Module - 5

Digital Communication through Fading Multipath Channels

Characterization of fading multipath channels

* If an extremely short pulse [impulse] is transmitted over a time varying multipath channel, the received signal might appear as a train of pulses as shown in figure 5.1.

* The characteristic of a multipath channel is
  i. Time spread → introduced in the signal that is transmitted through the channel.
  ii. Time variations → introduced in the structure of the medium, due to such variations, the nature of the multipath varies with time.

* Due to time variation, The received pulse train undergoes changes in the size of the individual pulses or changes in the relative delays among the pulses and ~~size of the~~ changes in the number of pulses in the received pulses as shown in fig 5.1

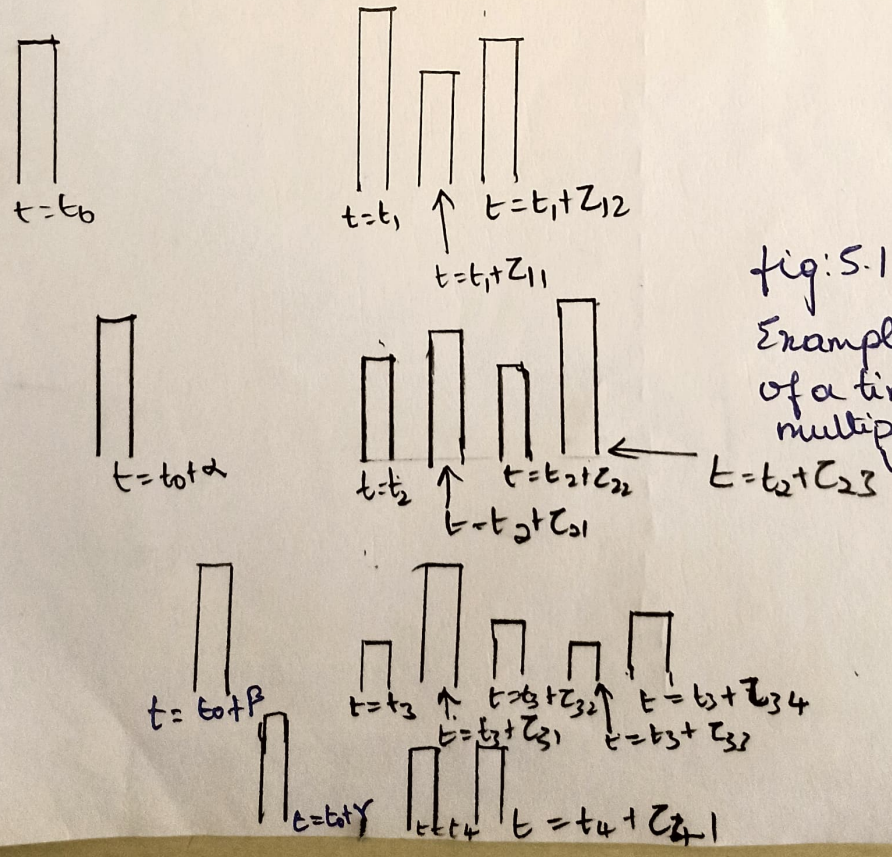


fig: 5.1
Example of the response of a time variant multipath channel to a very narrow pulse

* The time variations is unpredictable to the user of the channel
* Hence it is reasonable to characterize the time variant multipath channel statistically.

* Consider the transmission of the signal $s(t)$ over the radio channel

$$s(t) = Re\{s_1(t)\, e^{j2\pi f_c t}\} \quad \text{—} (1)$$

* Let assume there are multiple propagation paths. Each path is associated with propagation delay and an attenuation factor. Both propagation delays and the attenuation factors are time-variant.

