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(27th Aug 2024 - 18th Oct 2024)

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Algorithm Theory
Cost Function

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Recall we are searching for Beta values for a best-fit line.

$$\hat{y} = \sum_{i=0}^{n} \beta_i x_i$$

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How to choose these beta coefficients?

$$\hat{y} = \sum_{i=0}^{n} \beta_i x_i$$

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We've decided to define a "best-fit" as **minimizing the squared error.**

$$\hat{y} = \sum_{i=0}^{n} \beta_i x_i$$

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The residual error for some row ***j*** is:

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$$y^j - \hat{y}^j$$

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Squared Error for some row ***j*** is then:

$$\left(y^j - \hat{y}^j\right)^2$$

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Sum of squared errors for **m** rows is then:

$$\sum_{j=1}^{m} \left(y^j - \hat{y}^j\right)^2$$

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Average squared error for **m** rows:

$$\frac{1}{2m}\sum_{j=1}^{m}\left(y^{j}-\hat{y}^{j}\right)^{2}$$

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fining a cost function **J.**

$$J(\boldsymbol{\beta}) = \frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \hat{y}^j\right)^2$$

$$= \frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)^2$$

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A **cost function** is defined by some measure of error.

$$J(\boldsymbol{\beta})$$

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- This means we wish to **minimize** the cost function.

$$J(\boldsymbol{\beta})$$

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Our cost function can be defined by the squared error:

$$J(\boldsymbol{\beta}) = \frac{1}{2m} \sum_{j=1}^{m} \left(y^j - \hat{y}^j \right)^2$$

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Error between real **y** and predicted **ŷ**

$$J(\boldsymbol{\beta}) = \frac{1}{2m}\sum_{j=1}^{m}\left(\boxed{y^j - \hat{y}^j}\right)^2$$

Squaring corrects for negative and positive errors.

$$J(\boldsymbol{\beta}) = \frac{1}{2m} \boxed{\sum_{j=1}^{m}} \left(y^j - \hat{y}^j\right)^{\boxed{2}}$$

Summing error for m rows.

$$J(\boldsymbol{\beta}) = \frac{1}{2m} \sum_{j=1}^{\boxed{m}} \left(y^j - \hat{y}^j \right)^2$$

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Summing error for m rows.

$$J(\boldsymbol{\beta}) = \frac{1}{2m} \boxed{\sum_{j=1}^{m}} \left(y^j - \hat{y}^j\right)^2$$

Divide by **m** to get **mean**

$$J(\boldsymbol{\beta}) = \boxed{\frac{1}{2m}} \sum_{j=1}^{m} \left(y^j - \hat{y}^j \right)^2$$

Additional ½ is for convenience for derivative.

$$J(\boldsymbol{\beta}) = \frac{1}{\boxed{2}m} \sum_{j=1}^{m} \left(y^j - \hat{y}^j \right)^{\boxed{2}}$$

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$$\begin{aligned} J(\boldsymbol{\beta}) &= \frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \hat{y}^j\right)^2 \\ &= \frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)^2 \end{aligned}$$

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From calculus we know that to minimize a function we can take its derivative and set it equal to zero.

$$\begin{aligned}\frac{\partial J}{\partial \beta_k}(\boldsymbol{\beta}) &= \frac{\partial}{\partial \beta_k}\left(\frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)^2\right) \\ &= \frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)(-x_k^j)\end{aligned}$$

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$$\boxed{\frac{\partial J}{\partial \beta_k}}(\boldsymbol{\beta}) = \boxed{\frac{\partial}{\partial \beta_k}}\left(\frac{1}{2m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)^2\right)$$

$$= \frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)(-x_k^j)$$

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Recall from calculus to minimize a function we can take its derivative and set it equal to zero.

$$\frac{\partial J}{\partial \beta_k}(\boldsymbol{\beta}) = \frac{\partial}{\partial \beta_k}\left(\frac{1}{\boxed{2}m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)^{\boxed{2}}\right)$$

$$= \frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)(-x_k^j)$$

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- Unfortunately, it is not scalable to try to get an analytical solution to minimize this cost function.
- So we use **gradient descent** to minimize this **cost function.**

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Algorithm Theory
Gradient Descent

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we can describe this cost function through vectorized matrix notation and use **gradient descent** to have a computer figure out the set of Beta coefficient values that minimize the **cost/loss** function.

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Derivative of the cost function:

$$\frac{\partial J}{\partial \beta_k}(\boldsymbol{\beta}) = \frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)(-x_k^j)$$

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We can use a **gradient** to express the derivative of the cost function with respect to each β

$$\nabla_{\boldsymbol{\beta}} J = \begin{bmatrix} \frac{\partial J}{\partial \beta_0} \\ \vdots \\ \frac{\partial J}{\partial \beta_n} \end{bmatrix}$$

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$$\nabla_{\boldsymbol{\beta}} J = \begin{bmatrix} -\frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)x_0^j \\ \vdots \\ -\frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)x_n^j \end{bmatrix}$$

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We know we can vectorize our data:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1^1 & x_2^1 & \dots & x_n^1 \\ 1 & x_1^2 & x_2^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^m & x_2^m & \dots & x_n^m \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \qquad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$$

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$$\nabla_{\beta} J = \begin{bmatrix} -\frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)x_0^j \\ \vdots \\ -\frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right)x_n^j \end{bmatrix}$$

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and

$$\nabla_\beta J = \begin{bmatrix} -\boxed{\frac{1}{m}\sum_{j=1}^{m}}\left(y^j - \right.\boxed{\left.\sum_{i=0}^{n}\beta_i x_i^j\right) x_0^j} \\ \vdots \\ -\frac{1}{m}\sum_{j=1}^{m}\left(y^j - \sum_{i=0}^{n}\beta_i x_i^j\right) x_n^j \end{bmatrix}$$

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$$\nabla_{\beta} J = -\frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m} y^j x_0^j \\ \vdots \\ \sum_{j=1}^{m} y^j x_n^j \end{bmatrix} + \frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m} \sum_{i=0}^{n} \beta_i x_i^j x_0^j \\ \vdots \\ \sum_{j=1}^{m} \sum_{i=0}^{n} \beta_i x_i^j x_n^j \end{bmatrix}$$

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We can now calculate the gradient for any set of Beta values!

