Module I

Digital Modulation Techniques

Digital modulation formats, coherent binary modulation techniques, coherent quadrature-modulation techniques, Non coherent binary modulation techniques, Comparison of binary and quaternary modulation techniques, M-ary modulation, Power spectra, Bandwidth efficiency, M-ary modulation formats viewed in the light of the channel capacity theorem, Effect of intersymbol interference, Bit versus symbol error probabilities, Synchronization, Applications.

Digital formats

* Modulation is defined as the process by which some characteristic of a carrier is varied in accordance with a modulating wave.

* In digital communications, the digital data is usually in the form of binary. It can also be a block of n binary bits encoded into $M = 2^n$ possible discrete amplitudes. Such schemes are generally called as M-ary signalling.

* With a sinusoidal carrier, the feature that is used by the modulator to distinguish one signal from another is a step change in the amplitude, frequency or phase of the carrier. The result of this modulation process is amplitude-shift keying [ASK], frequency-shift keying [FSK], or phase-shift keying [PSK] respectively as shown in fig 1.

* Sometimes combination of both amplitude and phase of the carrier also changed to produce amplitude-phase shift keying [APK]. Such techniques are called as hybrid techniques.

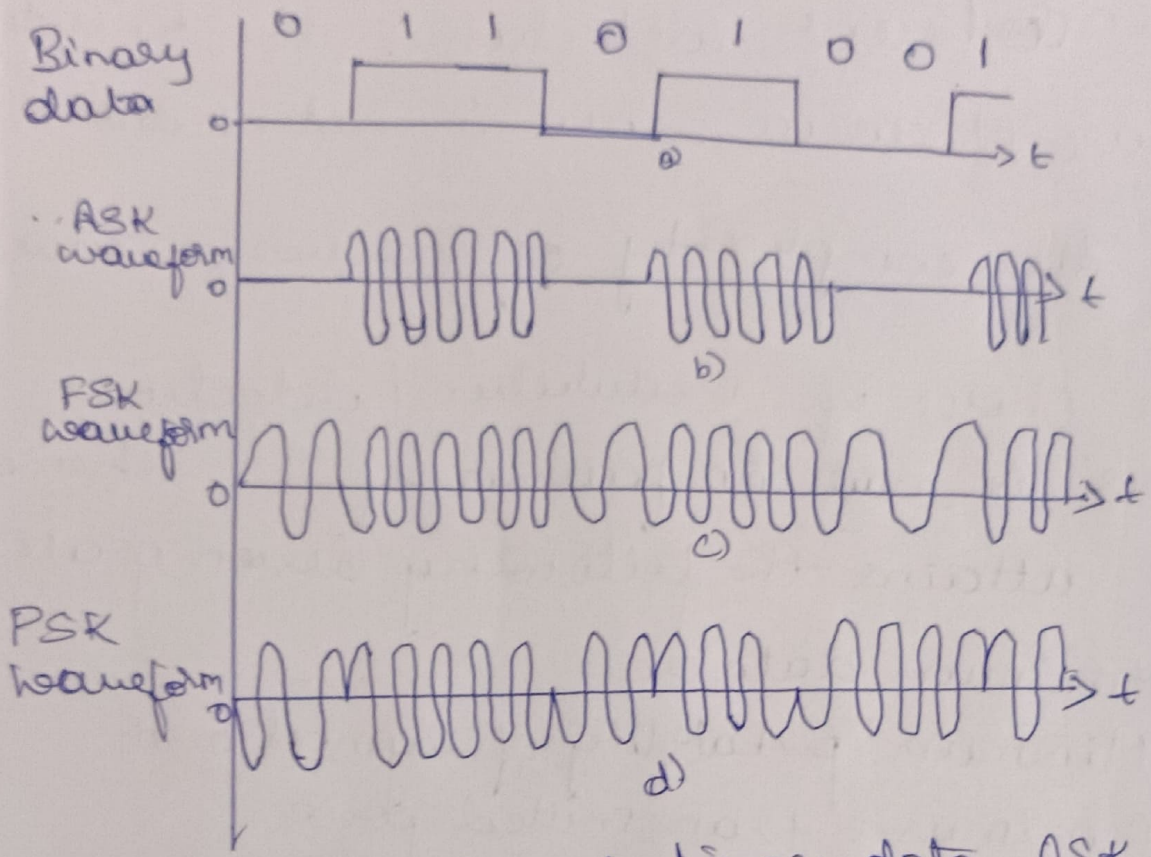


fig 1: waveform of binary data, ASK, FSK & PSK

* Demodulation is the inverse process of recovering the original binary waveform from the received modulated signal.

* Demodulation can be done using coherent or non coherent detection.

In the case of coherent detection, the receiver should have the exact knowledge of the carrier waves phase reference. The receiver is then said to be phase locked to the transmitter. In coherent detection the received signal is crosscorrelated with each one of the reference signals and the decision is based on the correlator output.

In non-coherent detection, the knowledge of phase of carrier wave is not needed.
Thus the complexity of receiver is reduced.

* The choice of modulation / detection schemes is made in favour of the scheme that attains the following design goals.

1. Maximum data rate
2. Minimum probability of symbol error.
3. Minimum transmitted power.
4. Minimum channel bandwidth
5. Maximum resistance to interfering signals.
6. Minimum circuit complexity.

Coherent Binary Amplitudue Shift Keying [ASK] or On - off Keying.

* The time domain expression for ASK is

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad , \quad 0 \le t \le T_b \quad \text{—(1)}$$

* In coherent binary ASK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbol 1 and 0 respectively.

ie
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t] \quad 0 \le t \le T_b \quad \text{—(2)}$$
$$s_2(t) = 0 \quad 0 \le t \le T_b$$

where E_b is the transmitted signal energy bit

* There is a single basis function $\phi_1(t)$ as given by
$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b \quad \text{—(3)}$$

$$\therefore \quad s_1(t) = \sqrt{E_b}\, \phi_1(t) \quad 0 \le t \le T_b \quad \text{—(4)}$$
$$s_2(t) = 0 \quad 0 \le t \le T_b$$

* The block diagram of ASK is shown in figure

* When the input binary symbol is 0, the product modulator o/p is $s_2(t)$ ie 0. If input

binary symbol is 1. The product modulator o/p is $S_1(t) = 1$.

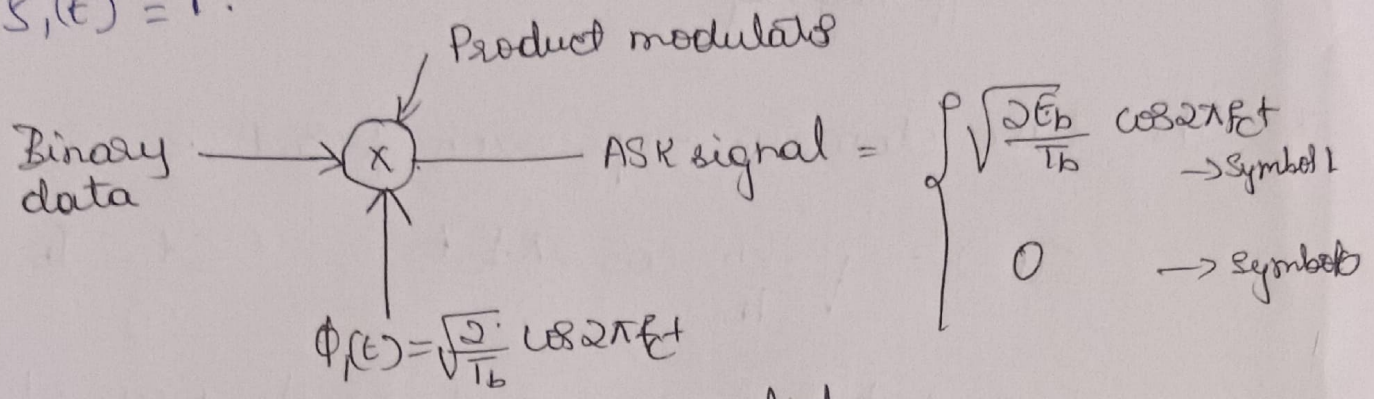


figure : ASK modulator.

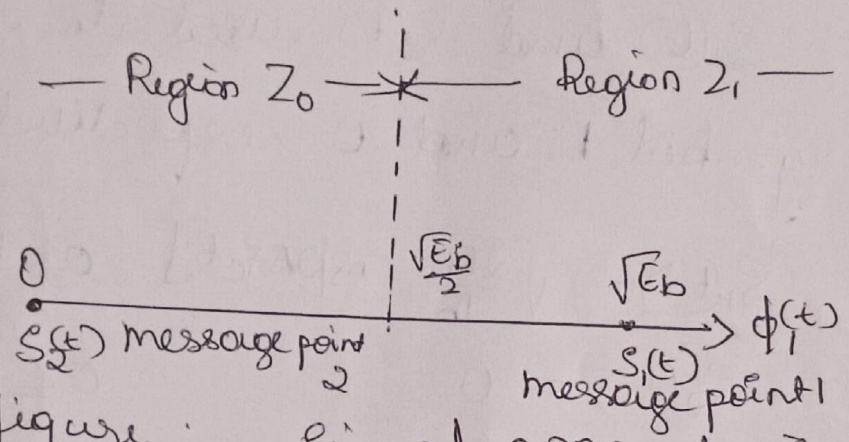


figure : Signal space diagram.

* figure shows signal space for the coherent binary ASK with one dimension [ie N=1].

* The distance between message points is $\sqrt{E_b}$ of symbol 0 and 1

* Decision region Z_0 corresponding to symbol 0 is a set of all points nearer to 0 and hence exists from $-\infty$ to $\frac{\sqrt{E_b}}{2}$ and the decision region Z_1 corresponding to symbol 1 is a set of all points nearer to $\sqrt{E_b}$ and hence exists from $\sqrt{E_b}/2$ to ∞

* coherent detection as shown in figure

The received signal $x(t)$ is cross correlated with local reference signal $\phi_1(t)$. J

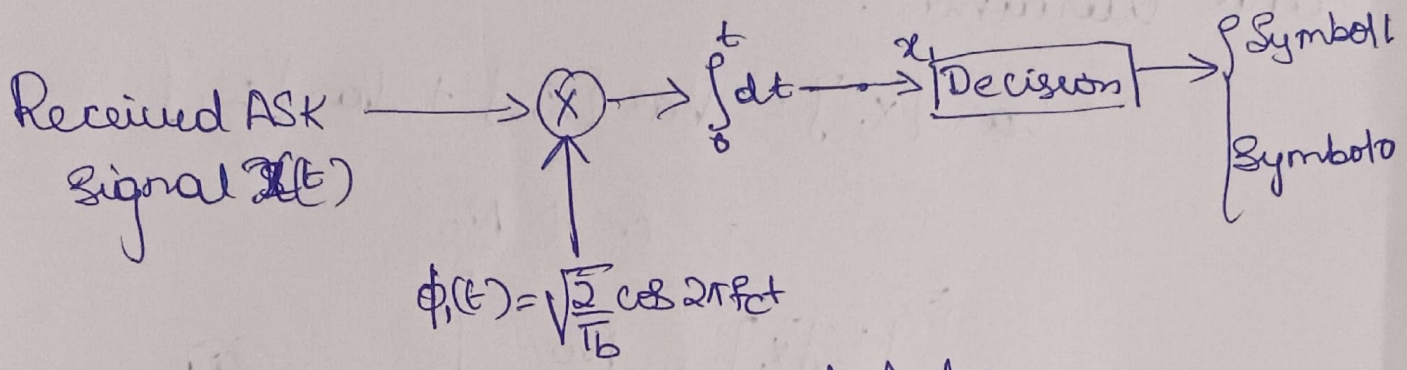


fig : ASK demodulator

Probability of Error calculation :

* Let $x(t)$ is received signal is given by

$$x(t) = s(t) + w(t) \quad \text{—— (5)}$$

where $w(t)$ is white Gaussian noise with zero mean and power spectral density $N_0/2$.

* $$x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt \quad \text{—— (6)}$$

when symbol 1 is transmitted,

$$x_1 = s_1 + w_1 \quad \text{—— (7)}$$

where $s_1 = \sqrt{E_b}$, is the signal component and w_1 is the noise component.

w_1 is a sample value of random variable W_1 having gaussian distribution with zero mean and variance $N_0/2$. x_1 is sample value of Gaussian random variable X_1

The Expected value or Mean of X_1 is given by

$$E[x_1] = E[S_1] + E[W_1]$$

$$= \sqrt{E_b} + 0$$

$$E[x_1] = \sqrt{E_b} \quad \text{——— (8)}$$

and variance.

$$Var[x_1] = Var[S_1] + Var[W_1]$$

$$= 0 + \frac{N_0}{2}$$

$$\boxed{Var[x_1] = \frac{N_0}{2}} \quad \text{——— (9)}$$

* The conditional PDF of x_1 given that symbol 1 is transmitted is given by

$$f_{x_1}(x_1|1) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(x_1 - \sqrt{E_b})^2}{N_0}\right]$$

Similarly

for $$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{x_1^2}{N_0}\right]$$

* If symbol 1 is transmitted and x_1 falls in the region Z_0, symbol error occurs when $x_1 < \frac{\sqrt{E_b}}{2}$ and decision is made in favour of symbol 0.

$\therefore$ $P_e(1)$ probability of error when symbol 1 is transmitted is given by

$$P_e(1) = P\left[x_1 < \frac{\sqrt{E_b}}{2}\right]$$

$$P_e(1) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\frac{\sqrt{E_b}}{2}} f_{x_1}(x_1|1)\, dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\frac{\sqrt{E_b}}{2}} \exp\left[-\frac{(x_1-\sqrt{E_b})^2}{N_0}\right] dx_1$$

let $z = \frac{x_1 - \sqrt{E_b}}{\sqrt{N_0}}$

$x_1 = -\infty$ $\qquad$ $x_1 = \frac{\sqrt{E_b}}{2}$

$dz = \frac{dx_1}{\sqrt{N_0}}$ $\qquad$ $z = -\infty$ $\qquad$ $z = -\frac{1}{2}\sqrt{\frac{E_b}{N_0}}$

$\Rightarrow dx_1 = dz\sqrt{N_0}$

$$\therefore P_e(1) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{-\frac{1}{2}\sqrt{\frac{E_b}{N_0}}} \exp[-z^2]\, dz$$

$$P_e(1) = \frac{1}{\sqrt{\pi}} \int_{\frac{1}{2}\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp[-z^2]\, dz$$

$$P_e(1) = \frac{1}{\sqrt{\pi}} \frac{2}{2} \int_{\frac{1}{2}\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp[-z^2]\, dz$$

$$P_e(1) = \frac{1}{2} \operatorname{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]$$

* Similarly

$$P_e(0) = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]$$

* The average probability error P_e is given by

$$\cancel{P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]}$$

$$P_e = P(1)\,P_e(1) + P(0)\,P_e(0)$$

$$P(0) = P(1) = 1/2$$

$$P_e = \frac{1}{2}\,\frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right] + \frac{1}{2}\,\frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right]}$$

Coherent Binary PSK

* In a coherent binary PSK system, the pair of signals, $S_1(t)$ and $S_2(t)$, used to represent binary symbols 1 and 0, respectively, are defined by

