and stack become)(((. Now when) come it pop all (((from stack and new stack become)). Again) comes and stack become))) . Now push ((((on stack and the stack becomes ((()())). Now pop one by one and get option **D** as the answer.

20 votes -- Mehak Sharma (1.2k points)

19.6 Trees (1)

19.6.1 Trees: TIFR2012-B-15

https://gateoverflow.in/25212

Let T be a tree of n nodes. Consider the following algorithm, that constructs a sequence of leaves $u_1, u_2 \ldots$. Let u_1 be some leaf of tree. Let u_2 be a leaf that is farthest from u_1. Let u_3 be the leaf that is farthest from u_2, and, in general, let u_{i+1} be a leaf of T that is farthest from u_i (if there are many choices for u_{i+1}, pick one arbitrarily). The algorithm stops when some u_i is visited again. What can u say about the distance between u_i and u_{i+1}, as $i = 1, 2, \ldots$?

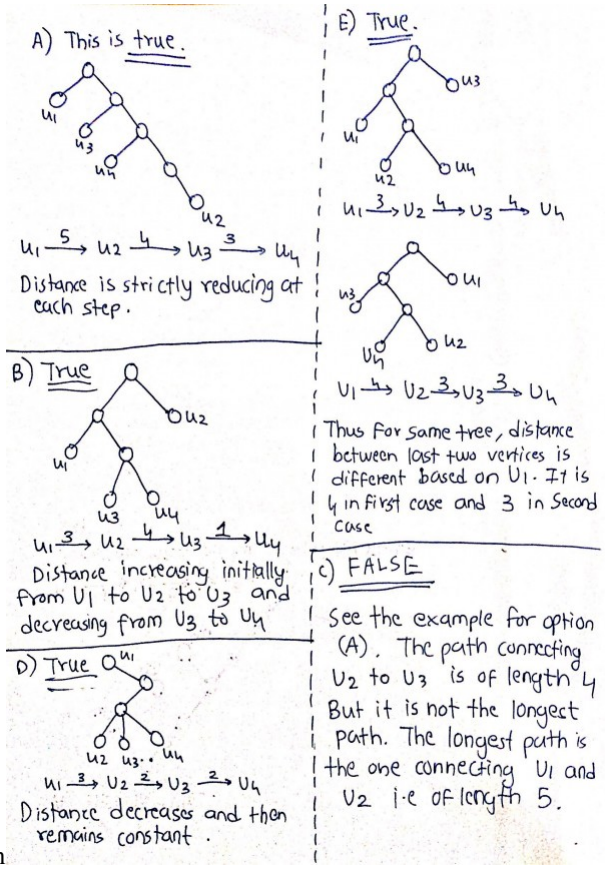
A. For some trees, the distance strictly reduces in each step.
B. For some trees, the distance increases initially and then decreases.
C. For all trees, the path connecting u_2 and u_3 is a longest path in the tree.
D. For some trees, the distance reduces initially, but then stays constant.
E. For the same tree, the distance between the last two vertices visited can be different, based on the choice of the first leaf u_1.

tifr2012 data-structures trees

Answers: Trees

19.6.1 Trees: TIFR2012-B-15

https://gateoverflow.in/25212



Hi. Here is my explanation

11 votes

-- Rohan Vaidya (575 points)

Answer Keys

19.1.1	B	19.2.1	A	19.3.1	A	19.3.2	C	19.3.3	C
19.3.4	B	19.3.5	D	19.4.1	C	19.5.1	D	19.6.1	C

20 Programming and DS: Programming (11)

Programming in C. Recursion.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	0	2	1	2	2	1	2	2	1	0	1.4	2
2 Marks Count	2	2	1	3	3	4	4	2	2	2	2.6	4
Total Marks	4	6	5	8	8	9	10	6	5	4	6.7	10

20.0.1 TIFR2011-B-24

https://gateoverflow.in/20402

Consider the program

```
x:=0; y:=0; (r1:=x; r2:=x; y:= if r1 = r2 then 1 ‖ r3:= y; x:= r3)
```

Note that $\|$ denotes the parallel operator. In which of the following cases can the program possibly result in a final state with $r1 = 0; r2 = r3 = 1$.

A. Such a transformation is not possible in Java.
B. Such a program transformation is possible in Java.
C. Possible in Pascal when the compiler appropriately translates the $\|$ operator to interleaved pascal statements.
D. Possible in all sequential programming languages when the compiler appropriately translates the $\|$ operator to interleaved statements in the sequential language.
E. None of the above.

tifr2011 programming non-gate

20.1 Loop Invariants (4)

20.1.1 Loop Invariants: TIFR2010-B-30

https://gateoverflow.in/19042

Consider the following program for summing the entries of the array b: array $[0..N-1]$ of integers, where N is a positive integer. (The symbol '<>' denotes 'not equal to').

```
var
    i, s: integer;
Program
    i:= 0;
    s:= 0;
[*] while i <> N do
        s := s + b[i];
        i := i + 1;
    od
```

Which of the following gives the invariant that holds at the beginning of each loop, that is, each time the program arrives at point $[*]$?

A. $s = \sum_{j=0}^{N} b[j]$ & $0 \leq i \leq N$
B. $s = \sum_{j=0}^{i-1} b[j]$ & $0 \leq i < N$
C. $s = \sum_{j=0}^{i} b[j]$ & $0 < i \leq N$
D. $s = \sum_{j=1}^{N} b[j]$ & $0 \leq i < N$
E. $s = \sum_{j=0}^{i-1} b[j]$ & $0 \leq i \leq N$

tifr2010 programming loop-invariants

20.1.2 Loop Invariants: TIFR2010-B-37

https://gateoverflow.in/19251

Consider the program where a, b are integers with $b > 0$.

```
x:=a; y:=b; z:=0;
while y > 0 do
    if odd (x) then
        z:= z + x;
        y:= y - 1;
    else y:= y % 2;
        x:= 2 * x;
    fi
```

Invariant of the loop is a condition which is true before and after every iteration of the loop. In the above program the loop invariant is given by

$$0 \leq y \text{ and } z + x * y = a * b$$

Which of the following is true of the program?

A. The program will not terminate for some values of a, b.
B. The program will terminate with $z = 2^b$
C. The program will terminate with $z = a * b$.
D. The program will not terminate for some values of a, b but when it does terminate, the condition $z = a * b$ will hold.
E. The program will terminate with $z = a^b$

tifr2010 programming loop-invariants

20.1.3 Loop Invariants: TIFR2017-B-5

https://gateoverflow.in/95683

Consider the following psuedocode fragment, where y is an integer that has been initialized.

```
int i=1
int j=1
while (i<10):
    j=j*i
    i=i+1
    if (i==y):
        break
    end if
end while
```

Consider the following statements:

i. $(i == 10)$ or $(i == y)$
ii. If $y > 10$, then $i == 10$
iii. If $j = 6$, then $y == 4$

Which of the above statements is/are TRUE at the end of the while loop? Choose from the following options.

A. i only B. iii only C. ii and iii only D. i, ii, and iii E. None of the above

tifr2017 programming loop-invariants

20.1.4 Loop Invariants: TIFR2019-B-9

https://gateoverflow.in/280486

Consider the following program fragment:

```
var x, y: integer;
x := 1; y := 0;
while y < x do
begin
   x := 2*x;
   y := y+1
end;
```

For the above fragment , which of the following is a loop invariant ?