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$$\nabla_{\boldsymbol{\beta}} J = -\frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m} y^j x_0^j \\ \vdots \\ \sum_{j=1}^{m} y^j x_n^j \end{bmatrix} + \frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m}\sum_{i=0}^{n} \beta_i x_i^j x_0^j \\ \vdots \\ \sum_{j=1}^{m}\sum_{i=0}^{n} \beta_i x_i^j x_n^j \end{bmatrix}$$

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What is the best way to "guess"?

$$\nabla_{\boldsymbol{\beta}} J = -\frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m} y^j x_0^j \\ \vdots \\ \sum_{j=1}^{m} y^j x_n^j \end{bmatrix} + \frac{1}{m}\begin{bmatrix} \sum_{j=1}^{m}\sum_{i=0}^{n} \beta_i x_i^j x_0^j \\ \vdots \\ \sum_{j=1}^{m}\sum_{i=0}^{n} \beta_i x_i^j x_n^j \end{bmatrix}$$

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Common mountain analogy

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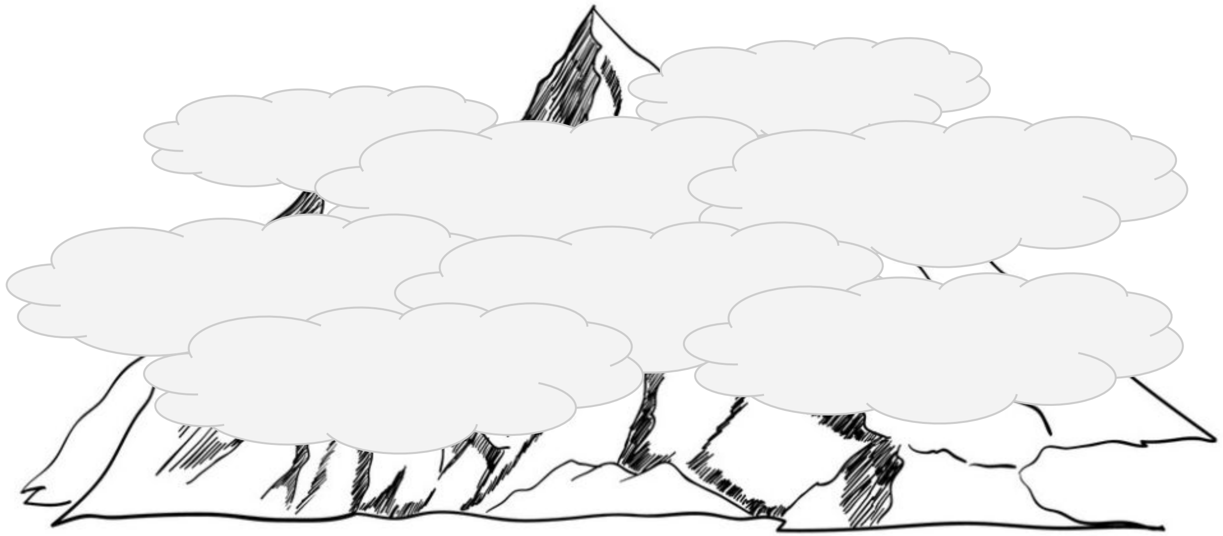

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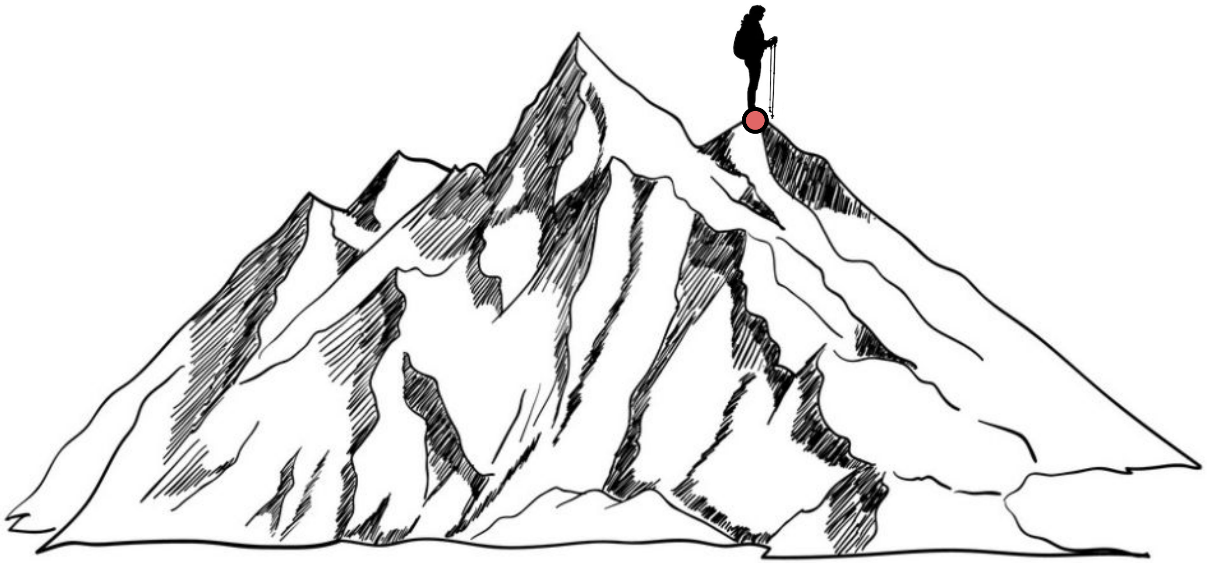

