Report Titled

Design and Simulation of A Cavity Resonator

Submitted

In partial fulfillment of

The requirements of the course

Electromagnetic Theory and Microwave Engineering

By

Shreyas Ravishankar

2016AAPS0180H

2018

Dr. Harish Vijay Dixit

Electrical and Electronics Engineering

2018

DECLARATION

I declare that this report titled **"Design and Simulation of A Cavity Resonator"** is my original work and represents my own ideas. Any inclusion of others' ideas and work has been accompanied with citation and reference of the original source.

I also declare that I have shown complete academic honesty while preparing this report. I am also aware of the penalty to be given in case of dishonesty.

Shreyas Ravishankar

2016AAPS0180H

31st November 2018

Abstract

The project titled 'Design and Simulation of a cavity resonator' was taken up to explore resonators and their behavior at various excitations . To that end, a simple rectangular cavity resonator with resonant frequency 3.7 GHz was designed and simulated. Two cases were considered for the cavity boundary- a perfect conductor and a conductor of finite conductivity. An 'eigen frequency study' and 'frequency domain study' was carried out. The former was used to find the various resonant frequencies of a cavity of given dimensions. The frequency domain study was done to find out the losses associated with surface currents and the Q factors for an input power of 200 KW. A similar analysis is carried out for a cylindrical cavity resonator.

ACKNOWLEDGEMENT

I would like to thank Electrical and Electronics Engineering Department, BITS-Pilani Hyderabad Campus and especially Dr. Harish V. Dixit for giving me the opportunity to work on this project and constantly guiding me.

Thanking You,

Shreyas Ravishankar

2016AAPS0180H

Table of contents

List of figures and tables

Figures

Tables

*All the images whose source has not been mentioned has been taken from comsol. Inc

Introduction

Resonators are structures which help to confine energy. They store energy in the form of electromagnetic fields at very high frequencies (>100MHz). Fundamentally they are similar to LC circuits. For high frequency applications, resonators are made of waveguide cavities, microstrip, transmission lines etc.
We go for such structures instead of LC circuits because-

1. Q factors obtained is upto order of 10^6 , whereas LC circuits can give only about 10^3 order.
2. The high resonant frequencies implies low values of L,C which are tough to manufacture.

Resonance and Q factors in LCR circuits

Consider an LCR circuit shown in Fig.1

Fig1

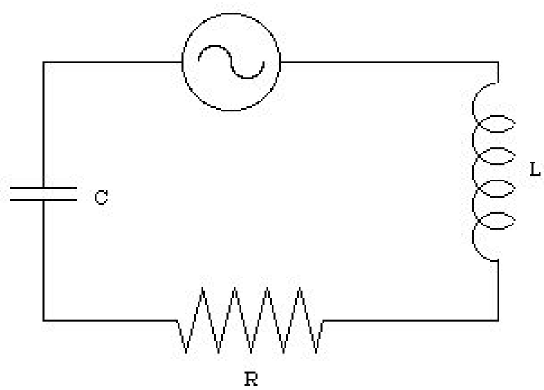


Fig.1 An LCR circuit
Source: Wikipedia

$$Z_{in}= R+ j\,\omega\, L -j(1/\,\omega\, C)$$

The complex power P_{in} is given by

$$P_{in}= 0.5V\, I^* = 0.5*Z_{in}\,|\, I\,|^2 = 0.5*(|\, I\,|^2)*(\, R+ j\,\omega\, L -j/\,\omega\, C\,)$$

The power lost through the resistor

$$P_{loss}= 0.5*\, R*|\, I\,|^2$$

The average stored magnetic and the average stored electric energy is given by-

$$W_m= 0.25*L*|\, I\,|^2$$

$$W_e = 0.25 * C * | V_c |^2$$

Resonance occurs when both magnetic and electric energies are equal. The input impedance at resonance will be the lowest.

$$Z_{in,reso} = R$$

The resonant frequency is given by

$$\omega_0 = 1/\sqrt{LC}$$

The most important parameter for characterizing a resonant circuit is it's Q factor.

$$Q = \omega \left(\frac{\textit{time average energy stored}}{\textit{energy loss per second}} \right)$$

$$= \omega (W_m + W_e) / P_{loss}$$

Q is a measure of loss of a resonator circuit. A higher Q means lower loss compared to a circuit with a lower value of Q. Since the LCR resonates only for certain frequencies, it can be thought of as a band pass filter. Higher the Q factor, the more frequency selective the circuit is.

