Experiment 8

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Aim:

To implement fuzzy set and fuzzy relations for a given problem.

Theory:

Fuzzy Relations

- Fuzzy relations relate elements of one universe (say X) to those of another universe (say Y) through the Cartesian product of the two universes.
- These can also be referred to as fuzzy sets defined on universal sets, which are Cartesian products.
- A fuzzy relation is based on the concept that everything is related to some extent or unrelated.
- A fuzzy relation is a fuzzy set defined on the Cartesian product of classical sets (X_1, X_2, ...,X_n} where tuples (x_1,x_2,...,x_n) may have varying degrees of membership μ_R (x1,x2,...,xn) within the relation. That is,

$$R(X_1, X_2, \ldots, X_n) = \int_{X_1 \times X_2 \times \cdots \times X_n} \mu_R(x_1, x_2, \ldots, x_n) | (x_1, x_2, \ldots, x_n), \quad x_i \in X_i$$

- A fuzzy relation between two sets X and Y is called binary fuzzy relation and is denoted by R(X, Y).
- A binary relation R(X,Y) is referred to as bipartite graph when X≠Y. The binary relation on a single set X is called directed graph or digraph. This relation occurs when X= Y and is denoted as R(X,X) or R(X^2).
- Let

$$\underset{\sim}{X} = \{x_1, x_2, \ldots, x_n\} \quad \text{and} \quad \underset{\sim}{Y} = \{y_1, y_2, \ldots, y_m\}$$

- Fuzzy relation R(X,Y) can be expressed by an n x m matrix as follows

$$\underset{\sim}{R}(\underset{\sim}{X},\underset{\sim}{Y}) = \begin{bmatrix} \mu_R(x_1,y_1) & \mu_R(x_1,y_2) & . & . & \mu_R(x_1,y_m) \\ \mu_R(x_2,y_1) & \mu_R(x_2,y_2) & . & . & \mu_R(x_2,y_m) \\ . & . & . & . & . \\ . & . & . & . & . \\ \mu_R(x_n,y_1) & \mu_R(x_n,y_2) & . & . & \mu_R(x_n,y_m) \end{bmatrix}$$

- The matrix representing a fuzzy relation is called fuzzy matrix.
- A fuzzy relation R is a mapping from Cartesian space X x Y to the interval (0,1) where the mapping strength is expressed by the membership function of the relation for ordered pairs from the universes [$\mu_g(x,y)$]
- A fuzzy graph is a graphical representation of a binary fuzzy relation. Each element in X and Y corresponds to a node in the fuzzy graph.
- The connection links are established between the nodes by the elements of X x Y with nonzero membership grades in R(X,Y).
- The links may also be present in the form of arcs. These links are labeled with the membership values as μ_g (x_i,y_j).
- When X≠Y, the link connecting the two nodes is an undirected binary graph called bipartite graph. Here, each of the sets X and Y can be represented by a set of nodes such that the nodes corresponding to one set are clearly differentiated from the nodes representing the other set.
- When X = Y, a node is connected to itself and directed links are used; in such a case, the fuzzy graph is called directed graph. Here, only one set of nodes corresponding to see X is used.
- The domain of a binary fuzzy relation R(X, Y) is the fuzzy set, dom RCX, Y), having the membership function as

$$\mu_{\text{domain}\,R}(x) = \max_{y\in Y} \mu_R(x,y) \quad \forall x \in X$$

- The range of a binary fuzzy relation R(X,Y) is the fuzzy set, ran R(X, Y), having membership function as:

$$\mu_{\text{range}\,R}(y) = \max_{x\in X} \mu_R(x,y) \quad \forall y \in Y$$

Cardinality of Fuzzy Relations

The cardinality of fuzzy sets on any universe is infinity; hence the cardinality of a fuzzy relation between two or more universes is also infinity. This is mainly the result of occurrence of partial membership in fuzzy sets and relations.