* Thus, received band-pass signal may be expressed as

$$x(t) = \sum_n \alpha_n(t)\, s[t - \tau_n(t)] \quad \text{—} (2)$$

where $\alpha_n(t)$ — attenuation factor

$\tau_n(t) \rightarrow$ propagation delay

Substituting (1) in (2)

$$x(t) = Re\left[\left\{\sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, s_1[t - \tau_n(t)]\right\} e^{j2\pi f_c t}\right] \quad \text{—} (3)$$

* The equivalent low-pass received signal is

$$r_1(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, s_1[t - \tau_n(t)] \quad \text{—} (4)$$

$c(\tau; t)$ is time variant impulse response

$$c(\tau; t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, \delta[\tau - \tau_n(t)] \quad \text{—} (5)$$

* For some channels, such as tropospheric scatter channel ie continuous multipath
The received signal $x(t)$ is expressed in the integral form

$$x(t) = \int_{-\infty}^{\infty} \alpha(\tau;t)\, s(t-\tau)\, d\tau \quad \text{— (6)}$$

Where $\alpha(\tau;t)$ denotes the attenuation of the signal components at delay τ and at 't' time instant 't'.

* Substituting eqn (1) into eqn (6) gives

$$x(t) = \mathrm{Re}\left\{\left[\int_{-\infty}^{\infty} \alpha(\tau;t)\, e^{-j2\pi f_c t}\, s_1(t-\tau)\, d\tau\right] e^{j2\pi f_c t}\right\} \quad \text{— (7)}$$

* Integral in eqn (7) represents the convolution of $s_1(t)$ with $c(\tau;t)$.

Hence $c(\tau;t) = \alpha(\tau;t)\, e^{-j2\pi f_c \tau}$ —— (8)

Where $c(\tau;t) \rightarrow$ the response of the channel at time t due to an impulse applied at time $t-\tau$

* Hence eqn (8) is the appropriate definition of the equivalent low pass impulse response when channel result in continuous multipath.
eqn (5) is appropriate for a channel that contains discrete multipath component.

* Let us consider the transmission of an unmodulated carrier at frequency f_c; then $s_1(t) = 1$ for all (t) and the received signal for discrete multipath is given by eqn (4) reduces to

$$x(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}$$

$$r_1(t) = \sum_n \alpha_n(t) e^{-j\theta_n(t)} \quad \text{— (9)}$$

Where $\theta_n(t) = 2\pi f_c \tau_n(t)$.

Thus the received signal consist of the sum of a number of time variant vectors having amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$.

* The delays $\tau_n(t)$ associated with different signal paths to change at different rates and in an unpredictable manner [random], which implies that received signal $r_1(t)$ in eqn (9) can be modeled as a random process.
* When there are a large number of paths, the central limit theorem can be applied, and $r_1(t)$ can be modeled as a complex-valued Gaussian random process.
* The multipath Propagation model for the channel embodied in the received signal $r_1(t)$ in eqn (9) results in signal fading.
* The fading phenomenon is primarily a result of the time variations in the phases $\{\theta_n(t)\}$
* A statistical treatment of the channel is suitable.

Some models are

* Rayleigh fading channel
* Rice fading channel
* Nakagami-m fading channel

* When the impulse response $c(\tau;t)$ is modeled as a zero mean complex valued Gaussian process, the envelope $|c(\tau;t)|$ at any instant t is Rayleigh - distributed and such channel is said to be Rayleigh fading channel.

* When there are fixed scatterers or signal reflectors in the medium, in addition to randomly moving scatterers, $c(\tau;t)$ can no longer be modeled as having zero mean; the envelope of $|c(\tau;t)|$ has a Rice distribution and the channel is said to be Ricean fading channel.

* Nakagami-m distribution is also another probability distribution function that has been used to model the envelope of fading signals.

Channel Correlation functions and Power Spectra

* Assume that $c(\tau;t)$ is wide sense-stationary.

then the autocorrelation function of $c(\tau;t)$ as

$$R_c(\tau_2, \tau_1; \Delta t) = E\left[c^*(\tau_1;t)\, c(\tau_2; t+\Delta t)\right] \quad (10)$$

* the attenuation and phase shift of the channel associated with path delay τ_1 is uncorrelated with the attenuation and phase shift associated with path delay τ_2. This is called uncorrelated scattering.

Assuming scattering at two different delays as uncorrelated, then eqn (10) can be written as

$$E\left[c^*(\tau_1;t)\, c(\tau_2; t+\Delta t)\right] = R_c(\tau_1;\Delta t)\, \delta(\tau_2-\tau_1) \quad (11)$$

* If $\Delta t = 0$, the resulting autocorrelation function $R_c(\tau; 0) \equiv R_c(\tau)$ is the average power output of the channel as a function of the time delay τ

Hence $R_c(\tau)$ is called the multipath intensity profile or the delay power spectrum of the channel.

