Module - 03

Linear & Adaptive Equalization :

Linear Equalization, Decision - feedback equalization Reduced complexity ML detectors, Iterative equalization and decoding - Turbo equaliz^n Adaptive linear equalizer, adaptive decision feedback equalizer, Recursive least square algorithms for adaptive equalization.

Introduction :

It is a technique used to combat ISI. An equalizer within a receiver compensates for average range of expected channel amplitude & delay characteristics

↳ Equalizer must be adaptive as channel is unknown & time-varying.

↳ It is a process of remove ISI & noise effects from channel. It is located at receiver end of the channel

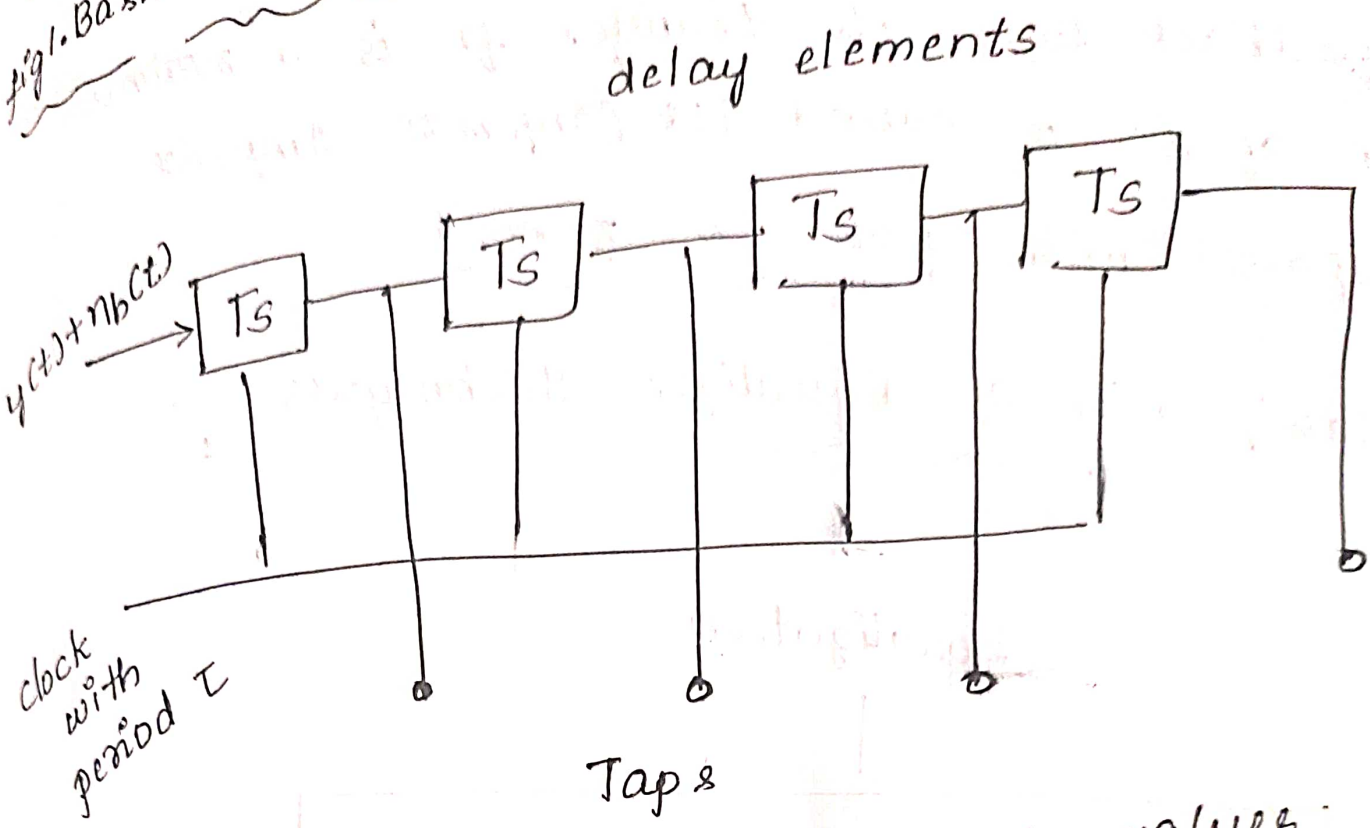
↳ Equalizer techniques can be classified as Linear & Non-linear equalization

↳ Linear equalizer : It d(t) is not used in the feedback path that equaliz^n is linear

↳ Non linear : It d(t) is fed back to change the o/p of equalizer.

Equalizer techniques.

fig/. Basic linear transversal equalizer structure



This determines that how the values of digital data bit being received & applies a slicing or thresholding operation in order to determine the value of d(t)