Operations on Fuzzy Relations

The basic operations on fuzzy sets also apply on fuzzy relations. Let and be fuzzy relations on the Cartesian space X x Y. The operations that can be performed on these fuzzy relations are described below

1. Union

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \max[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)]$$

2. Intersection

$$\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x, y) = \min[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)]$$

3. Complement

$$\mu_{\overline{\underset{\sim}{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y)$$

4. Containment

$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \leq \mu_{\underset{\sim}{S}}(x, y)$$

5. Inverse :The inverse of a fuzzy relation R on X x Y is denoted by R-1. It is a relation on Y x X defined by

$$R^{-1}(y, x) = R(x, y) \text{ for all pairs } (y, x) \in Y \times X.$$

6. Projection For a fuzzy relation R(X,Y), let (R ↓Y) denote the projection of R onto Y. Then (R ↓Y) is a fuzzy relation in Y whose membership function is defined by:

$$\mu_{[R \downarrow Y]}(x, y) = \max_{x} \mu_{\underset{\sim}{R}}(x, y)$$

Properties of Fuzzy Relations

Like classical relations, the properties of commutativity, associativity, distributivity, idem potency and identity also hold good for fuzzy relations. DeMorgan's laws hold good for fuzzy relations as they do classical relations.

The null relation ¢ and complete relation E_R are analogous to the null set ¢ and the whole set E, respectively in set theoretic form. The excluded middle laws are not satisfied in fuzzy relations as for fuzzy sets. This is because a fuzzy relation R is also a fuzzy set, and there exists an overlap between the relation and its complement. Hence,

$$\underset{\sim}{R} \cup \overline{\underset{\sim}{R}} \neq \underset{\sim}{E} \quad \text{(whole set)}$$
$$\underset{\sim}{R} \cap \overline{\underset{\sim}{R}} \neq \underset{\sim}{\phi} \quad \text{(null set)}$$

Fuzzy Composition

Before understanding the fuzzy composition techniques, let us learn about the fuzzy Cartesian product. Let A be a fuzzy set on universe X and B be a fuzzy set on universe. The Cartesian product over A and B results in fuzzy relation R and is combined within the entire (complete) Cartesian space, i.e.,

$$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R}$$

where

$$\underset{\sim}{R} \subset X \times Y$$

The membership function of fuzzy relation is given by

$$\boxed{\mu_{\underset{\sim}{R}}(x,y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x,y) = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)]}$$

The Cartesian product is not an operation similar to arithmetic product. Cartesian product R = A X B, is obtained in the same way as the of cross- product of two vectors. For example, for a fuzzy set-A that has three elements (hence column vector of size 3 x 1) and a fuzzy B, that has four elements (hence row vector of size 1 x 4), the resulting fuzzy relation R will be represented by a matrix of size 3 x 4, i.e, R will have three rows and four columns.

Now lets discuss the composition of the fuzzy relations. There are two types of fuzzy composition techniques:

1. Fuzzy max-min composition
2. Fuzzy max-product composition

There also exists fuzzy min-max composition method, bur the most commonly used technique is fuzzy max-min composition.

Let R be fuzzy relation on Cartesian space X x Y & S be fuzzy relation on Cartesian space Y x Z

The max-min composition of $R(X, Y)$ and $S(Y, Z)$, denoted by $R(X, Y) \circ S(Y, Z)$ is defined by $T(X, Z)$ as

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \circ \underset{\sim}{S}}(x, z) = \max_{y \in Y} \{\min [\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(y, z)]\}$$

$$= \bigvee_{y \in Y} [\mu_{\underset{\sim}{R}}(x, y) \wedge \mu_{\underset{\sim}{S}}(y, z)] \quad \forall x \in X, \quad z \in Z$$

The min-max composition of $R(X, Y)$ and $S(Y, Z)$, denoted as $R(X, Y) \circ S(Y, Z)$, is defined by $T(X, Z)$ as

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \circ \underset{\sim}{S}}(x, z) = \min_{y \in Y} \{\max[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(y, z)]\} = \bigwedge_{y \in Y} [\mu_{\underset{\sim}{R}}(x, y) \vee \mu_{\underset{\sim}{S}}(y, z)] \quad \forall x \in X, \ z \in Z$$