* $R_c(\tau; \Delta t)$ gives the average power output as a function of the time delay τ and the difference Δt in observation time.

* Practically, the function $R_c(\tau; \Delta t)$ is measured by transmitting very narrow pulses or equivalently a wideband signal and cross correlating the received signal with a delayed version of itself.

* Figure (1) shows the measured function $R_c(\tau)$

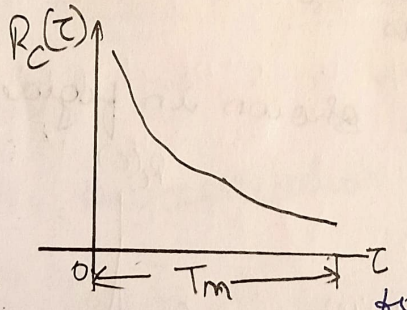


fig: Multipath intensity profile

* The range of value of τ over which $R_c(\tau)$ is essentially nonzero is called the multipath spread of the channel and is denoted by T_m.

* Equivalently in the fourier domain we have the time variant transfer function $C(f; t)$ defined as

$$C(f; t) = \int_{-\infty}^{\infty} c(\tau; t)\, e^{-j2\pi f \tau}\, d\tau \quad -12$$

* the autocorrelation function is defined as

$$R_c(f_2, f_1; \Delta t) = E\left[C^*(f_1; t)\, C(f_2; t+\Delta t)\right]$$

$$= R_c(\Delta f; \Delta t) \quad \text{— (13)}$$

where $\Delta f = f_2 - f_1$

$R_c(\Delta f; \Delta t)$ is the fourier transform of the multipath intensity profile.

* $R_c(\Delta f; \Delta t)$ is the spaced-frequency, spaced time correlation function of the channel.

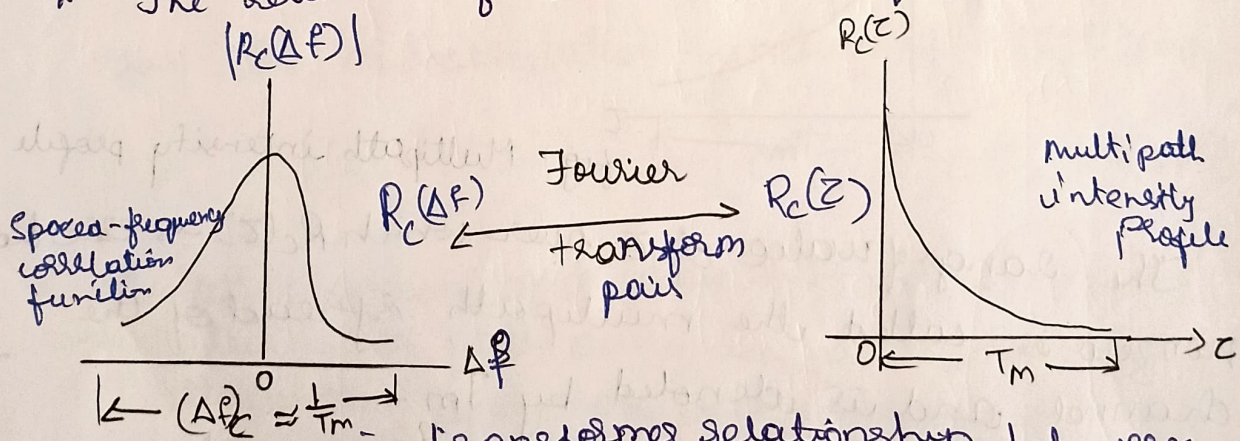
* If $\Delta t = 0$, then (13) gives

$$R_c(\Delta f; 0) \equiv R_c(\Delta f)$$

$$\& \quad R_c(\tau; 0) \equiv R_c(\tau)$$

ie $$R_c(\Delta f) = \int_{-\infty}^{\infty} R_c(\tau)\, e^{-j2\pi \Delta f \tau}\, d\tau \quad \text{— (14)}$$

* The relationship is shown in figure 1.2.