A. $x = y + 1$
B. $x = (y + 1)^2$
C. $x = (y + 1)2^y$
D. $x = 2^y$
E. None of the above, since the loop does not terminate

tifr2019 programming loop-invariants

Answers: Loop Invariants

20.1.1 Loop Invariants: TIFR2010-B-30

https://gateoverflow.in/19042

✔ Whenever we encounter the $[*]$, the variable s holds the sum of all elements $b[0]$ to $b[i-1]$.

When we first enter the loop, $i = 0$, and s doesn't have any elements summed up.

When we last enter the loop, $i = (N-1)$ and s contains the sum of elements $b[0]$ through $b[N-2]$.

We leave the loop when $i = N$, and s gets the sum of elements $b[0]$ to $b[N-1]$

The only option that matches this behavior is **option E.**

$$s = \sum_{j=0}^{i-1} b[j] \;\&\; 0 \leq i \leq N$$

👍 24 votes

-- Pragy Agarwal (18.3k points)

20.1.2 Loop Invariants: TIFR2010-B-37

https://gateoverflow.in/19251

✔ if x is odd then -

- $z = a * b$ will be the o/p

if x is even then -

case 1: if y is even then $x = 2 * x$ and $z = 0$ will be o/p

case 2: y is odd then loop will not terminate .

EDIT - When x and y are even, for some values of x, condition z= ab is not holding.
For example - when $a = 4\ and\ b = 4$, after loop terminates, $z = 8\ and\ ab = 16$.
$8 \neq 16$.
So, **A** should be the correct answer.

👍 13 votes

-- Saurav Shrivastava (1.3k points)

20.1.3 Loop Invariants: TIFR2017-B-5

https://gateoverflow.in/95683

✔ Consider the situations when loop gets broken -

1. If i becomes 10, or i becomes equal to y. $\implies$ (i) is correct.
2. If $y > 10$, the control will come out of the loop when i becomes 10. $\implies$ (ii) is correct.
3. During 1^{st} iteration, $j = 1, i = 2$
   - 2^{nd} iteration, $j = 2, i = 3$
   - 3^{rd} iteration, $j = 6, i = 4$

4^{th} iteration must not occur because j will become 24. Hence during 3^{rd} iteration break statement must have been executed and hence, y must be equal to 4 (equal to i during the 3^{rd} iteration).

This implies (iii) is also correct.

Hence, **Option D** is correct.

👍 20 votes

-- tarun_svbk (1.4k points)

20.1.4 Loop Invariants: TIFR2019-B-9

https://gateoverflow.in/280486

✔ A **loop invariant** is any condition which is true for the **start** of the loop, for **every iteration** of the loop and for the **exit** of the loop.

Before 1st iterations values of (x, y) are $(1, 0)$

After 1st iteration values of x and y are $(2, 1)$

After 2nd iteration values of x and y are $(4, 2)$

After 3rd iteration values of x and y are $(8, 3)$

$\vdots$

After nth iteration values of x and y are $(2^n, n)$

This loop will not terminate and that means we need not worry about the condition when loop exits ($p \rightarrow q$ is TRUE if p is false) So E is false

D is the right option

16 votes -- Mk Utkarsh (25.7k points)

20.2 Parameter Passing (2)

20.2.1 Parameter Passing: TIFR2011-B-32

https://gateoverflow.in/20619

Various parameter passing mechanisms have been in used in different programming languages. Which of the following statements is true?

A. Call by value result is used in language Ada.
B. Call by value result is the same as call by name.
C. Call by value is the most robust.
D. Call by reference is the same as call by name.
E. Call by name is the most efficient.

tifr2011 programming parameter-passing

20.2.2 Parameter Passing: TIFR2019-B-8

https://gateoverflow.in/280487

Consider the following program fragment:

```
var a,b : integer;
procedure G(c,d: integer);
begin
    c:=c-d;
    d:=c+d;
    c:=d-c
end;
a:=2;
b:=3;
G(a,b);
```

If both parameters to G are passed by reference, what are the values of a and b at the end of the above program fragment ?

A. $a = 0$ and $b = 2$ B. $a = 3$ and $b = 2$ C. $a = 2$ and $b = 3$ D. $a = 1$ and $b = 5$ E. None of the above

tifr2019 programming parameter-passing

Answers: Parameter Passing

20.2.1 Parameter Passing: TIFR2011-B-32

https://gateoverflow.in/20619

e. Call by name is the most efficient.

http://stackoverflow.com/questions/838079/what-is-pass-by-name-and-how-does-it-work-exactly

2 votes -- zambus (299 points)

20.2.2 Parameter Passing: TIFR2019-B-8

https://gateoverflow.in/280487

✔ Option B is correct as for the given inputs the program fragments does swapping of the values. Call-by-reference implies that the swapped values are reflected in the calling function as well.

PS: Procedure G is not doing proper swapping for all input values as for negative values, there is a chance of underflow in $c - d$.

10 votes -- Arjun Suresh (334k points)

20.3 Programming In C (2)

20.3.1 Programming In C: TIFR2018-A-7

https://gateoverflow.in/179276

```
Consider the following function definition.
void greet(int n)
{
    if(n>0)
```

```
    {
        printf("hello");
        greet(n-1);
    }
    printf("world");
}
```

If you run greet(n) for some non-negative integer n, what would it print?

A. n times "hello", followed by n+1 times "world"
B. n times "hello", followed by n times "world"
C. n times "helloworld"
D. n+1 times "helloworld"
E. n times "helloworld", followed by "world"

tifr2018 programming-in-c

20.3.2 Programming In C: TIFR2019-B-6

https://gateoverflow.in/280489

Given the following pseudocode for function $\text{printx}()$ below, how many times is x printed if we execute $\text{printx}(5)$?

```
void printx(int n)  {
    if(n==0){
        printf("x");
}
for(int i=0;i<=n-1;++i){
    printx(n-1);
}
}
```

A. 625 B. 256 C. 120 D. 24 E. 5

tifr2019 programming programming-in-c

Answers: Programming In C

20.3.1 Programming In C: TIFR2018-A-7

https://gateoverflow.in/179276

✔ Take n = 2:

```
void greet(2)
{
    if(2>0)
    {
        printf("hello");
//print number 1
        greet(1);
    }
    printf("world");
//3rd time print
}
```

```
void greet(1)
{
    if(1>0)
    {
        printf("hello");
//print number 2
        greet(0);
    }
    printf("world");
//2nd time print
}
```

```
void greet(0)
{
 if(0>0) fail {}
 printf("world");
//1st time print
}
```

Output = hellohelloworldworldworld i.e. n times hello and $n+1$ times world.