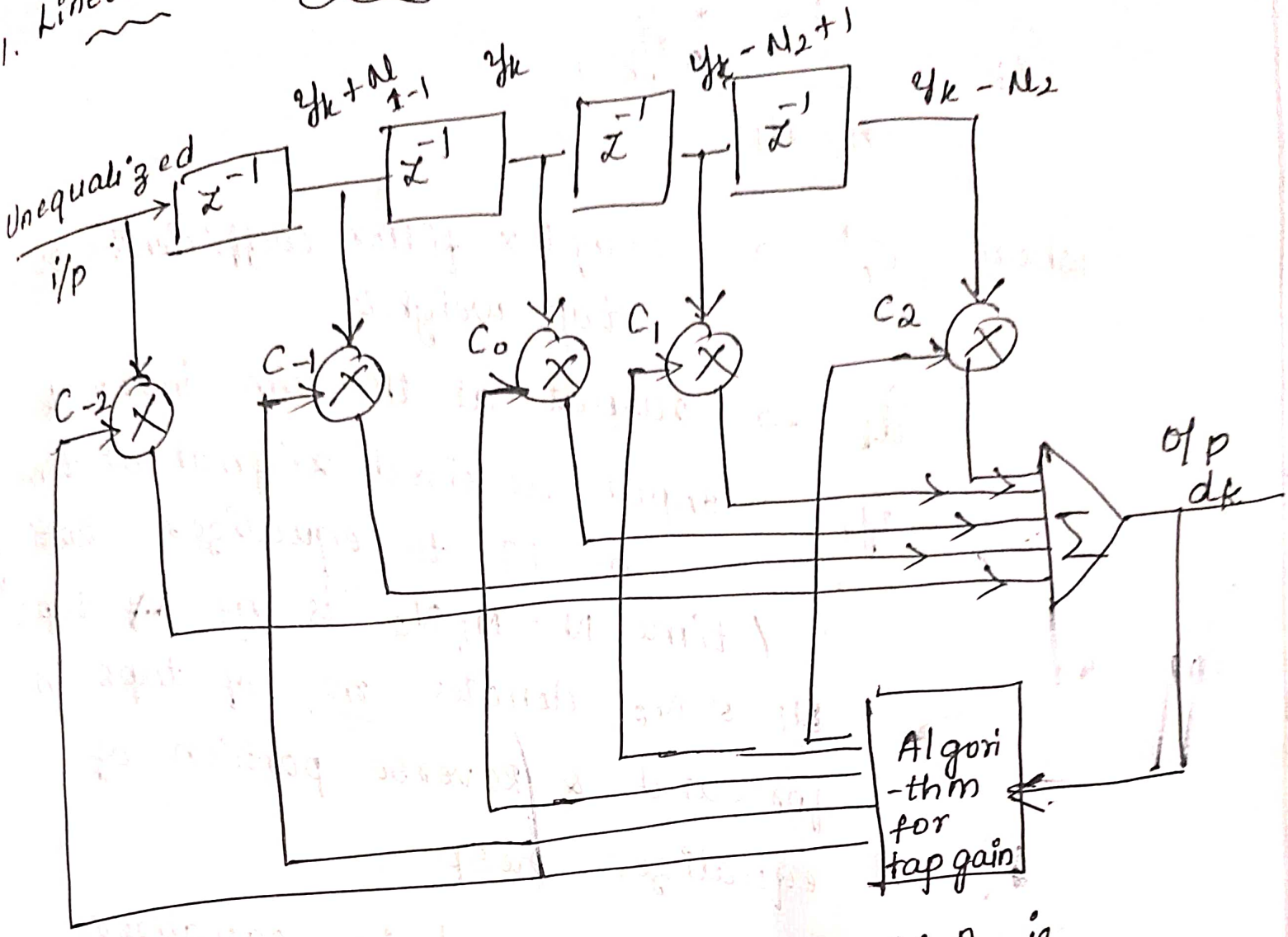
↳ LTE is made up of tapped delay line with tapping spaced a sym period (Ts) and it uses only feedforward taps & transfer function of equalizer filter. It has many zeroes but poles only at $z = 0$ & it is called as FIR (finite Impulse response) or transversal filter

↳ If the equalizer has both feedforward & feedback taps its transfer fn is a rational fn of z^{-1} is called IIR (Infinite Impulse response) with poles & zeros)

Classification of Equalizer Techniques

- Equalization
  - Linear
    - Transversal
      - Zero forcing
      - LMS
      - RLS
      - Fast RLS
      - Sq-Root RLS
    - Lattice
      - Gradient
      - RLS
  - Non-Linear
    - DFE
      - Transversal
        - LMS
        - RLS
        - Fast RLS
        - Sq-Root RLS
    - ML Symbol detector
      - Lattice
        - Gradient
        - RLS
    - MLSE
      - Transversal Channel Est
        - LMS
        - RLS
        - Fast RLS
        - Sq root RLS

1. Linear Equalization



↳ The linear filter used for equalizⁿ is transversal filter. In this current & past value of the received signal are linearly weighted by the filter coefficient & summed to produce o/p.

The output of this filter is gn by

$$\hat{d}_k = \sum_{n=N_1}^{N_2} C_n^* y_{k-n}$$

where C_n^* → complex filter coefficients or tap weights

$\hat{d}_k$ → output at the time index k

y_i → Input received signal at time to iT to equalizer starting time $N = N_1 N_2$ is no. of taps

N_1 & N_2 denotes no. of taps in forward & Reverse portion of equalizer resp.

↳ Two criteria have found in equalizer coefficients i.e., Peak distortion criteria and other is Mean square error criteria

Peak distortion Criteria (PDC)

Peak distortion criteria is defined as worst case ISI at the o/p of the equalizer. The minimizat^n of this performance index is called PDC.

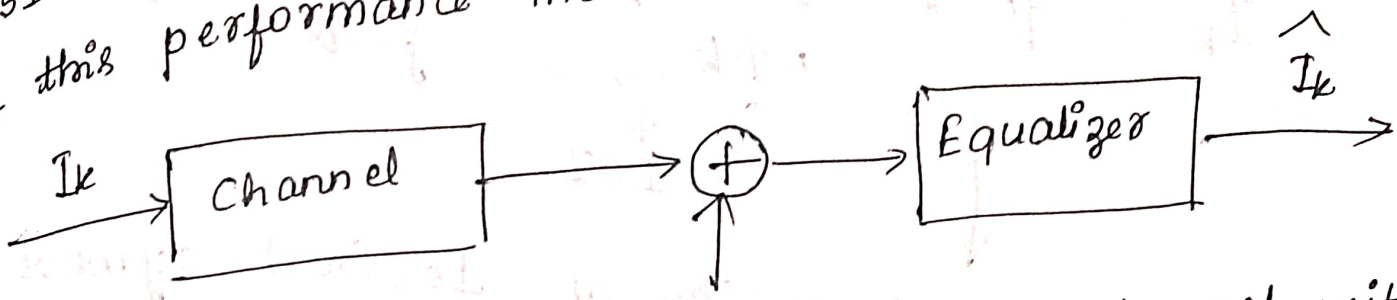


fig. Channel with zero-forcing equalizer

Consider cascade of discrete-time linear filter model having impulse response $\{f_n\}$ and an equalizer having impulse response $\{c_n\}$ can be represented by a single equivalent filter having impulse response

$$q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} \quad \text{—} \quad (1)$$

$q_n \rightarrow$ convolution of c_n & f_n

The equalizer is assumed to have an infinite no of taps.

Its o/p at kth sampling instant can be expressed in the form

$$\hat{I}_k = q_0 I_k + \underbrace{\sum_{n \neq k} I_n q_{k-n}}_{\text{ISI}} + \sum_{j=-\infty}^{\infty} c_j \eta_{k-j}$$

$\downarrow$

Scaled version of desired symbol

The peak distortion is

$$D(c) = \sum_{n=-\infty}^{\infty} |q_n|$$

$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left| \sum_{j=-\infty}^{\infty} c_j f_{n-j} \right|.$$

With an equalizer having infinite no. of taps, it is possible to select tap weights so that $D(c) = 0$, i.e., $q_n = 0$ i.e., ISI can be completely eliminated

then, the values of tap-weights is

$$q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

By taking z-transform

$$Q(z) = C(z)\, F(z) = 1$$

$$F(z) = 1/C(z)$$

or

$$\boxed{C(z) = 1/F(z)}$$

where $C(z)$ denotes z transform of c_j

ii] Mean-Square Error Criterion

In MSE tap-weight coefficients c_j of equalizer are adjusted to minimize MSE

$$\varepsilon_k = I_k - \hat{I}_k \quad \text{— (1)}$$

where I_k is information symbol transmitted in kth interval

$\hat{d}_k$ or $\hat{I}_k$ = estimate of symbol at o/p of equalizer

The performance index for MSE Criteria denoted by J is

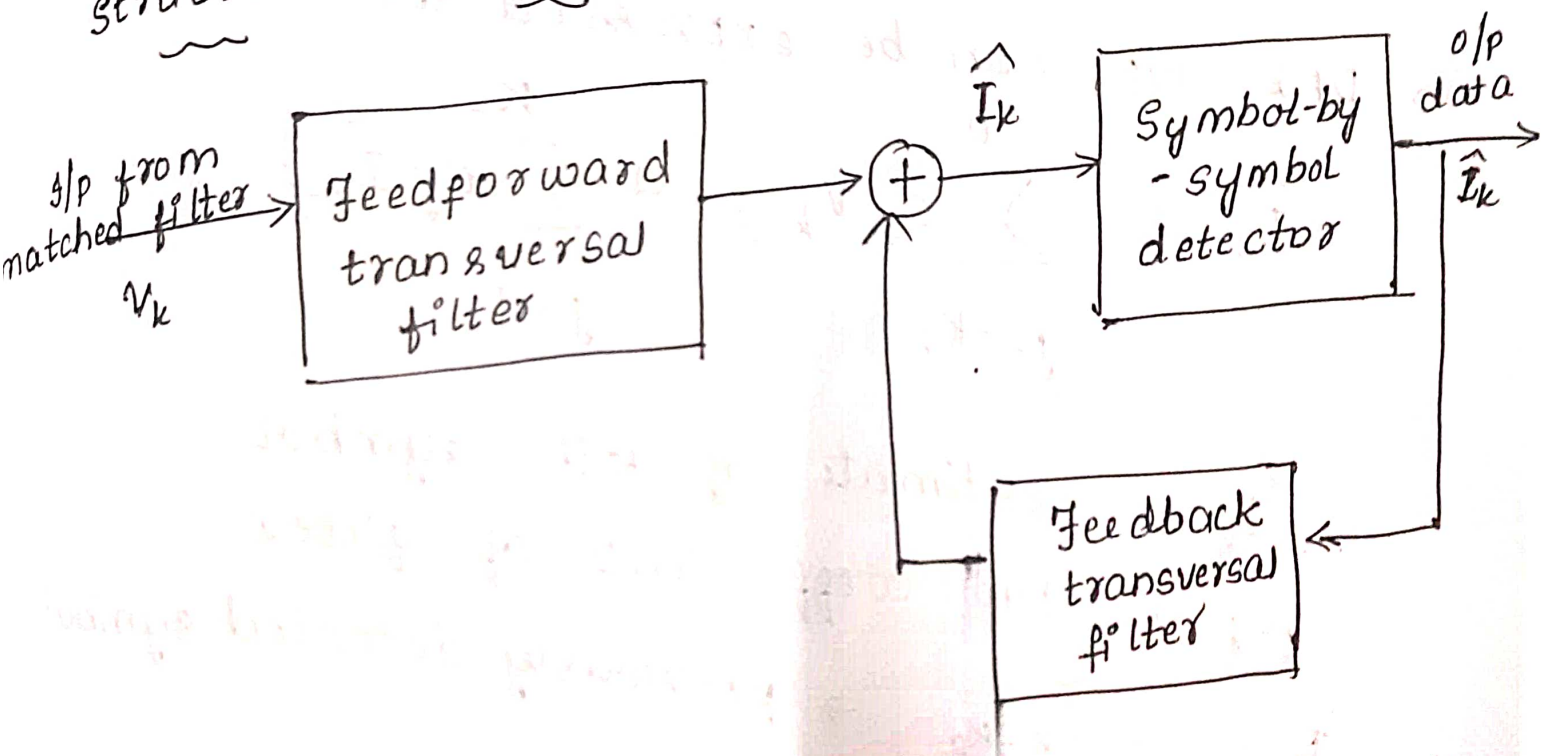
$$J = E|\varepsilon_k|^2 = E|I_k - \hat{I}_k|^2$$

The filter taps are adjusted such that MSE of ISI & noise power at equalizer o/p is minimized

ii. DECISION FEEDBACK EQUALIZATION (DFE)

↳ DFE is one of the nonlinear equalization technique where channel distortion is too severe.

structure of DFE



↳ The nonlinear equalizer consists of two filter feedforward filter & feedback filter and it is called DFE.

↳ The i/p to the feedforward filter is received signal sequence. The feedback filter has as its i/p the sequence of decision on detected symbols

↳ The feedback filter is used to remove part of ISI from present estimated symbol caused by previously detected symbols. Since the detector feeds hard decisions to the feedback filter, DFE is non-linear

↳ DFE o/p can be expressed as

$$\hat{I}_k = \sum_{j=-K_1}^{0} c_j V_{k-j} + \sum_{j=1}^{K_2} c_j \tilde{I}_{k-j}$$

$\hat{I}_k$ = estimate of kth symbol

c_j → tap coefficients of filter

$\tilde{I}_{k-1} \cdots \tilde{I}_{k-K_2}$ → previously detected symbol

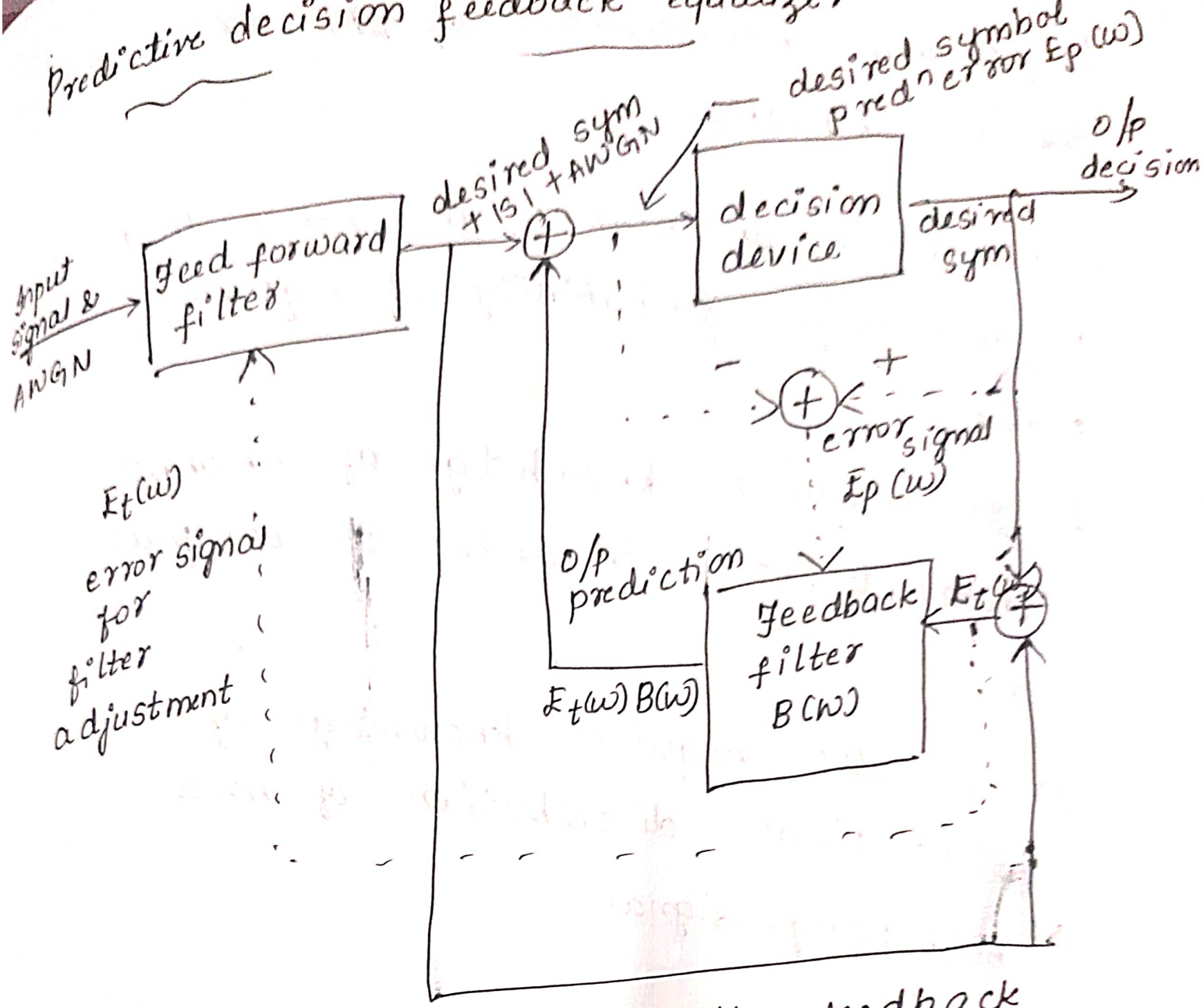
K_2 = feedback section

K_1+1 → feedforward section

↳ Minimum MSE of DFE is

$$E\{|e(n)|^2\}_{min} = \exp\left\{\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \ln\left[\frac{N_0}{|F(e^{j\omega T})|^2 + N_0}\right] d\omega\right.$$

Predictive decision feedback equalizer



↳ In the above figure the feedback filter is driven by an input sequence formed by the difference of the o/p of the detector and o/p of feedforward filter

↳ The FBF is called noise predictor because it predicts noise & residual ISI contained in the signal at FFF o/p and subtracts from it the detector o/p after some delay.

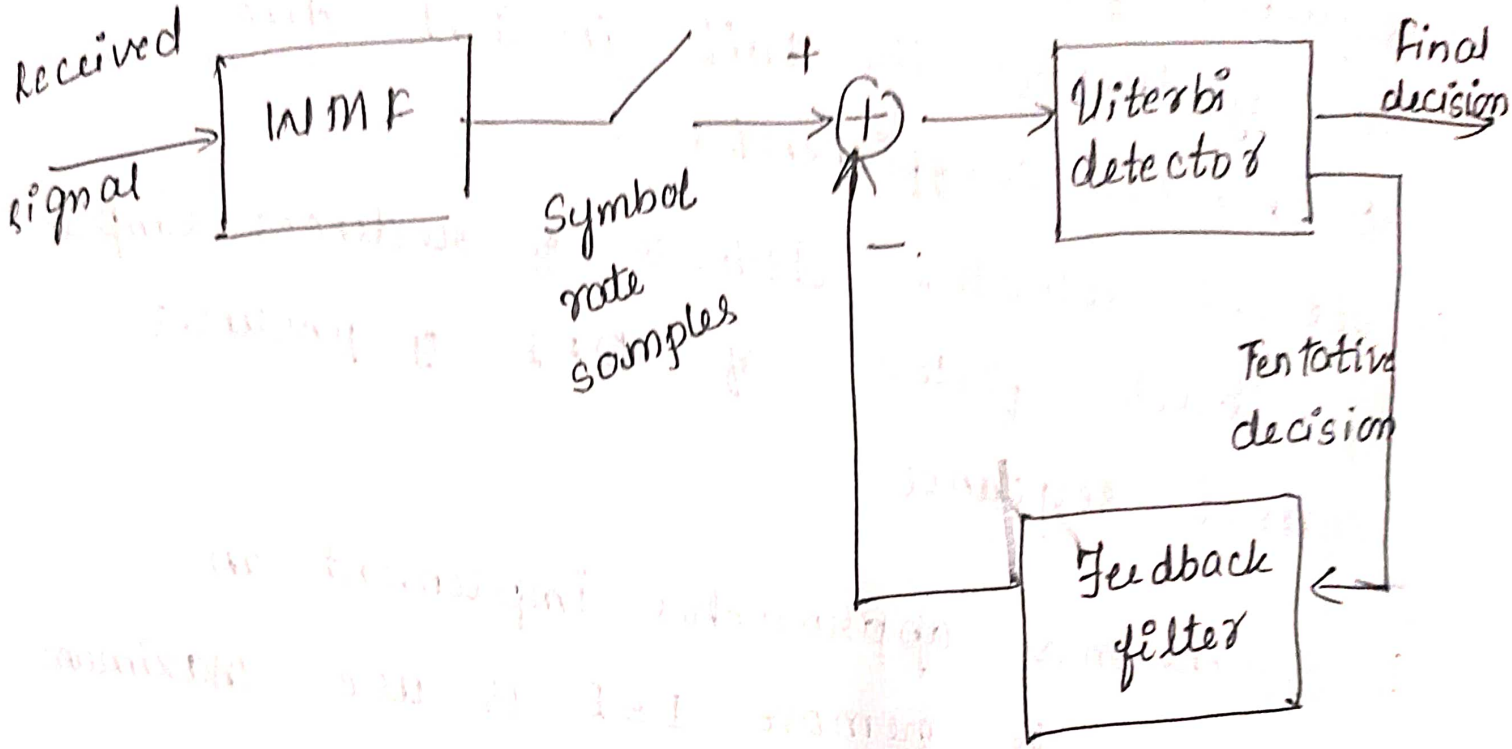
Performance characteristics of DFE or MLSE Equalizer

↳ MLSE is optimal in the sense that it minimizes probability of sequence error

↳ It requires knowledge of channel characteristics in order to compute decision

↳ It also requires knowledge of statistical distribution of noise corrupting signal

iii) REDUCED COMPLEXITY ML DETECTORS



In the above fig

↳ The filter preceding Viterbi detector is whitened matched filter. This reduces the channel to one that has a minimum phase characteristic.

↳ In the above fig the Viterbi detector employs feedback within Viterbi detector to reduce effective length of ISI from L terms to Lo terms.

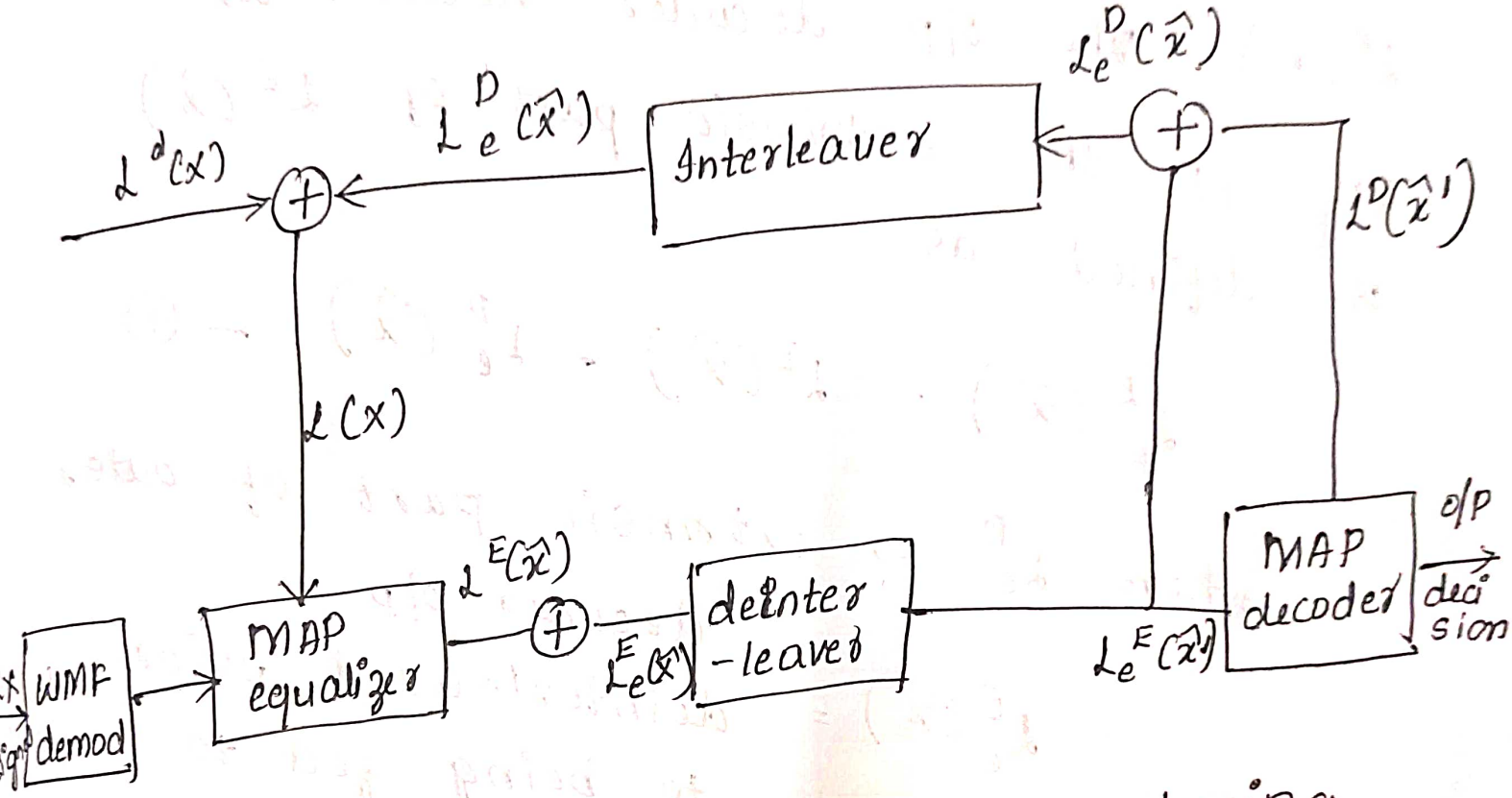
↳ The use of feedback where preliminary decisions obtained by using surviving sequence from Veterbi detector used to synthesize the tail in ISI due to channel coefficients.

↳ Veterbi detector detects & reduces simple feedback filter of DFE & produces correct sequence

↳ Conventional approaches implement an equalizer to remove ISE or use Maximum likelihood detection

iv. ITERATIVE EQUALIZATION & DECODING -

TURBO EQUALIZATION



↳ Turbo equalization is used for reducing the complexity of joint equalization & decoding where detection & decoding are performed in iterative fashion

↳ In the above fig the input to the MAP equalizer in the sequence v_k from WMF. The equalizer computes logarithm of likelihood ratio of coded bits denoted as $L^E(\hat{x})$. The o/p decoder receives as an i/p the extrinsic part of $L^E(\hat{x})$ is defined as

$$L_e^E(\hat{x}) = L^E(\hat{x}) - L_e^D(\hat{x}) \quad - (1)$$

where $L_e^D \rightarrow$ extrinsic part of outer decoder o/p

$L_e^E(\hat{x}) =$ deinterleaved prior to being fed to o/p decoder

The ~~o/p~~ outer decoder computes logarithm of likelihood ratio of coded bits denoted by $L^D(\hat{x}')$ and inform^n bits denoted as $L^D(\hat{I})$.

$L^D(\hat{x}')$ is extrinsic part denoted as $L_e^D(\hat{x}')$

is gn as

$$\boxed{L_e^D(\hat{x}') = L^D(\hat{x}') - L_e^E(\hat{x}')}$$

$L_e^D(\hat{x}')$ is interleaved to produce $L_e^D(\hat{x})$

and fed to MAP equalizer

~~the~~

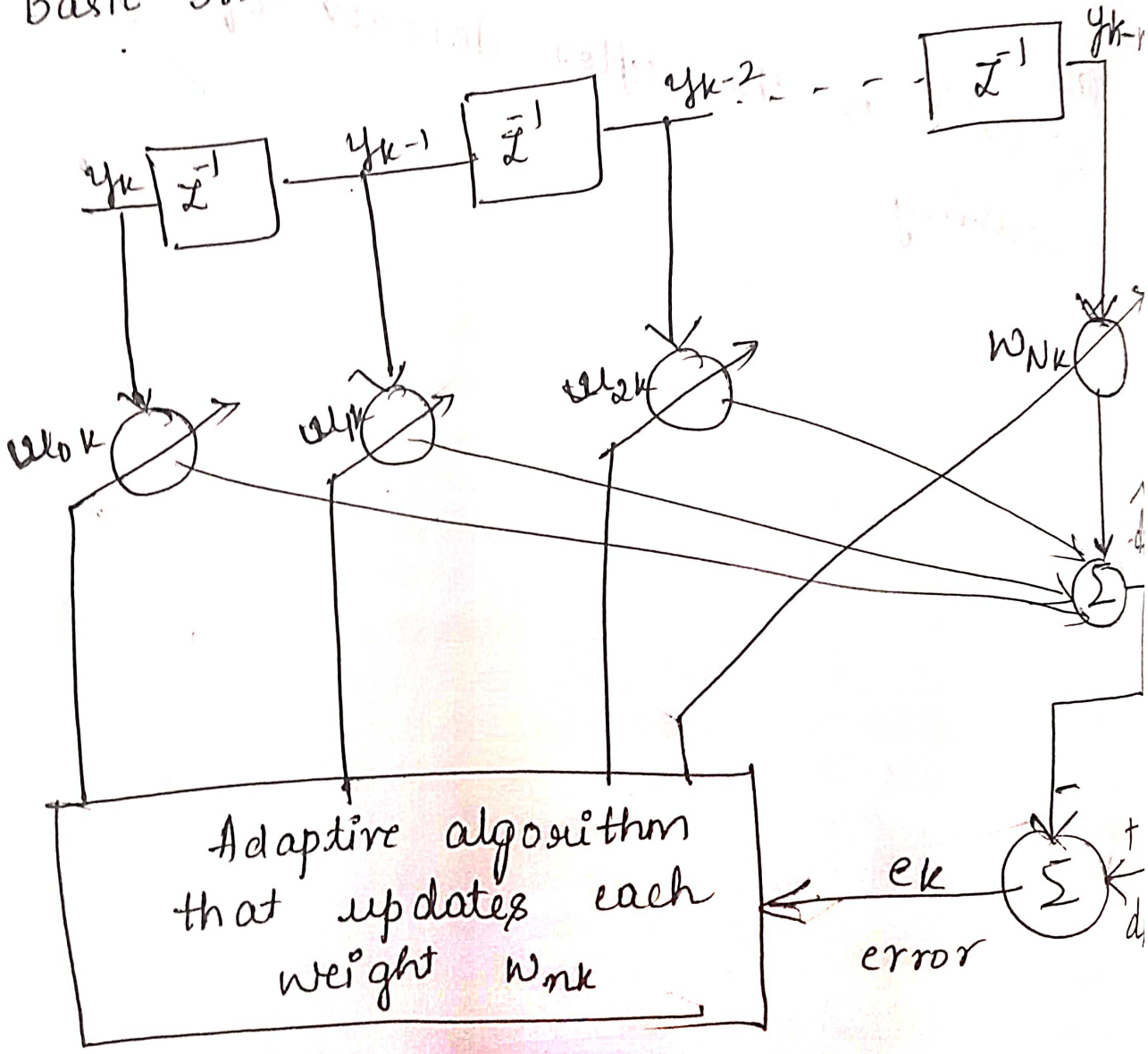
$L_e^E(\hat{x})$ is the after decoder after deinter

-leaving.

ADAPTIVE LINEAR EQUALIZER.

Adaptive equalizer is an equalizer that automatically adapts to the time varying properties of the communication channel.

Basic Structure of Adaptive Equalizer



Algorithm :

↳ The i/p signal to the equalizer is

$$y_k = [y_k, y_{k-1} \cdots y_{k-1}]^T$$

↳ The o/p of adaptive equalizer is gn by

$$\hat{d}_k = \sum_{n=0}^{N} w_{nk} y_{k-n}$$

↳ Weight vector can be writen as

$$w_k = [w_{0k}, w_{1k}, w_{2k} \cdots w_{Nk}]^T$$

↳ o/p in vector notation as

$$\hat{d}_k = y_k^T w_k = w_k^T y_k$$

↳ error signal e_k is gn by

$$e_k = d_k - \hat{d}_k = x_k - \hat{d}_k$$

↳ Putting the values of d_k & $\hat{d}_k$ the error signal is

$$e_k = x_k - y_k^T w_k = x_k - w_k^T y_k$$

↳ By considering Adaptive linear equalization there is a single i/p y_k at time instant the value of y_k depends upon the instantaneous state of radio channel and specific value of noise

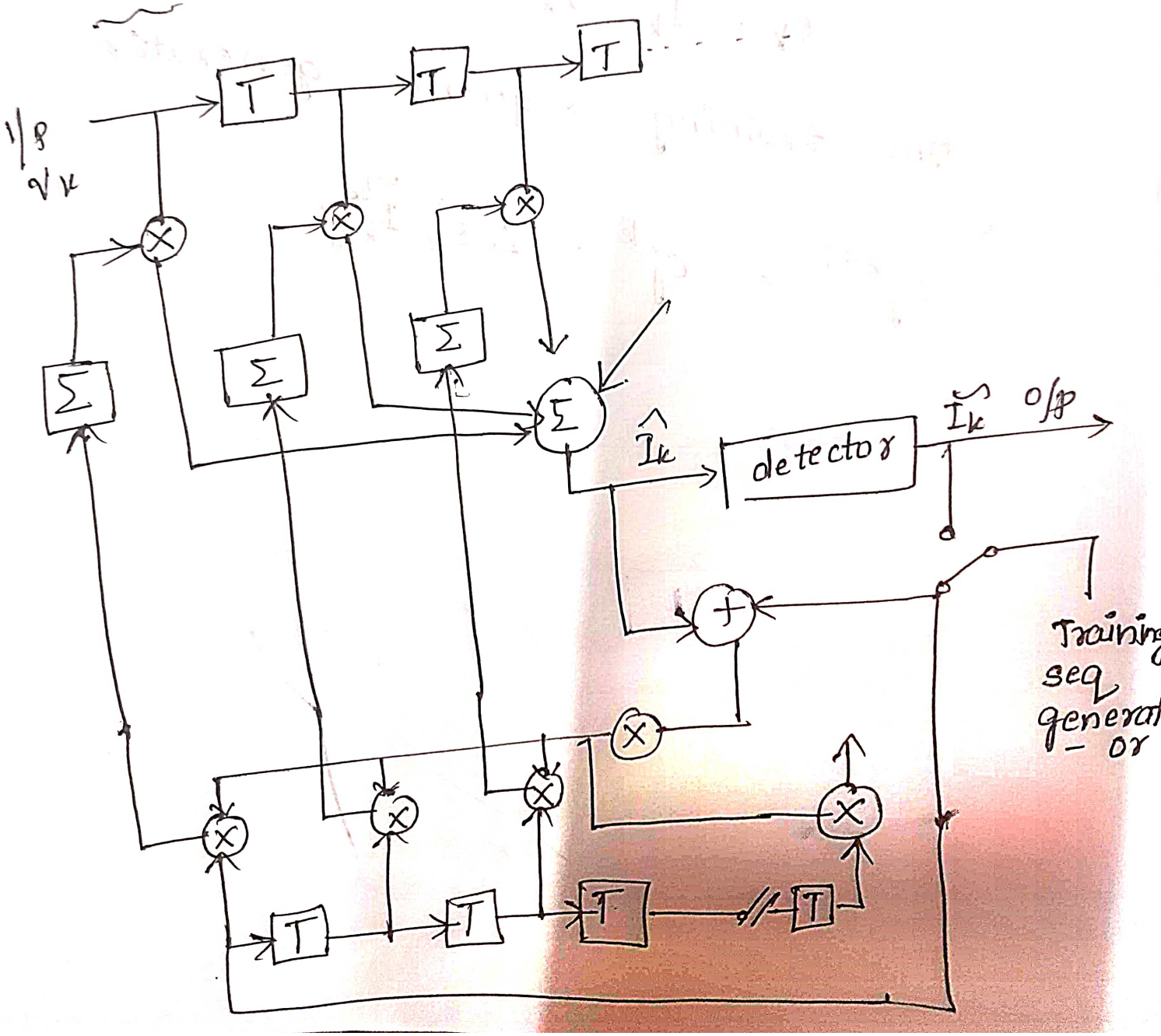
↳ The adaptive equalizer structure is called transversal filter has N delay elements, N I taps & N I tunable complex multipliers called weight

↳ It is controlled by the error signal e_k. This error signal is derived by comparing o/p of equalizer d_k

↳ There are three algorithms for performing optimization automatically & adaptively

- Zero-forcing algorithm
- Least Mean square Algorithm
- Recursive Least square Algorithm

i) Zero-Forcing Algorithm



↳ In zero forcing equalizer, the equalizer coefficients c_n are chosen to force the samples of combined channel and equalizer impulse response to zero at all but one of NT spaced sample point in tapped delay line filter

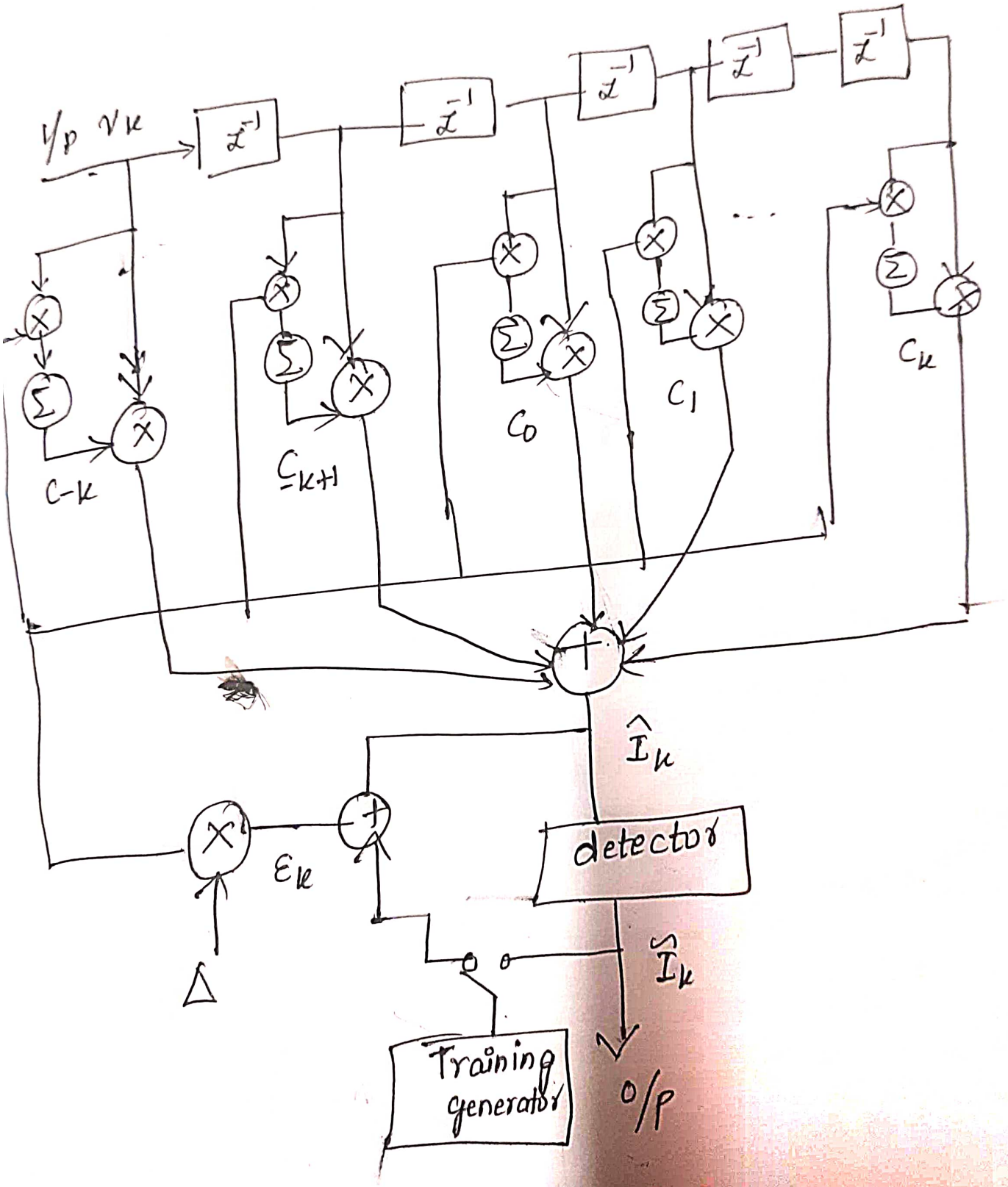
$$e_k = \tilde{I}_k - \hat{I}_k \quad - (1)$$