Modes in a resonator

The main modes we are concerned with here are TE (Transverse electric) and TM (Transverse magnetic modes. If the direction of propagation is considered to be along Z direction, then in a TE mode the electric field (E) will not have any Z component. Similarly TM mode means that the magnetic field strength (H) will not have Z component. These two modes constitute the various field patterns that can occur during resonance in a cavity resonator.

Report

Rectangular waveguide resonator

Consider a rectangular cavity as shown in Fig 2. Let the dimensions of the cavity be 'a' along x, 'b' along y and 'c' along z. This hollow structure is made of metal and is filled with a dielectric inside. Clearly, the cavity is nothing but a waveguide which is covered at both ends. Thus we expect to see standing waves along z direction as well. Therefore there will be no propagation and standing waves are formed along all the three directions.

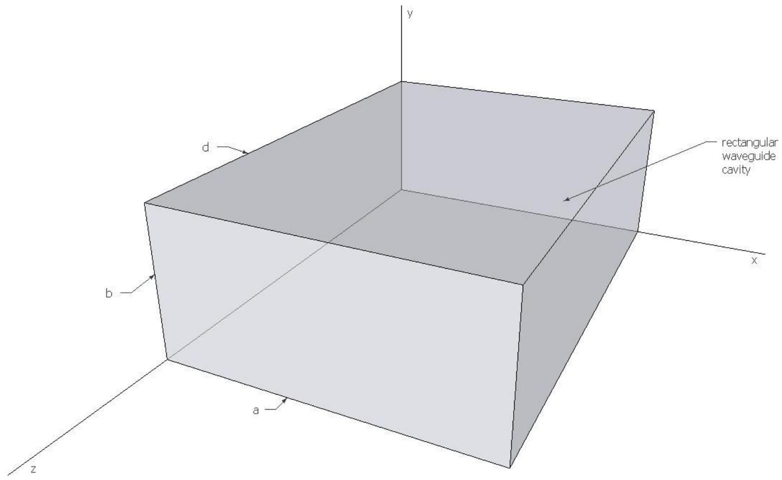


Fig.2 A rectangular cavity resonator
Source: Wikipedia

The electric and magnetic fields along all the directions are assumed to be of the following forms-

$$E_i(x,y,z) = X(x)Y(y)Z(z)$$

$$H_i(x,y,z) = X'(x)Y'(y)Z'(z) \qquad i \in (\hat{i}, \hat{j}, \hat{k})$$

TM mode - As discussed above, the variation of electric field along y direction will be -

$$X(x) = c_1\cos(k_x x) + c_2\sin(k_x x)$$
$$Y(y) = c_3\cos(k_y y) + c_4\sin(k_y y)$$
$$Z(z) = c_5\cos(k_z z) + c_6\sin(k_z z)$$

$$\text{Where } K^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon$$

The boundary conditions are

$$E_z = 0 \text{ at } x = 0,a$$

$$E_z = 0 \text{ at } y = 0,b$$

$$E_y = 0, \; E_x = 0 \text{ at } z = 0,d$$

Solving we get,

$K_x = m\pi/a$, $K_y = n\pi/b$, $K_z = p\pi/d$

Using the relations (1), (2), (3), (4)

$$E_x = -(\frac{\gamma}{h^2})\frac{(\partial Ez)}{\partial x} - (\frac{j\omega\mu}{h^2})\frac{(\partial Hz)}{\partial y} \quad \text{….(1)}$$

$$E_y = -(\frac{\gamma}{h^2})\frac{(\partial Ez)}{\partial y} - (\frac{j\omega\mu}{h^2})\frac{(\partial Hz)}{\partial x} \quad \text{….(2)}$$

$$H_x = -(\frac{\gamma}{h^2})\frac{(\partial Hz)}{\partial x} + (\frac{j\omega\mu}{h^2})\frac{(\partial Ez)}{\partial y} \quad \text{….(3)}$$

$$E_x = -(\frac{j\omega\mu}{h^2})\frac{(\partial Ez)}{\partial x} - (\frac{\gamma}{h^2})\frac{(\partial Hz)}{\partial y} \quad \text{….(4)}$$

