- Some NP-complete problems :
  - SATISFIABILITY
  - 0/1 Knapsack
  - PARTITION
  - Two-Processor Non-Preemptive Schedule Length
  - CLIQUE : An undirected graph G=(V, E) and a positive integer $J \leq |V|$

    Question : Does G contain a clique (complete subgraph) of size J or more?

# Proving NP-Completeness Results

- Example 1 : Show that the PARTITION problem is NP-C (NP-complete).

  Given a known NP-C problem - Sum of Subset Problem (SS), show that PARTITION problem is NP-C.

  SS Problem

  Instance : Given a finite set $A = \{a_1, a_2, \ldots, a_n\}$ of n positive numbers & a positive number M.

  Question : Is there a subset $A' \subseteq A$ such that $\Sigma A' = M$

## PARTITION Problem

Instance : Given a finite set $B=\{b_1, b_2, \ldots, b_m\}$ of m positive integers.

Question : Is there a subset $B' \subseteq B$ such that $\Sigma B' = \Sigma (B-B')$

(1) Show that: PARTITION is in NP

step#1: guess a subset B‘ of B (use one guess statement for each element) O(m)

step#2: verify $\Sigma B' = \Sigma (B-B')$? O(m)

Total O(m)

## (2) Show that: SS $\propto$ PARTITION

Given an arbitrary instance of SS,

i.e. $A = \{a_1, a_2, \ldots, a_n\}$ and M,

Construct an instance of PARTITION as follows :

$B=\{b_1, b_2, \ldots, b_n, \mathbf{b_{n+1}}, \mathbf{b_{n+2}}\}$ of $m = n+2$ positive numbers where

$$b_i = a_i \quad \text{for } 1 \leq i \leq n$$

$$\mathbf{b_{n+1} = M + 1}$$

$$\mathbf{b_{n+2} = \Sigma A + (1 - M)}$$

Note : $\Sigma\, b_i = 2\, \Sigma A + 2$. Also, the transformation can be done in polynomial time (based on input size of A & M)

To show the transformation is correct : The SS problem has a solution if and only if the PARTITION problem has a solution.

- If SS problem has a solution, then the PARTITION problem has a solution

  assume A' is the solution for SS problem then

  $Z' = A' \cup \{b_{n+2}\}$ and $Z\text{-}Z' = A\text{-}A' \cup \{b_{n+1}\}$

  $\Sigma Z' = M + \Sigma A + (1 - M) = \Sigma A + 1 = \Sigma (Z\text{-}Z')$

- If the PARTITION problem has a solution then the SS problem has a solution

if Z' is the solution then $\Sigma Z' = \Sigma A + 1$

$\rightarrow$ exactly one of $b_{n+2}$ or $b_{n+1} \in Z'$

$\rightarrow$ if $b_{n+2} \in Z'$ then $Z' - \{ b_{n+2} \} = A'$ and $\Sigma A' = M$

if $b_{n+1} \in Z'$, then use Z - Z' to obtain A'