You can use the internet to search for definitions. Please do not search for algorithms.

Question 1

Imagine a graph $G = (V, E)$. Furthermore, there is a mapping $L_E : E \to L$ from edges to a set of *labels* L. There is a designated initial vertex $v_0 \in V$, and a set of *final* vertices $V_f \subseteq V$.

Output the labels from the shortest path from v_0 to any $v_f \in V_f$. Please code this solution up.

Question 2

Imagine the same question as Question 1, but you are given multiple graphs $G_1, \ldots, G_n$. If $\mathcal{P}_i$ is the labels from all paths from initial to final for graph G_i, output the shortest labels in $\mathcal{P}_0 \cap \ldots \cap \mathcal{P}_n$. In other words, output the shortest labels that correspond to a valid path in every graph.

Hint: try to find a way to reduce this problem to Question 1. Please code this solution up.

Question 3

Imagine the same question as Question 1, but we also have a commutative semiring R where each edge is labelled by an element of the semiring $R_E : E \to R$. Find a path $e_1 \ldots e_n$ from v_0 to some $v_f \in V_f$ such that $e_1 \cdot \ldots \cdot e_n \neq 0$.

Question 4

Imagine the same as Question 3, but now we want to find a shared path, like we did in Question 2. However, we also want the product of *all* the semiring labels to not be zero. In other words, for each edge labelling, we will do a full product of all elements.