ML

③ $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

$|A - I\lambda| = 0$

$\left| \begin{bmatrix} 0-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} \right| = 0$

$-\lambda(3-\lambda) - (-2) = 0$

$-3\lambda + \lambda^2 + 2 = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$

$\lambda = 2 \quad , \lambda = 1$

For, $\lambda = 2$

$(A - \lambda I)v = 0$

$\begin{bmatrix} 0-2 & -1 \\ 2 & 3-2 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = 0$

$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = 0$

$-2V_x - V_y = 0$

$V_x = -\frac{V_y}{2}$

$2V_x + V_y = 0 \Rightarrow V_x = -\frac{V_y}{2}$ //.

For $\lambda = 1$,

$$\begin{bmatrix} 0-1 & -1 \\ 2 & 3-1 \end{bmatrix} \begin{bmatrix} W_x \\ W_y \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} W_x \\ W_y \end{bmatrix} = 0$$

$$-W_x - W_y = 0.$$

$$W_x = -W_y$$

$$2W_x + 2W_y = 0$$

$$W_x = -W_y //$$

Eigen vectors can

$$v_1 = k_1 \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} \qquad W = k_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} //$$

(2) $S = \{ v_1, v_2 \dots v_n \}$

$$v = \sum_{i=1}^{n} c_i v_i$$

for some constant c_i $i = 1, 2, \dots n$.

$$V = \sum_{i=1}^{n} c_i \sum_{i=1}^{n} v_i$$

soln:

$v_1 = c_1 v_1 + c_2 v_2 \dots c_n v_n$. — (1)

$v_2 = c_1 v_1 + c_2 v_2 \dots c_n v_n$.

$\vdots$

$v_n = \dots$

Since vectors are orthonormal

$v_1 . v_2 = 0$ & so on

Multiply ① by v_i

$v . v_i = c_1 v_1 v_i + c_2 v_2 v_i + \dots c_n v_n v_i$

~~$v_1 v_2 = c_1 [1 . v_1 v_1 = 1$ & $v_1 v_2 = 0 \dots v_1 v_n = 0]$~~

~~Similarly c_2~~ Every term except $v . v_i$ will ~~$c_2 c \, v_2 . v_2$~~ be 0

$\therefore \boxed{c_1 + c_2 + c_3 \dots c_n = v . v_i}$

$\therefore$ ~~$c_1 + c_2 + \dots c_n = c = \sum_{i=1}^{n} c$~~

~~$= v_1 v_1 + v_2 v_2 + \dots$~~

~~$= S . S$~~

$\therefore$ ~~$\boxed{\sum_{i=1}^{n} c = S . S}$~~

i.e. $\boxed{\sum_{i=1}^{n} c_i = v . v_i}$