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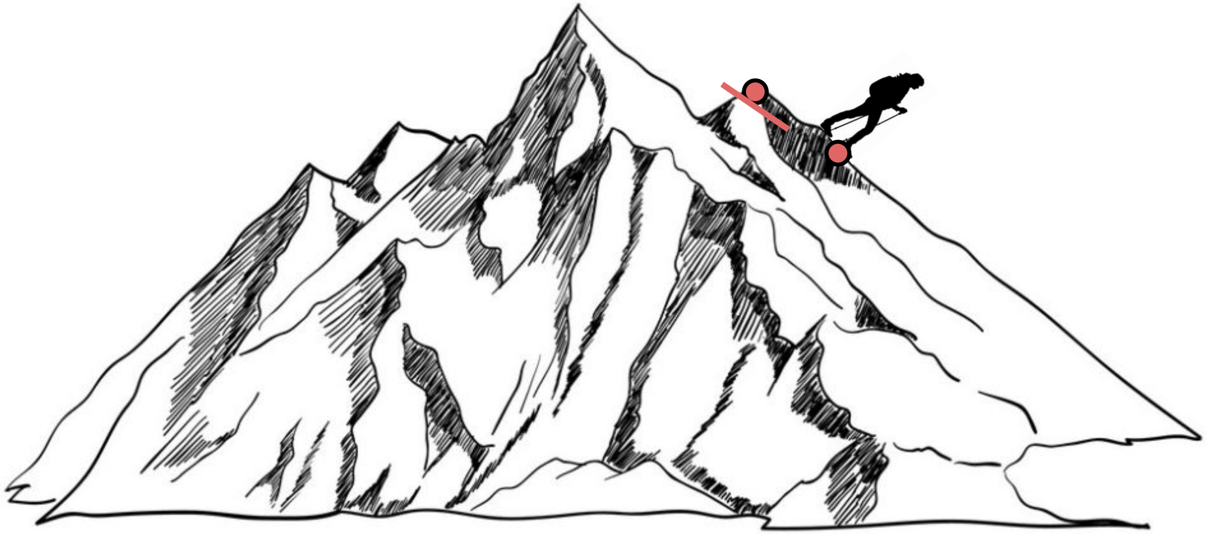

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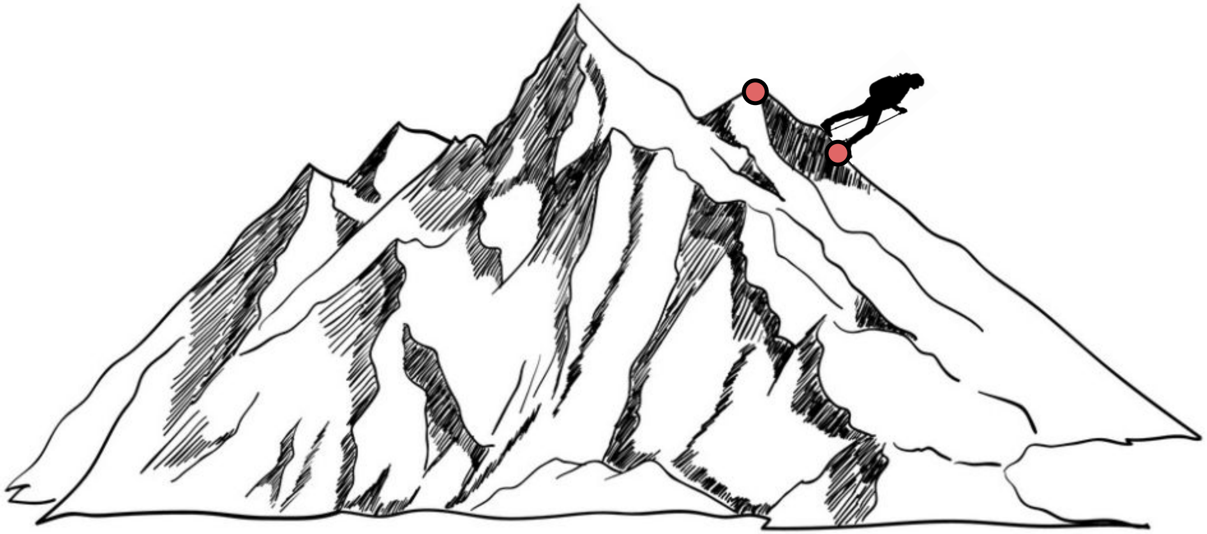

