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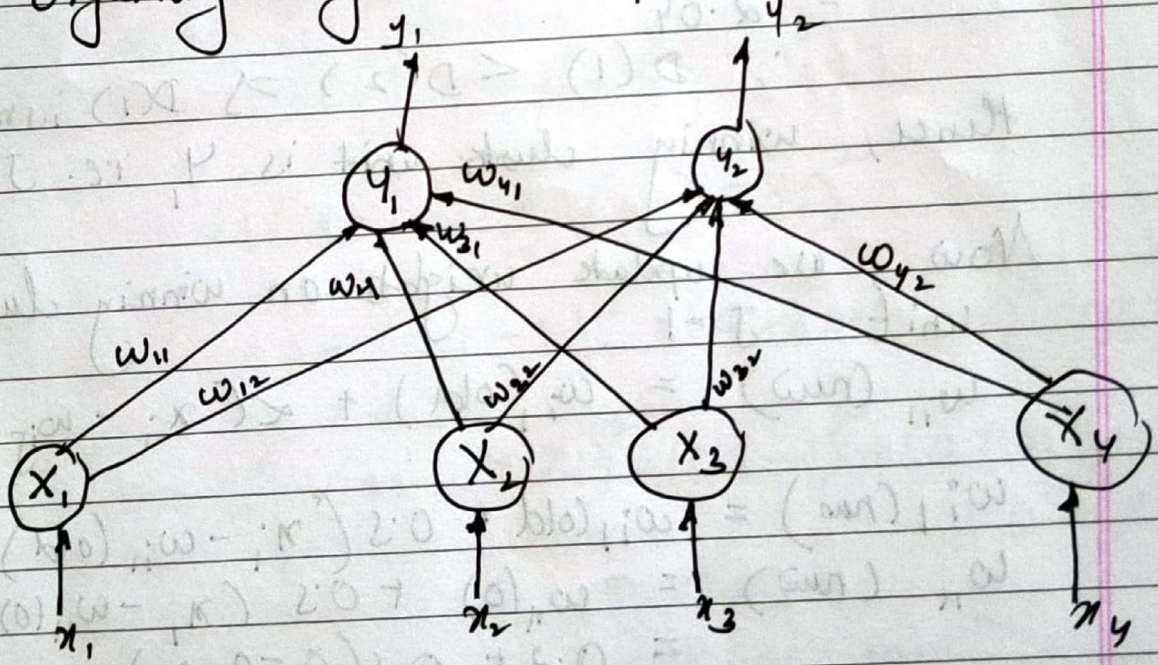
Q. Construct a Kohonen self organising map to cluster the 4 given vectors, [0 0 1 1], [1 0 0 0], [0 1 1 0] & [0 0 0 1]. The no. of cluster to be formed is 2. Assume initial learning rate of 0.5.

Sol^n: The no. of input vectors is 4 & no. of clusters to be formed is 2.

$\therefore n = 4, \quad m = 2.$

Following is the architecture of Kohonen self-organising feature map -



Step 0 : Initialize weights randomly between 0 & 1.

$$w_{ij} = \begin{bmatrix} 0.2 & 0.9 \\ 0.4 & 0.7 \\ 0.6 & 0.5 \\ 0.8 & 0.3 \end{bmatrix} ; \quad R = 0, \quad \alpha(0) = 0.5$$

For 1st input vector :

For $x = [0\ 0\ 1\ 1]$

Euclidian distance :

$$D(j) = \sum_i (w_{ij} - x_i)^2$$

$$D(1) = \sum_{i=1}^{4} (w_{i1} - x_i)^2$$

$$= (0.2-0)^2 + (0.4-0)^2 + (0.6-1)^2 + (0.8-1)^2$$

$\therefore D(1) = 0.4$

$$D(2) = \sum_{i=1}^{4} (w_{i2} - x_i)^2$$

$$= (0.9-0)^2 + (0.7-0)^2 + (0.5-1)^2 + (0.3-1)^2$$

$\therefore D(2) = 2.04$

$\because D(1) < D(2) \Rightarrow D(1)$ is min^m

Hence, winning cluster unit is 4, i.e. $J = 1$

Now, we update weights on winning cluster unit $J = 1$.

$$w_{ij}(new) = w_{ij}(old) + \alpha(x_i - w_{ij}(old))$$

$$w_{i1}(new) = w_{i1}(old) + 0.5(x_i - w_{i1}(old))$$

$$w_{11}(new) = w_{11}(0) + 0.5(x_1 - w_{11}(0))$$

$$= 0.2 + 0.5(0 - 0.2)$$

$$= 0.1$$

$$w_{21}(n) = w_{21}(0) + 0.5(x_2 - w_{21}(0))$$

$$= 0.4 + 0.5(0 - 0.4)$$

$$= 0.2$$

$$w_{31}(n) = w_{31}(0) + 0.5(x_3 - w_{31}(0))$$

$$= 0.6 + 0.5(1 - 0.6)$$

$$= 0.8$$

$w_{41}(n) = w_{41}(0) + 0.5(x_4 - w_{41}(0))$
$= 0.8 + 0.5(1 - 0.8)$
$= 0.9$

Updated weight matrix for 1st input pattern:

$$w_{ij} = \begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.7 \\ 0.8 & 0.5 \\ 0.9 & 0.3 \end{bmatrix}$$

