Module - 03

Digital modulation techniques

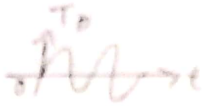
⇒ Coherent binary PSK - modulation :-

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t, \quad S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \pi]$$

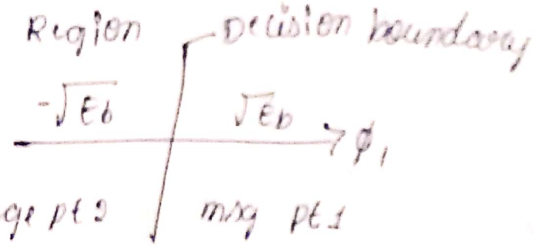
$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

basic function

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t, \quad 0 \le T_b \le t$$



$$S_1(t) = \sqrt{E_b}\,\phi_1(t) \quad -(1), \quad S_2(t) = -\sqrt{E_b}\,\phi_1(t) \quad -(2)$$



i.e. signal space diagram of coherent binary PSK

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t)\,dt$$

$$S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt$$

$$S_{11} = \sqrt{E_b}, \quad S_{21} = -\sqrt{E_b}$$

$z_1, \; 0 \le x \le \infty$

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - S_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{\left[-\frac{1}{N_0}[x_1 + \sqrt{E_b}]^2\right]}$$

assume, $z = x_1 + \frac{\sqrt{E_b}}{\sqrt{N_0}}$ —(1)

diff w.r.to x

$$dz = \frac{dx_1 + \sqrt{E_b}}{\sqrt{N_0}} \Rightarrow dz = \frac{dx}{\sqrt{N_0}} \Rightarrow dx = \sqrt{N_0}\,dz$$

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put $x = 0$

$$z = \sqrt{\frac{E_b}{N_0}}$$

$$x = \infty, \; z = \infty$$

probability,

$$P_e(0) = \int_0^{\infty} f_{x_1|x_1}(0)\, dx_1 \Rightarrow \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} \sqrt{N_0}\, dz$$

$\times$ly & $\div$ by $2/2$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \frac{2}{2} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} \sqrt{N_0}\, dz$$

$$P_e(0) = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz\right]$$

where, $\operatorname{erfc}(u) = \int_{ue}^{\infty} -u^2 du$

$$P_e(0) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}} \quad -(3)$$

lly, $P_e(1) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}} \quad -(4)$

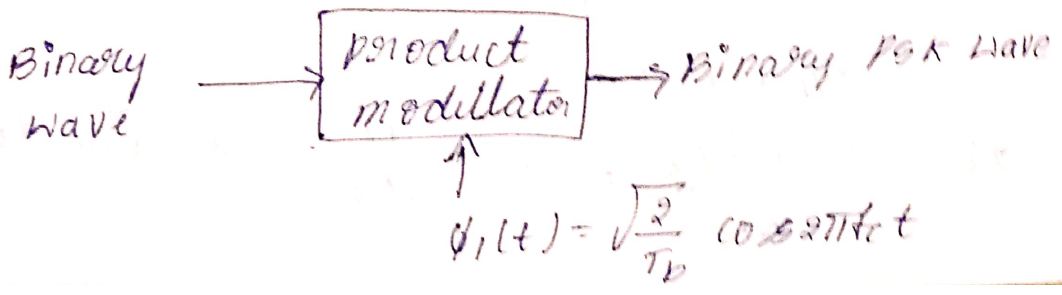
Total probability of error.

$$P = P(0) P_e(0) + P(1) P_e(1) \quad -(5)$$

$$= \frac{1}{2}\left[\tfrac{1}{2} \operatorname{erfc}\left(\sqrt{E_b/N_0}\right)\right] + \tfrac{1}{2}\left[\tfrac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}\right]$$

$$\left\langle P = \tfrac{1}{2} \operatorname{erfc}\left(\sqrt{E_b/N_0}\right) \right\rangle$$

binary PSK transmitter:



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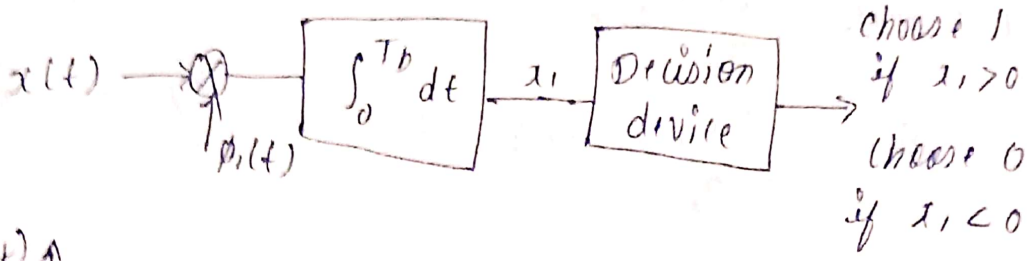
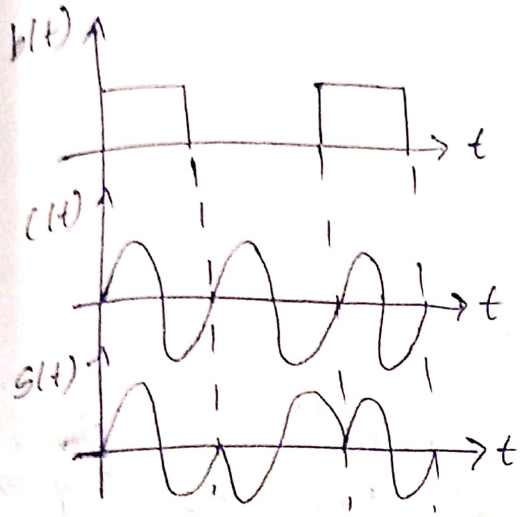
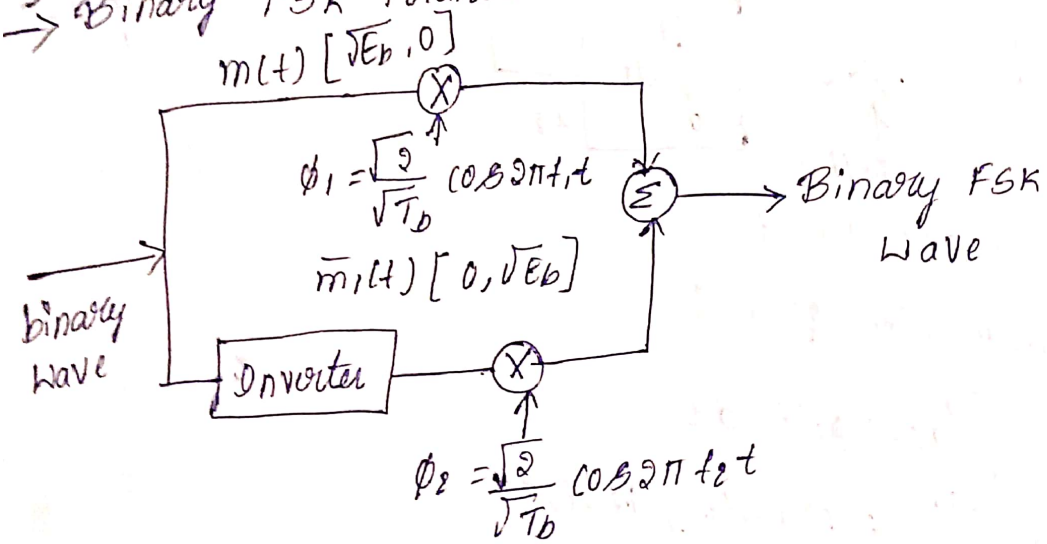
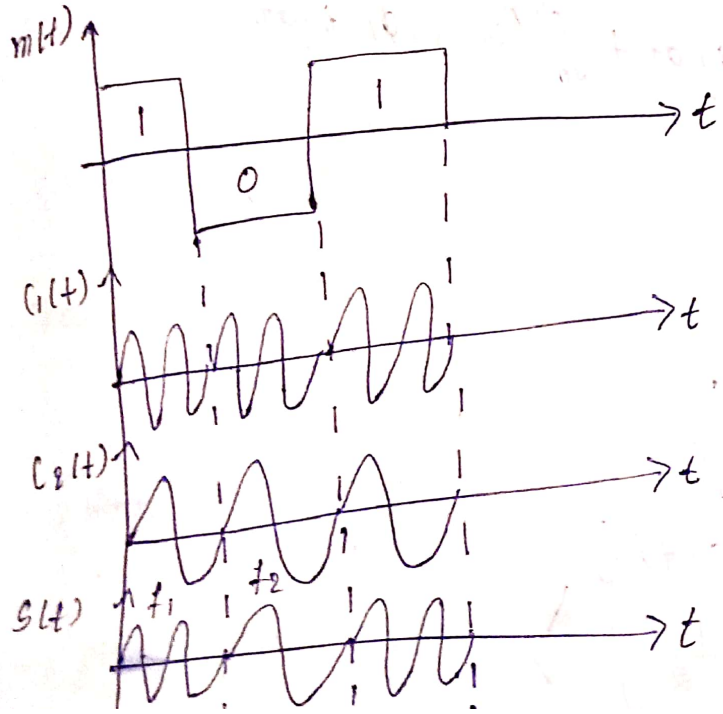




∴ Binary FSK transmitter :-





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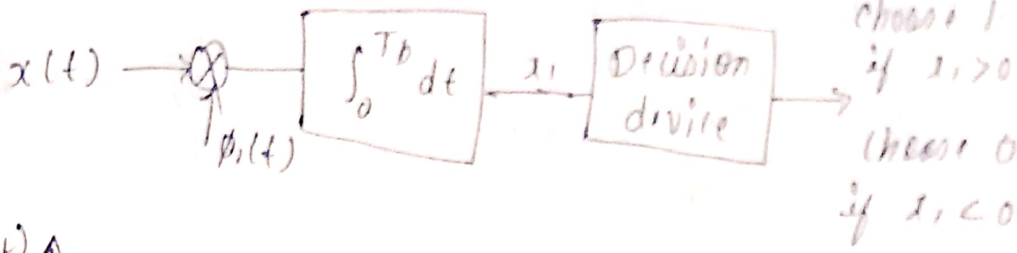
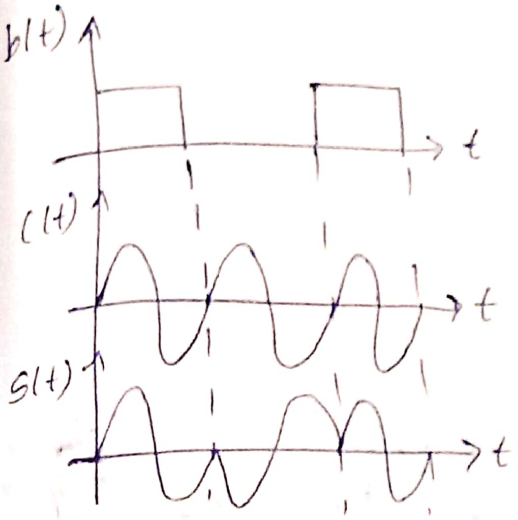
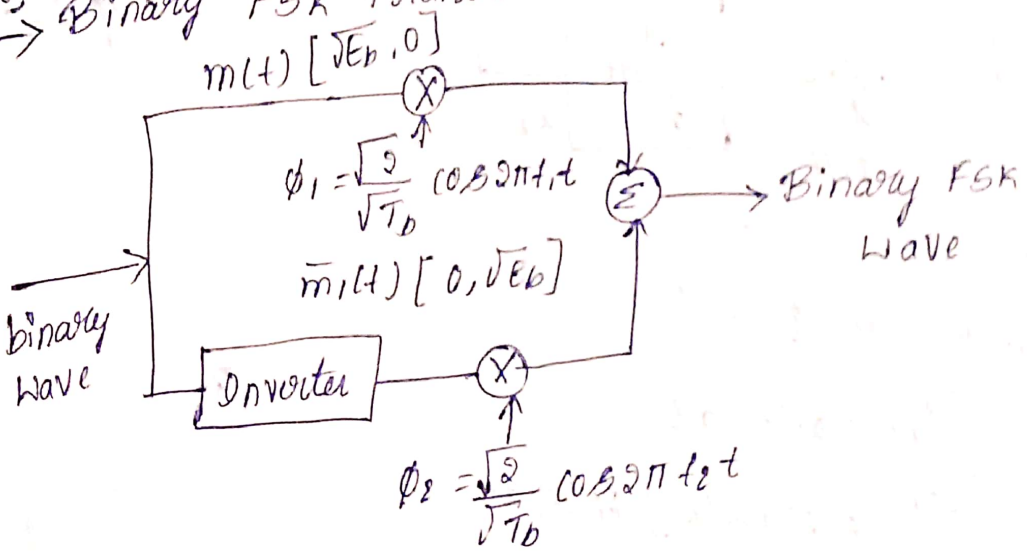
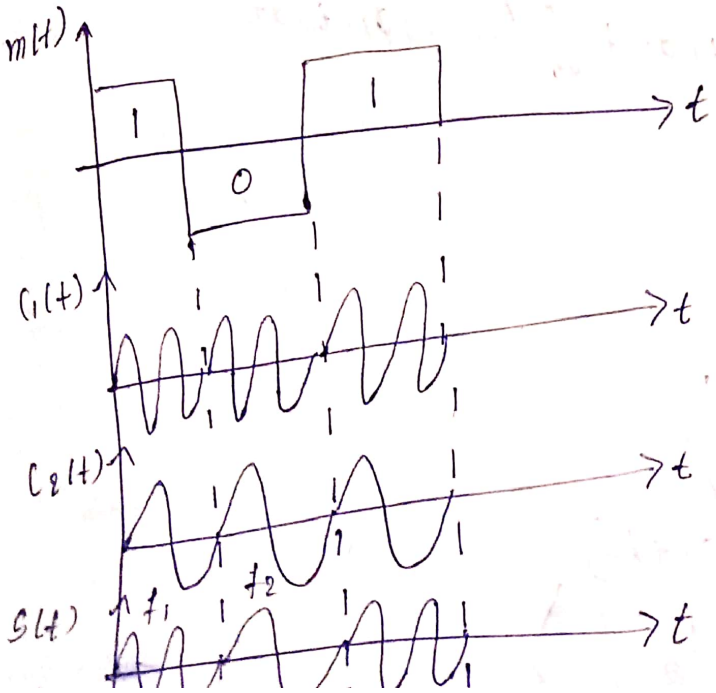