We get,

$$E_x = (\frac{j\omega\mu}{h^2})(\frac{n\pi}{b})H_0\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)\sin(\frac{p\pi}{d}z)$$

$$E_y = -(\frac{j\omega\mu}{h^2})(\frac{m\pi}{a})H_0\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\sin(\frac{p\pi}{d}z)$$

$$E_z = 0$$

$$H_x = (\frac{-1}{h^2})(\frac{m\pi}{a})(\frac{p\pi}{d})H_0\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\cos(\frac{p\pi}{d}z)$$

$$H_y = (\frac{-1}{h^2})(\frac{n\pi}{b})\frac{p\pi}{d}H_0\cos(\frac{m\pi}{a}y)\sin(\frac{n\pi}{b}y)\cos(\frac{p\pi}{d}z)$$

$$H_z = H_0\cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\sin(\frac{p\pi}{d}z)$$

Where m, n= 0,1,2,3 ; p= 1,2,3… and m=n $\neq$ 0

$h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$

TM_{mnp} mode

$$E_x = (\frac{-1}{h^2})(\frac{m\pi}{a})(\frac{p\pi}{d})E_0\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)\sin(\frac{p\pi}{d}z)$$

E_y= -($\frac{-1}{h^2}$)($\frac{n\pi}{b}$)($\frac{p\pi}{d}$)H_0sin($\frac{m\pi}{a}$ x)cos($\frac{n\pi}{b}$ y)sin($\frac{p\pi}{d}$ z)

E_z= E_0sin($\frac{m\pi}{a}$ x)sin($\frac{n\pi}{b}$ y)cos($\frac{p\pi}{d}$ z)

H_y= ($\frac{j\omega\varepsilon}{h^2}$)($\frac{n\pi}{b}$)H_0sin($\frac{m\pi}{b}$ x)cos($\frac{n\pi}{b}$ y)cos($\frac{p\pi}{d}$ z)

H_y= -($\frac{j\omega\varepsilon}{h^2}$)($\frac{n\pi}{b}$)H_0cos($\frac{m\pi}{a}$ y)sin($\frac{n\pi}{b}$ y)cos($\frac{p\pi}{d}$ z)

H_z= 0

Where m, n= 1,2,3 ; p= 0,1,2,3… and m=n$\neq$ 0
$h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$

The phase constant β is given by

$$\beta^2 = k^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + (\frac{p\pi}{d})^2$$

Resonant frequency f_c -

$$f_r = (\frac{1}{2\pi\sqrt{\mu\varepsilon}}) * ((\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + (\frac{p\pi}{d})^2)^{0.5} \quad ....(5)$$

Quality factor

We know that,

$$Q= 2\pi f (\frac{time\ average\ energy\ stored}{energy\ loss\ per\ second}) \quad ....(5.1)$$

$$= 2\pi (\frac{W}{P})$$

where
W is the total time average energy stored inside the electric and magnetic fields inside the cavity.
and P is time time-average power loss in the cavity.

If the dimensions if the box a,b,c, are such that a>b<c, then the dominant mode for TE will be TE_{101} . Q factor in that mode is given by

$$Q_{TE101} = \frac{(a^2 + c^2)abc}{\delta(2b(a^3 + c^3) + ac(a^2 + c^2))} \quad \text{....(6)}$$