For 2nd input vector:

for $x = [1\ 0\ 0\ 0]$

Euclidean distance:

$$D(j) = \sum_i (w_{ij} - x_i)^2$$

$$D(1) = \sum_{i=1}^{4} (w_{i1} - x_i)^2$$

$= (0.1 - 1)^2 + (0.2 - 0)^2 + (0.8 - 0)^2 + (0.9 - 0)^2$

$= 2.3$

$D(2) = \sum_{i=1}^{4} (w_{i2} - x_i)^2 = (0.9 - 1)^2 + (0.7 - 0)^2 + (0.5 - 0)^2 + (0.3 - 0)^2$

$= 0.84$

$\therefore D(2) < D(1)$ $\therefore D(2)$ is min^n

Hence, winning cluster is 2^{nd}, i.e. $J = 2$

Now, updating weights on $J = 2$

$w_{ij}(new) = w_{ij}(old) + \alpha(x_i - w_{ij}(old))$

$w_{i2}(new) = w_{i2}(old) + 0.5(x_i - w_{i2}(old))$

$w_{12}(n) = w_{12}(0) + 0.5(x_1 - w_{12}(0))$

$= 0.9 + 0.5(1 - 0.9)$

$= 0.95$

$w_{22}(n) = 0.7 + 0.5(0-0.7) = 0.35$

$w_{32}(n) = 0.5 + 0.5(0-0.5) = 0.25$

$w_{42}(n) = 0.3 + 0.5(0-0.1) = 0.15$

∴ Updated weight matrix after second input pattern:

$$w_{ij} = \begin{bmatrix} 0.1 & 0.95 \\ 0.2 & 0.35 \\ 0.8 & 0.25 \\ 0.9 & 0.15 \end{bmatrix}$$

3rd Input vector:

for $x = [0 \;\; 1 \;\; 1 \;\; 0]$

Euclidean distance:

$D(1) = (0.1-0)^2 + (0.2-1)^2 + (0.8-1)^2 + (0.9-0)^2$

$= 1.5$

$D(2) = (0.95-0)^2 + (0.35-1)^2 + (0.25-1)^2 + (0.15-0)^2$

$= 1.91$

∴ $D(1) < D(2) \Rightarrow D(1)$ is min^n

Hence, winning cluster unit is 4, i.e. $J = 1$

Now, Update weights on winning cluster unit $J = 1$:

$w_{11}(n) = 0.1 + 0.5(0-0.1) = 0.05$

$w_{21}(n) = 0.2 + 0.5(1-0.2) = 0.6$

$w_{31}(n) = 0.8 + 0.5(1-0.8) = 0.9$

$w_{41}(n) = 0.9 + 0.5(0-0.9) = 0.45$

∴ Updated weight matrix after 3rd Input pattern:

$$w_{ij} = \begin{bmatrix} 0.05 & 0.95 \\ 0.6 & 0.35 \\ 0.9 & 0.25 \\ 0.45 & 0.15 \end{bmatrix}$$

4th input vector :

for ~~inp~~ $x = [0 \ 0 \ 0 \ 1]$

Euclidean distance :

$$D(1) = \sum_{i=1}^{4} (w_{i1} - x_i)^2$$

$$= (0.05-0)^2 + (0.6-0)^2 + (0.9-0)^2 + (0.45-1)^2$$

$$= 1.175$$

$$D(2) = (0.95-0)^2 + (0.35-0)^2 + (0.2-0)^2 + (0.55-1)^2$$

$$= 1.011$$

$\because D(1) < D(2) \therefore D(1)$ is min^n

Hence, winning cluster unit is 4, $J = 1$

Now, update weights on winning cluster $J=1$

$w_{i1}(new) = w_{i1}(old) + \alpha[x_i - w_{i1}(old)]$

$w_{11}(n) = 0.05 + 0.5(0 - 0.05) = 0.025$

$w_{21}(n) = 0.6 + 0.5(0 - 0.6) = 0.3$

$w_{31}(n) = 0.9 + 0.5(0 - 0.9) = 0.45$

$w_{41}(n) = 0.45 + 0.5(1 - 0.45) = 0.725$

Thus final weight obtained after presentation of 4th input pattern is -

$$w_{ij} = \begin{bmatrix} 0.025 & 0.95 \\ 0.3 & 0.35 \\ 0.45 & 0.2 \\ 0.725 & 0.15 \end{bmatrix}$$

$\because$ all 4 given input patterns are presented, this is end of 1st iteration i.e. 1 epoch.

Now, learning rate is

$\alpha(t+1) = 0.5\,\alpha(t)$

$\alpha(1) = 0.5\,\alpha(0) = 0.5 \times 0.5 = 0.25$

With this rate, we can proceed for 100 iterations

Final Network: