Decision Trees

Gini Impurity

Gini Impurity

A mathematical measurement of how "pure" the information in a data set is.

We can think of this as a measurement of class uniformity.

Gini Impurity for Classification:

For a set of classes C for a given dataset Q:

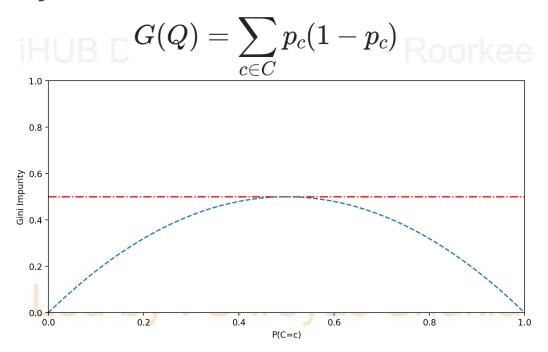
$$G(Q) = \sum_{c \in C} p_c (1-p_c)$$

Gini Impurity for Classification:

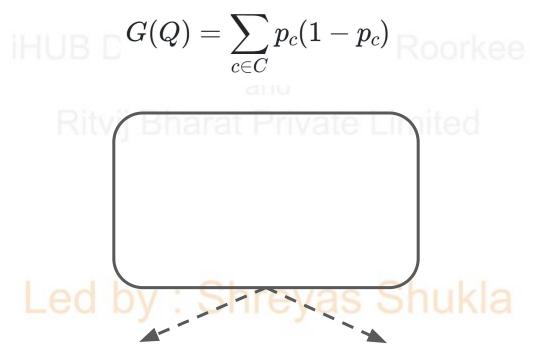
For a set of classes C for a given dataset Q, p_c is probability of class c.

$$p_c = rac{1}{N_Q} \sum_{x \in Q} \mathbb{1}(y_{class} = c) \hspace{0.5cm} G(Q) = \sum_{c \in C} p_c (1 - p_c)$$

Gini Impurity for Classification:

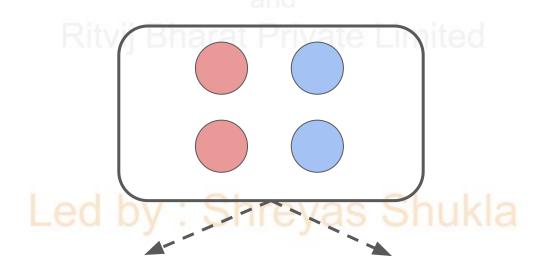


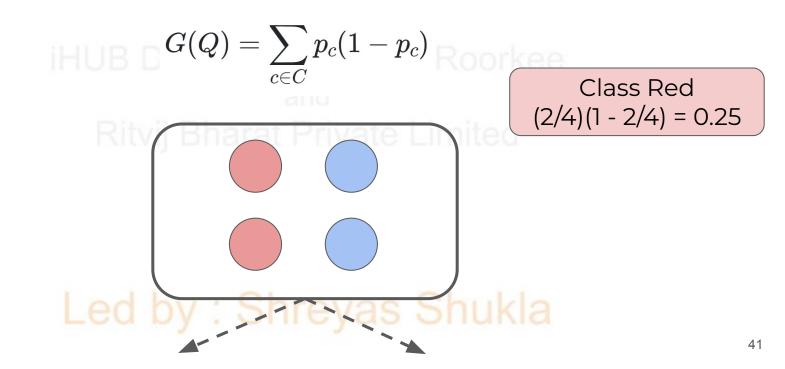
Gini Impurity for Classification:

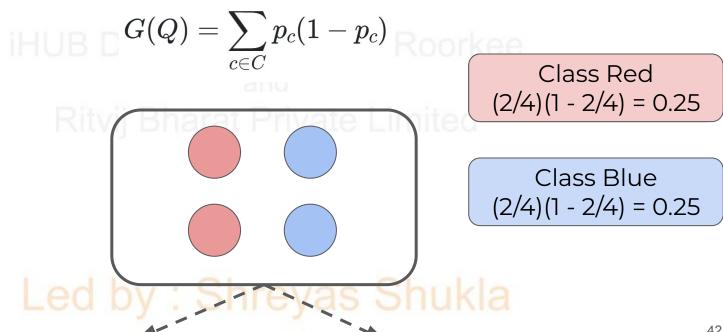


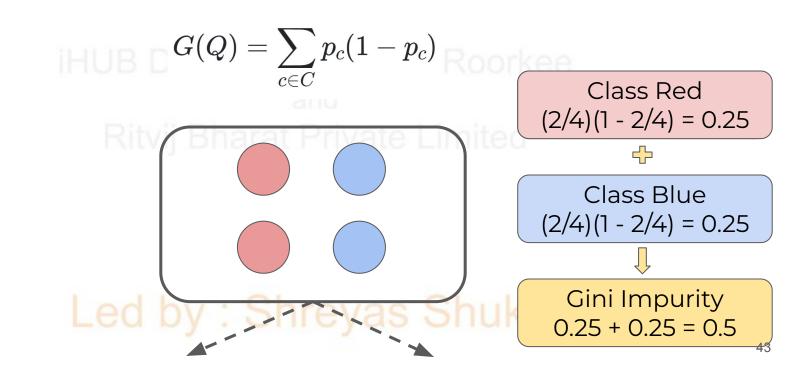
Gini Impurity for Classification:

$$G(Q) = \sum_{c \in C} p_c (1-p_c) \Big|_{\mathsf{Rookee}}$$

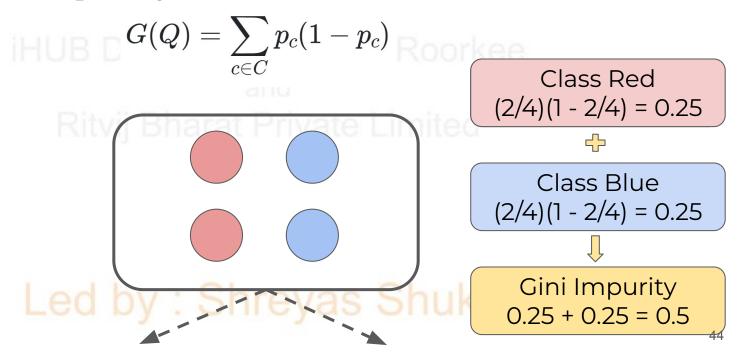




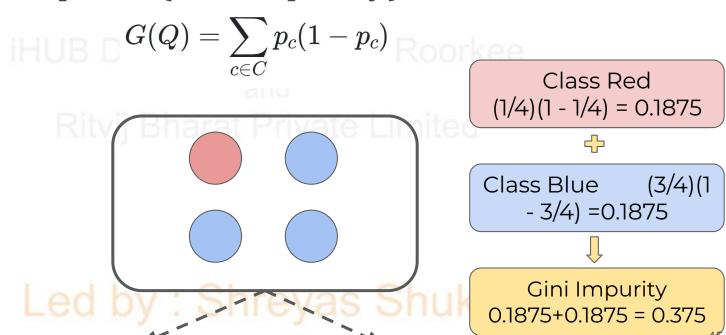




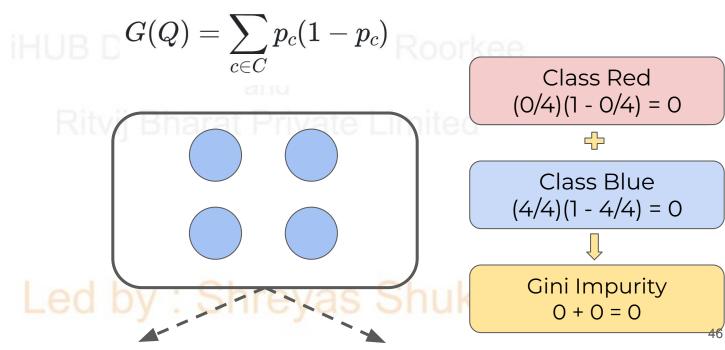
"Maximum" Impurity Possible



Data is more "pure" (less impurity)



Data is completely "pure" (no impurity)



Mastering Machine Learning with Pythor

If the goal of a decision tree is to separate out classes, we can use gini impurity to decide on data split values.

We want to minimize the gini impurity at leaf nodes.

Minimized impurity at leaf nodes means we are separating classes effectively

Decision Trees

Gini Impurity in Trees

For constructing a tree, we have to decide what feature will be root node.

Use gini impurity to compare the information contained within features for the training data.