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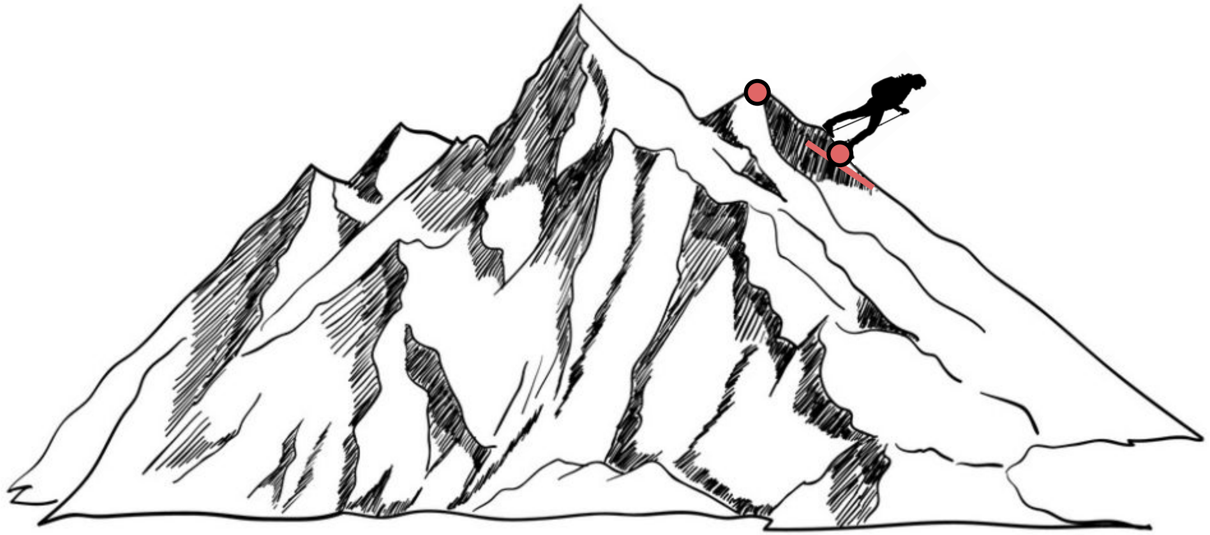

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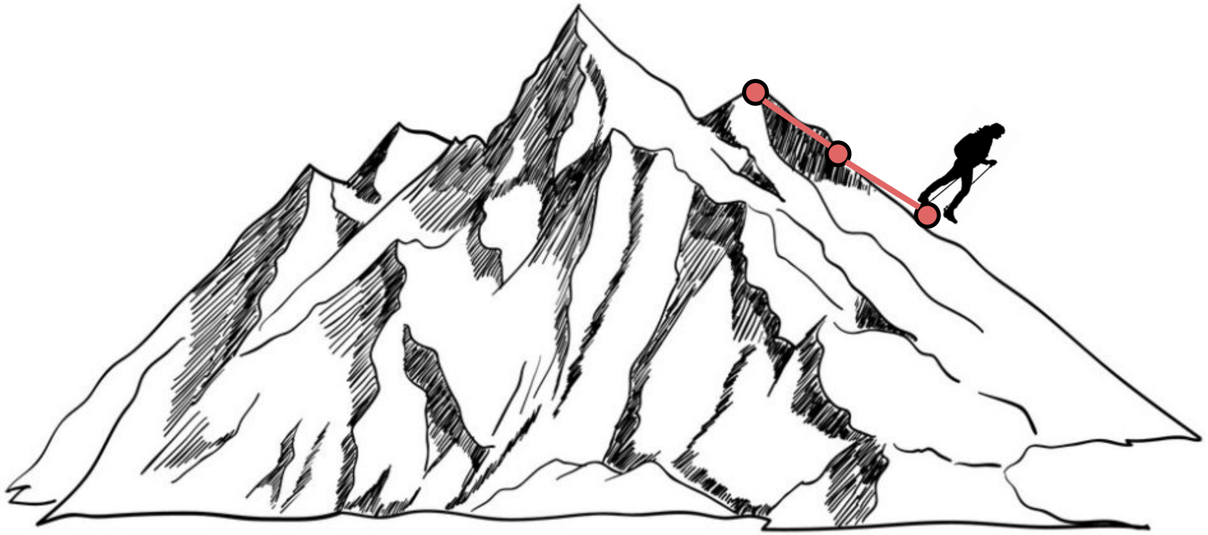

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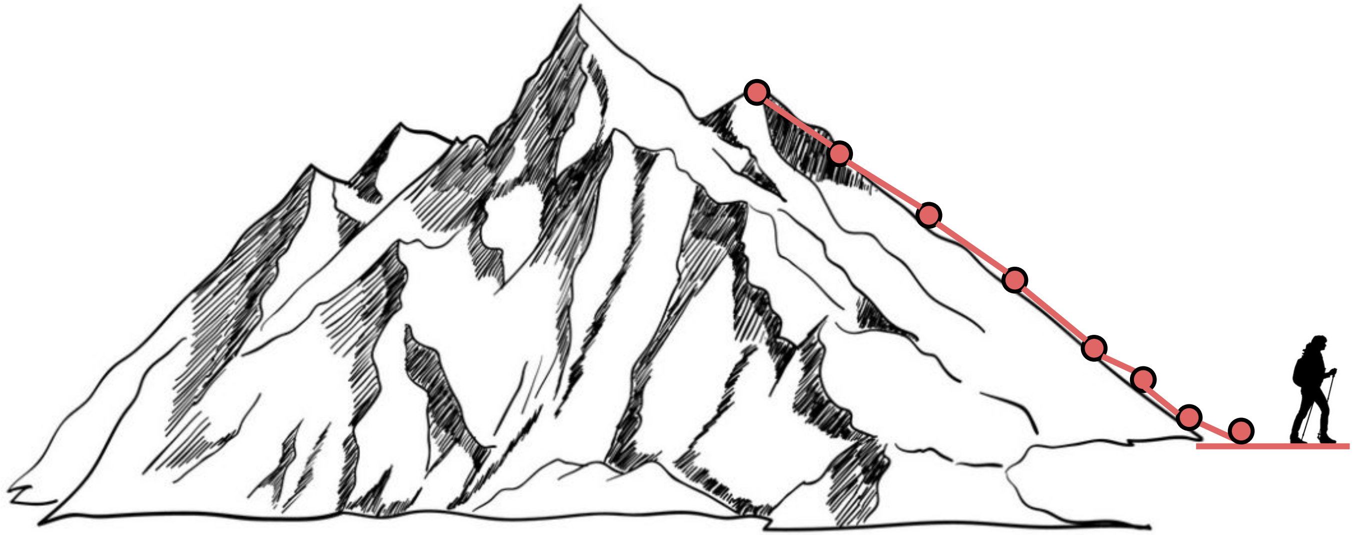