The training sequence generator is

$$c_j^{k+1} = c_j^k + \Delta \tilde{\varepsilon}_k \tilde{I}_{k-j}^*$$

Least Mean Square Algorithm

↳ The criterion used is the minimization of MSE between desired equalizer o/p & actual equalizer o/p

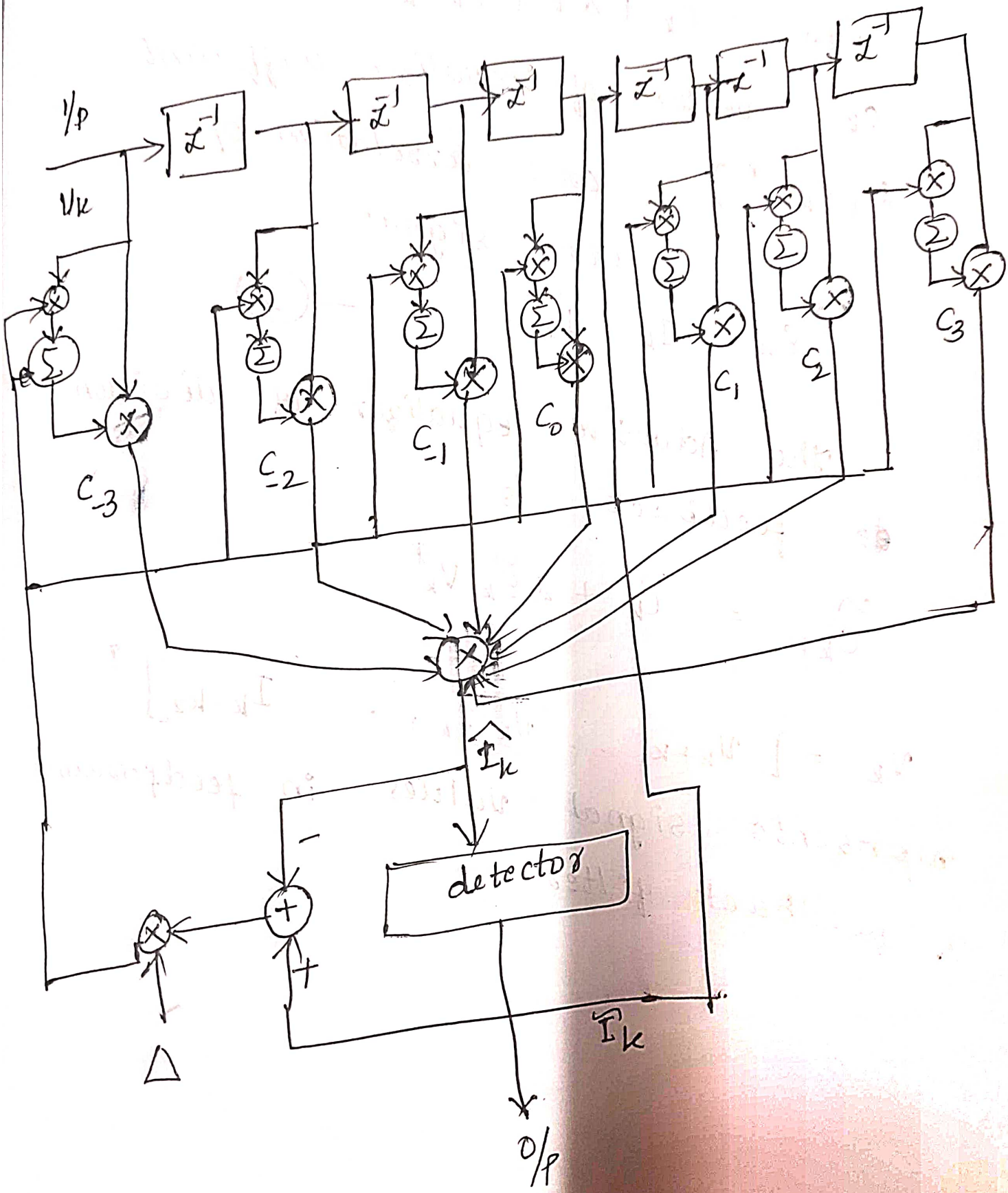


The LMS algorithm is gn by

$$\hat{C}_{k+1} = \hat{C}_k + \Delta \varepsilon_k v_k^*$$

↳ Convergence rate of LMS is slow

ADAPTIVE DECISION FEEDBACK EQUALIZER



In the case of Adaptive decision feedback equalizer

$$C_{k+1} = C_k + \Delta E(\varepsilon_k V_k^*) \quad -(1)$$

C_k = vector of equalizer coefficient

$E(\varepsilon_k V_k^*)$ = cross correlation of error signal

$$\varepsilon_k = I_k - \hat{I}_k \quad -(2)$$

The Adaptive equalizer of decision feedback is

$$\tilde{C}_{k+1} = \tilde{C}_k + \Delta \tilde{\varepsilon}_k V_k^*$$

$$V_k = [V_{k+K_1} \cdots V_k \; I_{k-1} \cdots I_{k-K_2}]^T$$

represents signal values in feedforward & feedback filter

RECURSIVE LEAST SQUARE FOR ADAPTIVE EQUALIZER

↳ In order to achieve faster convergence, complex algorithm which involve additional parameter are used

↳ The least sq error based on the time average is defined as

$$J(n) = \sum_{i=1}^{n} \lambda^{n-i} e^*(i,n)\, e(i,n)$$

λ is the weighted factor close to 1 but smaller than 1

$e^*(i,n)$ is complex conjugate of $e(i,n)$ & error $e(i,n)$ is gn as

$$e(i,n) = x(i) - y_N^T(i)\, w_N(n) \qquad 0 \le i \le n$$

& $y_N(i) = [y(i), y(i-1) \ldots y(i-N+1)]^T$

↳ $y_N(i)$ is data i/p vector at time i

$$\sum_{i=1}^{N} \lambda_i = y_N^T(n)\, y_N(n)$$