Gini Impurity for Classification:

For a set of classes C for a given dataset Q, p_c is probability of class c.

$$p_c = egin{array}{c} rac{1}{N_Q} \sum_{x \in Q} \mathbb{1}(y_{class} = c) & G(Q) = \sum_{c \in C} p_c (1 - p_c) \end{array}$$

Create a decision tree to predict spam.

X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	ed by Sh

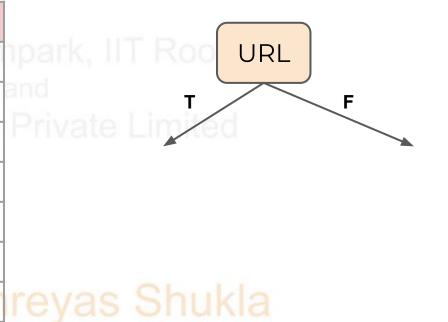
Only one X feature to use for a node.

X - URL Link	Y-Spam	
Yes	Yes	park, IIT Roo U
Yes	Yes	and Delicerte I lesite d
No	No	Private Limited
No	No	
No	Yes	
No	No	
Yes	ed by Sh	reyas Shukla

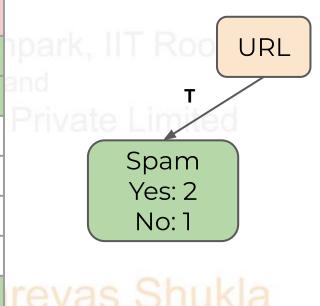


Predict if email is spam if it contains a URL:

X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	ed No Sh

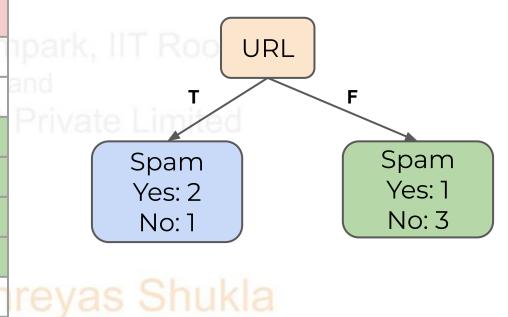


X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	No



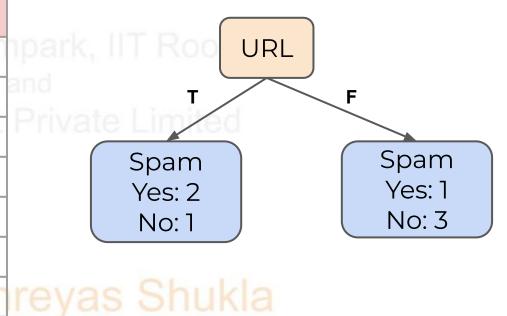
reyas Shukla

X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	ed byo: Sh



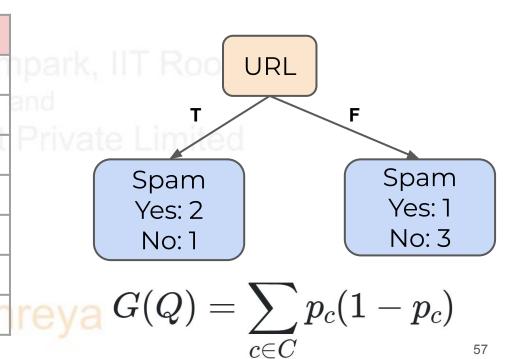
Predict if email is spam if it contains a URL:

X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	ed DNo Sh



Recall the gini impurity formula:

X - URL Link	Y-Spam
Yes	Yes
Yes	Yes
No	No
No	No
No	Yes
No	No
Yes	ed byo: Sh



Treat Yes Spam and No Spam as **c** classes:

Left Leaf Node:
$$(2/3)(1-2/3) + (1/3)(1-1/3)$$

Spam
Yes: 2
No: 1

No: 3

Led by : Shreya $G(Q) = \sum_{c \in C} p_c (1 - p_c)$

Treat Yes Spam and No Spam as **c** classes:

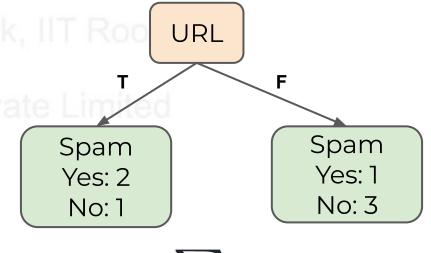
Left Leaf Node: DivyaSampark, IIT Roo

$$(\frac{2}{3})(1-\frac{2}{3}) + (\frac{1}{3})(1-\frac{1}{3})$$

Left Leaf Gini=0.44

Right Leaf Node:

$$(\frac{1}{4})(1-\frac{1}{4}) + (\frac{3}{4})(1-\frac{3}{4})$$



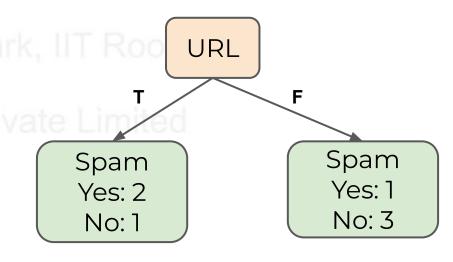
Right Leaf Gini=0.375
$$G(Q) = \sum_{c \in C} p_c (1-p_c)$$

Calculate gini impurity of URL feature.

Weighted Average of both:

Left Leaf Gini=0.44

Right Leaf Gini=0.375



Led by : Shreya
$$G(Q) = \sum_{c \in C} p_c (1 - p_c)$$

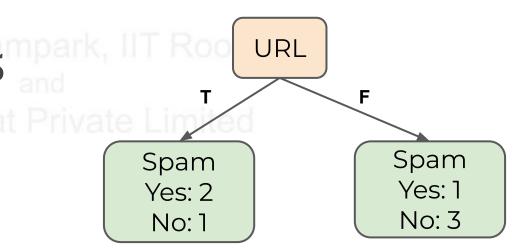
Total Emails: (2+1) + (1+3) = 7

Left Leaf Gini=0.44

Right Leaf Gini=0.375

Left Emails: 3 | Bharat Private Lin

Right Emails: 4



Led by : Shreya
$$G(Q) = \sum_{c} p_c (1 - p_c)$$

Total Emails:
$$(2+1) + (1+3) = 7$$

Left Leaf Gini=0.44

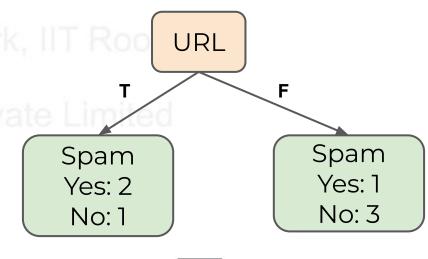
Right Leaf Gini=0.375

Left Emails: 3 | Bharat Private Lin

Right Emails: 4

(3/7)*0.44 + (4/7)*0.375

Gini Impurity: 0.403



Led by : Shreya
$$G(Q) = \sum_{c \in C} p_c (1 - p_c)$$

More issues to consider:

- Multiple Features
- Continuous Features
- Multi-categorical Features

We use the gini impurity to each of these issues to solve for best root nodes and best split parameters for leaves.

Decision Trees

Gini Impurity Part Two

Let's explore:

- Continuous numeric features
- Multi-categorical features (N>2)
- Choosing a root node feature

Imagine a continuous feature. Calculate the feature gini impurity:

X - Words in Email	Y-Spam
10	Yes
40	No
20	Yes
50	No
30	No

Sort data:

X - Words in Email	Y-Spam
10	Yes
40	No
20	Yes
50	No
30	No

Calculate potential split values for node

X - Words in Email	Y-Spam
10	Yes
20	Yes
30	No
40	No
50	No

ppark, IIT R Words ≤ N

Use averages between rows as values:

		a proper proper at
X - Wo	ords in Email	Y-Spam
15	10	Yes
	20	Yes
25	30	No
35	40	No
45	50	No

ppark, IIT R Words ≤ N

Perform all the potential split:

			bearing HT D.
X - Wo	rds in Email	Y-Spam	npark, III Ro
15	10	Yes	and
	20	Yes	Private Limi
25	30	No	
35	40	No	
45	50	No	

Words ≤ 15

Calculate gini impurity for each split:

X - Wo	ords in Email	Y-Spam
15	10	Yes
	20	Yes
	30	No
	40	No
	50	No

Words ≤ 15

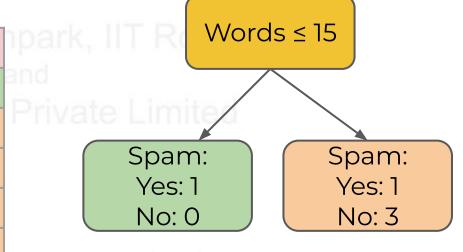
Calculate gini impurity for each split:

	11.11	
X - Wo	rds in Email	Y-Spam
15	10	Yes
13	20	Yes
	30	No
	40	No
	50	No

Words ≤ 15

Calculate gini impurity for each split:

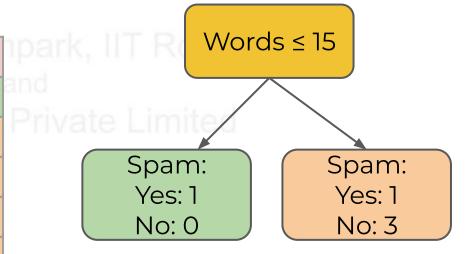
X - Words in Email		Y-Spam
15	10	Yes
	20	Yes
	30	No
	40	No
	50	No



Led by : Sh
$$G(Q) = \sum_{c \in C} p_c (1 - p_c)$$

Calculate gini impurity for each split:

X - Words in Email		Y-Spam
15	10	Yes
13	20	Yes
	30	No
	40	No
	50	No



$$G(Q) = (\%)(O+O) + (\%)((1/4)(1-1/4)+(3/4)(1-3/4)$$
 $= 0.3$

Do it for all possible splits:

X - Words in Email	Y-Spam	npark, IIT Roorkee
15 10	Yes	Cini-07
20	Yes	Gini=0.3 Gini=0
30	No	Gini=0.26
40	No	
50	No	Gini=0.4
	an hu · St	rovae Shukla

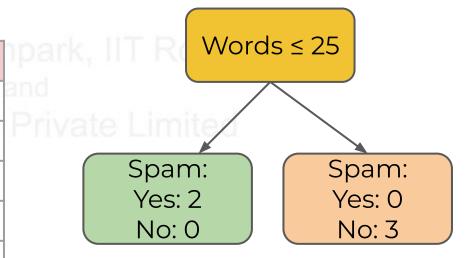
75

Choose lowest impurity split value

X - Wo	ords in Email	Y-Spam
	10	Yes
٦٢	20	Yes
25	30	No
	40	No
	50	No

Choose this as split value for node

TO THE TIME AND THE PARTY.			
X - Words in Email		Y-Spam	
	10	Yes	
25	20	Yes	
	30	No	
	40	No	
	50	No	



Led by : Sh
$$G(Q) = Q_{lukla}$$

Multicategorical feature

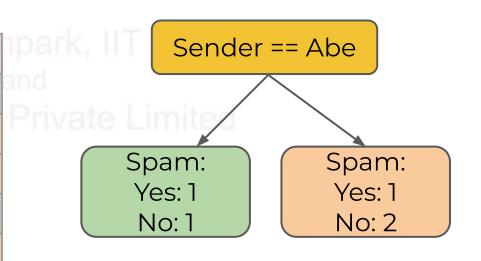
Calculate gini impurity for all combinations:

X - Sender	Y-Spam	
Abe	Yes	
Bob	Yes	
Claire	No	
Abe	No	
Bob	No	

npark, IIT Roorkee

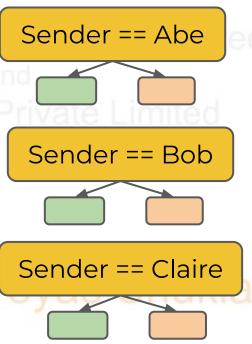
Calculate gini impurity for all combinations:

X - Sender	Y-Spam		
Abe	Yes		
Bob	Yes		
Claire	No		
Abe	No		
Bob	No		



Calculate gini impurity for all combinations

Sender			
Serider	Y-Spam	X - Sender	
and	Yes	Abe	
Private.	Yes	Bob	
Sende	No	Claire	
	No	Abe	
Condor	No	Bob	
Sender	ed by : Sh	L	



Calculate gini impurity for all combinations

Choose lowest impurity split combination

X - Sender	Y-Spam	Sender == Abe	Sender == Abe or Bob
Abe	Yes		
Bob	Yes	Private Limited	Sender ==
Claire	No	Sender == Bob	Claire or Bob
Abe	No		
Bob	No	Sender == Claire	Sender ==
	ed by · Sh	Serider Claire	Abe or Claire

Now we can split any type of feature.