Correct Answer: A

7 votes

-- Anu007 (14.4k points)

20.3.2 Programming In C: TIFR2019-B-6 https://gateoverflow.in/280489

✔ If we denote the number of times x being printed for input n by $C(n)$ we can write

$C(n) = n \times C(n-1)$ (Since the for loop runs n times and each iteration does a recursive call to $printx(n-1)$)

$C(0) = 1$

So, $C(5) = 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120.$

15 votes

-- Arjun Suresh (334k points)

20.4 Recursion (2)

20.4.1 Recursion: TIFR2010-B-31 https://gateoverflow.in/26484

Consider the following computation rules. **Parallel-outermost rule**: Replace all the outermost occurrences of F (i.e., all occurrences of F which do not occur as arguments of other F's) simultaneously. **Parallel - innermost rule**: Replace all the innermost occurrences of F (i.e.,all occurrences of F with all arguments free of F's) simultaneously. Now consider the evaluations of the recursive program over the integers.

```
F(x, y) <== if x = 0 then 0 else
            [ F(x + 1, F(x, y)) * F(x - 1, F(x, y))]
```

where the multiplication functions * is extended as follows:

```
0 * w & w * 0 are 0
a * w & w * a are w (for any non-zero integer a)
w * w is w
```

We say that $F(x,y) =$ w when the evaluation of $F(x,y)$ does not terminate. Computing $F(1,0)$ using the parallel - innermost and parallel - outermost rule yields

A. w and 0 respectively
B. 0 and 0 respectively
C. w and w respectively
D. w and 1 respectively
E. none of the above

tifr2010 programming recursion

20.4.2 Recursion: TIFR2011-B-38 https://gateoverflow.in/20923

Consider the class of recursive and iterative programs. Which of the following is false?

A. Recursive programs are more powerful than iterative programs.
B. For every iterative program there is an equivalent recursive program.
C. Recursive programs require dynamic memory management.
D. Recursive programs do not terminate sometimes.
E. Iterative programs and recursive programs are equally expressive.

tifr2011 recursion programming

Answers: Recursion

20.4.1 Recursion: TIFR2010-B-31 https://gateoverflow.in/26484

Decoding the question :-

Parallel-outermost rule :-Replace all the outermost occurrences of F.Now what does outermost occurrence mean? it means those occurrences of F which are not appeared inside F as argument.

$F(1,0)$ it's outermost occurrence as didn't appear inside any F.So replace it.See out recurrence relation this will be replaced by $F(2,F(1,0)) * F(0,F(1,0))$ Now here again Red color F and blue color F are outmost occurrences for each other as they are not occurred as argument of some other F.(other F which are uncolored are not included as 'outermost occurrence' as they are appeared as argument in some F)

So now replace only outermost occurrences(simultaneously). We don't have to replace other occurrences (uncolored) of F.

So overall it will become

$[F(3,F(2,F(1,0))) * F(1,F(2,F(1,0)))] * 0$ //$F(0,F(1,0))$ will become 0

Now we can observe some part will never terminate and some part will become 0 so overall we can say it would be something like $W * 0$ and which would result into 0. So i think Applying parallel-outermost rule will result into 0 .

Parallel-innermost rule:- Replace all the innermost occurrences of F. Here innermost occurrences mean those whose arguments are free from F.

Like in $F(1,0)$ argument of F are $1,0$ which are free from F so replace it by recurrence relation and hence it would become

$F(2,F(1,0)) * F(0,F(1,0))$ Here i colored another innermost occurrences of F which we have to replace again.uncolored F are not innermost occurrences as they contain such arguments which are not themselves function of F.

So overall it would become

$F(2,[F(2,F(1,0)) * F(0,F(1,0))]) * F(0,[F(2,F(1,0)) * F(0,F(1,0))])$

Here we can observe this that everytime we have to expand only function $F(1,0)$ because that's the only function whose arguments are free from any F So in that way , overall it would become $W * W$ type which would result into W.

So option A is the correct answer.

13 votes -- Rupendra Choudhary (11.4k points)

20.4.2 Recursion: TIFR2011-B-38

https://gateoverflow.in/20923

Answer is **option E.**

Computable function: those which can be incorporated in a program using for/while loops.

Total Function: Defined for all possible inputs

Well Defined: if its definition assigns it a unique value.

It was a belief in early 1900s that every Computable function was also Primitively Recursive. But the discovery of Ackermann function provided a counter to it.

The class of primitive recursive functions is a small subclass of the class of recursive functions. This means that there are some functions which are Well-Defined Total Functions and are Computable BUT **Not** primitively recursive; eg. Ackermann function.

This makes all options from option A to option D as True.

But **option E** as FALSE**.** As iterative programs are equivalent to only Primitively Recursive class.

21 votes -- Amar Vashishth (25.2k points)

Answer Keys

20.0.1	B	20.1.1	E	20.1.2	A	20.1.3	D	20.1.4	D
20.2.1	E	20.2.2	B	20.3.1	A	20.3.2	C	20.4.1	A
20.4.2	E								

21 Theory of Computation (28)

Regular expressions and finite automata, Context-free grammars and push-down automata, Regular and context-free languages, Pumping lemma, Turing machines and undecidability.

Mark Distribution in Previous GATE

Year	2021-1	2021-2	2020	2019	2018	2017-1	2017-2	2016-1	2016-2	Minimum	Average	Maximum
1 Mark Count	2	3	3	2	2	2	3	3	3	2	2.5	3
2 Marks Count	3	4	3	3	3	5	3	3	3	3	3.3	5
Total Marks	8	11	9	8	8	12	9	9	9	8	9.2	12

21.1 Closure Property (2)

21.1.1 Closure Property: TIFR2013-B-11

https://gateoverflow.in/25772

Which of the following statements is FALSE?

A. The intersection of a context free language with a regular language is context free.
B. The intersection of two regular languages is regular.
C. The intersection of two context free languages is context free
D. The intersection of a context free language and the complement of a regular language is context free.
E. The intersection of a regular language and the complement of a regular language is regular.

tifr2013 theory-of-computation closure-property

21.1.2 Closure Property: TIFR2014-B-14

https://gateoverflow.in/27321

Which the following is FALSE?

A. Complement of a recursive language is recursive.
B. A language recognized by a non-deterministic Turing machine can also be recognized by a deterministic Turing machine.
C. Complement of a context free language can be recognized by a Turing machine.
D. If a language and its complement are both recursively enumerable then it is recursive.
E. Complement of a non-recursive language can never be recognized by any Turing machine.

tifr2014 theory-of-computation closure-property

Answers: Closure Property

21.1.1 Closure Property: TIFR2013-B-11

https://gateoverflow.in/25772

✔ Context language are not closed under intersection so option **c.**

19 votes

-- Umang Raman (12.2k points)

21.1.2 Closure Property: TIFR2014-B-14

https://gateoverflow.in/27321

✔

A. True. For a recursive language, we have a TM which says for any string "w", "yes" if it belongs to the language and "no" if it does not. For the complement of a recursive language we just have to reverse 'yes' and 'no' conditions and this is possible with a slight modification to the original TM. So, the new language is also recursive.
B. True. Non-determinism does not add any recognizing power to a Turing machine though it can affect the time complexity in solving a problem.
C. True. Complement of CFL is recursive. and recursive language is recognized by Turing machine. So, complement of a context free language can be recognized by TM
D. True. If a language is r.e., we have a TM which says "yes" whenever a given string belongs to L. Similarly, if its complement is r.e., we have a TM which says "yes", whenever a string does not belong to the language. So, combining both we can always say "yes" if a string belongs to the language and "no" if a string does not belong to L $\implies$ L is recursive.
E. False.
Non-recursive Language can be:- (a) **r.e.** or (b) **Non r.e.**
As seen in option D, if the language is RE but not recursive, its complement cannot be r.e. (recognized by a TM). But, if the language is not r.e. its complement may be r.e. (which **Can be recognized by TM).** ***option E is false***.
Example:-

$L = \{\langle M, w\rangle \mid M \text{ is a TM and it does not halt on string } w\}$. //Non RE (complement of halting problem)
$L^c = \{\langle M, w\rangle \mid M \text{ is a TM and it halts on string } w\}$ //RE but not Recursive //recognized by TM (halting problem)

So, answer is (E).