Where

$$\delta = (\pi f_{TE101} \mu_0 \sigma_c)^{-0.5}$$

Cylindrical cavity resonator

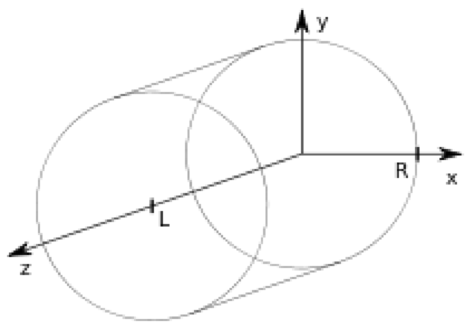


Fig3. A cylindrical cavity resonator
Source: Wikipedia

Given in Fig.3 is a cylindrical cavity resonator with radius a and length h. The derivation for the frequencies and Q factors is similar to the rectangular case. Here we directly write the expressions for the desired mode (TM_{010})

$$f_{TM010} = \left(\frac{1}{2\pi\sqrt{\mu\varepsilon}} \right) * \left(\frac{2.4092}{a} \right) \quad \text{....(6.1)}$$

Quality factor

$$Q_{TM101} = \frac{1.2025\eta}{Rs(1+\frac{a}{h})} \quad \text{....(6.2)}$$

Materials

The materials used to build the basic geometry are give in the table1

Table 1: materials

Material	Relative permeability	Relative Permittivity	Electrical Conductivity	Region
Air	1	1	0	Volume of cavity
Copper (Built in)	1	1	5.8e7	Surface of cavity
Dielectric	1	2	0	Coaxial cable

Steps for study

- First a resonator of appropriate dimensions was constructed to match a desired resonant frequency. (3.7GHz)
- An 'eigenfrequency study' was carried out over a range of frequencies to see if theoretical resonant frequency was realized in practice. This study also unearthed other modes/ resonant frequencies.
- The above steps were carried out in both a perfect conductor and a conductor with finite conductivity.
- The analytical Q factors were calculated. We have to excite the source as the power delivered in arbitrary
- Next a coaxial cable was attached to excite the resonator. A power of 200000W was given to the resonator using the coaxial cable. A 'frequency domain study' was then carried out to ensure that it resonates at the desired frequency.
- The losses, energy stored was calculated.
- Actual Q values were now found and it is approximately equal to the Q values that were calculated earlier.

Calculations

Eigen frequency study

Rectangular cavity

Consider a rectangular cavity with sides a, b and c. The mode that was considered for designing the resonator was TE_{101}.

The cavity is filled with air. ….(7)

Assuming a=c for simplicity ….(8)

Using (5), the equation simplifies to,

$$f_{TE101} = \left(\frac{c}{2\pi\sqrt{\mu r \varepsilon r}} \right) * \left(\left(\frac{m\pi}{a} \right)^2 + \left(\frac{p\pi}{d} \right)^2 \right)^{0.5}$$

`

Where $f_{TE101} = 3.7$ GHz

$\sqrt{\mu r \varepsilon r} = 1$

m=1, p=1

$c = 3*10^8 \quad ms^{-1}$

We obtain,

d=a ≈ 0.05733 m

b ≈ 0.02866 m

Case 1

The metal is assumed to be a perfect conductor. The boundary conditions that was applied was the 'Perfect Electric Conductor'.

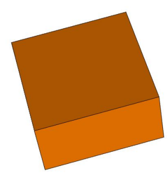


Fig4. Losses in a rectangular conducting cavity with infinite conducting boundary

As the air is a perfect dielectric (no conductivity) and the metal is considered to be of infinite conductivity. Therefore there will be no heating losses through the walls of the cavity.

As can be seen in the fig4. the cavity has no losses. This matches the theoretical values of Q as the skin depth is 0 due to infinite conductivity. Therefore, $Q_{TE101} \rightarrow \infty$

Case 2

The metal cavity was changed to copper with finite conductivity of $5.8*10^7$ S/m. Unlike the previous case we will have losses. The boundary condition used to simulate a this case was 'Impedance boundary condition'

We calculate Q values in 3 different ways-

1. $Q_{theoritical}$ - Q value which will be calculated using (6)
2. $Q_{definition}$- Q value which will be computed by the comsol solver using surface and volume integration definitions. In essence it will use a (5.1) .It

will be computed by using operators for surface and volume integration operators.

3. $Q_{computed}$ - Q value calculated by the solver in comsol using the boundary conditions.