∴ Binary FSK transmitter :-





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$$S(t) = S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t \quad -① \qquad 0 \le t \le T_b \quad \text{symbol 1}$$

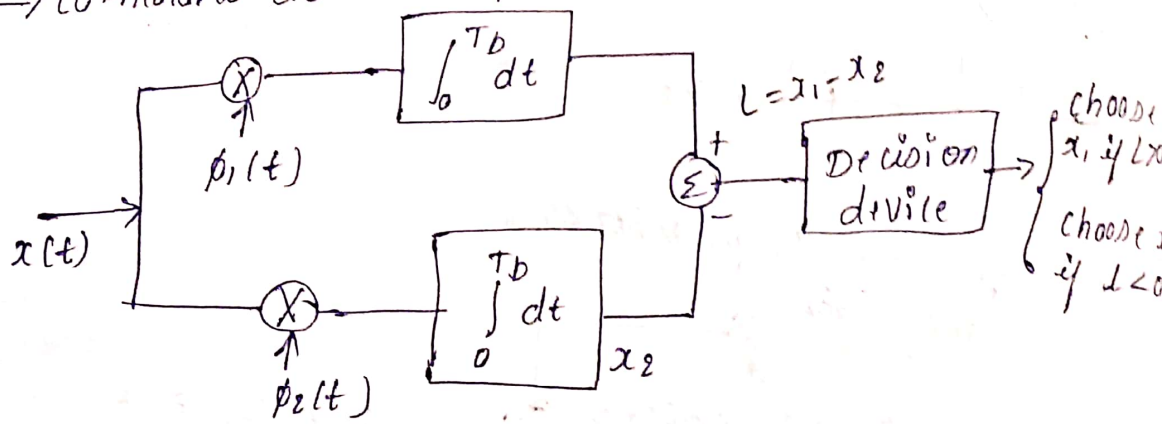
$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t \quad -② \qquad 0 \le t \le T_b \quad \text{symbol 0}$$

$$\phi(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f t.$$

$$\begin{cases} S_1(t) = \sqrt{E_b}\, \phi_1(t) \\ S_2(t) = \sqrt{E_b}\, \phi_2(t) \end{cases}$$

∴ Non-coherent

→ Co-herant detection of FSK :-



$$x(t) = S(t) + W(t)$$

Transmission of '0' :-

$$x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} [S(t) + W(t)]\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} S(t)\, \phi_1(t)\, dt + \int_0^{T_b} W(t)\, \phi_1(t)\, dt$$

$$\langle x_1 = S_{11} + W_1 \rangle -①$$

mean value :-

$$E[x_1] = E[S_{11} + W]$$

$$\langle E[x_1] = [0, 0] \rangle$$

$$var[x_1] = var[S_{11} + W_1]$$

$$\langle var[x_1] = [0, \frac{N_0}{2}] \rangle$$

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$x_2 = \int_0 \quad x_1(t) \cdot \phi_1(t)\, dt$

Transmission of '0' :-

$$x_2 = \int_0^{T_b} x_1(t) + \phi_2(t)\, dt$$

$$= \int_0^{T_b} [S_2(t) + W(t)]\, \phi_2(t)\, dt$$

$$= \int_0^{T_b} S_2(t)\, \phi_2(t)\, dt + \int_0^{T_b} W(t)\, \phi_2(t)\, dt$$

$$x_2 = S_{22} + W_2$$

$$\langle x_2 = \sqrt{E_b} + W_2 \rangle \quad -①$$

mean value :-

$$E[x_2] = E[\sqrt{E_b} + W_2]$$

$$E[x_2] = \sqrt{E_b}$$

variance value :-

$$Var[x_2] = Var[\sqrt{E_b} + W_2]$$

$$Var[x_2] = \frac{N_0}{2}$$

$$l = x_1 - x_2$$

$$E[l] = E[x_1] - E[x_2]$$

$$= 0 - \sqrt{E_b}$$

$$\langle E[l] = -\sqrt{E_b} \rangle$$

$$Var[l] = Var[x_1] + Var[x_2]$$

$$= \frac{N_0}{2} + \frac{N_0}{2}$$

$$\langle Var[l] = N_0 \rangle$$

→ probability density function [PDF] :-

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$$f[l/0] = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l+\sqrt{E_b})^2}{\sqrt{2N_0}}\right]$$

$$= \frac{1}{\sqrt{2\pi N_0}} e\left[\frac{-(l+\sqrt{E_b})^2}{\sqrt{2}\, N_0}\right] \quad -①.$$

Probability of 0 transmission.

$$P_e(0) = \int_0^{\infty} f_1(l/0)\, dl$$

$$z, = \quad 0 \le x \le \infty$$

$$\text{put } z = \frac{l+\sqrt{E_b}}{\sqrt{2N_0}} \quad -②.$$

$$dz = \frac{dl+\sqrt{E_b}}{\sqrt{2N_0}}$$

$$dz = \frac{dl}{\sqrt{2N_0}} \Rightarrow dl = \sqrt{2N_0}\, dz$$

for $l = 0$

$$z = \frac{\sqrt{E_b}}{\sqrt{2N_0}} = \sqrt{\frac{E_b}{2N_0}}$$

For $l = \infty$, $z = \infty$

Probability of '0' transmission

$$P_e(0) = \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi N_0}} e^{-z^2} dz . \sqrt{2N_0}$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} dz \Rightarrow \frac{1}{\sqrt{\pi}} \frac{2}{2} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} dz$$

x'y 2/2

$$P_e(0) = \frac{1}{2} \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} dz$$

$$\text{where } \operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp \sqrt{\frac{E_b}{2N_0}}$$

$$P_e(0) = 1/2 \operatorname{erfc}\sqrt{E_b/2N_0}$$

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$\text{Ily, } P_e(1) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$

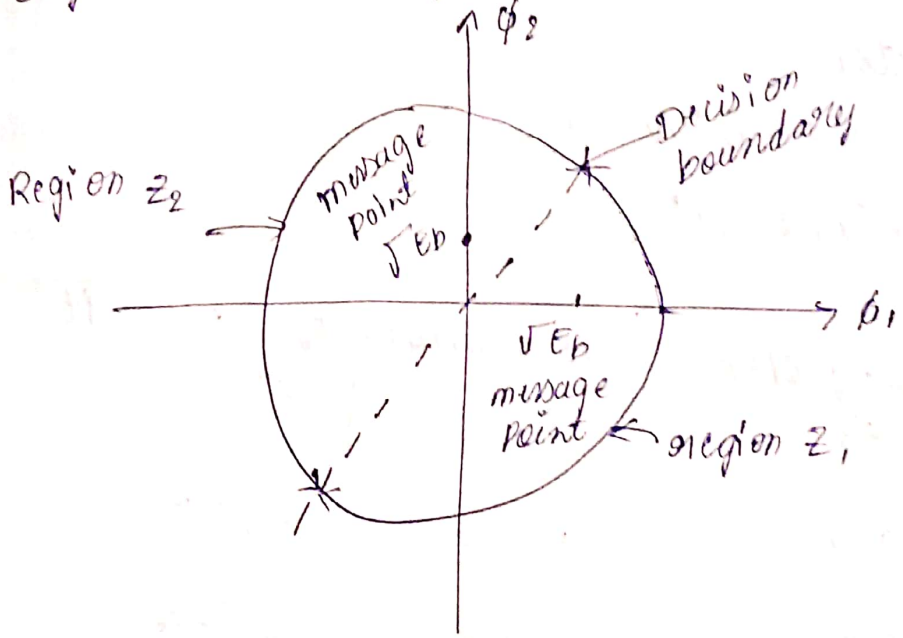
Total probability.

$$P = P(0)\,P_e(0) + P(1)\,P_e(1)$$

$$= \frac{1}{2}\left[\tfrac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}}\right] + \tfrac{1}{2}\left[\tfrac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}}\right]$$

$$\left\langle P = \tfrac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}} \right\rangle$$

→ Signal space diagram for FSK :-



$$S_{ij} = \int_0^{T_b} s_i(t)\,\phi_j(t)\,dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_i t * \sqrt{\frac{2}{T_b}}\cos 2\pi f_i t\,dt.$$

$$S_{ji} = \begin{cases} \sqrt{E_b} & \text{when } i=j \\ 0 & \text{when } i \neq j \end{cases}$$

$$\left\langle S_{ji} = \begin{bmatrix} S_{11} \\ S_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \right\rangle$$

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→ Quadriphase shift keying :

∵ $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$.

$$S_i(t) = \sqrt{\frac{2E}{T}} \{ \cos(\overset{A}{2\pi f_c t} + \overset{B}{(2i-1)\pi/4}) \quad -(1)$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t, \quad \phi_2(t) = \sqrt{2/T} \sin 2\pi f_c t.$$

$$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B.$$

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos 2\pi f_c t . \cos(2i-1)\pi/4 - \sqrt{\frac{2E}{T}} \sin 2\pi f_c t .$$

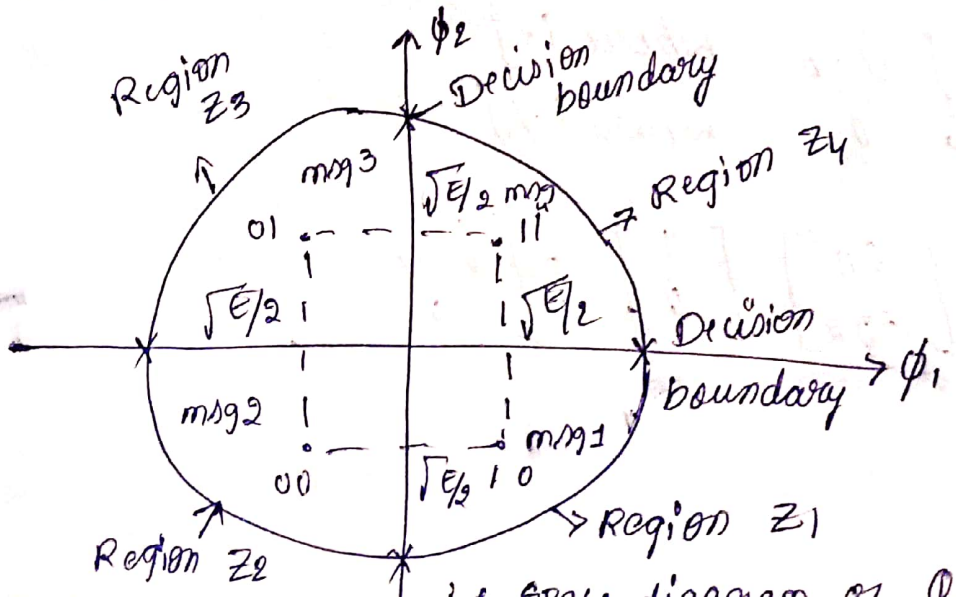
$\sin[2i-1]\pi/4$

symbol vectors.

$S_{i1} = \sqrt{E} \cos[2i-1]\pi/4$

$S_{i2} = -\sqrt{E} \sin[2i-1]\pi/4$.

i/p dibit	Phase of QPSK s/g	co-ordinates of message pt	
		S_{i1}	S_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$



signal diagram of QPSK.

→ Quadriphase shift keying :

∵ $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$.