* Hence fourier transformer relationship between $R_c(\Delta f)$ and $R_c(\tau)$. provides the coherence bandwidth of the channel

ie $$(\Delta f)_c \simeq \frac{1}{T_m}$$

where $(\Delta f)_c$ denotes the coherence bandwidth

* The two sinusoids with frequency separation greater than $(\Delta f)_c$ are affected differently by the channel.

* When an information bearing signal is transmitted through the channel, if $(\Delta f)_c$ is small in comparison to the bandwidth of the transmitted signal, the channel is said to be frequency-selective. In such cases, the signal is severely distorted by the channel.

* When an information bearing signal is transmitted through the channel, if $(\Delta f)_c$ is larger in comparison to the bandwidth of the transmitted signal, the channel is said to be frequency-non selective.

* In order to relate the doppler effects to the time variations of the channel.

It is defined that, The fourier transform of $R_c(\Delta f; \Delta t)$ with respect to the variable Δt to be the function $S_c(\Delta f; \lambda)$.

ie

$$S_c(\Delta f; \lambda) = \int_{-\infty}^{\infty} R_c(\Delta f; \Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

when $\Delta f = 0$; $S_c(0; \lambda) \equiv S_c(\lambda)$

then the eqn becomes

$$S_c(\lambda) = \int_{-\infty}^{\infty} R_c(0; \Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

* The function $S_c(\lambda)$ is a power spectrum that gives the signal intensity as a function of the Doppler frequency λ. hence $S_c(\lambda)$ is the doppler power spectrum of the channel.

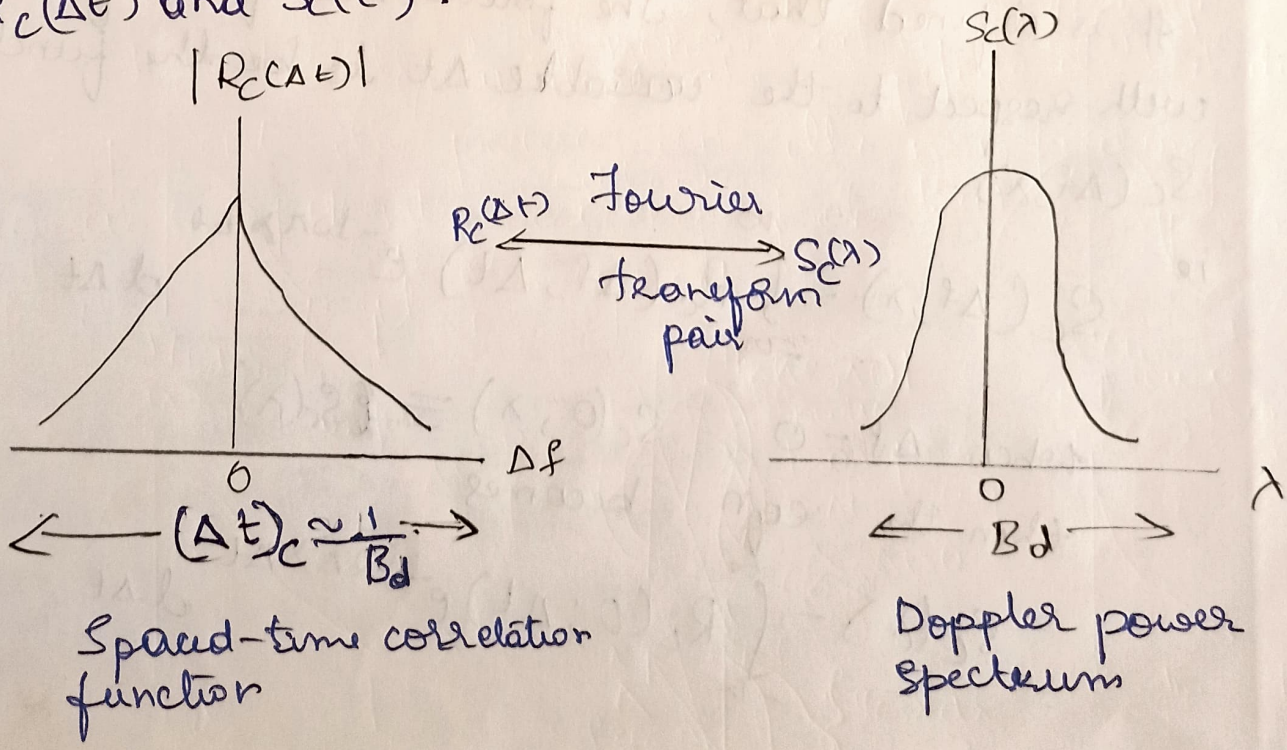
* The range of values of λ over which $S_c(\lambda)$ is essentially nonzero is called the Doppler spread B_d of the channel. The reciprocal of B_d is a measure of the coherence time of the channel.