The theoretical Q factor was calculated using (6). Applying assumptions (7) and (8) for the lossy case, Q simplifies to

$$Q_{theoritical} = a/(4\,\delta)$$

$$\delta = (\pi\, f_{TE101}\, \mu_0\, \sigma_c)^{-0.5}$$

Where $a = 0.05733$ m

$\sigma_c = 5.8*10^7$ S/m

$f_{TE101} = 3.7$ GHz

$Q_{theoritical} = 13193.34$

$Q_{computed} = 13187.89$

$Q_{definition} = 13187.25$

Therefore we can see that the Q values computed are in agreement with the Q values that were calculated theoretically. The losses have been indicated in Fig5.

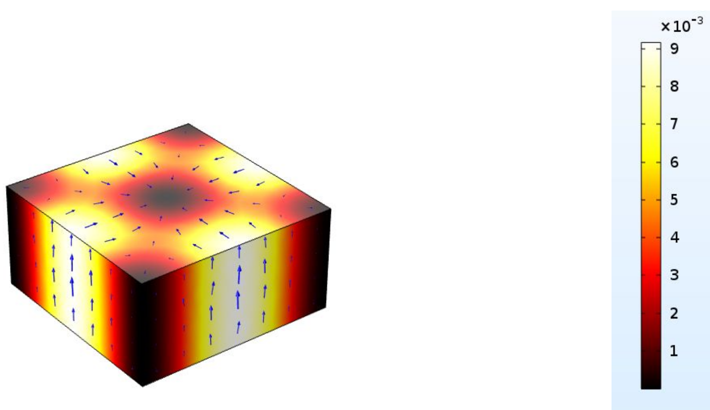


Fig5. Losses in a rectangular cavity with finite conductivity.

Cylindrical cavity

The equations (6.1) and (6.2) is used to obtain the required dimensions

$$f_{TM010} = \left(\frac{c}{2\pi\sqrt{\mu\varepsilon}}\right) * \left(\frac{2.4092}{a}\right)$$

Where f_{TM010}= 3.7 GHz

$\sqrt{\mu r \varepsilon r} = 1$

c= $3*10^8$ ms^{-1}

a= 0.0311m

For TM_{010} to be the dominant mode, it is required that ($\frac{h}{a}$) < 2.03. Therefore we arbitrarily choose h= 0.0466m, which satisfies the above inequality.

Case 1

Firstly the boundary condition that was applied was perfectly electrical boundary. The results obtained were as expected. There was no loss through the cylindrical walls as the conductivity was infinite. (Fig6.)

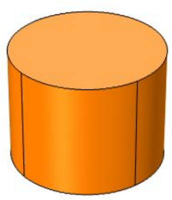


Fig6. Losses in a cylindrical conducting cavity with infinite conducting boundary.

Case 2

Now the impedance boundary condition was applied with a metal conductivity of $5.8 * 10^7$. The Q values that are defined in the previous section are calculated for the cylindrical case.

1. $Q_{theoritical}$ - Q value which will be calculated using (6.2)
2. $Q_{definition}$- Q value which will be computed by the comsol solver using surface and volume integration definitions. In essence it will use (5.1)
3. $Q_{computed}$ - Q value calculated by the solver in comsol using the boundary conditions.

$Q_{theoritical}$= 17150

$Q_{computed}$= 17102

$Q_{definition}$= 17092

The losses have been indicated in Fig7.

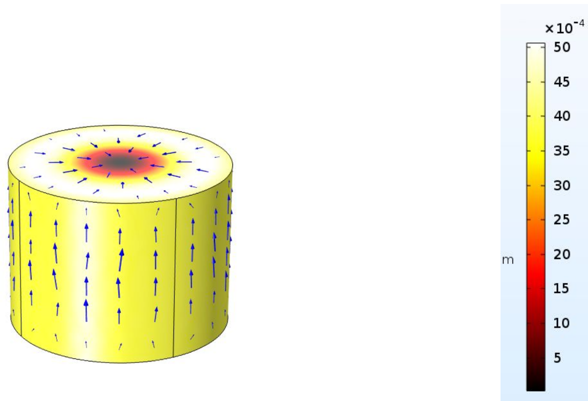


Fig7. Losses in a cylindrical cavity of finite conductivity.