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\pi/4\right) \quad -(1)$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t, \quad \phi_2(t) = \sqrt{2/T} \sin 2\pi f_c t.$$

$$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B.$$

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \cdot \cos(2i-1)\pi/4 - \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \cdot \sin[2i-1]\pi/4$$

symbol vectors.

$$S_{i1} = \sqrt{E} \cos[2i-1]\pi/4$$

$$S_{i2} = -\sqrt{E} \sin[2i-1]\pi/4.$$

i/p dibit	Phase of QPSK s/g	co-ordinates of message pt	
		S_{i1}	S_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
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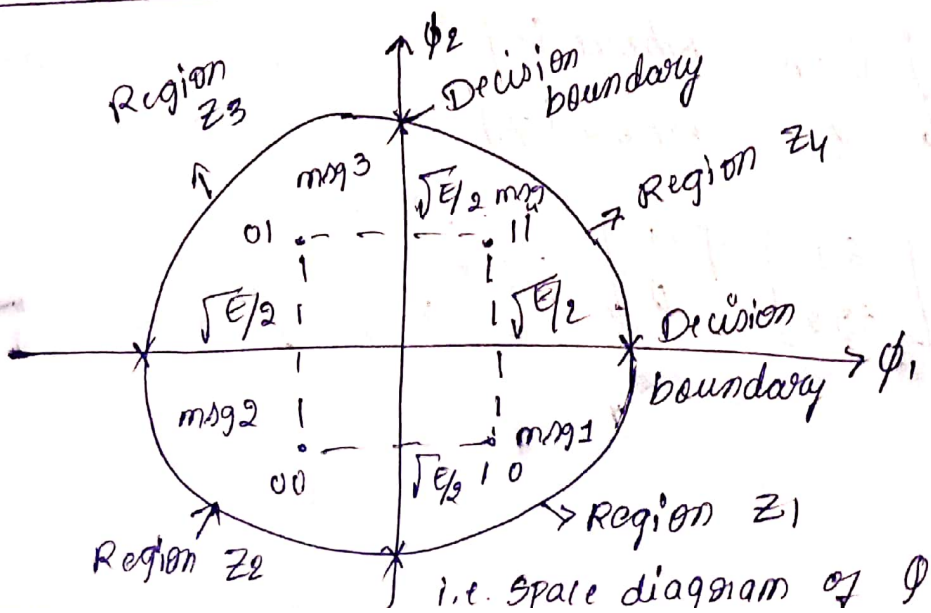
11	$7\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$



i.e. Space diagram of QPSK.

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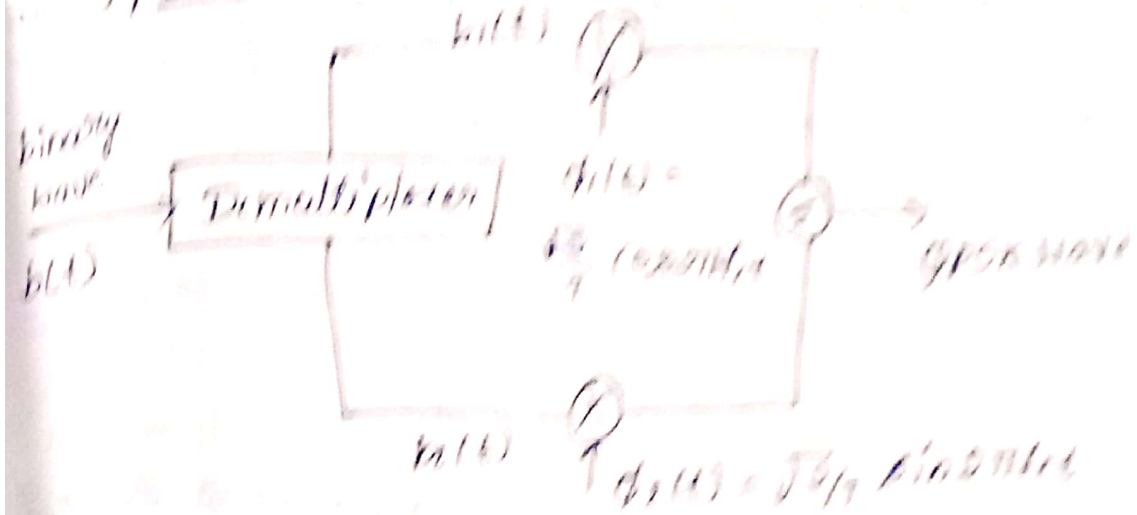
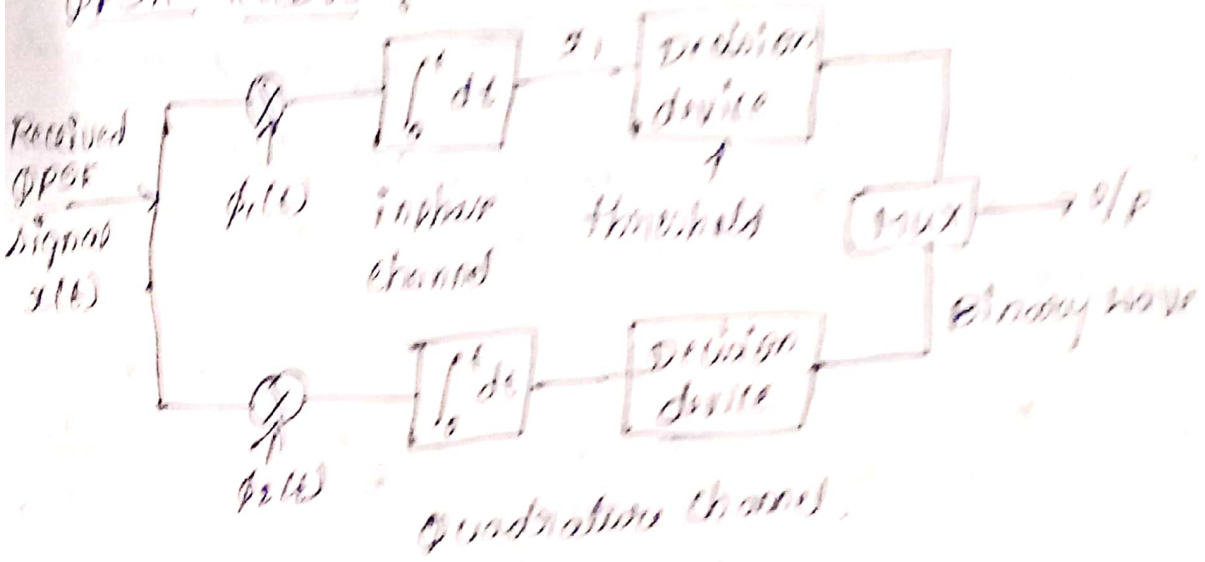
QPSK - Modulation :



QPSK Receiver :



→ probability of error :-

$E = 2E_b$, $E_b = E/2$, $\sqrt{E_b} = \sqrt{E/2}$

$P' = 1/2\, erfc\left[\sqrt{E_b/n_0}\right] \Rightarrow 1/2\, erfc\left[\sqrt{E/2n_0}\right]$ — ①

Probability of correct decision.

$P_c = [1 - P']^2 \Rightarrow \left[1 - 1/2\, erfc\sqrt{E/2n_0}\right]^2$

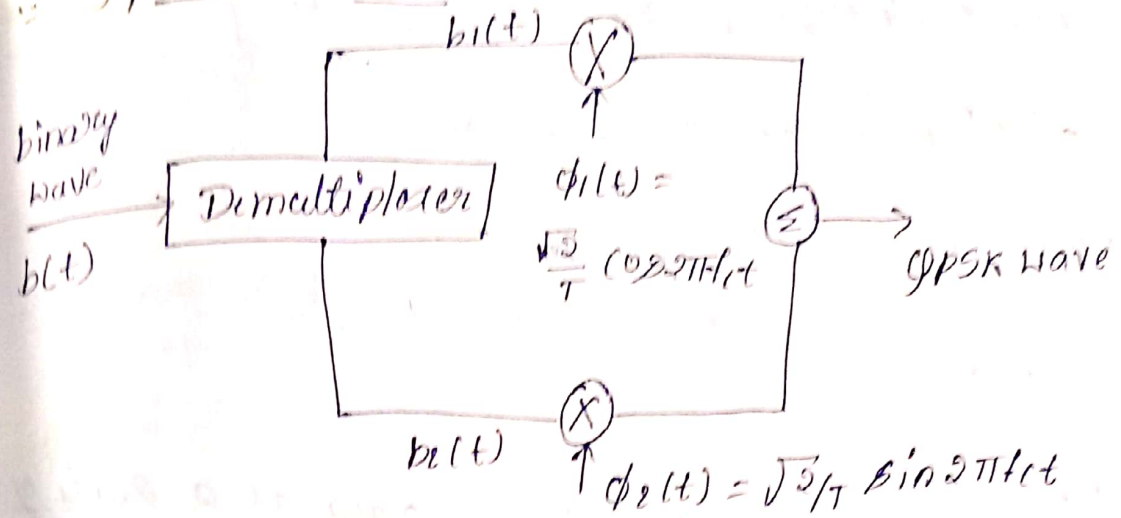
$\therefore (a-b)^2 = a^2 + b^2 - 2ab$

$P_c = 1 + 1/4\left[erfc^2\sqrt{E/2n_0}\right] - erfc\sqrt{E/2n_0}$ — ②

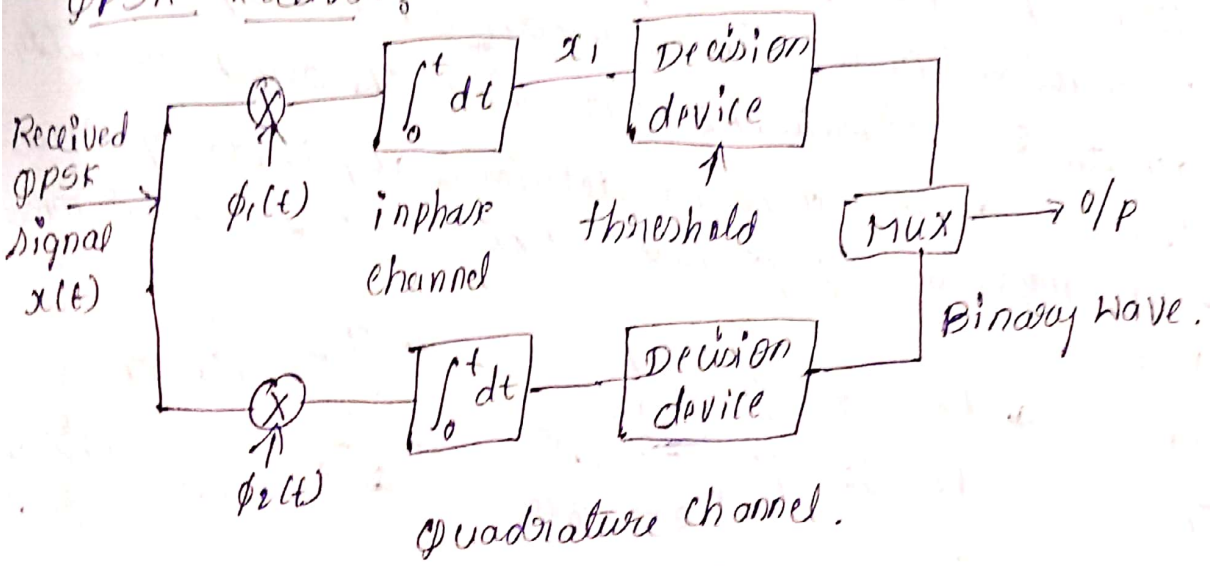
Avg probability of error of QPSK

$P_e = 1 - P_c$

QPSK Transmitter :-



QPSK Receiver :-



→ probability of error :-

$E = 2E_b$, $E_b = E/2$, $\sqrt{E_b} = \sqrt{E/2}$.

$P' = 1/2\, erfc\left[\sqrt{E_b/N_0}\right] \Rightarrow 1/2\, erfc\left[\sqrt{E/2N_0}\right]$ — ①

Probability of correct decision.

$P_c = [1 - P']^2 \Rightarrow \left[1 - 1/2\, erfc\sqrt{E/2N_0}\right]^2$

$\therefore (a-b)^2 = a^2 + b^2 - 2ab$.

$P_c = 1 + 1/4\left[erfc^2\sqrt{E/2N_0}\right] - erfc\sqrt{E/2N_0}$ — ②

Avg probability of error of QPSK

$P_e = 1 - P_c$

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$$P_e = 1 - \left[1 + \frac{1}{4}\,\mathrm{erfc}^2\sqrt{E/2N_0} - \mathrm{erfc}\sqrt{E/2N_0}\right]$$

$$P_e = -\frac{1}{4}\,\mathrm{erfc}^2\sqrt{E/2N_0} + \mathrm{erfc}\sqrt{E/2N_0}$$

where $\sqrt{E/2N_0} >> 0$,

$$\left\langle P_e = \mathrm{erfc}\sqrt{E/2N_0} \Rightarrow P_e = \mathrm{erfc}\sqrt{E_b/N_0} \right\rangle$$

Problems:

1) An FSK system transmitter binary data at a rate of 10^6 bits per second, assuming channel matrix additive white gaussian (AWGN) with 0 mean & power spectral density 2×10^{-20} watts/Hz. determine the avg probability of error. assume co-herant detector & amplitude of rec -ved sinosodial s/g for both symbol 1 & 0 to be 1.2 micro volts.

Ans.

$T_b = ?$

$E_b = ?$

$P_e = ?$

* $P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}$

$P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{7.2\times10^{-19}}{2\times4\times10^{-20}}}$

$\langle P_e = \frac{1}{2}\,\mathrm{erfc}[3] \rangle$ — ①

* $T_b = \frac{1}{\text{bit rate}}$

$\langle T_b = \frac{1}{10^6} = 10^{-6}\,s \rangle$

* s/g energy per bit

$E_b = \frac{1}{2}[\text{s/g amplitude}]^2 \times T_b$

$E_b = \frac{1}{2}[1.2\times10^{-6}]^2 \times 10^{-6}$

$\langle E_b = 7.2\times10^{-19}\,J \rangle$

* $W = \frac{N_0}{2}$

$N_0 = 2W$

$N_0 = 2\times$

$\langle N_0 = 4\times$

* $\mathrm{erfc}(U) = 1 - \mathrm{erf}[U]$ — ②

$\mathrm{erf}[3] = 0.99998$

eqn. ② become.

$\mathrm{erfc}[3] = 1 - \mathrm{erf}[U]$

$= 1 - 0.99998$

$\langle \mathrm{erfc}[3] = 0.00002 \rangle$

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$$P_e = \cancel{1} - \left[\cancel{1} + 1/4\, erfc\, 2\sqrt{E/2N_0} - erfc\sqrt{E/2N_0}\right]$$

$$P_e = -1/4\, erfc\, 2\sqrt{E/2N_0} + erfc\sqrt{E/2N_0}$$

where $\sqrt{E/2N_0} >> 0$,

$$\langle P_e = erfc\sqrt{E/2N_0} \Rightarrow P_e = erfc\sqrt{E_b/N_0} \rangle$$

Problems:

1) An FSK system transmitts binary data at a rate of 10^6 bits per second, assuming channel matrix is additive white gaussian (AWGN) with 0 mean & power spectral density 2×10^{-20} watts/Hz. determine the avg probability of error. assume co-herant detector & amplitude of recv -ved sinosodial s/g for both symbol 1 & 0 to be 1.2 micro volts.

Ans.

$T_b = ?$

$E_b = ?$

$P_e = ?$

* $P_e = 1/2\, erfc\sqrt{\frac{E_b}{2N_0}}$

$P_e = 1/2\, erfc\sqrt{\frac{7.2\times10^{-19}}{2\times4\times10^{-20}}}$

$$\langle P_e = 1/2\, erfc[3] \rangle \quad \text{—} (1)$$

* $T_b = \frac{1}{\text{bit rate}}$

$$\langle T_b = \frac{1}{10^6} = 10^{-6} s \rangle$$

* $N = \frac{N_0}{2}$

$N_0 = 2N$

$N_0 = 2\times2\times10$

$\langle N_0 = 4\times10 \rangle$

* s/g energy per bit

$E_b = 1/2\,[\text{s/g amplitude}]^2 \times T_b$

$E_b = 1/2\,[1.2\times10^{-6}]^2 \times 10^{-6}$

$$\langle E_b = 7.2\times10^{-19}\, J \rangle$$

* $erfc(U) = 1 - erf[U]$ — (5)

$erf[3] = 0.99998$

eqn. (5) become.

$erfc[3] = 1 - erf[U]$

$= 1 - 0.99998$

$$\langle erfc[3] = 0.00002 \rangle$$

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eqn ① becomes.

$P_e = 1/2\,[0.00002]$

$\langle P_e = 0.00001 \rangle_{\%} \simeq 1\times10^{-5}$

2] Binary data are transmitted at a rate of 10^6 bit/sec, over a microwave link assuming channel is noise is AWGN with 0 mean & power spectral density at the receiver i/p is 10^{-10} Watts/Hz. find the avg co-ver power required to maintain an avg probability of error $P_e \le 10^{-4}$ for co-herant binary FSK determine the minimum channel b/w required.

Ans:-

* $P_e = 1/2\, erfc\sqrt{\frac{E_b}{2N_0}}$

$10^{-4} = 1/2\, erfc\sqrt{\frac{E_b}{2N_0}}$

$erfc\sqrt{\frac{E_b}{2N_0}} = 2\times10^{-4}$

$erfc[U] = 1 - erf[U]$

$erf[U] = 1 - erfc[U]$

$= 1 - [2\times10^{-4}]$

$erf[U] = 0.9998$

$erf\left[\sqrt{\frac{E_b}{2N_0}}\right] = 0.9998$

$erf[2.6] = 0.9998$

$\sqrt{\frac{E_b}{2N_0}} = 2.6$

square on b/s

$\frac{E_b}{2N_0} = [2.6]^2$

$E_b = 6.76\times2\times10^{-1}$

$\langle E_b = 1.35\times10^{-9} \rangle_{\%}$

* Bit rate $= 10^6$ bit/sec.

Bit duration $= \frac{1}{\text{bit rate}}$

$BW = \frac{1}{10^6} = 10^{-6}$.

* avg transmitted power $= \frac{E_b}{T_b} = \frac{1.35\times10^{-9}}{10^{-6}} = 1.35\times10^{-3}$ Watt $\rangle_{\%}$

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→ Minimum shift keying:- $0 \le t \le T_b$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t + \theta(0) \rightarrow \text{symbol } 1$$

$$\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2(t) + \theta(0) \rightarrow \text{symbol } 2.$$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c(t) + \theta(t)$$

where, $f_c = 1/2\,(f_1 + f_2)$

$$\theta(t) = \theta(0) + \frac{\pi h}{T_b} t$$

where, $h \rightarrow$ Deviation ratio

$$h = T_b[f_1 - f_2]$$

$$\theta[T_b] = \theta(0) \pm \frac{\pi h}{T_b}$$

$$\theta[T_b] = \theta(0) = \pm \frac{\pi h}{T_b}$$

$$\theta[T_b] - \theta(0) = \begin{cases} +\pi h/T_b & ; \text{ for symbol } 1 \\ -\pi h/T_b & ; \text{ for symbol } 2. \end{cases}$$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)]$$

$$= \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \cdot \cos\theta(t) - \sqrt{\frac{2E_b}{T_b}} \sin 2\pi f_c t \cdot \sin\theta(t)$$

$$s(t) = s_I(t) \cos 2\pi f_c t - s_Q(t) \sin 2\pi f_c t.$$

where, $I \rightarrow$ inphase signal

$Q \rightarrow$ quadrature signal.

$$\theta[T_b] - \theta[0] = \pm \frac{\pi h}{T_b} \quad -(1)$$

put $h = 1/2$ $\quad T_b \le t \le +T_b$

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→ Minimum shift keying:- $0 \le t \le T_b$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t + \theta(0) \rightarrow \text{symbol } 1.$$

$$\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2(t) + \theta(0) \rightarrow \text{symbol } 2.$$

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put $h = 1/2$ $\quad T_b \le t \le +T_b$

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$$\sqrt{\frac{2E_b}{T_b}}\cos\theta(t) \Rightarrow \sqrt{\frac{2E_b}{T_b}}\cos\left[\theta(0) \pm \frac{\pi h}{T_b}\right]$$

for $\theta(0) \Rightarrow 1$

$\theta(\pi) \Rightarrow -1$

$$s_I(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left[\frac{\pi t}{2T_b}\right]$$

$$s_Q(t) = \sqrt{\frac{2E_b}{T_b}}\sin\theta(t) \Rightarrow \sqrt{\frac{2E_b}{T_b}}\sin\left[\theta(0) \pm \frac{\pi}{2T_b}t\right]$$

for $\theta(\pi/2) \Rightarrow 1$ $\qquad 0 \le t \le 2T_b$

$\theta(-\pi/2) \Rightarrow -1$

$$\left\langle s_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}}\sin\left[\frac{\pi}{2T_b}t\right]\right\rangle$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos\left[\frac{\pi}{2T_b}t\right]\cos 2\pi f_c t$$

$$\phi_2(t) = \sqrt{2/T_b}\sin\left[\frac{\pi}{2T_b}t\right]\sin 2\pi f_c t$$

for MSK signal.

$$s(t) = s_1\phi_1(t) + s_2\phi_2(t) \quad - (2).$$

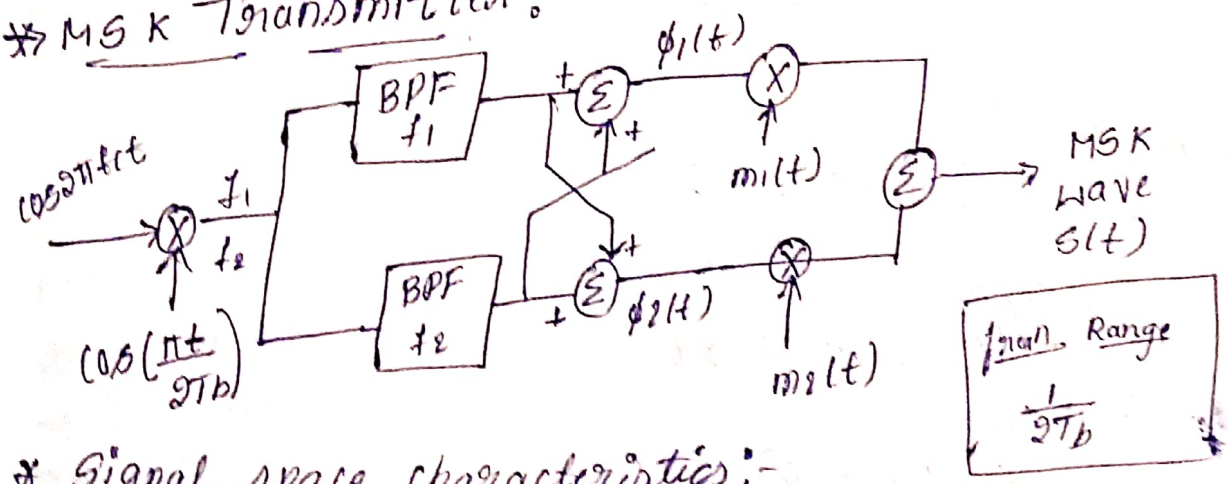
$s_1 \rightarrow$ inphase.

$$s_1 = \int_{-T_b}^{T_b} s(t)\phi_1(t)\,dt, \quad s_2 = \int_0^{2T_b} s(t)\phi_2(t)\,dt.$$

$$\left\langle s_1 = \sqrt{E_b}\cos\theta(0) \quad s_2 = -\sqrt{E_b}\cos\theta[t_b]\right\rangle$$

$0 \le t \le 2T_b$

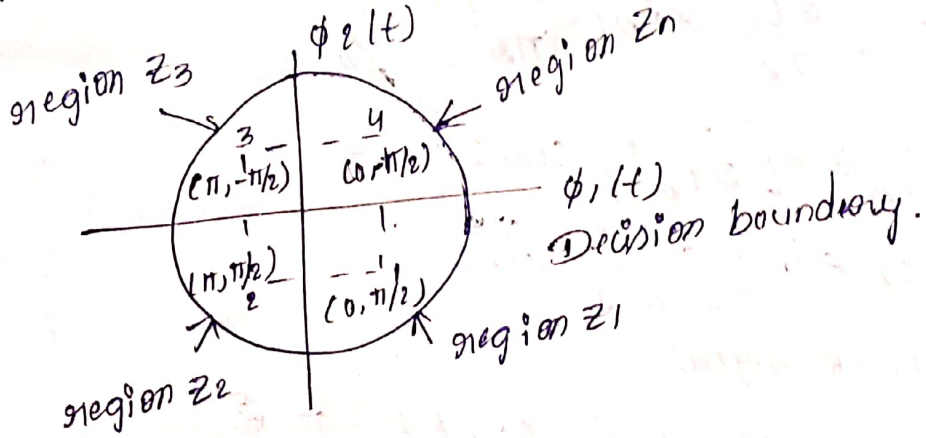
* MSK Transmitter :-



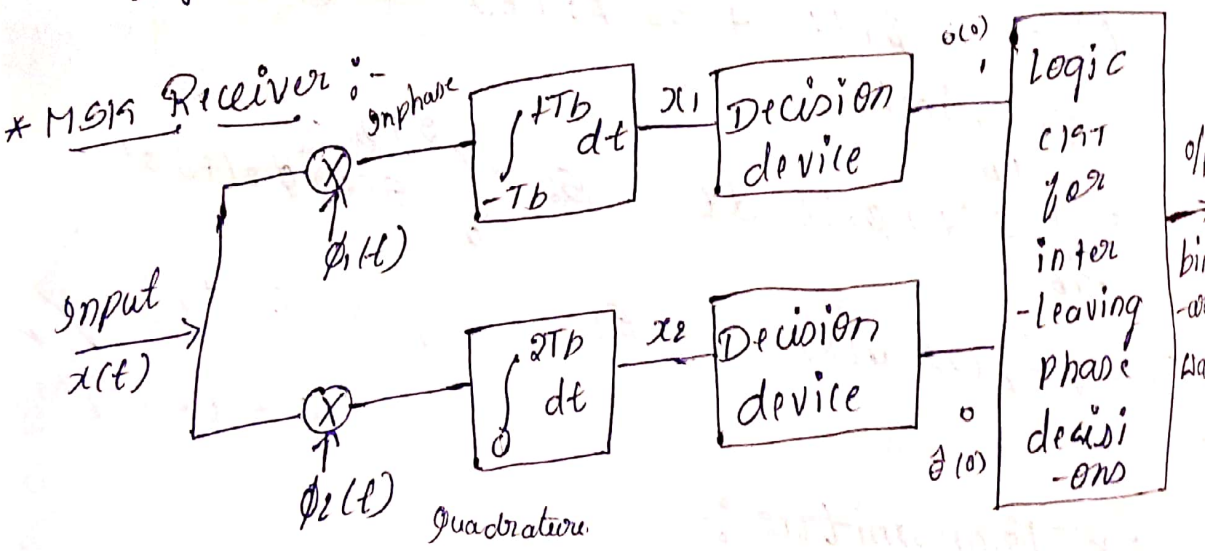
* Signal space characteristics :-

Transmitted binary symbol	Phase status		Co-ordinates of msg points	
	cos	sin	cos S_1	sin S_2
1	0	$\pi/2$	$+\sqrt{E_b}$	$-\sqrt{E_b}$
0	π	$\pi/2$	$-\sqrt{E_b}$	$-\sqrt{E_b}$
1	π	$-\pi/2$	$-\sqrt{E_b}$	$+\sqrt{E_b}$
0	0	$-\pi/2$	$+\sqrt{E_b}$	$+\sqrt{E_b}$

* Space diagram for MSK :-



* MSK Receiver :-



AWGS :-

$x(t) = S(t) + W(t)$ — ①

for inphase

$$x_1 = \int_{-T_b}^{T_b} x(t)\,\phi_1(t)\,dt$$

$x_1 = S_1 + W_1$ — ②

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$\cos\phi \rightarrow$ $x_1 > 0$ $\theta(0) \rightarrow 0$

$x_1 < 0$ $\theta(0) \rightarrow \pi$

for Quadrature.

$$x_2 = \int_0^{2T_b} x(t)\,\phi_2(t)\,dt$$

$x_2 = S_2 + W_2$

$x_2 > 0 \rightarrow \hat{\theta}(0) = -\pi/2$

$x_2 < 0 \rightarrow \hat{\theta}(0) = \pi/2$.

$\hat{\theta}(0) = 0$ & $\hat{\theta}(T_b) = -\pi/2$ } Transmissions of symbol 0

(or) $\hat{\theta}(0) = \pi$ & $\hat{\theta}(T_b) = \pi/2$

when, $\hat{\theta}(0) = \pi$ & $\hat{\theta}(T_b) = -\pi/2$ } transmission of symbol 1.

(or) $\hat{\theta}(0) = 0$ & $\hat{\theta}(T_b) = \pi/2$

CPFSK :-

$$P_e = erfc\sqrt{\frac{E_b}{N_0}} - \underbrace{\frac{1}{4}\,erfc^2\sqrt{\frac{E_b}{N_0}}}_{\frac{E_b}{N_0} > 71}$$

$$\left\langle P_e = erfc\sqrt{\frac{E_b}{N_0}} \right\rangle_{u}$$

PM

→ Problems on FSK :-

1) The binary FSK system transmitte data at a rate of 2M Bit/sec (or) 2MBPS over an AWGN channel. the noise is zero with power spectral density $\frac{N_0}{2} = 10^{-20}$ W/Hz the amplitude of received s/g in the absences of nois -e is 1 uV. determine the avg probability of error for co-herant detection of FSK?

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$x_1 > 0 \quad \theta(0) \to 0$

$x_1 < 0 \quad \theta(0) \to \pi$

Quadrature

$$x_1 = \int_0^{2T_b} x(t)\, \phi_2(t)\, dt$$

$= S_2 + W_2$

$x_1 > 0 \to \hat{\theta}(0) = -\pi/2$

$x_1 < 0 \to \hat{\theta}(0) = \pi/2$.

$\hat{\theta}(0) = 0$ & $\hat{\theta}(T_b) = -\pi/2$ } Transmissions of symbol 0

(or) $\hat{\theta}(0) = \pi$ & $\hat{\theta}(T_b) = \pi/2$

when, $\hat{\theta}(0) = \pi$ & $\hat{\theta}(T_b) = -\pi/2$ } Transmission of symbol 1.

(or) $\hat{\theta}(0) = 0$ & $\hat{\theta}(T_b) = \pi/2$

CPFSK :-

$$P_e = erfc\sqrt{\frac{E_b}{N_0}} - \underbrace{\frac{1}{4}\, erfc^2\sqrt{\frac{E_b}{N_0}}}_{\frac{E_b}{N_0} > 7 \text{ dB}}$$

$$\left\langle P_e = erfc\sqrt{\frac{E_b}{N_0}} \right\rangle$$

→ Problems on FSK :-

1) The binary FSK system transmitte data at a rate of 2M Bit/sec (or) 2MBPS over an AWGN channel. The noise is zero with power spectral density $\frac{N_0}{2} = 10^{-20}$ W/Hz the amplitude of received sig in the absences of noise is 1 μV. determine the avg probability of error for co-herent detection of FSK?

$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{2N_0}}$

$E_b = \frac{[amplitude]^2}{2} \times T_b \Rightarrow \frac{(1\times10^{-6})^2}{2} \times 500\times10^{-9} \Rightarrow 2.5\times10^{-13}$

where T_b = bit interval $= \frac{1}{\text{transmission rate}} \Rightarrow \frac{1}{2\times10^6}$

$= 500\times10^{-9}$

$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{2N_0}}$

$= \frac{1}{2} erfc \sqrt{\frac{2.5\times10^{-13}}{2\times2\times10^{-20}}}$

$= \frac{1}{2} erfc[2.5]$

$= \frac{1}{2} \times 0.00041$

$\langle P_e = 2.05\times10^{-4} \rangle$

$\frac{N_0}{2} = 10^{-20}$

$N_0 = 2\times10^{-20}$

$erfc(u) = 1 - erf(u)$

$= 1 - erf(2.5)$

$= 1 - 0.99959$

$\langle erfc(2.5) = 0.00041 \rangle$

2] A binary data is transmitted is the ASK over a AWGN channel at a rate of 2.5 MBPS. the carrier amplitude at the receiver is 1mV, noise power spectral density $\frac{N_0}{2} = 10^{-15}$ W/Hz. find the avg probability of error for coherant ASK?

Soln. $T_b = \frac{1}{\text{transmission rate}} = \frac{1}{2.5\times10^6} = 4\times10^{-7}$

$E_b = \frac{[amplitude]^2}{2} \times T_b \Rightarrow \frac{[1\times10^{-3}]^2}{2} \times 4\times10^{-7}$

$\langle E_b = 2\times10^{-13} \rangle$

$P_e = 1/2\, erfc\left[\frac{1}{2}\sqrt{\frac{E_b}{2N_0}}\right]$

$= 1/2\, erfc\left[\frac{1}{2}\sqrt{\frac{2\times10^{-13}}{2\times2\times10^{-15}}}\right]$

$\frac{N_0}{2} = 10^{-15}$

$N_0 = 2\times10^{-15}$

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$= \frac{1}{2} \, erfc[7.07]$

$= \frac{1}{2} \times 0.001$

$P_e = 0.5 \times 10^{-3}$

$erfc(u) = 1 - erf(u)$

$= 1 - erf(7.07)$

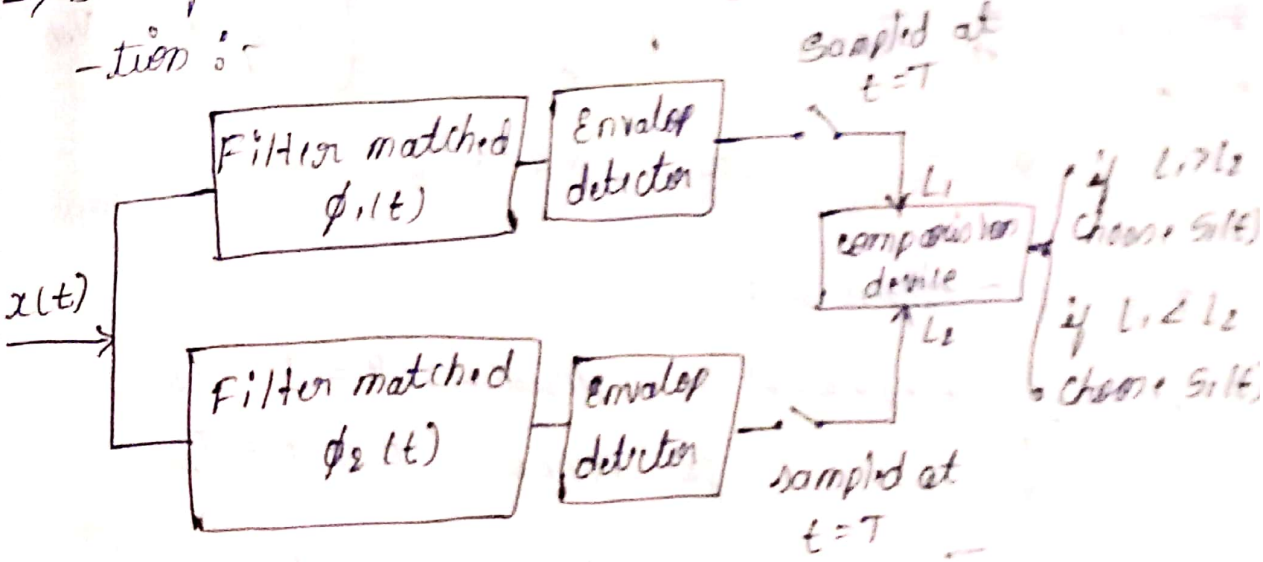
$= 1 - 0.999$

$= 0.001$

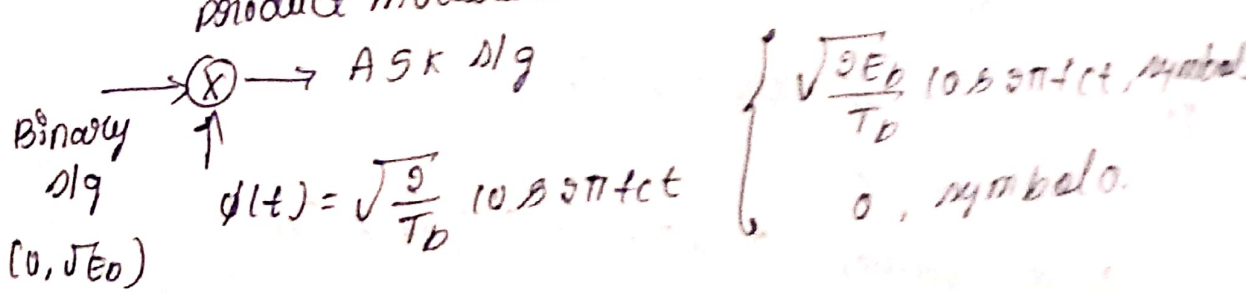
→ Non-coherent binary modulation :- don't have the envelope info.

$$x_1(t) = \begin{cases} g_1(t) + W_1(t) & \rightarrow \text{mean}, \frac{N_0}{2}, \quad 0 \le t \le T \\ g_2(t) + W_2(t) & \quad 0 \le t \le T \end{cases}$$

→ Binary Receiver for Non-coherent orthogonal modulation :-



→ ASK modulation :-



symbol 1, $S_1 = \sqrt{E_b}$

symbol 0, $S_2 = 0$

→ Space diagram :-

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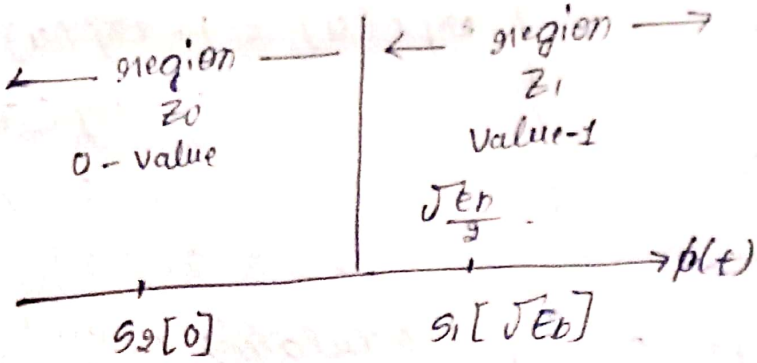
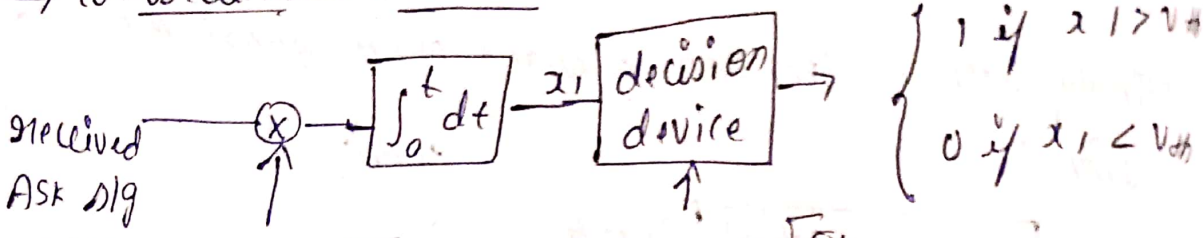


$-ve$ to $\sqrt{\frac{E_b}{2}}$ $\rightarrow$ 0 value. , $\sqrt{\frac{E_b}{2}}$ to $+\infty$ $\rightarrow$ 1 value

$\rightarrow$ <u>co-ordinates relationship</u> :-



$x(t) \rightarrow$ received sig.

$$x(t) = \begin{cases} s(t) + w(t) & \text{for symbol 1} \quad 0 \le t \le T_b \\ 0 & \text{for symbol 0} \quad 0 \le t \le T_b \end{cases}$$

W - mean value zero & varience value $\frac{N_0}{2}$

$$x_1 = \int_0^{T_b} x(t)\, \phi(t)\, dt$$

for symbol 1 is transmiting

$x_1 = s_1 + w_1$

mean value, $E[x_1] = E[s_1] + E[w_1]$

$= \sqrt{E} + 0 = \sqrt{E_b}$

varience for symbol 1

$var[x_1] = var[s_1] + var[w_1]$

$= 0 + \frac{N_0}{2} = \frac{N_0}{2}$

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Symbol 0 transmission

$x_1 = S_2 + W_2$

mean value, $E[x_1] = E[S_2] + E[W_2]$

$= 0 + 0 = 0$

variance value,

$var[x_1] = var[S_1] + var[W_2]$

$= 0 + \frac{N_0}{2} = \frac{N_0}{2}.$

→ PDF :-

$$f_L(L|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(L-\sqrt{E_b})^2}{N_0}\right]$$

Then, $$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\frac{\sqrt{E_b}}{2}} f_L(L|0)\,dL$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\frac{\sqrt{E_b}}{2}} \exp\left[-\frac{(L-\sqrt{E_b})^2}{N_0}\right]$$

put $z = \frac{L-\sqrt{E_b}}{\sqrt{N_0}}$

$dz = \frac{dL-\sqrt{E_b}}{\sqrt{N_0}}$

$dz = \frac{dL}{N_0}$

$dL = \sqrt{N_0}\, dz$

$z = \infty$, $L = -\infty$

$L = \frac{\sqrt{E_b}}{2}$

then, $$z = \frac{\sqrt{E_b}/2 - \sqrt{E_b}}{\sqrt{N_0}} \Rightarrow \frac{\frac{\sqrt{E_b} - 2\sqrt{E_b}}{2}}{\sqrt{N_0}}$$

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$$= \frac{-\sqrt{E_b}}{-2\sqrt{N_0}} \Rightarrow \frac{1}{2}\sqrt{\frac{E_b}{N_0}}$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{-1/2\sqrt{\frac{E_b}{N_0}}} e^{-z^2} \sqrt{N_0}\, dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{1/2\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz \Rightarrow \frac{1}{2}\frac{2}{\pi}\int_{1/2\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz$$

$$= \frac{1}{2} erfc\left(1/2\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e(1) = 1/2\, erfc\left(1/2\sqrt{\frac{E_b}{N_0}}\right)$$

× Avg probability :-

$$P = P(0)\, P_e(0) + P(1) + P_e(1)$$

$$P = P(0) + P(1) = 1$$

$$P(0) = 1/2 \quad , \quad P(1) = 1/2$$

$$P = \frac{1}{2}\left\{\frac{1}{2} erfc\left[1/2\sqrt{\frac{E_b}{N_0}}\right]\right\} + 1/2\left\{erfc\left[1/2\sqrt{\frac{E_b}{N_0}}\right]\right\}$$

$$\left\{P = \frac{1}{2} erfc\left[1/2\sqrt{\frac{E_b}{N_0}}\right]\right\}$$

→ Quadrature phase shift keying (QPSK) :-

QPSK, $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

$$s(t) = \sqrt{\frac{2E}{T}}\left\{\cos\left\{\underbrace{2\pi f_c t}_{A} + \underbrace{2i\,(-1)\,\pi/4}_{B}\right\}\right] \quad i = 1, 2, 3, 4 \ldots$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$$

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$s_1(t) = \sqrt{\frac{2E}{T}} \sin \omega_c t$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$s_i(t) = \sqrt{\frac{2E}{T}} \sin \omega_c t \cos(i-1)\pi/4 - \sqrt{\frac{2E}{T}} \cos \omega_c t \sin(i-1)\pi/4$

$s_i(t) = \sqrt{E} \sin(i-1)\pi/4$

symbol mapping

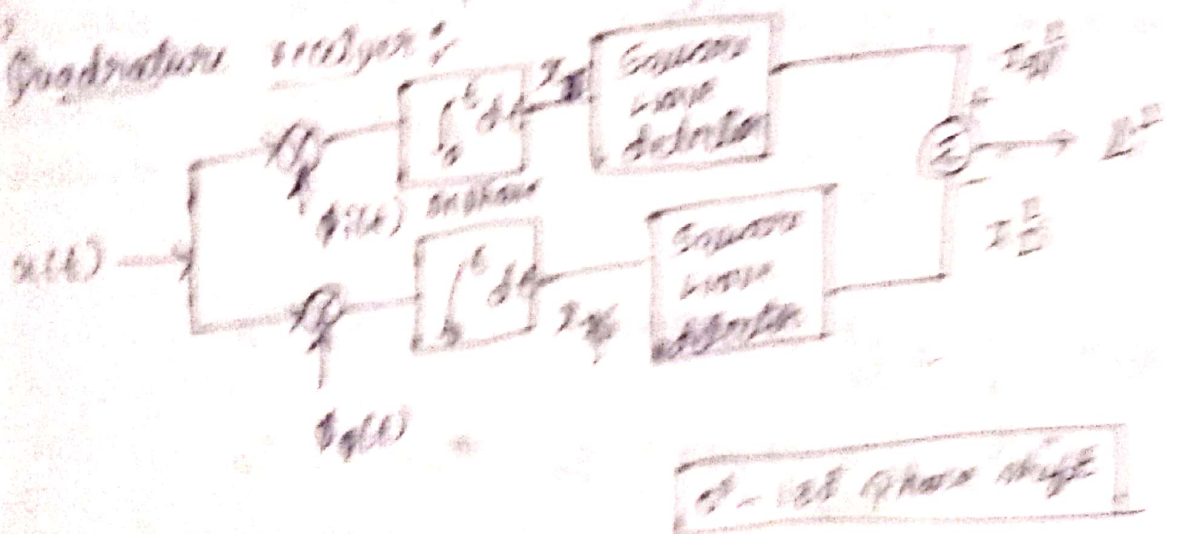
$s_1 = \sqrt{E} \cos(i-1)\pi/4$

$s_2 = \sqrt{E} \sin(i-1)\pi/4$

signal dibit	phase of QPSK wf	co-ordinates of message point	
		s_1	s_2
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$

→ Non-coherent orthogonal modulation :-

* Quadrature receiver :-



$s_i(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$\cos(A+B) = \cos A \cos B - \sin A \sin B.$

$s_i(t) = \sqrt{\frac{2E}{T}} \cos 2\pi f_c t . \cos(2i-1)\pi/4 - \sqrt{\frac{2E}{T}} \sin 2\pi$

$f_c t . \sin(2i-1)\pi/4$

symbol vector

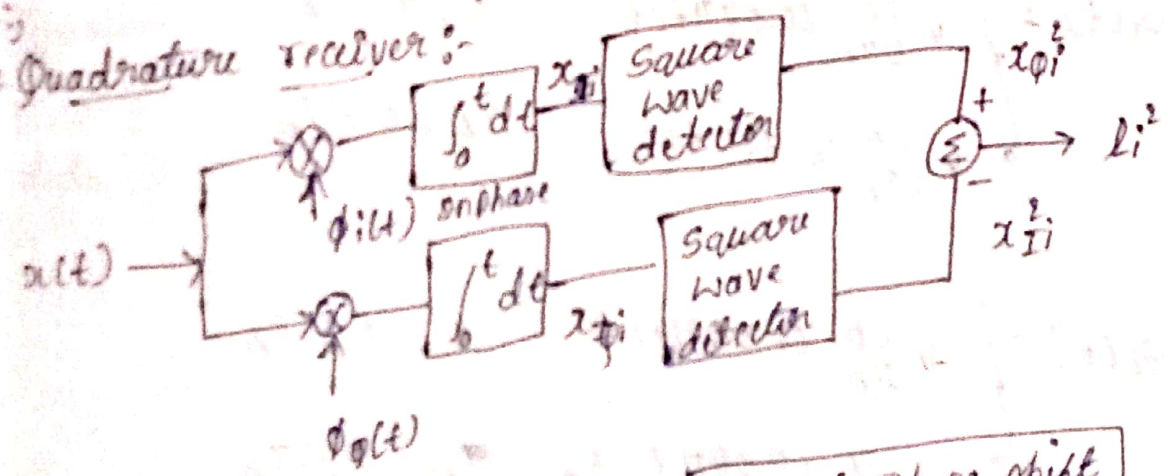
$s_1 = \sqrt{E} \cos(2i-1)\pi/4$

$s_2 = \sqrt{E} \sin(2i-1)\pi/4$

signal dibit	phase of QPSK sig	co-ordinates of message point	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$.

→ Non-coherent orthogonal modulation :-

* Quadrature receiver :-



$\boxed{0° - 180 \text{ Phase shift}}$

$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$\cos(A+B) = \cos A \cos B - \sin A \sin B.$

$s_i(t) = \sqrt{\frac{2E}{T}} \cos 2\pi f_c t . \cos(2i-1)\pi/4 - \sqrt{\frac{2E}{T}} \sin 2\pi$

$f_c t . \sin(2i-1)\pi/4$

symbol vector

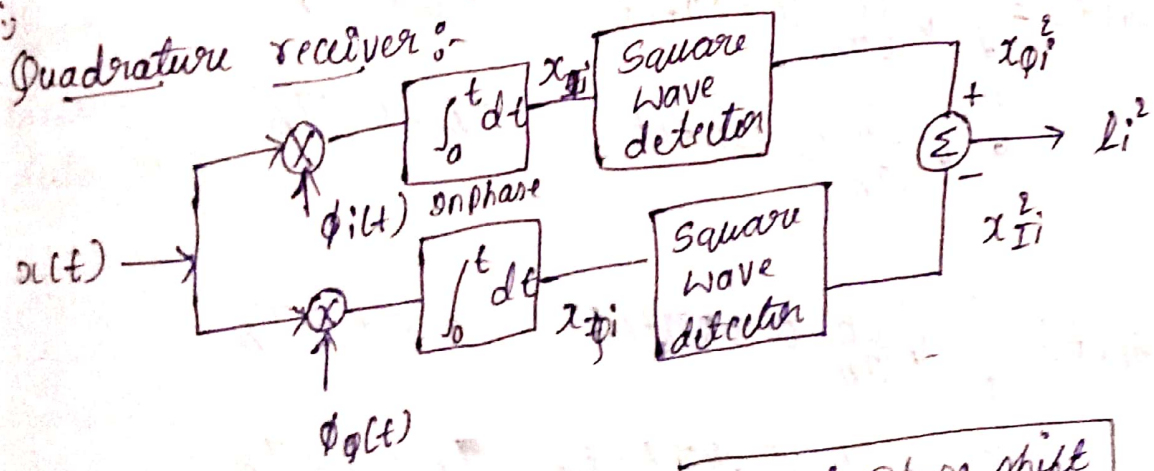
$s_1 = \sqrt{E} \cos(2i-1)\pi/4$

$s_2 = \sqrt{E} \sin(2i-1)\pi/4$

Input dibit	phase of QPSK sig	Co-ordinates of message point	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$.