28 votes -- srestha (85.3k points)

21.2 Decidability (3)

21.2.1 Decidability: TIFR2010-B-25

https://gateoverflow.in/18745

Which of the following problems is decidable? (Here, CFG means context free grammar and CFL means context free language.)

A. Given a CFG G, find whether $L(G) = R$, where R is regular set.
B. Given a CFG G, find whether $L(G) = \{\}$.
C. Find whether the intersection of two CFLs is empty.
D. Find whether the complement of CFL is a CFL.
E. Find whether CFG G_1 and CFG G_2 generate the same language, i.e, $L(G_1) = L(G_2)$.

tifr2010 theory-of-computation context-free-languages decidability

21.2.2 Decidability: TIFR2011-B-25

https://gateoverflow.in/20404

Let A_{TM} be defined as follows:

$A_{TM} = \{\langle M, w\rangle \mid \text{ The Turing machine } M \text{ accepts the word } w\}$

And let L be some $\text{NP} - \text{complete}$ language. Which of the following statements is FALSE?

A. $L \in \text{NP}$
B. Every problem in NP is polynomial time reducible to L.
C. Every problem in NP is polynomial time reducible to A_{TM}.
D. Since L is $\text{NP} - \text{complete}$, A_{TM} is polynomial time reducible to L.
E. $A_{TM} \notin \text{NP}$.

tifr2011 theory-of-computation decidability

21.2.3 Decidability: TIFR2020-B-2

https://gateoverflow.in/333121

Consider the following statements.

1. The intersection of two context-free languages is always context-free
2. The super-set of a context-free languages is never regular
3. The subset of a decidable language is always decidable
4. Let $\Sigma = \{a, b, c\}$. Let $L \subseteq \Sigma$ be the language of all strings in which either the number of occurrences of a is the same as the number of occurrences of b OR the number occurrences of b is the same as the number of occurrences of c. Then, L is not context-free.

Which of the above statements are true?

A. Only (1)
B. Only (1) and (2)
C. Only $(1), (2)$ and (3)
D. Only (4)
E. None of $(1), (2), (3), (4)$ are true.

tifr2020 theory-of-computation context-free-languages decidability

Answers: Decidability

21.2.1 Decidability: TIFR2010-B-25

https://gateoverflow.in/18745

✔

A. We don't have any standard algorithm to change CFG into CFL
from a given CFG deciding a language is finite is decidable but regular its undecidable (Ref: http://gatecse.in/wiki/Grammar:_Decidable_and_Undecidable_Problems)
B. From a given given CFG we can determine the CFL and CFL emptiness is Decidable.
C. Intersection of two CFL is undecidable coz it is not closed under intersection.

D. CFL is not closed under Complement so its undecidable.
E. CFL is not closed under equivalence so it is undecidable to compare 2 language.

Therefore, **B** is decidable and **A,C,D** and **E** are undecidable.

References

18 votes -- Umang Raman (12.2k points)

21.2.2 Decidability: TIFR2011-B-25 https://gateoverflow.in/20404

✔

A_{TM} is the language of the Halting Problem. It is undecidable, but Recursively Enumerable.
L is NPC

a. True. Any language in NPC is also in NP by definition.
b. True. By definition, any problem in NP is polynomial time reducible to any NPC problem.
c. True. A_{TM} is undecidable. Any language that is decidable is polynomial time reducible to A_{TM}!
d. **False. A_{TM} is undecidable. No Turing Machine can guarantee an answer in a finite time, let alone a polynomial time.**
e. True. A_{TM} is undecidable. It is certainly not in NP.

Hence, the correct answer is option d.

23 votes -- Pragy Agarwal (18.3k points)

21.2.3 Decidability: TIFR2020-B-2 https://gateoverflow.in/333121

1) CFL is not closed under intersection.
2) $\sum^*$ is the superset of every CFL which is regular
3) Not true
4) It is non deterministic CFL.

Option e) is correct answer

1 votes -- Ashwani Kumar (13.1k points)

21.3 Identify Class Language (10)

21.3.1 Identify Class Language: TIFR2010-B-22 https://gateoverflow.in/18622

Let L consist of all binary strings beginning with a 1 such that its value when converted to decimal is divisible by 5. Which of the following is true?

A. L can be recognized by a deterministic finite state automaton.
B. L can be recognized by a non-deterministic finite state automaton but not by a deterministic finite state automaton.
C. L can be recognized by a deterministic push-down automaton but not by a non-deterministic finite state automaton.
D. L can be recognized by a non-deterministic push-down automaton but not by a deterministic push-down automaton.
E. L cannot be recognized by any push-down automaton.

tifr2010 theory-of-computation identify-class-language

21.3.2 Identify Class Language: TIFR2010-B-35 https://gateoverflow.in/19247

Consider the following languages over the alphabet $\{0,1\}$.

$$L1 = \{x.x^R \mid x \in \{0,1\}^*\}$$
$$L2 = \{x.x \mid x \in \{0,1\}^*\}$$

Where x^R is the reverse of string x; e.g. $011^R = 110$. Which of the following is true?

A. Both $L1$ and $L2$ are regular.
B. $L1$ is context-free but not regular where as $L2$ is regualr.

C. Both $L1$ and $L2$ are context free and neither is regular.
D. $L1$ is context free but $L2$ is not context free.
E. Both $L1$ and $L2$ are not context free.

tifr2010 theory-of-computation identify-class-language

21.3.3 Identify Class Language: TIFR2012-B-18

https://gateoverflow.in/25216

Let a^i denote a sequence $a.a...a$ with i letters and let $\aleph$ be the set of natural numbers $1, 2, \ldots$. Let $L_1 = \left\{a^i b^{2i} \mid i \in \aleph\right\}$ and $L_2 = \left\{a^i b^{i^2} \mid i \in \aleph\right\}$ be two languages. Which of the following is correct?

A. Both L_1 and L_2 are context-free languages.
B. L_1 is context-free and L_2 is recursive but not context-free.
C. Both L_1 and L_2 are recursive but not context-free.
D. L_1 is regular and L_2 is context-free.
E. Complement of L_2 is context-free.

tifr2012 theory-of-computation identify-class-language

21.3.4 Identify Class Language: TIFR2014-B-13

https://gateoverflow.in/27320

Let L be a given context-free language over the alphabet $\{a, b\}$. Construct L_1, L_2 as follows. Let $L_1 = L - \{xyx \mid x, y \in \{a, b\}^*\}$, and $L_2 = L \cdot L$. Then,

A. Both L_1 and L_2 are regular.
B. Both L_1 and L_2 are context free but not necessarily regular.
C. L_1 is regular and L_2 is context free.
D. L_1 and L_2 both may not be context free.
E. L_1 is regular but L_2 may not be context free.

tifr2014 theory-of-computation identify-class-language

21.3.5 Identify Class Language: TIFR2015-B-8

https://gateoverflow.in/29865

Let $\sum_1 = \{a\}$ be a one letter alphabet and $\sum_2 = \{a, b\}$ be a two letter alphabet. A language over an alphabet is a set of finite length words comprising letters of the alphabet. Let L_1 and L_2 be the set of languages over $\sum_1$ and $\sum_2$ respectively. Which of the following is true about L_1 and L_2:

A. Both are finite.
B. Both are countably infinite.
C. L_1 is countable but L_2 is not.
D. L_2 is countable but L_1 is not.
E. Neither of them is countable.

tifr2015 identify-class-language

21.3.6 Identify Class Language: TIFR2017-B-14

https://gateoverflow.in/95825

Consider the following grammar G with terminals $\{[,]\}$, start symbol S, and non-terminals $\{A, B, C\}$:

$$S \rightarrow AC \mid SS \mid AB$$

$$C \rightarrow SB$$

$$A \rightarrow [$$

$$B \rightarrow]$$

A language L is called prefix-closed if for every $x \in L$, every prefix of x is also in L. Which of the following is FALSE?