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1 dimensional cost function (single Beta)

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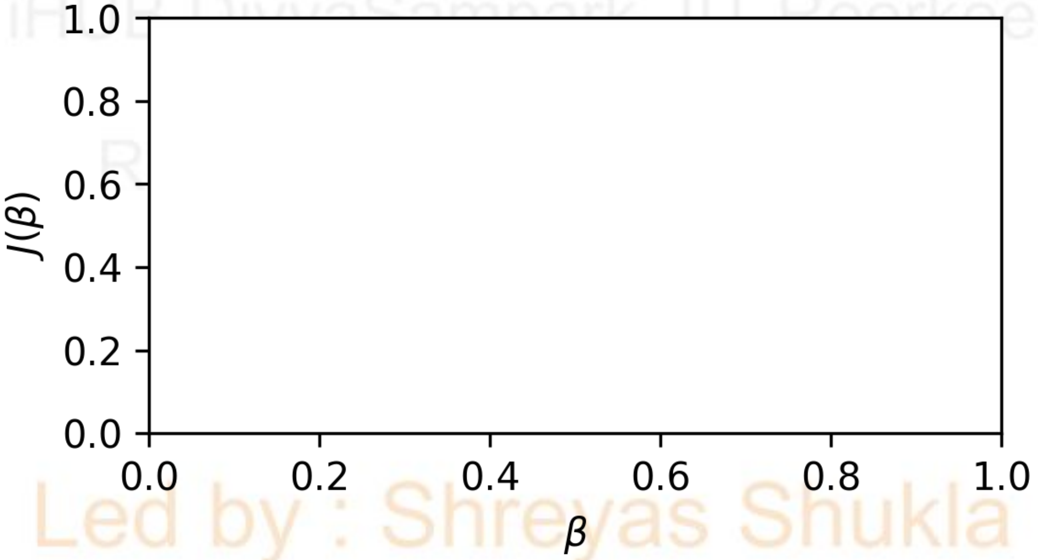

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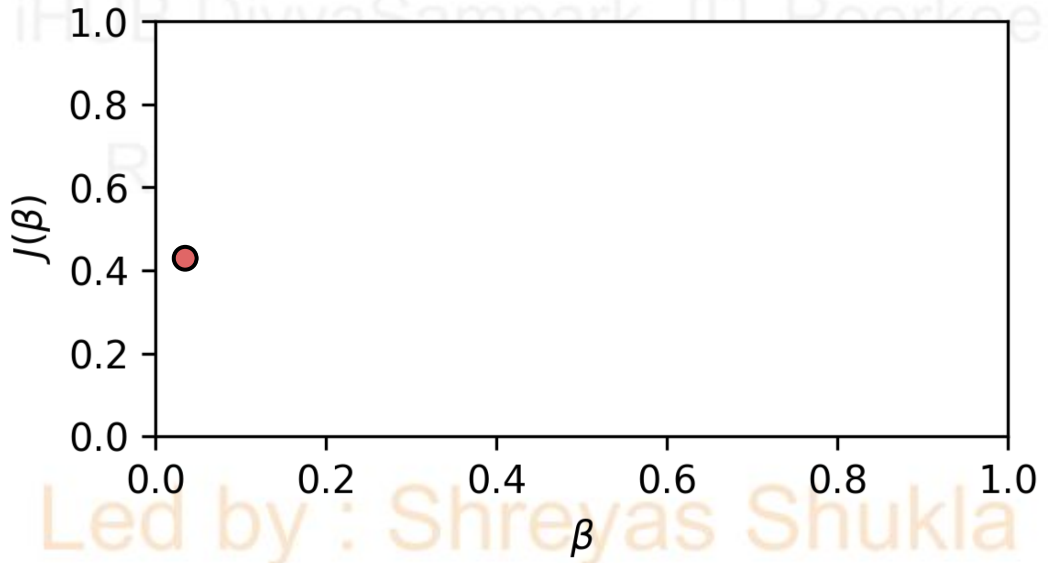
Choose a starting point