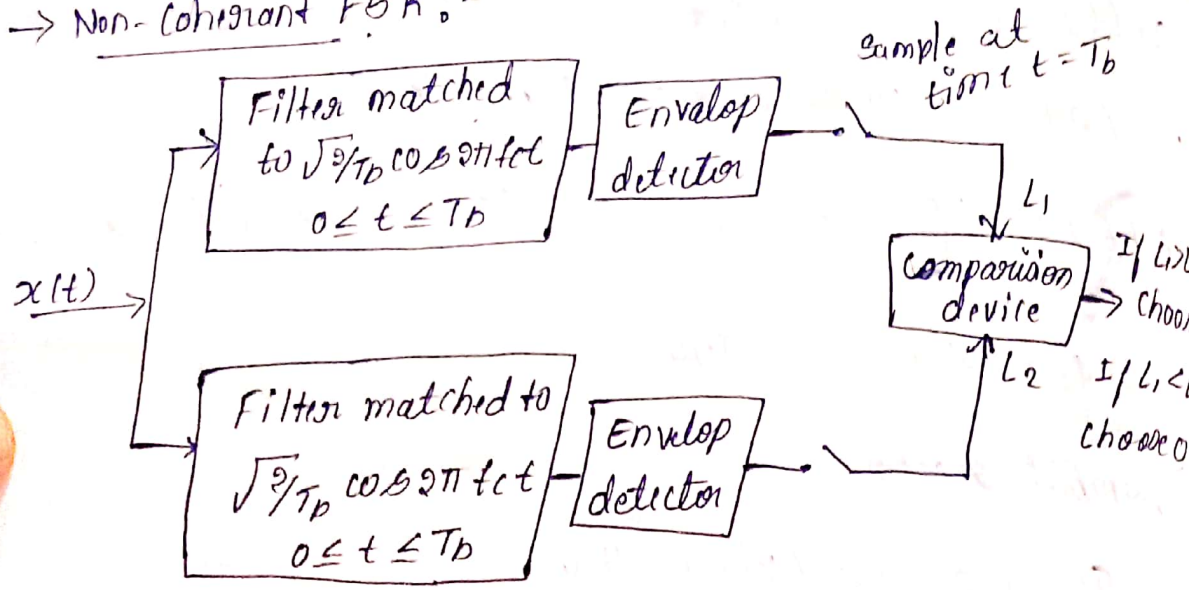
→ Non-coherant orthogonal modulation :-

* Quadrature receiver :-



$0° - 180$ Phase shift

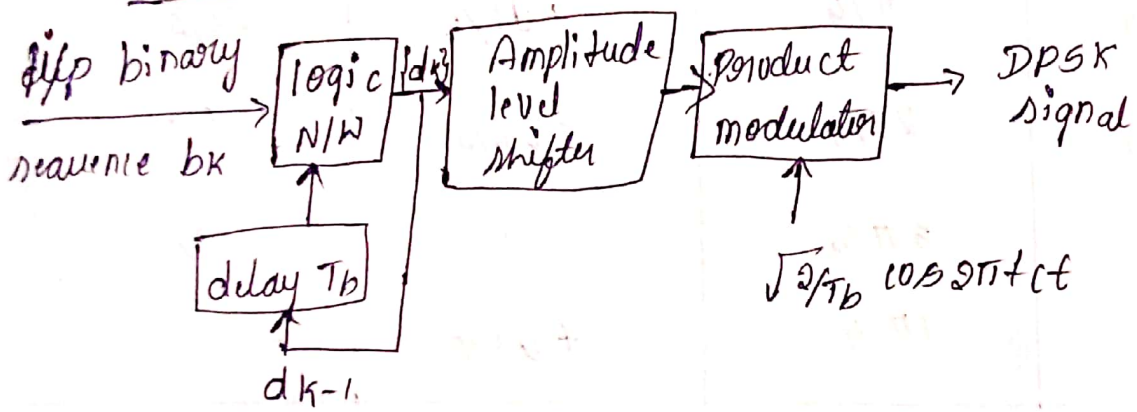
→ Non-Coherent FSK :-



$$\boxed{E = E_b,\ T = T_b \qquad P_e = \tfrac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}}$$

$$s_i(t) = \sqrt{\frac{2E}{T_b}}\cos 2\pi f_i t$$

→ Differential phase shift keying [DPSK] :-



i.e. DPSK Transmitter.

$$s_1(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}}\cos 2\pi f_c t & 0 \le t \le T_b \\ \sqrt{\frac{E_b}{2T_b}}\cos 2\pi f_c t & T_b \le t \le T_b \end{cases}$$

$$s_2(t) = \begin{cases} \sqrt{\frac{E_b}{T_b}}\cos 2\pi f_c t & 0 \le t \le T_b \\ \sqrt{\frac{E_b}{2T_b}}\cos 2\pi f_1 t & T_b \le t \le T_b \end{cases}$$

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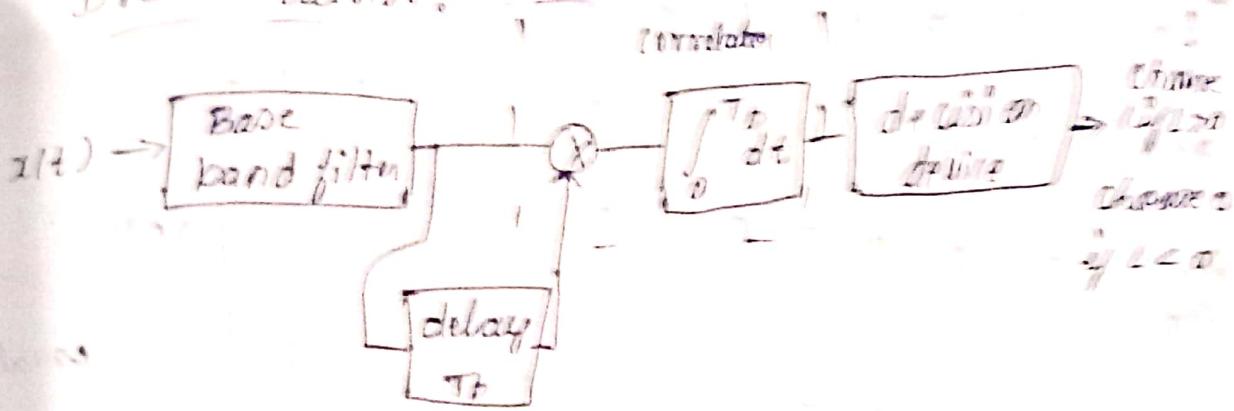
1) DPSK receiver :-



Problems :- $d_K = d_{K-1} \oplus b_K$

1) The bit-stream to be transmitted using DPSK determine the encoded sequence $\{d_K\}$, phase of the transmitted DPSK s/g & detected binary sequence.

$0 \Rightarrow \pi$
$1 \Rightarrow 0$

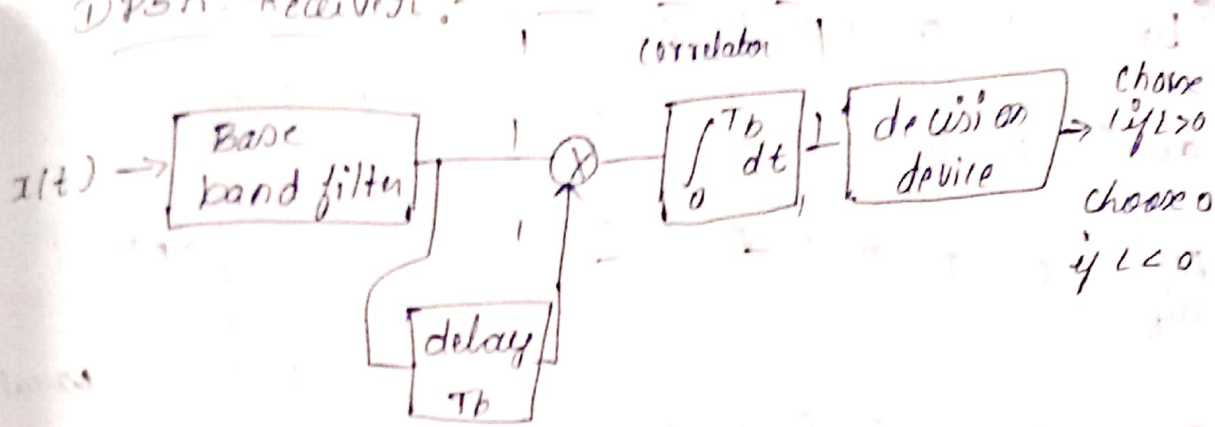
$b_K \Rightarrow$	0	0	1	0	0	1	1	0	0	1	1	1	0
$d_K = 1$	0	1	1	0	1	1	1	0	0	1	1	1	0
phase difference	π	0	0	π	0	0	0	π	0	0	0	0	π
$d_{K-1} \Rightarrow$	1	0	1	1	0	1	1	1	0	1	1	1	1
Phase difference	0	π	0	0	π	0	0	0	π	0	0	0	0
differential binary data $\{b_K\}$	0	0	1	0	0	1	1	0	0	1	1	1	0

XNOR

0	0	1
0	1	0
1	0	0
1	1	1

equal

DPSK receiver :-



Problems :- $d_K = d_{K-1} \oplus b_K$

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$b_K \Rightarrow$		0	0	1	0	0	1	1	0	0	1	1	1	0
$d_K = 1$	0	1	1	0	1	1	1	0	0	1	1	1	0	
Phase difference	π	0	0	π	0	0	0	π	0	0	0	0	π	
$d_{K-1} \Rightarrow$	1	0	1	1	0	1	1	1	0	1	1	1	1	
Phase difference	0	π	0	0	π	0	0	0	π	0	0	0	0	
differential binary data $\{b_K\}$		0	0	1	0	0	1	1	0	0	1	1	1	0

XOR

equal.

XNOR

0 0	1
0 1	0
1 0	0
1 1	1

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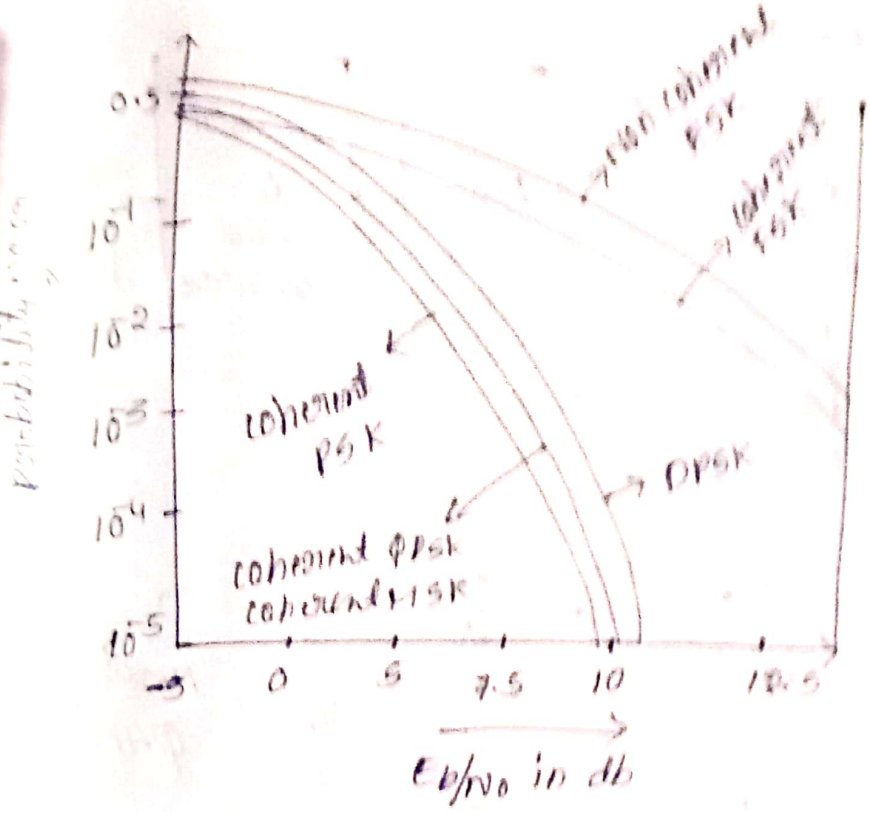
b_k =	1	0	1	1	1	0	0	0	1	1	1
d_{k-1}	1	0	0	0	0	1	0	1	1	1	1
Phase diff	0	π	π	π	π	0	π	0	0	0	0
d_{k-1}	1	1	0	0	0	0	1	0	1	1	1
phase diff	0	0	π	π	π	π	0	π	0	0	0
diff binary data $\{b_k\}$	1	0	1	1	1	0	0	0	1	1	1

XNOR

→ Binary & quaternary modulation techniques:-

Modulation	Detection method	Error probability
BPSK	coherent	$\frac{1}{2}\,\mathrm{erfc}\sqrt{E_b/N_0}$
FSK	coherent	$\frac{1}{2}\,\mathrm{erfc}\sqrt{E_b/2N_0}$
QPSK	— " —	$\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}$
MSK	— " —	$\mathrm{erfc}\sqrt{E_b/N_0}$
DPSK	non-coherent	$\frac{1}{2}\exp[-E_b/N_0]$
FSK	— " —	$\frac{1}{2}\exp[-E_b/2N_0]$



- M-ary modulation technique:

3 types

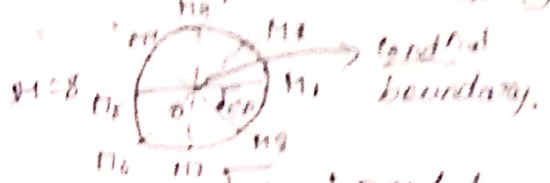
1. M-ary PSK.
2. M-ary ~~QAM~~ QAM.
3. M-ary FSK.

→ $S_i(t) = \sqrt{2E/T}\cos[2\pi f_c t + \theta_i]$

$M \to \log_2 \leftarrow \theta_i$ $\quad i = 0, 1, 2, \ldots M-1$

$$S_i(t) = \sqrt{\frac{2E}{T}}\cos 2\pi f_c t \cdot \cos\theta_i - \sqrt{\frac{2E}{T}}\sin 2\pi f_c t \cdot \sin\theta_i \quad \text{—(1)}$$

Where $\theta = \frac{2\pi i}{M}$,



$\theta_1(t) = \sqrt{2/T}\cos 2\pi f_c t$, $\quad \theta_2(t) = \sqrt{2/T}\sin 2\pi f_c t$

* Probability of error:

$P_e = erfc\left[\sqrt{E/N_0}\sin\left(\frac{\pi}{m}\right)\right]$ → M-ary PSK

For m-ary DPSK.

$P_e = erfc\left[\sqrt{2E/N_0}\sin(\pi/2m)\right]$

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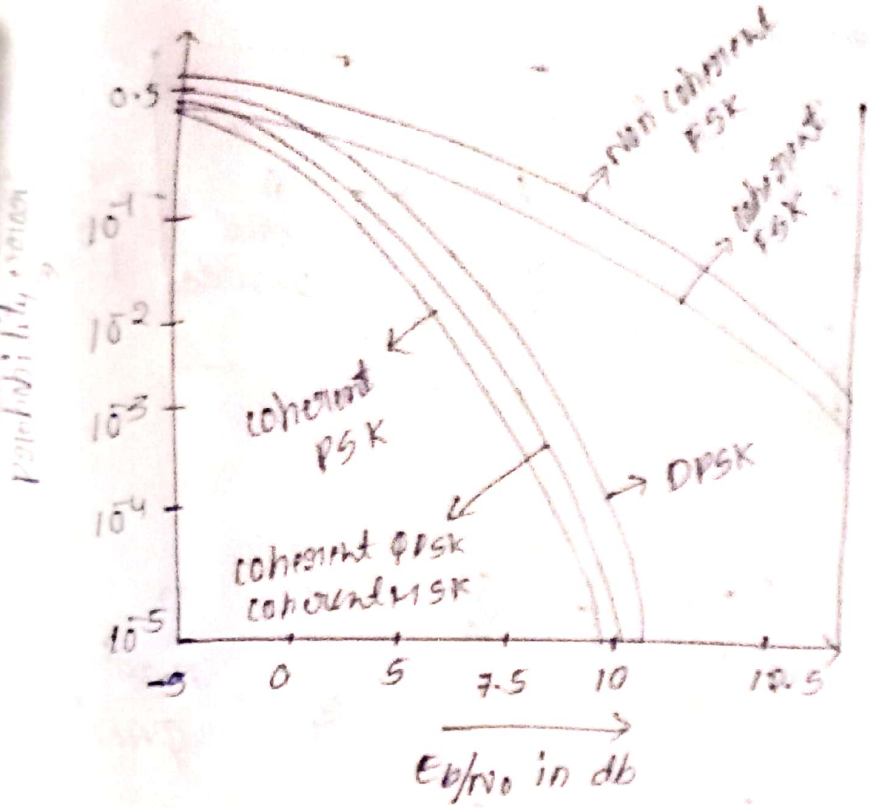


→ M-ary modulation technique:

→ types

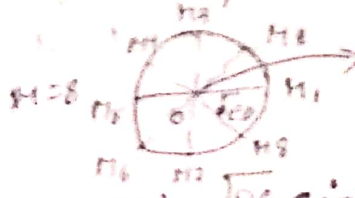
① M-ary PSK.
② M-ary ~~QAM~~ QAM.
③ M-ary FSK.

→ $S_i(t) = \sqrt{2E/T}\cos[2\pi f_c t + \theta_i]$

$M \to \log \leftarrow \theta_i \qquad i = 0, 1, 2, \ldots M-1$

$$S_i(t) = \sqrt{\frac{2E}{T}}\cos 2\pi f_c t \cdot \cos\theta_i - \sqrt{\frac{2E}{T}}\sin 2\pi f_c t \cdot \sin\theta_i \quad \text{—— ①}$$

where $\theta = \frac{2\pi i}{M}$,



$\theta_1(t) = \sqrt{2/T}\cos 2\pi f_c t$, $\quad \theta_2(t) = \sqrt{2/T}\sin 2\pi f_c t$

* Probability of error:

$P_e = erfc\left[\sqrt{E/N_0}\sin\left(\frac{\pi}{m}\right)\right]$ → M-ary PSK

For m-ary DPSK.

$P_e = erfc\left[\sqrt{2E/N_0}\sin(\pi/2m)\right]$

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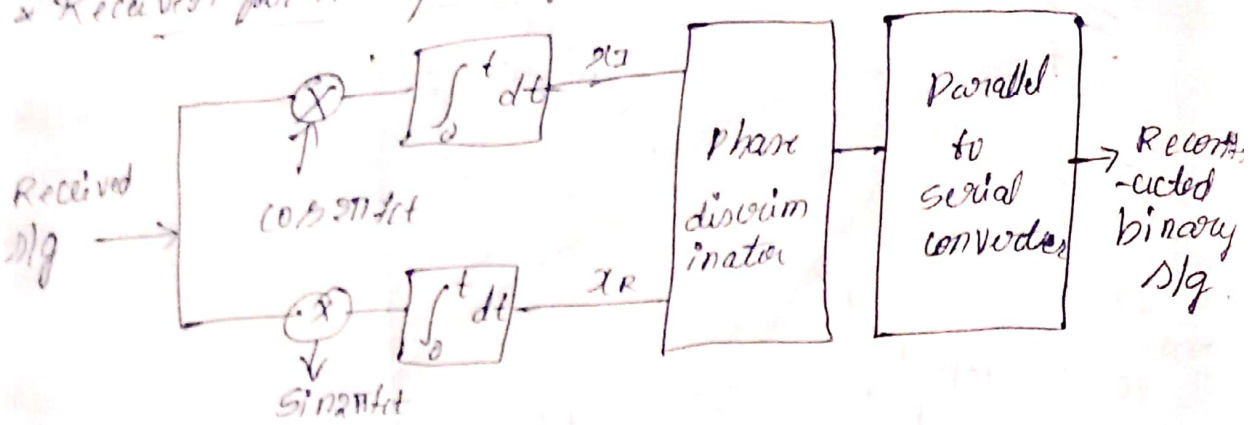
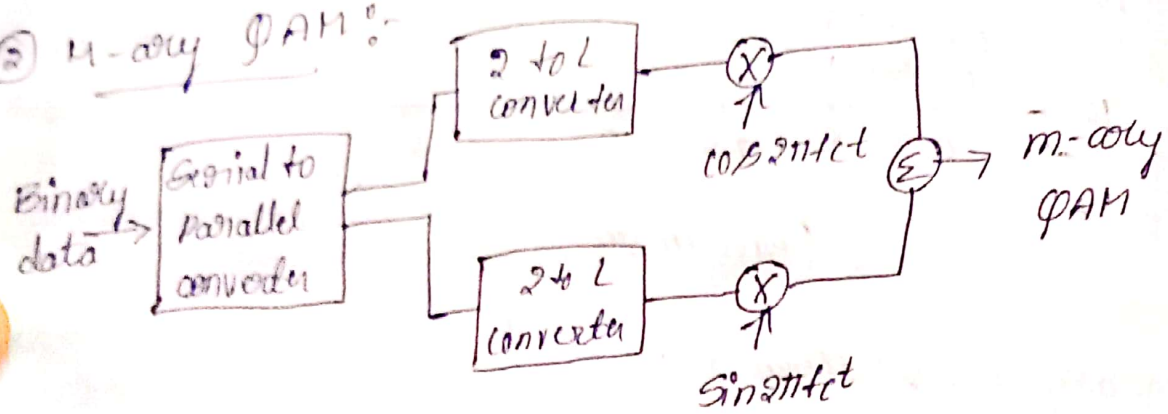
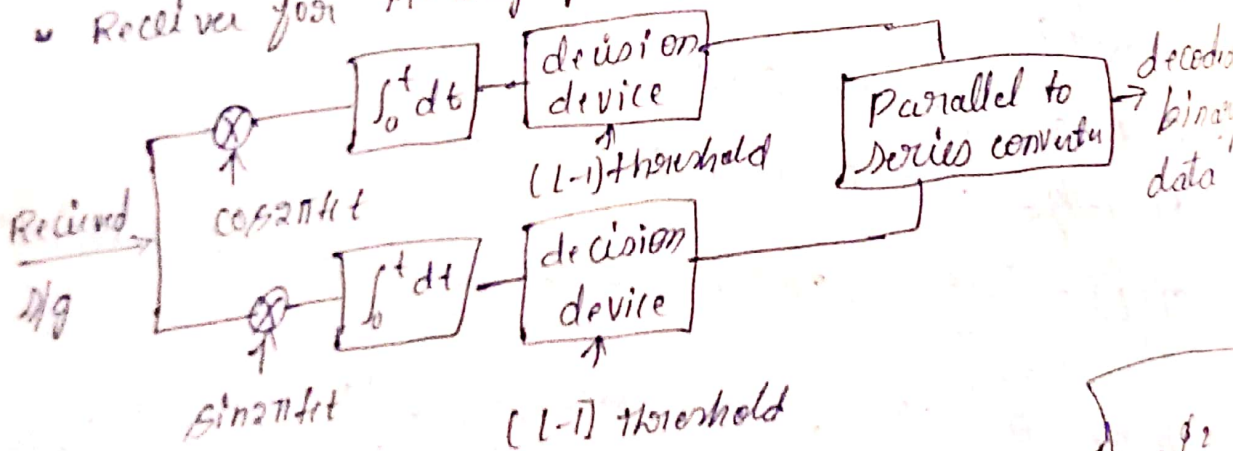
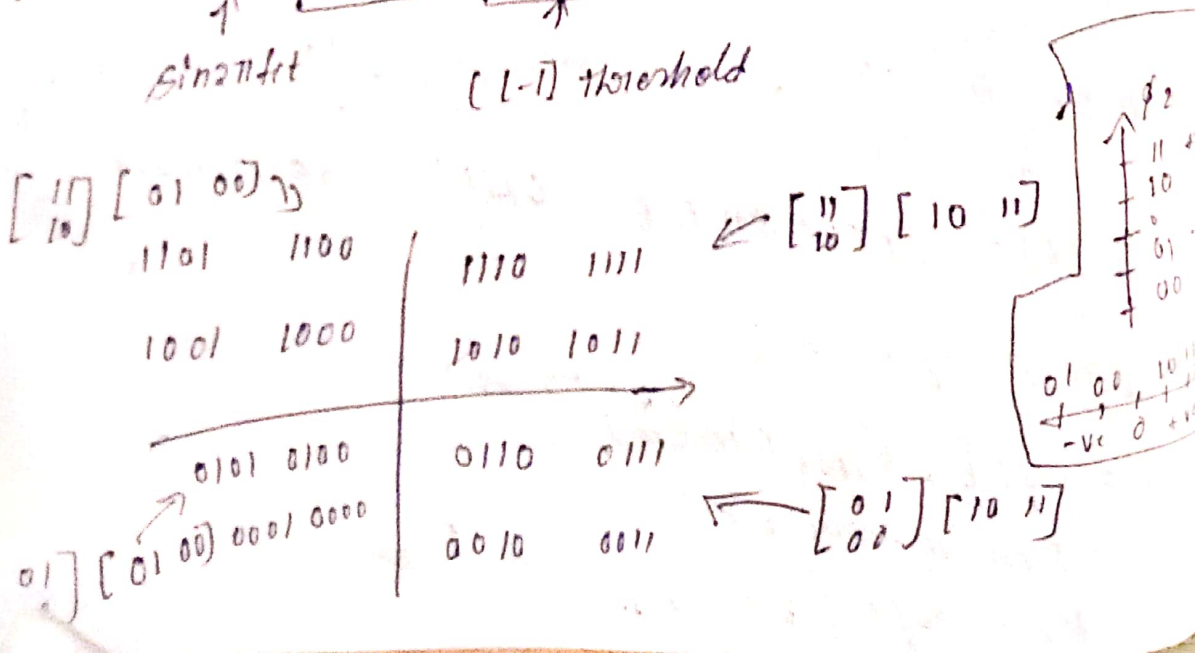
* Receiver for M-ary PSK :



② M-ary QAM :-



i.e. Transmitter.

* Receiver for M-ary QAM :-