Frequency domain study

Fig 8. Coaxial excitation geometry
a)rectangular *b) cylindrical*

- The coaxial cable input was constructed as shown in the figure below. The dimensions given correspond to the dimensions of the coaxial cable. (Fig8.(a))

Height of the outer cylinder = R
Radius of the outer cylinder = l
Height of the inner cylinder = r
Radius of the outer cylinder (rectangular)= l+0.1*H
Radius of outer cylinder (cylindrical)= l + 0.1*H'
The dimensions of the rectangular and cylindrical cavity is same as before.

Where R= 5mm
r= 1mm

l= 20mm
H= 0.05733mm,
H' =0.0466m

- A coaxial cable of same dimensions was added to both the cylindrical resonator cavity as well. (Fig8.(b))

Results

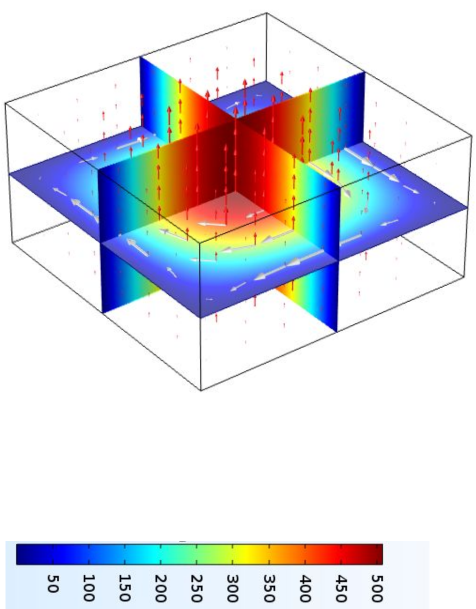
Eigenfrequency study

Lossy Case

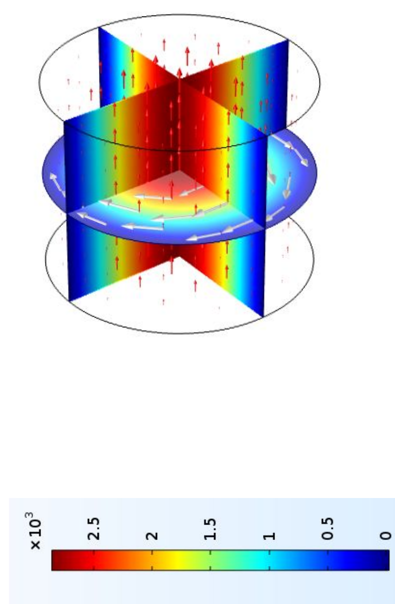
- Heat map is the norm of the electric field and the red arrows correspond to the direction of the electric field. The white arrows correspond to the magnetic field. Fig9. (a) - Rectangular cavity. Fig9.(b) - Cylindrical cavity.

Perfect conductor Case

- The heat map is similar to the previous case is similar to the previous one , only the losses will change.



(a)



(b)

Fig 9. Electric field norm and directions of electric and magnetic fields (a)rectangular (b) cyl

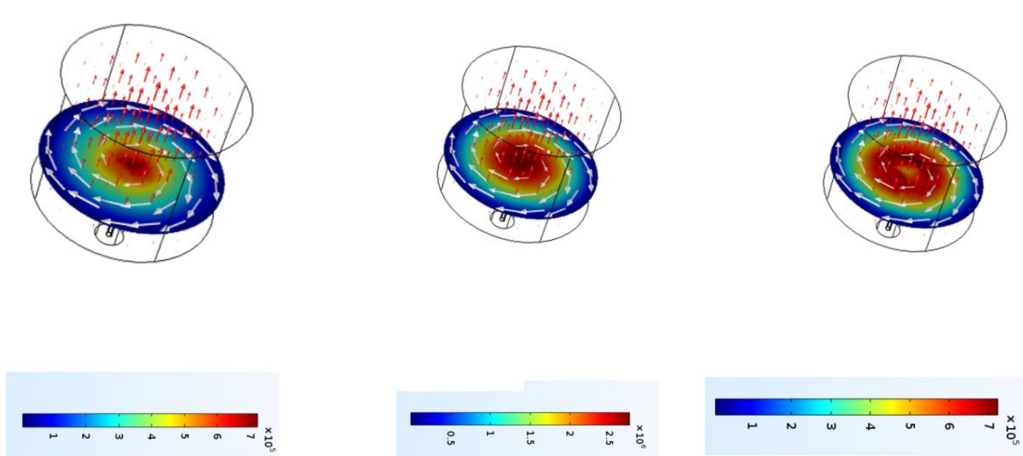
- The resonant frequencies were found to be complex with a small imaginary part. This is because of the losses present in the system.

Frequency domain study

Lossy case

- The electric field norm (heat map), magnetic field (white arrows) and electric field (red arrows) is shown for three different frequencies - 3.6 GHz, 3.7 GHz, 3.8 GHz

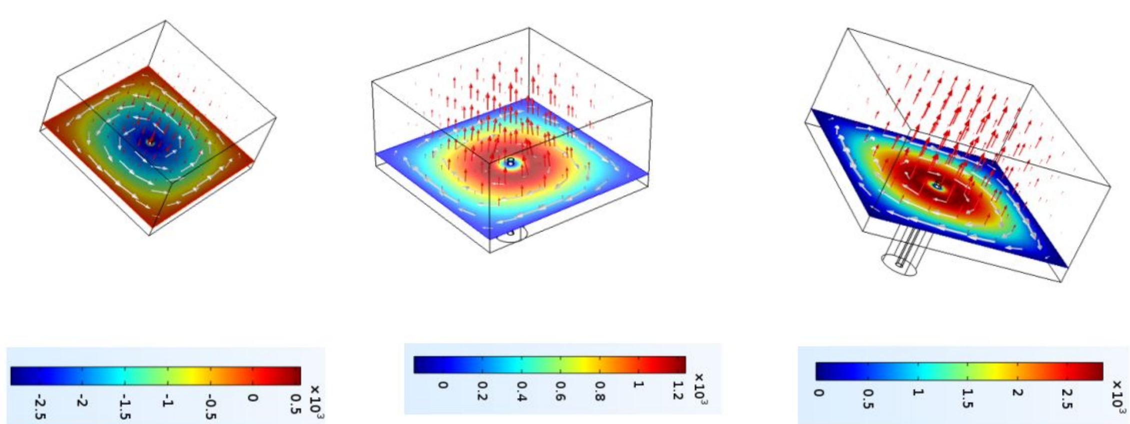
Fig10.(a), (b), (c) is for the cylindrical cavity. Fig11.(a), (b), (c) is for the rectangular cavity.



(a) (b) (c)

Fig10. Electric field norm and directions of electric and magnetic fields (cylindrical with source)

(a) 3.6 GHz (b) 3.7 GHz (c) 3.8GHz



(a) (b) (c)

Fig11. Electric field norm and directions of electric and magnetic fields (rectangular with source)

(a) 3.6 GHz (b) 3.8 GHz (c) 3.7GHz

- Comparing the electric field norms at the center , we can clearly see that at resonance, they take on greater values.

- An input power of 200KW was given. The stored energies and power losses were calculated for various frequencies after coupling with the source are shown below.

Table2: Rectangular cavity data

Frequency	Total Electrical Energy	Total Magnetic Energy	Time average energy stored	Energy loss per second	Q factor(measured at resonance)
3.6	0.00011169	0.0001878	0.000230341	397.47	
3.7	0.00032158	0.00032962	0.000650337	1162.5	13022.76
3.8	0.00004121	0.00049109	0.000090257 5	167.63	

Rectangular cavity (Fig12.)

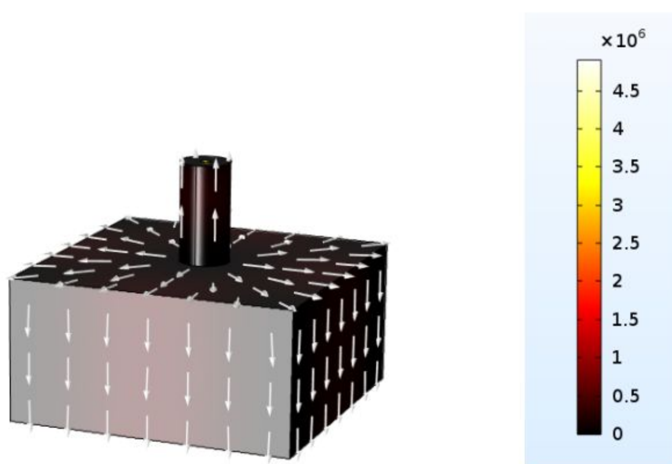


Fig12. Losses when source is present(rectangular)

Cylindrical cavity(Fig13.)

Table3 : Cylindrical cavity data

Frequency	Total Electrical Energy(J)	Total Magnetic Energy(J)	Time average energy stored(J)	Energy loss per second(W)	Q factor(measured at resonance)
3.6	0.00004029	0.00004870	0.00008994	140.98	
3.7	0.0006177	0.00066972	0.0013315	1842.6	16799.319
3.8	0.00003049	0.00003828	0.00006977	116.21	

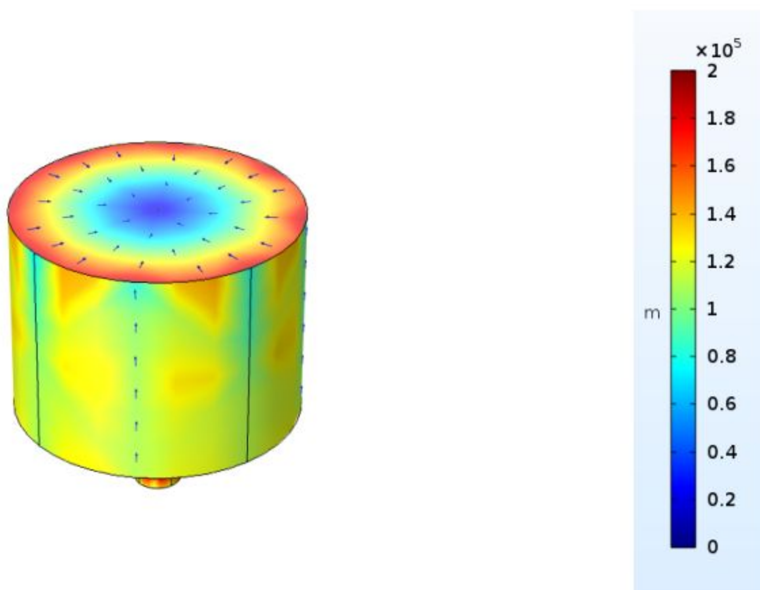


Fig13. Losses when source is present (cylindrical)

- Clearly the energy stored at resonance is much higher than at the non- resonant frequencies.
- $Q_{rectangular}$ factor is found to be 13883.027. Therefore the error from the ideal case is about 5%.
- $Q_{cylindrical}$ factor is found to be 16799.319. The error from the ideal case is about 2%.

- The Q values are lesser than those calculated from the eigen frequency analysis because now there are additional losses due to the coaxial source that was attached.

Conclusion and future work

A rectangular and a cylindrical cavity resonator were designed for the frequency 3.7 GHz and simulation was carried out successfully. An extension of this would be to thicken the wall of the cavity and apply transition boundary condition. Another extension would be to explore different ways of excitation and their calculate corresponding coupling constants etc. The above extensions were not completed due to time constraints.

References

1. Pozar, David M. *Microwave Engineering.* New York: John Wiley and Sons, 2011. 4th edition.
2. Sadiku, Matthew. Principles of Electromagnetics. New- Delhi: Oxford. 2009. 4th edition.
3. "Application Gallery." Computing Q factors and resonant frequencies of cavity resonators, COMSOL inc. ,2018, https://www.comsol.co.in/model/computing-q-factors-and-resonant-frequencies-of-cavity-resonators-9618.