From the above definitions it can be noted that

$$\boxed{\overline{R(X, Y) \circ S(Y, Z)} = \overline{R(X, Y)} \circ \overline{S(Y, Z)}}$$

The max-min composition is used, hence the problems discussed in this chapter are limited to max-min composition. The max-product composition of R(X, Y) and S(Y, Z), denoted as R(X, Y) . S(Y, Z) is defined by T(X, Z) as

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \cdot \underset{\sim}{S}}(x, z) = \max_{y \in Y} [\mu_{\underset{\sim}{R}}(x, y) \cdot \mu_{\underset{\sim}{S}}(y, z)]$$

$$= \bigvee_{y \in Y} [\mu_{\underset{\sim}{R}}(x, y) \cdot \mu_{\underset{\sim}{S}}(y, z)]$$

The properties of fuzzy composition can be given as follows:

$$\underset{\sim}{R} \circ \underset{\sim}{S} \neq \underset{\sim}{S} \circ \underset{\sim}{R}$$

$$(\underset{\sim}{R} \circ \underset{\sim}{S})^{-1} = \underset{\sim}{S}^{-1} \circ \underset{\sim}{R}^{-1}$$

$$(\underset{\sim}{R} \circ \underset{\sim}{S}) \circ \underset{\sim}{M} = \underset{\sim}{R} \circ (\underset{\sim}{S} \circ \underset{\sim}{M})$$

Solved Example:

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Q3. Two fuzzy relations are given by

$$\underset{\sim}{R} = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

&

$$\underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain fuzzy relation $\underset{\sim}{T}$ as a composition between the fuzzy relations.

Solⁿ The composition between 2 given fuzzy relations is performed in 2 ways as:

(a) Max-min composition

(b) Max-product composition

(a) Finding Max min composition.

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

The calculations for obtaining $\underset{\sim}{T}$ are as follows

$$\mu_{\underset{\sim}{T}}(x_1, z_1) = \max\left[\{\min(\mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{S}}(y_1, z_1))\}, \{\min(\mu_{\underset{\sim}{R}}(x_1, y_2), \mu_{\underset{\sim}{S}}(y_2, z_1))\}\right]$$

$$= \max[\min(0.6, 1), \min(0.3, 0.8)]$$

$$\mu_{\underset{\sim}{T}}(x_1, z_1) = \max(0.6, 0.3) = 0.6$$

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$$\mu_{\underset{\sim}{T}}(x_1, z_2) = \max[\min(0.6, 0.5), \min(0.3, 0.4)]$$
$$= \max(0.5, 0.3) = 0.5$$

$$\mu_{\underset{\sim}{T}}(x_1, z_3) = \max[\min(0.6, 0.3), \min(0.3, 0.7)]$$
$$= \max(0.3, 0.3) = 0.3$$

$$\mu_{\underset{\sim}{T}}(x_2, z_1) = \max[\min(0.2, 1), \min(0.9, 0.8)]$$
$$= \max(0.2, 0.8) = 0.8$$

$$\mu_{\underset{\sim}{T}}(x_2, z_2) = \max[\min(0.2, 0.5), \min(0.9, 0.4)]$$
$$= \max(0.2, 0.4) = 0.4$$

$$\mu_{\underset{\sim}{T}}(x_2, z_3) = \max[\min(0.2, 0.3), \min(0.9, 0.7)]$$
$$= \max(0.2, 0.7) = 0.7$$

(b) Finding Max product composition

$$\underset{\sim}{T} = \underset{\sim}{R} \bullet \underset{\sim}{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