i.e $(\Delta t)_c \approx \frac{1}{B_d}$

where $(\Delta t)_c \approx \frac{1}{B_d}$

where $(\Delta t)_c$ denotes the coherence time.

* a slowly changing channel has a large coherence time or equivalently, a small doppler spread. figure illustrates the relationship between $R_c(\Delta t)$ and $S_c(\lambda)$.



* $S(\tau;\lambda)$ is the fourier transform of $R_c(\tau;\Delta t)$ in the Δt variable.

ie $$S(\tau;\lambda) = \int_{-\infty}^{\infty} R_c(\tau;\Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

$S(\tau;\lambda)$ and $S_c(\Delta f;\lambda)$ are a fourier transform pair.

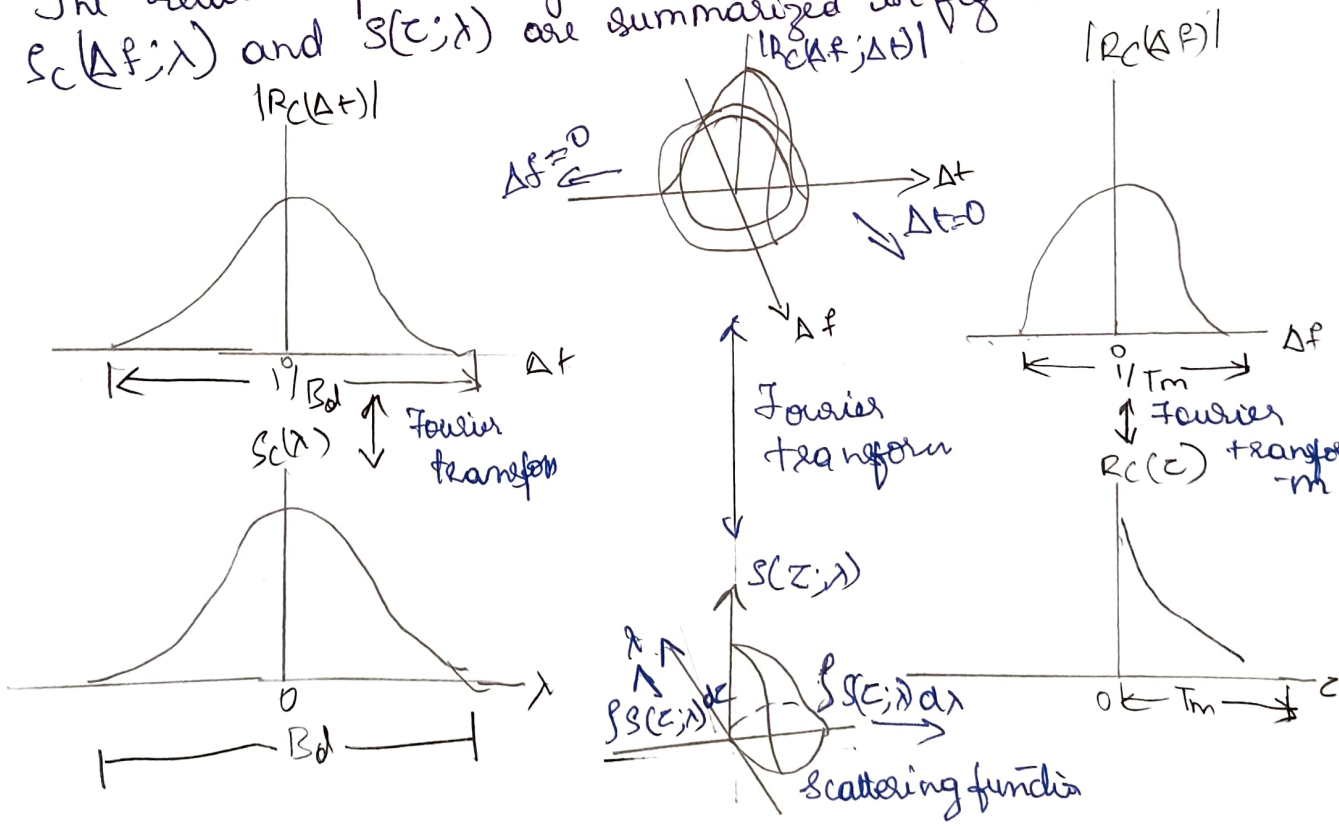
$$S(\tau;\lambda) = \int_{-\infty}^{\infty} S_c(\Delta f;\lambda)\, e^{j2\pi\tau\Delta f}\, d\Delta f$$

$S(\tau;\lambda)$ and $R_c(\Delta f;\Delta t)$ are related by the double F.T

$$S(\tau;\lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_c(\Delta f;\Delta t)\, e^{-j2\pi\lambda\Delta t}\, e^{j2\pi\tau\Delta f}\, d\Delta t\, d\Delta f$$

$S(\tau;\lambda)$ is the scattering function of the channel

* $S(\tau;\lambda)$ provides a measure of the average power output of the channel as a function of the time delay τ and the doppler frequency λ.
The relationships among the four functions $R_c(\Delta f;\Delta t)$, $R_c(\tau;\Delta t)$, $S_c(\Delta f;\lambda)$ and $S(\tau;\lambda)$ are summarized in figure.



Statistical Models for fading Channels

Several Probability distributions can be considered to model the statistical characteristics of the fading channel.

* Large number of scatterer in the channel that contribute to the signal at the receiver, Application of the central limit theorem leads to a Gaussian process model for the channel impulse response.

If the Process is zero-mean, then the envelope of the channel impulse response at any instant has a Rayleigh probability distribution and the phase is uniformly distributed in the interval $(0, 2\pi)$

ie:
$$P_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega} \quad r \geq 0$$

where, $\Omega = E(R^2)$

Rayleigh distribution is characterized by the single parameter $E(R^2)$.

* An alternative statistical model for the envelope of the channel response is the Nakagami-m distribution the PDF of this is

$$P_R(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{-mr^2/\Omega}$$

where $\Omega = E(R^2)$

This distribution provides more flexibility and accuracy in matching the observed signal statistics.

* The Nakagami-m distribution can be used to model fading channel conditions that are either more or less severe than the Rayleigh distribution.
* It includes the Rayleigh distribution as a special case $m=1$.

* The Rice distribution is also a two parameter distribution. It can be expressed by PDF,

ie
$$P_R(r) = \frac{r}{\sigma^2} e^{-(r^2+s^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right) \quad r \geq 0$$

s and σ^2 are the two parameters

s^2 is called the noncentrality parameter in the equivalent chi-square distribution.

It represents the power in the non fading signal components called spectral components of the received signal.

The effect of signal characteristics on the choice of a Channel model

* let $s_1(t)$ be the equivalent low pass received signal, exclusive of additive noise can be expressed in terms of the time domain variables $c(\tau;t)$ and $s_1(t)$ as

$$r_1(t) = \int_{-\infty}^{\infty} c(\tau;t)\, s_1(t-\tau)\, d\tau \quad \text{—— (1)}$$

in terms of frequency function $C(f;t)$ and $S_1(f)$ as

$$r_1(t) = \int_{-\infty}^{\infty} C(f;t)\, S_1(f)\, e^{j2\pi ft}\, df \quad \text{—— (2)}$$

* If transmitting digital information over the channel by modulating the basic pulse $s(t)$ at a rate $1/T$, where T is the signaling interval. Then.

* Then the time-variant channel characterized by the transfer function $C(f;t)$ distorts the signal $S(f)$.

If $S_1(f)$ has a bandwidth W greater than the coherence bandwidth $(\Delta f)_c$ of the channel, $S_1(f)$ is subjected to different gains and phase shifts across the band. In such cases the channel is said to be frequency selective.

* Additional distortion is caused by the time variations in $C(f;t)$ and termed as fading.

* Hence frequency selectivity and fading are two different types of distortion.

* The frequency selectivity depends on the multipath spread or equivalently on the coherence bandwidth of the channel relative to the transmitted signal bandwidth W.

* Fading depends on the time variations of the channel, which are characterized by the coherence time (Δt_c) or by the Doppler spread B_d.

* The effect of the channel on the transmitted signal $S(t)$ is a function of the choice of signal bandwidth and signal duration.

* If signaling interval T to satisfy the condition $T >> T_m$, the channel introduces a negligible amount of intersymbol interference.

* If bandwidth of the signal pulse $S(t)$ is $W \approx 1/T$, the condition $T >> T_m$ implies that

$$W << \frac{1}{T_m} \approx (\Delta f)_c$$

ie. the signal bandwidth W is much smaller than the coherence bandwidth of the channel. Hence the channel is frequency-nonselective or flat fading

* Hence all of the frequency components in $S(f)$ undergoes the same attenuation and phase

shifts in transmission through the channel.

* Within the bandwidth occupied by $S(f)$, the time variant transfer function $C(f;t)$ of the channel is a complex valued constant in the frequency variable.

* at $f=0$, $C(f;t) = C(0;t)$, then eqn ②

can be rewritten as

$$r_1(t) = C(0;t) \int_{-\infty}^{+\infty} S_1(f)\, e^{j2\pi f t}\, df$$

$$= C(0;t)\, S_1(t) \quad \text{——— ④}$$

where $C(0;t)$ is complex valued random process

* The transfer function $C(0;t)$ for a frequency non selective channel may be expressed as

$$C(0;t) = \alpha(t)\, e^{-j\phi(t)}. \quad \text{——— ⑤}$$

where $\alpha(t) \rightarrow$ envelope

$\phi(t) \rightarrow$ phase of the equivalent low pass channel

* When $C(0;t)$ is modeled as a zero mean complex valued gaussian random process, the envelope $\alpha(t)$ is Rayleigh distributed for any fixed value of t and $\phi(t)$ is uniformly distributed over the interval $(-\pi, \pi)$.

The rapidity of the fading on the frequency nonselective channel is determined from the correlation function $\phi_c(\Delta t)$ or from doppler

power spectrum. $S_c(\lambda)$.

* Let consider signal bandwidth W to satisfy the condition

$$W << (\Delta f)$$

and signalling interval T to satisfy the condn

$$T << (\Delta t)_c$$

Since T is smaller than the coherence time of the channel, the channel attenuation and phase shift are essentially fixed for the duration of atleast one signaling interval. Such channel is called as slowly fading channel.

* Signalling interval $T >> 1/B_d \approx (\Delta t)_c$ Such channel is fast fading channel
* When $W \approx 1/T$, the channel is frequency-nonselective and slowly fading. which implies the product of T_m and B_d must satisfy the condn

$$T_m B_d < 1$$

$T_m B_d \longrightarrow$ spread factor of the channel

* If $T_m B_d < 1$, the channel is said to be underspread. otherwised it is overspread.
* The multipath spread, the doppler spread and the spread factor are listed in table 1 for several channels.
* From table 1, several radio channels including moon when used as passive reflector are under spread.

Type of channel	Multipath duration 's	Doppler Spread Hz	Spread factor
1. Shortwave ionospheric propagation (HF)	$10^{-3} - 10^{-2}$	$10^{-1} - 1$	$10^{-4} - 10^{-2}$
2. Ionospheric propagation under distributed auroral condition (HF)	$10^{-3} - 10^{-2}$	$10 - 100$	$10^{-2} - 1$
3. Ionospheric forward scatter (VHF)	10^{-4}	10	10^{-3}
4. Tropospheric scatter (SHF)	10^{-6}	10	10^{-5}
5. Orbital scatter (X band)	10^{-4}	10^{3}	10^{-1}
6. Moon at max, libration ($f_0 = 0.4$ kmC)	10^{-2}	10	10^{-1}

<u>Frequency</u> – <u>Non Selective</u>, <u>slowly fading</u> <u>channel</u>

* Consider the transmitted signal is $s_1(t)$ and the received equivalent low pass signal in one signaling interval is

$$r(t) = \alpha e^{j\phi} s_1(t) + z(t) \quad , \quad 0 \le t \le T \quad \text{—} (1)$$

where $z(t)$ represents the complex valued white Gaussian noise process corrupting the signal.

