- Example 2 : Show that the Traveling Salesman (TS) Problem is NP-complete.

  Given a known NPC problem - Hamiltonian Circuit (HC), show that TS problem is NPC.

  Hamiltonian Circuit (HC) problem

  Instance : Give an undirected graph G=(V, E)

  Question : Does G contain a Hamiltonian circuit, i.e. a sequence $< v_1, v_2, \ldots, v_n >$ of all vertices in V which is a simple cycle.

Traveling Salesman (TS) Problem

Instance : Give an undirected complete graph $G=(V, E)$ with distance $d(i,j) \geq 0$ for each edge (i,j) for $i \neq j$ and a positive integer B.

Question : Is there a tour of all cities (a simple cycle with all vertices) having total distance no more than B.

- TS is in NP

  guess a tour, i.e. sequence of all vertices $O(|V|)$

  verify that it is a cycle covering all vertices and total distance $\leq B$ $O(|V|)$

- HC $\propto$ TS

  Given arbitrary instance of HC, i.e. $G=(V, E)$.

  Construct an instance of TS as follows :

  $G' = (V, E')$, where $(u,v) \in E'$

  for all $u, v \in V$ and $u \neq v$

  $d(u,v) = 0 \quad$ if $(u,v) \in E$

  $d(u,v) = 1 \quad$ if $(u,v) \notin E$

  and $B = 0$

  Note : The transformation can be done in polynomial time (based on input size of V and E)

To show the transformation is correct : The HC problem has a solution if and only if the TS problem has a solution.

- If HC problem has a solution, then TS problem has a solution

  Assume a $< v_1, v_2, \ldots, v_n >$ is the solution for HC →
  It is a simple cycle which contains all vertices →
  Each edge (u,v) in this cycle has d(u,v) = 0 →
  Total distance is 0 →
  Solution for TS

- If TS problem has a solution then HC problem has a solution

  Obvious to see.

- Example 3 : Show that the Vertex Cover (VC) Problem is NP-complete.

  Given 3SAT problem is NPC, show that VC problem is NPC.

  3SAT Problem

  Instance : Given a set of variables $U = \{u_1, u_2, \ldots, u_n\}$ and a collection of clauses $C = \{c_1, c_2, \ldots, c_m\}$ over U such that $|c_i| = 3$ for $1 \leq i \leq m$.

  Question : Is there a truth assignment for U that satisfies all clauses in C?

  Note : 3SAT problem is a restricted problem of SATISFIABILITY problem.

Vertex Cover (VC) Problem

Instance : Given an undirected graph $G=(V, E)$ and a positive integer $K \leq |V|$

Question : Is there a vertex cover of size K or less for G, i.e. a subset $V' \subseteq V$ such that $|V'| \leq K$ and, for each $(u,v) \in E$, at least one of u or $v \in V'$.

- VC is in NP

  guess a set of vertices $V' \subseteq V$ $O(|V|)$

  verify that $|V'| \leq K$ and,

  for each $(u,v) \in E$, $u \in V'$ or $v \in V'$ $O(|V|+|E|)$