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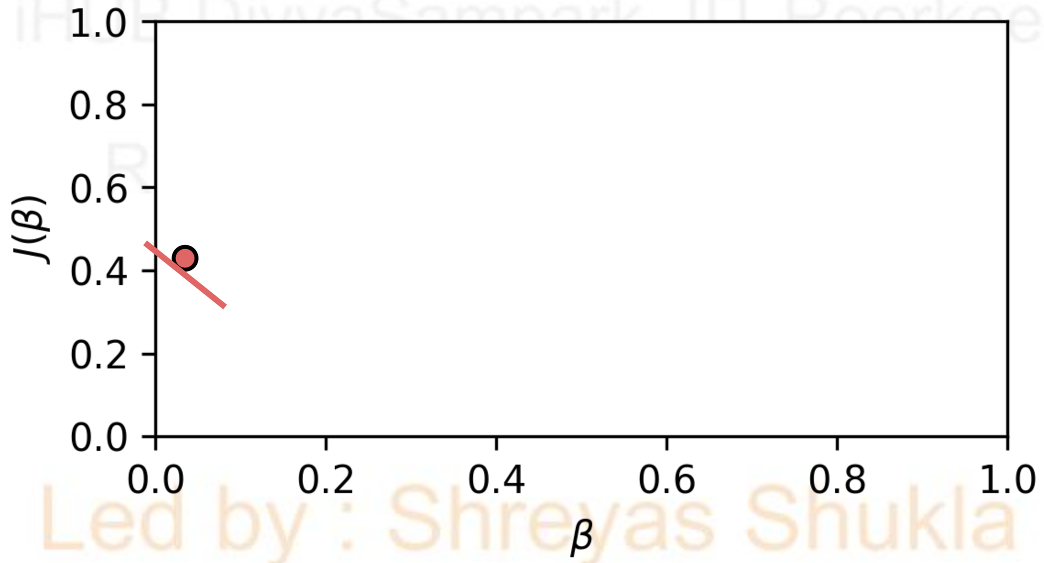
Calculate gradient at that point



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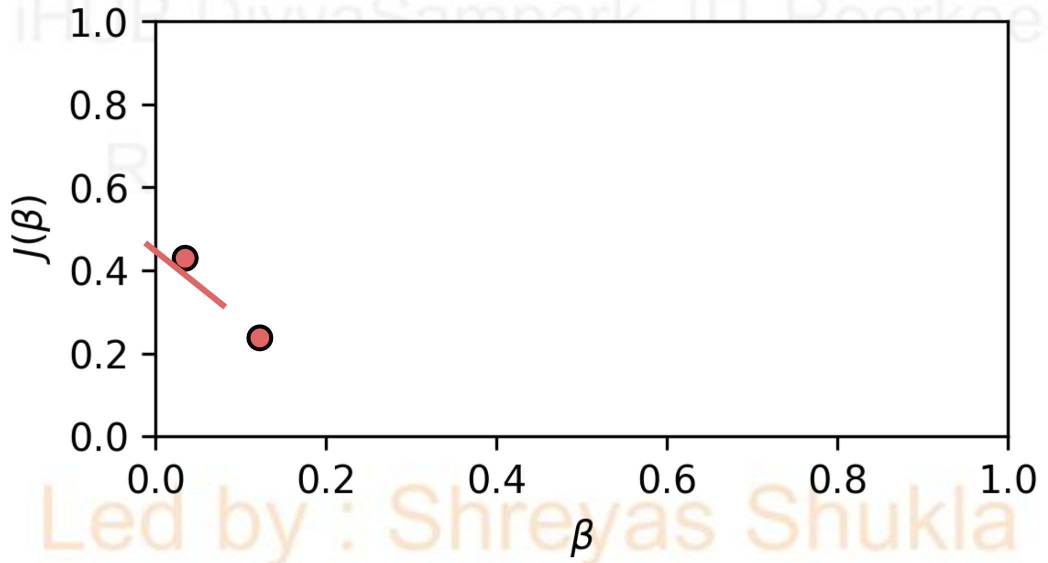
Step forward proportional to **negative** gradient



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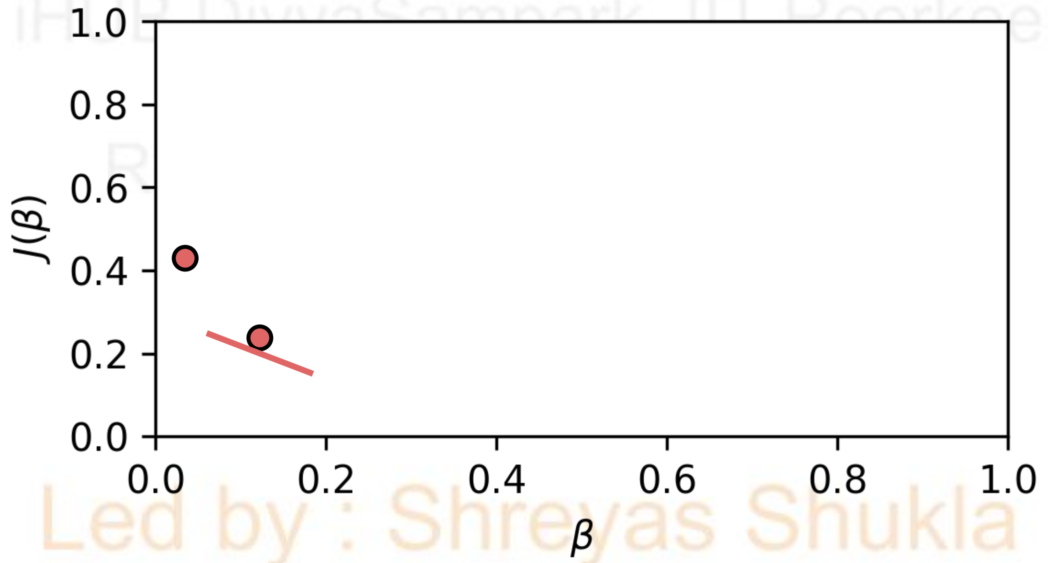
Repeat