* Assume phase shift ϕ is estimated without error for the received signal. Hence the ~~received~~ ideal coherent detection of the received signal can be achieved.

Thus the received signal can be processed by passing it through a matched filter in the case of binary PSK or through a pair of matched filter.

The expression for the error rate of binary PSK as a function of the received SNR γ_b is

$$P_b(\gamma_b) = Q\sqrt{2\gamma_b} \quad \text{—— (2)}$$

where $\gamma_b = \alpha^2 \mathcal{E}_b / N_0$ and α as fixed.

Similarly for FSK

$$P_b(\gamma_b) = Q\sqrt{\gamma_b} \quad \text{—— (3)}$$

eqn (2) and eqn (3) can be viewed as conditional error probabilities, where α is fixed.

In case when α is random, then error probability is evaluted as

$$P_b = \int_0^\infty P_b(\gamma_b)\, P(\gamma_b) \cdot d\gamma_b \quad \text{—— (4)}$$

where $P(\gamma_b)$ is the probability density function of γ_b when α is random

Frequency - Non selective, slowly fading channel with Rayleigh statistics

* Assume α is Rayleigh - distributed, α^2 has a chi - square probability distribution with two degree of freedom.

ie $$P(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\gamma_b/\bar{\gamma}_b}, \quad \gamma_b \geq 0$$

$\bar{\gamma}_b$ is the average signal to noise ratio and defined as

$$\bar{\gamma}_b = \frac{\mathcal{E}_b}{N_0} E(\alpha^2)$$

the term $E(\alpha^2)$ is simply the average value of α^2

* the bit error probability is given by

$$P_b = \begin{cases} \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) & : \text{for BPSK} \\ \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right. & : \text{for BFSK} \end{cases}$$

* The error probabilities for high SNR ie $\bar{\gamma}_b >> 1$ become

$$P_b \simeq \begin{cases} 1/4\bar{\gamma}_b & \text{for coherent PSK} \\ \frac{1}{2\bar{\gamma}_b} & \text{for coherent, orthogonal FSK} \\ 1/2\bar{\gamma}_b & \text{for DPSK} \\ 1/\bar{\gamma}_b & \text{for noncoherent, orthogonal FSK} \end{cases}$$

* The error rate P_b decreases only inversely with SNR, compare to a non fading channel. ie on a fading channel, the transmitter must transmit a large amount of power in order to obtain a low probability of error.

* figure shows the error probabilities.

Frequency - Non selective, slowly fading channel, with Nakagami - m statistics

* When envelope α is characterized by the Nakagami - m distribution, the distribution of $\gamma = \alpha^2 \mathcal{E}_b / N_0$ is

$$P(\gamma) = \frac{m^m}{\Gamma(m)\bar{\gamma}^m} \gamma^{m-1} e^{-m\gamma/\bar{\gamma}}$$

where $\bar{\gamma} = \mathcal{E}/N_0 E(\alpha^2)$.

If $m = 1$ corresponds to Rayleigh fading

$m > 1$ corresponds to fading less severe than Rayleigh.

$m < 1$ corresponds to fading more severe than Rayleigh.

Diversity Techniques for fading multipath channels

* Diversity techniques are based on the errors occurs in reception when the channel attenuation is large.

* The idea is to supply to the receiver several replicas of the same information transmitted over independently fading channels, then the probability of all the signal components will fade simultaneously is reduced considerably.

* If P is the probability that any one signal will fade below some critical value, then P^L is the probability that all L independently fading replicas of the same signal will fade below the critical value.

* General methods to achieve diversity are

i. Frequency diversity : the same information bearing signal is transmitted on L carriers, where the seperation between successive carriers equal or exceeds the coherence bandwidth $(\Delta f)_c$ of the channel.

ii Time diversity : L independently fading version of the same information bearing signal is to transmitt the signal in L different time slots, where the seperation between successive time slots equals or exceeds the coherence time $(\Delta t)_c$ of the channel.

iii Space diversity : by transmitting on L antennas with minimum separation between antennas greater than $(\Delta d)_c$

iv. Angle of arrival diversity

v. Polarization diversity.