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \ldots 0 \le t \le T_b \quad -(1)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) \quad \ldots 0 \le t \le T_b \quad -(2)$$

$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

where E_b is the transmitted signal energy per bit.

and carrier frequency $f_c = \frac{n_c}{T_b}$ n_c is fixed integer

* From eqn (1) and (2), there is only one basis function of unit energy is given by

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b \quad -(3)$$

* $S_1(t)$ & $S_2(t)$ in terms of $\phi_1(t)$ is given by

$$S_1(t) = \sqrt{E_b}\, \phi_1(t) \quad 0 \le t \le T_b \quad -(4)$$

$$S_2(t) = -\sqrt{E_b}\, \phi_1(t) \quad 0 \le t \le T_b \quad -(5)$$

* A coherent binary PSK system is characterized by having a signal space with one dimension ie $N=1$ and with two message points ie $M=2$

as shown in fig a

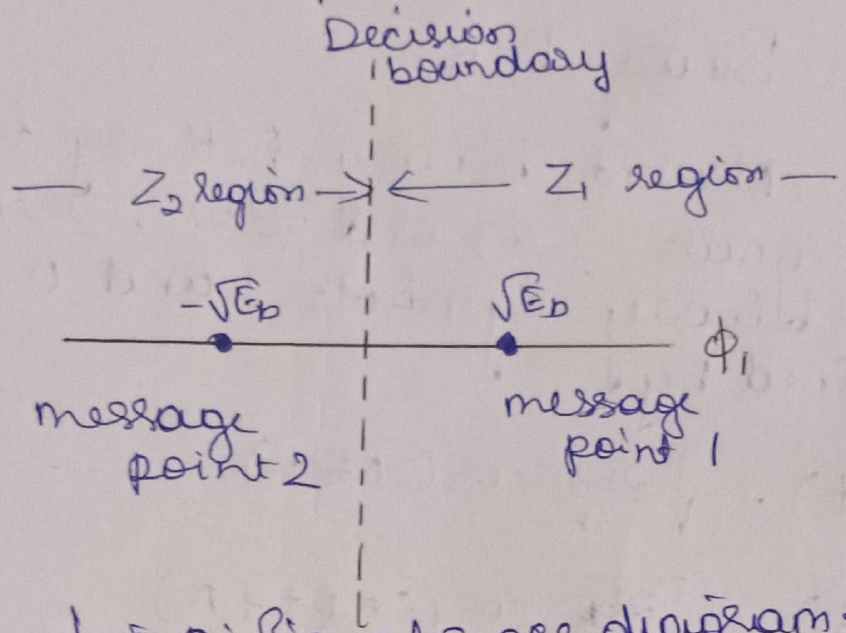


fig a: Signal space diagram for coherent binary PSK system.

* The coordinates of the message points are

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t)\,dt = +\sqrt{E_b} \quad \text{— (6)}$$

and

$$S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt = -\sqrt{E_b} \quad \text{— (7)}$$

Calculation of probability error

The decision region associated with symbol 1 or signal $S_1(t)$ is described by

$$Z_1 : 0 \le x_1 < \infty$$

where $$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt \quad \text{— (8)}$$

$x(t) \rightarrow$ received signal.

* The maximum likelihood function, when symbol 0 or $S_2(t)$ is transmitted is defined by

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - S_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \quad \text{--- (9)}$$

* The conditional probability of the receiver deciding in favour of symbol 1, given that symbol 0 was transmitted is given by

$$P_e(0) = \int_0^\infty f_{x_1}(x_1|0)\, dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \quad \text{--- (10)}$$

Putting $Z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$

$$dz = \frac{1}{\sqrt{N_0}}(dx_1 + 0) = \frac{dx_1}{\sqrt{N_0}}$$

$$dx_1 = dz\sqrt{N_0}$$

when $x_1 = 0$ $\qquad$ $x_1 = \infty$

$$Z = \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}} \qquad Z = \infty$$

$$Z = \frac{0 + \sqrt{E_b}}{\sqrt{N_0}} =$$

$$\boxed{Z = \sqrt{\frac{E_b}{N_0}}}$$

then eqn (10) can be rewrite as

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{E_b/N_0}}^{\infty} e^{-z^2}\, dz \sqrt{N_0}$$

~~Coherent Binary modulation techniques~~

$$P_e(0) = \frac{1}{\sqrt{\pi}} \times \frac{2}{2} \int_{\sqrt{E_b/N_0}}^{\infty} e^{-z^2} \cdot dz$$

$$P_e(0) = \frac{1}{2} \left[\frac{2}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} e^{-z^2} dz \right]$$

We have

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2} du$$

erfc - complementary error function

$$\boxed{\therefore P_e(0) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]} \quad \text{—— (11)}$$

* Similarly for $P_e(1)$, the conditional probability of receiver deciding in favor of symbol 0, given that symbol 1 was transmitted is given by

$$\therefore \boxed{P_e(1) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]} \quad \text{—— (12)}$$

The average probability of error for coherent binary PSK is $P_e = P(0) P_e(0) + P(1) P_e(1)$

$$P(0) \,\&\, P(1) = 1/2$$

$$\therefore P_e = \frac{1}{2} \cdot \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}} + \frac{1}{2} \cdot \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}} \quad \text{—— (13)}$$

Generation of binary PSK wave

* The binary wave and a sinusoidal carrier wave $\phi_1(t)$ are applied to a product modulator as shown in fig b. The desired PSK wave is obtained at the modulator output

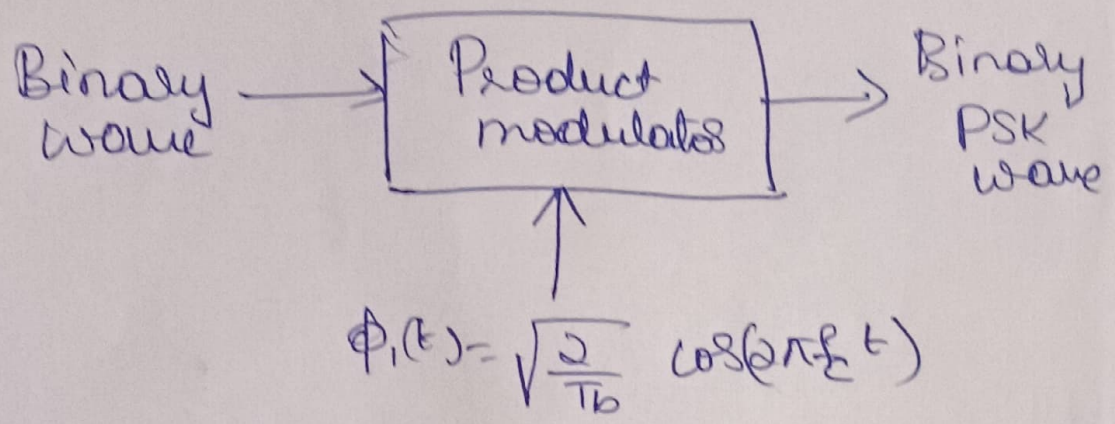


fig b: Block diagram of binary PSK transmitter.

Detection of original binary sequence

* To detect the original binary sequence of 1's and 0's, the noisy PSK wave $x(t)$ to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$ as shown in fig c.

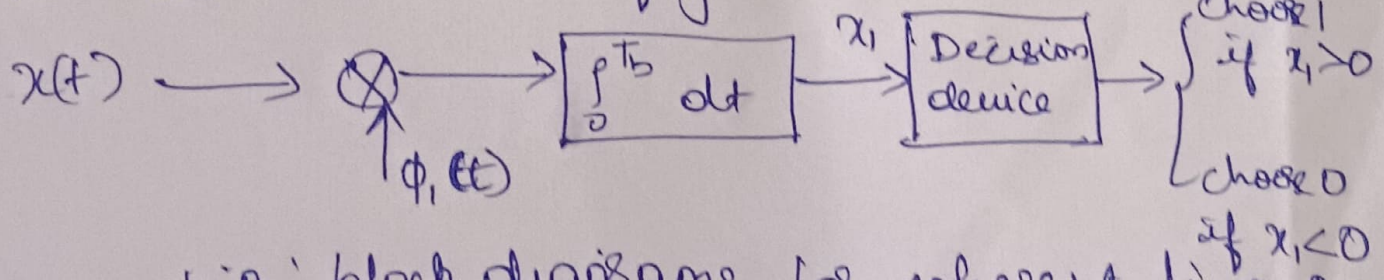


fig: block diagrams for coherent binary PSK receiver.

* If $r_1 > 0$, the receiver decides in favor of symbol 1.

If $r_1 < 0$ it decides in favor of symbol 0

Coherent Binary FSK [Frequency shift keying]

* In a binary FSK system, symbol 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount, which is given by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \quad -(1)$$

where $i = 1, 2$ and E_b is the transmitted signal energy per bit and transmitted frequency equals

$$f_i = \frac{n_c + i}{T_b} \quad -(2)$$

n_c is fixed integer, $i = 1, 2$

Thus symbol 1 is represented by $s_1(t)$ & symbol 0 is represented by $s_2(t)$

* The signal $s_1(t)$ and $s_2(t)$ are orthogonal and the orthonormal basis function is given by

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \quad -(3)$$

where $i = 1, 2$

* Correspondingly, the coefficient s_{ij} for $i = 1, 2$ and $j = 1, 2$ is defined as.

$$s_{ij} = \int_0^{T_b} s_i(t)\,\phi_j(t)\,dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\,dt$$

$$= \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \neq j \end{cases} \quad \text{—— (4)}$$

* Thus a coherent binary FSK system is characterized by having a signal space that is two dimensional ie $N=2$ with two message points ie $M=2$ as shown in fig a

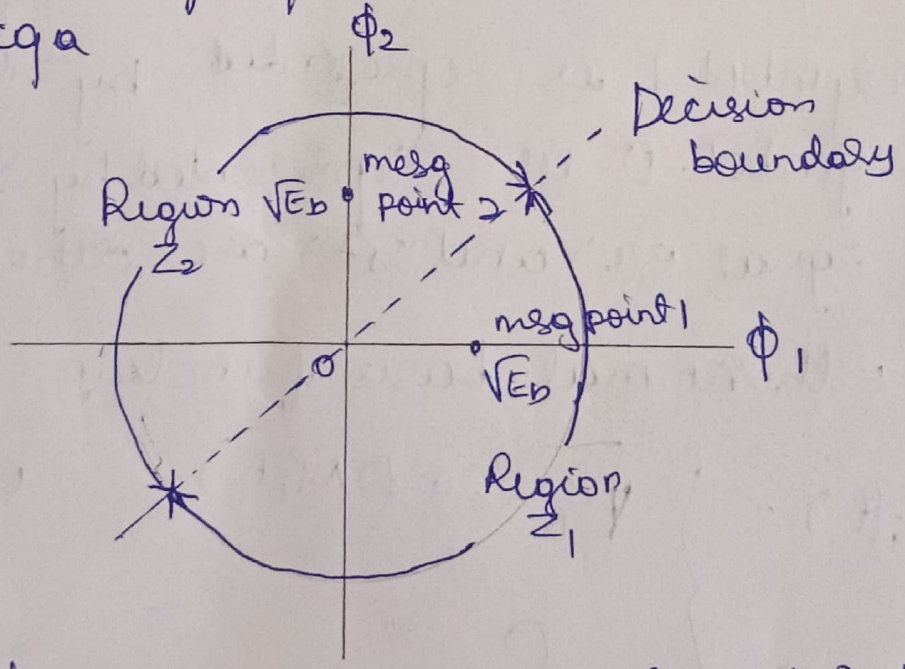


fig a: Signal space diagram for coherent binary FSK system.

* The two message points are defined by the signal vector is given by

$$s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad \text{—— (5)}$$

$$s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \quad \text{—— (6)}$$

* The distance between the two message points is equal to $\sqrt{2E_b}$.

* The observation vector x has two elements, x_1 and x_2 are defined by

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt \quad \text{—— (7)}$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt \quad \text{—— (8)}$$

where $x(t)$ is the received signal,

* If symbol 1 is transmitted

$$x(t) = s_1(t) + w(t) \quad \text{—— (9)}$$

where $w(t)$ is sample function of a white Gaussian noise process of zero mean & power spectral density $N_0/2$.

If symbol 0 is transmitted

$$x(t) = s_2(t) + w(t) \quad \text{—— (10)}$$

~~The mean of eqn (8) is given by~~
~~$E[x_2] = 0$~~

* Let L be the new Gaussian random variable L whose sample value l is equal to the difference between x_1 and x_2 thus

$$l = x_1 - x_2 \quad \text{——} \quad (11)$$

* The mean value of the random variable L depends on which binary symbol is transmitted

* If symbol 1 was transmitted, the Gaussian random variable X_1 and X_2 whose sample value are denoted by x_1 and x_2 has mean values ie $E[X_1] = \sqrt{E_b}$ & $E[X_2] = 0$ respectively.

Correspondingly, the conditional mean of the random variable L, given that symbol 1 was transmitted is given by

$$E[L|1] = E[X_1|1] - E[X_2|1]$$

$$= \sqrt{E_b} - 0$$

$$E[L|1] = \sqrt{E_b} \quad \text{——} \quad (12)$$

* If symbol 0 was transmitted, the random variables X_1 and X_2 have mean values ie $E[X_1] = 0$ & $E[X_2] = \sqrt{E_b}$ respectively,

Correspondingly, the conditional mean of the random variable L, given that symbol 0 was transmitted is given

$$E[L|0] = E[X_1|0] - E[X_2|0]$$

$$E[L|0] = 0 - \sqrt{E_b}$$

$$E[L|0] = -\sqrt{E_b} \quad \text{——— (13)}$$

* The variance of the random variable L is independent of which symbol was transmitted. Hence $Var[x_1]$ & $Var[x_2]$ is equal to $\frac{N_0}{2}$

$$Var[L] = Var[x_1] + Var[x_2]$$

$$= \frac{N_0}{2} + \frac{N_0}{2}$$

$$Var[L] = N_0 \quad \text{——— (14)}$$

* Suppose symbol 0 was transmitted, corresponding value of the conditional probability density function of the random variable L is given by

$$f_L[l|0] = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] \quad \text{——— (15)}$$

* The conditional probability of error, when symbol 0 was transmitted, the receiver making a decision in favor of symbol 1 is given by

$$P_e(0) = \int_0^\infty f_L(l|0)\, dl$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] dl \quad \text{——— (16)}$$

put $\frac{l + \sqrt{E_b}}{\sqrt{2N_0}} = z$

when $l = 0$ when $l = \infty$

$z = \frac{l + \sqrt{E_b}}{\sqrt{2N_0}}$ $z = \frac{l + \sqrt{E_b}}{\sqrt{2N_0}}$ $z = \infty$

$dz = \frac{dl + 0}{\sqrt{2N_0}}$ $z = \sqrt{\frac{E_b}{2N_0}}$

$dl = dz \cdot \sqrt{2N_0}$

then $$P_e(0) = \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \cdot e^{-z^2} \, dz \cdot \sqrt{2N_0}$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} \cdot dz$$

$$= \frac{1}{\sqrt{\pi}} \times \frac{2}{2} \int_{\sqrt{E_b/2N_0}}^{\infty} e^{-z^2} \, dz$$

Complementary error function is

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-u^2) \, du$$

then $$P_e(0) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right] \quad \text{— (17)}$$

* Similarly $P_e(1)$, the conditional probability of error, given that symbol 1 was transmitted and decision is taken in favour of

Symbol 0

$$P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right] \quad \text{—— (18)}$$

$$P_e(0) = P_e(1) = \frac{1}{2}$$

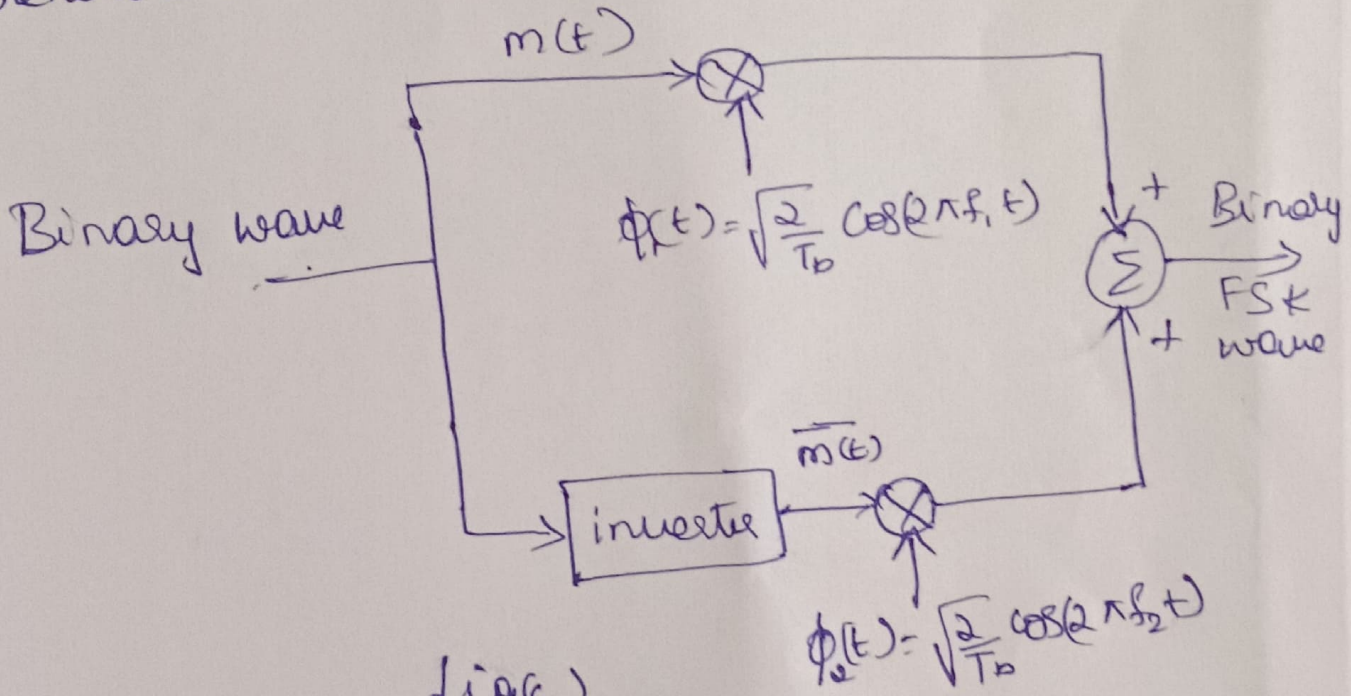
The average probability of error is

$$P_e = P(0)\,P_e(0) + P(1)\,P_e(1)$$

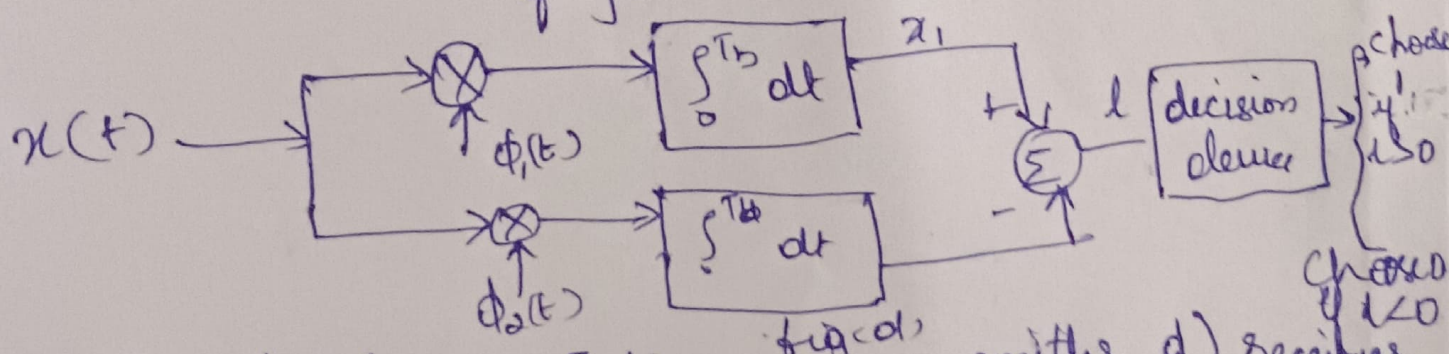
$$= \frac{1}{2}\left[\frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]\right] + \frac{1}{2}\,\frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right]$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}} \quad \text{—— (19)}$$

* Generation & detection of binary FSK



fig(c)



fig(d)

fig: Block diagram for c) binary FSK transmitter d) receiver

Non Coherent binary Modulation Techniques

a. Non coherent Orthogonal modulation

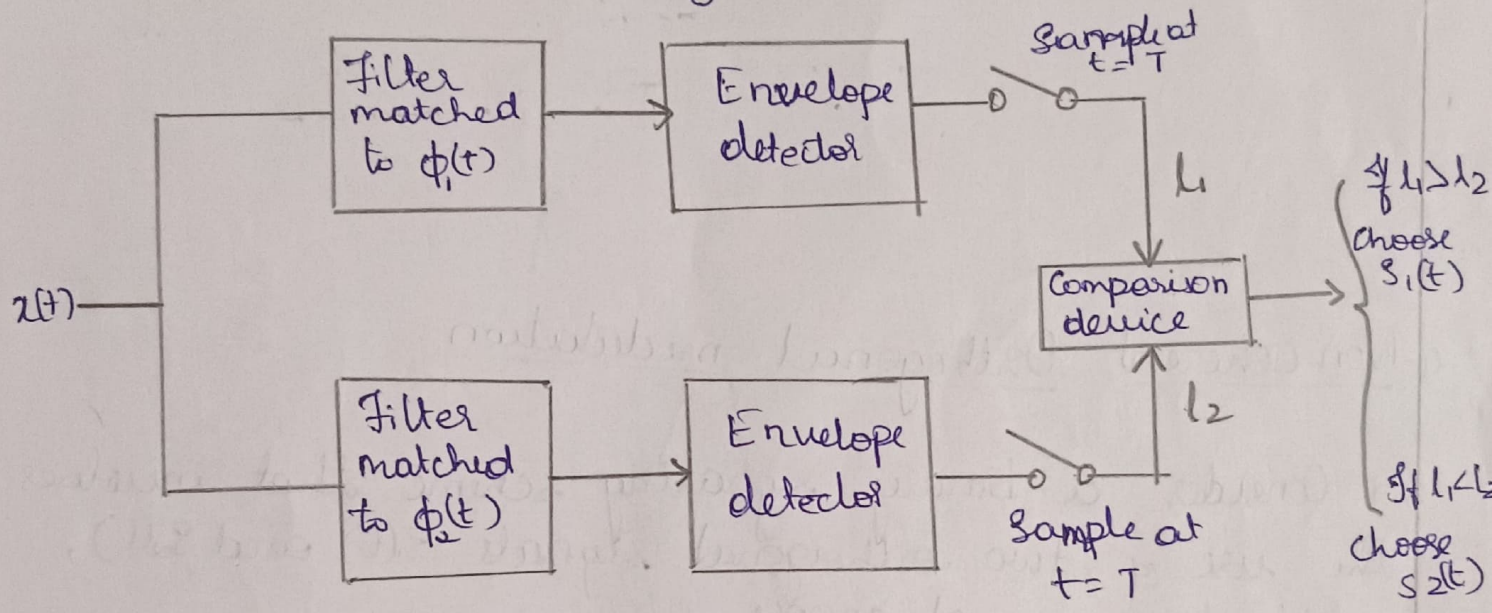
* Consider a binary signaling scheme that involves the use of two orthogonal signals $S_1(t)$ and $S_2(t)$, which have equal energy.
* During the interval $0 \leq t \leq T$, one of these two signals is sent over an imperfect channel that shifts the carrier phase by an unknown amount
* Let $g_1(t)$ and $g_2(t)$ denote the phase shifted versions of $S_1(t)$ and $S_2(t)$ respectively.
* It is assumed that signals $g_1(t)$ and $g_2(t)$ remain orthogonal and of equal energy, such a signaling scheme is reffered as noncoherent orthogonal modulation

* Channel introduces an additive white Gaussian noise $w(t)$ of zero mean and power spectral density $N_0/2$

* Received signal $x(t)$ is

$$x(t) = \begin{cases} g_1(t) + w(t) & 0 \le t \le T \\ g_2(t) + w(t) & 0 \le t \le T \end{cases} \quad \text{—(1)}$$

The requirement is to use $x(t)$ to discriminate between $s_1(t)$ and $s_2(t)$ regardless of the carrier phase



fig(a): Generalized binary receiver for non coherent orthogonal modulation.

* Fig a has an output amplitude l_1 greater than the output amplitude l_2 of the lower path, the receiver makes a decision in favour of $s_1(t)$.
If the l_2 is greater than l_1, the receiver makes a decision in favor of $s_2(t)$.
If the amplitude of l_1 and l_2 are equal, the decision is either $s_1(t)$ and $s_2(t)$.

* In any event, a decision error occurs when the matched filter that rejects signal component of the received signal $x(t)$ has a larger output amplitude

than the matched filter that passes it.

*

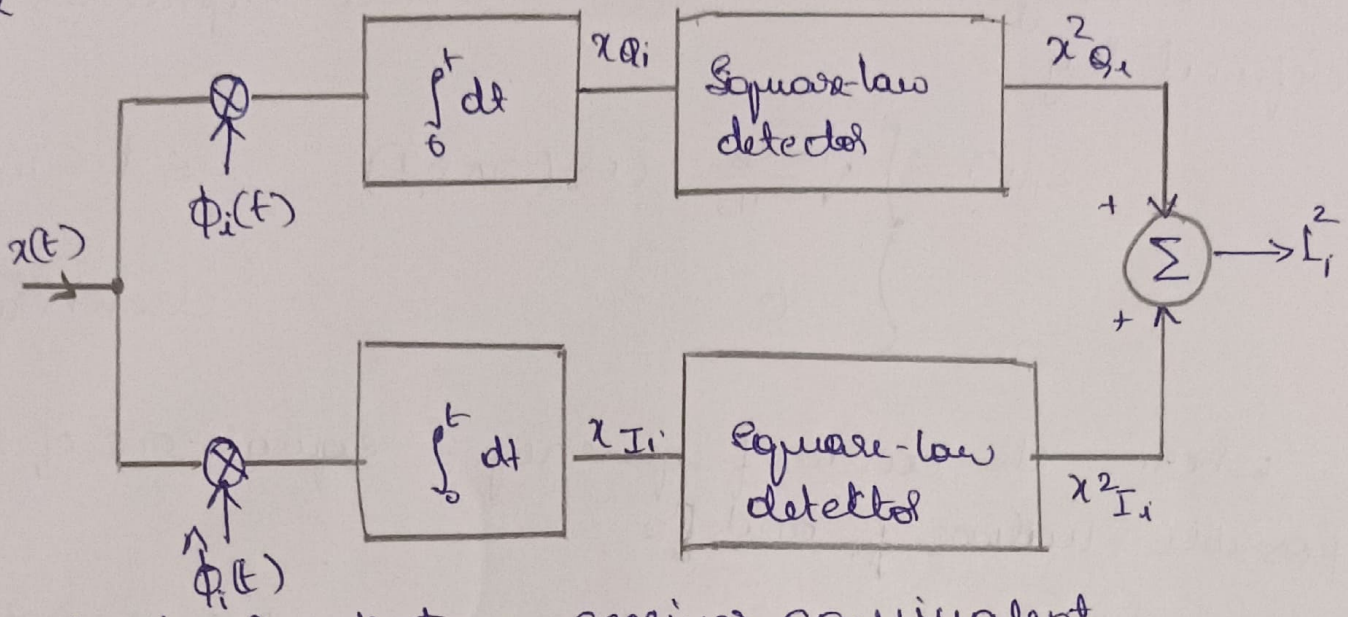


fig b. Quadrature receiver equivalent

* fig b is quadrature receiver. In the upper path called the in-phase path, the received signal $x(t)$ is correlated against the basis function $\phi_i(t)$, representing a scaled version of the transmitted signal $s_1(t)$ or $s_2(t)$ with zero carrier phase.

* In the lower path, called the quadrature path, on the other hand, $x(t)$ is correlated against another basis function $\hat{\phi}_i(t)$ representing the version of $\phi_i(t)$, that results from shifting the carrier phase by $-90°$. Naturally, $\phi_i(t)$ and $\hat{\phi}_i(t)$ are orthogonal to each other.

* The average probability error for the non coherent receiver is given as

$$P_e = \frac{1}{2} \exp\left(-\frac{E}{2N_0}\right)$$

Noncoherent binary FSK.

* In binary FSK, the transmitted signal is defined by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \quad \text{—(1)}$$

where the carrier frequency f_i equals one of two possible values f_1 and f_2

* The transmission of frequency f_1 represents symbol 1 and the transmission of frequency f_2 represents symbol 0.

* For the non coherent detection of FSK frequency modulated wave, the receiver consists of a pair of matched filters followed by envelope detectors as shown in fig a

* The filter in the upper path of the receiver is matched to $\sqrt{2/T_b} \cos(2\pi f_1 t)$ and the filter in the lower path is matched to $\sqrt{2/T_b} \cos(2\pi f_2 t)$ and $0 \le t \le T_b$.

* The resulting envelope detector outputs are sampled at $t = T_b$ and their values are compared. The upper and lower path envelope samples are l_1 and l_2, as shown in fig a respectively.

If $l_1 > l_2$, the receiver decides in favour of symbol 1

and

If $l_1 < l_2$, the receiver decides in favour of symbol 0

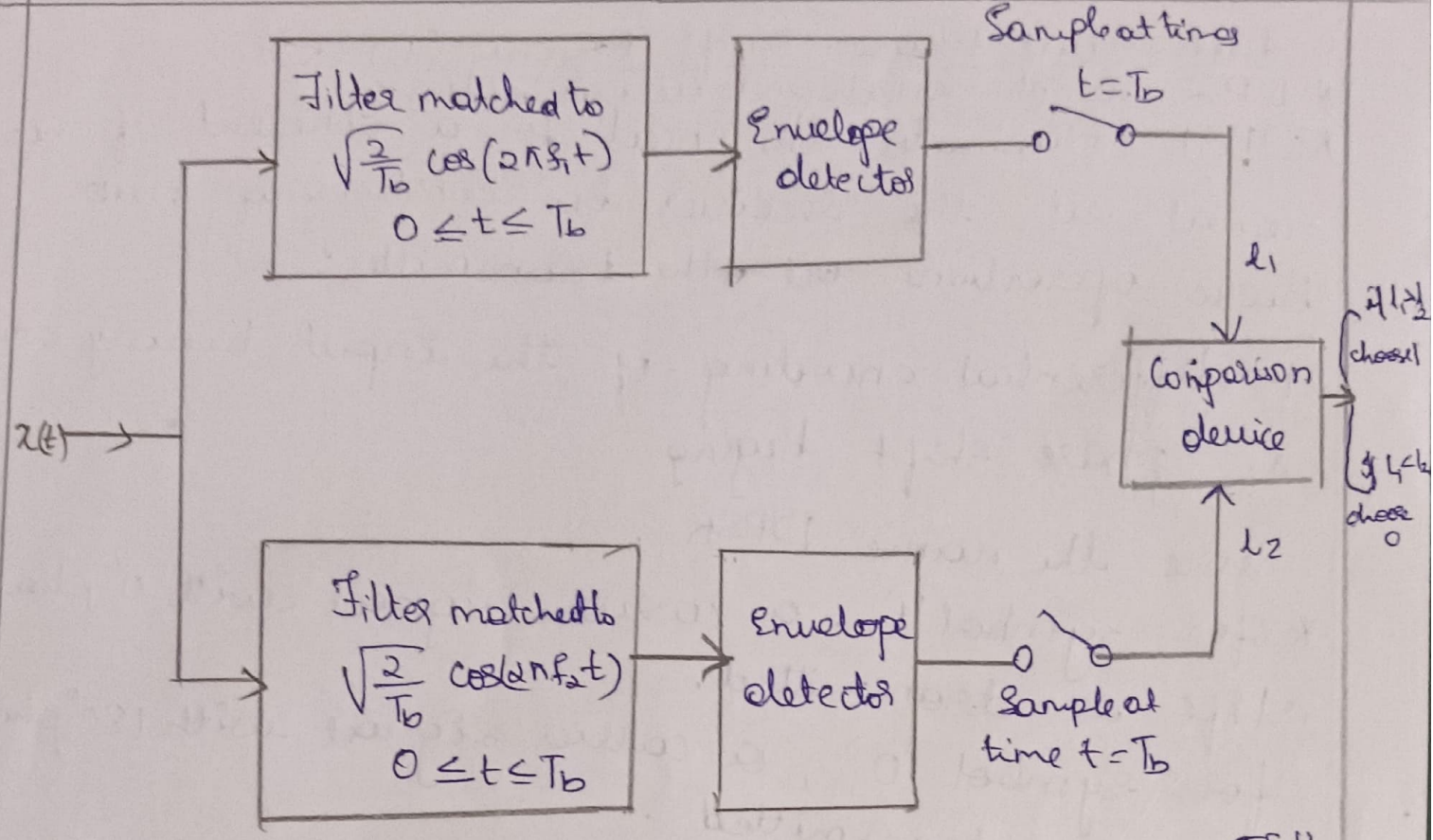


fig a: Noncoherent receiver for detection of binary FSK signals.

The noncoherent binary FSK is a special case of noncoherent orthogonal modulation with

$$T = T_b$$

and

$$E = E_b$$

where T_b is the bit duration and E_b is the signal energy per bit.

The average probability of error for non coherent binary FSK is given by

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

2. Differential Phase - Shift Keying (DPSK)

* DPSK is the noncoherent version of PSK.
* DPSK eliminates the need for a coherent refrence signal at the receiver by combining two basic operations at the transmitter:

1. differential encoding of the input binary wave
2. phase shift keying

hence the name DPSK.

* for symbol '1', a carrier signal with 0° phase shift is transmitted.
for symbol '0', a carrier signal with 180° phase shift is transmitted.

* In DPSK system, its transmitted signal span over two bit intervals.
* Let us assume the initial bit be '1' and transmitted over the interval $0 \leq t \leq T_b$.
* the phase of DPSK signal depends on what is transmitted over the next bit-interval during $T_b \leq t \leq 2T_b$.
* If symbol 1 appears over the second bit interval. ie $T_b \leq t \leq 2T_b$, phase of the carrier signal remains unchanged.
* If symbol 0 appears over the second bit interval, ie $T_b \leq t \leq 2T_b$, phase of the carrier signal is phase advanced by 180°.
* Suppose the transmitted DPSK signal, over the bit

bit interval $0 \le t \le T_b$, where T_b is the bit duration is $\sqrt{E_b/2T_b}\cos(2\pi f_c t)$

where E_b is signal energy per bit

* Let $S_1(t)$ denotes the transmitted DPSK signal for $0 \le t \le 2T_b$ for the case binary symbol 1 appearing at $T_b \le t \le 2T_b$

* The transmission of 1 leaves the carrier signal unchanged and $S_1(t)$ is given by

$$S_1(t) = \begin{cases} \sqrt{\frac{E}{2T_b}}\cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\frac{E}{2T_b}}\cos(2\pi f_c t) & T_b \le t \le 2T_b \end{cases} \quad -(1)$$

* Let $S_2(t)$ denotes the transmitted DPSK signal for $0 \le t \le 2T_b$ for the case when binary symbol 0 appearing at $T_b \le t \le 2T_b$.

* the transmission of 0 advances the carrier phase by $180°$ and $S_2(t)$ is given by

$$S_2(t) = \begin{cases} \sqrt{\frac{E}{2T_b}}\cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\frac{E}{2T_b}}\cos(2\pi f_c t + \pi) & T_b \le t \le 2T_b \end{cases} \quad -(2)$$

* from (1) & (2) the signals $S_1(t)$ and $S_2(t)$ are orthogonal over the two bit interval $0 \le t \le 2T_b$.

* DPSK is a special case of noncoherent orthogonal modulation with

$$T = 2T_b \quad \text{and} \quad E = 2E_b$$

Hence the average probability of error for DPSK is given by

$$\boxed{P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)}$$

{ Note: the average pobability of error for non coherent orthogonal modulation is given by

$$P_e = \frac{1}{2} \exp\left(-\frac{E}{2N_0}\right)$$

where $E = 2E_b$ for dPSK

by substituting it in above eqn

$$P_e = \frac{1}{2} \exp\left(-\frac{\cancel{2}E_b}{\cancel{2}N_0}\right)$$

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$$ }

* The differentially encoded sequence $\{d_k\}$ is generated using the logical expression

$$d_k = d_{k-1} b_k + \bar{d}_{k-1} \bar{b}_k \quad \text{modulo-2} \quad \text{—③}$$

where b_k is the input binary digit at time kT_b
d_{k-1} is the previous value of the differentially encoded digit.

* ~~Table illustrates the generation of DPSK signal~~

$\{b_k\}$	1	0	0	1	0	0	1	1
$\{\bar{b}_k\}$	0	1	1	0	1	1	0	0
$\{d_{k-1}\}$	1	1	0	1	1	0	1	1
$\{\bar{d}_{k-1}\}$	0	0	1	0	0	1	0	0
$\{b_k d_{k-1}\}$	1	0	0	1	0	0	1	1
$\{\bar{b}_k \bar{d}_{k-1}\}$	0	0	1	0	0	1	0	0
Differential encoded sequency $\{d_k\}$	1	1	0	1	1	0	1	1
Transmitted phase	0	0	π	0	0	π	0	0

* table 1 illustrates the logical operation involved in the eqn (3), assuming that reference bit added to the differentially encoded sequence $\{d_k\}$ is 1.

* the differentially encoded sequence $\{d_k\}$ thus generated is used to phase shift key a carrier with phase angles 0 & π radians.