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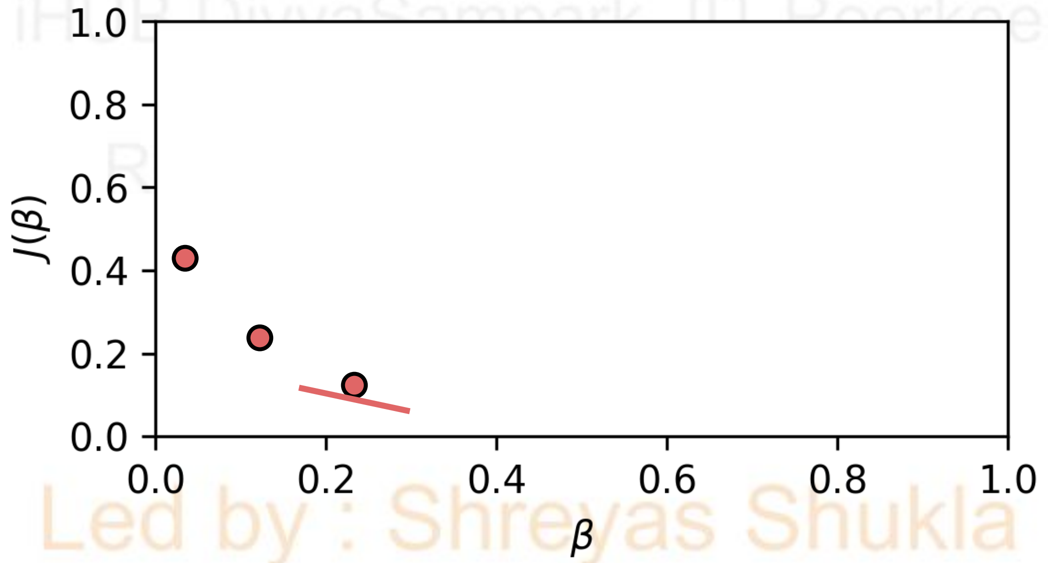
Repeat



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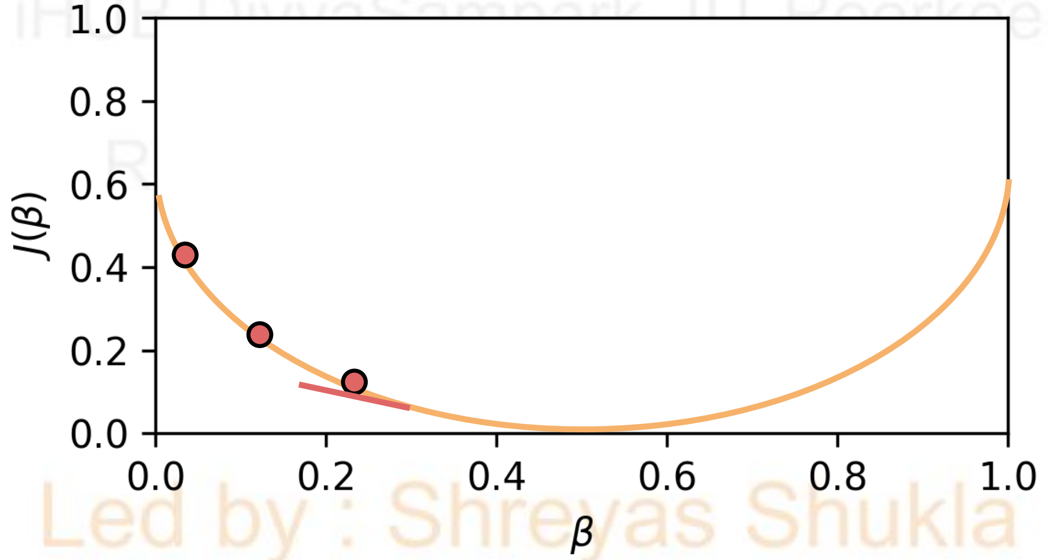

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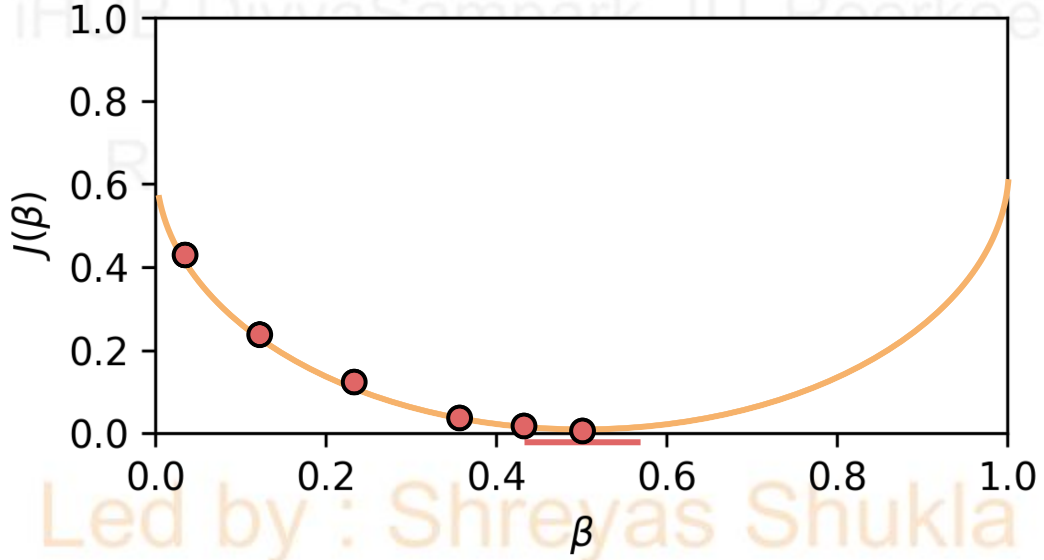
Eventually we will find the Beta that minimizes the cost function!