How does the decision tree decide on the root node of a multi-feature dataset?

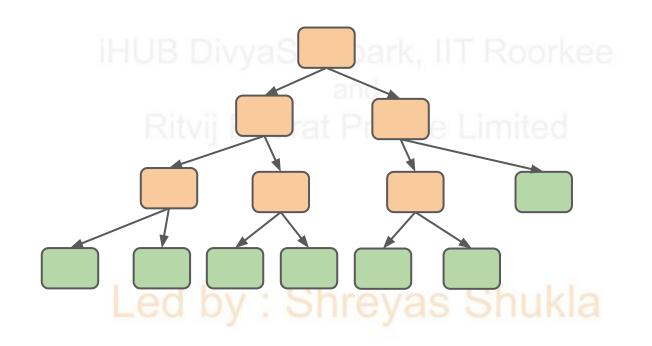
Calculate the gini impurity values of each feature and choose the lowest impurity value to split on first.

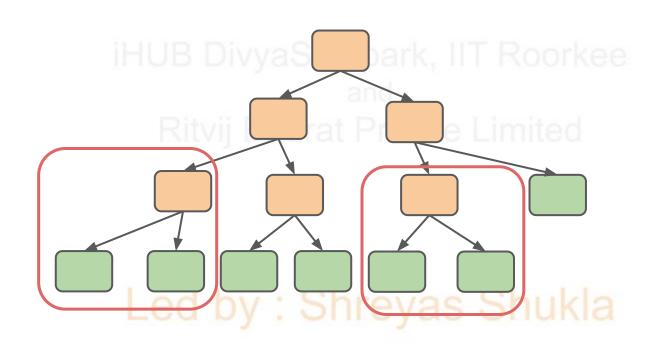
By choosing the feature with the lowest resulting gini impurity in its leaf nodes, we are choosing the feature that best splits the data into "pure" classes.

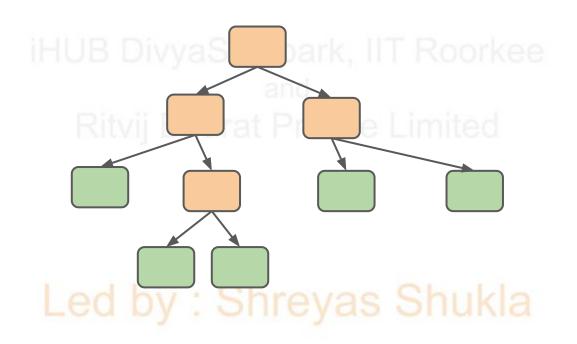
By using gini impurity as a measurement of the effectiveness of a node split, we can perform automatic feature selection by mandating an impurity threshold for an additional feature based split to occur.

A large overfitted tree.

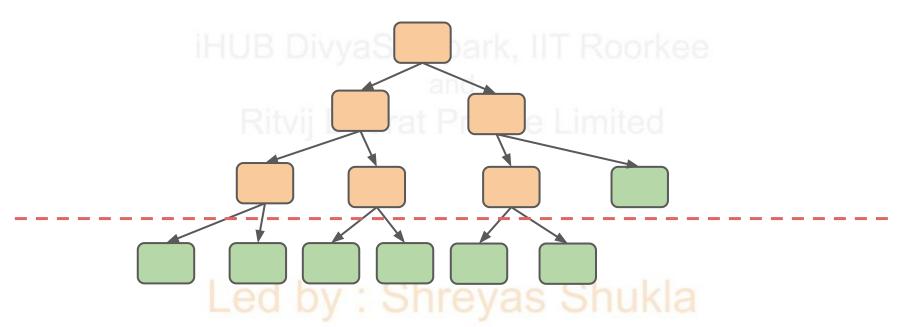
Add minimum gini impurity decrease

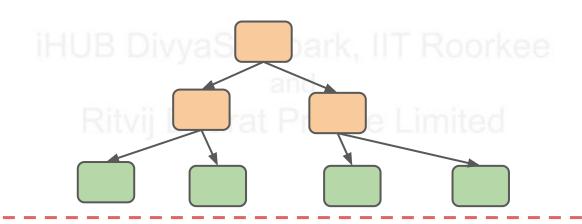






We can also mandate a max depth





iHUB DivyaSampark, IIT Roorkee
and
Ritvij Bharat Private Limited
Let's code !!