Calculations for obtaining above fuzzy relation $\underset{\sim}{T}$ by max-product composition is as follows:

$$\mu_{\underset{\sim}{T}}(x_1, z_1) = \max\{[\mu_{\underset{\sim}{R}}(x_1, y_1) \bullet \mu_{\underset{\sim}{S}}(y_1, z_1)], [\mu_{\underset{\sim}{R}}(x_1, y_2) \bullet \mu_{\underset{\sim}{S}}(y_2, z_1)]\}$$

$$\therefore \mu_{\underset{\sim}{T}}(x_1, z_1) = \max(0.6, 0.24) = 0.6$$

$$\mu_{\underset{\sim}{T}}(x_1, z_2) = \max[0.6 \times 0.5, 0.3 \times 0.4]$$
$$= \max(0.3, 0.12) = 0.3$$

$$\mu_{\underset{\sim}{T}}(x_1, z_3) = \max[0.6 \times 0.3, 0.3 \times 0.7]$$
$$= \max(0.18, 0.21) = 0.21$$

$$\mu_{\underset{\sim}{T}}(x_2, z_1) = \max[0.2 \times 1, 0.9 \times 0.8]$$
$$= \max(0.2, 0.72) = 0.72$$

$$\mu_{\underset{\sim}{T}}(x_2, z_2) = \max[0.2 \times 0.5, 0.9 \times 0.4]$$
$$= \max(0.10, 0.36) = 0.36$$

$$\mu_{\underset{\sim}{T}}(x_2, z_3) = \max[0.2 \times 0.3, 0.9 \times 0.7]$$
$$= \max(0.06, 0.63) = 0.63$$

Code:

```python
# Function to display result
def display(n, T):
    print('-'*65)
    print('|\t\t|\t', end='')
    for m in ['z'+str(i+1) for i in range(n)]:
        print(m + '\t|\t', end='')
    print('')
    print('-'*65)
    for i, m in enumerate(T):
        print('|\tx'+str(i+1), end='\t')
        for j in m:
            print('|\t', j, end='\t')
        print('|')
    print('-'*65)

# Function to find max min composition
def max_min_composition(R, S):
    T = []
    for i in range(len(R)):
        l = []
        for j in range(len(S[i])):
            min_l = []
            for k in range(len(S)):
                min_l.append(min(R[i][k], S[k][j]))
            l.append(max(min_l))
        T.append(l)
    return T

# Function to find max product composition
def max_product_composition(R, S):
    T = []
    for i in range(len(R)):
        l = []
        for j in range(len(S[i])):
            min_l = []
            for k in range(len(S)):
                min_l.append(round(R[i][k] * S[k][j], 2))
            l.append(max(min_l))
        T.append(l)
    return T

# Pg 287, Solved Example 3
R = [[0.6, 0.3], [0.2, 0.9]]
S = [[1, 0.5, 0.3], [0.8, 0.4, 0.7]]

print('R : ', R)
print('S : ', S,'\n')
print('*'*65)
print('\n\t\t        Max-Min Composition\n')
```

```
max_min = max_min_composition(R, S)
display(len(S[0]) ,max_min)
print('')
print('*'*65)
print('\n\t\t     Max-Product Composition\n')
max_pro = max_product_composition(R, S)
display( len(S[0]),max_pro)
print('')
print('*'*65)
```

Output:

```
R :  [[0.6, 0.3], [0.2, 0.9]]
S :  [[1, 0.5, 0.3], [0.8, 0.4, 0.7]]

*****************************************************************

                        Max-Min Composition

-----------------------------------------------------------------
|               |       z1       |       z2       |       z3       |
-----------------------------------------------------------------
|       x1      |        0.6     |        0.5     |        0.3     |
|       x2      |        0.8     |        0.4     |        0.7     |
-----------------------------------------------------------------

*****************************************************************

                     Max-Product Composition

-----------------------------------------------------------------
|               |       z1       |       z2       |       z3       |
-----------------------------------------------------------------
|       x1      |        0.6     |        0.3     |        0.21    |
|       x2      |        0.72    |        0.36    |        0.63    |
-----------------------------------------------------------------

*****************************************************************
```

Conclusion:

In this experiment, I have implemented the max-min and max-product composition of two fuzzy relations in python. I learnt about fuzzy sets and fuzzy compositions, fuzzy relations and its properties and operations.