An Introduction to Legendrian Knots

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Contact Structures

Definition

A contact structure on $\mathbb{R}^3$ is a method of assigning a plane to every point, such that these planes satisfy certain technical conditions.

Throughout, we will only consider the standard contact structure, which twists along the y-axis:

Legendrian Knots

Definition

A Legendrian knot K is a smooth knot which is, at every point, parallel to the plane at that point given by the contact structure.

We can use coordinates: if K is the image of $t \mapsto (x(t), y(t), z(t))$, then K is Legendrian if for all t,

$$z'(t) - y(t)x'(t) = 0$$

Equivalences of Legendrian Knots

Two Legendrian knots are equivalent if there is a continuous family of Legendrian knots between them. This is similar to the definition of smooth knot equivalence.

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