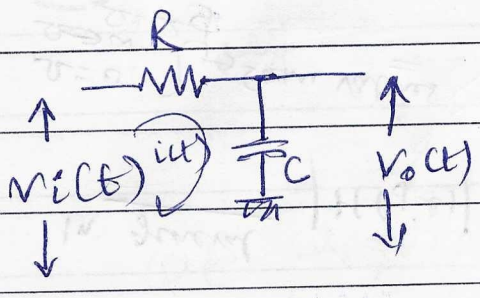
RC filter



KVL: $V_i(t) = iR + V_o(t) \rightarrow (1)$

WKT: $V_o(t) = V_c(t) = \frac{1}{C}\int i(t)\,dt$

Diff. Wrt t: $\frac{dv_o(t)}{dt} = \frac{1}{C} i(t)$

$$i(t) = C\frac{dv_o(t)}{dt} \rightarrow (2)$$

(2) in (1)

$$V_i(t) = RC\frac{dv_o(t)}{dt} + V_o(t)$$

in std form $x(t) = RC\,y' + y(t)$

LT ⇒ $V_i(s) = RCs\,V_o(s) + V_o(s)$

TF ⇒ $\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+1}$

$H(s) = \frac{1/RC}{s + 1/RC}$ $\qquad f_c = \frac{1}{2\pi RC}$

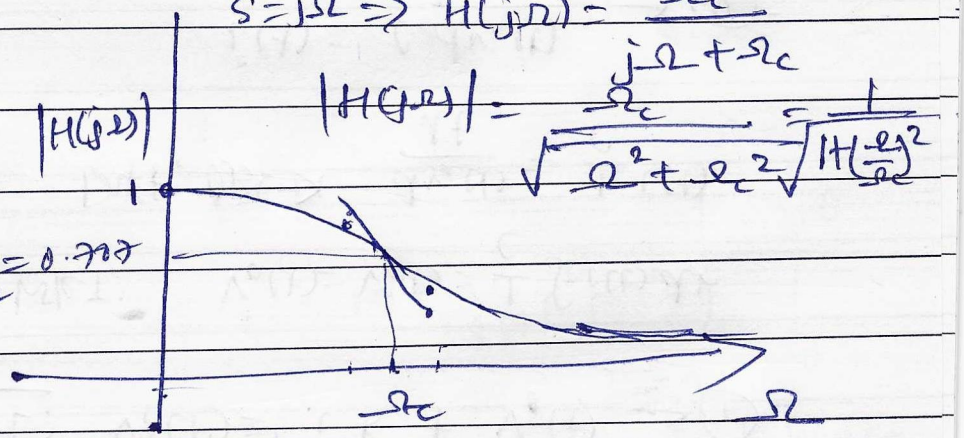
$$\boxed{H(s) = \frac{\Omega_c}{s+\Omega_c}} \rightarrow (3)$$

$2\pi f_c = \frac{1}{RC}$

$\Omega_c = \frac{1}{RC}$

$s = j\Omega \Rightarrow H(j\Omega) = \frac{\Omega_c}{j\Omega + \Omega_c}$

$|H(j\Omega)| = \frac{\Omega_c}{\sqrt{\Omega^2 + \Omega_c^2}} = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^2}}$



In general $|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$

$\Omega = 0$, $\Omega \Rightarrow \infty$, $\Omega = \Omega_c$ → Same values

1.21×1.21

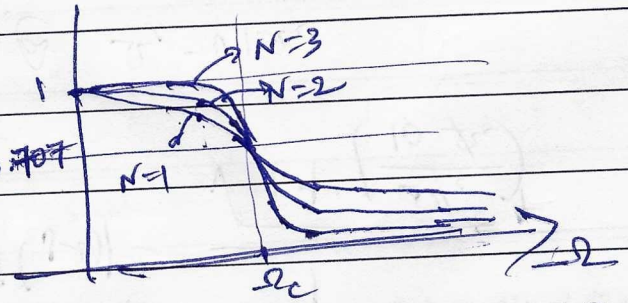
@ $\Omega_c + 0.1\Omega_c$ =

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega_c+0.1\Omega_c}{\Omega_c}\right)^4}} = \frac{1}{\sqrt{1+1.1^4}} = 1.4641$$

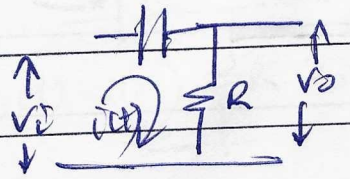
$= 0.68$

Ω	0	$\Omega_c - 0.1\Omega_c$	Ω_c	$\Omega_c + 0.1\Omega_c$	$\Omega \to \infty$
N=1	1	0.74	0.707	0.67	0
N=2	1	0.77	0.707	0.68	0
N=3	1	0.808	0.707	0.60	0

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}} = \frac{1}{\sqrt{1+(\Omega)^{2N}}}$$



RC HPF



KVL

$v_i(t) = v_c(t) + v_o$

WKT $V_o(t) = i(t)R$

$i(t) = \frac{V_o(t)}{R}$

$V_c(t) = v_i(t) - v_o(t)$

$v_i(t) = \frac{1}{C}\int i(t)\,dt + v_o(t)$

$i(t) = C\frac{dv_c}{dt} = C\frac{d(v_i - v_o)}{dt}$

$\therefore v_i(t) = v_o(t) + \frac{1}{C}\int i(t)\,dt$

$v_i(t) = v_o(t) + \frac{1}{C}\int \frac{v_o(t)}{R}\,dt$

$v_i(t) = v_o(t) + \frac{1}{RC}\int v_o(t)\,dt$

DIFF OBS $\frac{dv_i(t)}{dt} = \frac{dv_o(t)}{dt} + \frac{v_o(t)}{RC}$

$v_o(t) = RC\left[\frac{dv_i(t)}{dt} - \frac{dv_o(t)}{dt}\right]$

$V_o(s) = RCsV_i(s) - RCsV_o(s)$

$\frac{V_o(s)}{V_i(s)} = \frac{RCs}{1+RCs}$

$$H(s) = \frac{s}{s + 1/RC} = \frac{s}{s+\Omega_c}$$

$$H(s)\big|_{s=j\Omega} = H(j\Omega) = \frac{j\Omega}{j\Omega+\Omega_c}$$

$$|H(j\Omega)| = \frac{\Omega^2}{\sqrt{\Omega^2+\Omega_c^2}} = \frac{\Omega}{\sqrt{1+\left(\frac{\Omega_c}{\Omega}\right)^2}}$$

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega_c}{\Omega}\right)^{2n}}}$$

Ω	$\Omega=0$	$\Omega=0.9\Omega_c$	$\Omega=\Omega_c$	$\Omega=1.1\Omega_c$	$\Omega=\infty$
$n=1$	0		0.707		1
$n=2$	0		0.707		1
$n=3$	0		0.707		1
$n=5$	?	?	?	?	?

Find these values

Find these values