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Steps are proportional to negative gradient!
Steeper gradient at start gives larger steps.
Smaller gradient at end gives smaller steps.

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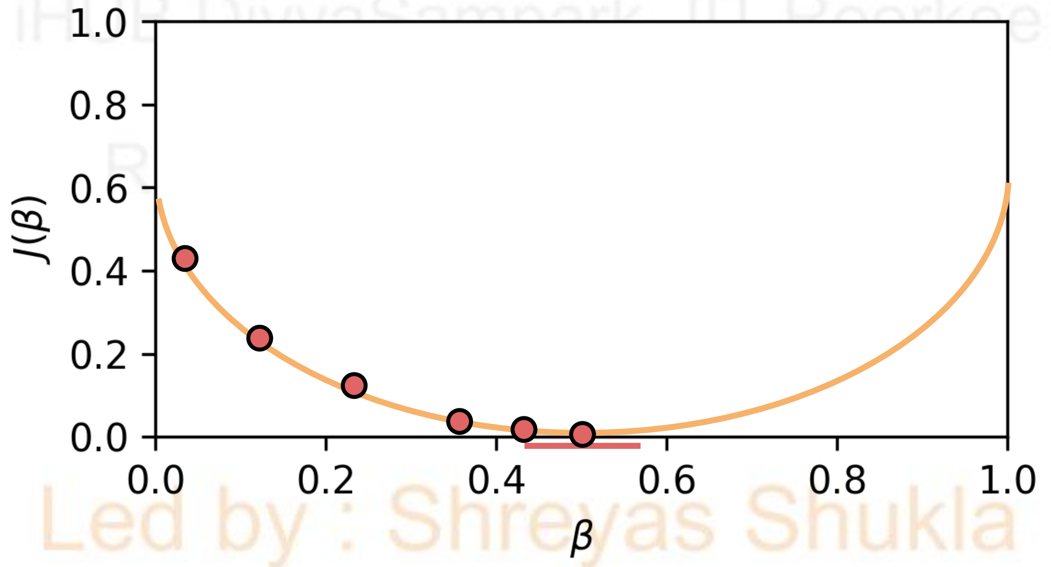

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To further understand this, let's visualize this gradient descent search for two Beta values.
Process is still the same:

- Calculate gradient at point.
- Move in a step size proportional to negative gradient.
- Repeat until minimum is found.

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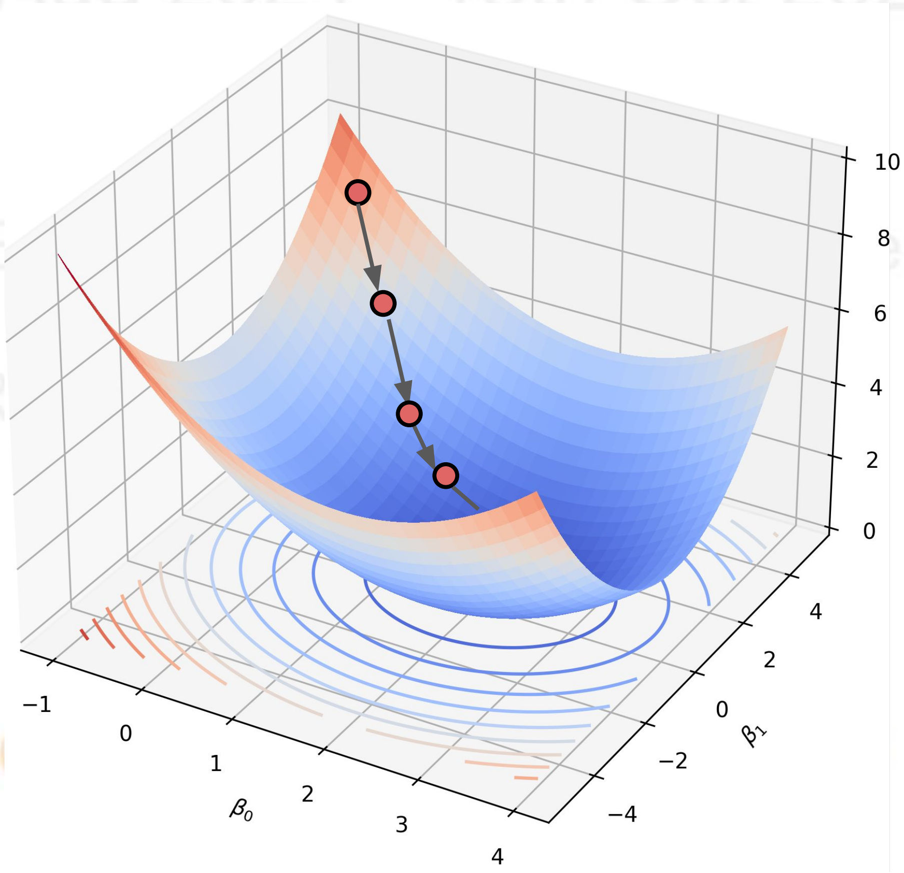

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Cost Function/Loss Function: It basically ***determines how well a machine learning model performs for a given dataset.*** It calculates the difference between the expected value and predicted value and represents it as a single term.

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Gradient Descent: "Gradient Descent is an optimization algorithm which is used for optimizing the cost function or error in the model." It enables the models to take the gradient or direction to reduce the errors by reaching to least possible error. Gradient descent is an iterative process where the model gradually converges towards a minimum value, and if the model iterates further than this point, it produces little or zero changes in the loss.

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