*

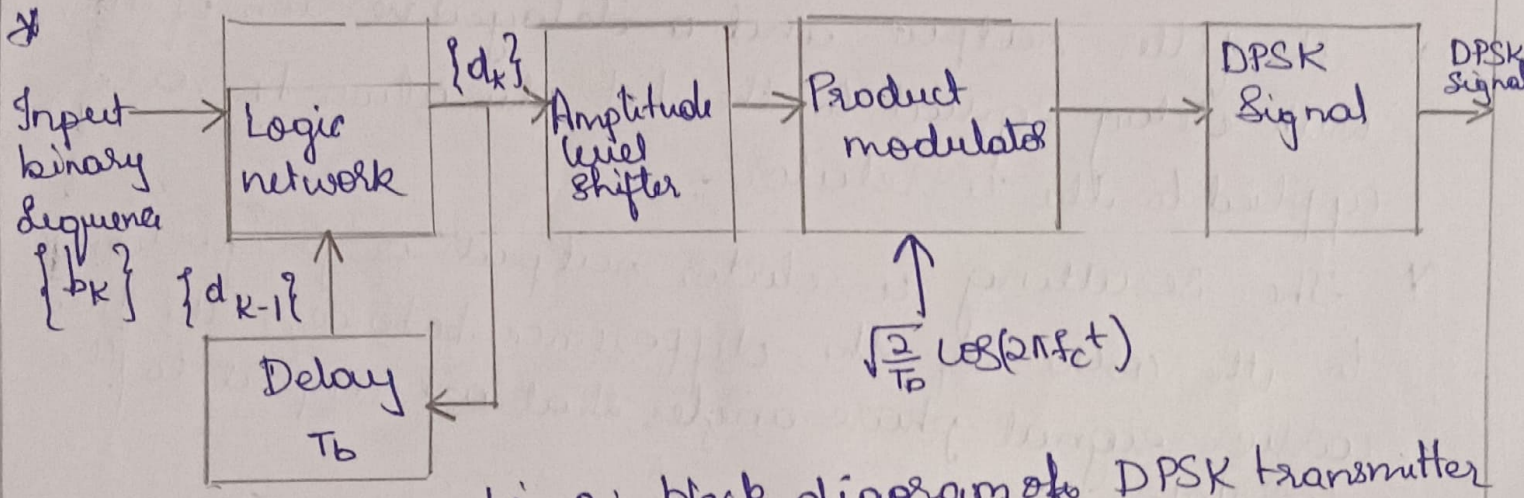


fig a: block diagram of DPSK transmitter

* the block diagram of a DPSK transmitter is shown in fig a: It consist of a logic network and a one bit delay element interconnected to convert an input binary sequence $\{b_k\}$ into a differentially encoded sequence $\{d_k\}$. This sequence is amplitude level shifted and then used to modulate a carrier wave of frequency f_c to produce desired dPSK signal.

*

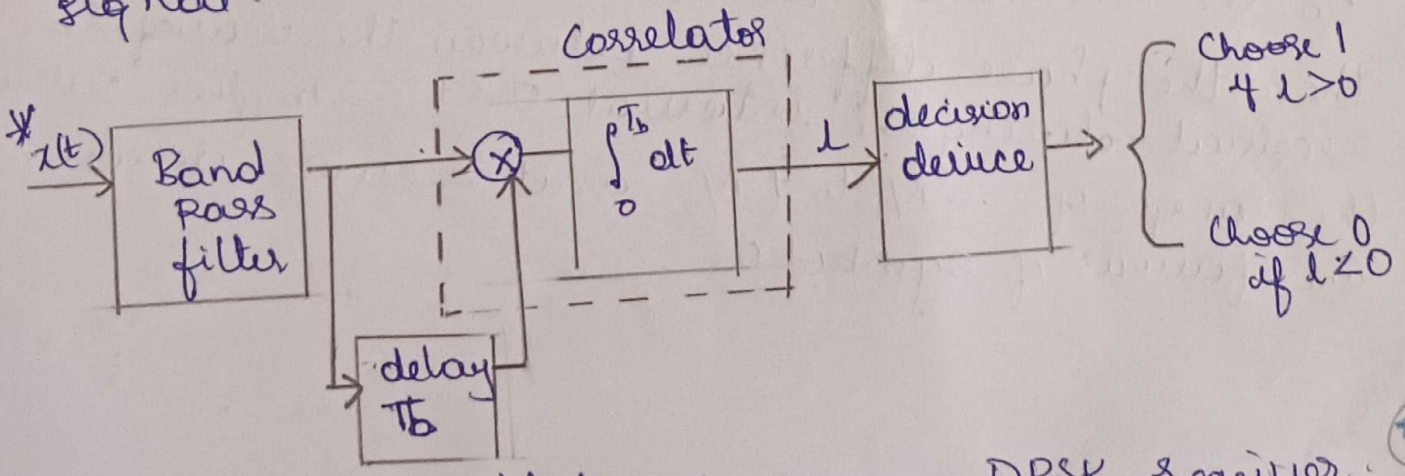


fig b: block diagram of DPSK receiver.

* The method of demodulating the DPSK signal is shown in fig b.

* At the receiver input, the received DPSK signal plus noise is passed through a band pass filter centered at the carrier frequency f_c. This operation is performed to limit the noise power.

* The filter output and a delayed version of it, with delay equal to the bit duration T_b are applied to the correlator.

* The resulting correlator output is proportional to the cosine of the difference between the two carrier signal phase angles that are input to the correlator.

* The correlator output is finally compared with a threshold of zero volts using a decision device

* If the correlator o/p difference is positive, then phase difference between the waveforms received during the interval $0 \le t \le 2T_b$ lies inside the range $-\pi/2$ to $+\pi/2$ and receiver decides in favour of symbol 1.

* On the other hand, the correlator output is negative, then the phase difference between the waveforms received during the interval $0 \le t \le 2T_b$ lies outside the range of $-\pi/2$ to $+\pi/2$ and receiver decides in favour of symbol 0.

Coherent binary FSK. Determine the minimum channel bandwidth required.

→ For binary FSK scheme

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

For $P_e = 10^{-4}$

$$10^{-4} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\operatorname{erfc} \sqrt{\frac{E_b}{2N_0}} = 2 \times 10^{-4}$$

$$\operatorname{erfc}(u) = 1 - \operatorname{erf}(u) \Rightarrow \operatorname{erf}(u) = 1 - \operatorname{erfc}(u)$$

$$u = \sqrt{\frac{E_b}{2N_0}}$$

$$\operatorname{erf}(u) = 1 - 2 \times 10^{-4}$$

$$\operatorname{erf}\left(\sqrt{\frac{E_b}{2N_0}}\right) = 0.9998$$

From error function table

$$u = 2.8 \text{ or } 2.9$$

$$\operatorname{erf}\left(\sqrt{\frac{E_b}{2N_0}}\right) = 0.9998$$

$$\frac{N_0}{2} = 10^{-10} \text{ W/Hz}$$

$$N_0 = 2 \times 10^{-10} \text{ W/Hz} \quad \therefore \sqrt{\frac{E_b}{2N_0}} = 2.8$$

$$\frac{E_b}{2N_0} = (2.8)^2$$

$$E_b = N_0 \times (2.8)^2$$

$$E_b = 2 \times 2 \times 10^{-10} \times (2.8)^2 = 29.2 \times 10^{-10} \text{ joule}$$

data rate is 10^6 bit/sec

the bit duration $T_b = 10^{-6}$ sec

The average transmitted power or average carrier power is

$$\frac{E_b}{T_b} = \frac{29.2 \times 10^{-10}}{10^{-6}} = 29.2 \times 10^{-4} \text{ watts}$$

Minimum channel bandwidth required

$$= \frac{1}{T_b}$$

$$= 10^6 \text{ Hz} = 1 \text{ MHz}.$$