A. $L(G)$ is context free
B. $L(G)$ is infinite
C. $L(G)$ can be recognized by a deterministic push down automaton
D. $L(G)$ is prefix-closed
E. $L(G)$ is recursive

tifr2017 theory-of-computation identify-class-language

21.3.7 Identify Class Language: TIFR2017-B-4

https://gateoverflow.in/95680

Let L be the language over the alphabet $\{1, 2, 3, (,)\}$ generated by the following grammar (with start symbol S, and non-terminals $\{A, B, C\}$):

$S \rightarrow A\ B\ CA \rightarrow (B \rightarrow 1\ B \mid 2\ B \mid 3\ BB \rightarrow 1 \mid 2 \mid 3C \rightarrow)$

Then, which of the following is TRUE?

A. L is finite
B. L is not recursively enumerable
C. L is regular
D. L contains only strings of even length
E. L is context-free but not regular

tifr2017 theory-of-computation identify-class-language

21.3.8 Identify Class Language: TIFR2018-B-11

https://gateoverflow.in/179295

Consider the language $L \subseteq \{a, b, c\}^*$ defined as

$$L = \{a^p b^q c^r : p = q \quad or \quad q = r \quad or \quad r = p\}.$$

Which of the following answer is TRUE about the complexity of this language?

A. L is regular but not context-free
B. L is context-free but not regular
C. L is decidable but not context free
D. The complement of L, defined as $\overline{L} = \{a, b, c\}^* \backslash L$, is regular.
E. L is regular, context-free and decidable

tifr2018 identify-class-language theory-of-computation

21.3.9 Identify Class Language: TIFR2018-B-14

https://gateoverflow.in/179298

Define the language $\text{INFINITE}_{DFA} \equiv \{(A) \mid A \text{ is a DFA and } L(A) \text{ is an infinite language}\}$, where (A) denotes the description of the deterministic finite automata (DFA).Then which of the following about INFINITE_{DFA} is TRUE:

A. It is regular.
B. It is context-free but not regular.
C. It is Turing decidable (recursive).
D. It is Turing recognizable but not decidable.
E. Its complement is Turing recognizable but it is not decidable.

tifr2018 identify-class-language

21.3.10 Identify Class Language: TIFR2019-B-10

https://gateoverflow.in/280485

Let the language D be defined in the binary alphabet $\{0, 1\}$ as follows:

$D := \{w \in \{0, 1\}^* \mid \text{substrings 01 and 10 occur an equal number of times in w}\}$

For example , $101 \in D$ while $1010 \notin D$. Which of the following must be TRUE of the language D ?

A. D is regular
B. D is context-free but not regular
C. D is decidable but not context-free
D. D is decidable but not in NP
E. D is undecidable

tifr2019 theory-of-computation identify-class-language

Answers: Identify Class Language

21.3.1 Identify Class Language: TIFR2010-B-22

https://gateoverflow.in/18622

✔ L can be recognized by a dfa. we have a dfa to accept all such strings which when interpretated as decimal number are divisible by n. Where n can be anything the dfa of such can be made by a trick.

States are equal to possible remainders

Transition Table

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

If you can see the symmetry in it. write states and make fill like $q_0\ q_1\ q_2\ q_3\ q_4\ q_0 \ldots$

Now, it is saying that it has to always start with 1 which the above dfa will not satisfy so make it a nfa by making a transition from q_0 on zero to a new dead state. now you have a nfa reduce it which will result in a deterministic **DFA** .

So, option is **A.**

19 votes -- Ravi Singh (11.8k points)

21.3.2 Identify Class Language: TIFR2010-B-35

https://gateoverflow.in/19247

✔ $\mathrm{L1} = \{\mathrm{x}.\mathrm{x}^{\mathrm{R}} \mid \mathrm{x} \in \{0,1\}^*\}$ Its even palindrome so it is CFL
$\mathrm{L2} = \{\mathrm{x}.\mathrm{x} \mid \mathrm{x} \in \{0,1\}^*\}$ Its string matching so it is CSL but not CFL

Option (D) $\mathrm{L1}$ is context free but $\mathrm{L2}$ is not context free.

18 votes -- Umang Raman (12.2k points)

21.3.3 Identify Class Language: TIFR2012-B-18

https://gateoverflow.in/25216

✔ L_1 - CFL, $S \rightarrow aSbb \mid abb$

L_2 - Not CFL ,we can't count i^2 using **CFL.**

a. False because L_2 is not **CFL.**
b. **True**. L_2 is recursive
c. False because L_1 is **CFL.**
d. False, L_1 not regular.
e. False, as complement of L_2 is also not context free. It still need to computer i^2 for checking for inequality.

Answer is **B.**

14 votes -- Akash Kanase (36.1k points)

21.3.4 Identify Class Language: TIFR2014-B-13

https://gateoverflow.in/27320

✔ L is a context free language over $\{a,b\}$
$L_1 = L - \{xyx \mid x,y \in \{a,b\}^*\}$
$= L - \{\text{all strings over} \{a,b\}\}$ [Note: all strings can be generated from y by putting $x = \epsilon$]
$= L - (a+b)^* = \{\}$, empty set. [Note : $L_1 - L_2 = \{\text{string in } L_1 \text{ but not in L2 }\}$]
So , L_1 is a Regular Language.
L is a context free language over $\{a,b\}$
$L_2 = L \cdot L$
Context free languages are closed under Concatenation.
So, L_2 is Context Free Language.

Option **C** is correct.

52 votes -- Praveen Saini (42k points)

21.3.5 Identify Class Language: TIFR2015-B-8

https://gateoverflow.in/29865

✔ Languages over alphabet set are uncountable so, answer should be (**E**).

15 votes -- Pooja Palod (24.1k points)

21.3.6 Identify Class Language: TIFR2017-B-14

https://gateoverflow.in/95825

✔ The given grammar generates balanced parenthesis.

Lets take a smallest string : [[ ]] (say x)

Prefixes of x are : [, [[, [[ ]

BUT they don't belong to the language generated by the given grammar.

So, the answer will be **Option D.**

Correct me if am wrong.

22 votes -- sarveswara rao vangala (1.4k points)

21.3.7 Identify Class Language: TIFR2017-B-4

https://gateoverflow.in/95680

✔ Language accepted by that grammar is:
$((1+2+3)^+)$
Hence, regular!

Correct Answer: C

20 votes -- Motamarri Anusha (8.6k points)

21.3.8 Identify Class Language: TIFR2018-B-11

https://gateoverflow.in/179295

$L1 = a^p b^q c^r : p = q$
$L2 = a^p b^q c^r : q = r$
$L3 = a^p b^q c^r : r = p$
$L = L1 \cup L2 \cup L3$
$L1, L2, L3$ **are CFL**

We know that union of CFL is CFL
Hence, L is Context free language

Answer is option B.

18 votes -- Rameez Raza (1.8k points)

21.3.9 Identify Class Language: TIFR2018-B-14

https://gateoverflow.in/179298

We can directly eliminate option A and B as we need to do some computation which is beyond (in this case) the capacity of the FSM and the PDA.

Let's understand what does 'encoding' of a DFA means. It is nothing but a string of 1's and 0's which on reading, you will get the complete idea about DFA, that is how many states it has, which is the start stare, which are final states, all transitions etc. Basically, we are 'encoding' whatever we know about the given DFA in 0's and 1's. There is no fixed procedure for how this encoding should be. You can do it in whichever way you like.

Now the question is on taking a string of 1's and 0's (The description of a DFA) as the input, can we write a C program which will print "YES" when the FSM corresponding to that string accepts the infinite language and prints "NO" otherwise. The answer is YES we can write such a program. We will be checking for the "loops" in the FSM corresponding to the given string as the existence of a loop and a valid final state implies that the FSM accepts infinite language. (We can use the logic of finding a cycle in the directed graphs which can be done in O(V+E) time for the graphs.) So this is a decidable problem.

10 votes -- Utkarsh Joshi (5.3k points)

21.3.10 Identify Class Language: TIFR2019-B-10

https://gateoverflow.in/280485

✔ Regular expression for the language D$= 0(0+1)^*0 + 1(0+1)^*1$

DFA: http://flolac.iis.sinica.edu.tw/flolac11/lib/exe/logic_computation_theory_hw2s.pdf

References

13 votes -- Manoja Rajalakshmi Aravindakshan (7.7k points)

21.4 Recursive And Recursively Enumerable Languages (2)

21.4.1 Recursive And Recursively Enumerable Languages: TIFR2010-B-40 https://gateoverflow.in/19048

Which of the following statement is FALSE?

A. All recursive sets are recursively enumerable.
B. The complement of every recursively enumerable sets is recursively enumerable.
C. Every Non-empty recursively enumerable set is the range of some totally recursive function.
D. All finite sets are recursive.
E. The complement of every recursive set is recursive.

tifr2010 theory-of-computation recursive-and-recursively-enumerable-languages

21.4.2 Recursive And Recursively Enumerable Languages: TIFR2012-B-19 https://gateoverflow.in/25218

Which of the following statements is TRUE?

A. Every turning machine recognizable language is recursive.
B. The complement of every recursively enumerable language is recursively enumerable.
C. The complement of a recursive language is recursively enumerable.
D. The complement of a context-free language is context-free.
E. The set of turning machines which do not halt on empty input forms a recursively enumerable set.

tifr2012 theory-of-computation recursive-and-recursively-enumerable-languages

Answers: Recursive And Recursively Enumerable Languages

21.4.1 Recursive And Recursively Enumerable Languages: TIFR2010-B-40 https://gateoverflow.in/19048

✔ (**B**) The complement of every recursively enumerable sets is recursively enumerable.

Because RE language are not closed under complement.

15 votes -- Umang Raman (12.2k points)

21.4.2 Recursive And Recursively Enumerable Languages: TIFR2012-B-19 https://gateoverflow.in/25218

✔

a. False. Turing recognizable language are recursive enumerable and recursive set is a **proper subset** of it.
b. False, Complement of r.e. **need not** be r.e.
c. **True**. Complement of recursive language is recursive and every recursive language is recursive enumerable.
d. False. Complement of CFL **need not** be CFL (but is guaranteed to be a CSL).
e. False. We cannot make a Turing machine to determine if a given Turing machine does not halt on empty string (for that matter any string). But the complement of this is r.e. (of course not recursive)

22 votes -- srestha (85.3k points)

21.5 Regular Expressions (5)

21.5.1 Regular Expressions: TIFR2010-B-34 https://gateoverflow.in/19047

Let r, s, t be regular expressions. Which of the following identities is correct?

A. $(r+s)^* = r^*s^*$
B. $r(s+t) = rs+t$
C. $(r+s)^* = r^* + s^*$
D. $(rs+r)^*r = r(sr+r)^*$

E. $(r^*s)^* = (rs)^*$

tifr2010 theory-of-computation regular-expressions

21.5.2 Regular Expressions: TIFR2015-B-7

https://gateoverflow.in/29861

Let a, b, c be regular expressions. Which of the following identities is correct?

A. $(a+b)^* = a^*b^*$
B. $a(b+c) = ab+c$
C. $(a+b)^* = a^* + b^*$
D. $(ab+a)^*a = a(ba+a)^*$
E. None of the above.

tifr2015 theory-of-computation regular-expressions

21.5.3 Regular Expressions: TIFR2017-B-9

https://gateoverflow.in/95705

Which of the following regular expressions correctly accepts the set of all $0/1$-strings with an even (possibly zero) number of 1s?

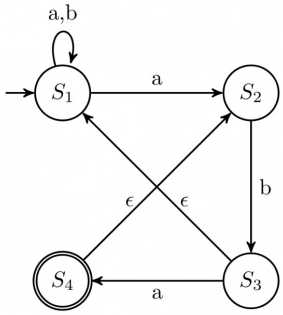
A. $(10^*10^*)^*$
B. $(0^*10^*1)^*$
C. $0^*1(10^*1)^*10^*$
D. $0^*1(0^*10^*10^*)^*10^*$
E. $(0^*10^*1)^*0^*$

tifr2017 theory-of-computation regular-expressions

21.5.4 Regular Expressions: TIFR2018-B-2

https://gateoverflow.in/179286

Consider the following non-deterministic automation, where s_1 is the start state and s_4 is the final (accepting) state. The alphabet is $\{a, b\}$. A transition with label ϵ can be taken without consuming any symbol from the input.



Which of the following regular expressions corresponds to the language accepted by this automation ?

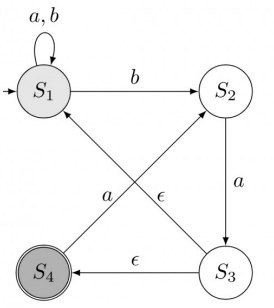
A. $(a+b)^*aba$ B. $aba(a+b)^*aba$ C. $(a+b)aba(b+a)^*$ D. $aba(a+b)^*$ E. $(ab)^*aba$

tifr2018 regular-expressions finite-automata

21.5.5 Regular Expressions: TIFR2019-B-11

https://gateoverflow.in/280484

Consider the following non-deterministic automaton,where s_1 is the start state and s_4 is the final (accepting) state. The alphabet is $\{a, b\}$. A transition with label ϵ can be taken without consuming any symbol from the input.



Which of the following regular expressions correspond to the language accepted by this automaton ?

A. $(a+b)^*aba$ B. $(a+b)^*ba^*$ C. $(a+b)^*ba(aa)^*$ D. $(a+b)^*$ E. $(a+b)^*baa^*$

tifr2019 theory-of-computation regular-expressions

Answers: Regular Expressions

21.5.1 Regular Expressions: TIFR2010-B-34

https://gateoverflow.in/19047

✔

a. $(r+s)^* = r^*s^*$ LHS can generate ' sr' but RHS not
b. $r(s+t) = rs+t$ LHS can generate ' rt' but RHS not
c. $(r+s)^* = r^*+s^*$ LHS can generate ' sr' but RHS not
d. $(rs+r)^*r = r(sr+r)^*$ They are equivalent
e. $(r^*s)^* = (rs)^*$ LHS can generate ' $rrrs$' but RHS not

So option **D** is correct answer.

23 votes -- Umang Raman (12.2k points)

21.5.2 Regular Expressions: TIFR2015-B-7

https://gateoverflow.in/29861

✔

a. $(a+b)^* = \{$ any strings of over $\{a,b\}\}$
$a^*b^* = \{$ any number of a's followed by any number of b's $\}$
False, as strings, ba, aba, bab, etc are not present in a^*b^*
b. $a(b+c) = \{ab, ac\}$
$ab+c = \{ab, c\}$
False
c. $(a+b)^* = \{$ any strings of over $\{a,b\}\}$
$a^*+b^* = \{$ any numbers of a's or any numbers of b's $\}$
False , as strings , ab, ba, aba, bab etc are not present in a^*+b^*
d. $(ab+a)^*a = a(ba+a)^*$, **True**
$p(qp)^* = p\{\epsilon, qp, qpqp, qpqpqp, \dots\} = \{p, pqp, pqpqp, pqpqpqp, \dots\} = \{\epsilon, pq, pqpq, pqpqpq, \dots\}p = (pq)^*p$
$(ab+a)^*a = (a(b+\epsilon))^*a = a((b+\epsilon)a)^* = a(ba+a)^*$

Correct Answer: D

36 votes -- Praveen Saini (42k points)

21.5.3 Regular Expressions: TIFR2017-B-9

https://gateoverflow.in/95705

✔ As, mentioned in the question, the regular expression must accept all strings of 0 and 1 but with even no of 1s (including no 1s). Hence, 00 must be in the language. Option A, B, C, D do not accept 00. Hence, option **E** is correct.

14 votes -- tarun_svbk (1.4k points)

21.5.4 Regular Expressions: TIFR2018-B-2

https://gateoverflow.in/179286

✔ The correct option is (A).

Other option can be easily eliminated by using string bbaba.

8 votes -- Subham Mishra (11.5k points)

21.5.5 Regular Expressions: TIFR2019-B-11

https://gateoverflow.in/280484

✔ On initial state S1, we have a loop labeled a,b which implies $(a+b)^*$

S1 on input b moves to S2. Then S2 on input a moves to S3 and S3 on ϵ reaches the final state S4.

Following these input transitions, we obtain $(a+b)^*ba$

But we have to account for the case when S4 on input a moves to S2.

Here we see, S4 on being given two successive a's forms a cycle/loop and comes back to the final state.

Hence, the regular expression corresponding to the language accepted by automaton would be $(a+b)^*ba(aa)^*$

PS: We do have an ϵ move from S3 to S1, but this is not important as S1 state captures $(a+b)^*$ meaning instead of moving to S3 and back to S1, there is an option to remain in S1 only. The given automata captures all binary strings ending with a "ba" followed by zero or more "aa".

10 votes -- Sayan Bose (5k points)

21.6 Regular Languages (6)

21.6.1 Regular Languages: TIFR2013-B-6 https://gateoverflow.in/25667

Let L and L' be languages over the alphabet Σ. The left quotient of L by L' is

$L/L' \stackrel{def}{=} \{w \in \Sigma^* : wx \in L \text{ for some } x \in L'\}$

Which of the following is true?

A. If L/L' is regular then L' is regular.
B. If L is regular then L/L' is regular.
C. If L/L' is regular then L is regular.
D. L/L' is a subset of L.
E. If L/L' and L' are regular, then L is regular.

tifr2013 theory-of-computation regular-languages

21.6.2 Regular Languages: TIFR2013-B-8 https://gateoverflow.in/25670

Which one of the following languages over the alphabet $0, 1$ is regular?

A. The language of balanced parentheses where $0, 1$ are thought of as $(,)$ respectively.
B. The language of palindromes, i.e. bit strings x that read the same from left to right as well as right to left.
C. $L = \left\{0^{m^2} : 3 \leq m\right\}$
D. The Kleene closure L^*, where L is the language in (c) above.
E. $\{0^m 1^n | 1 \leq m \leq n\}$

tifr2013 theory-of-computation regular-languages

21.6.3 Regular Languages: TIFR2014-B-12 https://gateoverflow.in/27314

Consider the following three statements:

i. Intersection of infinitely many regular languages must be regular.
ii. Every subset of a regular language is regular.
iii. If L is regular and M is not regular then $L \bullet M$ is necessarily not regular.

Which of the following gives the correct true/false evaluation of the above?

A. true, false, true.
B. false, false, true.
C. true, false, true.
D. false, false, false.
E. true, true, true.

tifr2014 theory-of-computation regular-languages

21.6.4 Regular Languages: TIFR2015-B-10 https://gateoverflow.in/30039

Consider the languages

$L_1 = \{a^m b^n c^p \mid (m = n \vee n = p) \wedge m + n + p \geq 10\}$

$L_2 = \{a^m b^n c^p \mid (m = n \vee n = p) \wedge m + n + p \leq 10\}$

State which of the following is true?

A. L_1 and L_2 are both regular.
B. Neither L_1 nor L_2 is regular.
C. L_1 is regular and L_2 is not regular.
D. L_1 is not regular and L_2 is regular.
E. Both L_1 and L_2 are infinite.

tifr2015 regular-languages

21.6.5 Regular Languages: TIFR2015-B-6 https://gateoverflow.in/29860

Let B consist of all binary strings beginning with a 1 whose value when converted to decimal is divisible by 7.

A. B can be recognized by a deterministic finite state automaton.
B. B can be recognized by a non-deterministic finite state automaton but not by a deterministic finite state automaton.

C. B can be recognized by a deterministic push-down automaton but not by a non-deterministic finite state automaton.
D. B can be recognized by a non-deterministic push-down automaton but not by a deterministic push-down automaton.
E. B cannot be recognized by any push down automaton, deterministic or non-deterministic.

tifr2015 theory-of-computation regular-languages

21.6.6 Regular Languages: TIFR2018-B-12

https://gateoverflow.in/179296

Consider the following statements:

i. For every positive integer n, let $\#n$ be the product of all primes less than or equal to n.
Then, $\#p + 1$ is a prime, for every prime p.
ii. π is a universal constant with value $\frac{22}{7}$.
iii. No polynomial time algorithm exists that can find the greatest common divisor of two integers given as input in binary.
iv. Let $L \equiv \{x \in \{0,1\}^* \mid x \text{ is the binary encoding of an integer that is divisible by } 31\}$
Then, L is a regular language.

Then which of the following is TRUE ?

A. Only statement **(i)** is correct.
B. Only statement **(ii)** is correct.
C. Only statement **(iii)** is correct.
D. Only statement **(iv)** is correct.
E. None of the statements are correct.

tifr2018 regular-languages

Answers: Regular Languages

21.6.1 Regular Languages: TIFR2013-B-6

https://gateoverflow.in/25667

✔

a. False because - $L = a^*b^*$, $L' = a^nb^n$ Here $L/L' = a^*$. L/L' is regular, but L' is not.
b. True. If L is regular, L/L' is prefix of language. Regular languages are closed under Qoutient/Prefix. So this is correct.
c. False $L' =$ Empty set. Then L/L' is Empty set whatever L is. Here L can be say a^nb^n. See defination of L/L' to see why L/L' should be empty set.
d. False because L/L' can accept prefixes of string of Language L, which may or may not be accepted by L itself. So L/L' is not subset. (It is not Superset either , because L' can be empty set)
e. False. Same explanation as **C.**

Answer is **B.**

17 votes -- Akash Kanase (36.1k points)

21.6.2 Regular Languages: TIFR2013-B-8

https://gateoverflow.in/25670

✔ Here, **option D** is **regular,** reason is as follows:

$L = \{0^{m^2} : m \geq 3\}$

Now, **in** L^* if we can **generate** 9 **continuous powers of zero**, then further every power can be generated by **concatenating** 0^9**.**

Here , $L = \{0^9, 0^{16}, 0^{25}, \dots\}$

So, here are 9 **continuous powers:**

$0^{120} : 0^{16}0^{16}0^{16}0^90^90^90^90^90^90^90^9$

$0^{121} : 0^{16}0^{16}0^{16}0^{16}0^{16}0^{16}0^{16}0^{25}$

$0^{122} : 0^{16}0^{16}0^90^90^90^90^90^90^90^90^90^9$

$0^{123} : 0^{16}0^{16}0^{16}0^{25}0^{25}0^{25}$

$0^{124} : 0^{16}0^{18}0^{18}0^{18}0^{18}0^{18}0^{18}$

$0^{125} : 0^{25}0^{25}0^{25}0^{25}0^{25}$

0^{126} : $0^{18}0^{18}0^{18}0^{18}0^{18}0^{18}0^{18}$ $\{0^{18}\text{can be generated as}0^90^9\}$

0^{127} : $0^{16}0^{16}0^{16}0^{16}0^90^90^90^90^90^90^9$

0^{128} : $0^{16}0^{16}0^{16}0^{16}0^{16}0^{16}0^{16}0^{16}$

Now, 0^{129} can be given as $0^{120}0^9$ and so on.

Every Further powers can be generated by concatenating 0^9.

23 votes -- Himanshu Agarwal (12.4k points)

21.6.3 Regular Languages: TIFR2014-B-12

https://gateoverflow.in/27314

✔

i. **False**
Regular Languages are not closed under Infinite Union and Intersection
$L_1 \cup L_2 \cup L_3 \cup L_4 \cup \ldots$
For example :
$ab \cup aabb \cup aaabbb \cup aaaabbbb \cup \ldots$
$= \{a^nb^n, n \geq 1\}$ which is not regular
So, Infinite Union is not closed
$L_1 \cap L_2 \cap L_3 \cap L_4 \cap \ldots$
$= (L_1' \cup L_2' \cup L_3' \cup L_4' \cup \ldots)'$
As Infinite Union is not closed, Infinite Intersection is also not closed

ii. **False.**
a^*b^* is regular
its subset $a^nb^n, n \geq 1$ is not regular
a^* is regular
a^p, p is prime, is not regular

iii. **False**
$L = \{\}$ is regular
M be non-regular like $\{0^n1^n \mid n > 0\}$.
$L.M = \{\}$, is regular

Correct Answer: D

35 votes -- Praveen Saini (42k points)

21.6.4 Regular Languages: TIFR2015-B-10

https://gateoverflow.in/30039

✔ L_2 is finite, so regular.

L_1 is non-regular.

(It seems CFL to me as I think it can be implement with the help of PDA , as stack can ensure $(m = n \vee n = p)$ and we can also ensure $(m + n + p \geq 10)$ with minimum states changes along with transitions).

L_2 is actually $\{c^p \mid p \leq 10\} \cup \{abc^p \mid p \leq 8\} \cup \{a^2b^2c^p \mid p \leq 6\} \cup \{a^3b^3c^p \mid p \leq 4\} \cup \{a^4b^4c^p \mid p \leq 2\} \cup \{a^5b^5\} \cup \{a^p \mid p \leq 10\} \cup \{a^pbc \mid p \leq 8\} \cup \{a^pb^2c^2 \mid p \leq 6\} \cup \{a^pb^3c^3 \mid p \leq 4\} \cup \{a^pb^4c^4 \mid p \leq 2\} \cup \{b^5c^5 \mid p \leq 10\}$

Correct Answer: D

23 votes -- Praveen Saini (42k points)

21.6.5 Regular Languages: TIFR2015-B-6

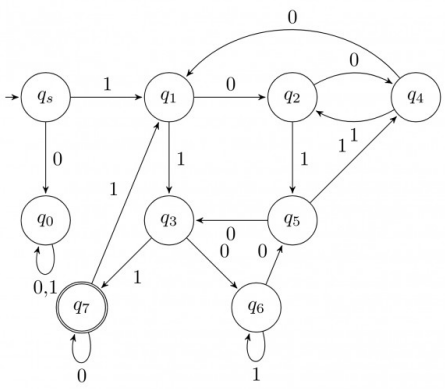
https://gateoverflow.in/29860

✔ The given language is the intersection of two regular languages.

1. Binary strings beginning with 1
2. Binary strings divisible by 7 (For any integer n, divisibility by n for a binary string can be checked using a finite automata making the language regular.)

Intersection of two regular language is regular (Regular set being closed under intersection).
Correct Answer: A.

18 votes -- Ahwan Mishra (10.2k points)



Answer will be (**A**).

If it start with 1 it goes to final state.

If start with 0 it will go to the reject state.

18 votes -- srestha (85.3k points)

21.6.6 Regular Languages: TIFR2018-B-12 https://gateoverflow.in/179296

✔ π is not equal to 22/7

https://math.stackexchange.com/questions/93222/is-22-7-equal-to-the-pi-constant

For (i), consider $n = 9$. Now, we get product of primes less than 9 as $\#n = 2 \times 3 \times 5 \times 7 \times 9 = 1890$. Now $\#n + 1 = 1891$ is not a prime as $1891 = 31 \times 61$. (This example might be difficult to arrive during exam, but this option could have been eliminated by good intuition or by seeing that other there are better other options).

We can construct dfa for binary string divisible by 31

https://stackoverflow.com/questions/21897554/design-dfa-accepting-binary-strings-divisible-by-a-number-n

Option 4

References

9 votes -- Rameez Raza (1.8k points)

Answer Keys

21.1.1	C	21.1.2	E	21.2.1	B	21.2.2	D	21.2.3	E
21.3.1	A	21.3.2	D	21.3.3	B	21.3.4	C	21.3.5	E
21.3.6	D	21.3.7	C	21.3.8	B	21.3.9	C	21.3.10	A
21.4.1	B	21.4.2	C	21.5.1	D	21.5.2	D	21.5.3	E
21.5.4	A	21.5.5	C	21.6.1	B	21.6.2	D	21.6.3	D
21.6.4	D	21.6.5	A	21.6.6	D				